2017 年度 大問 2

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1　問題

コイン投げ

2　解答

(1)

漸化式を解く。

$$\left(\frac{1}{2}-\frac{1-\theta_B}{2-\theta_A-\theta_B}\right)(-1+\theta_A+\theta_B)^{n-1}+\frac{1-\theta_B}{2-\theta_A-\theta_B}$$

(2)

漸化式を解く。

$$\frac{\theta_A+\theta_B-2\theta_A\theta_B}{2-\theta_A-\theta_B}$$

(3)

比較する。

$$\begin{aligned}\frac{\theta_A+\theta_B-2\theta_A\theta_B}{2-\theta_A-\theta_B}-\frac{\theta_A+\theta_B}{2}&=2(2-\theta_A-\theta_B)(\theta_A-\theta_B)^2\\&\geq 0\end{aligned}$$

3　おまけ

Listing 1 2

```
import random

theta_A = None
theta_B = None


def isHead(whichCoin: str):
    if whichCoin == "A":
        return random.random() < theta_A
    else:
        return random.random() < theta_B


def trial(n: int):
    whichCoin = ["A", "B"][random.randint(0, 1)]
    headCount = 0
    for _ in range(n):
        if isHead(whichCoin):
            headCount += 1
        else:
            whichCoin = "A" if whichCoin == "B" else "B"
    return whichCoin, headCount + int(isHead(whichCoin))


def problem1(n):
    return ((1 / 2) - ((1 - theta_B) / (2 - theta_A - theta_B)))
       * (
        (-1 + theta_A + theta_B) ** (n - 1)
    ) + ((1 - theta_B) / (2 - theta_A - theta_B))


def problem2():
    return (theta_A + theta_B - 2 * theta_A * theta_B) / (2 -
       theta_A - theta_B)


def main():
    global theta_A, theta_B

    for _theta_A in [0.3, 0.5, 0.7]:
        for _theta_B in [0.3, 0.5, 0.7]:
            theta_A = _theta_A
            theta_B = _theta_B
            NUM_OF_TRIAL = 10000
            n = 100
            NthCoinIsA = 0
```

```
            Hn = 0
            for _ in range(NUM_OF_TRIAL):
                whichCoin, headCount = trial(n)
                NthCoinIsA += int(whichCoin == "A")
                Hn += headCount
            NthCoinIsA /= NUM_OF_TRIAL
            Hn /= NUM_OF_TRIAL
            Hn /= n
            print(f"{theta_A=}, {theta_B=}")
            print(f"{problem1(n)=}, {NthCoinIsA=}")
            print(f"{problem2()=}, {Hn=}")


if __name__ == "__main__":
    main()
```

Listing 2 result

```
theta_A=0.3, theta_B=0.3
problem1(n)=0.5, NthCoinIsA=0.5025
problem2()=0.3, Hn=0.30355699999999997

theta_A=0.3, theta_B=0.5
problem1(n)=0.4166666666666667, NthCoinIsA=0.4172
problem2()=0.4166666666666667, Hn=0.420066

theta_A=0.3, theta_B=0.7
problem1(n)=0.30000000000000004, NthCoinIsA=0.2929
problem2()=0.5800000000000001, Hn=0.5850310000000001

theta_A=0.5, theta_B=0.3
problem1(n)=0.5833333333333334, NthCoinIsA=0.582
problem2()=0.4166666666666667, Hn=0.421361

theta_A=0.5, theta_B=0.5
problem1(n)=0.5, NthCoinIsA=0.506
problem2()=0.5, Hn=0.504502

theta_A=0.5, theta_B=0.7
problem1(n)=0.37500000000000006, NthCoinIsA=0.3812
problem2()=0.625, Hn=0.631972

theta_A=0.7, theta_B=0.3
problem1(n)=0.7, NthCoinIsA=0.6978
problem2()=0.5800000000000001, Hn=0.585776

theta_A=0.7, theta_B=0.5
problem1(n)=0.625, NthCoinIsA=0.6327
```

```
problem2()=0.625, Hn=0.630889

theta_A=0.7, theta_B=0.7
problem1(n)=0.5, NthCoinIsA=0.4996
problem2()=0.7, Hn=0.707874
```