METCS248 - HW#4

1. Let S = {1, 2, 3, 4, 5, 6, 7, 8}, and let |T| = 10.

a. How many different functions can we define from S to S?

Each preimage in the domain S has eight images in the codomain S.

8^8

b. How many injective functions can we define from S to T?

For 1 in S we have 10 choices in T.
For 2 in S we have 9 choices in T.
For 3 in S we have 8 choices in T.
⋮

$10P_8 = \frac{10!}{(10-8)!} = 1,814,400$

c. How many bijections can we define between S and S?

For 1 in S we have 8 choices in S.
For 2 in S we have 7 choices in S.
For 3 in S we have 6 choices in S.
⋮

$8! = 40,320$

2) You have a group of 10 men and 6 women.

a) In how many ways can you seat them in a row?

If assume all the men or women are distinctive.

$16! = 20,922,789,888,000$

If assume all the men or women are indistinctive.

$10C_6 = \frac{10!}{(10-6)!6!} = 210$

b) In how many ways can you choose a committee of 5 with at least 2 women?

Choose	2	women	$6C_2 \cdot 10C_3 =$	1,800
Choose	3	women	$6C_3 \cdot 10C_2 =$	900
Choose	4	women	$6C_4 \cdot 10C_1 =$	150
Choose	5	women	$6C_5 \cdot 10C_0 =$	6
				2,856 ways

3. A florist has roses, carnations, lilies, and snapdragons in stock. How many different bouquets of one dozen flowers can be made? Explain briefly.

Suppose we have 15 dots.
We choose 3 dots.
We assign roses from the leftmost to the 1st left.
We assign carnations from the 2nd left to the 2nd left.
We assign lilies from the 2nd left to the 3rd left.
We assign snapdragons from the 3rd left to the rightmost.

$15C_3 = 455$

4. How many distinct permutations of the characters in HORROR are there?

Out of 6, we assign one "H".
Out of 5, we assign two "O".
Out of 3, we assign three "R".

$6C_1 \cdot 5C_2 \cdot 3C_3 = 60$

5. Suppose you have a computer network with 60 switching nodes.

a) The network is designed to withstand the failure of any two nodes. In how many ways can such a failure occur?

We choose two nodes out of 60 nodes.

$60C_2 = 1,770$

b) In how many ways can one or two nodes fail?

One node fails $60C_1 = 60$
two nodes fails $60C_2 = 1,770$
1,830 ways

c) If one node failed, in how many ways can seven nodes be selected without encountering the failed node?

We choose seven nodes out of 59 nodes.

$59C_7 = \frac{59!}{(59-7)!7!} = 341,149,446$