Classiq QRISE 2024 Challenge

Implementing an Exponential Quantum Advantage Algorithm

Classiq Entanglers

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Motivation and Vision

- Throughout this challenge our aim was to practically demonstrate the quantum speedup of the algorithm by Babbush et al. [2023].
- We wanted to do this by running an instance of this algorithm which is proven by the authors to be **BQP complete** and try to ensure scalability to show practical demonstration on a **70+** qubit system, possibly showcasing quantum supremacy.
- We believe that we have managed to do that for a specific case which only takes polynomial number of resources and would fit within the depth range of $(\mathbf{10^4}, \mathbf{3} * \mathbf{10^4})$.

Problem Formulation and Utility

- We have selected the problem case as the following:

$$d = 1, E = 1, m_i = 1, x_i = 0, \dot{x}_i = (1, -1, 0, \dots),$$

$$k_{i,j} = 1, j > i, \forall (i,j) \in [1, N].$$

Hence the initial state will be:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2E}} \begin{bmatrix} \sqrt{M}\dot{\vec{x}}(0) \\ i\vec{\mu}(0) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \\ \vdots \end{bmatrix} \tag{1}$$

- We have done this for the following reasons:
  - This formulation according to the authors **satisfies BQP completeness**.
  - The formulation is quite **scalable** to high number of qubits with a **deterministic** implementation, **without needing Oracle** $\mathcal{S}$.
  - Hence it ensures that this can be ran on a **70+** qubits Quantum Computer with an appropriate depth.

Implementation and Novelty - Part 1

- There are many areas where the paper is ambiguous about implementation of this algorithm, we have **filled in these gaps** with possibly **some improvements** for our problem case and in general for some areas.
- For instance, as part of unitary $\mathcal{U}_H$ preparation, according to equation 'A6', one needs to map the following through some unspecified unitary:

$$|0\rangle^{\otimes n} \rightarrow \frac{1}{\sqrt{d}} \sum_{l=1}^{d} |l\rangle.$$

- We propose to address this by utilizing an efficient algorithm by Shukla and Vedula [2024]. This has a circuit depth or gate complexity of only $\mathcal{O}(\log_2 \ d)$ without any ancilla qubits or gates with multiple controls.
- Following the application of this algorithm, we just have to apply the Pauli-Z gate.

Implementation and Novelty - Part 2

- We meticulously understand the paper and painstakingly **derive the general Hamiltonian matrix** and formulate a class for it.
- Quantum Simulation of our Hamiltonian ($U(t) = e^{-iHt}$) is the biggest hurdle since our state preparation is done acccurately and deterministically with $\mathcal{O}(1)$ gates.
- Because these methods involve depth some ambiguity based on H, but we know H to be sparse, we'll analyse at the following methods:
  - Trotter-Suzuki method
  - Randomized Evolution (Qdrift)
  - Exponentiation function which generates an efficient higher-order Trotter-Suzuki quantum program
- Lastly, we compare the state fidelities and note down the depth of the circuits.

Classiq Integration

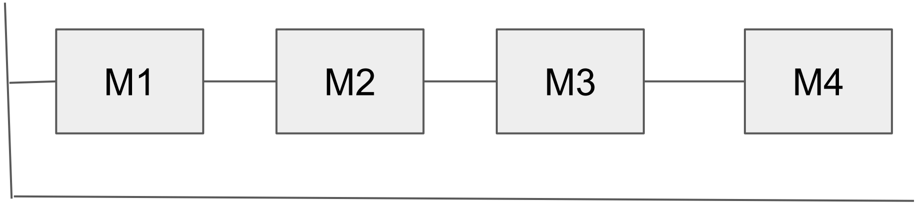
- We use an iterative approach to perform Hamiltonian simulation using the classiq framework.
- The first step is to understand classically the dynamics of the coupled spring mass system. We computed the lagrangian of the system and solved it for various time to get position and velocity.
- Next, we manually performed the approach in the paper for a simple 2 mass system by computing the hamiltonian using the equations given in paper.
- After that, we focusses our approach primarily in classiq due to the advantages of the synthesis framework in solving our problem.
- We first start with the computation of initial state vector using the masses and spring constants provided.
- This state vector is normalized and converted to quantum state using the classiq.

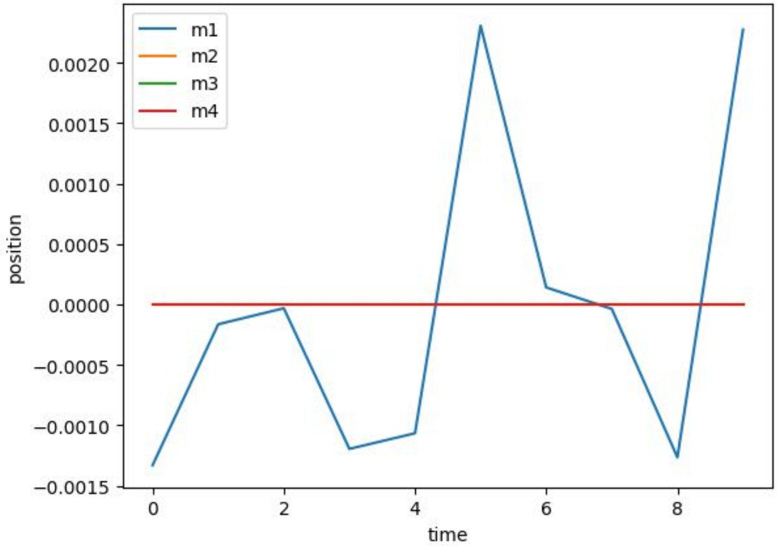
Classiq Integration

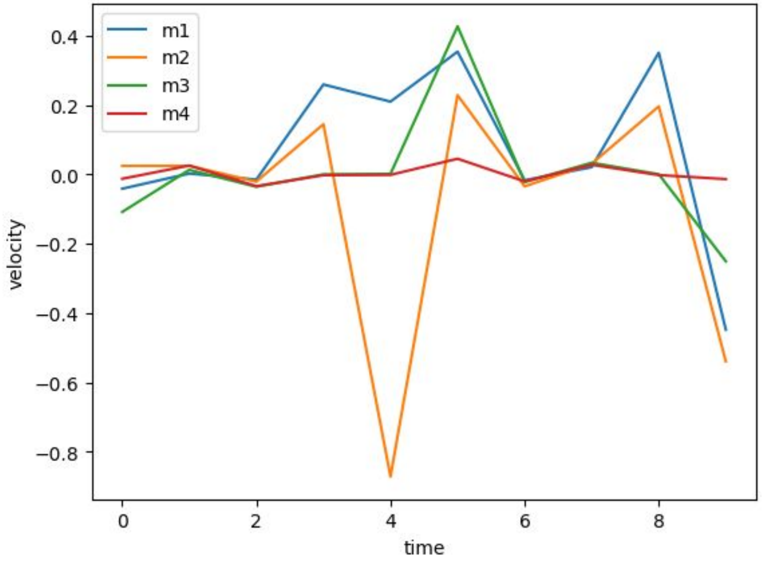
- Next we compute the Hamiltonian using the deterministic approach instead of the oracle approach stated in the paper. The novelty in our approach is that we classically compute the hamiltonian matrix making it a deterministic circuit instead of an oracle based one.
- This Hamiltonian matrix is converted to Sparse Pauli Ops to get the Pauli representation of the matrix.
- Once we have the initial state and Hamiltonian circuit, the next step is to perform time evolution. We use Suzuki Trotterization approach to find out the state after 1 time unit.
- Finally, we converted the state vector obtained from the simulation in position and velocity values for various masses.

Results

Considering a spring mass system as shown below



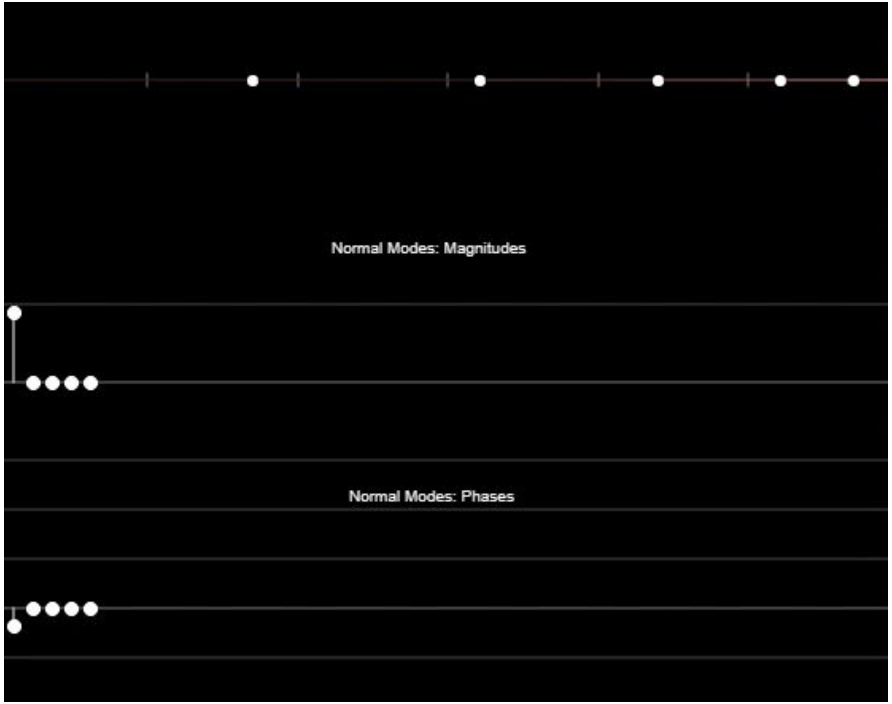




Results

Classical Simulation of similar system is done in this tool

[Coupled Oscillation Simulation (falstad.com)](falstad.com)



References

Ryan Babbush, Dominic W. Berry, Robin Kothari, Rolando D. Somma, and Nathan Wiebe. Exponential quantum speedup in simulating coupled classical oscillators. *Physical Review X*, 13 (4), December 2023. ISSN 2160-3308. doi: 10.1103/physrevx.13.041041. URL http://dx.doi.org/10.1103/PhysRevX.13.041041.

Alok Shukla and Prakash Vedula. An efficient quantum algorithm for preparation of uniform quantum superposition states. *Quantum Information Processing*, 23(2), January 2024. ISSN 1573-1332. doi: 10.1007/s11128-024-04258-4. URL http://dx.doi.org/10.1007/s11128-024-04258-4.

Thank you!