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(i)  $h(x) = f_1(x)$

- Cumple para (b) porque son datos linealmente separables, no cumple para los demás casos.

(ii)  $h(x) = f_x(f_1(x))$

- Como  $f_1$  y  $f_2$  son funciones lineales, su composición también lo será. Igual que el ítem anterior, solo cumple (b).

(iii)  $h(x) = f_2(g(f_1(x)))$

- puede suceder que  $f_2$  sea la función identidad, lo que conduce a  $f_2(g(f_1(x))) = g(f_1(x))$ , y esta función 'g' al ser NO LINEAL podría modelar regiones curvas. Para este caso cumple 'a', 'b' y 'c'.

(iv)  $h(x) = f_4(f_3(f_2(f_1(x))))$

- De forma similar a (i), la composición de funciones lineales sigue siendo lineal, esto solo resolvería 'a'.

(v)  $h(x) = g_2(g_1(x))$

- La composición de dos funciones no lineales, es no lineal. Puede resolver 'a', 'b' y 'c'.

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(a)  $\frac{\partial z}{\partial h_1} = \frac{\partial (f_1(x) \cdot f_2(x))}{\partial h_1} = \frac{\cancel{\partial h_1}}{\partial h_1} h_2 + h_1 \cancel{\frac{\partial h_2}{\partial h_1}} = h_2$

(b)  $\frac{\partial z}{\partial h_2} = \frac{\partial (f_1(x) \cdot f_2(x))}{\partial h_2} = \frac{\cancel{\partial h_1}}{\partial h_2} h_2 + h_1 \frac{\partial h_2}{\partial h_2} = h_1$

(c)  $\frac{\partial z}{\partial x} = \frac{\partial (f_1(x) \cdot f_2(x))}{\partial x} = \frac{\partial f_1(x)}{\partial x} f_2(x) + f_1(x) \frac{\partial f_2(x)}{\partial x}$   
 $= \frac{\partial f_1(x)}{\partial x} h_2 + h_1 \frac{\partial f_2(x)}{\partial x}$

(a)	$h_1$	$h_2$	$h_3$	$r_1$	$r_2$	$r_3$	$s$	$y_1$	$y_2$	$z$
	2	-10	3	2	0	3	3	0.27	0.73	1

(b)  $y_1$  y  $z$  se aproximan a 0.27 y 1 respectivamente.

(c) Sea  $g(x)$  la función sigmoide, su derivada  $\frac{dg}{dx} = g(x)(1-g(x))$

$$\bullet \frac{\partial h_1}{\partial w_{12}} = \frac{\partial (w_{11}x_1 + w_{12}x_2)}{\partial w_{12}} = x_2$$

$$\bullet \frac{\partial h_1}{\partial x_1} = \frac{\partial (w_{11}x_1 + w_{12}x_2)}{\partial x_1} = w_{11} = 6$$

$$\bullet \frac{\partial r_1}{\partial h_1} = \frac{\partial (\max(h_1, 0))}{\partial h_1} = \max\left(\frac{\partial h_1}{\partial h_1}, \frac{\partial 0}{\partial h_1}\right) = \max(1, 0) = 1$$

$$\bullet \frac{\partial y_1}{\partial r_1} = \frac{\partial \text{Softmax}(r_i)}{\partial r_i}$$

$$\text{Sea } p_j = \frac{e^{o_j}}{\sum_k e^{o_k}}$$

$$\frac{\partial p_i}{\partial o_i} = p_i(1-p_i) \quad \text{si } i=j$$

$$\frac{\partial p_i}{\partial o_i} = -p_i p_j \quad \text{si } i \neq j$$

Usando esas fórmulas:

$$\frac{\partial y_1}{\partial r_1} = y_1(1-y_1) = 0.1971$$

$$\bullet \frac{\partial y_1}{\partial s_1} = -y_1 y_2$$

(de acuerdo a la ecuación anterior)

$$= 0.1971$$

$$\bullet \frac{\partial z}{\partial y_1} = \frac{\partial (y_1 + y_2)}{\partial y_1} = \frac{\partial y_1}{\partial y_1} + \frac{\partial y_2}{\partial y_1} = 1$$

$$\bullet \frac{\partial z}{\partial x_1} = \frac{\partial (y_1 + y_2)}{\partial x_1} = \frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_1} = 2.1681 \quad (\text{ver } \alpha)$$

$$y_1 = \text{Softmax}(\max(x_1 w_{11} + x_2 w_{12}, 0))$$

$$y_2 = \text{Softmax}(\max(\max(x_1 w_{21} + x_2 w_{22}, 0), \max(x_1 w_{31} + x_2 w_{32}, 0)))$$

$$\bullet \frac{\partial s_1}{\partial r_2} = \frac{\partial (\max(r_2, r_3))}{\partial r_2} = \max\left(\frac{\partial r_2}{\partial r_2}, \frac{\partial r_3}{\partial r_2}\right) = \max(1, 0) = 1$$

$\alpha$ :

Sea  $O_1 = \max(x_1 w_{11} + x_2 w_{12}, 0)$

$$\frac{\partial O_1}{\partial x_1} = \frac{\partial \max(x_1 w_{11} + x_2 w_{12}, 0)}{\partial x_1}$$

$$\frac{\partial O_1}{\partial x_1} = \max(w_{11}, 0)$$

$$\Rightarrow \frac{\partial y_1}{\partial x_1} = \frac{\partial y_1}{\partial O_1} \frac{\partial O_1}{\partial x_1} = y_1 (1 - y_1) \max(w_{11}, 0)$$

$$\frac{\partial y_1}{\partial x_1} = 1.1826$$

Sea  $O_2 = \max(\max(x_1 w_{21} + x_2 w_{22}, 0), \max(x_1 w_{31} + x_2 w_{32}, 0))$

$$\frac{\partial O_2}{\partial x_1} = \max(\underbrace{\max(w_{21}, 0)}_4, \underbrace{\max(w_{31}, 0)}_5)$$

$$\Rightarrow \frac{\partial y_2}{\partial x_1} = \frac{\partial y_2}{\partial O_2} \frac{\partial O_2}{\partial x_1} = \frac{1}{2} (1 - y_2) \frac{\partial O_2}{\partial x_1} = 0.9855$$