Question 1

A、1. $(10011011)_2 = 1\times2^0 + 1\times2^1 + 1\times2^4 + 1\times2^7$

$$= \underbrace{1+2+8+16}_{27}+128$$

$$= (155)_{10} \ \#$$

2、$(456)_7 = 6\times7^0 + 5\times7^1 + 4\times7^2$

$$= \underbrace{6+35}_{41}+196$$

$$= (237)_{10} \ \#$$

3、$(38A)_{16} = 10\times16^0 + 8\times16^1 + 3\times16^2$

$$= \underbrace{10+128}_{138}+768$$

$$= (906)_{10} \ \#$$

4. $(2214)_5 = 4\times5^0 + 1\times5^1 + 2\times5^2 + 2\times5^3$

$$= \underbrace{4 + 5}_{9} + \underbrace{50 + 250}_{300}$$

$$= (309)_{10} \quad \#$$

B. 1. $(69)_{10}$

Recursively divide the quotinents and collect reminders.

$$\begin{array}{r|l l}
2 & 69 & \\
2 & 34 & 1 \\
2 & 17 & 0 \\
2 & 8 & 1 \\
2 & 4 & 0 \\
2 & 2 & 0 \\
  & 1 & 0
\end{array}$$

$(1000101)_2$ #

2. $(485)_{10}$

Recursively divide the quotinents and collect reminders

$$\begin{array}{r|l l}
2 & 485 & \\
2 & 242 & 1 \\
2 & 121 & 0 \\
2 & 60 & 1 \\
2 & 30 & 0 \\
2 & 15 & 0 \\
2 & 7 & 1 \\
2 & 3 & 1 \\
  & 1 & 1
\end{array}$$

$(111100101)_2$ #

3. $(6D1A)_{16} = (0110\ 1101\ 0001\ 1010)_2$

hexadecimal	binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
a	1010
b	1011
c	1100
d	1101
e	1110
f	1111

Each digits convert to binary.

$C.1. (11101011)_2 = \underline{(6B)_{16}}$ #

$6 = 0110_2$

$B_{16} = 1011_2$

2. $(895)_{10}$

Recursively divide the quotinents and collect the remainders.

$$\begin{array}{r|l} 16 & 895 \\ \hline 16 & 55 \quad 15 \\ \hline & 3 \quad\ 7 \end{array}$$

$\underline{(37F)_{16}}$ #

Question 2

1. $(7566)_8 + (4515)_8 = \underline{(14303)_8}$

$$
\begin{array}{r}
7566 \\
+\ 4515 \\
\hline
14303
\end{array}
$$

2. $(10110011)_2 + (1101)_2 = \underline{(11000000)_2}$

$$
\begin{array}{r}
10110011 \\
+\ 1101 \\
\hline
11000000
\end{array}
$$

Question 2

3. $(7A66)_{16} + (45C5)_{16} = \underline{(C02B)_{16}}$ //

$$
\begin{array}{r}
{}^{1}7\,{}^{1}A\,6\,6 \\
+\;4\,5\,C\,5 \\
\hline
C\,0\,2\,B
\end{array}
$$

4. $(3022)_5 - (2433)_5 = \underline{(34)_5}$ //

$$
\begin{array}{r}
\overset{2}{\cancel{3}}\,\overset{4}{\cancel{0}}\,\overset{1}{\cancel{2}}\,2 \\
-\;2\,4\,3\,3 \\
\hline
3\,4
\end{array}
$$

Question 3

A. 1. $(124)_{10}$

$$
\begin{array}{r|l l}
2 & 124 & \\
2 & 62 & 0 \\
2 & 31 & 0 \\
2 & 15 & 1 \\
2 & 7 & 1 \\
2 & 3 & 1 \\
 & 1 & 1
\end{array}
$$

We first obtain the binary representation of 124_{10}.

$\rightarrow (1111100)_2$

the 8-bits 2 complement representation of 124_{10}

is $(01111100)_{8\text{-bits 2 complement}}$ #

2. $(-124)_{10}$

$$(1\,0000\,0000)_2 - (01111100)_2 = (10000100)_2$$

$\therefore (10000100)_{8\text{-bits 2 complement}}$ #

Question 3

A. 3. $(109)_{10}$

2	109	
2	54	1
2	27	0
2	13	1
2	6	1
2	3	0
	1	1

We first obtain the binary representation of $(109)_{10}$

$\rightarrow (1101101)_2$

The 8-bits two's complement representation of $(109)_{10}$

is $(01101101)_{\text{8-bits 2 complement}}$ #

Question 3

A. 4. $(-79)_{10}$

First, we obtain the binary representation of $(79)_{10}$

$$\begin{array}{r|l l}
2 & 79 & \\
2 & 39 & 1 \\
2 & 19 & 1 \\
2 & 9 & 1 \\
2 & 4 & 1 \\
2 & 2 & 0 \\
 & 1 & 0
\end{array}$$

$\rightarrow (1001111)_2$

Then, it converts to 8 bit two's complement representation.

$(79)_{10} = (01001111)_2$

Flip the bits

$(10110000)_2$

Then, add 1

(10110001) 8-bits 2 complement

B. 1. $(0001\ 1110)_2 = 2^1 + 2^2 + 2^3 + 2^4$

$\quad\ \ 1\ 111$
$\quad 2^4\,2^3\,2^2\,2^1$

$$= \underbrace{2 + 4 + 8}_{14} + 16$$

$$= \underline{(30)_{10}}\ \#$$

2. $(1110\ 0110)_2 = -(0001\ 1010)_2$ ... Flip and add 1

$\quad 0001\ 1001$

$$= -(2^1 + 2^3 + 2^4)$$

$$= -(2 + 8 + 16)$$

$$= \underline{(-26)_{10}}\ \#$$

3. $(0010\ 1101)_2 = 2^0 + 2^2 + 2^3 + 2^5$

$$= \underbrace{1 + 4 + 8}_{13} + 32$$

$$= \underline{(45)_{10}}\ \#$$

4. $(1001\ 1110)_2 = 2^1 + 2^2 + 2^3 + 2^4 - 2^7$

$\quad 0110\ 0001$

$$= \underbrace{2 + 4 + 8 + 16}_{30} - 128$$

$$= \underline{(-98)_{10}}\ \#$$

Question 4

1. Exercise 1.2.4

(b)

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

(c)

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2. Exercise 1.3.4

(b)

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

(d)

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

1. Ex 1.2.7.

(b) $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$ //

(c) $B \vee (D \wedge M)$ //

2. Ex 1.3.7.

(b) $(S \vee g) \to p$ //

(c) $g \to p$ //

(d) $p \Leftrightarrow (S \wedge g)$ //

(e) $p \to (S \vee g)$ //

3. Ex 1.3.9

(c) $C \to p$ //

(d) $p \to C$ //

Question 6

1. Ex 1.3.6.

(b) If Joe maintains a B average, then he is eligible for the honors program.

(c) If Rajiv can go on the roller coaster, then he is at least four feet tall.

(d) If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Ex. 1.3.10. (p: T, q: F, r: ?)

(c) False

$\because$ $(p \vee r)$ is true, and $(q \wedge r)$ is false.

Then, $(p \vee r) \leftrightarrow (q \wedge r)$ is false.

(d) unknown

$\because$ $(p \wedge r)$ is True or False,

and $(q \wedge r)$ is False.

Then, $(p \wedge r) \leftrightarrow (q \wedge r)$ depends on $(p \wedge r)$.

Question 6

2. Ex. 1-3.10. (p: T, q: F, r: ?)

(e) unknown

$\because$ p is Truth, and $(r \vee q)$ is True or False.

Then, $p \to (r \vee q)$ depends on $(r \vee q)$

(f) True

$\because$ $(p \wedge q)$ is false, so $(p \wedge q) \Rightarrow r$ is True.

Question 7

Ex. 1.4.5.

(b) $\neg j \to (l \vee \neg r)$

$(r \wedge \neg l) \to j$

equivalent

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

(c) $j \to \neg l$

$\neg j \to l$

not equivalent

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

(d) $(r \vee \neg l) \to j$

$j \to r \wedge \neg l$

not equivalent

j	l	r	$(r \vee \neg l) \to j$	$j \to r \wedge \neg l$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8

1. Ex 1.5.2

(c) $(p \to q) \wedge (p \to r)$ hypothesis

$(\neg p \vee q) \wedge (\neg p \vee r)$ conditional identities

$\neg p \vee (q \wedge r)$ Distributive law

$p \to (q \wedge r)$ conditional identities

(f) $\neg (p \vee (\neg p \wedge q))$ hypothesis

$\neg ((p \vee \neg p) \wedge (p \vee q))$ Distributive law

$\neg (T \wedge (p \vee q))$ Complement law

$\neg (p \vee q)$ identity law

$\neg p \wedge \neg q$. De Morgan's law

(i) $(p \wedge q) \to r$ hypothesis

$\neg (p \wedge q) \vee r$ conditional identities

$(\neg p \vee \neg q) \vee r$ De Morgan's law

$(\neg p \vee r) \vee \neg q$ Associative law

$\neg (p \wedge \neg r) \vee \neg q$ De Morgan's law

$(p \wedge \neg r) \to \neg q$ conditional identities

2. Ex. 1.5.3.

(c) $\neg r \vee (\neg r \rightarrow p)$ hypothesis

$\neg r \vee (r \vee p)$ conditional

$(\neg r \vee r) \vee p$ Associative

$T \vee p$ complement

T Domination

(d) $\neg(p \rightarrow q) \rightarrow \neg q$ hypothesis

$(p \rightarrow q) \vee \neg q$ conditional

$(\neg p \vee q) \vee \neg q$ conditional

$\neg p \vee (q \vee \neg q)$ Associative

$\neg p \vee T$ complement

T Domination

Question 9

1. Ex 1.6.3.

(c) $\exists x\,(x = x^2)$

(d) $\forall x\,(x \leqq x^2 + 1)$

2. Ex. 1.7.4.

(b) $\forall x\,(\neg S(x) \wedge W(x))$

(c) $\forall x\,(S(x) \rightarrow \neg W(x))$

(d) $\exists x\,(S(x) \wedge W(x))$

Question 10

1. Ex 1.7.9

(c) T. example: a,

(d) T example: e,

(e) T

(f) T

(g) F counterexample: c,

(h) T

(i) T example: a,

2. Ex 1.9.2

(b) T Let $x=2$, then $Q(x,1)$, $Q(x,2)$, $Q(x,3)$ are True.

(c) T Let $x=1$, then $P(1,x)$, $P(2,x)$, $P(3,x)$ are all True.

(d) F

(e) F

(f) T when $y=1$, then $P(1,y)$, $P(2,y)$, $P(3,y)$ are True.

(g) F counter example: $(x,y)=(1,2)$

(h) T example: $(x,y)=(2,1)$

(i) T

Question 11

1. Ex 1.10.4

(c) $\exists x \exists y (x + y = xy)$

(d) $\forall x \forall y ((x > 0 \wedge y > 0) \rightarrow \frac{y}{x} > 0)$

(e) $\forall (x) ((x > 0 \wedge x < 1) \rightarrow \frac{1}{x} > 1)$

(f) $\neg \exists x \forall y (x <= y)$

(g) $\forall x \exists y ((x \neq 0) \rightarrow y = \frac{1}{x})$

2. Ex 1.10.7

(c) $\exists x (N(x) \wedge D(x))$

(d) $\forall x (D(x) \rightarrow P(\text{Sam}, x))$

(e) $\exists x \forall y (N(x) \wedge P(x, y))$

(f) .

$\exists x (N(x) \wedge D(x) \wedge \forall y ((x \neq y) \rightarrow \neg (N(y) \wedge D(y)))$

$\exists x \forall y (N(x) \wedge D(x) \wedge ((x \neq y) \rightarrow \neg (N(y) \wedge D(y))))$

3. Ex. 1.10-10.

(c) $\forall x \exists y \, (y \neq \text{Math101} \land T(x,y))$

(d) $\exists x \forall y \, (y \neq \text{Math101} \rightarrow T(x,y))$

(e) $\forall x \exists y \exists z \, ((x \neq \text{Sam}) \rightarrow ((y \neq z) \land T(x,y) \land T(x,z)))$

(f). $\exists x \exists y \, (((x \neq y) \land (T(\text{sam},x) \land T(\text{sam},y))) \land \forall z \, ((x \neq z) \rightarrow \neg T(\text{sam},z)))$

$\exists x \exists y \forall z \, (((x \neq y) \land T(\text{sam},x) \land T(\text{sam},y)) \land ((x \neq z \land y \neq z) \rightarrow \neg T(\text{sam},z)))$

//

Question 12

1. Ex 1.8.2

(b) $\forall x\,(D(x) \vee P(x))$

$\neg \forall x\,(D(x) \vee P(x))$

$\exists x\,(\neg D(x) \wedge \neg P(x))$

There is a patient who was not given the medication and the placebo

(c) $\exists x\,(D(x) \wedge M(x))$

$\neg \exists x\,(D(x) \wedge M(x))$

$\forall x\,(\neg D(x) \vee \neg M(x))$

Every patient was not given the medication or did not have migraines or both.

(d) $\forall x\,(P(x) \Rightarrow M(x))$

$\forall x\,(\neg P(x) \vee M(x))$

$\neg \forall x\,(\neg P(x) \vee M(x))$

$\exists x\,(P(x) \wedge \neg M(x))$

Some patients was given the placebo and didn't have migraines.

(e) $\exists x\,(M(x) \wedge P(x))$

$\neg \exists x\,(M(x) \wedge P(x))$

$\forall x\,(\neg M(x) \vee \neg P(x))$

Every patient didn't have migraines or was not given the placebo or both.

2. Ex 1.9.4.

(c) $\neg\left(\exists x \forall y\,(P(x,y) \rightarrow Q(x,y))\right)$

$\neg\left(\exists x \forall y\,(\neg P(x,y) \vee Q(x,y))\right)$

$\forall x \exists y\,(P(x,y) \wedge \neg Q(x,y))$

(d) $\neg\left(\exists x \forall y\,(P(x,y) \leftrightarrow P(y,x))\right)$

$\neg\left(\exists x \forall y\,(P(x,y) \rightarrow P(y,x) \wedge P(y,x) \rightarrow P(x,y))\right)$

$\neg\left(\exists x \forall y\,((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))\right)$

$\forall x \exists y\,(\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$

$\forall x \exists y\,((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

(e) $\neg\left(\exists x \exists y\, P(x,y) \wedge \forall x \forall y\, Q(x,y)\right)$

$\forall x \forall y\, \neg P(x,y) \vee \exists x \exists y\, \neg Q(x,y)$