Introduction to Data Mining

Lecture7 Mining Frequent Patterns, Association and Correlations

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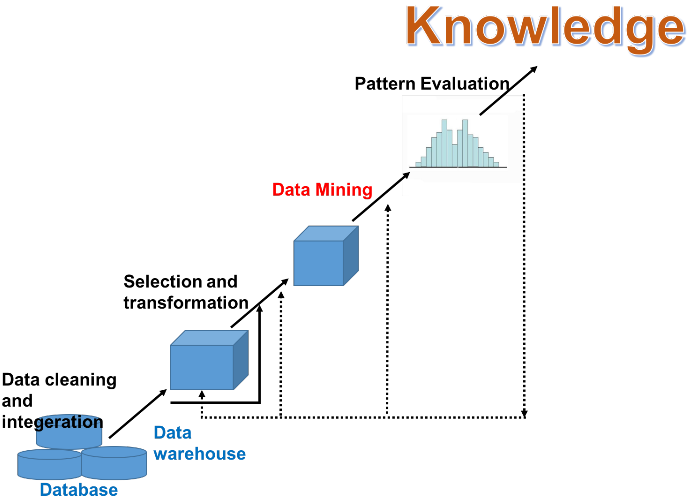
KDD Process

Data Mining-Core of Knowledge discovery process



Mining Frequent Patterns, Association and Correlations

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Mining Frequent Patterns

- Basic Concepts
- Boolean Association Rules
- FP-Tree
- Mining multilevel association rules
- Mining multidimensional association rules
- Sequential Patterns
- Summary

- Basic Concepts
- Mining single-dimensional Boolean association rules
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Basic Concepts

- **What are patterns?**
  - **Patterns**: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - **Patterns** represent intrinsic and important properties of datasets
- **Pattern discovery**: Uncovering patterns from massive data sets
- **Motivation examples:**
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What code segments likely contain copy-and-paste bugs?
  - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

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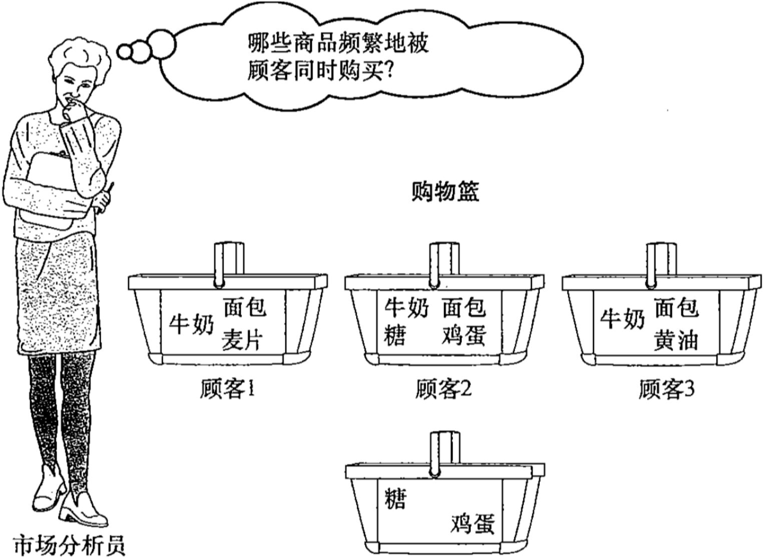
- Finding **inherent regularities** in a data set
- **Foundation** for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- **Broad applications**
  - Basket data analysis
  - Cross-marketing
  - Catalog design
  - Sale campaign analysis
  - Web log (click stream) analysis
  - DNA sequence analysis

Market Basket Analysis



What Is Association Rules Mining?

- **Association rules mining**
  - Finding frequent patterns, associations among sets of items or objects in transaction databases, relational databases, and other information repositories
- **Examples**
  - What products were often purchased together? —Beer and diapers?
  - What DNA segments often occur together in DNA sequences?
- **Where does the data come from?**
  - Supermarket transactions, membership cards, discount coupons, customer complaint calls

Basic Concepts

Transaction-ID	Items bought
10	A,B,D
20	A,C,D
30	A,D,E
40	B,E,F
50	B,C,D,E,F

- **Item collection** $X = \{x_1, ..., x_m\}$, e.g., {A,B,...,F}
- **Itemset**: a set of items, k-itemset
- **Transaction** $T \subseteq X$, each T associates a unique Tid and items bought by a customer
- **Rule** form $\alpha \geq \beta, \alpha \subset X, \beta \subset X, \alpha \cap \beta = \varnothing$

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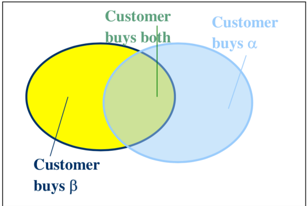
Basic Concepts

- **Support**, s, probability that a transaction contains α and β
  - support ($\alpha => \beta$) $= P(\alpha \cap \beta)$
- Frequent itemset, occurrence greater than a min_support
- Frequent itemset mining, find all the rules $\alpha \geq \beta$ satisfying min_support
- Let supmin $= 50\%$,
- frequent Itemsets A:3, B:3, D:4, E:3, AD:3
- support (A) $= 3/5 = 60\%$, support (AD) $= 3/5 = 60\%$



Basic Concepts

- **Support**, s, conditional probability that a transaction having α also contains β
- **Confidence** $(\alpha => \beta) = P(\beta|\alpha) = \frac{P(\alpha \cap \beta)}{P(\alpha)}$
- Measure of rule interestingness
- Rules satisfy **min_support** and **min_confidence** are strong
- Let supmin = 50%, confmin = 50%,
- frequent itemsets A:3, B:3, D:4, E:3, AD:3
- Association rules: $\alpha \Rightarrow \beta$ (support, confidence)
  - A => D (60%, 100%)
  - D => A (60%, 75%)

There Are Too Many Frequent Patterns

- A long pattern contains a **combinatorial number of sub-patterns**
- How many frequent itemsets does the following TDB1 contain?
  - **TDB1**: $T_1 : \{a_1, ..., a_{50}\}; T_2 : \{a_1, ..., a_{100}\}$
  - Assuming (absolute) minsup $= 1$
  - Let ' s have a try
  - 1-itemsets: $\{a_1\} : 2, \{a_2\} : 2, ..., \{a_{50}\} : 2, \{a_{51}\} : 1, ..., \{a_{100}\} : 1,$
  - 2-itemsets: $\{a_1, a_2\} : 2, ..., \{a_1, a_{50}\} : 2, \{a_1, a_{51}\} : 1..., ..., \{a_{99}, a_{100}\} : 1,$
  - ..., ..., ..., ...
  - 99-itemsets: $\{a_1, a_2, ..., a_{99}\} : 1, ..., \{a_2, a_3, ..., a_{100}\} : 1$
  - 100-itemset: $\{a_1, a_2, ..., a_{100}\} : 1$
- The total number of frequent itemsets: $2^{100} - 1$

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- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is closed if X is frequent, and there exists no super-pattern $Y \supset X$, with the same support as X
  - Let Transaction DB TDB1: $T_1 : \{a_1, ..., a_{50}\}$; $T_2 : \{a_1, ..., a_{100}\}$
  - Suppose minsup $= 1$. How many closed patterns does TDB1 contain?
    - Two: P_1 : "$\{a_1, ..., a_{50}\} : 2$"; P_2 : "$\{a_1, ..., a_{100}\} : 1$"
- **Closed pattern** is a **lossless** compression of frequent patterns
  - Reduces the # of patterns but does not lose the support information!
  - You will still be able to say: "$\{a_2, ..., a_{40}\} : 2$", "$\{a_5, a_{51}\} : 1$"

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- Solution 2: **Max-patterns**: A pattern X is a max-pattern if X is frequent and there exists no frequent super-pattern $Y \supset X$
- Difference from close-patterns?
  - Do not care the real support of the sub-patterns of a max-pattern
  - Let Transaction DB TDB1: $T_1 : \{a_1, ..., a_{50}\}; T_2 : \{a_1, ..., a100\}$
  - Suppose minsup $= 1$. How many max-patterns does TDB1 contain?
    - One: P : "$\{a_1, ..., a_{100}\} : 1$"
- **Max-pattern is a lossy compression!**
  - We only know one pattern is frequent, e.g., $\{a_1, ..., a_{40}\}$
  - But we do not know the real support of $\{a_1, ..., a_{40}\}$, ..., any more!
- Thus in many applications, **mining close-patterns is more desirable than mining max-patterns**

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Association Rule Mining

- **Boolean** vs. **quantitative** associations (based on the types of valued handled)
  - **Boolean association rules**, only concern presence or absence of items, buys(x,"SQLServer") and buys(x,"DMBook") $\Rightarrow$ buys(x,"DBMiner")[0.2%,60%]
  - **Quantitative association rules**, concern quantitative attributes, age(x,"30$\cdots$39") and income(x,"42$\cdots$48K") $\Rightarrow$ buys(x,"HD TV") [1%, 75%]
- **Single level** vs. **multiple-level** analysis (based on the levels of abstraction involved)
  - age(x,"30$\cdots$39") $\Rightarrow$ buys(x,"laptop computer")
  - age(x,"30$\cdots$39") $\Rightarrow$ buys(x,"computer")
- **Single dimension** vs. **multiple dimensional** associations (based on dimensions involved)
  - buys(X, "milk") => buys(X, "bread")
  - age(X,"19-25") and occupation(X,"student") => buys(X, "coke")

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Handling Exponential Complexity

- Given n transactions and m different items:
  - Number of possible association rules: $O(2^m)$
  - Computation complexity: $O(nm2^m)$
- Apriori Principle
  - Collect single item counts, find large items
  - Find candidate pairs, count them $=>$ large pairs of items
  - Find candidate triplets, count them $=>$ large triplets of items, And so on...
  - **Guiding Principle**: **Every subset of a frequent itemset has to be frequent**
    - Used for pruning many candidates

Apriori: A Candidate Generation and Test Approach

- Apriori uses prior knowledge of frequent itemsets
- Iterative approach, level-wise search
- The Apriori property (downward closure property, anti-monotone) of frequent patterns
  - **Any subset of a frequent itemset must be frequent**
  - **If any itemset is infrequent, its superset should not be generated/tested**
  - If {beer, diaper, nuts} is frequent, so is {beer, diaper}, every transaction having beer, diaper, nuts also contains beer, diaper
  - If {beer, diaper} is infrequent, {beer, diaper, nut} cannot be frequent at all

Apriori: A Candidate Generation and Test Approach

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- **Apriori Method**:
  1. Initially, scan DB once to get frequent 1-itemset
  2. Generate length $(k+1)$ candidate itemsets from length k frequent itemsets
  3. Test the candidates against DB
  4. Terminate when no frequent or candidate set can be generated

Example of Apriori Algorithm

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

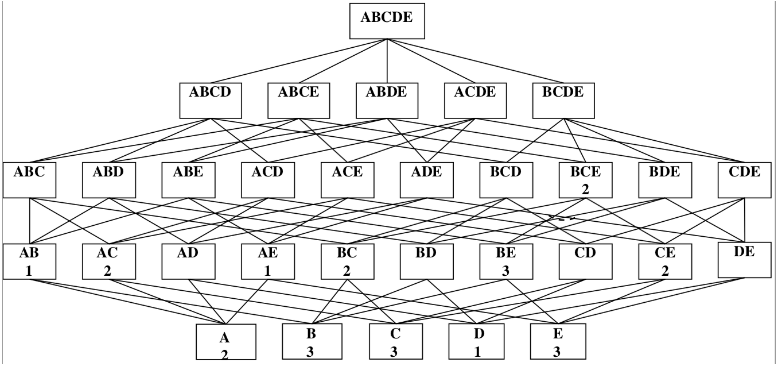
Itemset
{B, C, E}

3^{rd} scan

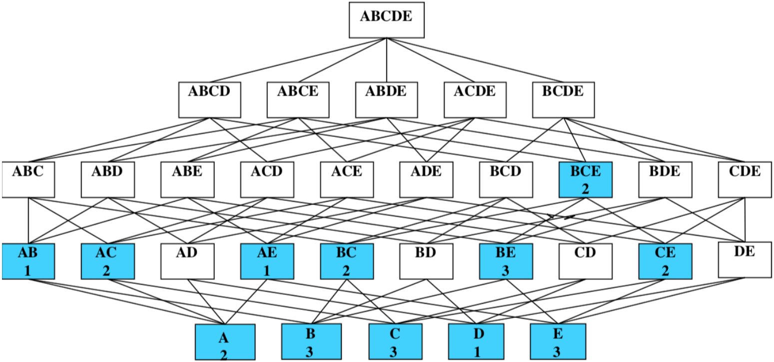
F_3

Itemset	sup
{B, C, E}	2

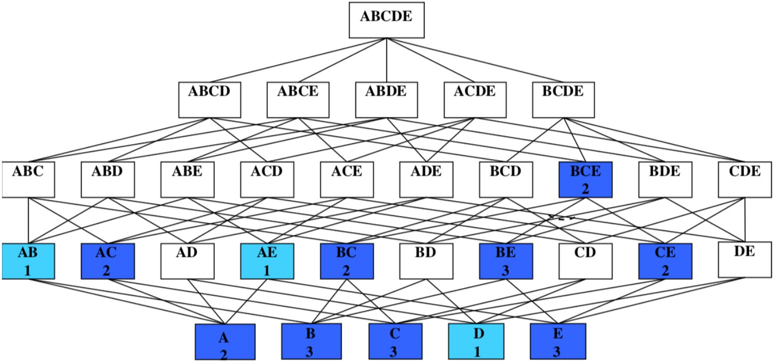
Example of Apriori Algorithm



Example of Apriori Algorithm



Example of Apriori Algorithm



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Apriori Algorithm
Pseudo-code

```
Ck: Candidate itemset of size k
Lk: frequent itemset of size k
Input: Database D, min_support
Output: frequent itemsets L
L1 = {frequent single items from D};
for (k = 2; Lk − 1! = ∅; k + +) do
   Ck = candidates generated from Lk−1;
   for each transaction t ∈ D do
      increment the count of all candidates in Ck which are
      contained in t
   end
   Lk = candidates in Ck with min_support
end
return L = ∪k Lk;
```

How to Generate Candidates?

- How to generate candidates?
  - **Step 1**: self-joining L_k
  - **Step 2**: pruning
- Example
  - $L_3 = \{abc, abd, acd, ace, bcd\}$
  - **Self-joining**: $L_3 \bowtie L_3$
    - abc and $abd \rightarrow abcd$, acd and $ace \rightarrow acde$
  - **Pruning**:
    - acde is pruned because ade is not in L_3
  - $C_4 = \{abcd\}$

How to Generate Candidates?

Suppose the items in L_{k-1} are listed in order
Step 1: self-joining L_{k-1}
for each itemset $l_1 \in L_{k-1}$
  for each itemset $l_2 \in L_{k-1}$
    if $(l_1[1] = l_2[1])$ and $(l_1[2] = l_2[2])$ and $\cdots$and
      $(l_1[k-2] = l_2[k-2])$ **then**
      $c = l_1$ join l_2
      pruning (c)
  end
end
Step 2: pruning
forall $(k-1)$-subsets s of c **do**
  if $(s$ is not in $L_{k-1})$ then delete c

How to Count Supports of Candidates?

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- **Why counting supports of candidates a problem?**
- The total number of candidates can be very huge
- One transaction may contain many candidates

- **Method:**
- Candidate itemsets are stored in a hash-tree
- Leaf node of hash-tree contains a list of itemsets and counts
- Interior node contains a hash table

Exercise

- A database has 9 transactions. Let min_sup $= 20\%$. Please present all the candidates and frequent itemsets at each iteration and frequent itemsets at each iteration.

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

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Challenges of Frequent Pattern Mining

- **Challenges**
- Multiple scans of transaction database
- Huge number of candidates
- Tedious workload of support counting for candidates

- **Improving Apriori**
- Reduce passes of transaction database scans
- Shrink number of candidates
- Facilitate support counting of candidates

Apriori: Improvements and Alternatives

- **Reduce passes of transaction database scans**
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)
- **Shrink the number of candidates**
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)
- **Exploring special data structures**
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Patition: Scan Database Only Twice

- A. Savasere, E. Omiecinski, and S. Navathe. **An efficient algorithm for mining association in large databases**. In VLDB' 95.
- Partitioning technique
  - Partition the data into N small partitions
  - **Phase 1**: find local frequent itemsets on each data partition. Record all local frequent itemsets.
  - **Phase 2**: Integrate all local frequent itemsets, scan database, find global frequent itemsets.
- **Correctness**: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions

Patition: Scan Database Only Twice

- Each partition can be fit into memory
- Scan database only **twice**! Reduce I/O cost!
- Execution time scales linearly
- Good for very large-scale database
- Applicable to **parallel/distributed** computing systems
  - Each processor performs FIM on its local data
  - Central server aggregates local frequent itemsets, broadcast potential global itemsets
  - Each processor scans local data to count the frequency
  - Central server aggregates the counts, find the global itemsets

Reduce the Number of Candidates

DHP:Direct Hashing and Pruning

- J. Park, M. Chen, and P. Yu. **An effective hash-based algorithm for mining association rules**. In SIGMOD' 95
- Hash-based technique
  - When scanning transactions to generate frequent k–itemsets, L_k, generate all $(k+1)$-itemsets for each transaction
  - Hash all $(k+1)$-itemsets into buckets, increase bucket count
  - If a $(k+1)$-itemset bucket count is below min_sup, it must be removed from $(k+1)$ candidate itemsets, C_{k+1}
- **Correctness**:A k-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent

Reduce the Number of Candidates
DHP:Direct Hashing and Pruning

- Example: At the 1st scan of TDB, count 1-itemset, and Hash 2-itemsets in the transaction to its bucket
  - $\{ab, ad, ce\}$
  - $\{bd, be, de\}$
  - $\cdots$
- At the end of the first scan,
- if minsup $= 80$, remove ab, ad, ce, since count{ab, ad, ce} < 80

Itemsets	Count
{ab, ad, ce}	35
{bd, be, de}	298
……	…
{yz, qs, wt}	58

Hash Table

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Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
- To find frequent itemset $i_1, i_2, ..., i_{100}$
  - \# of scans: 100
  - \# of Candidates: $(100^1) + (100^2) + ... + (100^{100}) = 2^{100} - 1 \approx 1.27 * 10^{30}$
- Bottleneck: candidate generation and test
- Can we avoid candidate generation?

Construct FP-Tree from a Transaction Database

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1. **Scan DB once**, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order L
3. Create the root of the tree, labeled with "null"
4. **Scan DB again**, sort each transaction in L order, a branch is created for each transaction
   - Increment the count of each node along a common prefix by 1
   - Create nodes for the items following the prefix
5. Build a header table, connect each item point in the tree

Construct FP-Tree from a Transaction Database

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TID	Items in the Transaction	Ordered, frequent itemlist
100	{f, a, c, d, g, i, m, p}	f, c, a, m, p
200	{a, b, c, f, l, m, o}	f, c, a, b, m
300	{b, f, h, j, o, w}	f, b
400	{b, c, k, s, p}	c, b, p
500	{a, f, c, e, l, p, m, n}	f, c, a, m, p

- Let min_support $= 3$
- 1-itemset: $f:4, a:3, c:4, b:3, m:3, p:3$
- $L = f \to c \to a \to b \to m \to p$

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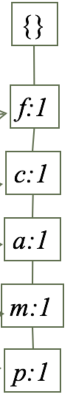
Construct FP-Tree from a Transaction Database

After inserting the 1st frequent Itemlist: *"f, c, a, m, p"*

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

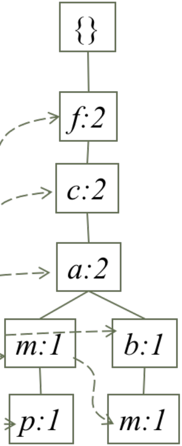


Construct FP-Tree from a Transaction Database

After inserting the 2nd frequent itemlist "*f, c, a, b, m*"

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



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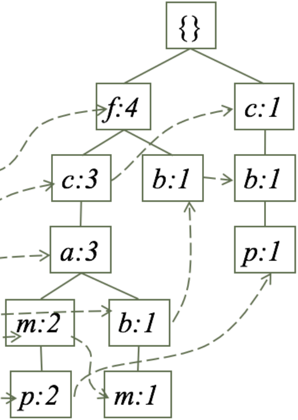
Construct FP-Tree from a Transaction Database

After inserting all the frequent itemlists

Header Table

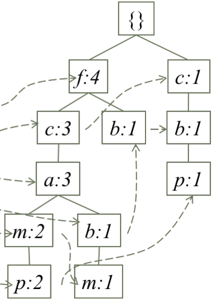
Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Construct FP-Tree from a Transaction Database

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



Conditional database of each pattern

Item	*Conditional database*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Benefits of the FP-tree Structure

- **Completeness**
- Preserve complete information for frequent pattern mining
- Never break a long pattern of any transaction

- **Compactness**
- Reduce irrelevant info—infrequent items are gone
- Items in frequency descending order: the more frequently occurring, the more likely to be shared
- Never be larger than the original database

Mining Frequent Patterns With FP-trees

procedure **FP_growth**(Tree, α)

if Tree contains a single path P then

  for each combination (denoted as β) of the nodes in the path P

    generate pattern $\beta \cup \alpha$ with support_count $=$ minimum support count of nodes in β

else

  for each α_i in the header of Tree {

    generate pattern $\beta = \alpha_i \cup \alpha$ with support_count $=$ α_i.support_count

    construct β's conditional pattern base and then β's conditional FP_tree Tree$_\beta$

    if Tree$_\beta$ **then**

      call **FP_growth**(Tree$_\beta$, β)}

Exercise

- **A database has 9 transactions. Let min_sup = 20%. Please present all the candidates and frequent itemsets at each iteration and frequent itemsets at each iteration.**

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

Solution

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
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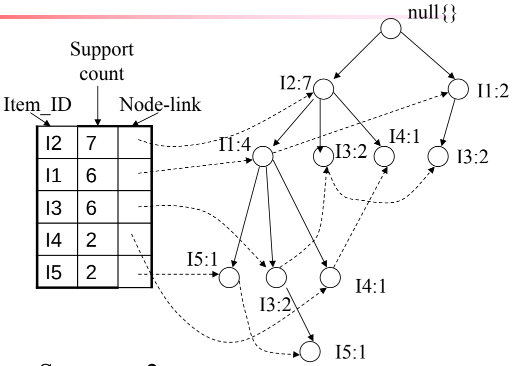
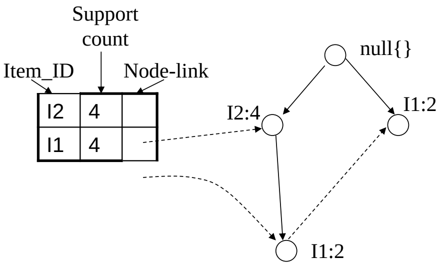
T900	I1,I2,I3



Support =2

Solution

item	*conditional pattern base*	*conditional FP-tree*	*frequent patterns generated*
I5	{{I2,I1: 1}, {I2,I1,I3: 1}}	⟨I2: 2, I1: 2⟩	{I2,I5: 2}, {I1,I5: 2}, {I2,I1,I5: 2}
I4	{{I2,I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2,I4: 2}
I3	{{I2,I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2,I3: 4}, {I1,I3: 4}, {I2,I1,I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2,I1: 4}



FP Tree vs. Apriori: Scalability With the Support Threshold

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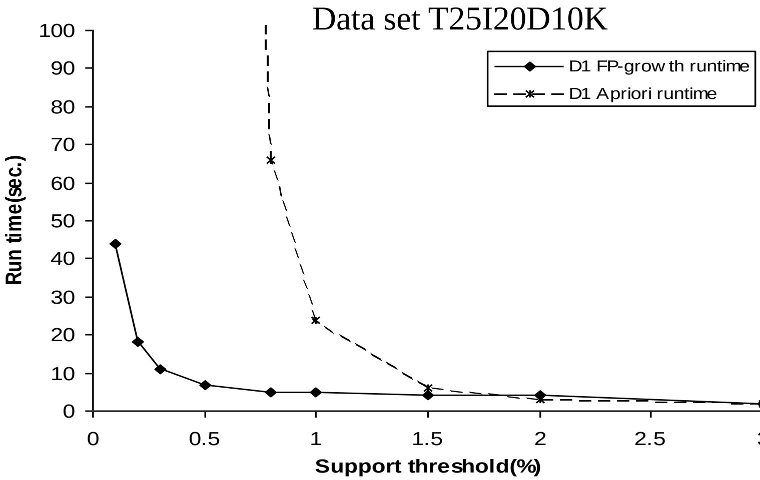

Why Is FP-Growth the Winner?

- **Divide-and-conquer:**
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Focus searching on smaller databases
- **Other factors**
  - No candidate generation, no candidate test
  - Compressed database: FP-tree structure
  - Two scans of entire database
  - Basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

Scaling FP-growth by Item-Based Data Projection

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- What if FP-tree cannot fit in memory? —Do not construct FP-tree
  - "Project" the database based on frequent single items
  - Construct & mine FP-tree for each projected DB
- **Parallel projection** vs. **partition projection**
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions

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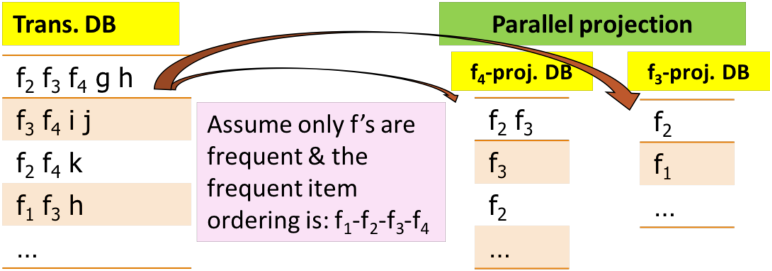
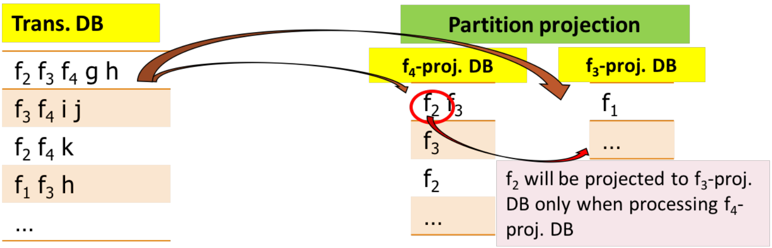
Scaling FP-growth by Item-Based Data Projection

- **Parallel projection**: Project the DB on each frequent item
  - Space costly, all partitions can be processed in parallel



Scaling FP-growth by Item-Based Data Projection

- **Partition projection**: Partition the DB in order
  - Passing the unprocessed parts to subsequent partitions



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A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

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- ECLAT: A depth-first search algorithm using set intersection [Zaki et al. KDD’ 97]
- **Tid-List**: List of transaction-ids containing an itemset
- **Vertical format**: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- **Properties of Tid-Lists**
  - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac) = t(d)$)
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using **diffset** to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}, t(ce) = \{T_{10}, T_{30}\} \rightarrow Diffset(ce, e) = \{T_{20}\}$

Mining Frequent Patterns, Association and Correlations

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Mining multilevel association rules

- Association rules at high concept levels may represent common sense knowledge
- Hard to find association rules at low concept level
- Items at the lower level usually have lower support, less than min_support threshold
- Mining association rules at multiple levels of abstraction
- Example: sales in AllElectronics store computer sector

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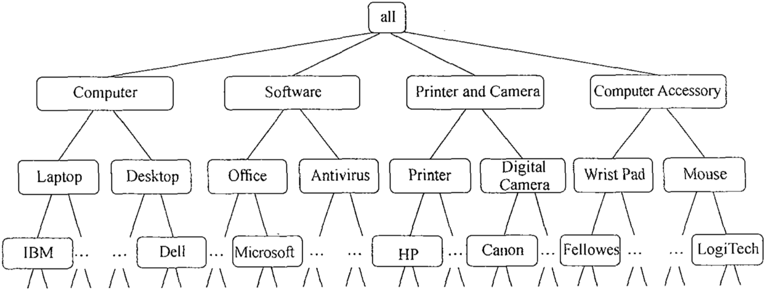
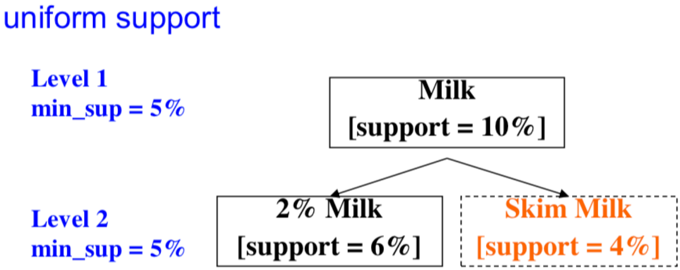
Example


all
Computer
Software
Printer and Camera
Computer Accessory
Laptop
Desktop
Office
Antivirus
Printer
Digital Camera
Wrist Pad
Mouse
IBM
Dell
Microsoft
HP
Canon
Fellowes
LogiTech

Mining multilevel association rules

- **Uniform support**
  - Top-down, level-wise
  - Use uniform minimum support for each level
  - Perform Apriori at each level
  - Optimization: if an ancestor is infrequent, the search on the descendants can be avoided



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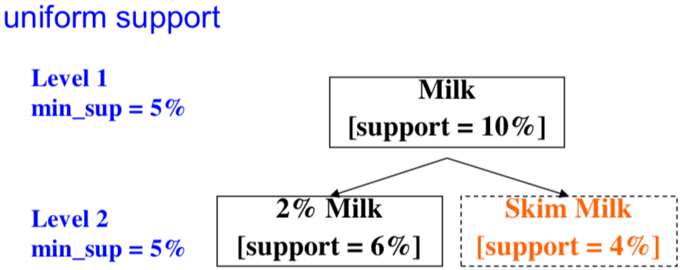
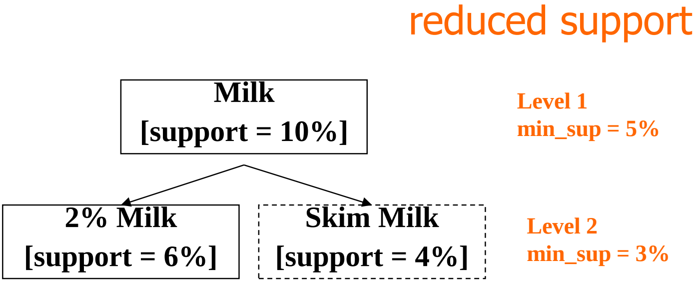
Mining multilevel association rules



- **Drawbacks**
  - Miss interesting associations with too high threshold
  - Generate too many uninteresting rules with too low threshold

Mining multilevel association rules

- **Reduced support**
  - Top-down, level-wise
  - Each concept level has its own minimum support threshold
  - The lower level, the smaller threshold
  - Perform Apriori at each level



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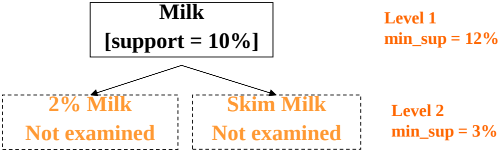
Mining multilevel association rules

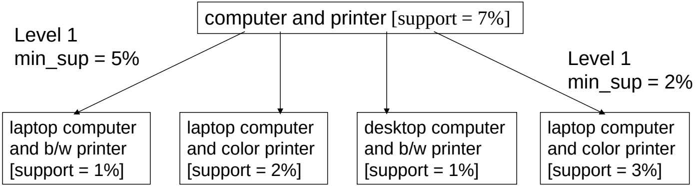
- **Reduced support**
  - Optimization – level-cross filtering by single item
  - An item at the ith concept level is examined iff its parent concept at the $(i-1)$th level is frequent
  - If a concept is infrequent, its descendents are pruned from the database
  - Drawbacks
    - Miss associations at low level items which are frequent based on a reduced min_support, but whose ancestors do not satisfy min_support

reduced support



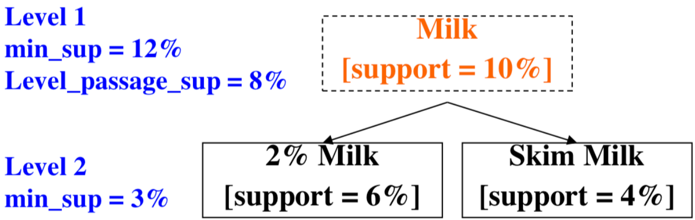
Mining multilevel association rules

- **Reduced support**
  - Optimization – level-cross filtering by k-itemset
    - Only the children of frequent k-itemsets are examined
    - Drawback: many valuable patterns may be filtered out



Mining multilevel association rules

- **Reduced support**
  - Optimization – Controled level-cross filtering by single item
    - next level min sup < level passage threshold < min sup
    - Allow the children of items that do not satisfy the min_sup to be examined if they satisfy the level passage threshold



Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to "ancestor" relationships between items
- Example
  - buys(X,"Laptop computer")=> buys(X,"HP printer") [support = 8%, confidence = 70%]
  - buys(X,"IBM laptop computer")=> buys(X,"HP printer") [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule
- A rule is redundant if its support is close to the "expected" value, based on the rule's ancestor

Mining Frequent Patterns, Association and Correlations

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Mining multidimensional association rules

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- Single-dimensional rules:
  buys(X, "milk") => buys(X, "bread")
- Multi-dimensional rules: $\geq$2 dimensions or predicates
  - Inter-dimension assoc. rules (no repeated predicates)
    age(X,"19-25") and occupation(X,"student") => buys(X, "coke")
  - hybrid-dimension assoc. rules (repeated predicates)
    age(X,"19-25") and **buys**(X, "popcorn") => **buys**(X, "coke")
- **Categorical Attributes**: finite number of possible values, no ordering among values
- **Quantitative Attributes**: numeric, implicit ordering among values —discretization, clustering approaches

Mining Quantitative Associations

Techniques can be used to **categorize numerical attributes**

- **Static discretization** based on predefined concept hierarchies
- **Dynamic discretization** based on data distribution
- **Clustering**: Distance-based association
  - one dimensional clustering then association

Multidimensional Association Rules and Static Discretization of Quantitative Attributes

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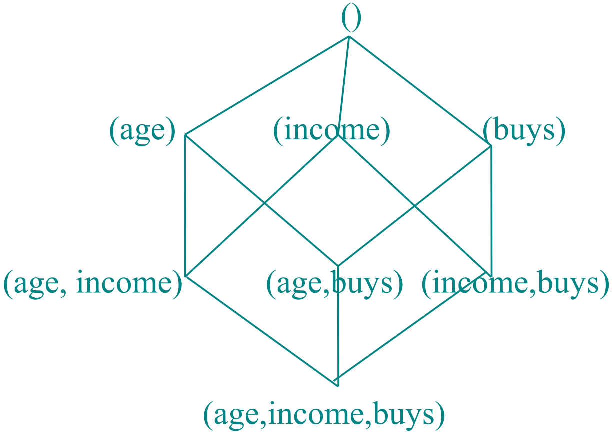
- Discretized prior to mining using **concept hierarchy**
- Numeric values are **replaced by ranges**
- In relational database, finding all frequent k-predicate sets will require k or $k+1$ table scans
- **Data cube** is well suited for mining
  - The cells of a n-dimensional: cuboid correspond to the dimensions
  - Mining from data cubes can be much faster

Multidimensional Association Rules and Static Discretization of Quantitative Attributes

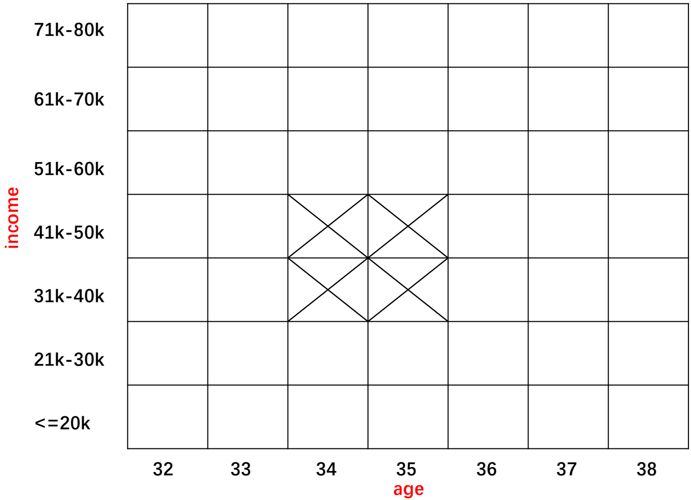


Quantitative Association Rules

- Numeric attributes are dynamically discretized
  - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules:
  A_{quan1} and $A_{quan2} => A_{cat}$
- Association rule clustering system (ARCS)
  - Binning: 2-D grid, manageable size
  - Finding frequent predicate sets: scan the database, count the support for each grid cell
  - Clustering the rules: cluster adjacent cells to form a rule

Quantitative Association Rules

Example: age and income => buy HD TV



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Quantitative Association Rules

- **Example**:
  - age(X,"34") and income(X,"31-40K") => buys(X,"HD TV")
  - age(X,"35") and income(X,"31-40K") => buys(X,"HD TV")
  - age(X,"34") and income(X,"41-50K") => buys(X,"HD TV")
  - age(X,"35") and income(X,"41-50K") => buys(X,"HD TV")
- => age(X,"34-35") and income(X,"31-50K") => buys(X,"HD TV")

Interestingness Measure: Correlations (Lift)

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- play basketball $=>$ eat cereal [40%, 66.7%] is misleading
- The overall percentage of students eating cereal is 75% $>$ 66.7%.
- Measure of dependent/correlated events:

$$lift(A, B) = \frac{P(A \cap B)}{P(A)P(B)}$$

- $lift(A, B) = 1$: A and B are independent
- $lift(A, B) > 1$: A and B are positive correlated
- $lift(A, B) < 1$: A and B are negative correlated

Interestingness Measure: Correlations (Lift)

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not Cereal	1000	250	1250
Sum (col.)	3000	2000	5000

- $lift(A, B) = \frac{2000/5000}{(3000/5000)*(3750/5000)} = 0.89$
- $lift(A, \bar{B}) = \frac{1000/5000}{(3000/5000)*(1250/5000)} = 1.33$
- $A \Rightarrow B$ [**support, confidence, correlation**]

Sequence Databases & Sequential Patterns

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- Sequential pattern mining has broad applications
  - Customer shopping sequences
  - Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
  - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
  - Weblog click streams, calling patterns, $\cdots$
  - Software engineering: Program execution sequences, $\cdots$
  - Biological sequences: DNA, protein, $\cdots$
- Transaction DB, sequence DB vs. time-series DB
- Gapped vs. non-gapped sequential patterns
  - Shopping sequences, clicking streams vs. biological sequences

Sequential Pattern and Sequential Pattern Mining

- Sequential pattern mining: Given a set of sequences, find the complete set of frequent subsequences (i.e., satisfying the min_sup threshold)

A *sequence database*

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

- A sequence: $<(ef)(ab)(df)cb>$
- An element may contain a set of items (also called events)
- Items within an element are unordered and we list them alphabetically
- $<a(bc)dc>$ is a subsequence of $<a(abc)(ac)d(cf)>$
- Given support threshold min_sup $=2$, <(ab)c> is a sequential pattern

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- Algorithm requirement: Efficient, scalable, finding complete set, incorporating various kinds of user-specific constraints
- The Apriori property still holds: If a subsequence s1 is infrequent, none of s1's super-sequences can be frequent
- Representative algorithms
  - GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)
  - Vertical format-based mining: SPADE (Zaki@Machine Leanining'00)
  - Pattern-growth methods: PrefixSpan (Pei, et al. @TKDE'04)
- Mining closed sequential patterns: CloSpan (Yan, et al. @SDM'03)
- Constraint-based sequential pattern mining (to be covered in the constraint mining section)

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GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All 8-singleton sequences
- $<a>, <b>, <c>, <d>, <e>, <f>, <g>, <h>$
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences
- Repeat (for each level (i.e., length-k))
  - Scan DB to find length-k frequent sequences
  - Generate length-$(k+1)$ candidate sequences from length-k frequent sequences using Apriori
  - set $k = k + 1$
- Until no frequent sequence or no candidate can be found

GSP: Apriori-Based Sequential Pattern Mining

Algorithm GSP($\mathcal{S}$)

$C_1 \leftarrow$ init-pass ($\mathcal{S}$)
$F_1 \leftarrow \{< \{f\} > | f \in C_1, f.count/n \geq min\_sup\}$
for $(k = 2; F_{k-1} \neq \varnothing; k++)$ **do**
    $C_k \leftarrow$ candidate-gen-SPM(F_{k-1})
    for each data sequence $s \in \mathcal{S}$ **do**
        for each candidate $c \in C_k$ **do**
            if c is contained in s **then**
                $c.count++;$
            end
        end
        $F_k \leftarrow \{c \in C_k | c.count/n \geq min\_sup\}$
    end
    return $F \leftarrow \cup_k F_k$
end

GSP: Apriori-Based Sequential Pattern Mining

Function candidate-gen-SPM(F_{k-1})

1. **Joint step**. Candidate sequences are generated by joining F_{k-1} with F_{k-1}. A sequence s_1 joins with s_2 if the subsequence obtained by dropping the first item of s_1 is the same as the subsequence obtained by dropping the last item of s_2. The candidate sequenc generated by joining s_1 with s_2 is the sequence s_1 extended with the last item in s_2. There are two cases:
   - the added item forms a separate element if it was a separate element in s_2, and is appended at the end of s_1 in the merged sequence
   - the added item is part of the last element of s_1 in the merged sequence

   When joining F_1 with F_1, we need to add the item in s_2 both as part of an itemset and as a separate element. That is, joining $<\{x\}>$ with $<\{y\}>$ gives us both $<\{x, y\}>$ and $<\{x\}, \{y\}>$. Note that x and y in $\{x, y\}$ are ordered.
2. **Prune step**. A candidate sequence is pruend if any one of its $(k-1)$-subsequences is infrequent (without minimum support)

GSP: Apriori-Based Sequential Pattern Mining

min_sup = 2

Cand.	sup
<a>	3
<b>	5
<c>	4
<d>	3
<e>	3
<f>	2
~~<g>~~	1
~~<h>~~	1

	<a>	<b>	<c>	<d>	<e>	<f>
<a>	<aa>	<ab>	<ac>	<ad>	<ae>	<af>
<b>	<ba>	<bb>	<bc>	<bd>	<be>	<bf>
<c>	<ca>	<cb>	<cc>	<cd>	<ce>	<cf>
<d>	<da>	<db>	<dc>	<dd>	<de>	<df>
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<f>	<fa>	<fb>	<fc>	<fd>	<fe>	<ff>

	<a>	<b>	<c>	<d>	<e>	<f>
<a>		<(ab)>	<(ac)>	<(ad)>	<(ae)>	<(af)>
<b>			<(bc)>	<(bd)>	<(be)>	<(bf)>
<c>				<(cd)>	<(ce)>	<(cf)>
<d>					<(de)>	<(df)>
<e>						<(ef)>
<f>						

GSP: Apriori-Based Sequential Pattern Mining

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

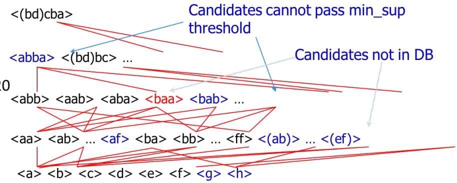
5th scan: 1 cand. 1 length-5 seq. pat.

4th scan: 8 cand. 7 length-4 seq. pat.

3rd scan: 46 cand. 20 length-3 seq. pat. 20 cand. not in DB at all

2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all

1st scan: 8 cand. 6 length-1 seq. pat.



Summary

- Frequent pattern mining —an important task in data mining
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Partition, DIC, DHP, etc.
  - Projection-based (FP-growth)
- Mining a variety of rules and interesting pattern

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