SM

PyR@TE 3.0

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Contents

1 Model

1.1 Gauge groups

Name	Type	Abelian	Coupling constant
U1Y	$U(1)$	True	$g_1 \to \sqrt{\frac{5}{3}} g_1$
SU2L	$SU(2)$	False	g_2
SU3c	$SU(3)$	False	g_3

1.2 Fermions

Name	Generations	U1Y × SU2L × SU3c
Q	3	$(+\frac{1}{6}, \mathbf{2}, \mathbf{3})$
L	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
u_R	3	$(+\frac{2}{3}, \mathbf{1}, \mathbf{3})$
d_R	3	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$
e_R	3	$(-1, \mathbf{1}, \mathbf{1})$

1.3 Scalars

Name	Complex	Expression	Generations	U1Y × SU2L × SU3c
H	True	$\frac{1}{\sqrt{2}} (\Pi + i\,\Sigma)$	1	$(+\frac{1}{2}, \mathbf{2}, \mathbf{1})$

2 Lagrangian

2.1 Definitions

$$\tilde{H}_i = \epsilon_{i,j} H_j^\dagger$$

2.2 Yukawa couplings

$$-\mathcal{L}_Y = \ + Y_{u f_1,f_2} \tilde{H}_i \overline{Q}_{f_1,i,a} u_{R f_2,a} + Y_{d f_1,f_2} \overline{Q}_{f_1,i,a} H_i d_{R f_2,a} + Y_{e f_1,f_2} \overline{L}_{f_1,i} H_i e_{R f_2} + \text{h.c.}$$

2.3 Quartic couplings

$$-\mathcal{L}_Q = +\lambda H_i^\dagger H_i H_{i_1}^\dagger H_{i_1}$$

2.4 Scalar mass couplings

$$-\mathcal{L}_{sm} = -\mu H_i^\dagger H_i$$

3 Renormalization Group Equations

3.1 Convention

$$\beta\left(X\right) \equiv \mu \frac{dX}{d\mu} \equiv \frac{1}{\left(4\pi\right)^2}\beta^{(1)}(X) + \frac{1}{\left(4\pi\right)^4}\beta^{(2)}(X) + \frac{1}{\left(4\pi\right)^6}\beta^{(3)}(X)$$

3.2 Gauge couplings

$$\beta^{(1)}(g_1) = \frac{41}{10}g_1^3$$

$$\beta^{(2)}(g_1) = +\frac{199}{50}g_1^5 + \frac{27}{10}g_1^3g_2^2 + \frac{44}{5}g_1^3g_3^2 - \frac{17}{10}g_1^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) - \frac{1}{2}g_1^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{3}{2}g_1^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)$$

$$\begin{aligned}\beta^{(3)}(g_1) = & -\frac{388613}{24000}g_1^7 + \frac{123}{160}g_1^5g_2^2 - \frac{137}{75}g_1^5g_3^2 + \frac{789}{64}g_1^3g_2^4 - \frac{3}{5}g_1^3g_2^2g_3^2 + \frac{297}{5}g_1^3g_3^4 + \frac{27}{50}g_1^5\lambda \\ & + \frac{9}{10}g_1^3g_2^2\lambda - \frac{9}{5}g_1^3\lambda^2 - \frac{2827}{800}g_1^5\mathrm{Tr}\left(Y_u^\dagger Y_u\right) - \frac{1267}{800}g_1^5\mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{2529}{800}g_1^5\mathrm{Tr}\left(Y_e^\dagger Y_e\right) \\ & - \frac{471}{32}g_1^3g_2^2\mathrm{Tr}\left(Y_u^\dagger Y_u\right) - \frac{1311}{160}g_1^3g_2^2\mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{1629}{160}g_1^3g_2^2\mathrm{Tr}\left(Y_e^\dagger Y_e\right) \\ & - \frac{29}{5}g_1^3g_3^2\mathrm{Tr}\left(Y_u^\dagger Y_u\right) - \frac{17}{5}g_1^3g_3^2\mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \frac{303}{40}g_1^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2 + \frac{339}{80}g_1^3\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) \\ & + \frac{3}{8}g_1^3\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right) + \frac{177}{20}g_1^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{51}{40}g_1^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)^2 \\ & + \frac{183}{80}g_1^3\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) + \frac{157}{20}g_1^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_e^\dagger Y_e\right) + \frac{199}{20}g_1^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\ & + \frac{99}{40}g_1^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)^2 + \frac{261}{80}g_1^3\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)\end{aligned}$$

$$\beta^{(1)}(g_2) = -\frac{19}{6}g_2^3$$

$$\beta^{(2)}(g_2) = +\frac{9}{10}g_1^2g_2^3 + \frac{35}{6}g_2^5 + 12g_2^3g_3^2 - \frac{3}{2}g_2^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) - \frac{3}{2}g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{1}{2}g_2^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)$$

$$\begin{aligned}\beta^{(3)}(g_2) = & -\frac{5597}{1600}g_1^4g_2^3+\frac{873}{160}g_1^2g_2^5-\frac{1}{5}g_1^2g_2^3g_3^2+\frac{324953}{1728}g_2^7+39g_2^5g_3^2+81g_2^3g_3^4+\frac{3}{10}g_1^2g_2^3\lambda+\frac{3}{2}g_2^5\lambda \\ & -3g_2^3\lambda^2-\frac{593}{160}g_1^2g_2^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{533}{160}g_1^2g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{51}{32}g_1^2g_2^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right) \\ & -\frac{729}{32}g_2^5\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{729}{32}g_2^5\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{243}{32}g_2^5\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-7g_2^3g_3^2\mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\ & -7g_2^3g_3^2\mathrm{Tr}\left(Y_d^\dagger Y_d\right)+\frac{45}{8}g_2^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2+\frac{57}{16}g_2^3\mathrm{Tr}\left(Y_u^\dagger Y_uY_u^\dagger Y_u\right)+\frac{27}{8}g_2^3\mathrm{Tr}\left(Y_u^\dagger Y_dY_d^\dagger Y_u\right) \\ & +\frac{45}{4}g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+\frac{45}{8}g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)^2+\frac{57}{16}g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_dY_d^\dagger Y_d\right) \\ & +\frac{15}{4}g_2^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_e^\dagger Y_e\right)+\frac{15}{4}g_2^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+\frac{5}{8}g_2^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)^2 \\ & +\frac{19}{16}g_2^3\mathrm{Tr}\left(Y_e^\dagger Y_eY_e^\dagger Y_e\right)\end{aligned}$$

$$\beta^{(1)}(g_3) = -7g_3^3$$

$$\beta^{(2)}(g_3) = +\frac{11}{10}g_1^2g_3^3+\frac{9}{2}g_2^2g_3^3-26g_3^5-2g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-2g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)$$

$$\begin{aligned}\beta^{(3)}(g_3) = & -\frac{523}{120}g_1^4g_3^3-\frac{3}{40}g_1^2g_2^2g_3^3+\frac{77}{15}g_1^2g_3^5+\frac{109}{8}g_2^4g_3^3+21g_2^2g_3^5+\frac{65}{2}g_3^7-\frac{101}{40}g_1^2g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\ & -\frac{89}{40}g_1^2g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{93}{8}g_2^2g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{93}{8}g_2^2g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-40g_3^5\mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\ & -40g_3^5\mathrm{Tr}\left(Y_d^\dagger Y_d\right)+\frac{9}{2}g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_uY_u^\dagger Y_u\right)+\frac{21}{2}g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)^2-3g_3^3\mathrm{Tr}\left(Y_u^\dagger Y_dY_d^\dagger Y_u\right) \\ & +21g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+\frac{9}{2}g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_dY_d^\dagger Y_d\right)+\frac{21}{2}g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)^2 \\ & +\frac{7}{2}g_3^3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\mathrm{Tr}\left(Y_e^\dagger Y_e\right)+\frac{7}{2}g_3^3\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\mathrm{Tr}\left(Y_u^\dagger Y_u\right)\end{aligned}$$

3.3 Yukawa couplings

$$\begin{aligned}\beta^{(1)}(Y_u) = & +\frac{3}{2}Y_uY_u^\dagger Y_u-\frac{3}{2}Y_dY_d^\dagger Y_u+3\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_u+3\mathrm{Tr}\left(Y_d^\dagger Y_d\right)Y_u+\mathrm{Tr}\left(Y_e^\dagger Y_e\right)Y_u-\frac{17}{20}g_1^2Y_u \\ & -\frac{9}{4}g_2^2Y_u-8g_3^2Y_u\end{aligned}$$

$$\begin{aligned}\beta^{(2)}(Y_u) = & +\frac{3}{2}Y_uY_u^\dagger Y_uY_u^\dagger Y_u-\frac{1}{4}Y_uY_u^\dagger Y_dY_d^\dagger Y_u-Y_dY_d^\dagger Y_uY_u^\dagger Y_u+\frac{11}{4}Y_dY_d^\dagger Y_dY_d^\dagger Y_u \\ & -\frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_uY_u^\dagger Y_u\right)Y_u-\frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_uY_u^\dagger Y_u+\frac{15}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u\right)Y_dY_d^\dagger Y_u\end{aligned}$$

$$
\begin{aligned}
&+\frac{3}{2}\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right) Y_u - \frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_u Y_u^\dagger Y_u - \frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) Y_u \\
&+\frac{15}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d Y_d^\dagger Y_u - \frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_u Y_u^\dagger Y_u + \frac{5}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_d Y_d^\dagger Y_u \\
&-\frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right) Y_u - 12\lambda Y_u Y_u^\dagger Y_u + 6\lambda^2 Y_u + \frac{223}{80} g_1^2 Y_u Y_u^\dagger Y_u + \frac{135}{16} g_2^2 Y_u Y_u^\dagger Y_u \\
&+16 g_3^2 Y_u Y_u^\dagger Y_u - \frac{43}{80} g_1^2 Y_d Y_d^\dagger Y_u + \frac{9}{16} g_2^2 Y_d Y_d^\dagger Y_u - 16 g_3^2 Y_d Y_d^\dagger Y_u + \frac{17}{8} g_1^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_u \\
&+\frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_u + 20 g_3^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_u + \frac{5}{8} g_1^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_u + \frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_u \\
&+20 g_3^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_u + \frac{15}{8} g_1^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_u + \frac{15}{8} g_2^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_u + \frac{1187}{600} g_1^4 Y_u \\
&-\frac{9}{20} g_1^2 g_2^2 Y_u + \frac{19}{15} g_1^2 g_3^2 Y_u - \frac{23}{4} g_2^4 Y_u + 9 g_2^2 g_3^2 Y_u - 108 g_3^4 Y_u
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}(Y_d) = &-\frac{3}{2} Y_u Y_u^\dagger Y_d + \frac{3}{2} Y_d Y_d^\dagger Y_d + 3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_d + 3\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d + \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_d - \frac{1}{4} g_1^2 Y_d \\
&-\frac{9}{4} g_2^2 Y_d - 8 g_3^2 Y_d
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}(Y_d) = &+\frac{11}{4} Y_u Y_u^\dagger Y_u Y_u^\dagger Y_d - Y_u Y_u^\dagger Y_d Y_d^\dagger Y_d - \frac{1}{4} Y_d Y_d^\dagger Y_u Y_u^\dagger Y_d + \frac{3}{2} Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d \\
&-\frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) Y_d + \frac{15}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_u Y_u^\dagger Y_d - \frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_d Y_d^\dagger Y_d \\
&+\frac{3}{2}\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right) Y_d + \frac{15}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_u Y_u^\dagger Y_d - \frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) Y_d \\
&-\frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d Y_d^\dagger Y_d + \frac{5}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_u Y_u^\dagger Y_d - \frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_d Y_d^\dagger Y_d \\
&-\frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right) Y_d - 12\lambda Y_d Y_d^\dagger Y_d + 6\lambda^2 Y_d - \frac{79}{80} g_1^2 Y_u Y_u^\dagger Y_d + \frac{9}{16} g_2^2 Y_u Y_u^\dagger Y_d \\
&-16 g_3^2 Y_u Y_u^\dagger Y_d + \frac{187}{80} g_1^2 Y_d Y_d^\dagger Y_d + \frac{135}{16} g_2^2 Y_d Y_d^\dagger Y_d + 16 g_3^2 Y_d Y_d^\dagger Y_d + \frac{17}{8} g_1^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_d \\
&+\frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_d + 20 g_3^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_d + \frac{5}{8} g_1^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d + \frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d \\
&+20 g_3^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_d + \frac{15}{8} g_1^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_d + \frac{15}{8} g_2^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_d - \frac{127}{600} g_1^4 Y_d - \frac{27}{20} g_1^2 g_2^2 Y_d \\
&+\frac{31}{15} g_1^2 g_3^2 Y_d - \frac{23}{4} g_2^4 Y_d + 9 g_2^2 g_3^2 Y_d - 108 g_3^4 Y_d
\end{aligned}
$$

$$
\beta^{(1)}(Y_e) = +\frac{3}{2} Y_e Y_e^\dagger Y_e + 3\mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_e + 3\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_e + \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_e - \frac{9}{4} g_1^2 Y_e - \frac{9}{4} g_2^2 Y_e
$$

$$
\begin{aligned}
\beta^{(2)}(Y_e) = &+\frac{3}{2} Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e - \frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) Y_e - \frac{27}{4}\mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_e Y_e^\dagger Y_e \\
&+\frac{3}{2}\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right) Y_e - \frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) Y_e - \frac{27}{4}\mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_e Y_e^\dagger Y_e
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right) Y_e - \frac{9}{4}\mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_e Y_e^\dagger Y_e - 12\lambda Y_e Y_e^\dagger Y_e + 6\lambda^2 Y_e + \frac{387}{80} g_1^2 Y_e Y_e^\dagger Y_e \\
&+ \frac{135}{16} g_2^2 Y_e Y_e^\dagger Y_e + \frac{17}{8} g_1^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_e + \frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_e + 20 g_3^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) Y_e \\
&+ \frac{5}{8} g_1^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_e + \frac{45}{8} g_2^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_e + 20 g_3^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) Y_e + \frac{15}{8} g_1^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_e \\
&+ \frac{15}{8} g_2^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) Y_e + \frac{1371}{200} g_1^4 Y_e + \frac{27}{20} g_1^2 g_2^2 Y_e - \frac{23}{4} g_2^4 Y_e
\end{aligned}
$$

3.4 Quartic couplings

$$
\begin{aligned}
\beta^{(1)}(\lambda) = &+ 24\lambda^2 - \frac{9}{5} g_1^2 \lambda - 9 g_2^2 \lambda + \frac{27}{200} g_1^4 + \frac{9}{20} g_1^2 g_2^2 + \frac{9}{8} g_2^4 + 12\lambda \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + 12\lambda \mathrm{Tr}\left(Y_d^\dagger Y_d\right) \\
&+ 4\lambda \mathrm{Tr}\left(Y_e^\dagger Y_e\right) - 6\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) - 6\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) - 2\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}(\lambda) = &- 312\lambda^3 + \frac{108}{5} g_1^2 \lambda^2 + 108 g_2^2 \lambda^2 + \frac{1887}{200} g_1^4 \lambda + \frac{117}{20} g_1^2 g_2^2 \lambda - \frac{73}{8} g_2^4 \lambda - \frac{3411}{2000} g_1^6 \\
&- \frac{1677}{400} g_1^4 g_2^2 - \frac{289}{80} g_1^2 g_2^4 + \frac{305}{16} g_2^6 - 144\lambda^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) - 144\lambda^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) \\
&- 48\lambda^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) + \frac{17}{2} g_1^2 \lambda \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{5}{2} g_1^2 \lambda \mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \frac{15}{2} g_1^2 \lambda \mathrm{Tr}\left(Y_e^\dagger Y_e\right) \\
&+ \frac{45}{2} g_2^2 \lambda \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{45}{2} g_2^2 \lambda \mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \frac{15}{2} g_2^2 \lambda \mathrm{Tr}\left(Y_e^\dagger Y_e\right) + 80 g_3^2 \lambda \mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\
&+ 80 g_3^2 \lambda \mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{171}{100} g_1^4 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{9}{20} g_1^4 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{9}{4} g_1^4 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) \\
&+ \frac{63}{10} g_1^2 g_2^2 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{27}{10} g_1^2 g_2^2 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \frac{33}{10} g_1^2 g_2^2 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) - \frac{9}{4} g_2^4 \mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\
&- \frac{9}{4} g_2^4 \mathrm{Tr}\left(Y_d^\dagger Y_d\right) - \frac{3}{4} g_2^4 \mathrm{Tr}\left(Y_e^\dagger Y_e\right) - 3\lambda \mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) - 42\lambda \mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right) \\
&- 3\lambda \mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) - \lambda \mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right) - \frac{8}{5} g_1^2 \mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) + \frac{4}{5} g_1^2 \mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) \\
&- \frac{12}{5} g_1^2 \mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right) - 32 g_3^2 \mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right) - 32 g_3^2 \mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right) \\
&+ 30\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u\right) - 6\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_d Y_d^\dagger Y_u\right) - 6\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger Y_u\right) \\
&+ 30\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d Y_d^\dagger Y_d\right) + 10\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e Y_e^\dagger Y_e\right)
\end{aligned}
$$

3.5 Scalar mass couplings

$$
\beta^{(1)}(\mu) = - \frac{9}{10} g_1^2 \mu - \frac{9}{2} g_2^2 \mu + 12\lambda\mu + 6\mu \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + 6\mu \mathrm{Tr}\left(Y_d^\dagger Y_d\right) + 2\mu \mathrm{Tr}\left(Y_e^\dagger Y_e\right)
$$

$$
\begin{aligned}
\beta^{(2)}(\mu) = &+ \frac{1671}{400} g_1^4 \mu + \frac{9}{8} g_1^2 g_2^2 \mu - \frac{145}{16} g_2^4 \mu + \frac{72}{5} g_1^2 \lambda \mu + 72 g_2^2 \lambda \mu - 60\lambda^2 \mu + \frac{17}{4} g_1^2 \mu \mathrm{Tr}\left(Y_u^\dagger Y_u\right) \\
&+ \frac{5}{4} g_1^2 \mu \mathrm{Tr}\left(Y_d^\dagger Y_d\right) + \frac{15}{4} g_1^2 \mu \mathrm{Tr}\left(Y_e^\dagger Y_e\right) + \frac{45}{4} g_2^2 \mu \mathrm{Tr}\left(Y_u^\dagger Y_u\right) + \frac{45}{4} g_2^2 \mu \mathrm{Tr}\left(Y_d^\dagger Y_d\right)
\end{aligned}
$$

$$+\frac{15}{4}g_2^2\mu\mathrm{Tr}\left(Y_e^\dagger Y_e\right)+40g_3^2\mu\mathrm{Tr}\left(Y_u^\dagger Y_u\right)+40g_3^2\mu\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-72\lambda\mu\mathrm{Tr}\left(Y_u^\dagger Y_u\right)$$
$$-72\lambda\mu\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-24\lambda\mu\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-\frac{27}{2}\mu\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right)-21\mu\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right)$$
$$-\frac{27}{2}\mu\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right)-\frac{9}{2}\mu\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)$$

3.6 Vacuum-expectation values

Definitions:

$$H\ :\ \frac{1}{\sqrt{2}}\Pi_2\to\frac{1}{\sqrt{2}}\left(\Pi_2+v\right)$$

RGEs:

$$\beta^{(1)}(v)=+\frac{9}{20}g_1^2v+\frac{3}{20}\xi g_1^2v+\frac{9}{4}g_2^2v+\frac{3}{4}\xi g_2^2v-3v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-3v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-v\mathrm{Tr}\left(Y_e^\dagger Y_e\right)$$

$$\begin{aligned}\beta^{(2)}(v)=&-\frac{1293}{800}g_1^4v+\frac{9}{200}\xi g_1^4v+\frac{9}{200}\xi^2g_1^4v-\frac{27}{80}g_1^2g_2^2v+\frac{9}{20}\xi g_1^2g_2^2v+\frac{9}{20}\xi^2g_1^2g_2^2v+\frac{271}{32}g_2^4v\\&+\frac{27}{8}\xi g_2^4v-\frac{17}{8}g_1^2v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{9}{10}\xi g_1^2v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{5}{8}g_1^2v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)\\&-\frac{9}{10}\xi g_1^2v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{15}{8}g_1^2v\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-\frac{3}{10}\xi g_1^2v\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-\frac{45}{8}g_2^2v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)\\&-\frac{9}{2}\xi g_2^2v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-\frac{45}{8}g_2^2v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{9}{2}\xi g_2^2v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)-\frac{15}{8}g_2^2v\mathrm{Tr}\left(Y_e^\dagger Y_e\right)\\&-\frac{3}{2}\xi g_2^2v\mathrm{Tr}\left(Y_e^\dagger Y_e\right)-20g_3^2v\mathrm{Tr}\left(Y_u^\dagger Y_u\right)-20g_3^2v\mathrm{Tr}\left(Y_d^\dagger Y_d\right)+\frac{27}{4}v\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger Y_u\right)\\&-\frac{3}{2}v\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger Y_u\right)+\frac{27}{4}v\mathrm{Tr}\left(Y_d^\dagger Y_d Y_d^\dagger Y_d\right)+\frac{9}{4}v\mathrm{Tr}\left(Y_e^\dagger Y_e Y_e^\dagger Y_e\right)-6\lambda^2v\end{aligned}$$

A Group theoretical information

A.1 Gauge groups

Group	Lie algebra	Dim.	Rank	Representations			
				Name / Dim.	Dynkin labels	Index	Reality
SU2	A1	3	1	$\mathbf{2}$	[1]	1/2	Pseudo-real
				$\overline{\mathbf{2}}$	[1, True]	1/2	Pseudo-real
SU3	A2	8	2	$\mathbf{3}$	[1, 0]	1/2	Complex
				$\overline{\mathbf{3}}$	[0, 1]	1/2	Complex