Report for 2024/07/30

Hiroki Hamaguchi

I. CG-Newton method

On the last time meeting, I have implemented the CG-Newton (I named it) method, non-linear conjugate gradient method with Newton's method based line search, and showed the result of the numerical experiment. Although I'm quite interested in this topic, since I couldn't find the corresponding method in literature, I temporarily suspended the research on this topic.

According to Takahashi-san, this method might be related to A coordinate gradient descent method for nonsmooth separable minimization, which adopt the following line search method (I modified little bit):

> Choose α_{init} and let α be the largest element of $\{\alpha_{\text{init}}\beta^i\}_i$ satisfying
>
> $$f(x+\alpha d) \leq f(x)+\alpha\sigma\left(\nabla f(x)^\top d + \frac{1}{2}d^\top \nabla^2 f(x) d\right)$$

Although this is not completely the same as the line search method I implemented, I think this is quite suggestive. I implemented as $\alpha_{\text{init}} = -\frac{d^\top \nabla f(x)}{d^\top \nabla^2 f(x) d}$ with normal Armijo rule, but in analysis α_{init} doesn't affect the convergence rate. Thus, we need to prove that the obtained α satisfies stronger condition than the normal Armijo rule like above one. Let us only consider the case of f is convex and backtracking will not occur. Since

$$\begin{aligned} f(x+\alpha d) - f(x) &\simeq \alpha \nabla f(x)^\top d + \frac{\alpha^2}{2} d^\top \nabla^2 f(x) d \\ &= -\frac{(d^\top \nabla f(x))^2}{2d^\top \nabla^2 f(x) d}, \end{aligned}$$

we can expect that

$$f(x_{k+1}) - f(x_k) \leq -\frac{(d_k^\top \nabla f(x_k))^2}{2d_k^\top \nabla^2 f(x_k) d_k}.$$

However, I don't have any idea what it means and how to utilize it in the analysis. It might necessary to consider the smoothness of $\nabla^2 f$ or something like that.

Additionally, I noticed that (at least) some of the existing methods are also optimal for the step-size α, not only for the direction β. To show that CG-Newton method's advantage, we might need to assume more conditions on the problem, which I don't know yet.

II. Graph Drawing

Based on the advice of Prof. Takeda and Prof. Marumo, I began to think that the best thing to do would be to do research that I enjoy the most, so I started a new research project on graph drawing. Of course, it is related to what I have been doing so far, and I believe that I can make the most of what I have been doing so far. I am sorry for the sudden change.

Originally, my former supervisor, Prof. Tanigawa, specializes in graph drawing, and you might think that I am talking about discrete optimization, but after consulting with Prof. Tanigawa, I think that this is rather a continuous optimization problem, so I decided to do research on this topic.

I am using the IEEE template simply because the two main prior studies were both IEEE papers.

On the last time meeting, I said that Random Subspace Method does work well for a narrow valley, and thus I proposed the CG-Newton method. But based on Prof.Takeda's advise, I considered what kind of problems can be well solved by Random Subspace Method, and realized that Random Subspace Method is very suitable for the problem with separable variable $X = (x_1, x_2, \ldots, x_n) \quad (x_i \in \mathbb{R}^2)$, i.e., the graph drawing problem. (Actually, one of the candidates for my graduation thesis theme was about graph drawing.)

$$\underset{X}{\text{minimize}} \qquad \sum_{i<j} f(x_i, x_j)$$

As you know, stochastic gradient decent works well for the separable problem ($f = \sum_i f_i$), and separableness of $X = (x_1, x_2, \ldots, x_n)(x_i \in \mathbb{R}^2)$ is corresponding to this property.

Let us consider the case a random matrix P is a permutation matrix like, which is a practical situation. Then, the separableness of X is necessary to terminate the algorithm in a few step.

Of course, a completely separable problem is too trivial (It is just a sum of independent problems), but I think that the graph drawing problem is a good example of the problem with a moderate separableness.

III. Next Meeting

Since currently I'm working on the graduation thesis with Prof. Tanigawa,and I have to go to Hokkaido to attend AQIS (conference) from 2024/08/26 to 2024/08/30, I would like to have the next meeting on September if possible. How about your schedule?