MET CS248 - HW #1

1) Decide which of the following are propositions (Y/N) and give the truth value:

a) 127 is an even integer

Yes, false.

b) All triangles have 4 sides or more

Yes, false.

2) Negate the following statements, using full sentences:

a) John is smart or Fred is not tall

John is not smart and Fred is tall.

b) Some animals are intelligent

No animals are intelligent.

3) Is $(\overline{P} \rightarrow Q) \leftrightarrow$ to$(P \rightarrow \overline{Q})$? ($\overline{Q}$ means not Q, $\leftrightarrow$ stands for equivalent). Justify your answer.

P	Q	$\overline{P}$	$\overline{P} \rightarrow Q$	$\overline{Q}$	$P \rightarrow \overline{Q}$
T	T	F	T	F	F
T	F	F	T	T	T
F	T	T	T	F	T
F	F	T	F	T	T

Not equivalent

4) Given the proposition P(n) " If n^3 is odd then n is odd "

a) Prove P(n) by contraposition:

Hypo : $n = 2k_1$

Concl : $n^3 = 2k_2$

Proof : $n = 2k_1 \Rightarrow n^3 = 2^3 \cdot l_1^3 \Rightarrow n^3 = 2 \cdot (4 l_1^3)$

$\Rightarrow n^3 = 2k_2$

b) Prove P(n) contradiction:

Hypo : $n = 2k_1$

Concl : $n^3 = 2k_2 + 1$

Proof : $n = 2k_1 \Rightarrow n^3 = 2 \cdot (4k_1^3)$

Contradiction

5) Prove by induction: $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$, $n \geq 1$

1- Base : $1^2 = \frac{1\cdot(1+1)\cdot(2\cdot 1+1)}{6} = 1$

2- Assume : $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

3- Proof : $1 + 2^2 + 3^2 + \cdots + n^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6} = \frac{(n+1)\{n(2n+1)+6(n+1)}{6}$$

$$= \frac{1}{6}(n+1)(2n^2+7n+6) = \frac{1}{6}(n+1)(2n+3)(n+2)$$

$$\begin{matrix} 2 & & 3 \\ 1 & \times & 2 \end{matrix}$$

$$= \frac{(n+1)\{(n+1)+1\}\{2(n+1)+1\}}{6}$$

6) Using the predicate symbols shown and appropriate quantifiers, write each English Language statement in symbolic form (The domain is the whole world).

J(x) is "x is a judge" *C(x) is "x is a chemist"*
L(x) is "x is a lawyer" *A(x, y) is "x admires y"* *W(x) is "x is a woman"*

a) There are some women lawyers who are chemists.

$\exists x \; W(x)\cdot L(x)\cdot C(x)$

b) No woman is both a lawyer and chemist.

$[\exists x \; W(x)\cdot L(x)\cdot C(x)]'$

c) Some lawyers admire only judges.

$(\exists x)[L(x)\cdot(\forall z)(A(x,z)\to J(z))]$

d) All women lawyers admire some judges.

$(\forall x)[L(x)\to(\exists z)(A(x,z)\cdot J(z))]$

e) Some women admire no lawyer.

$(\exists x)[W(x)\cdot(\forall z)(A(x,z)\to(L(z))')]$