問1

$$Y = \begin{bmatrix} x_{11}w_{11} + x_{12}w_{21} & x_{11}w_{12} + x_{12}w_{22} & x_{11}w_{13} + x_{12}w_{23} \\ x_{21}w_{11} + x_{22}w_{21} & x_{21}w_{12} + x_{22}w_{22} & x_{21}w_{13} + x_{22}w_{23} \end{bmatrix}$$

問2

$$\frac{\partial L}{\partial Y} = \begin{bmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{bmatrix}$$

問3

$$\frac{\partial y_{1i}}{\partial x_{11}} = w_{1i}, \quad \frac{\partial y_{2i}}{\partial x_{11}} = 0 \text{ より}$$

$$\frac{\partial L}{\partial x_{11}} = \sum_{j=1}^{2}\sum_{i=1}^{3} \frac{\partial L}{\partial y_{ji}} \frac{\partial y_{ji}}{\partial x_{11}} = w_{11}\frac{\partial L}{\partial y_{11}} + w_{12}\frac{\partial L}{\partial y_{12}} + w_{13}\frac{\partial L}{\partial y_{13}}$$

問4

$$\frac{\partial y_{1i}}{\partial x_{12}} = w_{2i}, \quad \frac{\partial y_{2i}}{\partial x_{12}} = 0 \text{ より}$$

$$\frac{\partial L}{\partial x_{12}} = w_{21}\frac{\partial L}{\partial y_{11}} + w_{22}\frac{\partial L}{\partial y_{12}} + w_{23}\frac{\partial L}{\partial y_{13}}$$

$$\frac{\partial y_{1i}}{\partial x_{21}} = 0, \quad \frac{\partial y_{2i}}{\partial x_{21}} = w_{1i} \text{ より}$$

$$\frac{\partial L}{\partial x_{21}} = w_{11}\frac{\partial L}{\partial y_{21}} + w_{12}\frac{\partial L}{\partial y_{22}} + w_{13}\frac{\partial L}{\partial y_{23}}$$

$$\frac{\partial y_{1i}}{\partial x_{22}} = 0, \quad \frac{\partial y_{2i}}{\partial x_{22}} = w_{2i} \text{ より}$$

$$\frac{\partial L}{\partial x_{22}} = w_{21}\frac{\partial L}{\partial y_{21}} + w_{22}\frac{\partial L}{\partial y_{22}} + w_{23}\frac{\partial L}{\partial y_{23}}$$

問5

問5

$$\frac{\partial L}{\partial X} = \begin{bmatrix} \frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} \\ \frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}\frac{\partial L}{\partial y_{11}} + w_{12}\frac{\partial L}{\partial y_{12}} + w_{13}\frac{\partial L}{\partial y_{13}} & w_{21}\frac{\partial L}{\partial y_{11}} + w_{22}\frac{\partial L}{\partial y_{12}} + w_{23}\frac{\partial L}{\partial y_{13}} \\ w_{11}\frac{\partial L}{\partial y_{21}} + w_{12}\frac{\partial L}{\partial y_{22}} + w_{13}\frac{\partial L}{\partial y_{23}} & w_{21}\frac{\partial L}{\partial y_{21}} + w_{22}\frac{\partial L}{\partial y_{22}} + w_{23}\frac{\partial L}{\partial y_{23}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ w_{13} & w_{23} \end{bmatrix} = \frac{\partial L}{\partial Y} W^T$$

問6

$$\frac{\partial L}{\partial W} = \begin{bmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} & \frac{\partial L}{\partial w_{13}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} & \frac{\partial L}{\partial w_{23}} \end{bmatrix}$$

(例えば $\frac{\partial L}{\partial w_{11}} = \sum_{j=1}^{2}\sum_{i=1}^{3}\frac{\partial L}{\partial y_{ji}}\frac{\partial y_{ji}}{\partial w_{11}}$ について
$\frac{\partial y_{j1}}{\partial w_{11}} = x_{j1}$, $\frac{\partial y_{j2}}{\partial w_{11}} = \frac{\partial y_{j3}}{\partial w_{11}} = 0$
を用いて計算できる.)

$$= \begin{bmatrix} x_{11}\frac{\partial L}{\partial y_{11}} + x_{21}\frac{\partial L}{\partial y_{21}} & x_{11}\frac{\partial L}{\partial y_{12}} + x_{21}\frac{\partial L}{\partial y_{22}} & x_{11}\frac{\partial L}{\partial y_{13}} + x_{21}\frac{\partial L}{\partial y_{23}} \\ x_{21}\frac{\partial L}{\partial y_{11}} + x_{22}\frac{\partial L}{\partial y_{21}} & x_{21}\frac{\partial L}{\partial y_{12}} + x_{22}\frac{\partial L}{\partial y_{22}} & x_{21}\frac{\partial L}{\partial y_{13}} + x_{22}\frac{\partial L}{\partial y_{23}} \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{bmatrix} = X^T \frac{\partial L}{\partial Y}$$