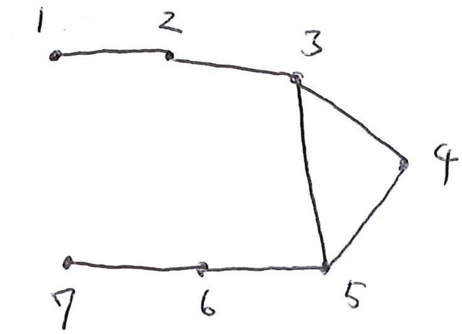
HW#5 - METCS 248

1. Given the undirected graph G = (V, E), where V = {1, 2, 3, 4, 5, 6, 7}, E = {{1, 2}, {2, 3}, {3, 4}, {3, 5},{4, 5},{5, 6},{6, 7}}



a) Is the graph simple?

Yes. No loops and no multiple edges.

b) Is the graph complete?

No. No edges between some nodes.

c) Is the graph connected?

Yes. There exist path between any nodes.

d) Can you find 2 paths from 3 to 6?

Yes. 3-5-6 and 3-4-5-6.

e) Can you find a cycle?

Yes. 3-4-5-3.

f) Can you find an edge whose removal makes the graph acyclic?

Yes. Remove (3,5).

g) Can you find an edge whose removal makes the graph not connected?

Yes. Remove (1,2),(2,3),(5,6), or (6,7).

h) Give the adjacency matrix of G.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

i) Is G a tree?

No. Undirected → Yes
Connected → Yes
Acyclic → No

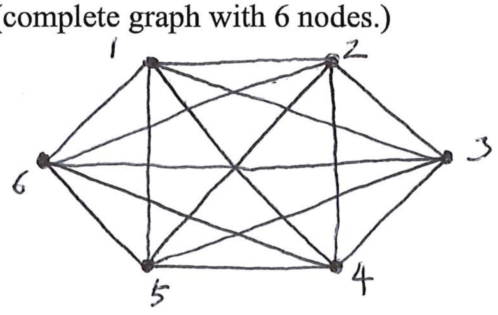
j) Does an Eulerian path exist?

No. 0 or 2 odd degree nodes? → No.

k) Does a Hamiltonian path exist?

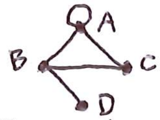
Yes. 1-2-3-4-5-6-7.

2. Draw K_6, (complete graph with 6 nodes.)



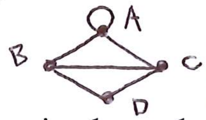
3. For each of the following characteristics, draw a graph or explain why such graph does not exist: (Graphs don't have to be simple).

a) Four nodes of degree 1, 2, 3, and 4, respectively



deg (A) = 4
deg (B) = 3
deg (C) = 2
deg (D) = 1

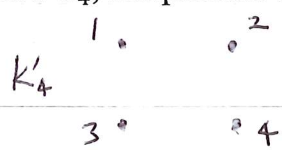
b) Four nodes of degree 2, 3, 3, and 4, respectively



deg(A) = 4
deg(B) = 3
deg(C) = 3
deg(D) = 2

4. If G is a simple graph, the complement of G, denoted G', is the simple graph with the same set of nodes as G, where nodes x-y are adjacent in G' if and only if they are not adjacent in G.

a) Draw K_4', complement of K_4.

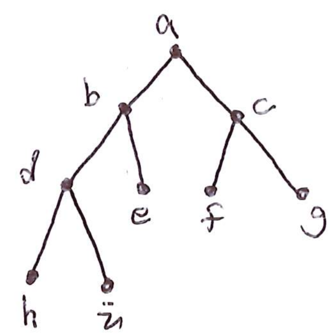


b) Given an adjacency matrix A for a simple graph G, describe the adjacency matrix for G'.

Adjacent matrix B for G'

$B = [b_{ij}]$ where $\begin{cases} 0 \text{ if } i \& j \text{ are adjacent in } G. \\ 1 \text{ if } i \& j \text{ are not adjacent in } G. \end{cases}$

5. Given the following graph (tree) with node a as the root and constructed from left to right: T = (V, E), where V= {a, b, c, d, e, f, g, h, i}, and E ={{a, b},{a, c},{b, d},{b, e},{c, f},{c, g},{d, h}, {d, i}}



a) Is this a binary tree?

Yes. Undirected, connected and acyclic.

b) Is it a full binary tree?

Yes. All nodes have 0 or 2 children.

c) Is it a complete binary tree?

Yes. Filled from the left bottom.

d) What is the parent of e?

b

e) What is the right child of e?

e has no right child.

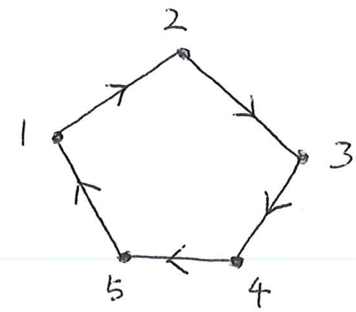
f) What is the depth of g?

2. Distance between the node and root.

g) What is the height of the tree?

3. Distance between the furthest node and root.

6. Given the following digraph: D = (V, E), where V = {1, 2, 3, 4, 5} and E = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)}:



a) By inspection is the graph strongly connected?

Yes. Every vertex is reachable from any vertex.

b) Compute M^2 that is $M.M$, where M is the adjacency of digraph D.

$$M^2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

c) Compute Warshall's first matrix after the adjacency matrix, $_1W$.

$$_0W = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$_1W = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$