MET CS662 - Assignment #5

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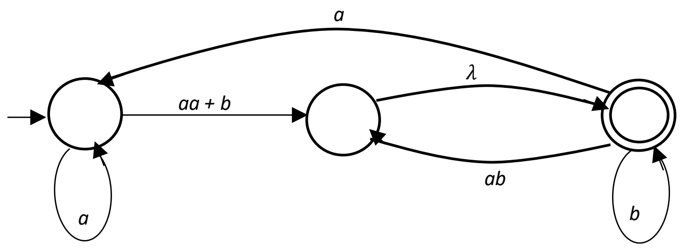
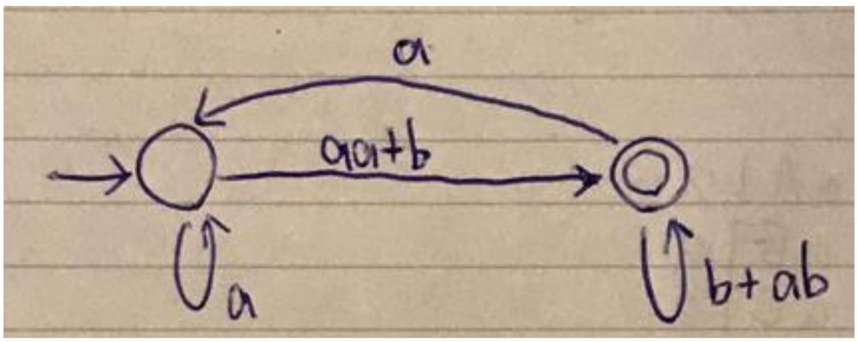
1. Consider the following generalized transition graph



a. Find an equivalent generalized transition graph with only two states.



b. What is the language accepted by this graph?

Regular expression

$a^*(aa+b)(b+ab)^*[aa^*(aa+b)(b+ab)^*]^*$

2. Construct right and left grammars for the language $L = \{a^n b^m \mid n \geq 2, m \geq 3\}$.

Right	Left
$S \to aaA$	$S \to Bbbb$
$A \to aA \mid bbbB$	$B \to Bb \mid Aaa$
$B \to bB \mid \lambda$	$A \to Aa \mid \lambda$

3. Construct a right linear grammar for the language $L((aab^*ab)^*)$.

$S \to A \mid \lambda$
$A \to aaB$
$B \to bB \mid abS$

4. Find a regular grammar that generates the language on $\Sigma = \{a, b\}$ consisting of all strings with no more than three a's.

$S \to bS \mid aA \mid \lambda$
$A \to bA \mid aB \mid \lambda$
$B \to bB \mid aC \mid \lambda$
$C \to bC \mid \lambda$

5. Find a regular grammar for the language $L = \{a^n b^m \mid n + m \text{ is even}\}$

$S \to S_1 \mid S_2$
$S_1 \to aaS_1 \mid A \mid \lambda$
$A \to bbA \mid \lambda$
$S_2 \to aaS_2 \mid aB$
$B \to bbB \mid b$

6. The symmetric difference of two sets S_1 and S_2 is defined as

$$S_1 \ominus S_2 = \{x \mid x \in S_1 \text{ or } x \in S_2, \text{but } x \text{ is not in both } S_1 \text{ and } S_2\}$$

Show that the family of regular languages is closed under symmetric difference.

Let S_1 and S_2 be regular sets.
$(S_1$ or $S_2)$ and (not $(S_1$ and $S_2))$
$= (S_1 \cup S_2) \cap \overline{(S_1 \cap S_2)} = S_1 \ominus S_2$
is regular, since regular sets are closed under union, intersection, and complement.

7. Suppose we know that $L_1 \cup L_2$ is regular and that L_1 is finite. Can we conclude from this that L_2 is regular?

$L_2 = (L_1 \cup L_2) - (L_1 - L_2)$
$L_1 \cup L_2$ is regular by assumption
$L_1 - L_2$ is regular since it's finite minus something
$(L_1 \cup L_2) - (L_1 - L_2)$ is regular, since regular sets are closed under set difference.

8. Which of the following are true for all regular languages and all homomorphisms?

a) $h(L_1 \cup L_2) = h(L_1) \cup h(L_2)$

Assume $L_1 = \{00, 11\}$ and $L_2 = \{10, 01\}$ and $h(0) = c$, $h(1) = cd$

$h(L_1) = h\{(00, 11)\} = \{h(00), h(11)\} = \{cc, cdcd\}$

$h(L_2) = h\{(10, 01)\} = \{h(10), h(01)\} = \{cdc, ccd\}$

$h(L_1) \cup h(L_2) = \{cc, cdcd\} \cup \{cdc, ccd\}$

$= \{cdc, cdcd, cc, ccd\}$

$h(L_1 \cup L_2) = h\{(00, 11) \cup (10, 01)\}$

$= h\{10, 11, 00, 01\}$

$\cancel{= h\{}$

$= \{h(10), h(11), h(00), h(01)\}$

$= \{cdc, cdcd, cc, ccd\}$

Hence, it is true

b) $h(L_1 \cap L_2) = h(L_1) \cap h(L_2)$

Assume $L_1 = \{00, 11\}$ and $L_2 = \{00, 01\}$ and $h(0) = c$, $h(1) = cd$

$h(L_1) = h\{(00, 11)\} = \{cc, cdcd\}$

$h(L_2) = h\{(00, 01)\} = \{cc, ccd\}$

$h(L_1) \cap h(L_2) = \{cc\}$

$\neq h(L_1 \cap L_2) = h\{(00, 11) \cap (00, 01)\}$

$= \{cc\}$

Hence, it is true.

c) $h(L_1 L_2) = h(L_1) h(L_2)$

Assume $L_1 = \{00, 11\}$ and $L_2 = \{00, 01\}$ and $h(0) = c$, $h(1) = cd$

$h(L_1) = h\{(00, 11)\} = \{cc, cdcd\}$

$h(L_2) = h\{(00, 01)\} = \{cc, ccd\}$

$h(L_1) h(L_2) = \{cc, cdcd\}\{cc, ccd\}$

$= \{cccc, cccdcd, ccdcc, ccdcdcd\}$

$h(L_1 L_2) = h\{(00, 11)(00, 01)\}$

$= h\{0000, 0011, 0100, 0111\}$

$= \{cccc, cccdcd, ccdcc, ccdcdcd\}$

Hence, it is true.

9. If L is a regular language, prove that $L_1 = \{uv | \, u \in L, \; |v| = 2\}$ is also regular.

From the definition of L_1, we have that $L_1 = LL'$, where $L' = \{v : |v| = 2\}$.
L' is regular since we can construct a DFA that accepts strings with two symbols.
Thus, $L_1 = LL'$ is regular since the family of regular language is closed under concatenation.
