MET CS248 - HW #1

- 1) Decide which of the following are propositions (Y/N) and give the truth value:
 - a) 127 is an even integer

b) All triangles have 4 sides or more

- 2) Negate the following statements, using full sentences:
 - a) John is smart or Fred is not tall

b) Some animals are intelligent

3) Is $(\overline{P} \to Q) \leftrightarrow to(P \to \overline{Q})$? $(\overline{Q}$ means not $Q \leftrightarrow stands$ for equivalent). Justify your answer.

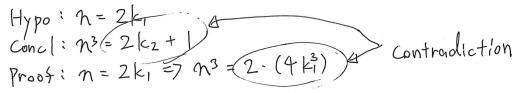
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- 4) Given the proposition P(n) " If n³ is odd then n is odd "
 - a) Prove P(n) by contraposition:

Hypo:
$$N = 2k_1$$

Concl: $N^3 = 2k_2$
Proof: $N = 2k_1 \Rightarrow N^3 = 2^3 \cdot l_1^3 \Rightarrow N^3 = 2 \cdot (4l_1^3)$
 $=> N^3 = 2k_2$

b) Prove P(n) contradiction:



5) Prove by induction:
$$1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{n(n+1)(2n+1)}{6}$$
, $n \ge 1$
 $1 - \beta_{0} \le e^{-\frac{1}{2}}$: $1^{2} = \frac{1 \cdot (1+1) \cdot (2 \cdot 1+1)}{6} = 1$
 $2 - Assume$: $1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{n(n+1)(2n+1)}{6}$
 $3 - Proof$: $1 + 2^{2} + 3^{2} + \dots + n^{2} + (n+1)^{2} = \frac{n(n+1)(2n+1)}{6} + (n+1)^{2}$
 $= \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^{2}}{6} = \frac{(n+1) \le n(2n+1) + 6(n+1)}{6}$
 $= \frac{1}{6} (n+1) (2n^{2} + 7n + 6) = \frac{1}{6} (n+1)(2n+3)(n+2)$
 $= \frac{(n+1) \le (n+1) + 1 \le 2(n+1) +$

6) Using the predicate symbols shown and appropriate quantifiers, write each English Language statement in symbolic form (The domain is the whole world).

$$J(x)$$
 is "x is a judge"
 $L(x)$ is "x is a lawyer"

$$C(x)$$
 is "x is a chemist"
 $A(x, y)$ is "x admires y"

W(x) is "x is a woman"

a) There are some women lawyers who are chemists.

b) No woman is both a lawyer and chemist.

c) Some lawyers admire only judges.

$$(\exists x) \vdash L(x) \cdot (\forall s) (A(x,s) \rightarrow J(s))$$

d) All women lawyers admire some judges.

e) Some women admire no lawyer.