311001

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B. Tech. III Sem. (Main) Exam., Dec. - 2019

Aeronautical Engineering

3ME2-01 Advanced Engineering Mathematics-I

AE, AG, CR, CE, EC, EIC, ME, MH

Time: 3 Hours **Maximum Marks: 120**

Instructions to Candidates:

Part – A: *Short answer questions (up to 25 words) 10 × 2 marks = 20 marks. All ten questions are compulsory.*

Part – B: *Analytical/Problem Solving questions 5 × 8 marks = 40 marks. Candidates have to answer five questions out of seven.*

Part – C: *Descriptive/Analytical/Problem Solving questions 4 × 15 marks = 60 marks. Candidates have to answer four questions out of five.*

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting materials is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART - A

Q.1 Prove that : $\Delta\,[\log f(x)] = \log\left[1+\frac{\Delta f(x)}{f(x)}\right]$.

Q.2 State the fundamental theorem of finite differences and hence find the value of

$\Delta^6\,(ax-1)\,(bx^2-1)\,(cx^3-1)$

Q.3 Write the Stirling's central interpolation formula.

Q.4 If function f (x) is a continuous function in the interval $(-\infty, \infty)$ and $\bar{f}(s)$ is the Fourier transform of f (x), then prove that $F\{f'(x)\} = -is\,\bar{f}(s)$.

Q.5 Write modified Euler's method to solve the ordinary differential equation.

Q.6 Find the Laplace transform of $F(t) = te^{t} \sin t$.

Q.7 State & prove the change of scale property for Laplace transform.

Q.8 Find $L^{-1}\left[\frac{1}{(s-1)(s+2)}\right]$.

Q.9 Find the Fourier sine transform of $f(x) = \begin{cases} 1 & ; \quad 0 < x < a \\ 0 & ; \quad x > a \end{cases}$

Q.10 Find the Z – transform of $u_n = c^n \cosh an, n \geq 0$.

PART – B

Q.1 Use Newton – Raphson method to find a real root of $(x) = x^3 - 3x - 5 = 0$.

Q.2 Using Lagrange's interpolation formula, find the value of y (5) from the following table:

x	:	1	2	3	4	7
y	:	2	4	8	16	128

Q.3 Use Milne's Predictor – Corrector method to find y (0.8), given that:

$$\frac{dy}{dx} = x - y^2 \; ; \; y(0) = 0, \; y(0.2) = 0.02, \; y(0.4) = 0.0795, \; y(0.6) = 0.1762$$

Q.4 Prove that $L\{t F(t)\} = -\frac{d}{ds} f(s)$ and hence find $L^{-1}\left[\log\left(\frac{s+2}{s+1}\right)\right]$.

Q.5 Find the Laplace transform of $\sin\sqrt{t}$. Hence show that $L\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{4s}}$.

Q.6 Find the Fourier sine transform of $f(x) = e^{-x}, x \geq 0$ and hence show that:

$$\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m} \; ; \; m > 0$$

Q.7 Find the inverse Z – transform of following function:

$f(z) = \frac{1}{(z-1)(z-2)}$, if ROC is

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

PART – C

Q.1 (a) Use Simpson's $\frac{1}{3}$ and $\frac{3}{8}$ rule to evaluate the integral $\int_0^1 \frac{1}{1+x^2}\,dx$. Hence obtain the approximate value of π.

(b) Given the following data:

x	:	0.0	0.2	0.4	0.6	0.8
y	:	0.3989	0.3910	0.3683	0.3332	0.2897

Evaluate the value of y (0.25) and y (0.62).

Q.2 Use Runge - Kutta method of fourth order to find y (1.2), given that:

$$\frac{dy}{dx} = x^2 + y^2 \ ; \ y(1) = 0.$$

Q.3 (a) Prove that $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ and hence find $L\left(\frac{\sin at}{t}\right)$. Does $L\left(\frac{\cos at}{t}\right)$ exists?

(b) Solve the differential equation, using Laplace transform:

$(D^2 + 3D + 2)\,y = 1$; $y(0) = 0$, $y'(0) = 0$; $D \equiv \frac{d}{dt}$.

Q.4 Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & ; \ |x| < 1 \\ 0 & ; \ |x| > 1 \end{cases}$.

Also evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\,dx$.

Q.5 (a) Use Convolution theorem to find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$.

(b) Using Z – transform solve the difference equation:

$U_{n+2} + 5u_{n+1} + 6u_n = 0$, given that $u_0 = 0$, $u_1 = 1$
