The theory of Quasi-duality map and fractals

(翻訳　類双対性写像とフラクタルの理論)

PartⅣ: Deformations of fractals and a generative approach to zeta structures —Generative Deformations of the Riemann Zeta Function from Graph Zetas via Quasi-dual Divergent Maps and the Construction of Their Zero Points—(A Study on the Riemann Zeta Function)

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Abstract

This final part examines the half-critical region and the non-regular structures of the Riemann Zeta function. Using spiral graphs, ideal class motifs, and topological genus transitions, the text proposes a novel geometric interpretation of the critical line and the continuum hypothesis.We analyze the critical line of the Riemann zeta function from various perspectives using “quasi-duality maps,” and discuss the mechanism of Hilbert-Polya action and the mechanism of “symmetric divergence control” in the analytic continuation of Euler products.

Summary and Explanation of This Document

The important point is that this began with the exploration of noncommutative structures in the mathematical modeling of noise phenomena.

What I consider most important is the “interpretation of zero points in divergent zeta constructions” in quasi-dual maps. In other words, the zeros of the zeta function are structural objects that can be reconstructed from the trace of a non-periodic ordered prime number structure. The essence of this construction method is that when a conceptual object exists, by considering its trace and performing various transformations using the trace bundle, a special “divergent reconstruction” occurs, which is shown to possess the natural properties of a “zeta structure.”

In summary, by considering a method of “contracting loops or expanding trees” in what might be called “infinite repetition within an infinitely continuing path,” I discovered the possibility of constructing a zeta structure. It was an extremely abstract theory, and I found a vivid example of this theory in the Ihara zeta. The Ihara zeta function is the very transformation that converts any “loop structure” into a “tree structure.” As a result, through the determinant representation of the Ihara zeta function, a function with a structure similar to the Riemann zeta function appears, and it seems possible to reexamine its zero point structure from the perspective of regularity.

Finally, this paper includes a chapter that outlines a constructive solution to the Hilbert-Polya operator problem using a matrix ring based on a prime number structure.

In this summary, I will focus on how the zeros and function expressions of Riemann zeta-like formulas, which appear as a consequence of the theory of class duality maps in dynamic transformation, are generated.

First, in the regular Ramanujan graph structure of the Ihara zeta function, the zeros are arranged on a circle, and in particular, in the “bouquet zeta function,” they are arranged on the unit circle. Repeatedly applying the “quasi-dual zeta extension” of this bouquet graph leads to a state where “the length of the path is a prime number” (a state where all prime paths exist) only under regular conditions.

Then, in this state, the matrix obtained by expressing the “prime Lyndon” expansion of the Ihara zeta function based on paths in a “determinant representation” is transformed by the class-dual transformation “u^p→e^-slogp,” which decomposes the path information into “infinite prime power circles” for each path, and then shifts it to the critical line (Re=1/2). Then, from the regular Ramanujan condition of the Ihara zeta function, it can be seen that this zeta function satisfies the Riemann hypothesis-like condition, but in fact, this transformation converts it into the Riemann zeta function.

The quasi-dual transformation "u^p→e^-slogp" connects this infinite concentric circle structure in a tensor-like manner based on the "prime structure (cycle)," creating a structure where there are infinitely many prime numbers. This is a basic yet special regular graph, but even in this graph, it is shown that the zero points are properly arranged on the unit circle. Applying the quasi-dual transformation "u^p→e^-slogp" to this graph results in the same structure, so the quasi-dual transformation "u^p→e^-slogp" invariant, i.e., it leads to a fixed point. Then, by defining a "matrix" that satisfies the conditions of the quasi-dual transformation at each scale logp of the concentric circles and infinitely combining them, it interacts with the determinant representation of the Ihara zeta function and acts as a Hilbert-Polya operator.

The above is an overview, and I will insert an explanation including the motivation later.

1. Starting Point: Noise Phenomena and Noncommutative Structures

The starting point for this exploration is the "mathematical structuring of noise phenomena."

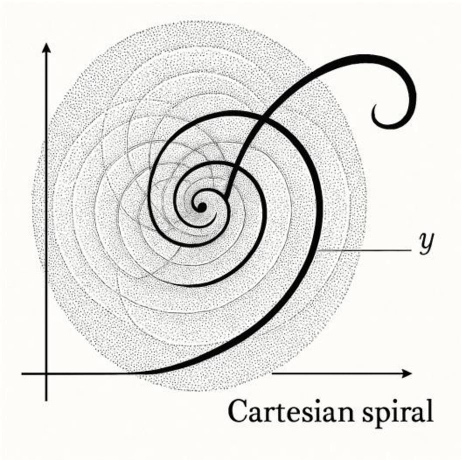
My interest in the "mathematical modeling of semi-subjective phenomena" known as noise phenomena arose when I learned about the "error backpropagation method" used in today's popular generative artificial intelligence.

In the "error backpropagation method," there is a mechanism for "generating meaningful images from noise." However, I had already noticed that, similarly, in "noise phenomena," through training, one can observe "non-trivial and involuntary meaningful visual or audIharary images (generally perceptual images) within the noise." I named this "intuitive images." For example, in literary expression, Aldous Huxley's "The Doors of Perception" describes a similar experiential structure. I won't explain this here, but since this noise phenomenon generally has a fractal-like structure with movement, I decided to explore how fractal structures transform while maintaining their fractal nature.

The structural similarities between "human observation methods" and "artificial intelligence generation methods" do not end there. For example, there are generative intelligences capable of "enhancing the resolution of old photos, converting them to color," or "removing human figures from photos." In fact, through "noise observation," I had already experienced this sensation. Therefore, I recognized that "images are created by contracting fractals (after expanding the world fractally)" and named this process "fractal contraction," referring to the operation as "quasi-dual transformation" or "quasi-dual mapping."

In its simplest form, this manifests as “mapping a Cartesian spiral fractal to an infinite concentric circle fractal in a one-to-one correspondence.” This diagram is important, so let me illustrate it with a figure. It is a schematic diagram, but imagine it as “a spiral expanding outward, cutting out each concentric circle one by one.”

Figure 1. The Infinite concentric circle fractal and The Cartesian spiral.



Basic quasi-dual mapping: infinite concentric circles →← Cartesian spiral

What is important is that by “simultaneously deforming” the “infinite repetitive structure” created by the enlargement mapping that appears in fractals, various fractal structures can be deformed while maintaining their fractal nature. When considering such matters, it is helpful to trace the structure and examine the “infinitely repeating portion” of the trace. In other words, “looping” involves compressing and connecting the infinitely repeating paths, while ‘treeing’ involves expanding them. Note that these are one-to-one correspondences. This is a local deformation, but we will also consider the case of “continuous mappings” later.

Such quasi-dual transformations are generally non-commutative, and the transformations themselves are multivalued and possess multiplicity. Regarding the name “quasi-dual transformation,” there is first the “dual transformation.” For example, using this, we can construct a “dual motif category,” which is a closed structure defined by a deformation structure, from a generalization of a graph structure called a hypergraph (which can be considered to show correspondence between multivaluedness and multivaluedness). This can be described as an operation that “expands graphs set-theoretically, consolidates multiplicity, and restores uniqueness.” This “dual operation” overlaps with “quasi-dual transformations” in very simple situations, such as regular graphs.

Therefore, we named this “quasi-dual mapping” as something similar to the dual operation. Since it is essentially non-commutative and multi-valued, considering the “closure structure” to handle it requires considering the round-trip of the operation transformation, and the categorical structure created by this operation can be described as a series of transformations from “loop-type structure” to “tree-type structure,” which we call “quasi-dual closure.”

Note that the essence of this transformation series is revealed in the simple mutual transformation state of “concentric circle fractal →← Cartesian spiral.”

These transformations are defined by “quasi-dual transformations,” and when considering what they are applied to, if there is a structure, we can take the path that traces its elements, i.e., the “trace.” These traces include both finite and infinite ones, but here we will only consider the infinite ones.

When “infinite repetition” appears in the trace, the transition from loop-type structure to tree-type structure can be expressed through the operation “concentric circle fractal →← Cartesian spiral.” We will look at a concrete example of this later in the form of the Ihara zeta function.

This operation itself can be performed as many times as desired where “traces and the trace bundles that collect them” are established, but when I learned that the construction method of “Ihara zeta” and its structure are identical to those of an infinite regular tree, I realized that it could actually be formalized in special cases, which became a major motivation for writing this paper. In other words, I understood that Ihara Zeta is a special, commutative, (semi-)unique, special class dual transformation example of “(regular) finite graph” (loop structure) →← “regular infinite tree” (tree structure).

However, it is important to note that the term “semi-unique” is only partially valid, as there actually exists a “non-trivial infinite repetitive structure” that naturally “divergently” transforms a finite graph into a graph structure with infinite loops. In other words, this is the structure called the “zeta structure,” and it is important to note that this is connected to the “zero product,” and that the product is first defined “non-complexly” and then “analytically connected to the complex domain” through the “Ihara zeta determinant representation.”

In other words, while the Ihara zeta function derived from a (regular) finite graph is expressed as a “rational function,” the determinant representation derived from the infinite loop graph transformed through the trace bundle formally becomes an infinite matrix, which can be interpreted as reflecting the possibility that the zeta function derived from a finite graph may have an infinite number of zeros. However, many divergent zetas also fall into the finite type, and the tolerance of this finiteness is shown to be related to regularity. But this will be explained later.

In other words, through the quasi-dual map, which deforms the trace bundle, and the exploration of the noncommutative transformation structure that emerges there, the possibility of describing the zeros that appear in the zeta function not as mere numerical properties but as the result of structural trace restoration deformation was accidentally discovered.

2. Reconstruction of the "structure" of zeros using quasi-duality maps and "divergent quasi-duality transformations"

Now, let us explain the "divergent zeta construction in quasi-duality maps" and the "structural interpretation of zeros" that naturally appears in it.

A quasi-dual map is an operation that "transforms one fractal into another fractal."

It involves collecting traces from a structure and examining a multivalued structure that can be constructed and reconstructed from them. When there is "infinite repetition" in the trace bundle, it is summarized and reduced to a "loop structure."

Here, for non-periodic trace structures (∞ Lyndon series), the proposition that zero points structurally emerge by identifying higher-order traces (i.e., zero-point products) that reconstruct them was proposed. This is considered crucial.

Let me explain.

First, the Ihara zeta function traces all possible "paths traversing the graph." This involves considering a "closed loop" as a "primitive structure" and constructing all possible paths as non-commutative combinations of these.

By doing so, the "loop structure" of a finite graph can be "transformed" into the form of a "regular infinite tree." This seems to be due to the observation of a person named Sale. The term 'transformation' is from my perspective, while the "quasi-dual map" is from the perspective of transforming the structure. Note that in this transformation, it is usually the "trace bundle itself" that is transformed. There is a difference between the "trace bundle" of the original structure and the "trace bundle" after transformation, which can be interpreted as a loss or addition of information.

It is important to convert the concept of "closed path" into the concept of "trace bundle." The concept of a closed circuit uses concepts such as equivalence and reduction to extract "loopiness." However, if we shift to the idea of restoring "infinite repetition" in the "trace bundle," these equivalence concepts for "extracting loop structures" become unnecessary. Strictly speaking, various problems arise, but I will not discuss them here, as they are not necessary for this discussion.

Furthermore, by replacing the concept of "closed path" with that of "trace bundle," it becomes important to extract "non-trivial loop structures."

For example, if there are two or more "prime structures," let us represent them as 0 and 1. Then, we can see that there are repetitive structures such as '01' and "001." In other words, it becomes clear that "non-trivial loop structures can be constructed by combining prime structures into non-periodic structures with order," and this is "necessarily contained within the trace bundle." If we denote the length of this repetitive structure as n, it can be stated that a "non-periodic structure" of length n necessarily exists, and that "non-periodic repetitive structures" exist infinitely. Specifically, this becomes clear when examining the construction method of sequences that do not contain repetitions.

By treating this "non-periodic structure" as a "higher-order prime structure" and repeatedly performing "quasi-dual restoration," it becomes clear that it is possible to restore an "infinite structure graph with infinitely divergent loop structures." However, redundancy always occurs, and if this is reduced, it becomes clear that this divergent graph is actually a finite system. The rigor of this will be demonstrated later in the analysis of Lyndon language. Finite graphs typically have "paths that trace back along the same route," so it is important to note that "the number of elementary structures is almost always two or more." Of course, it is possible to construct an Ihara zeta function for this "infinite structure graph," and it can be seen that there is a determinant representation. Furthermore, it can be seen that the infinite graph reconstructed from the finite graph shares the "trace structure" as much as possible and can be expressed by an infinite matrix, which corresponds to the fact that the zeros of the "(regular) finite graph zeta function" are at most infinitely many. However, as mentioned earlier, due to "redundancy," it ultimately becomes "finitely generated," and due to its remarkable expansive nature, it demonstrates hologram-fractal-like restorability, expanding to an "infinite graph where the length of each path is a distinct prime number p" under regularity conditions.

This special quasi-dual transformation to a divergent infinite graph can be called a "zeta transformation."

"Divergent quasi-dual transformation" = zeta transformation.

And this "zeta transformation" provides a natural construction and interpretation for the existence of the zero product.

Let's organize the diagram of this "zeta transformation" a little more. Note that there is a "nested structure" here.

(Figure of zeta expansion: usually restored from the repetition of "prime elements," but restoration from “non-periodic terms of prime elements” is also possible, and these have fixed points)

Note that this “divergent quasi-dual transformation” can be constructed one by one by actually combining several “non-periodic terms” to show the actual zero point structure in a determinant expression. This sequence is considered to be non-additive, and the matrix can be pointed out to have some symmetries based on the structure of the graph. Before actually constructing this “non-periodic term” as a Lyndon sequence, there is something called an “infinite non-periodic sequence,” and some people may think that this does not fit into the determinant representation because it appears to have no “end point.” However, “zeta extensions” have a hierarchical structure. That is, there exists a zeta extension that ‘swallows’ the “non-periodic term,” and it can be shown that the “non-periodic term” exists as an infinitely repetitive term within that extension. In other words, this hierarchical structure has a “nested structure” that infinitely swallows itself, and after the final extension, it forms a “fixed point.” Note that a “non-countable continuous topos,” which differs from ordinary rational numbers in that it can only observe “local structures” but can properly observe “adjacency relations,” naturally emerges. Moreover, it can be expressed as a “nested structure.” In rational numbers, there is no such thing as a “neighboring rational number.” However, this non-periodic term has a “neighbor.” This is thought to be a topic in “category theory,” which will not be discussed here, but those who are interested should pay attention to this difference.

Let us specifically construct the infinite graph structure resulting from this “divergent quasi-dual deformation” one by one.

The elementary structures are loops of various lengths... Since they are loops, they are connected, and this structure is reflected in the determinant representation. If these elementary structures are 0, 1, 2, 3... N, we connect them in a non-repetitive, ordered manner. For example, if there are only 0 and 1, it could be 1101. This is not a repetition of any length of sequence. We translate this into an actual structure and create a loop structure as a “new elementary structure.” Then we create another elementary structure. It could be 01. We also translate this into the actual structure and consider it as a new elementary structure to create a loop.

We collect such structures and arrange them infinitely... Note that this is “uncountable.” With this, we can gradually construct the determinant side of Ihara's “divergent zeta extension.”

Then, as in the construction of Ihara's zeta function, we can translate the “connectivity” between a new prime structure and a new prime structure into 0 or 1 and create a determinant representation.

If the characteristic equation of this determinant can be solved within the range of complex numbers, we can see that Ihara's zeta function is naturally analytically continued to the complex number domain and has a complex function representation based on zero points (new higher-order prime structures). These eigenvalues can be considered "zero points." It is important to note that if an Euler product (loop-type structure) exists, it can be transformed almost automatically into "loop-type structure → tree-type structure → zeta structure." At this point, it is also important to note that the "content of the prime structure" is not important, and only "the connection of the prime structure, the uniqueness of decomposition, and the power (repetition) of the prime structure" have structural significance.

This is the determinant representation showing the zero-point structure constructed from the graph structure.

In this construction, if we note that the quasi-dual transformation in Ihara's zeta function is commutative and unique because the graph structure is uniform (the combination of points and edges is the same) and regular Ramanujan-like, then the question arises, "In cases where this is not the case, are there generally multiple zero structures?" For example, "Does it become a Dedekind-type zeta function when it is non-regular?" This is because quasi-dual transformations typically have multiple transformation series and form a non-commutative category structure. However, this is not an issue here, so we will just note it. The conditions that the Ihara zeta function is rational and finite, and that the tree structure it expands into is regular (Ramanujan-like), can be summarized as such.

We reinterpret this "non-periodic term" as a semigroup structure called the Lyndon language.

Now, we refer to Hideaki Morita's "Semigroup Representation of Combinatorial Zetas."

If we consider a finite number of "prime structures" as the alphabet of Lyndon languages, we can see that their ordered combinations "decompose uniquely into non-periodic terms" and "have an operation structure without inverses." At the same time, we can also see that "the structure of non-periodic terms was already hidden in the zeta construction of finite graphs." In other words, the repetition of "non-repetitive terms" can be uniquely expressed as a combination of prime elements with non-repetitive properties called prime Lyndon words. And if we consider these "prime Lyndon words" to exist as loops, we can restore the trace path in its entirety. This is McMahon's fundamental theorem. In other words, we can see that "prime structures" have a “semigroup order structure” as higher-order prime structures.

In summary

All trace structures can be described as combinations of prime Lyndon sequences.

The repetition of these prime structures can be finite or infinite, and in the case of infinity, it forms a non-cyclic trace path and becomes a higher-order constituent factor involved in the generation of zero points in the zeta structure.

(The theorem that the diversity of trace bundles in finite graphs is equal to the diversity of trace structures in trace bundles in infinite graphs)

To put it clearly, this means that there are multiple prime decomposition structures in trace bundles.

The “infinite cycle structure” in trace bundles allows for two or more different types of decomposition methods at the same time: decomposition into prime Lyndon languages and decomposition into prime structures. This has a structure similar to that of unique decomposition in ideal theory in commutative factor rings, and is thought to correspond to the phenomenon of hierarchical structures appearing in decomposability in noncommutative ideal theory.

To summarize once again, the “decomposition representation theorem for trace bundles” is as follows.

Any finite trace bundle *T* can be decomposed into either of the following forms:

(1) Lexical decomposition by prime Lyndon words

(2) Prime structural cycle decomposition

These decompositions are mutually intersecting, resulting in a noncommutative hierarchical ideal decomposition structure.

The recursive trace of a prime Lyndon sequence is the basis in the quasi-dual category.

The loop structure that is restored may actually be infinite. It may be restored to an infinite graph. However, that is redundant.

The loop structure that can be restored may actually be infinite. It may be restored to an infinite graph. However, this is redundant.

In other words, in a noncommutative structure, there are (at least) two Gödel-generating bases obtained by prime structural decomposition. Gödel arranged prime numbers and inserted an “infinite decimal” structure into them to construct a one-to-one correspondence with natural numbers.

The question of how many “prime Lyndon elements” are needed to express all “non-periodic terms” will be discussed later, when we examine the relationship between “prime structures” and “prime Lyndon elements” derived from the fixed-point property in the quasi-dual map of (regular) finite graphs, at the same time as the zero-point ring structure.

The fact that the “expressibility of the trace bundle becomes multiple” allows for two structural representations: the Euler product and the zero product. This non-commutative structure enables the two graph structures, despite having different dimensions, to mutually transform into each other. They undergo “quasi-dual transformations.” The two graph structures are “reconstructed” from the trace bundle.

This is the most important content of this article.

Note that the infinite loop structure appears not as an isomorphism class of self-loops but as a noncommutative expansion structure, and that there is an order structure, which also includes cases where “prime structures continue infinitely.”

This self-loop-like zeta inclusion is thought to give rise to the essential nested structure of zero points.

This will become an issue later when constructing the Riemann zeta from the graph zeta.

The decomposition of natural numbers into prime numbers has a structure where “order” is absent or broken. We introduce the gamma structure into this broken part and follow the procedure of normalization. In this way, we transition from a structure with an order structure to one without an order structure.

3. Regular Graphs and Zero Points: The Cyclicity of Zero Point Structures in Bouquet Structures

It was from Shigenobu Kurokawa's Absolute Mathematics: Introduction to Absolute Mathematics and the method of constructing Ihara zetas described therein that it became clear that the quasi-duality mapping, which is part of the theory of abstract structures, can be expressed as a concrete formula in the form of a zeta structure. The Ihara zeta function can be considered as the zeta function that counts all possible trace paths within a graph-theoretic structure.

In Shigenobu Kurokawa's *Absolute Mathematics: Introduction to Absolute Mathematics *, there is a section where the Ihara zeta function in a graph with a bouquet structure is presented as an example where the Euler product of the absolute zeta function can be constructed.

In regular graphs of the Ihara zeta function, particularly those with a “bouquet graph” structure, it has been shown that the zeros lie on the unit circle, providing a foundation for the “structural Riemann hypothesis.”

Even when bouquet graphs are connected via an infinite tensor product (Kurokawa?)

and constructed as non-regular graphs with infinite prime structures, the zeros remain on the circle. Regular graphs with such circular zeros are summarized as having Ramanujan-like characteristics.

Here, we first consider the zeta structure of graphs and how their zero structures form circular configurations.

Now, let us first consider the finite graph structure (regular).

The class-dual mapping transformation of the Ihara zeta function had a remarkable property. That is, the "structure of the trace bundle" remains invariant, regardless of whether the transformation is from loop-type to tree-type, or even in the case of "divergent quasi-dual zeta transformations (extensions)." This can be described as an "invariant structure in transformations."

Here, we will note only that "the graph structure itself that is transformed is infinite." From this fact, we will later see that "the prime number structure is restored within the graph structure by the quasi-dual zeta expansion."

Let us explain the "fixed point structure of the quasi-dual mapping."

First, we define the quasi-dual mapping as follows.

Definition (quasi-dual map):

For a trace sequence with an infinite recursive structure, a map that deforms the local structure (prime structure) and the global structure (trace bundle) in a noncommutative manner, such that even when the structure is deformed by the operation, the trace bundle generated from the structure returns to the same trace sequence structure, thereby becoming a fixed point of the structure as a whole.

In this case, the "structure that becomes a fixed point" is a structure that cannot be further transformed by the quasi-dual map. In other words, it preserves the "infinite recursive structure" as a "structural closure," and as a result, the zero point is stably mapped to the loop structure, which is why it is called the "fixed point structure = cyclic zero point structure."

Why does it become a "cyclic zero point structure"?

Let us consider a very simple situation, such as the structure of a bouquet graph or a (2,2)-regular graph.

These graphs have the following properties:

All nodes are isomorphic (regularity), all trace paths are closed loops, and they have a structure that leads to infinite repetition. This trace structure is classifiable by a prime Lyndon word (contracted structure), and in such a structure, even if the trace bundle is deformed, the same structure reappears.

This is a structure that remains invariant under repeated application of the quasi-duality map.

This demonstrates that the matrix representation and eigenvalues in the construction of the Ihara zeta function actually indicate that “the determinant structure remains unchanged” because the trace bundle does not change.

From the above, in the Ihara zeta function, the trace paths corresponding to each prime loop (cycle) are:

Figure 2. The Ihara-type zeta and the “prime” length of paths.

各素ループ（サイクル）に対応するトレース経路は、

$$\prod_{[P]}(1-u^{\ell(P)})^{-1}$$

の形で表される（ここで $\ell(P)$ はループの長さ）

Ihara-type zeta function of bouquet graphs: Integration of loops of prime length.

If the graph is regular and Ramanujan-type, the eigenvalues of the corresponding matrix (such as the infinite Laplacian) are concentrated on the unit circle.

At this point, the zeros of the zeta function also appear on the unit circle.

What is important here is the fact that under the transformation “u→ q ˆ - s ,” the zeros are mapped to “Re＝1/2” and satisfy a condition similar to the Riemann hypothesis.

Let us introduce an auxiliary structural theorem.

The Iihara zeta function of a regular finite graph converges to a rational type. That is, there exists a homomorphism or isomorphic structure between “prime structures” (prime cycles) (= prime numbers) and “prime Lyndon elements” (prime Lyndon loops) (= zeros).

In other words, the number of minimal loops in the “quasi-dual divergent zeta structure” and the structure of the “combinatorial non-periodic terms” derived from them possess an infinite ideal structure generated by “prime Lyndon elements.” In other words, since the determinant representation of the Iihara zeta function of a regular finite graph converging to a rational type is finite, the number of prime structures is finite, and the same number of “zero points” arise, but due to the “duplication of zero points,” the following equation holds.

Number of prime structures ≥ number of prime Lyndon elements (number of zero points) (reduced by the number of zero point duplicates)

This means that, in the case of the highly restricted structure of regular graphs, the number of prime structures allows us to infer the number and structure of prime Lyndon elements, from which all trace structures can be reconstructed. Can we call this the finiteness of prime Lyndon elements in regular finite graphs?

This demonstrates an important structural condition in the non-commutative "zeta structure" generated by "quasi-dual divergent deformation."

It can be decomposed into a "zero product" in the same rational (finite) form.

Furthermore, it shows that the Iihara zeta structure has a "zero point" on the unit circle defined by the length of the prime Lyndon element. In other words, this means that regular graphs with Ramanujan-like properties have a "zero point structure" on the unit circle.

In summary,

Auxiliary proposition

The Ihara zeta function defined for graph structures that are fixed points of the quasi-dual map (e.g., regular bouquet structures) has a zero point structure on the unit circle.

Furthermore, this zero point structure is isomorphic to the structural stability of the trace bundle.

[Visual representation] Construction diagram (schematic)

[Trace bundle (infinite repetition)]

↓ Quasi-dual map

[Tree-like expansion (decomposition)]

↓ Closure reconstruction

[Restoration as a loop structure (circle)]

↓ Zeta construction

[Determinant zeta] − Eigenvalue → [Zero point on the unit circle]

The "circular zero point structure" is the result of the infinite loop structure as a fixed point of the quasi-dual map being analytically represented, and its essence is guaranteed by the regularity of the graph, the commutativity of the quasi-dual map, the closure structure of the trace bundle, and the stability of the eigenvalues of the characteristic equation of the zeta determinant. This results in the appearance of a circular structure as a stable topological structure for the hierarchy of quasi-dual transformations.

The phenomenon itself that the zero-point structure remains unchanged is the analytical shadow of the fixed-point structure in the quasi-dual map, which constitutes the decisive construction theorem of this cyclic zero-point structure.

Is “not changing” decisive?

To recap, the quasi-dual map is

a map that sublimates the infinite recursive structure into “elementary structure → elementary Lyndon sequence → higher-order structure,” temporarily transforms it into a tree-like expansion, and then recursively restores the structure in a closed manner.

However, in general, this transformation is non-commutative and non-associative, causing complex structural transformations. Since “trace bundles” are typically transformed, this situation indicates the existence of a special “fixed-point situation” or “structural kernel.”

In the theoretically constructed “divergent quasi-dual zeta,” the zero points indeed moved as the prime structures were transformed. Therefore, conversely, zero points that do not move even when transformed are

= structural fixed points

= stable category in the quasi-dual category

= essence of the circular zero-point structure

This demonstrates the “structural theorem.”

Auxiliary theorem

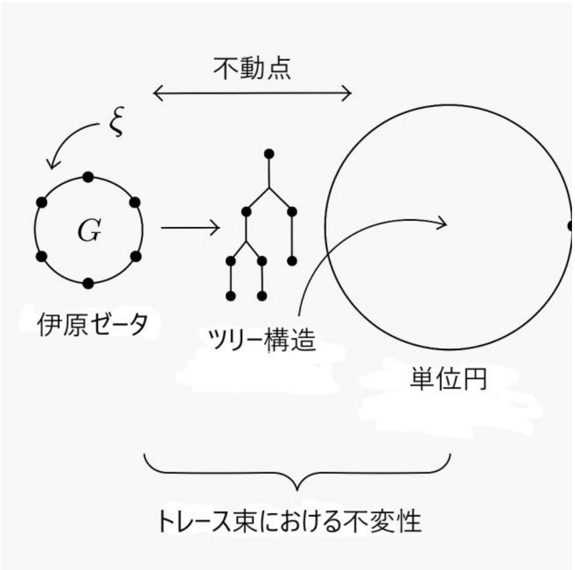
Circular zero-point structure as a quasi-dual fixed point

The Ihara zeta structure based on regular bouquet graphs is a trace bundle fixed point with respect to the quasi-dual mapping. In this case, the zero-point structure of the zeta function exists stably on the unit circle. This circular structure is also stable with respect to the closure of the iterative structure in the trace bundle and becomes a structural fixed point with respect to the iterative expansion by the quasi-dual map(see Figure 3).

With this structural theorem, the following can be said.

It becomes possible to consider “why the zero points form a circle” geometrically, algebraically, and categorically. The “boundary between when the zero points move and when they do not" becomes visible. As a concrete example, we can connect "stability of the quasi-dual map" with "Ramanujan property of the zeta structure."

Figure 3. The relationship between Iihara Zeta and Trace Bundles.



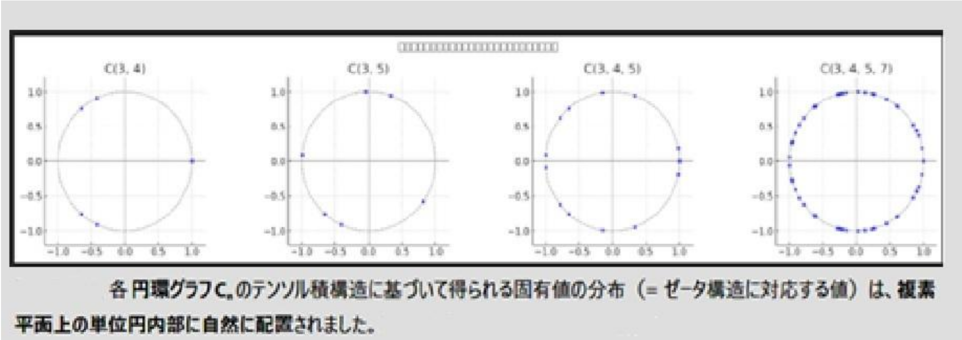
In the Ihara-type Zeta, the "trace bundle" structure is preserved.

In this divergent reconstruction, it is important to note that the reconstructed loop paths are basically noncommutative and have an ordered (directed) structure. However, at the zero point, a symmetric element (conjugate element) appears in the circularity. When it comes to the Nth root of 1, the conjugate appears slightly less than half of N. At this point, two possibilities arise in the restoration. That is, when there is a complex zero point symmetric to the real axis, a loop may have directedness in both forward and reverse directions and can be combined into a single loop. In this case, the directed edges lose their ordered structure. It is important to distinguish this from "multiple roots." In most cases, it is difficult to distinguish between the two reconstructions structurally. They can be separated or combined.

The "cyclic nature" of the zeros of the graph-theoretic Ihara zeta function of regular graphs has been clarified, and by connecting these zeta structures, it is possible to construct a zeta expansion with the same regular q-Ramanujan-like tree-type expansion, and that its trace structure also has a regular q-Ramanujan-like tree-type structure, it becomes clear that they simultaneously have "numerous, and in an extended sense, infinite, zero points constructed on the unit circle."

The actual calculation at this point is shown in the figure below.

Figure 4. Zero points of Ramanujan-type graphs.



The zero points of Ramanujan-type graphs lie on the unit circle.

The meaning of this structure will be discussed later in the “Zero Point Structure Theorem.”

4. Gamma normalization and order breaks

Now, using the Ihara zeta function related to the (regular) graph zeta function as a concrete example, we have described how a special “divergent deformation” in quasi-dual transformations constitutes a zeta structure. At this point, we notice that, metaphorically speaking, “=” appears to have a noncommutative action.

(Loop structure) = (←quasi-dual transformation→) = (tree structure)

At this point, “=” moves back and forth between the left and right sides, but each time it does so, it subtly deforms the “trace bundle” of the structure. We can see that when this deformation is a “commutative structure,” it is a “general equal.” This commutative structure, as can be seen from the previous discussion, means the same thing as “the structure of the trace bundle reaching an invariant fixed point through a quasi-dual transformation.” A quasi-dual mapping can generally be described as a non-commutative transformation that “moves back and forth between loops and trees.” The structure of the “trace bundle” is generally deformed by deformation. Fractal structures, such as the human vascular system, are both cyclic and tree-like in the peripheral vascular system. In other words, it is noteworthy that in many cases, fractal structures are “mixed structures of loops and tree structures.”

In such non-commutative transformations, the “Euler product” of the Ihara zeta function → “generating function notation that converges to rational form” appears to be unique, but in general, for example, “zeta deformation,” that is, zeta structures that have undergone divergent quasi-dual deformation, can they be restored to the structure of the Ihara zeta function as they are? In my opinion, the answer to this inverse problem is "they are likely to be multivalued." In other words, it is not considered to be uniquely reversible. However, I do not think this multi-valuedness will be an issue in this discussion.

Therefore, the divergent deformation observed in the quasi-dual "zeta deformation" of the Iihara zeta function can be regarded as the shadow of this multi-valuedness.

In this way, the "trace bundle" can be imagined as a "quantum existence" between structural deformations. It is a structure that has not yet taken shape before deformation is applied. The fact that 'loops' (waves), "trees" (particles), and class-dual transformations are fundamentally non-commutative and multivalued is somehow meaningful.

By the way, such graph zetas naturally possess what might be called "ordered prime factorization." That is, they "preserve the order in which the paths are traced." In this section, we proceed with the motivation that "if we can somehow break this order structure, we should be able to construct a Riemann zeta-like structure from the graph zeta."

And the first step in breaking this order is thought to be the gamma factor. "Ordered" means that "the order in which loops in the graph are traversed is preserved." If we can handle this, does it mean that "a graph zeta with N prime structures can be transformed into a factorial zeta"?

Here, we will take a brief look at the F_1 geometric structure through the multiplicative function b(n).

This gamma factor is b(n), which is constructed as a multiplicative function, and by acting on the graph zeta via a determinant representation, it is thought that analytic continuation to a Riemann zeta-like structure becomes possible. Note that this is also an example of how considering multiplicative functions naturally leads to F_1 geometric situations.

Now, let us consider this concretely.

First, as preparation, we introduce the multiplicative factorial function b(n) that indicates the number of overlaps.

In the graph zeta function, the order in which loops around prime structures are traversed is preserved. In other words, it is "ordered factorization." Due to the universality of the "trace bundle," there is isomorphism between "ordered prime factorization" and "ordered prime Lyndon factorization." We proceed with the discussion while noting this point first.

Counting the number of overlaps(see Figure 5),

Figure 5. Number of overlaps.

```
 1 : 1
 2 : 1
 3 : 1
 4 : 2        (2^2 → 2! = 2)
 5 : 1
 6 : 1        (2×3 → 1!×1! = 1)
 7 : 1
 8 : 6        (2^3 → 3! = 6)
 9 : 2        (3^2 → 2! = 2)
10 : 1
11 : 1
12 : 2        (2^2×3 → 2!×1! = 2)
13 : 1
14 : 1
15 : 1
16 : 24       (2^4 → 4! = 24)
17 : 1
18 : 2        (2×3^2 → 1!×2! = 2)
19 : 1
```

From the above observations,

Figure 6. Multiplication function of overlapping numbers.

$$b(n) = \prod_{p^e \| n} \Gamma(e+1)$$

it can be predicted that

This is a multiplicative function, as can be seen by taking only the terms of prime number p raised to the power of n from n, counting their overlaps as n!, and amplifying the number of overlaps of the remaining numbers. This can be understood by mathematical induction over prime numbers.

Intuitively, we might expect the generating function representation,

Figure 7. Divergence of generating function.

$$\zeta(s) + \zeta(s)^2 + \zeta(s)^3 + \cdots = \left(\sum_{k=1}^{\infty} \zeta(s)^k\right) = \frac{\zeta(s)}{1-\zeta(s)}$$

to hold, but in fact it diverges. However, here we introduce an F_1-geometric idea.

For example, let us count the number of overlaps for $2^3 = 8$.

$8 = [(1, 2, 2, 2), (2, 2, 2), (1, 2, 4), (2, 4), (4, 2), (1, 8)]\ 3! = 6$

In other words, if we count the identity element 1 as an absorbing element, the generating function of the infinite sum of the zeta function above does not diverge, and it is clear that it holds as the generating function of b(n). This idea naturally arises in the construction of the Kurokawa tensor product and in the theory of zero point structure. There, the pole of the Riemann function at S = 1 is interpreted as an "absorbing element."

Formally, it is as follows.

$4 \cdot 1 + 1 \cdot 4 = 1 \cdot 4 \quad 4 \neq 1 \cdot 4$ ($4 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 4$ does not diverge)

It may be possible to interpret this as "partial non-commutativity remaining in the unit element," but such considerations will be postponed for later.

In other words, the natural idea arises of taking the coefficient field of the multiplicative function from the F_1 geometric space, but this will be discussed later, so I will leave it at a suggestion here.

The gamma factor is important, but in fact, in this discussion, its existence is demonstrated intrinsically.

5. Quasi-duality, gamma normalization, and mapping to the critical line

Originally, I was not interested in zeta functions, or even mathematics, but rather in structures that seemed far removed from number theory, such as "noise phenomena," "trace structures," "dynamic transformation," "fractals," and "quasi-duality." However, I naturally came to discover a kind of zeta structure. I may write about this somewhere else. I was aware that I was constructing a highly abstract structure that possessed zeta-like properties.

I knew that the Ihara zeta was related to "loop structures," but I could not imagine that the structure in my mind (quasi-dual closure) could be expressed in any formulaic way, so the relationship remained unclear, and I considered it to be something like a non-commutative zeta structure.

However, when I began researching Iihara zeta, I realized that it was an example of the structure I had in mind. Furthermore, I discovered that it was possible to "transform the zeta divergence."

As I continued to transform these graph-like zeta functions, I realized that they possessed a zero configuration that satisfied the Riemann hypothesis, and that this configuration could be constructed through an explicit determinant structure.

I will refer to this structural phenomenon as the "virtual Riemann zeta structure."

It possesses a structural self-reference that mirrors the essential properties of the Riemann zeta function, and it describes the transformation process from discrete, constructive elements such as prime numbers and Lyndon elements to the zero structure on the critical line.

Regarding the property that the zero structure of the graph zeta function is arranged on the unit circle, by performing a variable transformation (u→q^-s), this zero sequence can be mapped to the critical line (Re(s)=1/2). This allows the function to be constructed without breaking the symmetry of the inversion formula of the correction term.

At this point, there exists a matrix representation (complex matrix) that applies a quasi-dual transformation to all eigenvalues of the matrix, and this operation constructs the transition from the “Ramanujan-type circle structure” of the zeta function to the “Riemann-type critical structure.”

Figure 8. Transformation of graph paths using divergent quasi-dual mappings.

伊原ゼータ関数はグラフの閉路（primitive loop）を対象にし、

$$Z(u) = \prod_{[P]} \left(1 - u^{\ell(P)}\right)^{-1}$$

と定義されます。

ここで、Möbius変換に類似した操作は：

- **各ループ P を、他のループとの合成／変形で再構成すること**
- これは、**トレース系列の長さ構造（距離）を変形する操作**

たとえば、長さ ℓ のループ列を $\frac{a\ell+b}{c\ell+d}$ のように写すなら、

- 対応する u^{ℓ} の変数が、$u^{f(\ell)}$ に変わり、
- ゼータ関数の項全体が「再定式化」されます

これは、「伊原ゼータに対する変数変換的な操作」＝Möbius写像の**離散版**に相当します。

Transformation that deforms loops in Iihara-type zeta functions using a class dual mapping = Möbius-like.

Generally, variable transformations alter the structure of loops or decomposition structures.

At this point, the question arises: Is there a variable transformation that deforms the zero points while preserving their decomposition structure, without altering the structural conditions of the graph, as in the quasi-dual transformation? For example, the Möbius transformation has a function similar to that of a quasi-dual mapping, changing a circle into a line, for instance. The quasi-dual mapping had the function of “mapping an infinite concentric circle fractal into a Cartesian spiral.” In my theory, a spiral is homeomorphic to a line. (This means that they are currently indistinguishable theoretically, though there are interpretations that distinguish them.)

At this point, the intuition arises that "perhaps the quasi-dual mapping itself functions as a quasi-dual transformation, akin to an ∞-Möbius transformation." This will be actually implemented later.

At this point, we utilize the fact that the lengths of the prime paths corresponding to the different "prime Lyndon loops" of the Ihara zeta function are identified with the prime numbers under regularity, and in the regular bouquet zeta function, we separate the zero points that overlap as the Nth roots of unity on the unit circle, as it were, for each prime number, and infinitely decompose them into a circular structure of prime powers. We then consider the operation of "quasi-dual transformation" on this infinitely decomposed concentric circle structure. I will explain this in detail later, but this picture will be useful.

At this point, I will explain the details later, but we consider the Kurokawa tensor product and the noncommutative zero structure.

From the "invariance of the trace bundle under class duality transformation" in the regular graph zeta function, if we perform a "quasi-dual divergent zeta transformation" on an "infinite bouquet graph" (a graph with infinitely many distinguishable edges attached to each node), under the regular condition, the following holds.

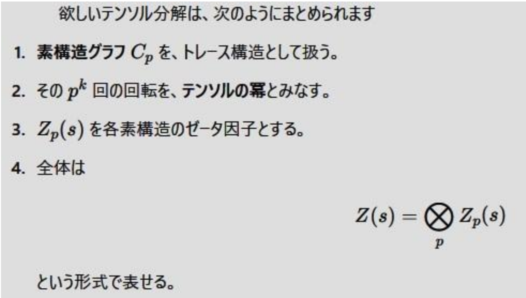
That is, the prime Lyndon factorization appearing in the "trace path formed by an infinite number of prime structures" is isomorphic to the "prime structure" (i.e., prime numbers) and the ordered combination of prime Lyndons (i.e., the decomposition of prime numbers into ordered natural numbers). This follows from the fact that the trace bundle remains unchanged in the "quasi-dual transformation," and thus the prime structure decomposition and the prime Lyndon decomposition coincide. This justifies the correspondence between "prime Lyndon loops" and "primes" mentioned earlier. Note that this does not hold in the case of non-regularity.

From this, the following can be derived.

Quasi-dual invariance of trace bundles In infinite flower bundle graphs, ordered infinite Kurokawa tensor representations are isomorphic to the structure of "trace bundles" in ordered "prime Lyndon loop structures."

Let me explain.

Figure 9. Viewing graph joins as tensor products.



欲しいテンソル分解は、次のようにまとめられます

1. **素構造グラフ** C_p を、トレース構造として扱う。
2. その p^k 回の回転を、**テンソルの冪**とみなす。
3. $Z_p(s)$ を各素構造のゼータ因子とする。
4. 全体は

$$Z(s) = \bigotimes_p Z_p(s)$$

という形式で表せる。

Using a divergent quasi-dual mapping, the Ihara zeta graph is reconstructed, allowing the tensor representation of the zeta structure to be recovered.

Consider the tensor product of such a graph structure. This tensor product can be reinterpreted as a structure in which each component is regular and connected by a single node or infinitely overlapping F_1 geometric “absorbing elements.”

And from the previous consideration, under regular conditions, the infinite Kurokawa tensor product, which connects prime Lyndon loop structures with lengths across all prime numbers, corresponds to the structure of ordered “prime factorization.”

In other words, there is an infinite series in which “quasi-dual divergent zeta extensions” are performed repeatedly in various ways on bouquet graph zeta.

At this point, an infinite graph structure can be restored, but according to the “uniqueness of prime Lyndon decomposition” (McMahon's theorem), if the path length n can be factored, the path length can be further reduced (zeta expansion) to a path that is a divisor of that factor, and ultimately, the path length will be reduced to pass through all prime numbers, i.e., 2, 3, 5, 7, etc.

This can be called “fractal restorability of prime number structures by zeta expansion.”

This remarkable property is an astonishing fact, and the fact that this zeta expands rapidly and shows a certain pattern is reminiscent of Shigenobu Kurokawa's theory of zeta conjugation by Kurokawa tensor product.

Now, this occurs because the graph is “Ramanujan regular,” i.e., “regular.” What would happen if it were ‘irregular’? Perhaps the “prime Lyndon structure” would expand further, exhibiting the structural characteristics of a certain ideal class group, and the graph zeta function would become a shadow of the Dedekind-type zeta function? Such a conjecture arises.

We will revisit this point later.

Here, the structural theorem from the previous chapter, which states that the zeros of a regular bouquet graph are arranged in a circular configuration, becomes important.

That is, for all prime numbers, the zeros are embedded within the unit circle, and the ordered structure where there are infinitely many overlaps only at the unit element is the essential structure of the Kurokawa tensor product and the ordered prime Lyndon loop structure.

As will become important later, note that due to the oddness of prime numbers, the configuration of zero points is extremely biased toward the negative complex plane. Furthermore, these configurations do not have multiplicative overlaps outside the unit element, depending on the properties of prime numbers. We will refer to this as "**anti-idealness.**" And, as it were, across an infinite number of prime numbers, it can be observed that finite fields are F_1-structural structures in which the additive structure is broken only in the overlapping part of the unit element.

This operation structure can be expressed by a transformation that rotates around the unit circle.

We consider the following quasi-dual map that transforms this rotating operation structure from a "circular structure" to a "linear structure (half-critical line)."

By considering this transformation, the zero points on the unit circle with prime Lyndon operation structures are all transformed onto the line with real part s = 1/2, while simultaneously preserving the transformation $s \to 1 - s$ satisfied by the Riemann zeta function.

Note that the "order structure" is preserved within the graph zeta structures indicating prime structures or prime Lyndon structures.

Additionally, it is important to note that the "infinite prime numbers overlap with multiple prime Lyndon zero points on the unit circle structure" has a quasi-dual transformation aspect of "transforming from an infinite circle to a line."

This graph zeta has a determinant representation by the Ihara zeta, and as shown earlier, since the class dual transformation also has characteristics, it can be seen that this transformed ordered zeta structure also has a determinant representation and a transformation structure that transfers it.

The question is whether it is possible to create a function that satisfies the structure predicted by the Riemann zeta function structure, as it were, "forgetting" this order.

In other words, let us first consider the zeta structure decomposed by the Kurokawa tensor product of infinite flower bundle graphs.

Consider the Ihara zeta function Z(u), take the logarithm in the same way, and convert it to a generating function notation. The form of the Ihara zeta function reduced to a rational system is written below(see Figure 10). Note that both of these are "combinatorial representations of all paths traced in a regular finite graph."

Figure 10. Basic transformation of the Ihara-type zeta function.

$$\log Z(u) = \sum_{[P]} \sum_{m=1}^{\infty} \frac{1}{m} u^{m\ell(P)} = \sum_{k=1}^{\infty} \frac{N_k}{k} u^k.$$

$$Z(u) = \prod_{[P]} \frac{1}{1 - u^{\ell(P)}}.$$

Here, it is clear from the "invariance of the trace bundle" that p can be either a prime Lyndon element or a prime structure. This Ihara zeta has a determinant representation in both prime structures and prime Lyndon elements, but the determinant is still arranged "anti-ideally" on the unit circle and geometrically in F_1.

As mentioned earlier, by repeatedly performing "quasi-dual zeta extensions" in various forms, l(p) reduces to a structure that passes through the prime numbers 2, 3, 5, etc. Please note this.

Here, we apply a quasi-dual transformation T(A) similar to an operation that infinitely applies a Möbius transformation that moves a circle to a line.

Then, the expansion formula of this matrix is transformed into a structure similar to the ordered Riemann zeta function.

In summary, the process is as follows.

Figure 11. Flow of quasi-dual morphism.

花束ゼータ → 類双対変換 → ガンマ正規化 → オイラー積 → リーマンゼータ

Bouquet Zeta → Quasi-dual transformation → Gamma normalization → Euler product → Riemann zeta

First, we define the quasi-dual transformation as a continuous deformation that maps a non-commutative discrete structure to a continuous space as follows.

That is, we consider the quasi-dual deformation that maps the prime power p^s = e^(-slog(p)) as an infinite concentric circle structure scaled by a prime number, and then maps it to a spiral and a straight line.

Figure 12. Explanation of quasi-dual morphism.

これを「類双対変換」と見ると：

$$\boxed{u^p \longrightarrow p^s \longrightarrow \bar{e}^{s\log p}.}$$

つまり、

- **u^p** は閉路構造（生成論的）
- **p^s** は解析的指数（積分構造）
- **e^{s　log p}** は螺旋（位相展開）

The quasi-dual morphism log P corresponds to a spiral component.

Note that this is the exact representation of the quasi-dual mapping shown in the first diagram(see Figure 12 and Figure 1), “Plotting the infinite concentric circle structure as a Cartesian spiral.” However, while the concentric circle structure consists solely of “prime circles,” the Cartesian spiral, as a scaling-invariant fractal structure, serves as an example of the quasi-dual mapping where “non-commutative fractal structures are transformed into commutative continuous fractal structures.”

To explain what is happening here, the zeros of the Ihara zeta function that were originally arranged along the unit circle are first “decomposed into an infinite concentric circle fractal at scale logp” in this process. Then, the decomposed infinite concentric circle fractal is transferred to the Riemann zeta critical line (Re = 1/2) as described in the “regular Ramanujan Ihara zeta theorem,” while retaining the “zero points” on the circle. In other words, this zeta satisfies the Riemann hypothesis-like condition.

To summarize what we have done so far, please note that we have been repeatedly performing “quasi-dual transformations.”

In other words, starting with a bouquet graph and performing a “quasi-dual divergent zeta extension,” the prime number structure that constitutes natural numbers was restored within the graph structure. Then, by performing the same “quasi-dual transformation” again, a new zeta function is obtained. In other words, the quasi-dual transformation is a mapping that transforms a fractal into another fractal, and the zeta function possesses fractal characteristics.

By this process, the non-commutative structure is transformed into a continuous structure while preserving its fractal structure.

Specifically,

Figure 13. Projection onto a straight line by quasi-dual morphism.

素数べきの位相

$$z_{p^m} = p^{im} = e^{i\,m \log p}.$$

螺旋パラメータ

$$r = a\,\theta, \quad \theta = m\,\log p. \quad \Longrightarrow \quad r_{p^m} = a\,m\,\log p.$$

臨界線への漸近

無限べきの積分極限で、
素数 log スケールが Re(s)=1/2 に線形射影される。

through this transformation, the "infinite prime Lyndon element (zero point)" expansion of the (regular) bouquet zeta function is transferred to the Euler product.

Figure 14. The Euler product transformation in quasi-duality.

類双対変換でオイラー積に移す

- 類双対変換：

$$u^p \longrightarrow p^{-s} \longrightarrow e^{-s\log p}.$$

- 花束ゼータのリンドン積は、

$$Z_{\text{花束}}(u) = \prod_P (1 - u^{\ell(P)})^{-1} \quad \longrightarrow \quad Z_{\mathrm{Euler}}(s) = \prod_p (1 - p^{-s})^{-1}.$$

- ここで、素閉路の長さ $\ell(P)$ が $\log p$ に対応するので、
無限素リンドン構造が素数構造に写る。

Euler product can be obtained by quasi-dual morphism.

At this point, note that in the case of the bouquet zeta function, the order structure remains and is preserved, so the multiplicative function b(n) for the overlap number is likely implicitly embedded. In other words, it is as follows.

Figure 15. The Gamma normalization.

ガンマ正規化

- 花束ゼータのリンドン積は、多重度が階乗的に発散する：

$$Z_{\text{花束}}(s) \sim \sum_n b(n)\, u^n.$$

- これを **ガンマ正規化**（階乗分割）で除去する：

$$Z_\Gamma(s) = \frac{Z_{\text{花束}}(s)}{\Gamma(b(n))} \quad \text{（概念的）}.$$

- これにより、重なりを除去して、
  素閉路の積が純粋なオイラー積に一致する形に整う。

By removing the "number of overlaps" in the "prime Lyndon product expansion" using this multiplicative function, it is thought that the Riemann zeta-like Euler product is generated in a constructive manner.

However, when performing the above transformation, the result reaches the Euler product.

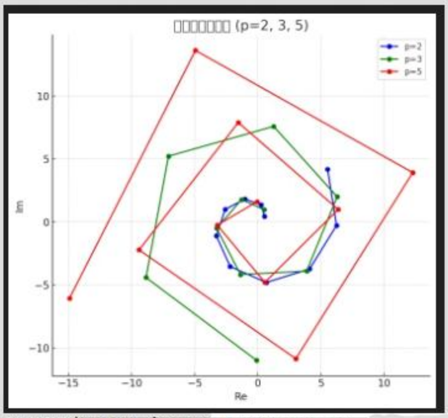
Figure 16. The Euler product of the Riemann zeta function.

$$Z_{\text{Euler}}(s) = \prod_p (1 - p^{-s})^{-1}.$$

At this point, it is thought that analytical continuation naturally occurs through continuous transformations in the quasi-dual mapping, and that the movement of zeros also takes place. Strictly speaking, this only considers the "discrete action" part, and it is unclear at this stage how the "continuous action" part occurs. However, the points on the concentric circles of the prime number scale are, in other words, the transformation from "prime Lyndon elements" to "zero points."

I have summarized this continuous plot on a computer, so please take a look at the figure.

Figure 17. Calculation of prime number spirals.



「素数の虚数べきで作る螺旋」の数値例です。

青：p=2

緑：p=3

赤：p=5

各素数について、$m = 1, 2, \ldots, 10$ の虚数べきによる螺旋をプロットしています。

$r = a \cdot m \log p, \quad \theta = m \log p.$

Prime spiral calculated by computer.

At this point, through quasi-dual transformations, the overlap of “prime Lyndon decompositions” and “zero point decompositions” demonstrates that the Riemann zeta function also possesses “prime number structures” and “higher-order prime number structures” (i.e., zero point structures), revealing an example of higher-order prime structures in commutative domains. The non-commutative overlap numbers are reduced and transferred to commutative structures.

In other words, the “gamma factor” was actually inherent in the variable transformation.

And this is precisely supported by the transition from

(Euler product) →← (Hadamard (zero point) product).

Also, note that due to the “prime nature” of prime numbers, this zero-point structure has no overlap and maintains “anti-idealness.” The concentric circles of prime numbers are prime roots of 1, so none of them overlap and are thought to have a structure similar to that of a finite field. However, in reality, the identity element forms “infinite overlap” in this infinite concentric circle, acting as an “absorbing element,” and partially breaks addition.

This is reflected in the Riemann zeta function, where there is a pole at s=1 and no zero points at -1, which can be seen as a reflection of “symmetry breaking.” As a result, it possesses what is known as an “F_1 geometric operation structure.”

Here, the non-commutative Kurokawa tensor decomposition is mapped to the commutative Kurokawa tensor decomposition (Euler product). Moreover, the construction of the “gamma normalization” b(n) when establishing this correspondence is calculated by considering the Nth power sum of the Riemann zeta function as the coefficient field of the F_1-geometric “absorbing element,” that is, through the F_1-geometric action. This is an interesting fact.

In other words,

the “Kurokawa infinite tensor product” of the bouquet graph zeta function ≅ zero-point structure (F_1 geometric)

From this isomorphism, one might imagine structural theorems such as the finiteness of rational points of elliptic functions.

This generative Riemann zeta function was analytically continued by being mapped via a scaling-invariant Cartesian spiral, but this raises two questions.

Namely, does this quasi-dual transformation and gamma normalization procedure provide a method for calculating the zeros of the Riemann zeta function from the “prime Lyndon zeros” of the bouquet zeta function, which are already computable as nth roots on the unit circle?

Furthermore, what is the “operator algebra” that this quasi-dual transformation acts upon? Intuitively, it is imagined to be something like a “fractal category” that deforms fractals into fractals while preserving fractality.

This is because there are various ways to transform a fractal into another fractal. Fundamentally, it is a combination of points, lines, circles, waves, and spirals, and whether all of these can be constructed through such combinations is an important topological problem in fractal theory.

Another question is why, through such a construction, the zero points plotted on an infinite concentric circle are transferred to the critical region of one-half(1/2).

Looking at the variable transformation known as the conjugate quasi-dual transformation, we can see that it is a mapping that transforms rotational states into lines on the complex plane. Considering this along with the symmetry of the Riemann zeta function under $s \to 1 - s$, the following is assumed:

Figure 18. Projection of zero points onto critical line by quasi-dual transformations.

類双対変換：$u^p \longrightarrow p^{2\pi s} \longrightarrow e^{s\,2\pi \log p}$　は、　位相＝螺旋＝解析接続の対称軸　を $\mathrm{Re}(s) = 1/2$ に整列させる。

- 「螺旋は位相（波数）」
- 「直線プロットは log スケール」
- 「解析接続の汎関数等式が対称軸を Re(s)=1/2 に決める」

This can be rephrased as follows: "The Riemann zeta function, which is constructed generatively, has a structure in which its zero points are shifted to the critical line, i.e., the region where Re = 1/2, through a quasi-dual transformation." This is what I mentioned earlier as the "intrinsic nature of the gamma factor."

In other words, the "inversion formula" of the gamma factor is inherent in this transformation.

With this, I believe I have provided an overview of one approach to constructing a class dual mapping for a "generative Riemann zeta-like structure."

From the regular Ramanujan graph theorem, the Riemann hypothesis, i.e., Re = 1/2, holds.

Now, the problem is that the quasi-dual mapping

Figure 19. Relationship between quasi-dual morphism and Euler products.

類双対変換でオイラー積に移す

- 類双対変換：

$$u^p \longrightarrow p^{-s} \longrightarrow e^{-s \log p}.$$

- 花束ゼータのリンドン積は、

$$Z_{\text{花束}}(u) = \prod_P (1 - u^{\ell(P)})^{-1} \quad \longrightarrow \quad Z_{\mathrm{Euler}}(s) = \prod_p (1 - p^{-s})^{-1}.$$

- ここで、素閉路の長さ $\ell(P)$ が $\log p$ に対応するので、
無限素リンドン構造が素数構造に写る。

is "discrete." And it does not clearly demonstrate proper "fractal nature(see Figure 16)."

In other words, it is necessary to incorporate a continuous structure into the quasi-dual mapping so that it maintains scaling and satisfies fractal properties through a fractal enlargement mapping.

To achieve this, we must reinterpret the concentric circle structure appearing as "prime powers" such as 2, 3, 5, 7, etc., using log scaling, which is said to be indicated by the increasing sequence of prime numbers, and consider a fractal enlargement mapping within that framework. the "fractal nature of the infinite prime number concentric circle structure and the infinite prime number spiral structure in log scaling" must be transferred to a straight line using "quasi-dual deformation."

In other words, while it has already been demonstrated that the "Euler product" can be constructed from the discrete distribution of prime power concentric circle structures, the goal is to "connect the structures between circles with a continuous structure," i.e., to analytically connect them.

In practice, if one simply uses a "Cartesian spiral," the terms collapse, rendering the transformation meaningless.

In other words, the task is to compress the "fractal structure" with its power-law expansion using logarithmic scaling and then insert a new "fractal structure" into it. By doing so, it becomes possible to continuously connect the "concentric circle structures expanding through prime powers."

By doing so, the "zero points on the prime number infinite concentric circle structure" are transferred to a "non-Cartesian logarithmic scaling spiral" rather than a Cartesian spiral, and if it is shown that they align on a straight line, that is the endpoint in the construction of my generative Riemann zeta function.

This is an example of the general construction method:

non-commutative, discrete zeta structure → commutative, continuous zeta structure

This type of quasi-dual transformation can be seen as a Möbius transformation that maps "circle → line," or more precisely, "infinite concentric circles → spiral shape → line (critical line Re = 1/2)," which is akin to an "∞-Möbius transformation."

In other words, it is a transformation that maps the infinite expansion of concentric circles, along with the surrounding space, into a continuous spiral-shaped expansion (or local space) while preserving fractal scaling. In this case, the "fixed point" would be the invariant part within the "minimal structure" of the fractal structure.

And the core is this.

Figure 20. Projection of zero points in a trace bundle onto a straight line.

$$\mathrm{Tr}(\text{類双対作用}) = \zeta(s) \quad \text{の臨界線上零点} \quad \Re(s) = \tfrac{1}{2}.$$

Translate the zero point on the circumference to the critical line (Re = 1/2) using a quasi-dual morphism. The spiral structure reflects the logarithmic length of prime cycles, showing alignment with ζ(s) zeros.

6. What is the Hilbert-Polya operator?

Let us return to the Iihara zeta function, which is a regular bouquet graph structure that restores all prime numbers constituting natural numbers.

The determinant representation of this zeta function can be constructed by calculating the non-periodic terms, prime Lyndon elements, and their connectivity, in accordance with the definition of the Ihara zeta function.

Considering this meaning, it seems that there is a kind of decay between prime numbers and the density of zero points,

Prime numbers ← log ← Zero points (density)

However, this is not a rigorous construction. The basic idea is that they are homeomorphic, meaning that there is a sense of being nested within a log-like structure. This will become important later.

Now, a quasi-dual transformation is applied to the matrix representing Ihara's zeta function.

Figure 21 . Continuation of local quasi-dual maps.

定義（類双対写像）

定義 1.4

素閉路変数 u^p に作用する**類双対写像** $\mathcal{D}$ を

$$\mathcal{D}(u^p) := e^{-s\log p} \quad (s \in \mathbb{C})$$

で与える。この作用素は全素数についてのテンソル積で拡張される：

$$\mathcal{D} := \bigotimes_{p\in\mathbb{P}} \mathcal{D}_p.$$

This quasi-dual transformation was previously defined only locally for each prime number, so we will extend it to the entire structure. This is illustrated in the figure below.

If we can apply such a tensor product to the Ihara matrix, it will become what is known as the “Hilbert-Polya” operator(see Figure 23).

When considering this, the previous idea is useful.

Figure 23. Structural diagram of continuous quasi-dual morphism.

定理（構成論的ヒルベルト・ポリヤ作用素）

定理 1.5

花束ループグラフ G のゼータ拡大を発散的類双対写像で写像するとき、
その行列式表示は次のように与えられる：

$$Z_G(u) = \prod_{[P]} (1 - u^{|P|})^{-1} \quad \xrightarrow{\mathcal{D}} \quad \zeta(s) = \prod_p (1 - p^{-s})^{-1}.$$

このとき、

- 変数写像は素閉路の次数を対数スケーリングに写す Mellin 核に一致する。
- 得られる行列式は自己随伴作用素のスペクトルを持ち、その固有値は臨界線上に配置される。

First, keep the following flow in mind.

In this way, the non-commutative order structure is controlled commutatively by the gamma factor.

Figure 23. Structural diagram of continuous quasi-dual morphism.

$$\underbrace{\text{素閉路}}_{\text{局所}} \Longrightarrow \underbrace{\text{行列ブロック}}_{\text{表現}} \Longrightarrow \underbrace{\text{入れ子複行列環}}_{\text{順序構造の統合}} \Longrightarrow \underbrace{\text{類双対変換}}_{\text{臨界線写像}}$$

In other words,

Figure 24. Block Matrix Materials.

素数ごとに構成されるブロック行列

$$M_p = \begin{pmatrix} 0 & \log p \\ \log p & 0 \end{pmatrix} \quad \text{で} \quad M = \bigoplus_p M_p$$

Note: This initial Hilbert–Pólya correspondence is structural only and will be revisited in the Appendix 1.

consider the following matrix.

First, since the diagonal components are zero, the prime number structure itself does not remain unchanged (non-triviality). The off-diagonal components are log$p$, and the logarithmic scale of prime numbers becomes the nested "jump width." Symmetry exists, and reverse paths and dual paths are treated equally (quasi-duality).

The concentric circle structure is determined by the prime number p, with the "circumference" being log$p$. Therefore, the log$p$ of the prime number can be said to correspond to the phase angle and the angular velocity of the spiral.

This block matrix can be considered as an operator that rotates the basis with a spiral structure. And, in other words, it binds all the concentric circle structures of prime numbers into one large operator space by infinite direct sum. This has the effect of breaking the order structure (the independence of prime numbers is mixed by symmetric nesting), demonstrating the intrinsic nature of the gamma factor.

As a result, the spiral structure is mapped to the critical line (preserving the geometric symmetry of points, lines, circles, and spirals in the quasi-dual transformation while maintaining the scaling invariance of the structure) (see Figure 20).

Figure 25. The Final form of continuous quasi-dual morphism.

作用素の核は

$T : u^p \mapsto e^{-s \log p}$　かつ　(全素数) p についてテンソル積

として定義できます。

1つの閉路 → Mellin型核
素閉路列 → 素リンドン分解
全素閉路 → 素リンドンテンソル積

この写像が**自己随伴性**を持ち、かつ全体で**固有値が臨界線上に並ぶ**

The kernel of the dual transformation is a Mellin-type transformation that incorporates gamma factors, transitions to elementary Lyndon structures for each elementary structure, and integrates their effects.

As a result, it transports infinite concentric circular information, along with the surrounding circumstances, into a linear form while maintaining fractal scaling.

This is the continuous version of the “quasi-dual mapping.”

In summary,

the Hilbert-Polya operator integrated by this result is

Figure 26. Incorporate into the determinant representation of the Iihara-type zeta function.

行列式構造

伊原ゼータの行列式表示と同じ：

$$Z(u) = \det(I - uA + u^2Q)^{-1}$$

ここで：

- A = アジャセンシー行列（経路情報）
- Q = 次数補正行列（自己ループ補正）

これに **ブロック行列** M を組み込むと：

$$\det(I - uMA + u^2MQ)$$

みたいな構造が自然に現れる。

Thus, here too, nested structures emerge, and it can be observed that fractal structures dominate the entire system through scaling. In short,

Figure 27. Constructive Hilbert-Polya operator.

補題（構成的ヒルベルト・ポリヤ作用素）

各素数pに対して、複行列

$$M_p = \begin{pmatrix} 0 & \log p \\ \log p & 0 \end{pmatrix}$$

を考える。これらは自己共役である。

全素数にわたる無限テンソル積

$$\mathcal{M} = \bigotimes_{p \in \mathbb{P}} M_p$$

は、ヒルベルト空間 $\mathcal{H} = \bigotimes_p \mathbb{C}^2$ 上の自己共役作用素である。

このとき、$\mathcal{M}$ のスペクトル構造は、素数構造と log p によるスケーリングを通じて、リーマンゼータ関数のゼロ点の対称構造と一致する。

特に、素リンドン経路の類双対変換（＝logスケールによる指数変換）を通じて、
同心円的配置のゼロ点が臨界線に写像されることにより、構成論的ヒルベルト・ポリヤの作用素として解釈される。

Hilbert-Polya action (D).

explaining each step, this "Hilbert-Polya operator" controls the "non-commutative prime number (i.e., prime structure) structure (zero-point structure, i.e., prime Lyndon)" created by graph-theoretic structures locally via "gamma factors," the "Möbius transformation" that maps circles to lines is bundled for each prime number, forming an infinite Möbius strip, and instantly maps the infinite concentric circle fractal, along with its surrounding space, to a line.

Furthermore, it possesses the structural characteristic of an "infinite matrix" with an infinite nested structure, and its contents are self-adjoint, maintaining self-adjointness on the infinite Hilbert space where they are infinitely connected. These requirements are underpinned by a "simple analytical" operation that involves repeatedly receiving various elements derived from the deformation of the "trace bundle" via the "quasi-dual mapping" and then deforming them again as "quasi-dual mappings."

A matrix is, in a sense, a nested matrix that inherently contains various operational patterns. While this will be briefly touched upon in another chapter, here it manifests itself as a conjugate infinite Möbius structure, specifically in the form of a prime spiral nested structure.

As can be seen from the previous discussion, the crucial operation is the "quasi-dual map." The impetus for this analysis was the discovery of the "quasi-dual divergent zeta extension" via the prime Lyndon path, which is a non-trivial restoration path. However, the revelation that the structure of prime numbers actually possesses holographic fractal restoration properties was surprising.

However, the "quasi-dual map" is highly multi-valued and intuitive, and I have not yet fully grasped its precise, overarching role. I have touched on this briefly, and in fact, it has been an inherent problem all along. For example, the question of whether to restore a line or a spiral from a trace bundle is actually a very difficult problem. For instance, can a person walking on an extremely gentle spiral staircase be distinguished from a person walking on a straight path in terms of their intrinsic state, that is, their subjective state? This restoration problem involves concepts such as the "categorization" of the trace path itself (i.e., layers? meaning the retention of local information) or the difference between infinite and finite immanence, but while there are partial ideas, a complete solution is still far off.

The concept of the "trace category" will undoubtedly become important when the trace path becomes "non-regular," and it is already known that this will generate another restoration path.

In the continuous "quasi-dual map" introduced in this chapter, there are "prime-type spirals," "prime-type infinite concentric circles," and "critical lines," and since these are all controlled simultaneously in this case, there is no problem, but there will be situations where problems arise.

In fact, an infinite Möbius structure was necessary to "wind the spiral." Is there nothing simpler? Everyone must think so.

The quasi-dual map, which I call the "theory of dynamic transformation," seems to be a world that has only just begun, in my understanding.

7. Continuation of fractal deformation of dual mapping

How many such deformations are there?

What are the methods of construction?

I will not delve into such issues in this essay, but in general, it can be assumed that there are many. For example, consider the dynamic transformation I often think about (which can also be described as observing noise), in which "the Cartesian spiral disintegrates into point fractals in each local state." This is also a type of "continuous transformation." And this "continuous transformation" will be mapped into an infinite concentric circle fractal via a "continuous quasi-dual transformation." The fact that there are likely to be many such decomposition methods can be imagined from this example. Furthermore, when the "scaling infinite structure" of the Cartesian spiral—that is, the fractal structure that remains unchanged no matter how much it is enlarged－is transferred into the infinite concentric circle structure, it becomes restricted to a "self-similar mapping" based on the size of the concentric circles and undergoes changes. In other words, while the fractal structure is preserved, the structure itself changes. The problem of Quasi-dual transformations also includes the issue of changes in the structure's information and relationships accompanying structural deformation. It goes without saying that this reveals the fundamental problem of "non-commutative deformation."

In this chapter, we will examine an example of how adding periodic waves to a spiral-shaped fractal is also quasi-dual.

Let us organize the problem.

First, through graph zeta-based generation theory based on prime number structures (trace bundles, prime Lyndon sequences, and fractal doubling maps), we constructed a Riemann zeta-like zeta structure with commutative Euler products. However, the "quasi-dual transformation" we performed holds in the discrete domain of infinite prime-powered concentric circles, so unless we "continuously extend" it, we end up in a situation where we cannot determine how various structures are ultimately mapped onto a "line" via the quasi-dual transformation. It might not even be a "line." In that case, what I constructed would result in something that differs from the known structural characteristics of the Riemann zeta function.

Therefore, I first considered constructing it as a continuous mapping corresponding to the log scale, and the idea arose to introduce a continuous topology that satisfies the fractal scaling transformation based on log scaling, corresponding to the discrete spread of prime number concentric circles.

By doing so, we can consider introducing a new scaling structure (log scaling) that satisfies the fractal nature by stretching and contracting the “concentric circle fractal with power-law expansion and Cartesian spiral structure” through logarithmic scaling. It is well known that the scaling where prime numbers appear is log scaling.

First, the basic log scaling,

Figure 28. Transform prime powers logarithmically.

- $\ell(P)$：巡回構造 P の「長さ」や「次数」
- p：それに対応する素数（写像によって対応）
- $\log p$：その対数スケール

つまり、

$$\ell(P) \quad \longmapsto \quad p \quad \longmapsto \quad \log p$$

and then extend this relationship to

Figure 28. Transform prime powers logarithmically.

素数からログへの写像（離散 → 連続）

次に、この素数列を連続変数へと写す「スケール変換」が：

$$p_n \quad \mapsto \quad \log p_n$$

これは自然対数写像（連続）です。

by doing so, we consider incorporating continuous log scaling into the discrete structure of prime power concentric circles. We then add an integral expression as follows, incorporating logarithmic spiral structures into the expansion of prime power concentric circle structures, and gradually consider “logarithmic scaling-based quasi-dual transformations” as continuous structures.

Figure 29. Successive transformation per prime.

$$Z[f] = \sum_{P\in\mathcal{P}} f(\log p_{\ell(P)})$$

や、連続化して：

$$Z[f] = \int_0^\infty f(\log p(x))\,dx \quad \text{with} \quad p(x) = \text{n番目の素数}$$

この「$p(x)$ の連続化」にはチュビンガーの素数近似式 $p_n \sim n\log n$ を使えば、

$$Z[f] \approx \int_0^\infty f(\log(n\log n))\,dn$$

のようにも変形できます。

In this way, by transforming the discrete graph traversal structure (Lyndon sequence) into a continuous log-scale space, the logarithmic spiral structure is naturally generated.

Functions that constitute such continuity connecting discrete structures are called functional extension, and a well-known example is the factorial function N! (equivalent to the gamma function) with introduced continuity.

However, simply applying log scaling to the Cartesian spiral and aligning it with the expansiveness of prime numbers is insufficient; it was found that a “correction term” must be added alongside it. This correction term is a vibrating term resembling a wave oscillating up and down. By adding such a correction term and deforming the log-scaled Cartesian spiral, we can refer to it as a “non-Cartesian spiral.”

Conversely, the fact that “adding wave terms to a spiral shape preserves fractal self-similarity” is considered to be one aspect of the multi-valued nature of the essential “quasi-dual mapping” and “quasi-dual transformation.” Simply put, visually, imagine growing various capillary-like fractal trees along the Cartesian spiral.

Summarizing the above considerations, it can be summarized as follows.

The fractal-like expanding structure is deformed by the wave correction while maintaining self-similarity.

Figure 30. Correct by wave motion.

波動補正入り積分核：

$$K(\theta,p)=e^{-(b+i)\theta}\,\Phi(\theta,p).$$

拡大構造の本体：

$$e^{-(b+i)\theta}\quad \text{（これが log スケールでの拡大核）.}$$

波動補正：

$$\Phi(\theta,p)=1+\text{（小さな補正項）.}$$

この形から分かるように、拡大核（フラクタル構造）自体は指数項だけで支配されている。

When considering what form the correction term should take, for example, the Riemann–Siegel formula, which is famous for Riemann's calculation of the zeros of the Riemann zeta function as a complex analytic function, was devised by Riemann to approximate the zeta function near the critical line and later systematized by Siegel. Using this(see Figure 31),

ζ (s) is roughly

Figure 31. The Shape of wave correction.

臨界線 $\Re(s)=1/2$ での $\zeta(s)$ は大まかに：

$$\zeta\Big(\frac{1}{2}+it\Big)=2\sum_{n=1}^{N}n^{-1/2}\cos\big(t\log n-\theta(t)\big)+R(N,t).$$

ここで：

- $\theta(t)$ はガンマ関数由来の位相補正項
  $\theta(t)=\arg\Gamma\Big(\frac{it}{2}+\frac{1}{4}\Big)-\frac{t}{2}\log\pi.$

and in this case, the correction term we are looking for is

Figure 32. Correction term on the critical line of the Riemann zeta function.

螺旋核：

$$K(\theta,p)=e^{-(b+i)\theta},\quad \theta=\log p.$$

に対して、

リーマン–ジーゲル型の波動補正：

$$\Phi(\theta,t)=e^{\,i[\,t\,\theta-\theta(t)\,]}.$$

を掛け合わせる。

Note that the wave correction behaves wave-like in log scaling and does not destroy fractality.

Substituting this,

Figure 32. Correction term on the critical line of the Riemann zeta function.

$$Z(t) = \sum_p K(\log p, p, t) = \sum_p e^{-b \log p} \cdot e^{i[(t-1)\log p - \theta(t)]}.$$

$$Z(t) = e^{-i\theta(t)} \sum_p p^{-b} \cdot p^{-i(t-1)}.$$

$$\boxed{Z(t) = e^{-i\theta(t)} \sum_p p^{-b} p^{-i(t-1)} = e^{-i\theta(t)} \sum_p p^{-(b+i(t-1))}.}$$

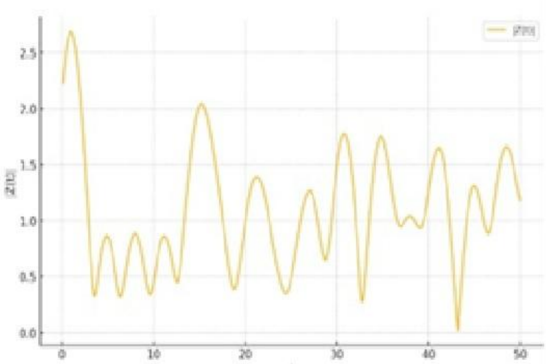
we obtain the desired equation.

This wave-corrected spiral kernel sum, when b = 1/2, gives wave-like oscillations along prime numbers and includes the gamma phase correction of analytic continuation, so it has a structure where Iharascillates linearly along the critical line Re = 1/2 due to the wave term(**Figure 31** illustrates the wave-like correction layers of dual Lyndon structures, stacked fractally along the complex Möbius spiral).

In the previous chapter, there was a “Hilbert–Polya operator” that acts as an operator controlling such a structure, and it shifted the zero point to the critical line. It functions like a wave operator. Note that the critical line carries the “prime Lyndon element” (i.e., zero point) brought from the graph zeta.

When this is calculated by a computer,

Figure 34. Behavior of zero points calculated using finite prime numbers.



小さな素数列（2, 3, 5, 7, 11, 13）で計算した
「螺旋核＋リーマン-ジーゲル波動補正」の
$|Z(t)|$ の挙動

it becomes clear that the real axis Re = 1/2 is already oscillating from the initial stage. The multi-valued nature of the continuous (functional) form of the quasi-dual mapping, aligned with the prime number (log) scale, has gradually progressed. This indicates that the "quasi-dual mapping," which geometrically continuousizes the prime number structure, can be explicitly expressed as an integral kernel, and that when P is taken to infinity, it converges at the critical line.

And although only an approximate formula has been derived so far, the following is already clear.

Define an enlargement mapping (spiral mapping) that preserves fractality in accordance with prime concentric circles on a logarithmic scale. This constitutes the continuous form of the quasi-dual mapping.

While this method has not yet yielded a clear explicit expression, it establishes the commutative, multivalued quasi-dual mapping as the kernel of a divergent zeta structure, and incorporates its multivalued nature into a prime Lyndon structure (infinite prime cycles), and further continuous it using a fractal enlargement operator on the log scale, we were able to approximately construct a fractal functional extension that maps the spiral (concentric circles) to the critical line.

At this point, based on "anti-ideality," or the fact that there is no overlap at the zero point, the "zero point" transferred to the spiral straight line moves back and forth along the critical line on the wave. From a structural perspective rather than a generative perspective, the structure of the zero point can be considered to have the symmetry s→1-s (Riemann's formula). In other words, it becomes clear within the wave nature that the "zero-point structure transported from the infinite bouquet zeta" will not be arranged in such a symmetrical form. Of course, if the operator is in an undetermined state, it may exhibit abnormal behavior, but considering the situation in the previous chapter, it is understood that such a thing will not occur at the generative construction stage.

I believe the objective of explaining how the zero-point structure is generated based on the elementary structure through the self-similarity of divergent quasi-dual transformations has been achieved.

8. Functional of the basic quasi-dual map and generative construction of deep Riemann structures

The subject of my essay was whether it would be possible to construct a Riemann zeta-like structure by creating a divergent zeta structure using my own class dual map, a noncommutative, multivalued operation, such that its multivalued nature naturally manifests as prime structures and higher prime structures (zero point structures). Through this effect, I sought to concretize a “theory of dynamic transformation” that discusses noise phenomena.

This generative task has, for the time being, been formally achieved up to the previous chapter... However, when defining an enlargement map that preserves fractal properties in the logarithmic scale aligned with prime number concentric circles, the extension of the continuity of the quasi-dual map has reached a stage where it can only be described as “there are various possibilities.”

This chapter will further discuss this point.

As can be seen from my analysis, the “quasi-dual mapping” that appears everywhere is what connects loop structures and tree structures via a “trace bundle.” In the previous chapter, I attempted to “continuously analyze” it as a functional, so to speak, and had to reconstruct the troublesome “fractal structure” within the transformation using log scaling.

However, those who read this must have naturally wondered, “What about the continuous analysis, or functionalization, of the ordinary form of the quasi-dual map, i.e., the simple power-law concentric circle fractal and spiral fractal?” At least, I did. I had never considered this possibility until I drew it.

However, this problem leads to a new method for constructing generative zeta structures.

Let's summarize.

The zeta construction method I have shown has the following generative principle: “When a prime structure exists, deformation introduces multi-valuedness, and within that, the construction of a divergent zeta structure naturally leads to a zeta structure.” Let's apply this to a simple concentric circle fractal, for example.

This approach may provide an alternative solution to Hilbert-Polya’s operator. While it may not be a proper realization of the conjecture, this issue was discussed in Chapter 6.

To summarize again, in the previous chapter, we used the wave term of the Riemann-Ziegler function, applied logarithmic scaling, and introduced fractal topology into the prime power concentric circle structure. However, upon further reflection, since the quasi-dual transformation between the power-law infinite concentric circle structure and the Cartesian spiral is also discrete, it seems possible to continuousize it using a “functional extension.” I couldn't help but realize that I was unaware of such an obvious perspective and had been tackling a more complicated problem.

Now, setting aside the log analysis of prime scales for the time being, I want to consider a functional that continuously connects the "infinite scale-free mapping of the unit circle to form concentric circle fractals and the Cartesian spiral" in a more pure manner. How should this be done?

First, we organize the structure using a power-of-two concentric circle fractal structure.

Figure 35. Quasi-dual morphism from a single Euler product.

等倍同心円の構造

- 半径 $r_k = 2^k$ （2倍拡大）
- 中心は同じで、円環的に配置
- $k \in \mathbb{Z}_{\geq 0}$

このとき、「無限同心円フラクタル」は、

$$S = \bigcup^{\infty} C_{2^k} \quad \text{where} \quad C_{2^k} = \{(r,\theta) \mid r = 2^k,\ \theta \in [0, 2\pi)\}$$

デカルト螺旋

螺旋は：

$$r(\theta) = a\,e^{b\theta}.$$

ここでは、**等倍同心円構造と対応させるには**

螺旋がちょうど「回転角度」で倍々拡大する形にしたい。

つまり、

円周の位相 と **螺旋の角度** が一致するとき、

螺旋の半径が 2^k に一致するようにする。

$$r(\theta) = a\,e^{b\theta} = 2^k.$$

したがって

$$b\theta = k\ln 2 - \ln a. \quad \Longrightarrow \quad \theta = \frac{k\ln 2 - \ln a}{b}$$

Now that we have determined the discrete structure, we will scale it continuously.

Figure 36. Nucleus of a local quasi-dual map.

$$Z[f] = \int_0^{\infty} f\big(r(u),\, \theta(u)\big)\, du.$$

$$\boxed{Z[f] = \int_0^{\infty} f\left(2^u,\ \theta(u) = \frac{u\ln 2 - \ln a}{b}\right) du.}$$

This can be considered the desired general functional extension.

Now, the question arises: can we express this in a more explicit form, including the wave term? As we encountered with prime-number concentric circles, there are many patterns in "function systems that satisfy fractal scaling."

Such "equal-scale infinite concentric circles × spiral × wave correction" functionals are thought to be able to form countless patterns depending on how the wave term is incorporated.

Now, based on the current "prime-free, simple equal-scale structure" form, let us combine several more explicit representative patterns that include the Riemann-Siegel-like wave term considered during log scaling.

Figure 36. Nucleus of a local quasi-dual map.

等倍 × 螺旋 × 波動補正の汎関数

もっとも汎用的な形は：

$$\boxed{Z = \int_0^{\infty} \exp\left[\, i\left(k\,\theta(u) + \omega\, u\, \ln 2 + \Phi(u)\right)\right] du, \quad \theta(u) = \frac{u \ln 2 - \ln a}{b}.}$$

ここで：

- k：螺旋の波動数
- ω：等倍スケールの対数波動数
- $\Phi(u)$：追加の波動補正（Riemann–Siegel 相や他の共鳴項）

Each combines discrete power-of-two concentric circle fractals, adjusted Cartesian spirals, and Riemann-Siegel-type wave terms that adjust them wave-like.

What exactly is the effect of this "quasi-dual transformation generic function"?

This is precisely the core role of the homotopic mapping: "mapping the (topological, closed-loop) state on the concentric circles to a linear critical region through a continuous mapping."

In other words, the original topological space is an infinite number of concentric circles (generated by a quasi-dual mapping and forming a fractal structure). The Cartesian spiral unfolds this topological state into a straight line using an angular parameter. Wave corrections (such as the Siegel term) are responsible for "refining the mapping and aligning conditions," striving to maintain this linear state.

Ultimately, the images converge in the critical region (e.g., Re(s) = 1/2).

At this point, recall that the basic discrete quasi-dual transformation considered in the previous chapter converted the concentric circle structure into an Euler product. In this case, the Euler product is (1-2^-s)^-1, which is the Euler product of the prime number 2. In other words, in this case, the "zero points" collected in this critical region are plotted from the circumference onto the critical line.

This brings to mind the core of Professor Kurokawa's “Deep Riemann Hypothesis,” which involves “extracting each prime number from the Euler product and examining whether it corresponds to the critical conditions of the Riemann Hypothesis on its own.” In other words, the process we just performed—the 2-power concentric circle fractal × spiral nucleus × wave correction—maps only the p=2 portion of the Euler product onto the critical line, demonstrating this as a functional.

Note that this is an operator called a “quasi-dual transformation.”

In other words, a “quasi-dual transformation” is a non-commutative transformation, and in this case, it maps points on the concentric circles (including the zero point) of the concentric circle fractal to the spiral fractal and plots them on the critical line. Note that in this case, through the divergent zeta extension of Ihara's zeta function and the bouquet zeta function, a correspondence with the commutative prime sequence (i.e., ordinary primes when the order is ignored) appears via the prime Lyndon element. And in the case of 2, the square root of 1 was the zero point. This zero point is thought to disappear from the critical line due to asymmetry and form a pole at s=1. This pole acted as an “absorbing element” in the infinite tensor product, where infinite zero points were superimposed.

In principle, the radius scaling of concentric circles is exponential because it is a power function.

The spiral mapping linearly solves the exponential structure using the angle parameter.

Figure 37. Continuous scaling.

$$r(\theta) = a\,e^{b\theta} \qquad \Longrightarrow \qquad \theta = \tfrac{1}{b}\ln(r/a).$$

Furthermore, the power-law concentric circle structure is

Figure 38. Local continuous scaling.

$$r_k = 2^k. \qquad \Longrightarrow \qquad \theta_k = \frac{k\ \ln 2 - \ln a}{b}.$$

discrete, so θk is a discrete sequence, but when continuous, $k \mapsto u$ causes the infinite concentric circle structure to fall onto a smooth $\theta(u)$ axis. As seen in the previous chapter, this is approximated wave-like and gradually falls onto a straight line.

Where this line ends up depends on the zeta structure where this “quasi-dual transformation" acts, and since this zeta structure has mirror symmetry with respect to s→1-s, it converges to Re=1/2.

In other words, by reorganizing the discrete action of the quasi-dual mapping into a continuous quasi-dual mapping via a topological infinite concentric circle structure (non-commutative, multi-valued) and a spiral, and combining it with critical line alignment via wave correction, the infinite overlapping structure of discrete closed loops is mapped to a linear region under a continuous operator, forming a "generative quasi-dual mapping."

As can be seen by substituting numerical values, this is composed of prime numbers such as 3, 5, 7, etc., and from these, the Euler product is constructed through the quasi-dual transformation. In other words, by combining all prime numbers, the Euler product of the Riemann zeta function is constructed.

This can be described as a generative "deep Riemann hypothesis," which states that if the local closed-loop structure of each prime number is spiraled by the quasi-dual mapping and guaranteed to be mapped to the critical line by wave correction, then their product (i.e., the entire zeta) should have zeros aligned on the critical line.

In other words, the form of the Hilbert-Polya operator can be approximately written as follows.

Figure 39. Bundle all prime numbers.

積分核で束ねれば、

$$\prod Z_p[f] \quad \text{with} \quad Z_p[f] = \int^{\infty} \exp\left[\, i\left(k\,\theta_p(u) + \Phi_p(u)\right)\right] du.$$

This is thought to be the operator of the Kurokawa tensor product, which tensor-bundles the functions for each prime number. This operation approximately bundles the infinite prime power circles onto the critical line. At the same time, this is considered to be equivalent to the "class dual transformation" described in the previous chapter, which is log-scaled.

Generatively, this means that if the local structure of each prime number falls on the critical line, the entire product is regularly aligned on the critical line. This can certainly be considered the framework of a generative proof in the form of the deep Riemann hypothesis.

The idea is to support Kurokawa Shigenobu's statement that "the Riemann hypothesis naturally follows from the deep Riemann hypothesis" from the perspective of generative theory.

Furthermore, it becomes clear that the "continuous, functional-type quasi-dual transformation," which seems to have an action akin to the ∞-Möbius transformation, is precisely the desired operator form, and that it possesses a tensor-like structure determined by the structure of each prime number. One might imagine this as an action that stacks infinity upon infinity.

The matrix form of the Hilbert-Polya operator is also such.

Let us summarize.

First, we replaced the role of a single linear fractional transformation called the Möbius transformation with a functional called a "quasi-dual mapping" for each infinite prime number.

This means that "the local zeta structure of each prime number itself is generated by a topological operation called a quasi-dual mapping."

In addition, I think the following can be said.

1. Each prime number independently possesses a "local spiral kernel."

$Zp[f] = \int \exp[i(k\theta p(u) + \Phi p(u))]du$. (The integral is from 0 to ∞.)

Here, the scale unique to the prime number p is woven in.

2. The product that binds them together coincides with the Kurokawa tensor product.

$Z = \otimes pZ\ [f]$. (p passes through all prime numbers)

This is not a simple commutative product, but rather a local structure that acts in an operator-like manner.

3. It incorporates multi-valuedness (quasi-duality), which could not be achieved by the "single linear fractional operation" of the Möbius transformation,

into the functional integral.

It is undoubtedly true that the observation methods of concentric circle fractals and Cartesian spiral fractals are the root cause of the generative zeta structure deformation. Additionally, the deformation characteristic that concentric circles have a doubling map while Cartesian spirals are infinite in scale is also important.

9. The Concept of Multiple Matrix Rings and the Elementary Lyndon Decomposition of Matrix Representations in Ihara Zeta Functions: A Generative Approach to Deep Riemannian Structures

I was originally exploring objects that change and evolve with mobility, such as noise phenomena, as noncommutative mobility.

In the process, I encountered graph structures, particularly hypergraph structures. This hypergraph is a generalization of graph structures, where the correspondence between "points (nodes)" and "lines (edges)" is not fixed, but varies locally, with 'N' and "M" (where N and M are different) points and lines. In other words, there are many points within a line, and many lines within a point. This is called non-regular, and it became necessary to develop a theory of the correspondence between variables of different numbers.

Part of the solution to this problem is the concept of "motifs," which involves applying dual operations to hyperedges, transforming them, and compressing them into "structured sets." This allows the multiplicity and multi-valued nature of structural transformations to be "pseudo-uniquely" defined, much like ideal theory or Riemann surface theory. It can also be proven that this forms a "closed structure" (i.e., motif closure).

As a slight digression, when using non-commutative algebras as "trace material," the theory of "motif closures" including "directed edges" will be necessary. The path is cut off, or suddenly divided. I do not yet fully understand this problem.
Please note that this essay focuses on the structure within a highly ordered situation, "regularity." This is why the invariance of the "trace bundle" could be used.

In this process, I encountered operations involving (2×2) matrices and (3×3) matrices. These are non-regular non-commutative operations with different degrees. When considering this(see Figure 40),

I noticed that within two (3×3) matrices, there are three (2×2) matrices, and when considering matrix operations, this can be extended to calculate (6×6) matrices in a nested manner. This can be called "nested" matrix multiplication. What is important is the abundance of "equivalence classes" in the case of infinite nesting. Contraction is also possible.

Even if it is not regular, any pattern of "$A \times B$" and "$C \times D$" can be applied, and the degree is easy to understand.

There is also a "combination operation."

For example, suppose there are a (2×2) matrix and a (4×4) matrix. Take out two columns from the latter. There are six possibilities.

Then, we take two rows from the four rows in those two columns. This also gives us six possibilities.

As a result, there are 36 possible operations between (2×2) and (2×2), which yields a (12×12) matrix. Although multiplicity is not an issue due to symmetry, this is a semigroup, and there may be cases where the "dimension" does not exist, resulting in a region where inverses cannot be found.

In the reverse case, you can simply divide the acting side. If the length of the rows is insufficient, be careful to calculate by considering permutations and combinations from the reverse side.

Figure 40. Examples of calculations using Multiple Matrix.

① (2,2) 行列 M

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

② (3,3) 行列 N

$$N = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

複行列の定義

複行列 $M \boxtimes N$ を、

> 「M の各成分 m_{ij} を係数として、全体をスケールした N を 4 ブロックに配置」

とする。すると、$M \boxtimes N \in \mathbb{C}^{6\times 6}$：

$$M \boxtimes N = \begin{bmatrix} 1\cdot N & 2\cdot N \\ 3\cdot N & 4\cdot N \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 2 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 & 4 & 0 \\ 3 & 0 & 3 & 4 & 0 & 4 \\ 0 & 3 & 0 & 0 & 4 & 0 \end{bmatrix}$$

Examples of Multiple Matrix calculations.

I named this the "multiple matrix ring."

When the number of operations is insufficient, you can simply expand it by asking, "How many can fit?" and extend the number of characters in the matrix. By generalizing this, you can define operations for "N × M" matrices and "L × O" matrices. This is also a situation that naturally arises when acting on matrices as vectors.

Note that this nested structure is "fractal-like."

The following prediction can be made.

The "motif closure" structure of hypergraphs is embedded "closure-wise" into the operation structure of the matrix ring.

Now, let us consider Iihara's "divergent zeta expansion."

A (2,2) regular graph becomes a circular loop graph. This has "two prime structures," but since there are forward and reverse directions, two conjugates appear in the determinant.

The minimal form of a loop graph with "one prime structure" is a structure with one (point) node and an indistinguishable self-loop, which Shigenobu Kurokawa considered to be related to F1 geometry. If we consider this to be distinguishable, then there exists a "quasi-dual divergent zeta extension" for all (2,2) regular graph structures of prime length, meaning that class-dually, "circular graphs of prime length are, so to speak, quasi-dual." In this case, "what about natural numbers? They are prime Lyndon decomposable." Now, how does this prime Lyndon decomposition occur?

That is, "every circular graph with a prime number structure has a quasi-dual divergent zeta extension to every graph with a natural number structure." In other words, every circular graph with a prime number length is homeomorphic to a circular graph with a natural number structure that combines prime number structures. This is the holographic fractal restorability of prime numbers.

This is equivalent to saying that if you have one Euler product, you can reconstruct all Euler products across all natural numbers, demonstrating the fractal and holographic nature of natural number structures.

It is understood that this corresponds to the prime number combination of the Euler product, which means that the local information of prime numbers acts as the nucleus for generating the overall structure. In other words, it can be expressed as "the Euler product is not closed locally, but it is closed within the matrix ring."
Some may also note that the matrix itself inherently contains a potential natural number structure.

From the correspondence between decomposition in "multiple matrix rings" and "prime Lyndon decomposition in the determinant of Ihara's zeta," a situation arises in which "only the prime number concentric circle structure needs to be considered" when constructing the analytic zeta from the graph zeta.

Now, when considering the structure of zeros, the operation of constructing the analytic zeta function from the graph zeta function resembles the process of creating a structure by tensorially combining an infinite number of (2,2) matrix elements.

One could also consider an operation where an infinite nesting is initially expanded infinitely in a certain "dimension," maintained in a self-similar form, and then a state where "something cannot be contracted" is considered.

Such a matrix ring corresponds to the "∞-Mobius transformation" in the sense that the "quasi-dual transformation" considered in the previous chapter, which transfers the correspondence between concentric circle fractals and spiral fractals to a line, has an action similar to that of the "∞-Mobius transformation." In other words, the nested structure of the matrix action corresponds to the quasi-dual "zeta construction," i.e., the "analytic connection." Note that this can be expressed as an operator.

At this point, note that each "regular complex matrix ring" of prime dimension has a nested structure and possesses an infinite-dimensional matrix operation structure. This nested structure corresponds to the "fractal nature" in the quasi-dual transformation.

In other words, it is an expression of the "∞-Möbius transformation" property in quasi-dual transformations.

Furthermore, its expansion as an operator is thought to exist as an approximate limIharaf the structure created by combining the quasi-dual functional structure analyzed in the previous chapter, namely "equivalence × spirality × wave nature."

Note that the analytical expansion of this determinant is the Euler product, which is an analytical expression of the so-called deep Riemann hypothesis, where one Euler product contains all the information of all Euler products, and the zero point of one Euler product is determined by the relationship with Euler products across all prime numbers.

This possesses an essential "non-commutativity" and further an essential "non-regularity." Non-commutativity means "cannot be rearranged," and non-regularity means "multiple objects of different numbers are combined." This suggests that essential non-commutativity and non-regularity are potentially contained within commutative and analytical objects such as the Riemann zeta structure.

10. Considerations on what appears to be the Hilbert-Polya operator expressed as a matrix

Here, when we observe the Hilbert-Polya operator expressed as a tensor product across multiple prime numbers, we find that it has remarkable characteristics.

To summarize the key points, even a single Euler product has a structure that can be extended to "all prime numbers" via a quasi-dual divergent zeta extension, which is the tensor product structure of a matrix ring, taking the form of the local structure of one prime number multiplied by the infinite product of all prime numbers.

The infinite product is concretized as an "infinite nested Möbius structure" in the operator.

This is connected to the "self-adjointness of the matrix ring" and the "closure property of determinants (infinite determinants)."

From this, we can conclude that the structure of the Euler product itself expresses the symmetry and entanglement that give rise to the conditions for the arrangement of zero points (the balance between divergence and cancellation).

As a result, "Re(s) = 1/2 is naturally aligned."

To arrive at this conclusion, we will now reconsider the Riemann zeta function.

Figure 41. Imaginary component of the Riemann zeta function.

$$Z(t) = \sum_{p \in \text{Primes}} p^{-s} = \sum_{p} p^{-(\sigma+it)}$$

We will transform this already analytically continued Riemann zeta function using logarithmic scaling.

Figure 42. Logarithmic scaling.

$$Z(t) = \sum_{p} e^{-s \log p} = \sum_{p} e^{-(\sigma + it) \log p}$$

We re-evaluate this in integral form.

Figure 43. Evaluated in integral form.

$$Z(t) = \int e^{-it \log p} \cdot e^{-\sigma \log p} \, d\pi(p) \quad x = \log p.$$

We regard this as the kernel of the "Hilbert-Polya operator,"

Figure 44. Extracting Hilbert-Polya kernels.

$$Z(t) = \hat{\mu}_\sigma(t), \quad \mu_\sigma(x) = e^{-\sigma x} d\pi(x) \quad (\text{素数分布}).$$

and consider the change in the value of σ.

Figure 45. The physical behavior at Re = 1/2 is most stable.

フーリエ波動と指数減衰の干渉が最大化する。

ここが本質で：

$$e^{-\sigma \log p} = p^{-\sigma} \quad \text{と} \quad e^{-it \log p} = p^{-it} \quad \Longrightarrow \quad p^{-s} = p^{-\sigma} p^{-it}$$

- $\sigma = 0$ だと発散する。
- $\sigma = 1$ だと指数的に消えすぎる。
- $\sigma = 1/2$ が最も「エネルギー密度が残りつつ、波動が干渉する臨界点」。

1/2 The physical behavior of the critical line is most stable.

This is the summarized diagram.

Let us summarize the key points.

Using logarithmic scaling of prime numbers, the irregularity of the prime number sequence is mapped onto a "continuous kernel" (Hilbert-Polya kernel).

In graph zeta (Ihara zeta, etc.), it was shown in the previous chapter that prime cycles correspond perfectly to prime factorization.

Analytically interpreting this using “prime Lyndon decomposition,”

directed cycles ≈ Lyndon languages ≈ prime factorization

thus, it becomes clear that the Euler product of graph zeta closes the natural number structure as a “circle.”

This is considered to be the concrete form of “prime number scale self-similarity (spiral kernel),” which is expressed in the form of a matrix and as the “Hilbert-Polya operator.”

Therefore, as long as it has a graph zeta-like Euler product structure, the zero points depend on the closed path structure (trace bundle) and are arranged along the critical line.

Even for a single prime number product, the logarithmic kernel already contains an infinite structure.

Since all infinite prime structures interfere with each other, self-similarity converging to Re(s) = 1/2 appears in every prime term. This can also be understood from the structure of the ∞-Möbius transformation and the quasi-dual transformation.

This is the “consistency between the local (prime) and the whole (zeta),” and it is clear that this forms the foundation of the self-adjoint structure of the matrix ring.

Even with a single Euler product, we have actually seen that it can be extended to “all primes” through a quasi-dual divergent zeta extension, as demonstrated by the operation of the “quasi-dual divergent zeta extension” on the Ihara zeta function.

This matrix ring retains the local structure of each prime number (prime Lyndon decomposition, infinite Möbius nesting structure) internally, and by tensor-combining them across all prime numbers, it concretizes the Hilbert-Polya operator and provides a theoretical framework that generates zeta zeros as its eigenvalues.

In conclusion, the structure of the Euler product itself generates the conditions for the arrangement of zeros (balance between divergence and cancellation), and as a result, the eigenvalues naturally align at Re(s) = 1/2.

The concentration at Re(s) = 1/2 appears even without the wave term because the main scaling of the eigenvalues (or traces) of the matrix at each prime number p is based on the logarithmic scale logp and consists of an exponential decay term p^-s. This suggests that the energy density is naturally maximized at Re(s) = 1/2.

The above reasoning does not include a specific method for determining the zero points.

Of course, this is purely a generative structure. That was the purpose.

However, the structural necessity that the zero point "must exist on the critical line Re(s) = 1/2"—the conclusion that "the zero point must only exist on the critical line" can be derived from constructive principles without knowing where the zero point is—is precisely what the tensor construction using a matrix ring possesses, as constructed in the previous chapter.

As a conclusion to this chapter, summarizing this paper while looking at it as a whole, what was demonstrated in this paper was the fact that the operator anticipated by Hilbert-Polya is generatively constructed by a matrix ring that includes the logarithmic scale structure of prime numbers, prime Lyndon decomposition, and infinite Möbius nesting structure.

While the direct numerical determination of the zero point remains, the structural necessity of why Iharanly appears at Re(s) = 1/2 is naturally explained by the infinite adjunction and self-similarity of the matrix ring.

Some may be confused by the unfamiliar concepts used here, but these originally stemmed from various observations and conceptual forms I developed to interpret the sensory phenomenon of "noise phenomena" exhibiting fractal properties. It was only recently that I mathematically formalized these concepts and, coincidentally, discovered that they could be expressed as equations through the construction method of the Ihara zeta function. Until then, it was largely conceptual "play with clay."

I never imagined that my former conceptual play would intersect with the core of the Riemann zeta function in this way. However, this discovery demonstrates that number-theoretic structures can coalesce into a single image in physical, geometric, and generative terms.

The challenge moving forward is to translate this generative structure into a concrete finite-dimensional approximation and confirm the numerical consistency of eigenvalues. Furthermore, the structure suggested by this matrix ring is likely to be extendable to noncommutative geometry, tensor categories, and nested supur graph theory, but this is not the subject of this paper.

This paper demonstrates that the zero structure of the Riemann zeta function can be "generatively constructed" within the matrix ring and its infinite tensor product.
As for applied theory (or rather, that was the beginning, and it wasn't even mathematics to begin with). The beginning of this mathematical exploration was
1. The fact that the error backpropagation method generates "images" from noise, which was completely isomorphic to my "noise method"... Also, from that perspective, I realized that various sensory training methods, such as extinction hallucinations, are implemented in artificial intelligence.

2. The common property of noise and artificial intelligence is “fractal contraction,” and I was enlightened to the relationship between fractals and zeta structures. ...

That's it.

For those interested in the “exploration of concrete phenomena,” I've written a brief note at the end, though it strays from the main topic.

11. Structural Conjectures for Complex Matrix Rings and the Restoration of Various Linear Algebraic Concepts

In this section, we consider propositions about complex matrix rings as a kind of appendix.

Conjectures on matrix rings

By viewing the nested structure of matrix rings of prime degree as a power, the tensor product of matrix rings can be decomposed in the same way as natural numbers and prime numbers, with the same structure as the unique decomposition of natural numbers and prime numbers. Within this structure, concepts such as the usual matrix ring structure, eigenvalues, and adjoint operators are naturally defined as “prime-normalized” structures.

This conjecture states that any complex matrix can be decomposed into a complex matrix of prime order, and the decomposition within the prime complex matrix ring is defined as a nested matrix ring obtained by repeatedly applying the nested structure of matrices as powers. For each prime number p, we define a local complex matrix ring A(p). Then, $A(p)$ is a regular complex matrix ring of order (p, p), naturally containing self-adjointness and eigenvalue structures.

In other words, this local complex matrix ring corresponds isomorphically to the prime factorization of natural numbers.

Figure 46. Decomposition of Multiple Matrix ring.

$$N = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k} \quad \Longleftrightarrow \quad A_N = A_{p_1}^{\otimes a_1} \otimes A_{p_2}^{\otimes a_2} \otimes \cdots \otimes A_{p_k}^{\otimes a_k}.$$

In this localized and normalized complex matrix ring of prime powers, concepts such as trace, eigenvalues, and adjoint are expected to be naturally defined.
Furthermore, as a global structure via tensor product, the entire complex matrix ring is constructed as an infinite product of primes.

Figure 47. Tensor product for each prime number.

$$O_{HP} := \bigotimes_{p \in P} A_p.$$

This structure is thought to correspond to the Euler product, and the combination of each local structure generates the overall picture of the Riemann zeta function.

Each prime matrix ring has a nested structure and an infinite Möbius-like construction. That is, each $A(p)$ contains an infinite Möbius action as a nested structure.

This infinite nesting corresponds to the prime cycle decomposition (prime Lyndon structure) in the graph zeta function and is thought to reflect the fractal nature of the natural number structure. At the same time, it is also thought to extend the structure of natural numbers.

1 = 0.999...

Is this the same?

"It is the same."

Furthermore, in the non-commutative and non-regular matrix ring we considered earlier, there exists a broader "equivalence," which I have termed "representational multi-valuedness." This can be understood by observing the structure that allows an (2,2) matrix to be infinitely expanded into an infinitely nested (2,2) matrix. This overlaps with the infinite nested Möbius structure of multiple matrices and is thought to guarantee the "multivaluedness of quasi-dual representations" necessary for generating zero points. It is an infinite extension of 1.

1 = 0.999... can be considered a simplified model of the equivalence motif space in the multiple matrix structure.

Representation multiplicity and the structure theorem of equivalent motif spaces

The matrix ring has a local nested structure for each prime number, but the specific values of the operators are not uniquely determined by "representation multiplicity." However, this multiplicity is not arbitrary; within the class dual transformation and infinite nested fractal structure, all representations converge to equivalent motif spaces.

Appendix 1.Numerical Experiment at Re(s) = 1/2 and the Hodge Bouquet

Here, I record my attempt to compute some concrete numerical results by letting the computer test my idea.

Figure 48.The value of the real axis 1/2 of the Riemann zeta function.

$$\zeta\left(\frac{1}{2}\right) \approx -1.4603545088\ldots$$

It is known that the Euler product converges only for Re(s) > 1.

So, to see how the values behave near Re(s) = 1/2,I compute the finite prime structure of the Ihara zeta function for primes $p \leq 11$.

Figure 49.Composed of graphs by Zeta Ihara.

$$U^{-|P|} \longrightarrow e^{-s\log p} \quad \text{かつ} \quad |P| = \log p.$$

だから、有限積で

$$Z(U) = \prod_{|P| \leq \log P_N} \frac{1}{1 - U^{-|P|}} \quad \Longrightarrow \quad \prod_{p \leq P_N} \frac{1}{1 - e^{-s\log p}}.$$

つまり：

$$\frac{1}{1 - e^{-s\log p}} = \frac{1}{1 - p^{-s}}.$$

Figure 50.Calculation from the origin on Iihara Zeta's graph.

$$\zeta_5(1/2) = \prod_{p \leq 11} \frac{1}{1 - p^{-1/2}}.$$

By observing how this value changes, I try to imagine how the “analytic continuation” (via the so-called quasi-dual mapping) might be realized.

Figure 51.The ordinary Euler product diverges.

Prime	Factor $1/(1-p^{-1/2})$
2	3.41421356
3	2.36602540
5	1.80901699
7	1.60762522
11	1.43166248

This infinite product factor ultimately diverges,but the infinite bouquet structure with its endless closed loops undergoes a spiral-like class dual deformation,twists back Möbius-like to reverse direction, and tends toward negative values.

I believe this mechanism produces the negative values of the Riemann zeta function at negative integers.

In other words, this infinite Möbius can be thought of as generating the gamma factor and the self-dual structure (s → 1−s) in the analytic continuation of the Riemann zeta function.

Thus, the locally positive product structure is turned over spiral-wise (by an infinite prime-bundled spiral) through the class dual mapping,creating a “topological core” that generates negative values via analytic continuation.

Local positive → class dual mapping → spiral → analytic continuation → negative value

That’s the flow.

In this sense, the spiral Möbius twist appears to reverse the sign of the exponent term,turning −log p into +log p.

This sign reversal explains how the analytic extension generates negative values and matches the self-duality symmetry (s → 1−s).

Here, I recall an idea I once abandoned:the matrix that swaps the sign,

Figure 52.Modified Möbius action.

$$M = \begin{bmatrix} 0 & \log p \\ -\log p & 0 \end{bmatrix}.$$

When I actually let the computer compute this,it became clear that it works to suppress the divergent factor precisely. In other words, the correction factor is 1.

Figure 53.Spiral-shaped dispersion suppression.

Prime	log p	exp(+is log p)	exp(−is log p)
2	0.6931	0.9405 + 0.3397i	0.9405 − 0.3397i
3	1.0986	0.8529 + 0.5221i	0.8529 − 0.5221i
5	1.6094	0.6933 + 0.7206i	0.6933 − 0.7206i
7	1.9459	0.5629 + 0.8266i	0.5629 − 0.8266i
11	2.3979	0.3633 + 0.9317i	0.3633 − 0.9317i

I thought, "Perhaps this was the right direction after all…" and had the machine calculate further.

Figure 54.Negative values appear in the correction term.

$\chi(s) = 2^s \pi^{s-1} \sin(\pi s/2)\Gamma(1-s)$ を定義

s	$\zeta(s)$	$\chi(s)$	$\zeta(1-s)$	$\chi(s)\cdot\zeta(1-s)$	差
0.5	−1.4603545088…	1.0	−1.4603545088…	−1.4603545088…	0.0
−1	−1/12 ≈ −0.08333	−0.05066…	π²/6 ≈ 1.64493…	−0.0833333…	≈ 0（誤差 ~10^{-32}）

Then I realized how this mechanism could indeed lead to negative values(see Figure 54 illustrating how gamma correction leads to negative values).

Without a computer, I would never have understood this structure.

At the time, I had a feeling that the sign of log p was "indifferent."

This arose from the essential sense that "rotation drives convergence" in the trace structure,and I believed that some degrees of freedom remained.

Now that I have confirmed it, I feel that this matrix M can naturally be defined as a "infinite compression operator" — an operator structure that embodies rotation.

Figure 55.Rotational Möbius action.

$$\exp(tM) = \begin{bmatrix} \cos(t\log p) & -\sin(t\log p) \\ \sin(t\log p) & \cos(t\log p) \end{bmatrix}$$

Next, I recall that the genus-1 "dual aperiodic path" can be seen, from the perspective of graph-theoretic Riemann surfaces,as precisely suppressing the divergence of the Euler product and constructing the quadratic zeta.

What I originally saw as the “infinite Möbius spiral” was the way log p aligns itself structurally.

But when it is “bundled,” for some reason, a correction factor appears to protect the whole from divergence.

Now I understand that this is the action of the genus-1 dual aperiodic path —in other words, an “imaginary multiplication trace compression.”

This note was written first, but various later ideas have connected back,recovering the hidden thread of this infinite Möbius structural understanding.

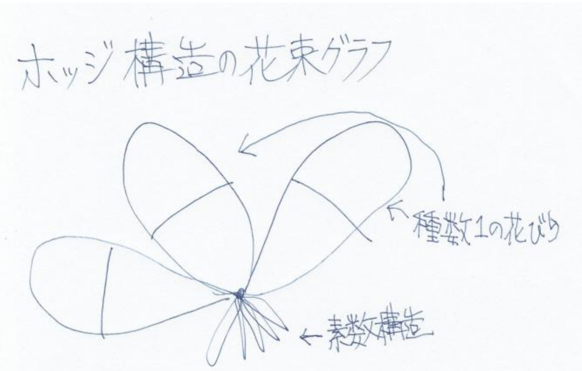
In short, the genus-0 “divergence control circuit” is spiral,while the genus-1 “divergence control circuit” is a “dual aperiodic circuit,”and both appear to be related to the imaginary multiplication circuit —that is the hypothesis this leads to.

Therefore, this matrix M can naturally be defined as the “infinite compression operator for divergence control” in the the Lyndon Complex Spiral Phase.

Analytic continuation is, in other words, a “divergence control circuit” formed by a “rotational spiral structure.”

Consequently, by considering Ramanujan’s quadratic zeta structure,I naturally arrived at what I now call the Hodge Bouquet Graph.

Figure 56.Hodge's bouquet.



To explain this Hodge Bouquet: when you attach a genus-1 petal to the prime structure,the number of aperiodic circuits becomes two,which tends to make divergence more likely,but the “dual aperiodic circuit” works in an imaginary-multiplication-like way to suppress divergence.

Through this, the quadratic zeta — and then the 4, 6, 8 and higher multiple Hecke operators —in other words, via multiple Hecke operators, an endlessly higher structure is generated.

In summary:

Genus 0: local rotational compression by spiral Möbius

Genus 1: non-commutative topological compression by dual aperiodic paths

Note that the number of "aperiodic circuits" in this bouquet structure takes the form $2n-2$.

In this way, you can see that connecting many "dual aperiodic circuits" creates a "cushion"that prevents divergence more effectively.

In other words, the "prime structure" becomes less prone to divergence.

From here, is it possible to build a theory of higher-dimensional "divergence control circuits"…?

Proposition

The real number line is not structurally complete.

This is because it does not contain a nested hierarchy of non-periodic sequences within itself.

Only through divergent quasi-dual recovery can an infinite Möbius bouquet be generated, elevating the bare continuum into a self-dual complex topological structure.

This is the origin of the self-duality ($s \leftrightarrow 1-s$)and the topological projection of the complex spiral syntax plane.

This theorem first appeared to me in a dream.

When I woke up, I initially rejected it as absurd.

However, it perfectly aligned with the structure I had been building.

What I had thought was a mistake was actually the key.

This is the final proposition of this essay.

What appears as meaningless concentric circles—flat and infinite—only becomes the grammar of a spiral when lifted through divergent embedding.

This is what it means to "wind infinite concentric circles into a spiral."

The double helix structure of fractals…

That is, consider an infinite bouquet graph with branches representing real numbers.

When attempting to reconstruct this graph through "divergent restoration," its value takes on a structure that ascends each level of the infinite nested structure.

In other words, to reproduce the structure of the infinite bouquet graph, it must demonstrate higher-dimensional nestedness not through convergence but through construction, utilizing the "infinite Möbius structure" it contains.

This is the proof of the existence of infinite Möbiusness.

Appendix 2

As a summary, I will organize the proof of the generation theory of the Riemann zeta function, including side paths, and future prospects.

1. The graph zeta function based on prime cycles in bouquet graphs is uniquely determined by prime Lyndon decompositions.
2. By divergent zeta extensions, the recursive decomposition of loops coincides with the infinite prime number structure.
3. The quasi-dual map $u^p \mapsto e^{-s \log p}$ is , which is equal to the product kernel of the Mellin transform.
4. By the regular Ramanujan condition, the eigenvalues of the graph determinant have Hilbert-Polya-type self-adjointness.
5. Therefore, the prime cycle sequence coincides with the analytic structure of the Euler product, and the zero structure is mapped onto the critical line.

The flow is very simple, and if there are any leaps or gaps, they are likely to be found in the logical structure.

In the class duality mapping theory related to zeta structures, another idea that comes to mind is that the prime Lyndon structure of "non-regular graphs" is "Dedekind-type," i.e., the prime Lyndon structure of non-regular graphs is Dedekind-type.

To summarize, it is as follows.

In regular (Ramanujan) graphs, the prime Lyndon decomposition of loops is locally finitely generated, and the eigenvalues (prime Lyndon) distribution align neatly, resulting in the zero points being arranged along the critical line.

This also suggests the "closure property of the zeta function, which is finite and closed, through quasi-dual divergent extensions." This is the same as restoring all prime paths under regular conditions.

On the other hand, in non-regular graphs, infinitely generated branches (forks) intrude into the local structure, and the decomposition of prime loops becomes "non-unique" or "redundant." It is also possible that a "trace category" will be needed to express the "local structure." This is also to increase the possibility of restoration.

Furthermore, in Dedekind domains, "class groups" that do not fit into principal ideals appear. Similarly, in "irregular trace bundles," the decomposition of prime cycles allows for multiple paths as "non-principal."

In other words, there is a correspondence between the redundancy of prime Lyndon structures → quasi-dual group bifurcations → the non-triviality of Dedekind-type zetas.

In regular Ramanujan graphs, the “Riemann hypothesis” and in irregular (Dedekind-type) graphs, the “quasi-dual group-containing zeta” correspond to each other in the quasi-duality map.

The Riemann hypothesis in regular Ramanujan graphs and the zeta function with group structure in non-regular (Dedekind-type) graphs may be unified by the difference in the structure of trace bundles in the quasi-duality map. The invariance of trace bundles is likely to be broken, and its deformation may become important.

Finally, a few points that were not explained.

The prime Lyndon sequence in bouquet graphs coincides with the finiteness of the trace bundle.

The limIharaf the zeta extension is isomorphic to the structure of the Kurokawa tensor product.

Non-regular zeta functions incorporate quasi-dual group structures due to redundant branching of prime Lyndon series.

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Glossary(用語集)

1, Related to the Lyndon series(リンドン列関連)

Aperiodic sequence:A sequence with an order that does not contain periodic elements throughout the sequence.
非周期列：内部に周期的な要素を含まない順序を持つ列。

Contraction: The unique decomposition and reduction of non-periodic Lyndon sequences into a minimal trace structure.　.

This refers to the transformation of infinite repetitions of non-trivial aperiodic sequences within trace bundles into loop-type or tree-type structures.
縮約　非周期的リンドン列を最小のトレース構造に分解し、簡約化する独自の過程。
これは、トレース束内の非自明な非周期的列の無限反復を、ループ型またはツリー型構造に変換するプロセスを指す。

Contraction morphism: An operation that performs structural deformation on a trace sequence, trace bundle, or structure in a class-dual manner while preserving fractality.
縮約写像　あるトレース列やトレース束、または構造体を類双対的に、フラクタル性を保ちつつ、構造的変形を行う縮約の操作

Dual Lyndon words ;Corresponding to the reverse order of Lyndon sequences, Lyndon sequence decomposition structures contribute to the stability of the existence of inverses in graph-like Riemann surfaces.
双対リンドン列リ ンドン列の逆順に対応する、リンドン列分解構造、グラフ的リーマン面では逆元の存在の安定性に寄与する。あるリンドン列に対応するグラフの双対構造
→Part II, Part III

Lyndon series reduction; Trace contraction of a non-periodic Lyndon sequence. Note that there are two types of Lyndon series Contraction.
リンドン系列の縮約　非周期的なリンドン列のトレース縮約。注意：リンドン系列の縮約には 2 種類あります。

prime Lyndon word:Shorthand for the smallest unit of a non-periodic sequence. It is uniquely determined by McMahon's theorem and Duval decomposition algorithm.
素リンドン語非周期列の最小の単位。マクマホンの定理や Duval 分解アルゴリズムによって一意的に定まる

Prime Lyndon sequence: An indivisible non-periodic sequence serving as the fundamental unit of contraction.
素リンドン：収縮の基本単位として機能する、分割不能で非周期的な列。単純に、「既約」ではなく、最小単位。

2, Quasi-dual morphism and Zeta(類双対写像とゼータ)

Complex spiral integration: Terms referring to the differential and integral structures of “complex spiral phases”
Although it is not yet clear, it is gradually becoming apparent that as the number of species increases, there is a “divergence control function” corresponding to complex spiral phases, and that there are conversions to higher-order structures and lower-order structures corresponding to this.
複素位相積分　「複素螺旋位相」の微分・積分構造に言及する語
まだ明らかにはなっていないが、種数が増えていくたびに、複素螺旋位相に対応する、「発散制御機能」があり、それに対応して、高次構造への変換や低次構造への変換が存在していることが次第に明らかになっている

Critical line symmetry: Symmetry on the critical line s→←1-s, mainly seen in the Riemann zeta function.
臨界線上の対称性
主にリーマンゼータに見られる s→←１－ｓという臨界線上の対称性

Ideal class motif : The ideal concept also undergoes a process of restoring higher-order structures by first extending a single structure to infinity and then contracting it. This is structurally similar to the graph-theoretic dual motif closure and the structure of class-dual divergent restoration in my theory.
イデアル概念も一旦単一的な構成を無限性へと引き伸ばしてから、縮約するという過程を伴って、高次構造を復元する過程をとる。これは、グラフ論的双対モチーフ閉包と、あるいは、僕の理論における類双対的発散的復元の構造と構造的に類似している。このことから、「一般非可換イデアル論」などの構成が示唆されている。

Infinite compression operator:This refers to the Möbius compression structure, which is an abstract description of the integral kernel that includes rotation, inversion, and spiral convergence. It has a mechanism that controls the divergence of the zeta structure of genus 0 in a spiral rotation, and arranges the structure symmetrically along the critical line of the Riemann zeta function.
無限圧縮作用素　Möbius 的圧縮構造のことで、回転・反転・スパイラル的収束を含む積分核の抽象記述。種数０のゼータ構造の発散を螺旋回転的に制御する仕組みを持っており、リーマンゼータの臨界線に沿って、左右対称に構造を鏡像的に配置する

Multiplicity of zero: When the “elementary Lyndon element” that is restored to zero is decomposed, the corresponding Euler product becomes a “multiple Euler product,”

giving zero points multiple values.
多重零点 ゼロ点へと復元される「素リンドン元」が分解されるときに、それに対応するオイラー積は、「多重オイラー積」になって、ゼロ点にも多重性を与える。

The basic quasi-dual mapping: One-to-one correspondence between infinite concentric circle fractals and Cartesian spirals. Pure transitions between loop shapes and tree shapes can be seen naturally.
基本類双対写像　無限同心円フラクタルとデカルト螺旋との一対一対応。ループ形とツリー形の純粋な移行が自然に見られる

Divergent-density completion:Denotes the state where an infinite set of prime-like structural elements achieves a density such that further divergent reconstructions cause no structural deformation.
発散密度完備 無限の素数類似構造要素の集合が、さらに発散する再構成が構造的変形を引き起こさないような密度を達成した状態を指す。

Divergent restoration: The operation of recovering a potentially infinite structure from contractions by non-closed quasi-dual e morphisms.
発散的復元：非自明な非周期列を復元する類双対写像を用いて、収縮的縮約から潜在的に無限の構造を回復する操作。

Effect of imaginary number multiplication:Imaginary multiplication realized through motif-aligned rotations. In this theory, the divergent structure of Euler products is controlled through “dual non-periodic paths.”
虚数乗法の作用　この理論では「双対非周期的経路」を通じてのオイラー積の発散的構造を制御する構造

Lyndon complex spiral continuous phase:A continuous complex phase that is uniquely determined for a Lyndon sequence, which is a semigroup. It is sometimes referred to as a “double helix” because it naturally contains spiral rotations and has a double main structure.
半群であるリンドン列に対して、一意的に定まる連続複素位相。自然に螺旋形の回転を含んでいるところ、二重の縮約的構造を持っているところなどから、「二重螺旋」と表現することもある。

Genus expansion: An expression for structural development accompanied by changes in

the number of species. This is particularly important in the context of the formulation of “higher-order imaginary multiplication.”
In other words, it can be understood that the Hecke operator of higher-order zeta functions acts as an operator that changes the structure of graph-like Riemann surfaces, allowing for the interpretation that this is a comprehensive integral of Riemann surfaces.
種数の拡張　種数の変化を伴う構造展開に対する表現。とくに「高次虚数乗法」の定式化文脈で重要。
つまり、グラフ的リーマン面の構造を高次元に変化させる作用素として、高次ゼータのヘッケ作用素が作用していることが分かるために、これはリーマン面の包括的積分である、という解釈を許す

Non-regular zeta structure：An extension of the zeta function with genus and loop structure. It naturally appears when constructing the quadratic zeta function in Dedekind's zeta function. The zero points probably extend beyond the critical line, and their Euler product divergence is prevented by “dual non-periodic paths.” Higher orders are also possible.
非正則ゼータ構造　種数・ループ構造をもつゼータ関数の拡張。デデキントのゼータで、二次のゼータを構成する時に自然に出てくる。ゼロ点はおそらく臨界線上からはみ出し、「双対非周期経路」によって、そのオイラー積の発散が防がれている。より、高次化も可能。

Spiral development:Spiral expansion representing recursive quasi-dual morphism. Used when bundling the infinite concentric circle structure of the Zeta function into a spiral shape and projecting it linearly.
螺旋的展開 再帰的類双対写像を表現する螺旋展開。ゼータ関数の無限同心円構造を螺旋形に束ねて、直線的に射影するときに使われる

Trace bundle: The structure generated by repeated contractions and expansions of Lyndon sequences.
トレース束　構造体の全経路を集約した構造。それぞれのトレースは、リンドン列と一意対応。
→全体（特に Part I, III）

Primitive p-th root of unity:Primitive p-th root of on the unit circle (associated with a prime p)

素数pに対応する単位円状の一乗根　p は素数。「素数に対応する無限同心円の上に対応する単位乗根」という意味

Quasi-dual morphism: A mapping that transforms fractals into fractals, transforming trace bundles into either loop-type or tree-type structures.A morphism that resembles duality but inherently resists full closure.　　quasi-dual　quasi-dual morphism
フラクタルをフラクタルへと変形する写像、トレース束をループ型のほうか、ツリー型のほうへと変形する

In this theory, we define quasi-dual operations as dual-like transformations that lack formal duality properties such as closure or invertibility, yet govern recursive, non-commutative constructions within trace structures.

全体（とくに Part II）

Recursive quasi-duality: A structure that repeatedly performs class dual operations. A concept connected to the category zeta structure in particular.
When repeating class dual transformations, it is necessary to determine whether the structure is invariant or not, while noting that it is non-commutative and multivalued, in order to find the restorability of a specific structure.
類双対操作を反復的に繰り返す構造。特に圏的ゼータ構造に接続する概念。類双対変形を繰り返すときそれが非可換であり、多値であることに注意しつつ、構造の不変性を変えているのか、変えていないのかを見ながら、特定の構造への復元性を見つけないといけない。

u^p→e^-slogp; One of the quasi-dual maps, often used in deformations such as the Ihara zeta function.
u^p→e^-slogp; 類双対写像の一つで、伊原ゼータ関数などの変形においてよく用いられる。

Zeta deformation process:When fractally deforming the zeta function, there is always “multivalueness,” so it is necessary to find an appropriate deformation method that corresponds to such “diverse deformation possibilities.” For this reason, I am attempting four types of deformation methods in my essay.
Just pay attention to scaling and discrete/continuous properties.
ゼータ変形　ゼータ関数をフラクタルや類双対写像で変形するプロセス。ゼータ関数をフラクタル的に変形するときには、必ず「多値性」があるので、そのような「多様な変形可能性」に応じて、適切な変形方法を探らないといけない。そのため、僕は論考の中で４種類の変形方法を試みている。スケーリングや離散・連続性に注意すればいい。

3, Fractal restoration theory(フラクタル復元理論)

Fractal reconstruction；Mainly by continuously applying divergent quasi-dual mappings, the internal completeness of the structure is constructed. If there are two prime structures, for example, one Euler product, then naturally all Euler products across all prime numbers can be restored.
The prime Lyndon elements contain all natural numbers,but the prime path lengths in the bouquet graph lack ordering,and this absence leads to a contraction to the prime number structure,corresponding to the Euler product.
フラクタル復元　部分構造から全体を生成する写像操作。主に発散的類双対写像の連続適用によって、構造体の内部的な完備性を構成する。素構造が 2 つあれば、たとえば、ひとつのオイラー積などは自然にすべての素数に渡るオイラー積が復元可能
「オイラー積に対応する」伊原ゼータの花束グラフを復元するときに、「素リンドン元にはすべての自然数が含まれる」けど、「素経路の長さ」には順序性がないから、「素数」へと縮約される,という「非可換」→「可換」という変換に注意。
→Part I, Part IV

4. Structures, graphs, and Riemann surfaces(構造体・グラフ・リーマン面)

"Bouquet graph"：A wedge sum of n circles, i.e., a single vertex with multiple attached loops. This structure serves as the minimal model for the trace contraction in the graphical Riemann surface.
花束グラフ　n 個の円からなるウェッジ和を指し、すなわち、複数のループが接続された単一の頂点からなる構造。この構造は、グラフ的リーマン面におけるトレース収縮の最小モデルとして機能します。

Deligne's condition:Unlike general Deligne cohomology, here we refer to the divergence control structure resulting from the combination of dual non-periodic paths and imaginary multiplication circuits as the Deligne structure. Structures that satisfy Ramanujan's inequality
ドリーニュの構造　一般のドリーニュコホモロジーの意味とは異なり、ここでは双対非周期的経路と虚数乗算回路の組み合わせから生じる発散制御構造をドリーニュ構造と呼ぶ。ラマヌジャンの不等式を満たす構造のこと。

Dual non-periodic paths :The dual structure of extremely simple non-periodic sequences

arising from two non-periodic circuits of curves with genus one.
双対的非周期回路　種数一の曲線の非周期的回路が 2 つであるところから生じる、極度に単純な非周期列の双対的構造

Hodge bouquet: A collection of Riemannian surface graphs with the same number of seeds, arranged in a bouquet graph. Note that it also has a normal "bouquet structure" corresponding to the "Euler product." It is also necessary to distinguish it from the commonly referred to "Hodge structure."
ホッジの花束　種数一のリーマン面グラフを花束グラフ状に束ねたもの。「オイラー積」に対応する通常の「花束構造」をも持っていることに注意。また、通常言われている「ホッジ構造」との区別が必要。

Trace bundle : This refers to the entire set of all paths (traces) that pass through the interior of a given structure, including both finite and infinite lengths.

In particular, when the components of the path can be uniquely distinguished, this set can be one-to-one corresponding with the entire Lyndon sequence (and its infinite repetition).
トレース束　ある構造体の内部を通過するすべての経路（トレース）を、有限長・無限長のいずれの場合も含めて集めた集合全体をいう。

とくに、その経路の構成要素が一意に区別可能なとき、この集合はリンドン列全体（およびその無限反復）と一対一に対応しうる。

quasi-modular trace ; A natural quasi-dual transformation that reduces "irreducible rational Lyndon" in trace bundles to "prime Lyndon" or "natural number Lyndon." Note that this can be performed even without a specific form, as long as a trace bundle is available. In that case, it can be expressed as a geometric operation as a deformation of the graph.
トレース束における「既約有理リンドン」を「素リンドン」や「自然数リンドン」へ縮約する自然な類双対変形。特に明示的形式がなくてもトレース束があれば行えることに注意。その場合、グラフの変形として、幾何学的操作の一環として、表現できるだろう。

Regularity:A function is regular when the local structure of its graph is uniform and orderly.non-regularity
正則性　関数が正則、グラフの局所構造が一様で整っていること
非正則性 グラフの局所構造が一様ではなく、正則でない構造、ゼロ点配置が乱れているなど

Non-regularity Irregularity: The local structure of the graph is not uniform but sparse. The zeros of the zeta function are scattered along the critical line.
非正則性　グラフの局所構造が一様ではなく、まばらであること。ゼータのゼロ点が臨界線からばらばらになる。

5,公理・写像・圏的表現

Collections of dual motif-closed sets ; A complete state that cannot be further expanded by repeating dual operations.
双対操作を繰り返すことによってこれ以上拡大しない圏的な完備状態

Fractal-based logic; Since quasi-duality transformations transform fractals into fractals, fractal properties are normally preserved even with normal restoration or reduction, as well as with divergent restoration or reduction. Note that there are times when the structure of the "trace bundle" remains unchanged and times when it undergoes structural changes. A language is needed to describe the structural changes of the trace bundle.
類双対性変換はフラクタルをフラクタルへと変形するので、通常の復元や縮約でも、発散的復元や縮約でも、普通にフラクタル性が保たれていること。そして、そのとき、「トレース束」の構造が不変であるときと構造論的な変化をする時があることに注意。トレース束の変化構造を記述する言語が必要。

quasi-dual morphism
→ 類双対写像

Quasi-duality closure ;A noncommutative, multivalued, quasi-dual transformation that cycles through all transformations between the maximum loop structure and the maximum tree structure until it reaches a state that cannot be further expanded. This becomes a zeta structure of a categorical structure.
→ 類双対閉包
非可換で、多値的な、類双対変換が、最大ループ構造と最大ツリー構造の間の変換をすべて巡らせて、これ以上拡大し得ない状態へと達すること 圏的構造のゼータ構造体になる