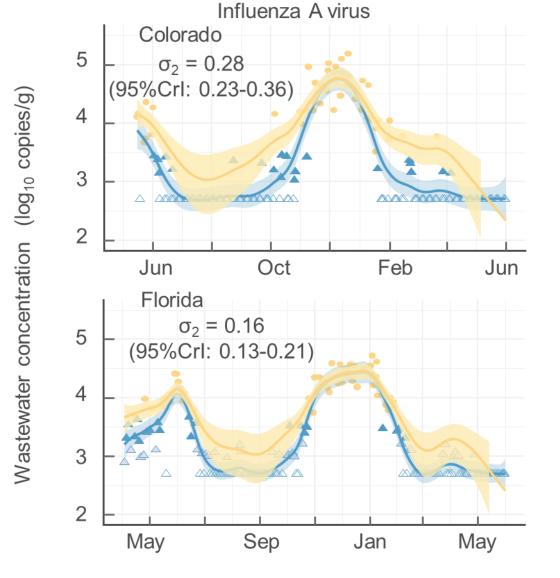
Brief guide for using the state-space model





Content

Name of used R and stan files (state_space_model_with_logistic.R) (state_space_model_with_logistic.stan)

- Install and download (slide: 2 ~ 3)
- Import wastewater-based data (slide: 4 ~ 9)
- Set prior distribution (slide 10)
- Implement analysis (slide 11 ~ 13)

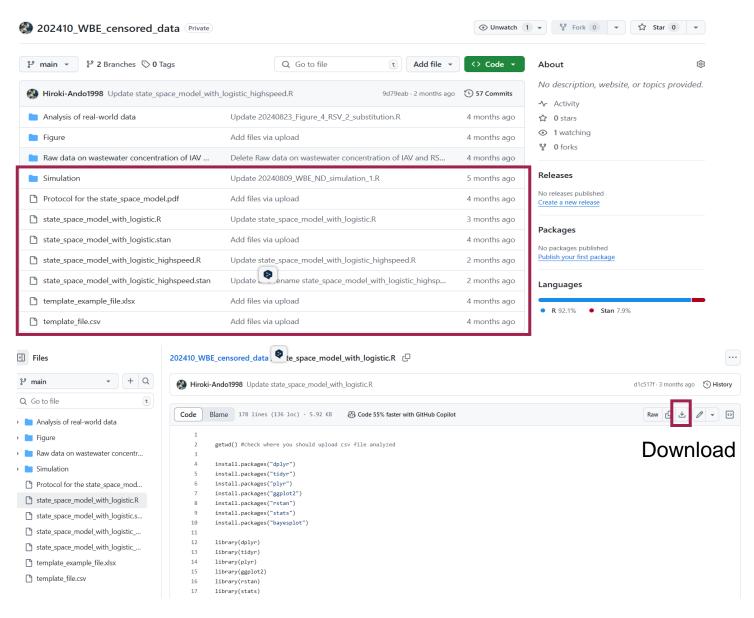
Name of used R and stan files (state_space_model_with_logistic_highspeed.R) (state_space_model_with_logistic_highspeed.stan)

High speed analysis

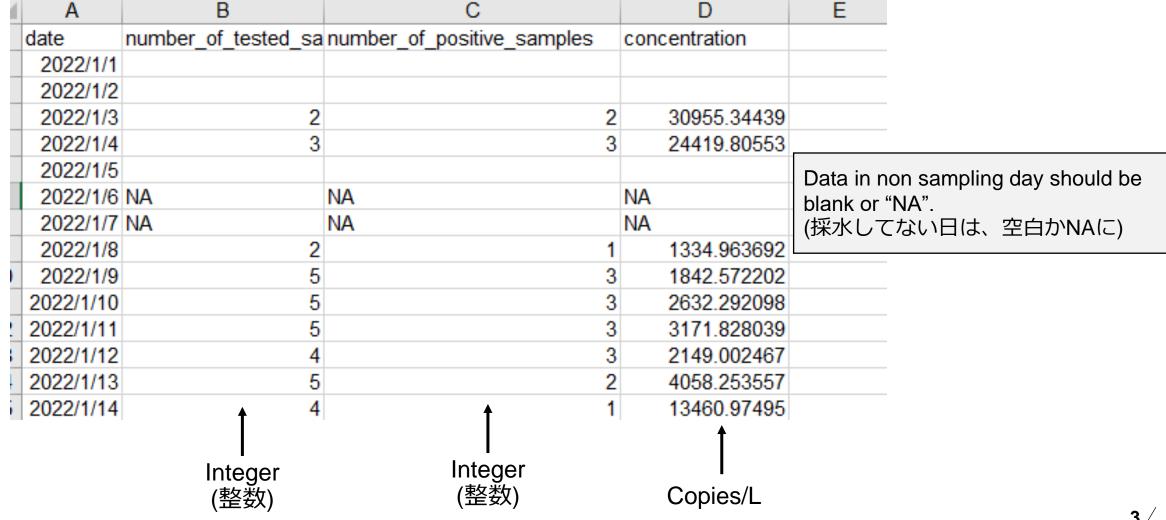
Contact: hirokiando@arizona.edu

• Install "R" and "Rstudio"
https://rstudio-education.github.io/hopr/starting.html

 Download R file, csv file, and stan file from the github.



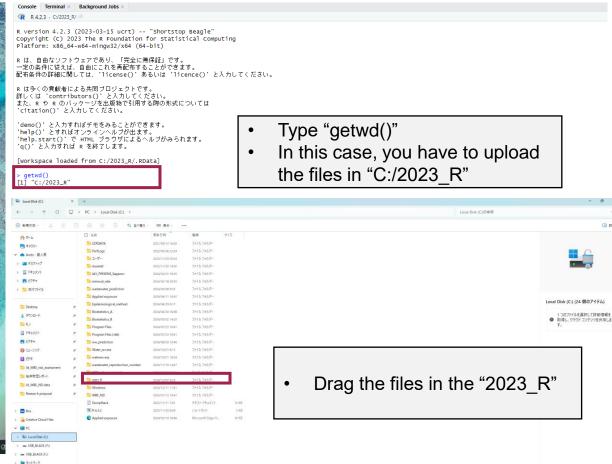
- Open the CSV file downloaded in your laptop
- Input wastewater-based data in the template CSV file



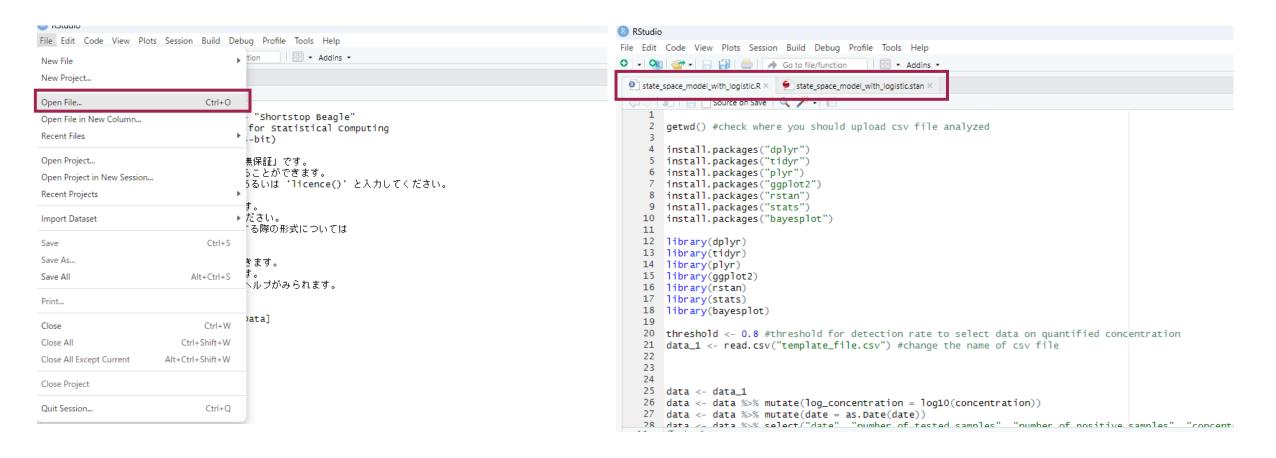
Open "Rstudio" in your desktop



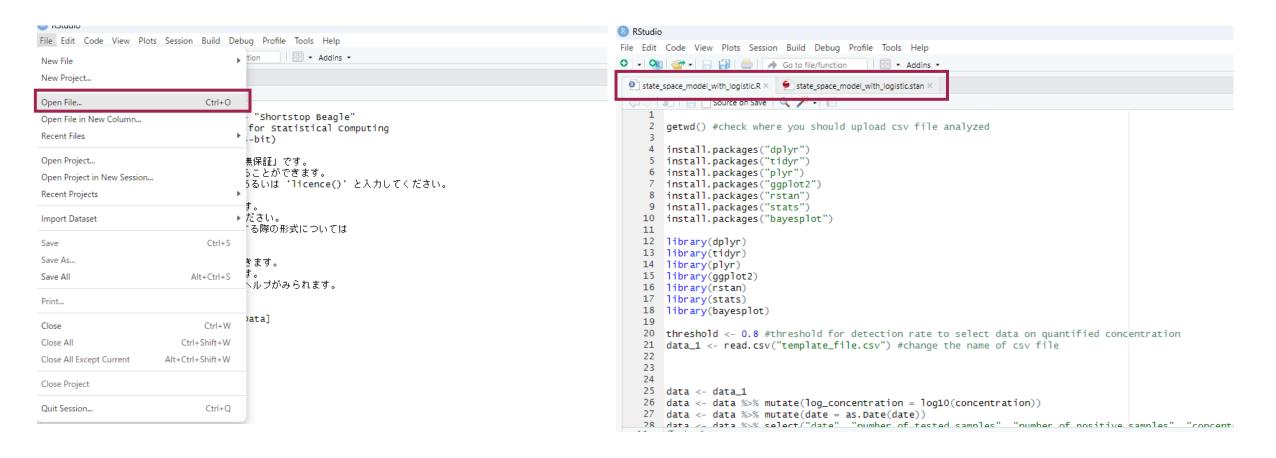
 Check where you have to upload CSV file, R and Stan file.



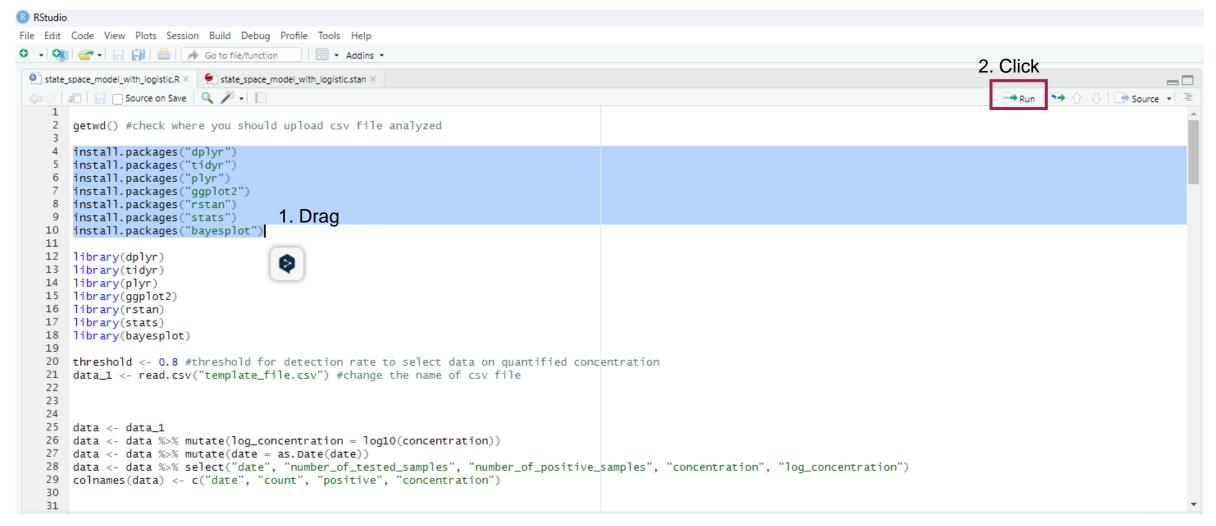
Open R file and Stan file in Rstudio



Open R file and Stan file in Rstudio

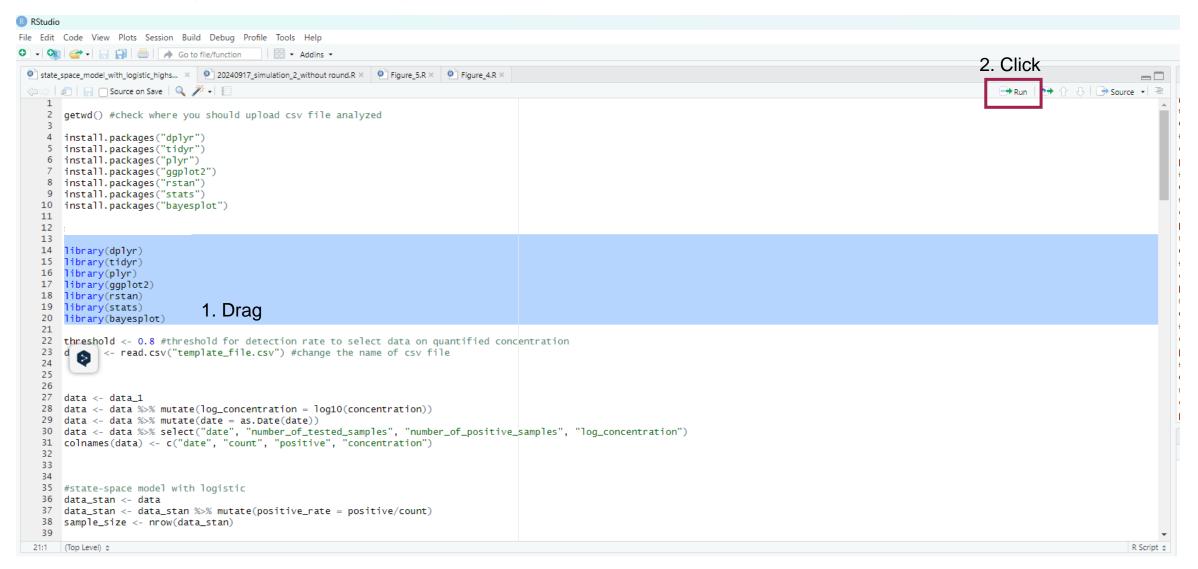


Install R packages used in the analysis



Note: You no longer need to repeat this process once the packages are installed.

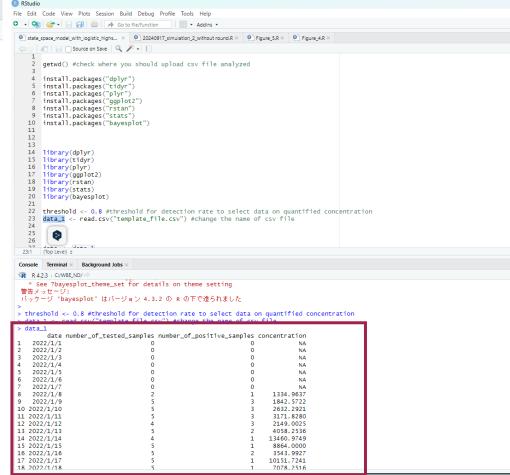
Upload library used in the analysis



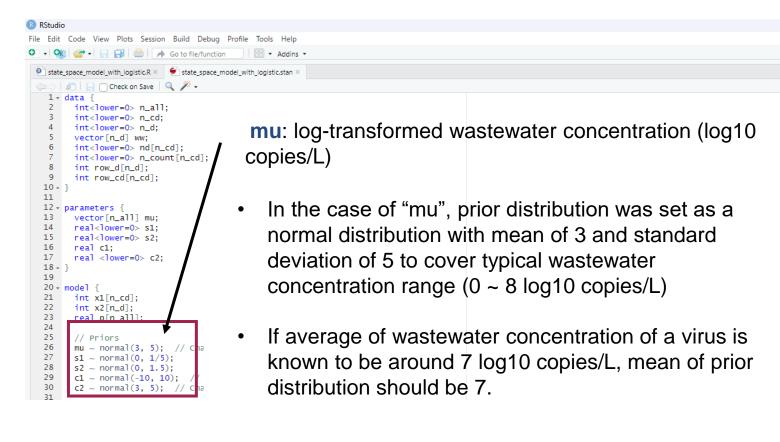
- Set arbitrary threshold (0.7 ~ 1.0)
- Import data

```
state_space_model_with_logistic.R ×
                           state_space_model_with_logistic.stan >>
     getwd() #check where you should upload csv file analyzed
      install.packages("dplyr")
      install.packages("tidyr")
      install.packages("plyr")
      install.packages("ggplot2")
      install.packages("rstan")
      install.packages("stats")
      install.packages("bayesplot")
  11
      library(dplyr)
      library(tidyr)
      library(plyr)
      library(ggplot2)
     library(rstan)
      library(stats)
      library(bayesplot)
  19
      threshold <- 0.8 #threshold for detection rate to select data on quantified concentration
      data_1 <- read.csv("template_file.csv") #change the name of csv file
```

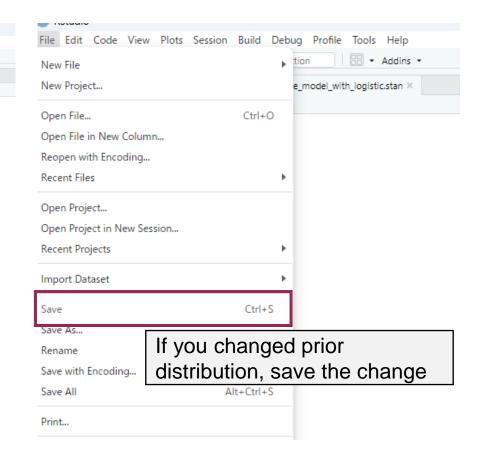
Check the name of CSV file you uploaded



 Prior distribution is important for convergence of parameters (Prior distribution should be decided from available information)



- S₁ is a parameter of the state-formula.
- S₂ is a parameter of the observation formula (i.e., measurement error).
- C₁ and C₂ are parameters of logistic function.



Drag the row from 25-87 and click Run

```
state_space_model_with_logistic.R × 🦸 state_space_model_with_logistic.stan ×
data <- data %>% mutate(log_concentration = log10(concentration))
      data <- data %>% mutate(date = as.Date(date))
      data <- data %>% select("date", "number_of_tested_samples", "number_of_positive_samples", "concentration", "log_concentration")
      colnames(data) <- c("date", "count", "positive", "concentration")</pre>
  30
  31
  32
      #state-space model with logistic
      data_stan <- data
      data_stan <- data_stan %>% mutate(positive_rate = positive/count)
      sample_size <- nrow(data_stan)
  37
      #vector of row number used for the analysis
  38
      #pick row numbers for censored data
      data_row_D <- data.frame(true = which((data_stan$positive_rate >= threshold)))
      sample_size_D <- nrow(data_row_D)</pre>
                                                                                          mcmc <- stan(
      #pick row numbers for censored data
                                                                                            file = "state_space_model_with_logistic.stan",
      data_row_CD <- data.frame(true = which(data_stan$count > 0))
                                                                                            data = data_list_ww,
                                                                                            seed = 1,
                                                                                            chain = 4,
                                                                                                                MCMC condition can be changed according to your
                                                                                      65
                                                                                            iter = 200000.
         Donice out pare
                                                                                      66
                                                                                            warmup = 100000
                                                                                                                data.(https://mc-stan.org/rstan/reference/stan.html)
      print(mcmc, pars = c("c1", "c2", "s1", "s2"), probe = c(0.025, 0.50, 0.975))
                                                                                            thin = 4
  71
                                                                                      68
      #check traceplots if you want
      #mcmc_combo(mcmc, pars = c("c1", "c2", "s1", "s2"))
  74
  75
  76
      #MCMC samples and 95% credible intervals
      mcmc_sample <- rstan::extract(mcmc)</pre>
     state_name <- "mu"
      result <- data.frame(t(apply(
       X = mcmc_sample[[state_name]],
  81
       MARGIN = 2,
        FUN = quantile,
  83
        probs = c(0.025, 0.5, 0.975) #credible interval can be changed
  84
       )))
  85
      colnames(result) <- c("low", "median", "upr")
      data_estimated_concentration <- cbind(data, result)</pre>
```

Check the convergence of parameters in the state-space model

```
> print(mcmc, pars = c("c1", "c2", "s1", "s2"), probe = c(0.025, 0.50, 0.975))
Inference for Stan model: anon_model.
4 chains, each with iter=4000; warmup=2000; thin=4;
post-warmup draws per chain=500, total post-warmup draws=2000.
                               25% 50% 75% 97.5% n eff Rhat
                  sd
                       2.5%
    mean se mean
            0.11 3.76 -32.08 -27.04 -24.37 -21.96 -17.60 1208 1.00
c1 -24.55
c2
    8.04
                     5.82 7.21
                                    8.01 8.82
                                                 10.39
                                                       1565 1.00
            0.03 1.18
                                                        272 1.02
51
    0.04
            0.00 0.01 0.02 0.03 0.04 0.05 0.07
            0.00 0.15
                                                       1805 1.00
52
    0.96
                       0.71 0.85 0.94 1.04
                                                  1.31
```

- Rhat should be lower than 1.10 (https://mc-stan.org/misc/warnings.html#bulk-ess)
- N_eff should be higher than 400 (ideally, 1000)

To improve Rhat and n_eff

- Fix prior distribution
- Increase chain, iter (warmup), and thin
- Conduct re-parameterization (Slide number 13 15)

Check the estimation of wastewater concentration

```
🖭 state_space_model_with_logistic.R × 📗 🗲 state_space_model_with_logistic.stan ×

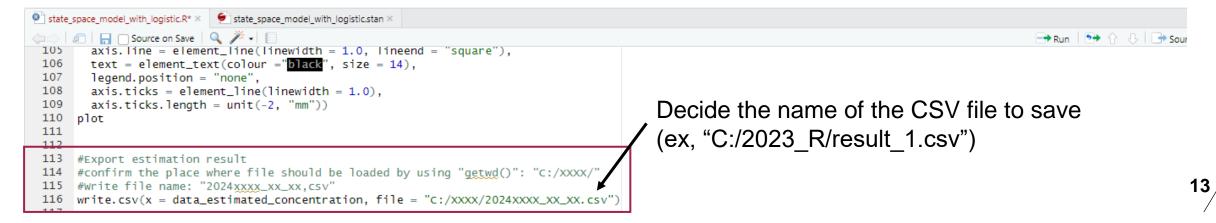
⟨□□⟩ | Ø□ | □ | □ Source on Save | Q  
Ø ▼ | □ |

                                                                                                                                                  ➡ Run | 🍽 🏗 🕒 | 🖶 Source 🕶
      #figure
      data_fig <- data_data_estimated_concentration
      data_fig <- data_fig %>% mutate(posi = if_else(count < 1, "no", if_else(positive >= threshold*count, "yes", "no")))
      plot <- ggplot(data_fig, aes(x = as.Date(date)))</pre>
     plot <- plot + geom_point(aes(y = concentration, color = posi, shape = posi))</pre>
  96 plot <- plot + scale_shape_manual(values = c(24, 16))
  97 plot <- plot + scale_color_manual(values = c("#FEC44F", "#0570B0"))
  98 plot <- plot + geom_ribbon(aes(ymin = low, ymax = upr), fill = "#369000", alpha = 0.3)
  99 plot <- plot + geom_line(aes(y = median), color = "#0570B0", size = 1)
 100 plot <- plot + labs(x = "Date", y = "wastewater concentration")
 101 _plot <- plot + scale x date(limits = c(as.Date("2022-01-01"), as.Date("2023-06-01")), date breaks = "2 months", date labels = "%b") #change the date
 plot <- plot + scale_y_continuous(limits = c(2.0, 5.5), breaks = seq(2.0, 5.5, 1)) #change the scale of y axis
 103 plot <- plot + theme_bw()
 104 plot <- plot + theme(
                                                                          Change
       axis.line = element_line(linewidth = 1.0, lineend = "square"),
       text = element_text(colour ="black", size = 14),

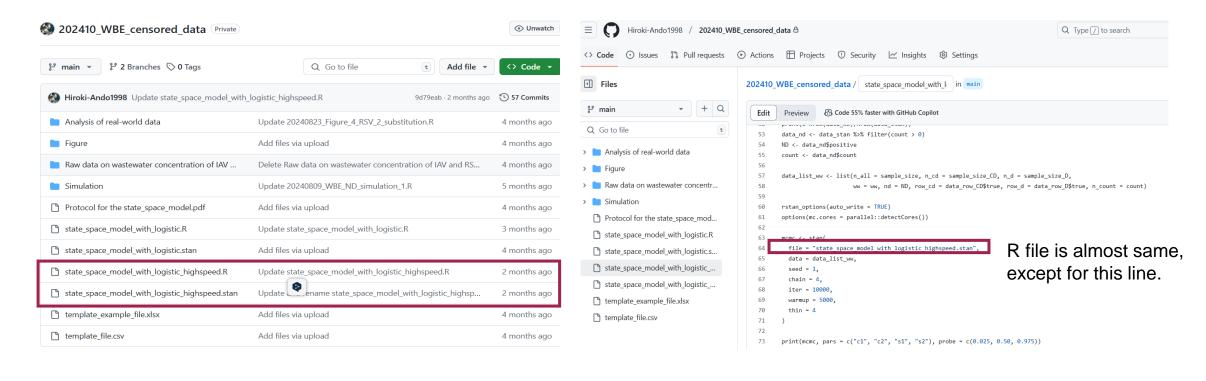
    X axis (date)

       legend.position = "none",
 108
        axis.ticks = element_line(linewidth = 1.0),
                                                                            Y axis (log10 wastewater concentration)
       axis.ticks.length = unit(-2, "mm"))
 110
      plot
                                                                          https://ggplot2.tidyverse.org/reference/scale continuous.html
 111
 112
```

Export the result in CSV file



- The above approach (file name: state_space_model_with_logistic.R & state_space_model_with_logistic.stan)
 require a lot of analytical time.
 (上の方法では、解析時間が長いのが難点)
- This problem is solved by using re-parameterization approach. (再パラメータ化というモデリング技術を使って、解析時間を短縮する)
 - https://mc-stan.org/docs/2_18/stan-users-guide/reparameterization-section.html
 - https://mc-stan.org/docs/stan-users-guide/reparameterization.html



Re-parameterization enable to standardize prior distributions of parameters (再パラメータ化では、全てのパラメータの事前分布のスケールを同一にする)

state_space_model_with_logistic_highspeed.stan (Stan file)

```
16
      real<lower=-4, upper=4> c1_raw;
17
      real<lower=-4, upper=4> c2_raw;
18 -
19
20
    transformed parameters(
21
          vector[n_all] mu;
22
      for (i in 1:n_all) {
23
        mu[i] = 3.5 + 1.5 * mu_raw[i];
24
25
26
27
       s1 = exp(s1_raw - 2);
28
        real s2:
29
        s2 = exp(s2\_raw - 2);
30
31
        real c1;
32
         c1 = -22 + 5*c1_raw;
33
34
        real c2;
35
        c2 = 6 + 3*c2_raw;
36
37 △
38
39
40 -
```

 $c2_raw \sim normal(0, 1);$

// Autoregressive prior for mu

vector<lower=-4, upper=4>[n_all] mu_raw;

real<lower=-4, upper=4> s1_raw;

real<lower=-4, upper=4> s2_raw;

12 → parameters

14

15

41

48

Range of parameters

Parameters are converted into reasonable scale values, which is expected from previous studies.

- In this case, typical range of wastewater concentration is expected to be 0.5 to 6.5 log₁₀ copies/L.
 - Typically, S_1 and S_2 are positive values, ranging from 0 to 1.
- C_1 and C_2 are probably in this range, according to a previous study.

These values can be changed, according to your dataset. (Do not forget to save file after changing values.)

 $mu_raw \sim normal(0, 1);$ $s1_raw \sim normal(0, 1);$ The same prior distribution $s2_raw \sim normal(0, 1);$ $c1_raw \sim normal(0, 1);$

mu[3:n_all] ~ normal(2 * mu[2:n_all-1] - mu[1:n_all-2], s1);



- Run MCMC sampling from R file
- Check R-hat and n_eff
- Confirm wastewater concentration