Problem 1(a)(i)

$$p(t_2 | t_1) = \lambda e^{-\lambda t_2}$$

$$p(t_2) = \lambda e^{-\lambda t_2}$$

$$p(t_2) = p(t_2 | t_1)$$

$$\Leftrightarrow p(t_2) p(t_1) = p(t_2 | t_1) p(t_1)$$

$$\Leftrightarrow p(t_2) p(t_1) = p(t_1, t_2)$$

$$p(t_i | t_1, t_2 \ldots t_{i-1}) = \lambda e^{-\lambda t_i} = p(t_i)$$

therefore

$$p(t_1, t_2, \ldots) = \prod_{i=1} p(t_i)$$ and

They are independent.

1, a, ii

$(T_1, T_2 \dots)$ are not independent

I prove this by showing (T_1, T_2) are not independent.

$$p(T_2) = \int_0^{T_2} \lambda e^{-\lambda(T_2 - t)} \times \lambda e^{-\lambda t} \, dt$$

however

$$p(T_2 \mid T_1) = \lambda e^{-\lambda(T_2 - T_1)}$$

therefore.

$$p(T_1, T_2) \neq p(T_1) p(T_2) \qquad (Q.E.D)$$

1. (a), (iii)

if they are independent,

$$p(t_N | t_1, t_2, \dots, t_{N-1}) = p(t_N) \quad (1)$$

if $\sum_{i=1}^{N-1} t_i = T'$,

$$p(t_N = (T - T' + 1) | t_1, t_2, \dots, t_{N-1}) = 0$$

because N spikes were observed in $[0\ T]$

but

$$p(t_N = T - T' + 1) = \lambda e^{-\lambda(T - T' + 1)}$$

$$(T - T' + 1 < T)$$

Therefore equation (1) is not true. and they are dependent

1.(a).(iv)

$(T_1, T_2 \ldots T_N)$ are dependent

eq

I prove this by showing (T_1, T_2) are not independent. given $T_2 < T$.

$$p(T_2) = \int_0^{T_2} \lambda e^{-\lambda(T_2 - t)} \times \lambda e^{-\lambda t} \, dt$$

however

$$p(T_2 \mid T_1) = \lambda e^{-\lambda(T_2 - T_1)}$$

therefore.

$$p(T_1, T_2) \neq p(T_1) p(T_2) \qquad (\text{Q.E.D})$$

1.(b).(i)

they are dependent.

Define $\lambda(t)$, $(0<t)$

$$p(t_2 | t_1) = \lambda(t_1+t_2)\, e^{-\lambda(t_1+t_2)}$$

$$p(t_2) = \int_0^{\infty} \lambda(t) e^{-\lambda(t)t} \times \lambda(t+t_2) e^{-\lambda(t+t_2)} dt$$

$$p(t_2) \neq p(t_2 | t_1)$$

therefore $(t_1, t_2 \ldots)$ are dependent

1. (b), (ii)

$$p(T_2) = \int_0^{T_2} \lambda(t) e^{-\lambda(t) \times t} \times \lambda(T_2 - t) e^{-\lambda(T_2 - t) \times (T_2 - t)} dt$$

however

$$p(T_2 | T_1) = \lambda(T_2 - T_1) e^{-\lambda(T_2 - T_1) \times (T_2 - T_1)}$$

therefore,

$$p(T_2) \neq p(T_2 | T_1)$$

They are not independent.

1.(b).(iii)

since $(t_1, t_2 \dots t_N)$ aren't independent,
from 2.(b).(i), although N spikes were observed, still they are independent.

Let's think about $N=2$ condition,
from the given condition.

$t_1 + t_2 < T$.

$$p(t_2 | t_1) = \lambda(t_1+t_2)\, e^{-\lambda(t_1+t_2)}$$

$$p(t_2) = \int_0^T \lambda(t) e^{-\lambda(t)t} \times \lambda(t+t_2) e^{-\lambda(t+t_2)} dt$$

$$p(t_2) \neq p(t_2|t_1)$$

therefore $(t_1, t_2 \dots)$ are dependent

1 (b) (iv)

$(T_1, T_2 \ldots T_N)$ are not independent by proving (T_1, T_2) aren't independents when $N=2$

$$p(T_2|T_1) = \lambda(T_2)\, e^{-\lambda(T_2)T_2}$$

$$p(T_2) = \int_0^{T_2} \lambda(t)\, e^{-\lambda(t)t} \times \lambda(T_2-t)\, e^{-\lambda(T_2-t)(T_2-t)}\, dt$$

therefore

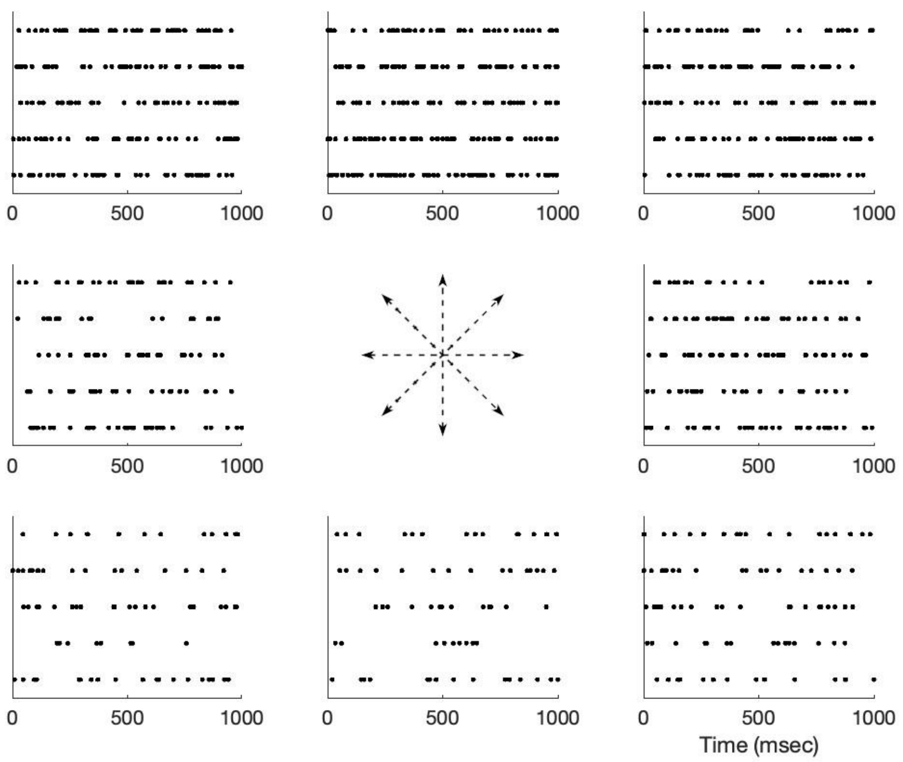
$p(T_2) \neq p(T_2|T_1)$ and.

(T_1, T_2) aren't independent.

(Q.E.D)

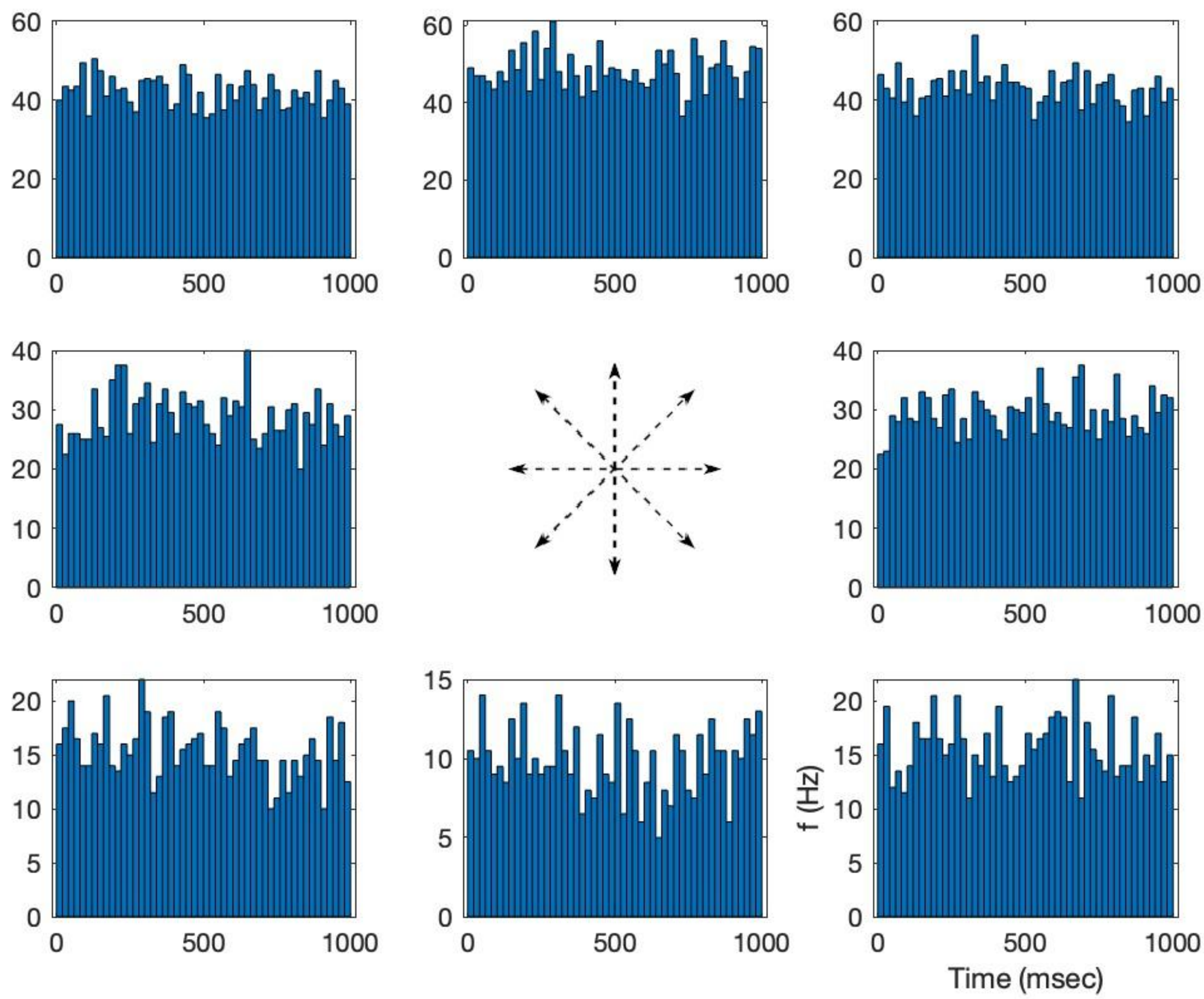
Problem 2(a)

```
r_0 = 30;
r_max = 50;
s_max = pi / 2;
tc = @(s) r_0 + (r_max - r_0) * cos(s - s_max);
% (a)
s = [0, 1, 2, 3, 4, 5, 6, 7] * pi / 4;
positions = [6, 3, 2, 1, 4, 7, 8, 9];
spikes = cell(1, 8);
figure;
for i = 1:8
   subplot(3, 3, positions(i))
   spikes{i} = zeros(100, 1000);
   for j = 1:100
       spikeN = poissrnd(tc(s(i)));
       % generate spike times
       spikeTimes = rand(1, spikeN);
       % convert to spike indices
       spikeIndices = ceil(spikeTimes * 1000);
       spikes{i}(j, spikeIndices) = 1;
   end
   for j = 1:5
       X = find(spikes{i}(j, :));
       scatter(X, j * ones(1, length(X)), 'k.')
       hold on
   end
   % remove the ticks
   set(gca, 'YTick', []);
   ylim([0.5 5.5]);
   if i == 8
       xlabel("Time (msec)")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q2.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



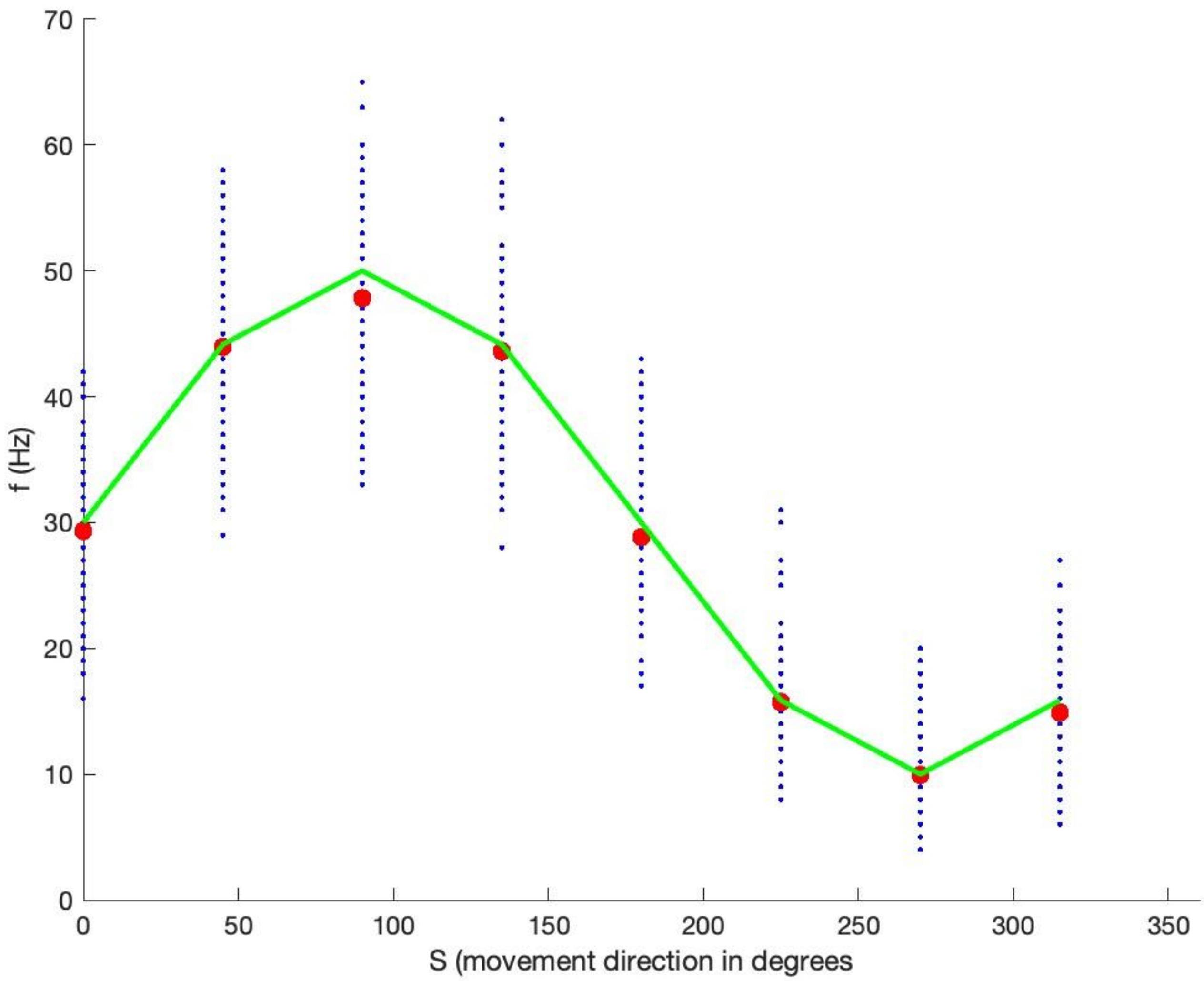
Problem 2(b)

```
spikeCounts20ms = cell(1, 8);
figure;
for i = 1:8
    subplot(3, 3, positions(i))
    spikeCounts20ms{i} = zeros(100, 50);
    for j = 1:100
        spikeCounts20ms{i}(j, :) = sum(reshape(spikes{i}(j, :), 20, 50), 1);
    end
    X = 10:20:1000;
    Y = mean(spikeCounts20ms{i}, 1);
    Y = Y / 20 * 1000;
    bar(X, Y, 1);
    if i == 8
        ylabel("f (Hz)")
        xlabel("Time (msec)")
    end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q2.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



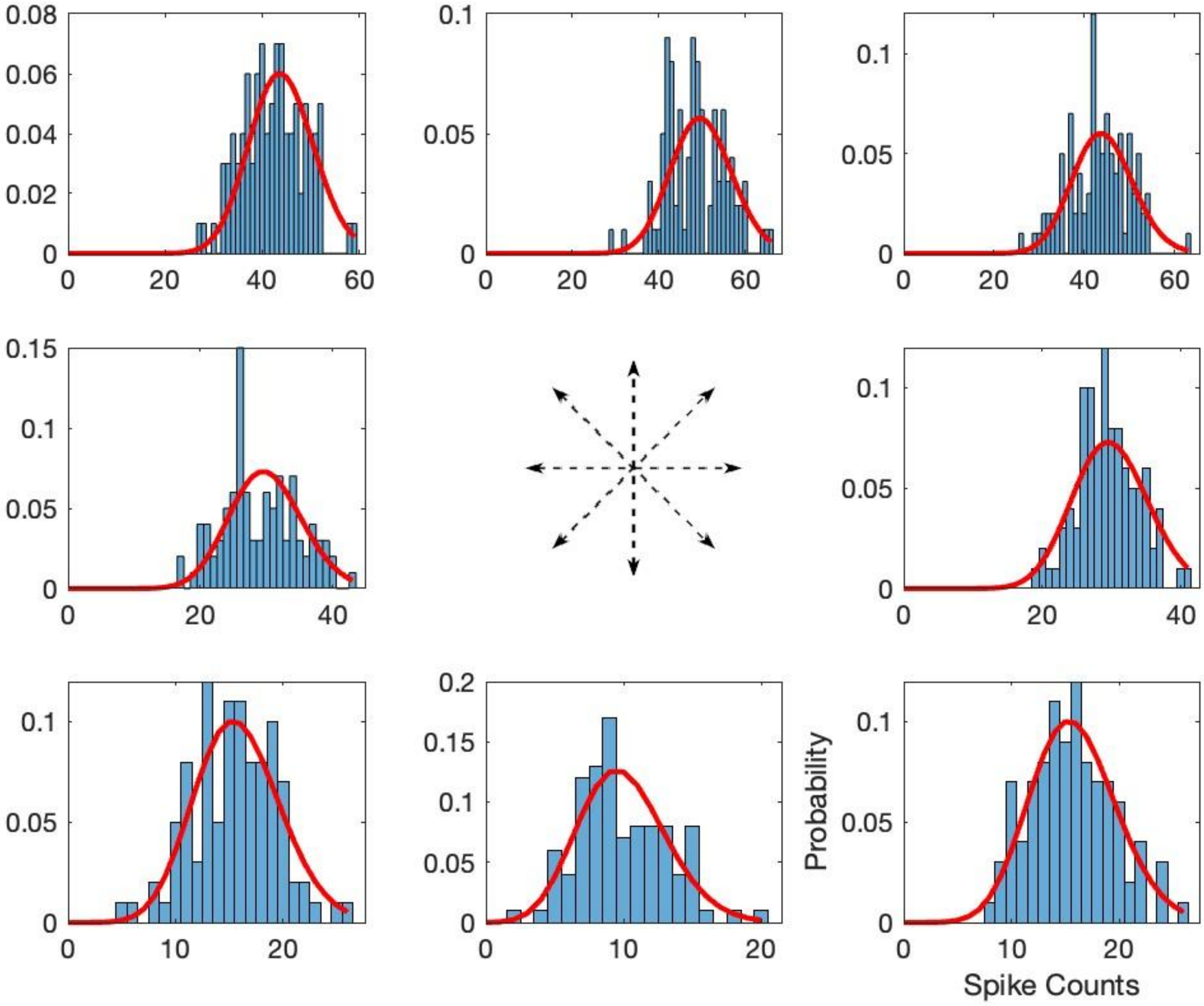
Problem 2(c)

```
spikeCounts = zeros(8, 100);
figure;
for i = 1:8
   spikeCounts(i, :) = sum(spikes{i}, 2);
   % plot all spike counts in one figure (x-axis: reaching angle, y-axis: spike counts)
   % opacity: 0.1
   scatter(s(i) * 180 / pi * ones(1, 100), spikeCounts(i, :), 20, 'b.', 'MarkerFaceAlpha', 0.1,
'MarkerEdgeAlpha', 0.1); hold on;
end
Y = mean(spikeCounts, 2);
X = s * 180 / pi;
plot(X, Y, 'r.', 'MarkerSize', 20)
hold on
plot(X, tc(s), 'g', 'LineWidth', 2)
xlim([0 360])
xlabel("S (movement direction in degrees")
ylabel("f (Hz)")
```



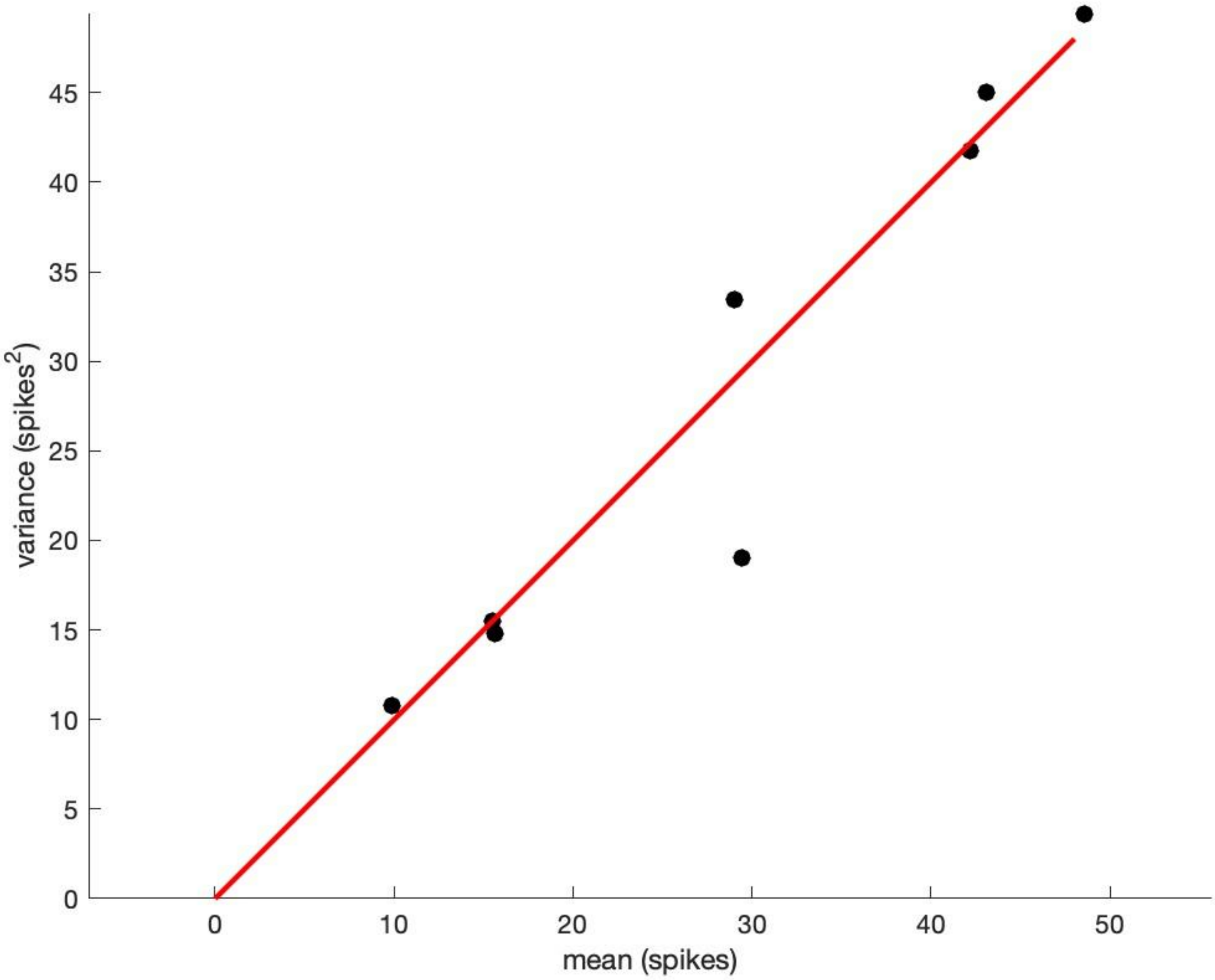
Problem 2(d)

```
figure;
for i = 1:8
   subplot(3, 3, positions(i))
   % plot normalized histogram, y-axis: normalized count
   histogram(spikeCounts(i, :), 'Normalization', 'probability');
   hold on
   x = 0:1:max(spikeCounts(i, :));
   y = poisspdf(x, tc(s(i)));
   plot(x, y, 'r', 'LineWidth', 2)
   if i == 8
       xlabel("Spike Counts")
       ylabel("Probability")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q2.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



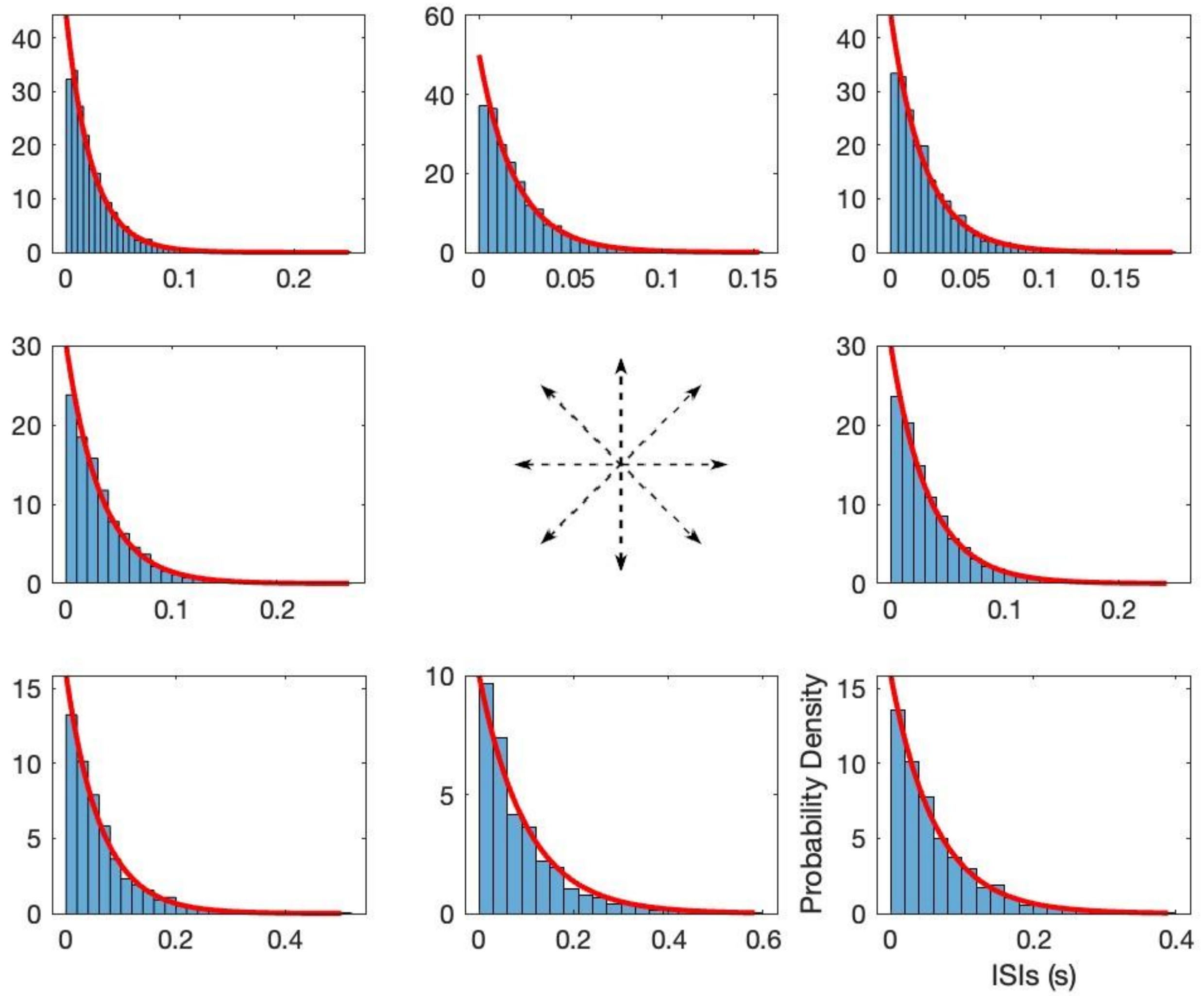
Probelm 2(e)

```
figure;
meanSpikeCounts = zeros(1, 8);
varSpikeCounts = zeros(1, 8);
for i = 1:8
   meanSpikeCounts(i) = mean(spikeCounts(i, :));
   varSpikeCounts(i) = var(spikeCounts(i, :));
end
scatter(meanSpikeCounts, varSpikeCounts, 50, "k", "filled");
hold on;
xlabel("mean (spikes)")
ylabel("variance (spikes^2)")
axis equal
% add x = y line
x = 0:1:max(meanSpikeCounts);
plot(x, x, 'r', 'LineWidth', 2);
```



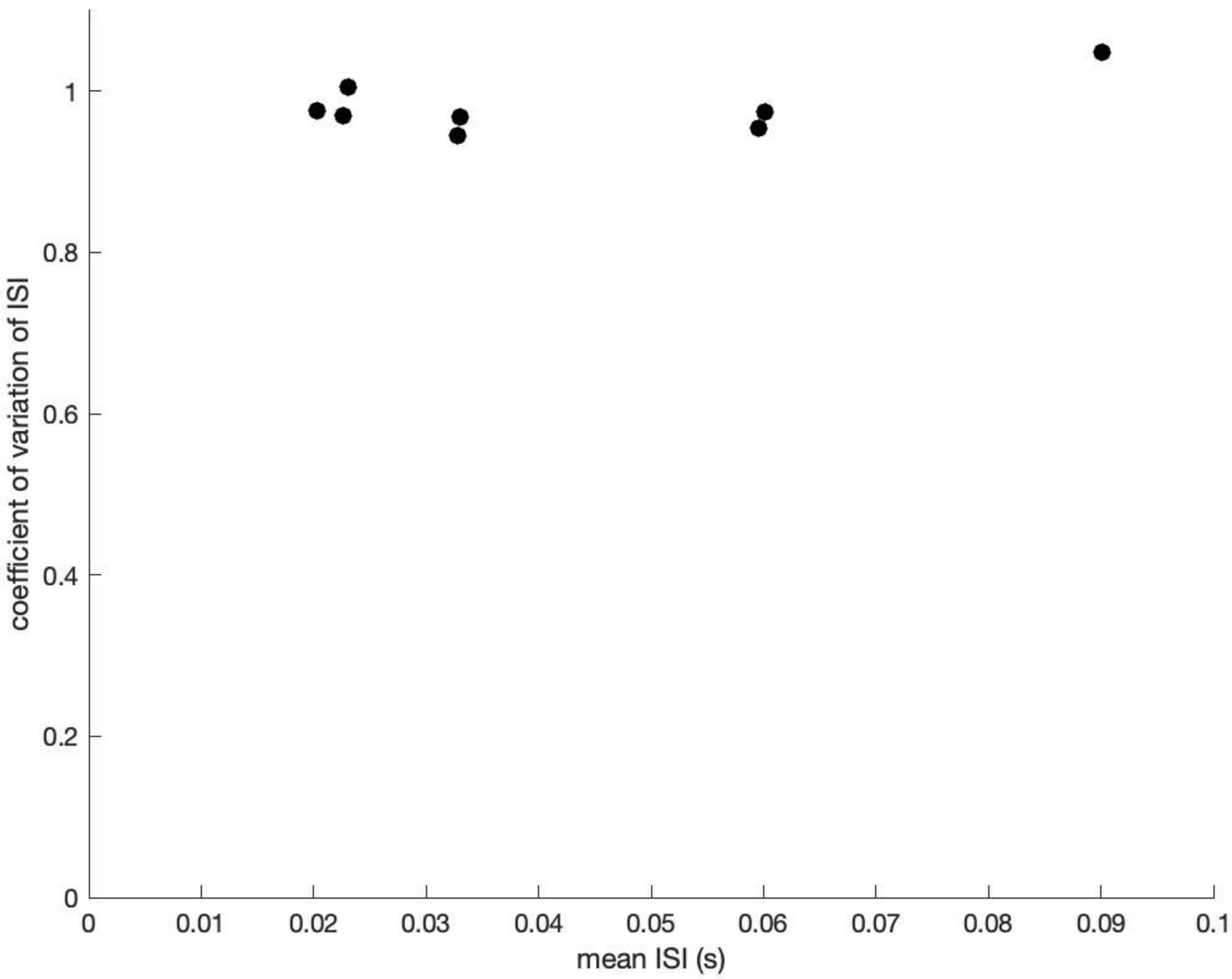
Problem 2(f)

```
figure;
for i = 1:8
    subplot(3, 3, positions(i))
    ISIs = [];
    for j = 1:100
        ISIs = [ISIs, diff(find(spikes{i}(j, :)))];
    end
    ISIs = ISIs / 1000;
    histogram(ISIs, 'Normalization', 'pdf')
    hold on
    x = 0:0.001:max(ISIs);
    % y is the pdf of normalized exponential distribution with mean 1/tc(s(i))
    y = exppdf(x, 1/tc(s(i)));
    plot(x, y, 'r', 'LineWidth', 2)
    if i == 8
        xlabel("ISIs (s)")
        ylabel("Probability Density")
    end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q2.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



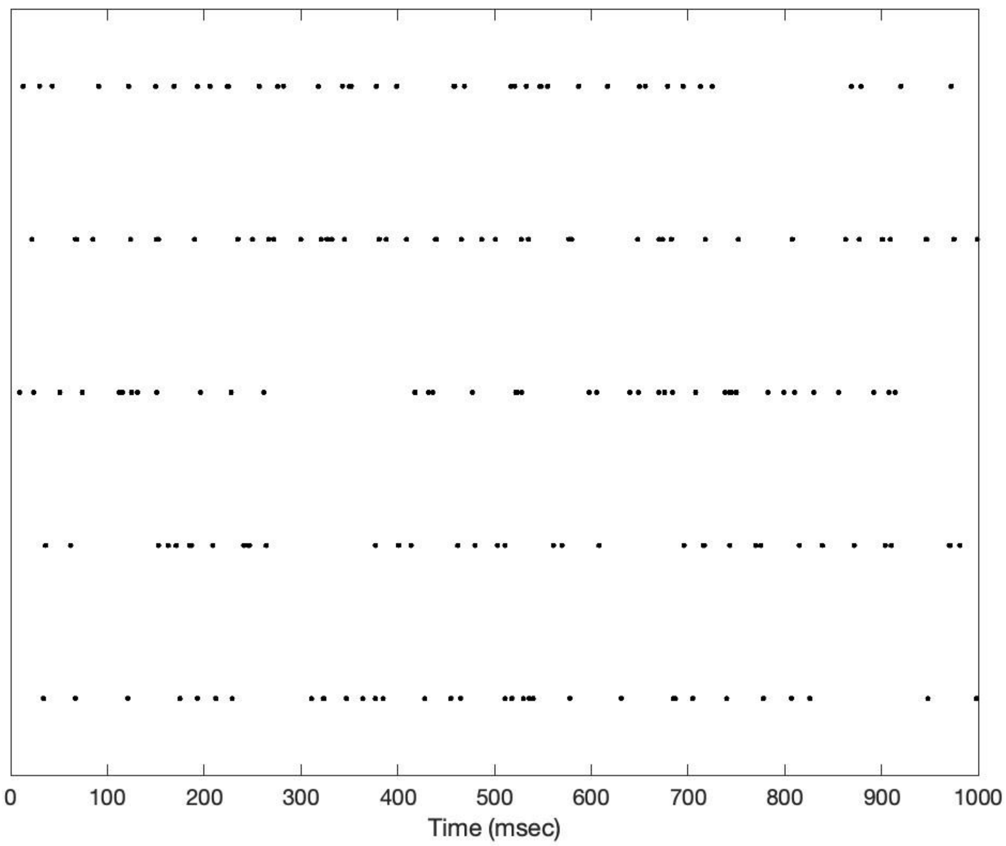
Problem 2(g)

```
figure;
for i = 1:8
   ISIs = [];
   for j = 1:100
       ISIs = [ISIs, diff(find(spikes{i}(j, :)))];
   end
   ISIs = ISIs / 1000;
   scatter(mean(ISIs), std(ISIs) / mean(ISIs), 50, 'k', 'filled')
   hold on
end
ylim([0 1.1])
xlim([0 0.1])
xlabel("mean ISI (s)")
ylabel("coefficient of variation of ISI")
```



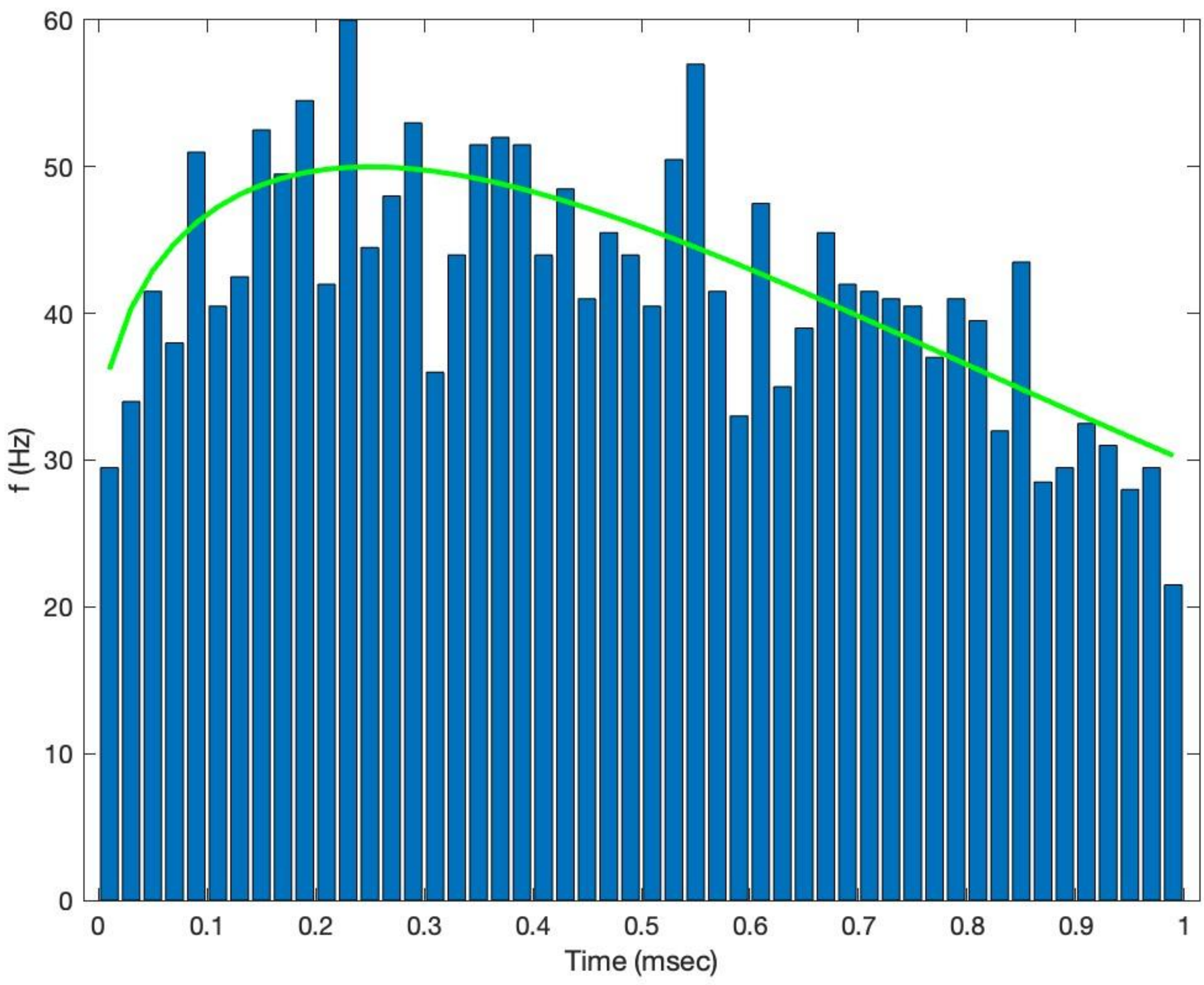
Problem 3(a)

```
r_0 = 30;
r_max = 50;
s_max = pi / 2;
tc = @(s) r_0 + (r_max - r_0) * cos(s - s_max);
s_t = @(t) sqrt(t) * pi;
s_inhom = zeros(1, 1000);
for i = 1:1000
   s_inhom(i) = s_t(i / 1000);
end
spikes_inhom = zeros(100, 1000);
% get max lambda;
max_lambda = max(tc(s_inhom));
for i = 1:100
   spikeN = poissrnd(max_lambda);
   spikeTimes = rand(1, spikeN);
   for j = 1:spikeN
       U = rand;
       if U < tc(s_inhom(ceil(spikeTimes(j) * 1000))) / max_lambda
           spikes_inhom(i, ceil(spikeTimes(j) * 1000)) = 1;
       end
   end
end
figure;
for i = 1:5
   X = find(spikes_inhom(i, :));
   plot(X, i * ones(1, length(X)), 'k.')
   hold on
end
set(gca, 'YTick', []);
ylim([0.5 5.5]);
xlabel("Time (msec)")
```



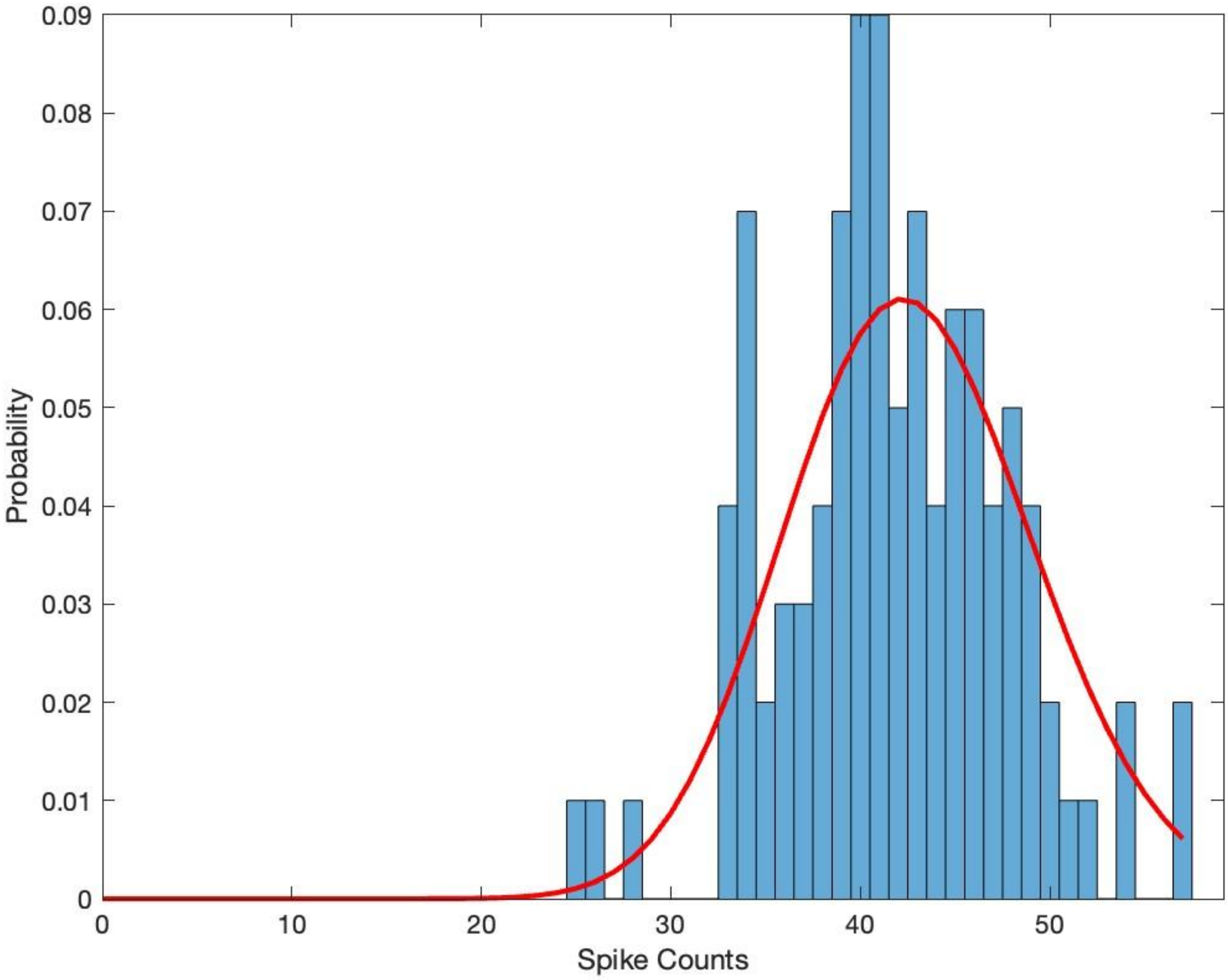
Problem 3(b)

```
spikeCounts20msInhom = zeros(100, 50);
for i = 1:100
   spikeCounts20msInhom(i, :) = sum(reshape(spikes_inhom(i, :), 20, 50), 1);
end
Y = mean(spikeCounts20msInhom, 1) * 1000 / 20;
X = (10:20:1000) / 1000;
figure;
bar(X, Y)
hold on
plot(X, tc(s_inhom(10:20:1000)), 'g', 'LineWidth', 2)
ylabel("f (Hz)")
xlabel("Time (msec)")
```



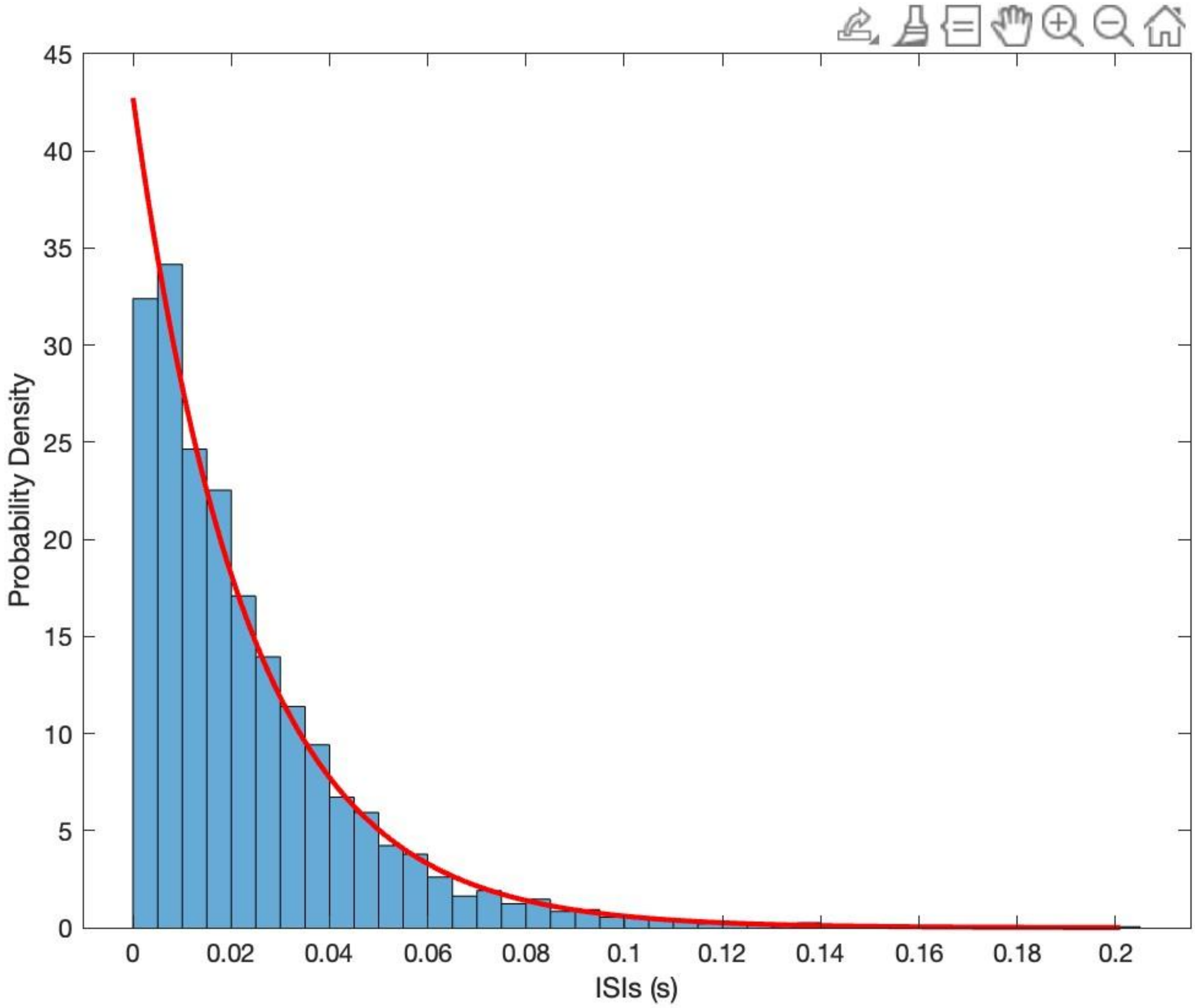
Problem 3(c)

```
spikeCountsInhom = sum(spikes_inhom, 2);
figure;
histogram(spikeCountsInhom, 'Normalization', 'probability');
hold on
x = 0:1:max(spikeCountsInhom);
y = poisspdf(x, mean(tc(s_inhom)));
plot(x, y, 'r', 'LineWidth', 2)
xlabel("Spike Counts")
ylabel("Probability")
```



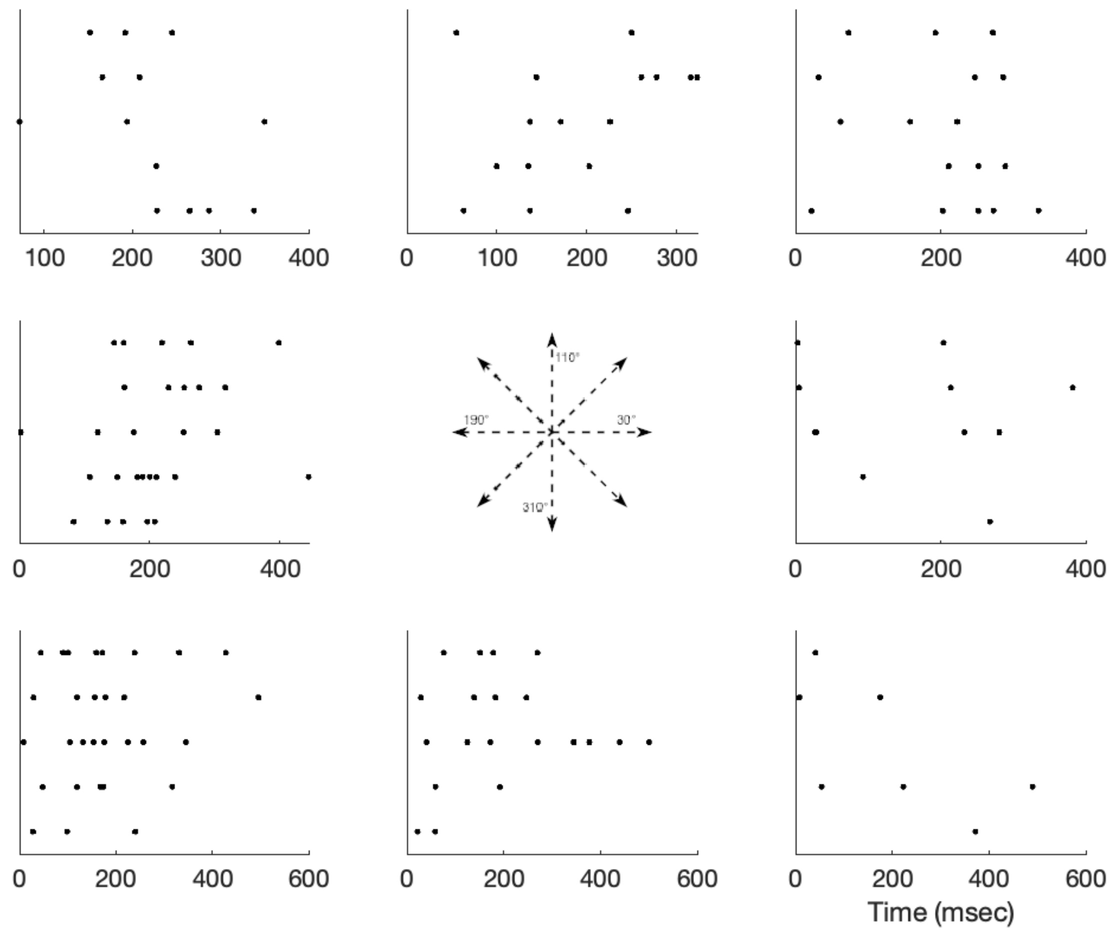
Problem 3(d)

```
ISIsInhom = [];
for i = 1:100
   ISIsInhom = [ISIsInhom, diff(find(spikes_inhom(i, :)))];
end
ISIsInhom = ISIsInhom / 1000;
figure;
histogram(ISIsInhom, 'Normalization', 'pdf')
hold on
x = 0:0.001:max(ISIsInhom);
y = exppdf(x, 1 / mean(tc(s_inhom)));
plot(x, y, 'r', 'LineWidth', 2)
xlabel("ISIs (s)")
ylabel("Probability Density")
```



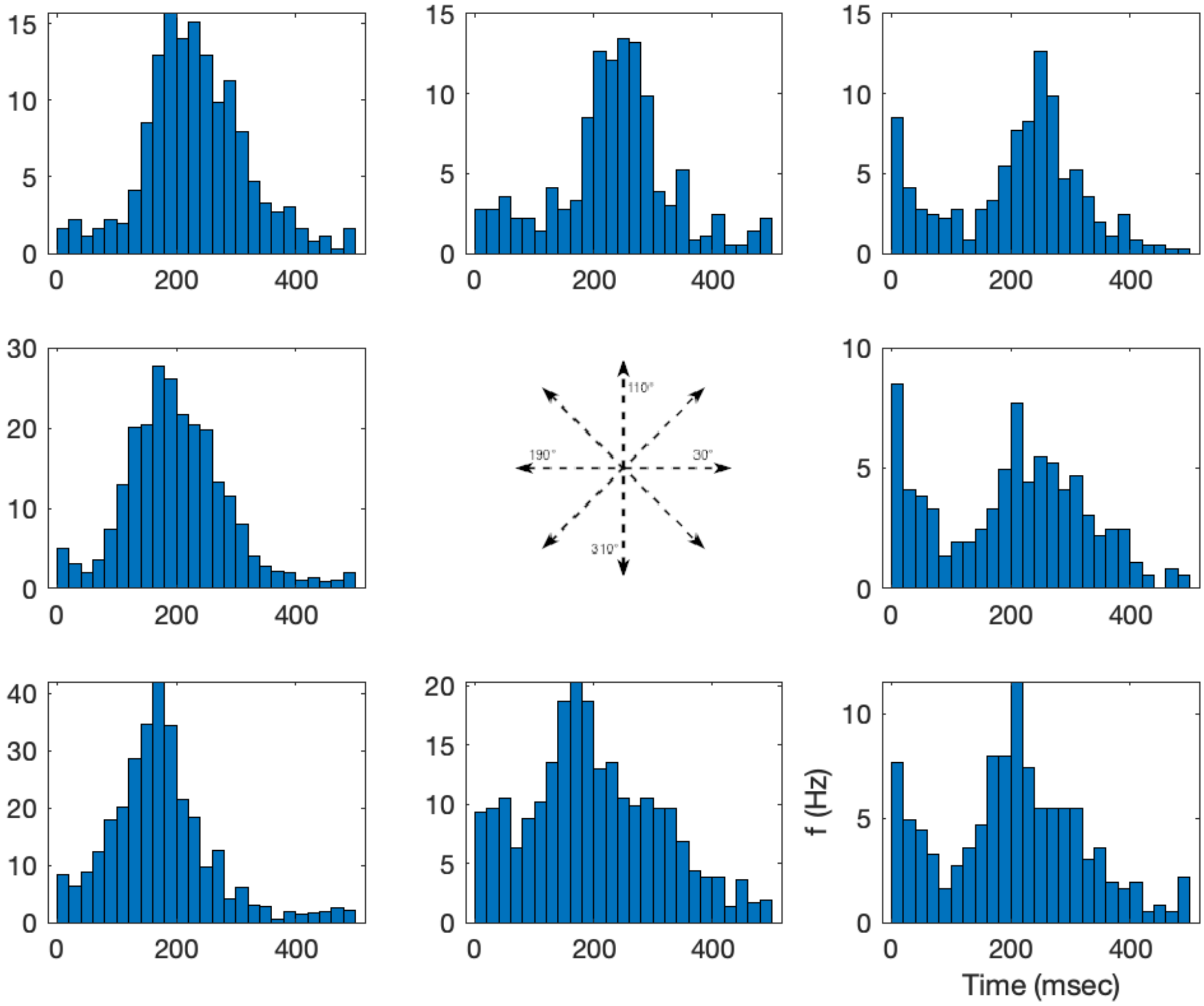
Problem 4(a)

```
load('ps2_data.mat');
ntrials = 182;
nangles = 8;
nneurons = 100;
angles = [30/180 * pi, 70/180 * pi, 110/180 * pi, 150/180 * pi, 190/180 * pi, 230/180 * pi, 310/180 *
pi, 350/180 * pi];
positions = [6, 3, 2, 1, 4, 7, 8, 9];
figure;
for i = 1:nangles
   subplot(3, 3, positions(i));
   for j = 1:5
       X = find(trial(j, i).spikes);
       Y = ones(1, length(X));
       scatter(X, j*Y, 'k.');
       hold on;
   end
   % remove the ticks
   set(gca, 'YTick', []);
   ylim([0.5 5.5]);
   if i == 8
       xlabel("Time (msec)")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q4.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



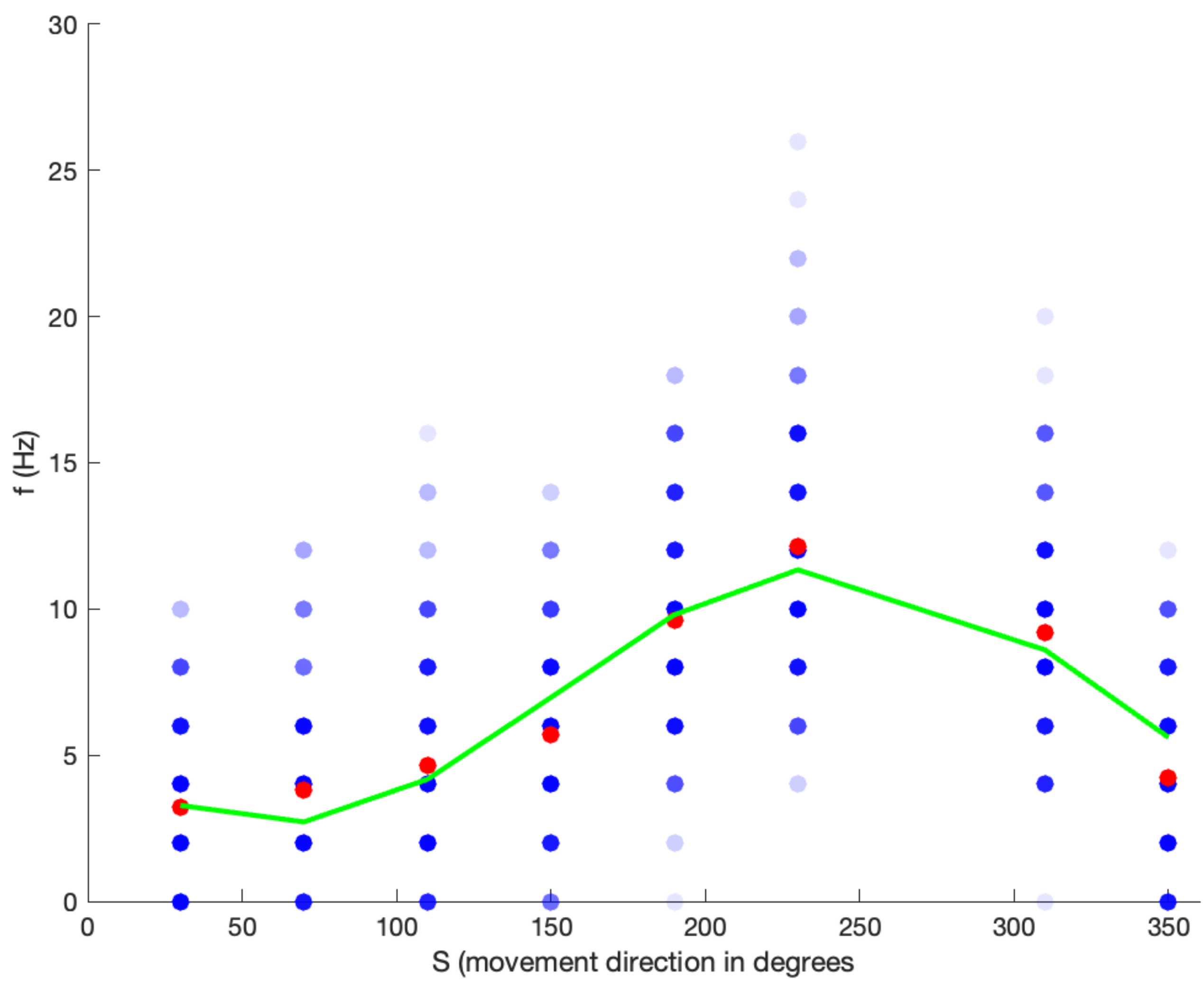
Problem 4(b)

```
figure;
spikeCounts20ms = zeros(ntrials, nangles, 500/20);
for i = 1:nangles
   subplot(3,3, positions(i));
   for j = 1:ntrials
       spikeTrain = trial(j, i).spikes;
       spikeCounts20ms(j, i, :) = sum(reshape(spikeTrain, 20, 25), 1);
   end
   meanSpikeCounts20ms = mean(spikeCounts20ms(:, i, :), 1);
   meanSpikeCounts20ms = squeeze(meanSpikeCounts20ms) * 1000 / 20;
   bar(10:20:500, meanSpikeCounts20ms, 1);
   % plot(10:20:500, meanSpikeCounts20ms, "k", "LineWidth", 1.5);
   hold on;
   if i == 8
       ylabel("f (Hz)")
       xlabel("Time (msec)")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q4.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



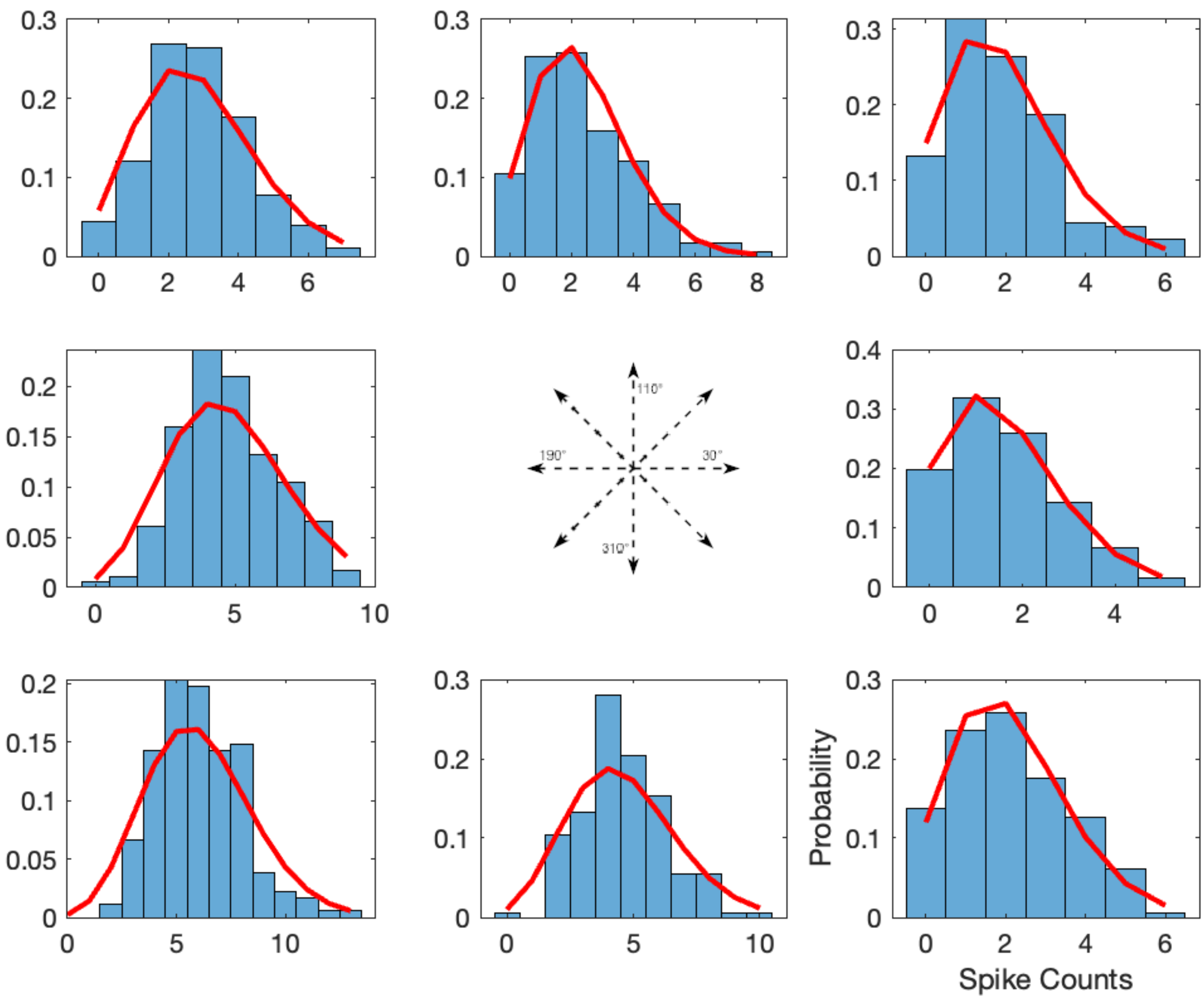
Problem 4(c)

```
figure;
spikeCounts = zeros(ntrials, nangles);
meanFireRates = zeros(1, 8);
for i = 1:nangles
   for j = 1:ntrials
       spikeTrain = trial(j, i).spikes;
       spikeCounts(j, i) = sum(spikeTrain);
       scatter(angles(i) * 180 / pi, spikeCounts(j, i), 50, 'blue', 'filled', 'MarkerFaceAlpha',
0.1); hold on;
   end
   meanFireRates(i) = mean(spikeCounts(:, i)) / 0.5;
end
scatter(angles * 180 / pi, meanFireRates, 50, 'red', 'filled'); hold on;
SEs = @(x) sum((meanFireRates - x(1) - (x(2) - x(1)) * cos(angles - x(3))).^2);
x0 = [0,1,0];
theta = fminsearch(SEs, x0);
r_0 = theta(1);
r_max = theta(2);
s_max = theta(3);
plot(angles * 180 / pi, r_0 + (r_max - r_0) * cos(angles - s_max), 'g', "LineWidth", 2);
xlim([0 360])
xlabel("S (movement direction in degrees")
ylabel("f (Hz)")
```



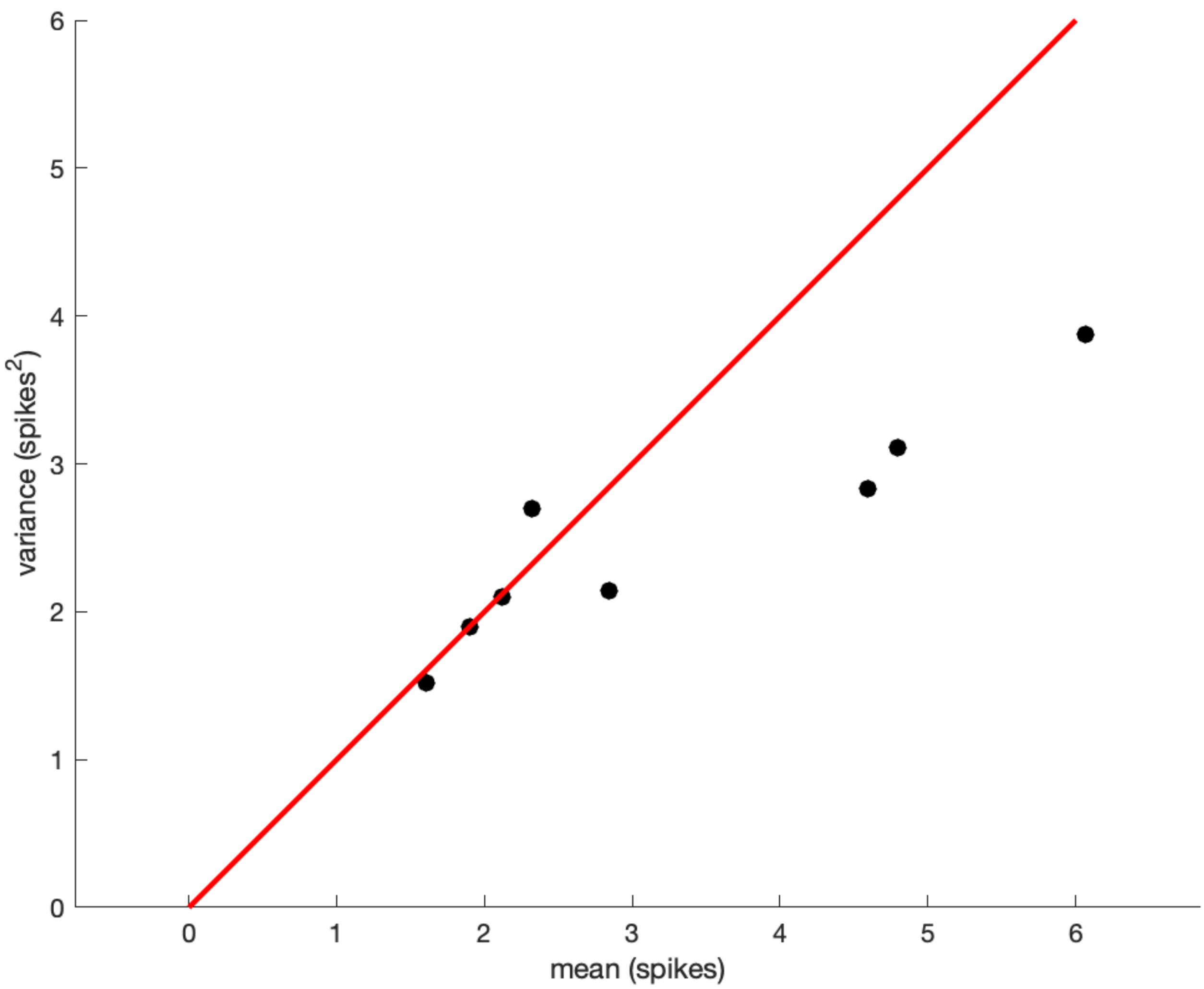
Problem 4(d)

```
figure;
for i = 1:nangles
   subplot(3, 3, positions(i));
   histogram(spikeCounts(:, i), 'Normalization', 'probability');
   hold on;
   x = 0:1:max(spikeCounts(:, i));
   lambda = mean(spikeCounts(:, i));
   plot(x, poisspdf(x, lambda), 'r', 'LineWidth', 2);
   if i == 8
       xlabel("Spike Counts")
       ylabel("Probability")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q4.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```



Problem 4(e)

```
figure;
meanSpikeCounts = zeros(1, 8);
varSpikeCounts = zeros(1, 8);
for i = 1:nangles
   meanSpikeCounts(i) = mean(spikeCounts(:, i));
   varSpikeCounts(i) = var(spikeCounts(:, i));
end
scatter(meanSpikeCounts, varSpikeCounts, 50, "k", "filled");
hold on;
xlabel("mean (spikes)")
ylabel("variance (spikes^2)")
axis equal
% add x = y line
x = 0:1:max(meanSpikeCounts);
plot(x, x, 'r', 'LineWidth', 2);
```



Problem 4(f)

```
figure;
for i = 1:nangles
   subplot(3, 3, positions(i));
   ISIs = [];
   for j = 1:ntrials
       ISIs = [ISIs, diff(find(trial(j, i).spikes))];
   end
   ISIs = ISIs / 1000;
   histogram(ISIs, 'Normalization', 'pdf')
   hold on
   x = 0:0.001:max(ISIs);
   y = exppdf(x, 1 / mean(spikeCounts(:, i)));
   plot(x, y, 'r', 'LineWidth', 2)
   if i == 8
       xlabel("ISIs (s)")
       ylabel("Probability Density")
   end
end
subplot(3, 3, 5);
axis off;
I = imread('arrows-Q4.png', 'BackgroundColor', [1 1 1]);
imshow(I);
```