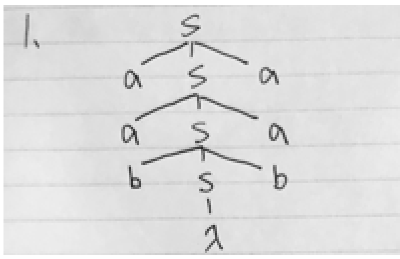
MET CS662 - Assignment #6

1. Draw the derivation tree corresponding to the derivation in Example 5.1, that is

$$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa$$

where the grammar is $S \to aSa$ $\quad S \to bSb$ $\quad S \to \lambda$



2. Find context-free grammars for the language $L = \{a^n b^m c^k \mid n = m \; or \; m \le k\}$, (with also, $n \ge 0, m \ge 0, k \ge 0$)

2. $S \to AC \mid XY$
$A \to aAb \mid \lambda$
$C \to cC \mid \lambda$
$X \to aX \mid \lambda$
$Y \to bYc \mid Yc \mid \lambda$

3. Let $L = \{a^n b^n \mid n \ge 0\}$.

a) Show that L^2 is context-free.

3.a. L^2 can be derived by
$S \to AB$
$A \to aAb \mid \lambda$
$B \to aBb \mid \lambda$

b) Show that L^k is context-free for any given $k \ge 1$.

3.b. L^k can be derived by
$S \to S_1 S_2 \cdots S_k$
$S_1 \to aS_1b \mid \lambda$
$S_2 \to aS_2b \mid \lambda$
$\vdots$
$S_k \to aS_kb \mid \lambda$

c) Show that $\bar{L}$ and L^* are context-free.

3. c. i) $\overline{L} = \{a^n b^m \mid n \neq m\} \cup \{(a \cup b)^* ba (a \cup b)^*\}$

Let $\{a^n b^m \mid n \neq m\}$ be P and $\{(a \cup b)^* ba (a \cup b)^*\}$ be Q

$P \rightarrow aPb \mid X \mid Y$

$X \rightarrow aX \mid a$

$Y \rightarrow bY \mid b$

$Q \rightarrow MbaM$

$M \rightarrow a \mid b \mid M \mid \lambda$

Using the grammars above,

$S \rightarrow P \mid Q$

ii) L^* can be derived by

$S \rightarrow aSb \mid SS \mid \lambda$

4. Find an s-grammar for the language $L(aaa^*b + b)$.

4. $S \rightarrow b \mid aA$

$A \rightarrow aBC$

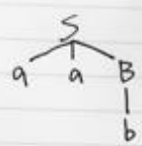
$B \rightarrow aB$

$C \rightarrow b$

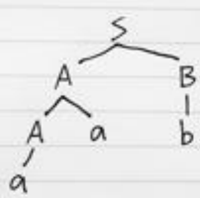
5. Show that the following language is ambiguous.

$$S \rightarrow AB \mid aaB \qquad A \rightarrow a \mid Aa \qquad B \rightarrow b$$

5. To derive aab,



or



6. Construct an unambiguous grammar equivalent to the grammar

$$S \rightarrow AB \mid aaB \qquad A \rightarrow a \mid Aa \qquad B \rightarrow b$$

6. $S \rightarrow aA$

$A \rightarrow aA \mid b$

7. Prove the following result. Let $G = (V, T, S, P)$ be a context-free grammar in which every $A \in V$ occurs on the left side of at most one production. Then G is unambiguous.

7. Consider lefmost productions. Since the variable to be expanded occurs on the left side of only one production, there is never a choice.

8. Transform the grammar with productions $S \to abAB,\ A \to bAB|\lambda,\ B \to BAa|A|\lambda$

into Chomsky normal form.

8. Remove λ productions

A: $S \to abAB \mid abB,\ A \to bAB \mid bB,\ B \to BAa \mid A \mid \cancel{\lambda} \mid \cancel{Ba}$

B: $S \to abAB \mid abA \mid ab,\ A \to bAB \mid bB \mid bA \mid b,$

$B \to BAa \mid A \mid Ba \mid Aa \mid a$

Remove unit production

$S \to abAB \mid abB \mid abA \mid ab,\ A \to bAB \mid bB \mid bA \mid b,$

$B \to BAa \mid bAB \mid bB \mid bA \mid b \mid Ba \mid Aa \mid a$

Convert the grammar into Chomsky normal form

Introduce new variables S_a, S_b for each $a, b \in T$

$S \to S_aS_bAB \mid S_aS_bB \mid S_aS_bA \mid S_aS_b$

$A \to S_bAB \mid S_bB \mid S_bA \mid b$

$B \to BAS_a \mid S_bAB \mid S_bB \mid S_bA \mid S_b \mid BS_a \mid AS_a \mid S_a$

$S_a \to a$

$S_b \to b$

Introduce additional variables to set the first two productions into normal form and we get the final result

$S \to S_aU \mid S_aX \mid S_aY \mid S_aS_b$

$A \to S_bV \mid S_bB \mid S_bA \mid S_b$

$B \to BZ \mid S_bV \mid S_bB \mid S_bA \mid S_b \mid BS_a \mid AS_a \mid S_a$

$U \to S_bV$

$V \to AB$

$X \to S_bB$

$Y \to S_bA$

$Z \to AS_a$

$S_a \to a$

$S_b \to b$

9. Convert the grammar $S \to aSb|bSa$ | a | b into Greibach form.

9. $S \to aSB \mid bSA \mid a \mid b$

$A \to a$

$B \to b$