Respuestas Examen Final

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1 Pregunta 1

1a.

$$\begin{bmatrix} 79.1959595 & -4.9 \\ 0.7959595 & 4.9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 195.0 \\ 194.968 \end{bmatrix}$$

$$x^{(0)} = \begin{bmatrix} -5.0500000 \\ 80.0500000 \end{bmatrix}$$

1b. Por el método de Descenso Rápido:

k	$v_1^{(k)}$	$v_2^{(k)}$	t	$x_1^{(k+1)}$	$x_2^{(k+1)}$	$Error$
0	987.1846255	−193.2577374	0.0129488	7.7328160	77.5475551	12.7828162
1	−37.4247610	−191.1703465	0.1455925	2.2840500	49.7145787	27.8329764
2	257.7139080	−50.4517650	0.0129488	5.6211253	49.0612909	3.3370754
3	−9.7700887	−49.9068318	0.1455925	4.1986733	41.7952285	7.2660624
4	67.2786596	−13.1709117	0.0129488	5.0698485	41.6246815	0.8711752
5	−2.5505743	−13.0286517	0.1455925	4.6985039	39.7278070	1.8968745
6	17.5637321	−3.4383914	0.0129488	4.9259324	39.6832841	0.2274285
7	−0.6658516	−3.4012531	0.1455925	4.8289894	39.1880871	0.4951971
8	4.5851788	−0.8976247	0.0129488	4.8883618	39.1764639	0.0593724
9	−0.1738269	−0.8879294	0.1455925	4.8630539	39.0471880	0.1292759
10	1.1970044	−0.2343334	0.0129488	4.8785536	39.0441537	0.0154997
11	−0.0453792	−0.2318024	0.1455925	4.8719467	39.0104050	0.0337487
12	0.3124894	−0.0611750	0.0129488	4.8759931	39.0096129	0.0040463
13	−0.0118467	−0.0605142	0.1455925	4.8742683	39.0008025	0.0088104
14	0.0815783	−0.0159703	0.0129488	4.8753246	39.0005957	0.0010563
15	−0.0030927	−0.0157978	0.1455925	4.8748744	38.9982956	0.0023000
16	0.0212968	−0.0041692	0.0129488	4.8751501	38.9982416	0.0002758
17	−0.0008074	−0.0041242	0.1455925	4.8750326	38.9976412	0.0006004
18	0.0055597	−0.0010884	0.0129488	4.8751046	38.9976271	0.0000720
19	−0.0002108	−0.0010767	0.1455925	4.8750739	38.9974703	0.0001568
20	0.0014514	−0.0002841	0.0129488	4.8750927	38.9974667	0.0000188
21	−0.0000550	−0.0002811	0.1455925	4.8750847	38.9974257	0.0000409
22	0.0003789	−0.0000742	0.0129488	4.8750896	38.9974248	0.0000049

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4.8750896 \\ 38.9974248 \end{bmatrix}$$

N Iteraciones=23

1c. Por el método de Gradiente Conjugado:

$$\begin{bmatrix} 6272.63355265128 & -384.16 \\ -384.16 & 48.02 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15598.3984810892 \\ -0.158358764648483 \end{bmatrix}$$

k	$v_1^{(k)}$	$v_2^{(k)}$	$Error$
0	7.3445986	79.1311898	78027.2082908
1	4.8750885	38.9974100	979.4468146

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4.8750885 \\ 38.9974100 \end{bmatrix}$$

1e. Recomiendo usar el método de Gradiente Conjugado, ya que converge más rápido y da un valor más cercano.

2 Pregunta 2

2a.

$$F(x) = \begin{bmatrix} x_1x_2 - 72 \\ x_1x_2 - 3x_1 + 2x_2 - 78 \end{bmatrix}$$

$$JF(x) = \begin{bmatrix} x_2 & x_1 \\ x_2 - 3 & x_1 + 2 \end{bmatrix}$$

$$[JF(x)]^{-1} = \begin{bmatrix} \frac{-x_1(3-x_2)-x_1(x_2-3)+x_2(x_1+2)}{x_2(-x_1(x_2-3)+x_2(x_1+2))} & -\frac{x_1}{-x_1(x_2-3)+x_2(x_1+2)} \\ \frac{3-x_2}{-x_1(x_2-3)+x_2(x_1+2)} & \frac{x_2}{-x_1(x_2-3)+x_2(x_1+2)} \end{bmatrix}$$

$$x^{(0)} = \begin{bmatrix} 3.0000000 \\ 6.0000000 \end{bmatrix}$$

$$tol = 0.0000100$$

2b.Usando el método de Newton:

k	$x_1^{(k)}$	$x_2^{(k)}$
0	3.0000000	6.0000000
1	7.7142857	14.5714286
2	6.1686183	12.2529274
3	6.0019831	12.0029746
4	6.0000003	12.0000004
5	6.0000000	12.0000000

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6.0000000 \\ 12.0000000 \end{bmatrix}$$

2c.Usando Homeotopia:

$$F(x) = \begin{bmatrix} x_1x_2 - 72 \\ x_1x_2 - 3x_1 + 2x_2 - 78 \end{bmatrix}$$

$$JF(x) = \begin{bmatrix} x_2 & x_1 \\ x_2 - 3 & x_1 + 2 \end{bmatrix}$$

$$[JF(x)]^{-1} = \begin{bmatrix} \frac{-x_1(3-x_2)-x_1(x_2-3)+x_2(x_1+2)}{x_2(-x_1(x_2-3)+x_2(x_1+2))} & -\frac{x_1}{-x_1(x_2-3)+x_2(x_1+2)} \\ \frac{3-x_2}{-x_1(x_2-3)+x_2(x_1+2)} & \frac{x_2}{-x_1(x_2-3)+x_2(x_1+2)} \end{bmatrix}$$

$$x^{(0)} = \begin{bmatrix} 3.0000000 \\ 6.0000000 \end{bmatrix}$$

$$tol = 0.0100000$$

i	$x_1^{(i)}$	$x_2^{(i)}$	$F_1(x^{(i)})$	$F_2(x^{(i)})$	error
0	3.0000000	6.0000000	−54.0000000	−57.0000000	−
1	3.6082272	7.1266265	−46.2855121	−48.8569407	48.8569407
2	4.1215055	8.1108297	−38.5711703	−40.7140275	40.7140275
3	4.5714462	9.0000264	−30.8568633	−32.5711490	32.5711490
4	4.9752725	9.8200516	−23.1425679	−24.4282822	24.4282822
5	5.3435971	10.5868242	−15.4282771	−16.2854199	16.2854199
6	5.6835074	11.3109753	−7.7139885	−8.1425599	8.1425599
7	6.0000142	12.0000214	0.0002990	0.0002990	0.0002990

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6.0000142 \\ 12.0000214 \end{bmatrix}$$

2e. Recomiendo usar el método de Newton, ya que necesita menos iteraciones para llegar al valor exacto, en cambio homeotopía demora muchisimo más en converger.

3 Pregunta 3

3a.

$$A = \begin{bmatrix} 0.7000000 & 0.2000000 & 0.1000000 \\ 0.3000000 & 0.6000000 & 0.1000000 \\ 0.1000000 & 0.3000000 & 0.6000000 \end{bmatrix}$$

$$y = \begin{bmatrix} 0.0000000 \\ 1.0000000 \\ 1.0000000 \end{bmatrix}$$

$$Ay = \begin{bmatrix} 0.3000000 \\ 0.7000000 \\ 0.9000000 \end{bmatrix}$$

$$A^2y = \begin{bmatrix} 0.4400000 \\ 0.6000000 \\ 0.7800000 \end{bmatrix}$$

$$A^3y = \begin{bmatrix} 0.5060000 \\ 0.5700000 \\ 0.6920000 \end{bmatrix}$$

$$\begin{bmatrix} 0.506 \\ 0.57 \\ 0.692 \end{bmatrix} + b_1 \begin{bmatrix} 0.44 \\ 0.6 \\ 0.78 \end{bmatrix} + b_2 \begin{bmatrix} 0.3 \\ 0.7 \\ 0.9 \end{bmatrix} + b_3 \begin{bmatrix} 0.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

3b. Por el método de Krylon:

$$f(x) = x^3 + -1.9000000x^2 + 1.1000000x + -0.2000000$$

3c. Por el metodo de Potencia, la tabla es:

k	y_1	y_2	y_3	λ	x_1	x_2	x_3	Err
0	–	–	–	–	1.0000000	0.0000000	0.0000000	–
1	0.7000000	0.3000000	0.1000000	0.7	1.0000000	0.4285714	0.1428571	0.451
2	0.8000000	0.5714286	0.3142857	0.7999999999999999	1.0000000	0.7142857	0.3928571	0.379
3	0.8821429	0.7678571	0.5500000	0.882142857142857	1.0000000	0.8704453	0.6234818	0.278
4	0.9364372	0.8846154	0.7352227	0.9364372469635627	1.0000000	0.9446606	0.7851275	0.177
5	0.9674449	0.9453091	0.8544747	0.9674448767833981	1.0000000	0.9771194	0.8832283	0.103
6	0.9837467	0.9745944	0.9230728	0.9837467042052107	1.0000000	0.9906965	0.9383236	0.056
7	0.9919717	0.9882503	0.9602031	0.9919716716712169	1.0000000	0.9962485	0.9679744	0.030
8	0.9960471	0.9945465	0.9796592	0.9960471356078358	1.0000000	0.9984934	0.9835470	0.015
9	0.9980534	0.9974508	0.9896762	0.9980533881605883	1.0000000	0.9993962	0.9916065	0.008
10	0.9990399	0.9987984	0.9947828	0.999039890692029	1.0000000	0.9997582	0.9957388	0.004
11	0.9995255	0.9994288	0.9973707	0.999525527783183	1.0000000	0.9999033	0.9978442	0.002
12	0.9997651	0.9997264	0.9986775	0.9997650699378258	1.0000000	0.9999613	0.9989122	0.001
13	0.9998835	0.9998680	0.9993357	0.9998834750583449	1.0000000	0.9999845	0.9994521	0.000
14	0.9999421	0.9999359	0.9996666	0.9999421178612532	1.0000000	0.9999938	0.9997245	0.000
15	0.9999712	0.9999687	0.9998328	0.9999712121130833	1.0000000	0.9999975	0.9998616	0.000
16	0.9999857	0.9999847	0.9999162	0.9999856675870317	1.0000000	0.9999990	0.9999306	0.000
17	0.9999929	0.9999925	0.9999580	0.9999928584691096	1.0000000	0.9999996	0.9999652	0.000
18	0.9999964	0.9999963	0.9999790	0.9999964391204464	1.0000000	0.9999998	0.9999826	0.000
19	0.9999982	0.9999982	0.9999895	0.9999982235184545	1.0000000	0.9999999	0.9999913	0.000
20	0.9999991	0.9999991	0.9999947	0.9999991133434805	1.0000000	1.0000000	0.9999956	0.000
21	0.9999996	0.9999995	0.9999974	0.9999995573056802	1.0000000	1.0000000	0.9999978	0.000
22	0.9999998	0.9999998	0.9999987	0.9999997789064754	1.0000000	1.0000000	0.9999989	0.000
23	0.9999999	0.9999999	0.9999993	0.9999998895547066	1.0000000	1.0000000	0.9999995	0.000
24	0.9999999	0.9999999	0.9999997	0.9999999448179445	1.0000000	1.0000000	0.9999997	0.000
25	1.0000000	1.0000000	0.9999998	0.9999999724252095	1.0000000	1.0000000	0.9999999	0.000
26	1.0000000	1.0000000	0.9999999	0.9999999862190999	1.0000000	1.0000000	0.9999999	0.000
27	1.0000000	1.0000000	1.0000000	0.9999999931121479	1.0000000	1.0000000	1.0000000	0.000
28	1.0000000	1.0000000	1.0000000	0.9999999965571134	1.0000000	1.0000000	1.0000000	0.000
29	1.0000000	1.0000000	1.0000000	0.9999999982789723	1.0000000	1.0000000	1.0000000	0.000

La solucion del valor y vector propios son $\lambda_1 = 1.0000000$ y $x_1 = \begin{bmatrix} 1.0000000 \\ 1.0000000 \\ 1.0000000 \end{bmatrix}$

Por el metodo de Potencia inversa, la tabla es:

k	y_1	y_2	y_3	λ	x_1	x_2	x_3
0	–	–	–	–	1.0000000	0.0000000	0.0000000
1	1.6500000	-0.8500000	0.1500000	1.6500000000000001	1.0000000	-0.5151515	0.0909091
2	1.8636364	-1.9242424	0.8030303	−1.9242424242424239	-0.9685039	1.0000000	-0.4173228
3	-1.9645669	2.9566929	-1.8464567	2.956692913385827	-0.6644474	1.0000000	-0.6245007
4	-1.4214381	2.7396804	-2.1737683	2.739680426098536	-0.5188335	1.0000000	-0.7934386
5	-1.1473876	2.6496962	-2.4560146	2.6496962332928313	-0.4330261	1.0000000	-0.9269042
6	-0.9791122	2.6034530	-2.6833815	−2.6833814830100415	0.3648800	-0.9702135	1.0000000
7	0.8386481	-2.4990857	2.7764349	2.7764348517084927	0.3020593	-0.9001060	1.0000000
8	0.7034456	-2.3019678	2.7004096	2.700409621015928	0.2604959	-0.8524513	1.0000000
9	0.6134213	-2.1689467	2.6489032	2.648903150441974	0.2315756	-0.8188094	1.0000000
10	0.5505640	-2.0753985	2.6126052	2.6126052489105174	0.2107337	-0.7943789	1.0000000
11	0.5051811	-2.0076004	2.5862700	2.5862700083727264	0.1953319	-0.7762532	1.0000000
12	0.4716116	-1.9573512	2.5667403	2.566740333409842	0.1837395	-0.7625825	1.0000000
13	0.4463323	-1.9194727	2.5520143	2.552014281843771	0.1748941	-0.7521403	1.0000000
14	0.4270384	-1.8905475	2.5407674	2.540767359434162	0.1680746	-0.7440853	1.0000000
15	0.4121614	-1.8682382	2.5320922	2.532092179695293	0.1627751	-0.7378239	1.0000000
16	0.4005996	-1.8508978	2.5253490	2.5253489787889905	0.1586314	-0.7329275	1.0000000
17	0.3915592	-1.8373381	2.5200759	2.5200758732994197	0.1553759	-0.7290805	1.0000000
18	0.3844565	-1.8266845	2.5159329	2.5159328518332815	0.1528087	-0.7260466	1.0000000
19	0.3788554	-1.8182830	2.5126656	2.5126656013135804	0.1507783	-0.7236470	1.0000000
20	0.3744253	-1.8116380	2.5100814	2.5100814218455216	0.1491686	-0.7217447	1.0000000
21	0.3709133	-1.8063699	2.5080328	2.508032751104796	0.1478901	-0.7202338	1.0000000
22	0.3681239	-1.8021859	2.5064056	2.506405621468372	0.1468732	-0.7190320	1.0000000
23	0.3659053	-1.7988579	2.5051114	2.5051114014861535	0.1460635	-0.7180750	1.0000000
24	0.3641385	-1.7962077	2.5040808	2.50408077818671	0.1454180	-0.7173122	1.0000000
25	0.3627302	-1.7940953	2.5032593	2.503259302511233	0.1449032	-0.7167038	1.0000000
26	0.3616069	-1.7924104	2.5026040	2.502604047121195	0.1444923	-0.7162181	1.0000000
27	0.3607104	-1.7910656	2.5020811	2.5020810700403993	0.1441642	-0.7158304	1.0000000
28	0.3599945	-1.7899918	2.5016635	2.5016634713222654	0.1439021	-0.7155206	1.0000000
29	0.3594227	-1.7891340	2.5013299	2.5013298921668476	0.1436926	-0.7152731	1.0000000
30	0.3589657	-1.7884486	2.5010633	2.5010633480797755	0.1435252	-0.7150753	1.0000000
31	0.3586005	-1.7879008	2.5008503	2.5008503167913734	0.1433914	-0.7149172	1.0000000
32	0.3583086	-1.7874629	2.5006800	2.5006800221396586	0.1432845	-0.7147907	1.0000000
33	0.3580752	-1.7871128	2.5005439	2.5005438697744347	0.1431989	-0.7146896	1.0000000
34	0.3578886	-1.7868329	2.5004350	2.5004350011859904	0.1431305	-0.7146088	1.0000000
35	0.3577393	-1.7866090	2.5003479	2.500347940407013	0.1430758	-0.7145441	1.0000000
36	0.3576200	-1.7864299	2.5002783	2.500278313590999	0.1430321	-0.7144924	1.0000000
37	0.3575245	-1.7862868	2.5002226	2.5002226260888554	0.1429971	-0.7144511	1.0000000
38	0.3574481	-1.7861722	2.5001781	2.5001780850125375	0.1429691	-0.7144180	1.0000000
39	0.3573871	-1.7860806	2.5001425	2.5001424578621867	0.1429467	-0.7143915	1.0000000
40	0.3573382	-1.7860073	2.5001140	2.500113959795962	0.1429288	-0.7143704	1.0000000
41	0.3572991	-1.7859487	2.5000912	2.500091163681172	0.1429144	-0.7143534	1.0000000
42	0.3572679	-1.7859018	2.5000729	2.500072928285573	0.1429030	-0.7143399	1.0000000
43	0.3572429	-1.7858643	2.5000583	2.500058340926577	0.1428938	-0.7143291	1.0000000
44	0.3572229	-1.7858343	2.5000467	2.5000466716521146	0.1428865	-0.7143204	1.0000000
45	0.3572069	-1.7858103	2.5000373	2.500037336624667	0.1428806	-0.7143134	1.0000000
46	0.3571941	-1.7857911	2.5000299	2.5000298688536526	0.1428759	-0.7143079	1.0000000
47	0.3571838	-1.7857757	2.5000239	2.5000238947974385	0.1428722	-0.7143035	1.0000000
48	0.3571756	-1.7857634	2.5000191	2.5000191156552445	0.1428692	-0.7142999	1.0000000
49	0.3571691	-1.7857536	2.5000153	2.5000152924072663	0.1428668	-0.7142971	1.0000000
50	0.3571638	-1.7857457	2.5000122	2.500012233850979	0.1428648	-0.7142948	1.0000000
51	0.3571596	-1.7857395	2.5000098	2.50000978703289	0.1428633	-0.7142930	1.0000000
52	0.3571563	-1.7857344	2.5000078	2.5000078295956607	0.1428621	-0.7142915	1.0000000
53	0.3571536	-1.7857304	2.5000063	2.5000062636569114	0.1428611	-0.7142904	1.0000000
54	0.3571514	-1.7857272	2.5000050	2.500005010912975	0.1428603	-0.7142894	1.0000000

La solucion del valor y vector propios son $\lambda_2 = 0.4000000$ y $x_2 = \begin{bmatrix} 0.1428572 \\ -0.7142857 \\ 1.0000000 \end{bmatrix}$

Por el metodo de Potencia inversa desplazado, la tabla es:

k	y_1	y_2	y_3	
0	–	–	–	
1	-2771445924535687.5000000	13857229622678452.0000000	-19400121471749836.0000000	−1.9400121
2	-2375525078173448.0000000	11877625390867234.0000000	-16628675547214124.0000000	−1.6628675

La solucion del valor y vector propios son $\lambda_3 = 0.4000000$ y $x_3 = \begin{bmatrix} 0.1428571 \\ -0.7142857 \\ 1.0000000 \end{bmatrix}$

3d. Debido a que todos los valores propios cumplen, si es una renta estable.

4 Pregunta 4

4a.
Coeficientes del polinomio

$$P(x) = 0.0091199x^7 + -0.1098769x^6 + 0.6090926x^5 + 0.0600000x^4 + -13.5000000x^3 + 60.0000000x^2 + -116.000000$$

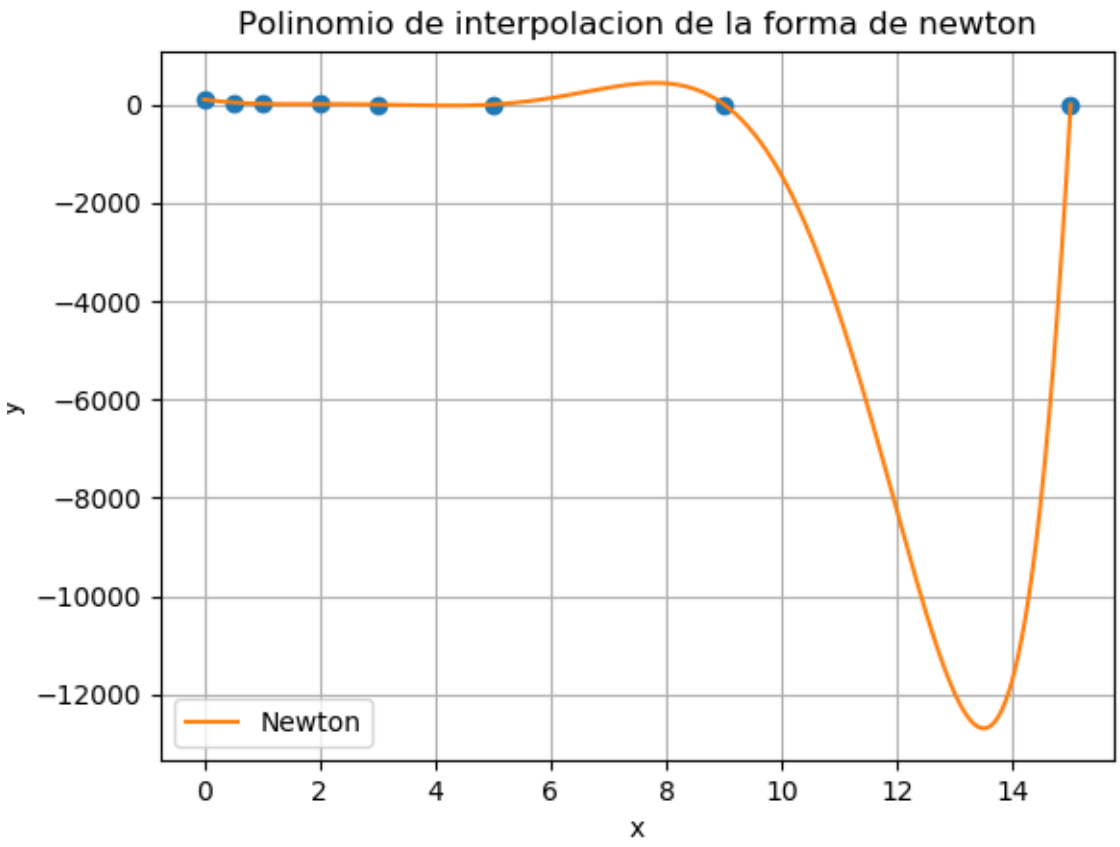
Polinomio de Newton

$$P(t) = 0.00911991815996465t\,(t-9.0)\,(t-5.0)\,(t-3.0)\,(t-2.0)\,(t-1.0)\,(t-0.5) - 0.109876925770308t\,(t-5.$$

4b.

$$P(5.5) = 498.2918625898433$$

4e.
Newton:


Polinomio de interpolacion de la forma de newton
0
−2000
−4000
−6000
−8000
−10000
−12000
y
Newton
0
2
4
6
8
10
12
14
x

Spline: