Faster Fruchterman–Reingold Algorithm by Random Subspace Method

Hiroki Hamaguchi

***Abstract*—The abstract goes here.**

***Index Terms*—Graph drawing, Fruchterman–Reingold algorithm, Random Subspace method.**

I. Introduction

DRAWING graph is one of the most fundamental task in computer science.

One of the graph drawing algorithms is force-directed algorithm. Davidson and Harel proposed algorithm using simulated annealing [1]. Kamada and Kawai [2] proposed an algorithm. Fruchterman–Reingold algorithm [**?**], as known as spring embedder algorithm, is one of the most popular algorithm. The algorithm is based on the physical model of a system of particles and springs.

To mitigate this heavy computational cost, several methods have been proposed.

One of the strategies is to approximate the N-body simulation by a hierarchical method, such as the fast multipole method [3], the Barnes–Hut approximation [4], and multilevel approach [5] or stress majorization [6].

The other method is to just speed up the optimization algorithm as it is, which has the same spirit of our work. use Stochastic Gradient Decent [7]. GPU parallel architecture [8], and numerical optimization techniques [**?**].

In this paper, we will use the Random Subspace method [9], [10], [**?**], [**?**] to accelerate the Fruchterman–Reingold algorithm.

The rest of the paper is organized as follows. In Section II, we define an optimization problem for Fruchterman–Reingold algorithm. In Section III, we introduce the Random Subspace method. In Section IV, we show the experimental results. Finally, we conclude the paper in Section V.

II. Preliminary

In this section, we define an optimization problem for Fruchterman–Reingold algorithm. Let $\mathbb{R}_+$ be a set of positive real numbers and $A = (a_{i,j}) \in \mathbb{R}_+^{n\times n}$ be an adjacency matrix of a graph $G = (V, E)$. Each vertex $v_i \in V$ is assigned a position $x_i \in \mathbb{R}^d$ and we define $x = (x_1, \dots, x_n) \in \mathbb{R}^{d\times n}$ as a matrix of positions. For an optimal distance k, and a distance d between two vertices v_i and v_j, Fruchterman and Reingold defined the power of attraction $F^a_{i,j} : \mathbb{R}_+ \to \mathbb{R}$ and the power of repulsion $F^r : \mathbb{R}_+ \to \mathbb{R}$ as

$$F^a_{i,j}(d) \coloneqq \frac{a_{i,j}d^2}{k}, \quad F^r(d) \coloneqq -\frac{k^2}{d}.$$

The energy for these powers $E^a_{i,j}, E^r$ and the total energy $E_{i,j}$, stress of the graph, are defined as

$$E^a_{i,j}(d) \coloneqq \int_0^d F^a_{i,j}(r)\,\mathrm{d}r = \frac{a_{i,j}d^3}{3k},$$
$$E^r(d) \coloneqq \int_\infty^d F^r(r)\,\mathrm{d}r = -k^2\log d,$$
$$E_{i,j}(d) \coloneqq \begin{cases} E^a_{i,j}(d) + E^r(d) & \text{if } i \neq j, \\ 0 & \text{if } i = j. \end{cases}$$

The energy function $E_{i,j}$ is convex for $a_{i,j} \in \mathbb{R}_+$ and minimized when $d = k\sqrt[3]{a_{i,j}}$. Based on these energies, the optimization problem for Fruchterman–Reingold algorithm is defined as

$$\underset{x\,\in\,\mathbb{R}^{d\times n}}{\text{minimize}} \quad f(x) \coloneqq \sum_{i,j} E_{i,j}(\|x_i - x_j\|) \tag{1}$$

In order to introduce the Random Subspace method, we define a function $f_i : \mathbb{R}^d \to \mathbb{R}$ for $1 \le i \le n$ as

$$f_i(x_i) \coloneqq \sum_{1\le j\le n} E_{i,j}(\|x_i - x_j\|).$$

The gradient and the Hessian of f_i are

$$\nabla f_i(x_i) = \sum_{j\neq i}(x_i - x_j)\left(\frac{a_{i,j}\|x_i - x_j\|}{k} - \frac{k^2}{\|x_i - x_j\|}\right),$$
$$\nabla^2 f_i(x_i) = \sum_{j\neq i}\left(\frac{a_{i,j}\|x_i - x_j\|}{k} - \frac{k^2}{\|x_i - x_j\|}\right)I_d +$$
$$\sum_{j\neq i}\left(\frac{a_{i,j}}{k\|x_i - x_j\|} + \frac{k^2}{\|x_i - x_j\|^3}\right)(x_i - x_j)(x_i - x_j)^\top.$$

As pointed out in [**?**], the Fruchterman–Reingold algorithm is a gradient descent method for the energy function f.

III. Algorithm

IV. Experiment

We used dataset from [11] and MatrixMarket [12].
https://reference.wolfram.com/language/tutorial/GraphDrawingIntroduction.html

V. Conclusion

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