Supplemental Material For 10/02 Book Reading Seminar

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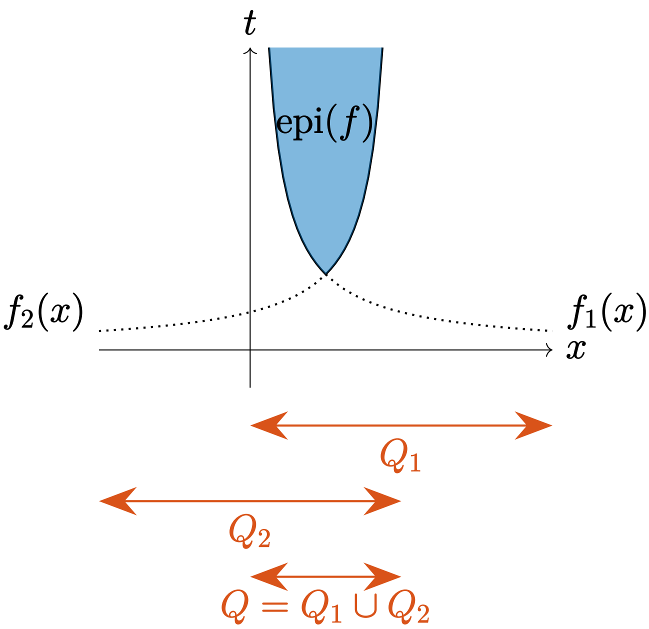
Let's have a break!

5 or 10 minutes break. Please feel free to ask me any questions.

p.147 Thm 3.1.5

3



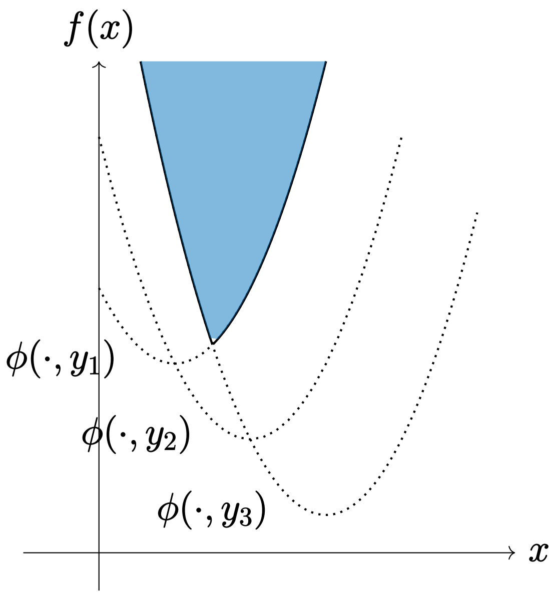
p.148 Thm 3.1.6

affine-invariant. whiteboard.

p.149 Thm 3.1.7

inf of $\phi(x, y)$. whiteboard.

p.149 Thm 3.1.8



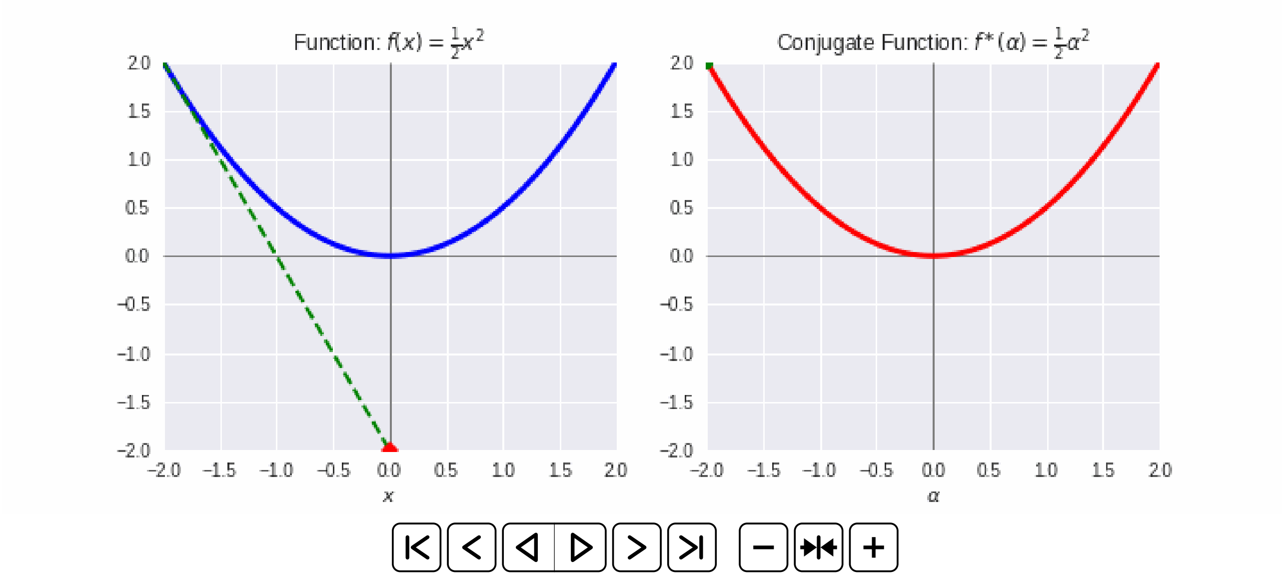
p.150 Thm 3.1.9

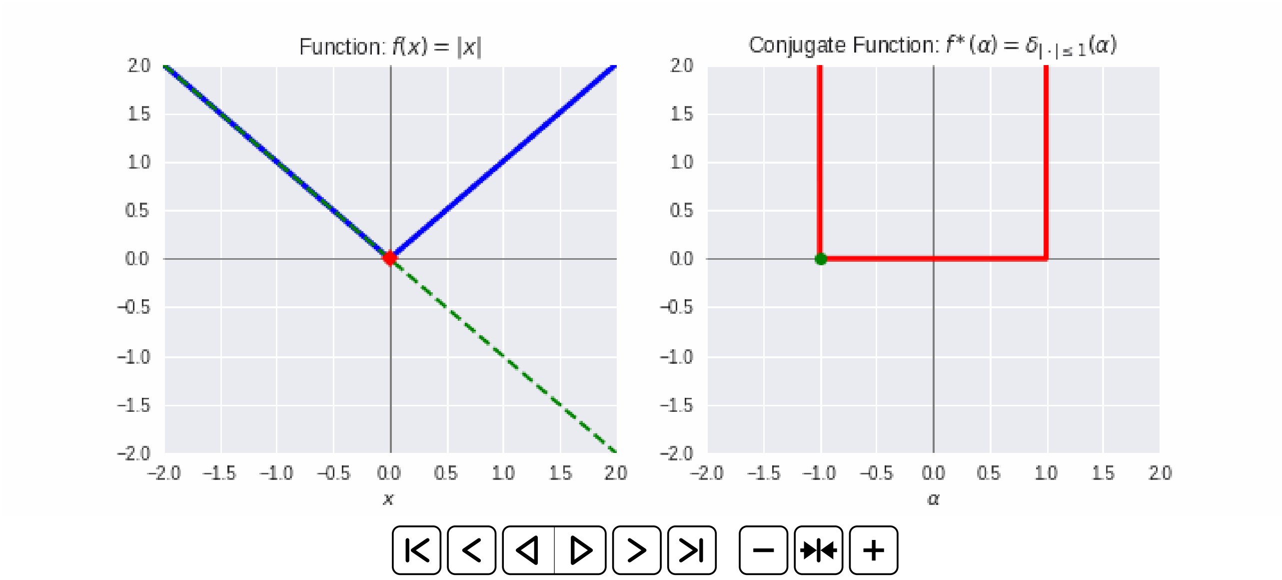
composition of convex functions. whiteboard.

p.150 Example 3.1.2

1

My favorite explanation of Fenchel conjugate: link



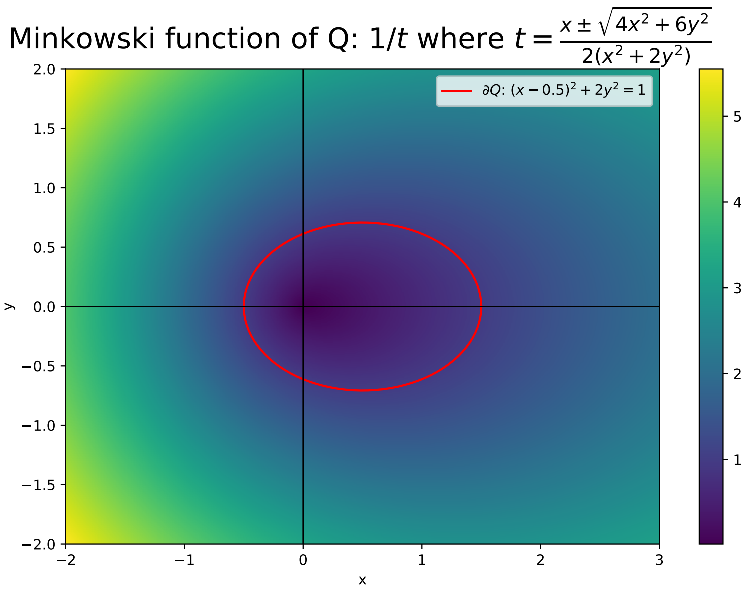


4

Minkowski function.

For example, let us consider the case the convex function Q is $Q = \{(x, y) \in \mathbb{R}^2 | (x - 0.5)^2 + 2y^2 \leq 1\}$. The Minkowski function is defined as

$$
\begin{aligned}
f(x, y) &= \min_{\tau \geq 0}\{\tau : (x, y) \in \tau Q\} \\
&= \min_{\tau \geq 0}\{\tau : (1/\tau x - 0.5)^2 + 2(1/\tau y)^2 \leq 1\} \\
&= \max_{t \geq 0}\{t : (tx - 0.5)^2 + 2(ty)^2 \leq 1\} \\
&= \max_{t \geq 0}\{t : t^2(x^2 + 2y^2) - tx - 0.75 \leq 0\} \\
&= \frac{\max(x \pm \sqrt{4x^2 + 6y^2})}{2(x^2 + 2y^2)}
\end{aligned}
$$

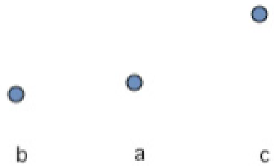


This is actually a convex function.

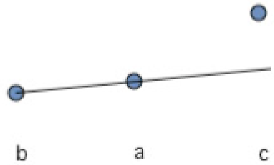
p.153 Lem 3.1.4

Please compare with lem 3.1.4.1.
visual proof link:

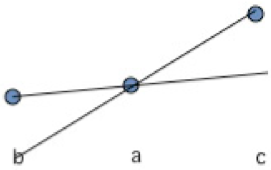
Suppose you want to prove continuity at a. Choose points b, c on either side. (This fails at an endpoint, in fact the result itself fails at an endpoint.)



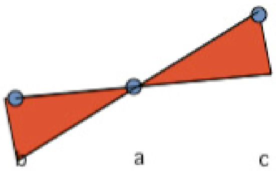
By convexity, the c point is above the a, b line, as shown:



Again, the b point is above the a, c line, as shown:



The graph lies inside the red region,



so obviously we have continuity at a.