**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS. The service manager plans to have work begin 10 minutes after the car is dropped off. Therefore, the time available for servicing the transmission is 60 minutes - 10 minutes = 50 minutes.

x = 50

µ = 45

= 8

Z 🡪 (50-45)/8

Z 🡪 5/8

P(Z>5/8) = 0.2643 or 26.43%.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS. import scipy.stats as stats

import seaborn as sns

import numpy as np

q2\_less\_than\_38 = stats.norm.cdf(38, loc = 38, scale = 6)

q2\_less\_than\_44 = stats.norm.cdf(44, loc = 38, scale = 6)

q2\_betweeen\_38\_and\_44 = q2\_less\_than\_44 - q2\_less\_than\_38

print(f"{q2\_betweeen\_38\_and\_44:.4f}")

0.3413

q2\_morethan\_44 = 1-stats.norm.cdf(44, loc = 38, scale = 6)

print(f"{q2\_morethan\_44:.4f}")

0.1587

False. Because the probability for employees at the processing center are more between 38 and 44 than older than 44.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS. False.

import scipy.stats as stats

mean = 38

std\_dev = 6

sample\_value = 36

z\_score = (sample\_value - mean) / std\_dev

probability = stats.norm.cdf(z\_score)

threshold = 0.5

expected = probability > threshold

print(f"The z-score is approximately {z\_score:.2f}")

print(f"The probability is approximately {probability:.4f}")

print(f"The statement is {expected}")

Output : The z-score for 36 is approximately -0.33.

The probability is approximately 0.3694.

The statement is False.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS. Summary:

a. For 2X1​:

Distribution: Normal (N(2μ,4σ^2))

Mean: 2μ

Variance: 4σ2

b. For X1+X2​​:

Distribution: Normal (N(2 μ ,2σ^2))

Mean: 2μ

Variance: 2σ^2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

A. import scipy.stats as stats

import numpy as np

confidence\_level = 0.99

a, b = stats.norm.interval(confidence\_level, loc=100, scale=20)

print(f"The two values of a and b are {a:.2f},{b:.2f}")

Output : The two values of a and b are 48.48, 151.52.

Option D.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS. Rupee range: 99.01 to 980.99

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS. 5th percentile of profit (in Rupees): 169.91

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS. Division 1 has a larger probability of making a loss.