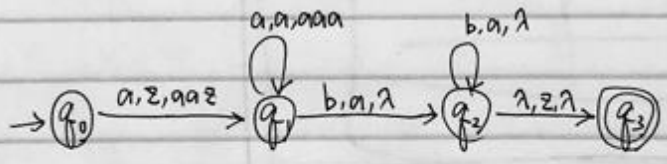
MET CS662 - Assignment #7

1. Construct an npda's that accept the language $L = \{\omega | \, n_a(\omega) = n_b(\omega) + 1\}$ on $\sum = \{a, b, c\}$,

1. $\delta(q_0, \lambda, z) = \{(q_1, bz)\}$
$\delta(q_1, a, z) = \{(q_1, az)\}$
$\delta(q_1, a, a) = \{(q_1, aa)\}$
$\delta(q_1, a, b) = \{(q_1, \lambda)\}$
$\delta(q_1, b, z) = \{(q_1, bz)\}$
$\delta(q_1, b, a) = \{(q_1, \lambda)\}$
$\delta(q_1, b, b) = \{(q_1, bb)\}$
$\delta(q_1, c, z) = \{(q_1, z)\}$
$\delta(q_1, c, a) = \{(q_1, a)\}$
$\delta(q_1, c, b) = \{(q_1, b)\}$
$\delta(q_1, \lambda, z) = \{(q_2, z)\}$ $q_2 \in F$

2. Show that $L = \{a^n b^{2n} | \; n \geq 1\}$ is a deterministic context-free language.

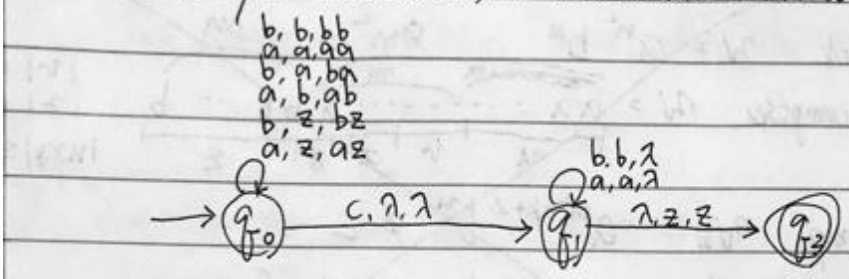
2.



We are able to construct a deterministic PDA. Hence, this language is a deterministic context-free language.

3. Is the language $L = \{\omega c \omega^R | \ \omega \in \{a, b\}^*\}$ deterministic?

3. A language is said to be deterministic context-free if and only if there exists a deterministic PDA for it.

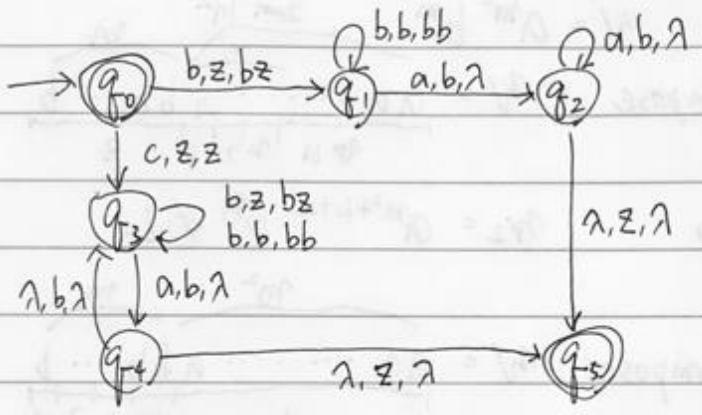


We are able to construct a deterministic PDA. Hence, this language is a deterministic context-free language.

4. Give an example of a deterministic context-free language whose reverse is not deterministic.

4. $L = \{b^n a^n\} \cup \{c\, b^{2n} a^n\}$

This language is deterministic.



The reverse is $L = \{a^n b^n\} \cup \{a^n b^{2n} c\}$

This language is not deterministic because the first a's are handled in the same way but b's need to be handled in different ways.

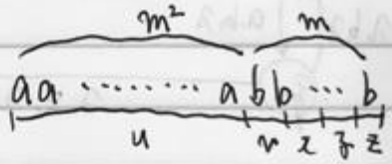
5. Show that the language $L = \{a^n b^j |\ n \leq j^2\}$ on $\sum = \{a, b, c\}$ is not context-free.

5. 1) Assume L is context-free.
2) m exists.
3) Pick $w = a^{m^2} b^m$
4) Decompose $w =$ 

$|v| = k$
$|y| = \ell$
$|vxy| \leq m$

5) Pump $w_2 = a^{m^2+k+\ell} b^m \notin L$

or,

4') Decompose $w =$ 

$|v| = k$
$|y| = \ell$
$|vxy| \leq m$

5') Pump $w_0 = a^{m^2} b^{m-k-\ell} \notin L$

Hence, this language is not context-free

6. Consider the language $L = \{a^n b^n c^m |\ n \geq 0, m \geq 0\}$. Show that this language is linear.

6. $S \to Sc \mid A \mid \lambda$
$A \to aAb \mid \lambda$
This grammar is linear, hence the language is linear.

7. Show that the family of linear languages is not closed under intersections

7. Consider the two linear grammars.
$S \to aS \mid B$
$B \to bBc \mid bc$
$S' \to Sc \mid A$
$A \to aAb \mid ab$
The intersection of these languages is
$\{a^i b^i c^i \mid i > 0\}$, which is not even context-free
Hence, the family of linear language is not closed under intersection.