EE1390: Matrix Project

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Contents

Question

Original

For the hyperbola $9x^2 - 16y^2 = 144$, find the equation of the normal at point P$(8, 3\sqrt{3})$.

Matrix form

Find the equation of the normal to the hyperbola $x^T \begin{bmatrix} 9 & 0 \\ 0 & -16 \end{bmatrix} x = 144$ drawn at the point P $\begin{bmatrix} 8 \\ 3\sqrt{3} \end{bmatrix}$.

Theoretical Solution

Equation of hyperbola in matrix form: $x^T \begin{bmatrix} 9 & 0 \\ 0 & -16 \end{bmatrix} x = 144$

Now equation of tangent of any second degree curve:

$(P^T V + u^T)x + P^T u + F = 0 \; u^T = 0$

$P = \begin{bmatrix} 8 \\ 3\sqrt{3} \end{bmatrix}$ and $V = \begin{bmatrix} 9 & 0 \\ 0 & -16 \end{bmatrix}$

So direction vector of normal at point P is $P^T V = \begin{bmatrix} -72 \\ 83.1384 \end{bmatrix}$

Thus, the normal vector for normal at point P:

$$\begin{bmatrix} -72 \\ 83.1384 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -83.1384 \\ -72 \end{bmatrix} = n$$

Final equation for the normal:

$n^T(x - P) = 0 \implies n^T x = n^T P = -1039.2304$

$\implies n^T x = -1039.2304$

Plotted Diagram

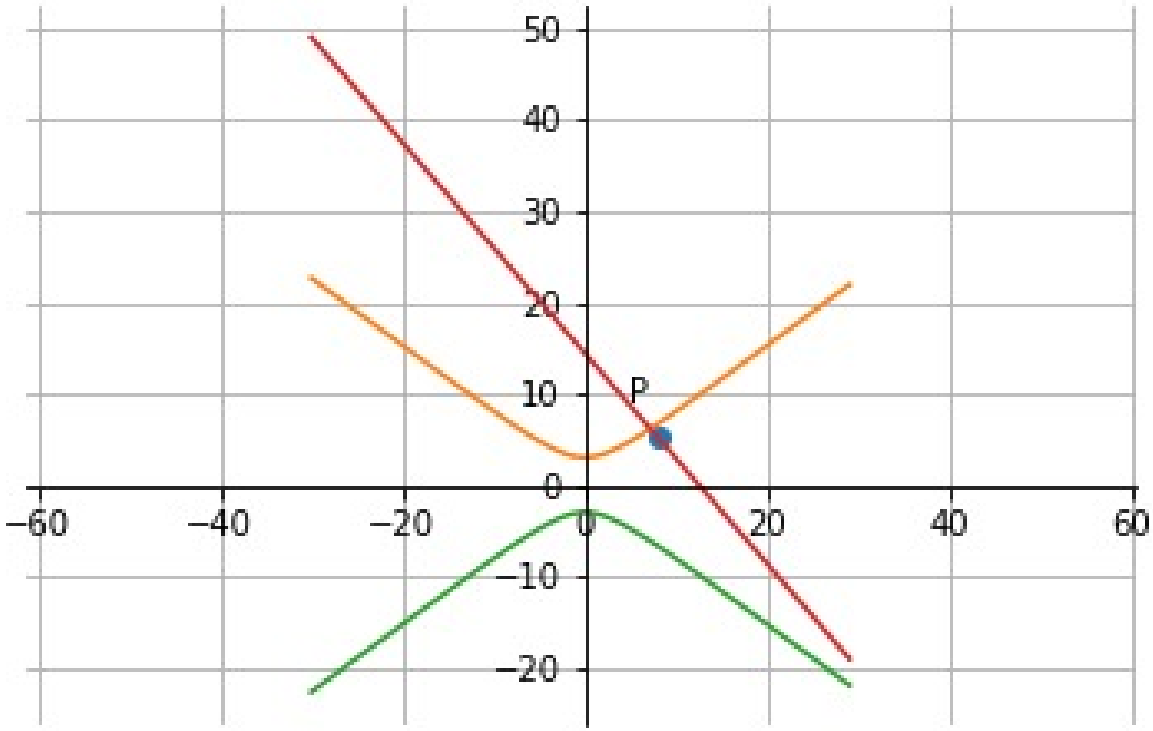

Figure: Graphical plot of the curve