School of Computer Science

Faculty of Science

National University of Engineering

Midterm Exam

Topics: **geometric transformations; curves and programming in Python**

Subject: Computational Mathematics   **Period: 2020-2**

1. (3 pts.) In a sheet of paper: reflect the point $P := (2, -1, 3)$ in the plane through the vector $P_0 := (1, 2, -1)$ with normal $(-1, 5, 2)$.

2. (3 pts.) Let U be the unit cube with vertices (a, b, c), where each component is 0 or 1. In the same sheet of paper above: find the co-ordinates of the vertices of U rotated $2\pi/3$ clockwise around the line from $P_0 := (0, 0, 0)$ to $(1, 1, 1)$.

3. Let $P_0 := (-2, 1)$, $P_1 := (0, -4)$, $P_2 := (3, 2)$ and $P_3 := (5, 0)$. In a *.ipynb file:

   (a) (3 pts.) Find the parametric description $P(t)$ of the cubic Bézier curve with control points: P_0, P_1, P_2 and P_3.

   (b) (1 pt.) Print the Bézier curve above with its control points.

   (c) (3 pts.) Find the parametric description $P(t)$ of the uniform quadratic B-spline using control points: P_0, P_1, P_2 and P_3.

   (d) (1 pt.) Print the B-spline above with its control points.

4. Let $P_0 := (-1, 0)$, $P_1 := (1, 4)$, $P_2 := (3, -2)$, $P_3 := (4, 3)$ and $P_4 := (6, 1)$. In the same *.ipynb file above:

(a) (3 pts.) Construct a uniform cubic B-spline using the control points P_0, P_1, P_2, P_3 and P_4. Find the parametric expressions for the coordinates x and y.

(b) (1 pt.) Print the curve above.

(c) (2 pts.) Verify by finding the derivatives that, at the joining point between the fourth and fifth segments, the first and second derivatives match.

December 23, 2020