

# Answers to Homework 1

Hongyi Zheng  
CS-UY 4533 Interactive Computer Graphics

January 12, 2021

- (a) The first midpoint to check is the midpoint between first  $N = (r, 1)$  and  $NW = (r-1, 1)$ , which is  $(r - \frac{1}{2}, 1)$ . To avoid floating-point computations, we define  $F(x, y) = 4(x^2 + y^2 - r^2)$ . Thus,  $D_{start} = F(M_{start}) = 5 - 4r$ .

Then, we follow those steps to update  $D$ :

$$D_{new} = D_{old} + F(M_{new}) - F(M_{old})$$

If  $N$  is chosen ( $D_{old} < 0$ ),

$$\begin{aligned} D_{new} &= D_{old} + 4(x^2 + (y+1)^2 - r^2) - 4(x^2 + y^2 - r^2) \\ &= D_{old} + 8y + 4 \end{aligned}$$

Here,  $y$  is the  $y$ -coordinate of the  $M_{old}$ , and is also the  $y$ -coordinate of the chosen  $N$ .

If  $NW$  is chosen ( $D_{old} \geq 0$ ),

$$\begin{aligned} D_{new} &= D_{old} + 4((x-1)^2 + (y+1)^2 - r^2) - 4(x^2 + y^2 - r^2) \\ &= D_{old} + 8y - 8x + 8 \end{aligned}$$

Here,  $y$  is the  $y$ -coordinate of the  $M_{old}$ , and is also the  $y$ -coordinate of the chosen  $NW$ . However,  $x = x_{M_{old}} = x_{NW} + \frac{1}{2}$ . Therefore,

$$\begin{aligned} D_{new} &= D_{old} + 8y - 8x + 8 \\ &= D_{old} + 8y_{NW} - 8(x_{NW} + \frac{1}{2}) + 8 \\ &= D_{old} + 8y_{NW} - 8x_{NW} + 4 \end{aligned}$$

In conclusion,

$$D_{new} = \begin{cases} D_{old} + 8y_N + 4 & (D_{old} < 0) \\ D_{old} + 8y_{NW} - 8x_{NW} + 4 & (D_{old} \geq 0) \end{cases}$$

- (b) In OpenGL's screen coordinate system, the origin is on the bottom-left corner, the  $x$ -axis increase to the right and the  $y$ -axis increase to the top. It is not possible to display pixels at  $(x, y)$  where  $x < 0$  or  $y < 0$ ;