P3 :

$P_0 = (1, 4, 8)$
$P_1 = (2, 2, -1)$
$P_2 = (5, 4, 0)$

El plano determinado por los 3 puntos, en coordenadas homogéneas es:

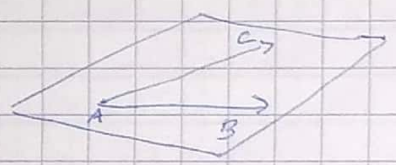
$$(x, y, z) = (1, 4, 8)r + (2, 2, -1)s + (5, 4, 0)t$$

con r, s y $t \in \mathbb{R}$.

En coordenadas cartesianas:

$$ax + by + cz + d = 0$$

$$\vec{AB} = (B_x - A_x, B_y - A_y, B_z - A_z)$$
$$\vec{AC} = (C_x - A_x, C_y - A_y, C_z - A_z)$$



$$\vec{AB} \times \vec{AC} = (a, b, c)$$

$$a = (B_y - A_y)(C_z - A_z) - (C_y - A_y)(B_z - A_z)$$
$$b = (B_z - A_z)(C_x - A_x) - (C_z - A_z)(B_x - A_x)$$
$$c = (B_x - A_x)(C_y - A_y) - (C_x - A_x)(B_y - A_y)$$
$$d = -(aA_x + bA_y + cA_z)$$

$$\Rightarrow \quad 16x - 28y + 8z + 32 = 0$$
$$n = (16, -28, 8)$$

Ahora tenemos los 3 planos:

$$\begin{aligned} 2x + 3y - 6z &= 2 \\ -x + 4y + z &= 7 \\ -8x + y - z &= 10 \end{aligned} \quad \Rightarrow \quad \left(\begin{array}{ccc|c} 2 & 3 & -6 & 2 \\ -1 & 4 & 1 & 7 \\ -8 & 1 & -1 & 10 \\ 16 & -28 & 8 & -32 \end{array}\right)$$

No existe solución

pero si solo consideramos los 3 planos:

$$\left(\begin{array}{ccc|c} 2 & 3 & -6 & 2 \\ -1 & 4 & 1 & 7 \\ -8 & 1 & -1 & 10 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 2 & 3 & -6 & 2 \\ 0 & \frac{11}{2} & -2 & 8 \\ 0 & 0 & -\frac{223}{11} & -\frac{10}{11} \end{array}\right)$$

$$\Rightarrow \quad \bar{x} = \left(-\frac{239}{223}, \frac{328}{223}, \frac{10}{223}\right) \quad \Rightarrow \quad -\frac{239}{223}x + \frac{328}{223}y + \frac{10}{223}z = 0$$