PC2 Sánchez Sauñe Cristhian Wiki

(3)

(1)

$$1.01x + 0.99y = 2$$
$$0.99x + 1.01y = 2$$

(a) Usando Gauss - Jordan (se omitió escribir todos los decimales)

$$\left(\begin{array}{cc|c} 1.01 & 0.99 & 2 \\ 0.99 & 1.01 & 2 \end{array}\right) \xrightarrow{f_1/1.01} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0.99 & 1.01 & 2 \end{array}\right)$$

$\to 0.03960396\ldots$

$$\xrightarrow{f_2-0.99f_1} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0 & 0.039 & 0.039 \end{array}\right) \xrightarrow{f_2/0.0396} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{f_1-0.98f_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right) \Rightarrow \begin{array}{l} x=1 \\ y=1 \end{array}$$

(b) Usando solo dos decimales y redondeo (con las mismas operaciones fila)

$$\left(\begin{array}{cc|c} 1.01 & 0.99 & 2 \\ 0.99 & 1.01 & 2 \end{array}\right) \xrightarrow{f_1/1.01} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0.99 & 1.01 & 2 \end{array}\right)$$

$$\xrightarrow{f_2-0.99f_1} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0 & 0.04 & 0.04 \end{array}\right) \xrightarrow{f_2/0.0396\ldots} \left(\begin{array}{cc|c} 1 & 0.98 & 1.98 \\ 0 & 1.01 & 1.01 \end{array}\right)$$

$$\xrightarrow{f_1-0.98f_2} \left(\begin{array}{cc|c} 1 & -0.01 & 0.99 \\ 0 & 1.01 & 1.01 \end{array}\right) \quad \begin{array}{l} \tilde{y}=1 \\ \tilde{x}=1 \end{array}$$

(c) $$A^{-1} = \frac{\text{Adj}\,A}{|A|} = \begin{pmatrix} 25.25 & -24.75 \\ -24.75 & 25.25 \end{pmatrix}$$

(d) El residuo obtenido en (b) $\quad x-\tilde{x} = 1-1 = 0$ con 2 decimales redondeados
$\quad y-\tilde{y} = 1-1 = 0$

1)

a) A será positiva si $\forall x \in \mathbb{R}^{n\times 1}$ con $n \in \mathbb{Z}^+$ se cumple

$$x^T A x > 0$$

$$(x_1 \cdots x_n)\begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \\ 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 2 \end{pmatrix}_{n\times n} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n\times 1} = (x_1 \cdots x_n)\begin{pmatrix} 2x_1 - x_2 \\ -x_1 + 2x_2 - x_3 \\ \vdots \\ -x_{n-1} + 2x_n \end{pmatrix}$$

$$= 2x_1^2 - x_1x_2 - x_1x_2 + 2x_2^2 - x_2x_3 \cdots - x_{n-1}x_n + 2x_n^2$$

$$= 2(x_1^2 + \cdots + x_n^2) - 2\sum_{i\neq j}^{n} x_i x_j = 2\left(\sum_{i=1}^{n} x_i^2 - \sum_{i\neq j}^{n} x_i x_j\right)$$

b) Para $n=3$ y $n=4$, verificaremos que se mantiene la estructura trigonal.

$$n=3: \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \qquad n=4: \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

fact. Cholesky:

$$A = LL^T$$

$$L_{jj} = \sqrt{A_{jj} - \sum_{k=1}^{j-1} L_{jk}^2}$$

$$L_{ij} = \frac{1}{L_{jj}}\left(A_{ij} - \sum_{k=1}^{j-1} L_{ik}L_{jk}\right) \quad \forall i > j$$

$$n=3: \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1393/985 & 0 & 0 \\ -985/1393 & 1079/881 & 0 \\ 0 & -881/1079 & 1351/1170 \end{pmatrix} \times \begin{pmatrix} 1393/985 & -985/1393 & 0 \\ 0 & 1079/881 & -\frac{881}{1079} \\ 0 & 0 & 1351/1170 \end{pmatrix}$$

$$n=4: \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1393/985 & 0 & 0 & 0 \\ -985/1393 & 1079/881 & 0 & 0 \\ 0 & -\frac{881}{1079} & \frac{1351}{1170} & 0 \\ 0 & 0 & -\frac{1170}{1351} & \frac{2889}{2584} \end{pmatrix} \times \begin{pmatrix} 1393/985 & -985/1393 & 0 & 0 \\ 0 & 1079/881 & -881/1079 & 0 \\ 0 & 0 & 1351/1170 & -\frac{1170}{1351} \\ 0 & 0 & 0 & \frac{2889}{2584} \end{pmatrix}$$

No se cumple, es bigonal.

④

| | | | | | |
|---|---|---|---|---|---|
| $x = t$ | 1 | 2 | 3 | 4 | 5 |
| $y$ = Pérdida | 9 | 7.5 | 4.2 | 3 | 2.1 |

f: función de pérdida de peso en función del tiempo.

ⓐ $f_1(t) = mt + p$

Sea $D(m,p) = \Sigma\left(y_i - (mt_i + p)\right)^2 = \left(Y - f(t)\right)^2$

Se busca mimimizar D.

$$\Rightarrow \begin{array}{l} \dfrac{\partial D}{\partial m} = 0 \\ \dfrac{\partial D}{\partial p} = 0 \end{array} \iff \underbrace{\begin{pmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} p \\ m \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{pmatrix}}_{B}$$

$\Rightarrow$

| | $x$ | $y$ | $x^2$ | $xy$ |
|---|---|---|---|---|
| | 1 | 9 | 1 | 9 |
| | 2 | 7.5 | 4 | 15 |
| | 3 | 4.2 | 9 | 12.6 |
| | 4 | 3 | 16 | 12 |
| | 5 | 2.1 | 25 | 10.5 |
| $\Sigma$ | 15 | 25.8 | 55 | 59.1 |

$A \quad X = B$

$$\Rightarrow \begin{pmatrix} 5 & 15 \\ 15 & 55 \end{pmatrix} \begin{pmatrix} p \\ m \end{pmatrix} = \begin{pmatrix} 25.8 \\ 59.1 \end{pmatrix} \quad (*)$$

resolviendo (*) $p = 10.65 \quad m = -1.33$

ⓑ Para la regresión cuadrática

$f_2(t) = mt^2 + pt + q$

$$\begin{array}{l} \dfrac{\partial D}{\partial m} = 0 \\ \dfrac{\partial D}{\partial p} = 0 \\ \dfrac{\partial D}{\partial q} = 0 \end{array} \iff \begin{pmatrix} n & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{pmatrix} \begin{pmatrix} q \\ p \\ m \end{pmatrix} = \begin{pmatrix} \Sigma y_i \\ \Sigma y_i x_i \\ \Sigma y_i x_i^2 \end{pmatrix}$$

| | $x^3$ | $yx^2$ | $x^4$ |
|---|---|---|---|
| | 1 | 9 | 1 |
| | 8 | 30 | 16 |
| | 27 | 37.8 | 81 |
| | 64 | 48 | 256 |
| | 125 | 52.5 | 625 |
| $\Sigma$ | 225 | 177.3 | 979 |

$$\Rightarrow \begin{pmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{pmatrix} \begin{pmatrix} q \\ p \\ m \end{pmatrix} = \begin{pmatrix} 25.8 \\ 59.1 \\ 177.3 \end{pmatrix} \quad (*)$$

resolviendo $(*)$
$q = 12.3$
$p = -3.24$
$m = 0.24$

$$D(m,p) = (9-8.82)^2 + (7.5-6.99)^2 + (4.2-5.16)^2 + (3-3.33)^2 + (2.1-1.5)^2 = 1.68$$

$$D(m,p,q) = (9-9.3)^2 + (7.5-6.78)^2 + (4.2-4.74)^2 + (3-3.18)^2 + (2.1-2.1)^2 = 0.9324$$

$$F_1(2.5) = -1.83(2.5) + 10.65 = 6.075 \text{ Kg}$$

$$f_2(2.5) = 0.24(2.5)^2 - 3.24(2.5) + 12.3 = 5.7 \text{ Kg}.$$

$\therefore$ La regresión cuadrática se ajusta mejor porque tiene el menor error.

5

$$A = \begin{pmatrix} 3 & -2 & 3 \\ -2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

a) La matriz es simétrica, por lo que

$$\|\cdot\|_1 = \|\cdot\|_\infty = 8$$

$$\|\cdot\|_2 = \sqrt{\rho(A^*A)} = \sqrt{\rho(A^TA)} = \sqrt{\rho(A^2)}$$

Los autovalores son:

$$\lambda_1 = -3.08 \quad \lambda_2 = 2.85 \quad \boxed{\lambda_3 = 5.23}$$

$$\Rightarrow \|\cdot\|_2 = 2{,}287.$$

b) $n = \|A\|\,\|A^{-1}\|$

$$A^{-1} = \frac{Adj(A)}{|A|} \quad \text{con } |A| \neq 0$$

$$A^{-1} = \begin{pmatrix} 3/46 & -4/23 & 7/46 \\ -4/23 & 3/23 & 6/23 \\ 7/46 & 6/23 & 1/46 \end{pmatrix}$$

Usando $\|\cdot\|_1$

$$\|A^{-1}\|_1 = \max\{0.044, 0.217, 0.435\} = 0.435$$

$$n = 8 \times 0.435 = 3.48$$