MET CS248 - HW #2

1) List all elements of set A, where $A = \{x \mid x \in N \text{ and } -3 \leq x \leq 5\}$, assume $0 \in N$.

$\{0, 1, 2, 3, 4, 5\}$

2) Fill in the blanks: $\cup$*: union,* $\cap$*: intersection,* $\emptyset$*: empty set,* $\mathscr{P}$*: power set,* $-$*: set difference,* $\times$*: cartesian product.*

a) $A \cup A = A$

b) $A \cap A = A$

c) $A \cup \emptyset = A$

d) $\mathscr{P}(\mathscr{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

e) $\mathscr{P}(\{a, \{b\}\}) = \{\emptyset, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$

f) $\{a, \{b\}\} - \{a, \{b\}, c\} = \emptyset$

g) $\{1, a\} \times \{a\} = \{(1, a), (a, a)\}$

h) $A \times \emptyset = \emptyset$

3) Let $A = \{2, 4, 5, 6, 8\}$ $B = \{1, 4, 5, 9\}$ $C = \{x \mid x \in \mathbf{Z} \text{ and } 2 \leq x < 5\}$ be subsets of $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Find

a) $A \cup B = \{1, 2, 4, 5, 6, 8, 9\}$

b) $A \cap B = \{4, 5\}$

c) $A \cap C = \{2, 4\}$

d) $C \cup B = \{1, 2, 3, 4, 5, 9\}$

e) $A - B = \{2, 6, 8\}$

f) $A' = \{0, 1, 3, 7, 9\}$

g) $A \cap A' = \emptyset$

h) $(A \cap B)' = \{0, 1, 2, 3, 6, 7, 8, 9\}$

i) $C - B = \{2, 3\}$

j) $(C \cap B) \cup A' = \{0, 1, 3, 4, 7, 9\}$

k) $(C' \cup B)' = \{2, 3\}$

l) $|B \times C| = 12$

4) Let $A = \{a,\ \{a\},\ \{\{a\}\}\}$, $B = \{\emptyset, \{a\}, \{a, \{a\}\}\}$, $C = \{a\}$ be subsets of $S = \{\emptyset, a,\ \{a\},\ \{\{a\}\}, \{a, \{a\}\}\}$. Find

a) $(B \cup C) \cap A = \{\emptyset, a, \{a\}, \{a, \{a\}\}\} \cap A$

$= \{a, \{a\}\}$

b) $A' \cap B = \{\emptyset, \{a, \{a\}\}\} \cap B$

$= \{\emptyset, \{a, \{a\}\}\}$

c) $\emptyset \cap B = \emptyset$

d) $\{\emptyset\} \cap B = \{\emptyset\}$

e) $C' \cap B = \{\emptyset, \{a\}, \{\{a\}\}, \{a, \{a\}\}\} \cap B$

$= \{\emptyset, \{a\}, \{a, \{a\}\}\}$

5) Is the infinite set of all words, meaningful or not, of any finite length, made with the 26 letters of the English alphabet countable? Justify. (Length of a word is the number of characters in it)

I assign $\Sigma(26^{n-1} \cdot d)$ to i,

i : positive number

n : digit of alphabet

d : order of alphabet

ex.

a	$= 26^{1-1} \cdot 1$	$= 1$
z	$= 26^{1-1} \cdot 26$	$= 26$
a a	$= 26^{1-1} \cdot 1 + 26^{2-1} \cdot 1$	$= 27$
a z	$= 26^{1-1} \cdot 26 + 26^{2-1} \cdot 1$	$= 52$
b a	$= 26^{1-1} \cdot 1 + 26^{2-1} \cdot 2$	$= 53$