MET CS662 - Assignment #2

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1. Let $L = \{ab, aa, baa\}$. Which of the following strings are in L^*.
   $abaabaaabaa,\ aaaabaaaa,\ \ baaaaabaaaab,\ baaaaabaa$?
   Ans:
   $abaabaaabaa,\ aaaabaaaa,\ baaaaabaa$

2. Find grammars for $\sum = \{a, b\}$ that generate the set of:

   a. All strings with exactly one *a*.
      Ans:
      S → BaB
      B → bB | λ
      In the process of derivation, S is called only once, hence the generated sentence contains exactly one a.

   b. All strings with no more than three *a's*.
      Ans:
      S → B | BaB | BaBaB | BaBaBaB
      B → bB | λ
      In the process of derivation, S is called only once, hence the generated sentence contains no more than three a's.
      Note that the textbook solution would say "S → BaB | BaBaB | BaBaBaB" since λ is not supposed to be in the language in the textbook, but the professor said in the class told us the language can contain λ, so this answer is correct.

   In each case, give convincing arguments that the grammar you give indeed generate the indicated language.

3. Give a simple description of the language generated by the productions:

   $S \rightarrow aA$
   $A \rightarrow bS$
   $S \rightarrow \lambda$
   Ans:
   S -> aA -> abS -> abaA -> ababS -> ababA -> $\cdots$
   -> λ -> ab -> abab

Hence, L = { $(ab)^n$ | n >= 0 }

4. What language does the grammar with these productions?

$S \rightarrow Aa$
$A \rightarrow B$
$B \rightarrow Aa$

Ans:

S -> Aa -> Ba -> Aaa -> Baa -> Aaaa -> $\cdots$

This grammer doesn't generate any string.

5. Find a grammar that generates the language

$$L = \{\omega\omega^R | \omega \in \{a, b\}^+\}$$

Ans:

S -> aAa | bAb

A -> aAa | bAb | λ

6. Are the two grammars with respective productions

$$S \rightarrow aSb \mid ab \mid \lambda$$

and

$$S \rightarrow aAb \mid ab$$
$$A \rightarrow aAb \mid \lambda$$

equivalent? Assume that S is the start symbol in both cases.

Ans:

Not equivalent. The counter example is as follows.

Grammer 1: S -> λ

Grammer 2: S -> (cannot generate λ).

7. So far we have given examples of only relatively simple grammars; every production had a single variable on the left side. As we will see, such grammars are very important, but the definition of grammar allows more general forms.

Consider the grammar $G = (\{A, B, C, D, E, S\}, \{a\}, S, P)$, with productions

$$S \rightarrow ABaC$$
$$Ba \rightarrow aaB$$
$$BC \rightarrow DC|E$$
$$aD \rightarrow Da$$

$$AD \rightarrow AB$$
$$aE \rightarrow Ea$$
$$AE \rightarrow \lambda$$

Derive three different sentences in $L(G)$. From these, make a conjecture about $L(G)$.

Ans:

S -> ABaC -> AaaBC -> AaaE -> AaEa -> AEaa -> λaa -> aa

S -> ABaC -> AaaDC ->AaDaC -> ADaaC -> ABaaC -> AaaBaC -> AaaaaBC -> AaaaaE -> AaaaEa -> AaaEaa -> AaEaaa -> AEaaaa -> λaaaa -> aaaa

S =>* aaaaaaaa

We can conjecture that L(G) = { a^{2^n} | n >= 1 }
