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Temporal $1/f^{\alpha}$ Fluctuations from Fractal Magnetic Fields in Black-Hole Accretion Flow

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Abstract

Rapid fluctuation with a frequency dependence of $1/f^{\alpha}$ (with $\alpha \simeq$ 1–2) is characteristic of radiation from black-hole objects. Its origin remains poorly understood. We examined three-dimensional magnetohydrodynamical simulation data, finding that a magnetized accretion disk exhibits both $1/f^{\alpha}$ fluctuation (with $\alpha \simeq 2$) and a fractal magnetic structure (with the fractal dimension of $D \sim 1.9$). The fractal field configuration leads to reconnection events with a variety of released energy and duration, thereby producing $1/f^{\alpha}$ fluctuations.

Key words: accretion, accretion disks — black hole physics — fractal — magnetohydrodynamics — $1/f$ fluctuations

1. Introduction

Apparently random temporal fluctuations from galactic black-hole candidates (BHCs; van der Klis 1995) and from active galactic nuclei (AGNs; Ulrich et al. 1997) have led many astronomers to recognizing how complex is the behavior of nature. The light curves are neither periodic nor random around some mean. Rather, they are seemingly composed of shot events with a variety of peak intensities and durations (Negoro et al. 1995). A number of analyses of X-ray light curves and optical AGN light curves show that the power spectral density (PSD) is flat at lower frequencies (f), and is a power-law ($\propto f^{-\alpha}$ with $\alpha \simeq$ 1–2) at higher frequencies. The break frequency corresponds to a few seconds for BHCs and to a few years for AGNs. Moreover, $1/f^{\alpha}$ fluctuations are ubiquitous in natural behavior, although their origins have been unsolved (Tajima, Shibata 1997, §2.6; Cable, Tajima 1997). The significance of $1/f^{\alpha}$ noise is that it contains a long-term memory (Press 1978). It has been a puzzle how $1/f^{\alpha}$ fluctuations can arise in black-hole accretion flows (or disks) under realistic circumstances.

Among a number of suggestions for a possible mechanism of variability, the most promising one is magnetic flares (Wheeler 1977; Galeev et al. 1979). It has been established through X-ray observations by the Yohkoh satellite that solar flares are triggered by magnetic reconnection (Shibata 1996). In fact, the solar soft X-ray variation exhibits $1/f^{\alpha}$ fluctuations (UeNo et al. 1997). Similarly, sporadic magnetic reconnection events which can occur in accretion flow may be responsible for the variability of black-hole objects, as well (Mineshige et al. 1995).

To prove this conjecture, we examined the three-dimensional data of global, magnetohydrodynamical (MHD) disk calculations first made by Machida, Hayashi, and Matsumoto (2000). They calculated how a magnetic field evolves in a rotating disk initially threaded by toroidal (B_{φ}) fields. Since no cooling is taken into account in the computations, the simulated disk is advection-dominated (Kato et al. 1998), rather than radiation-dominated, as in the standard disk. Then, the system which we analyzed corresponds to BHCs in the hard (low) state, in which fluctuations are largely enhanced and whose spectra can well be reproduced by advection-dominated flow (Narayan et al. 1996). Magnetic fields are amplified with time via a number of MHD instabilities together with differential rotation. The maximum field strength is determined either by the field dissipation by reconnection or field escape from accretion flow

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via the Parker instability. As a result, the mean plasma β, the ratio of the gas pressure to the magnetic pressure, finally reaches ~ 10, irrespective of the initial values of β. Locally, however, even low-β (< 1) regions appear; an inhomogeneous structure consequentially arises [see also similar discussion by Abramowicz et al. (1992), but for non-magnetic cases].

In the present paper, we thus analyze the temporal and spatial behaviors of accretion flow in order to clarify the origin of $1/f^\alpha$ fluctuations in black-hole objects. The results of the analysis are presented in the following section. The final section is devoted to discussions.

2. Temporal and Spatial Analysis of Accretion Flow

Figure 1 displays the light curves (A) and their PSDs (B) of a simulated disk obtained in the quasi-stationary state. Here, assuming that magnetic reconnection events greatly contribute to the energy output, we calculated time variation of $\int \eta j^2 dV$ (with η and j being the electric resistivity and electric current density) integrated over almost the whole disk. The high-frequency sides of the PSDs show a nearly power-law decline with an index of $\alpha \sim 2$, in agreement with the observations. It is, in a sense, amazing that without any fine-tuning of the parameters or special assumptions, an MHD disk naturally gives rise to $1/f^\alpha$ fluctuations over three orders of frequency range. On the other hand, the low-frequency sides flatten at frequencies lower than the reciprocal of several rotation timescale at a reference radius, which is also consistent with the observations (van der Klis 1995; Ulrich et al. 1997).

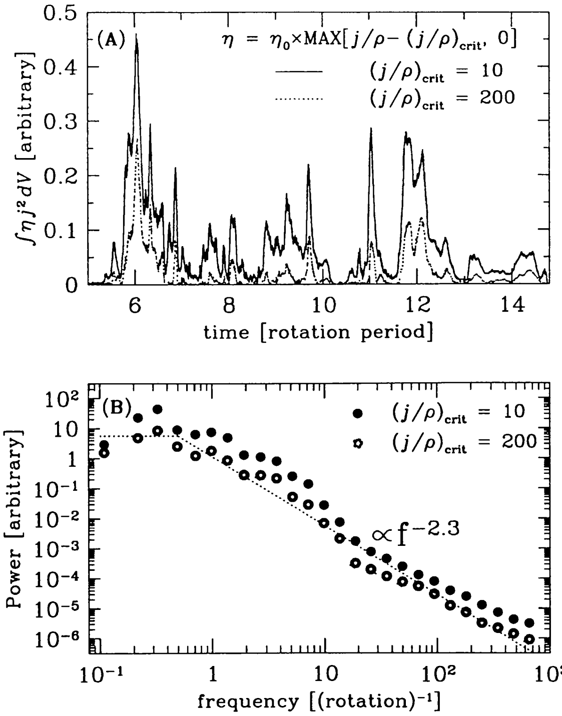

Fig. 1. (A) Typical light curves of the simulated MHD disk. Here, we assume that the radiation is predominantly due to field dissipation by magnetic reconnection, thus plotting the temporal variation of ηj^2 integrated over almost the whole disk. The electric resistivity, η, is assumed to be $\eta_0 \times \mathrm{MAX}[j/\rho - (j/\rho)_{\rm crit}, 0]$, since the magnetic reconnection seems to occur where local j/ρ is larger than some critical value. (B) The power-spectral densities of the above light curves. The dotted line is a least-squares fit with a broken power-law function for $(j/\rho)_{\rm crit}$ of 200 (open circles). It is of great importance to note that the general behavior does not depend on the values of $(j/\rho)_{\rm crit}$. The unit of time is the rotation period at a reference radius where the center of the initial torus is located. The rotation timescale used here corresponds to a few seconds for BHCs and a few years for AGNs in a realistic situation.

The appearance of $1/f^\alpha$ fluctuation, or more specifically the presence of a long-term time correlation, implies a long-distance spatial correlation in the distribution of the magnetic fields. It is thus tempting to examine the spatial magnetic-field distribution. We specially pick up the quantity j/ρ, which is the ratio of the absolute value of the electric current density to the matter density, since it is a good indicator regarding a trigger of reconnection (Parker 1994; Ugai 1999). In fact, it has been shown by MHD simulations that a fast reconnection, as is observed in solar flares, occurs when the electric resistivity becomes anomalously high in localized regions (Tajima, Shibata 1997, p237). Such a local, anomalous resistivity can be achieved where the electron drift velocity (which is proportional to j/ρ) exceeds a critical value (Yokoyama, Shibata 1995). Hence, any regions with high j/ρ values are all good candidates for a subsequent reconnection site.

We give in figure 2 (Plate 23) a snapshot of the spatial j/ρ distribution on a horizontal plane slightly above the equatorial plane. The panel roughly demonstrates to what extent the reconnected area expands, once reconnection is initiated somewhere. We point out that the distribution is quite inhomogeneous; patchy patterns are visible everywhere in figure 2 (Plate 23). Importantly, there seems to be no typical size of each patch. The presence of a fractal structure is suspected.

To confirm this idea, we conducted a fractal analysis for a three-dimensional MHD disk with mesh-point numbers of $(N_x, N_y, N_z) = (100,100,25)$; namely, we first marked the sites where j/ρ exceeds some critical value, $(j/\rho)_{\rm crit}$, and named any assembly of the marked sites clusters. We then counted the numbers of clusters according to the cluster sizes (i.e., volume) for the data sets at five different timesteps and averaged the counts.

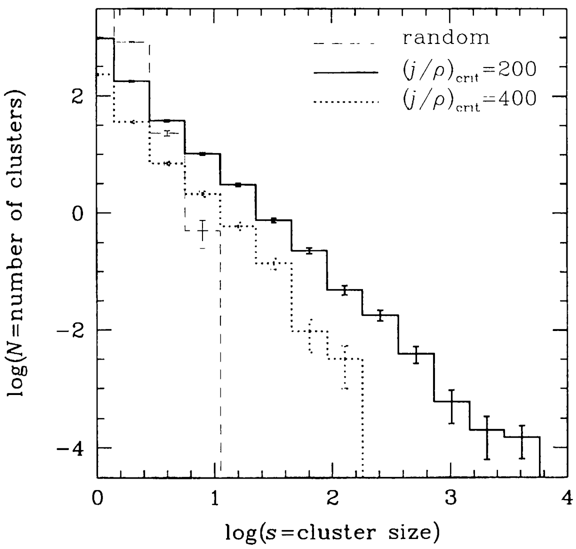

Fig. 3. Histograms of clusters with $j/\rho > (j/\rho)_{\rm crit} = 200$ (by the thick solid lines) and those with $j/\rho > 400$ (by the thick dotted lines) as functions of the cluster size (volume). A power-law distribution is realized over three orders of cluster sizes; roughly, $D(s) \propto s^{-2}$. For a comparison, we also plot the same, but for a random distribution (by the thin dashed lines). Obviously, large clusters are missing.

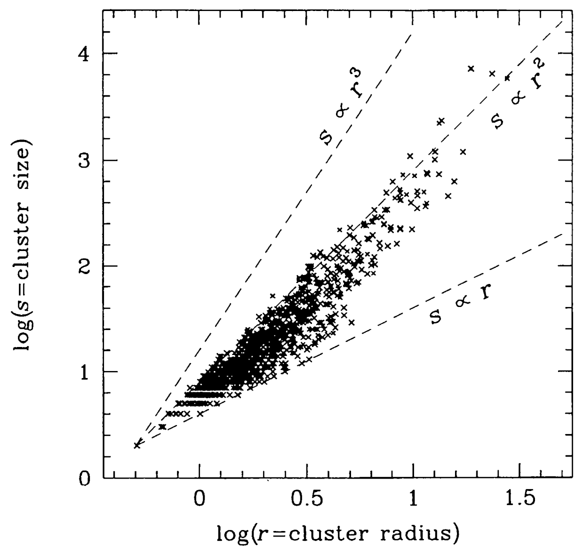

Fig. 4. Relation between the volume (s) and mean radius (r) of each cluster. The lines of $s \propto r$, $s \propto r^2$, and $s \propto r^3$ are also depicted. The least-squares fit shows $s \propto r^D$ with $D \sim 1.9$.

The resultant distribution is plotted in figure 3 with thick lines. Surprisingly, the cluster size is distributed in a power-law fashion over three orders of magnitudes of cluster sizes, from a few larger clusters to numerous smaller clusters.

For a comparison, we calculated the random distribution in the following way: we first evaluated what fraction of the entire volume is covered with the indicated sites in figure 2 (Plate 23), finding about 5.5% for $j/\rho > 200$. We next assigned a random number between 0 and 1 for each site of the three-dimensional box, and marked the sites where the number exceeded 0.945. We then repeated the same procedure as that done for the MHD disk, and plotted the resultant histogram in figure 3 with the thin dashed line. Clearly, there are no very big clusters in the random model. In other words, a long-distance j/ρ correlation is lost there.

Magnetic fields in the disk have a fractal structure. To find the fractal dimension, we plotted the size (volume) of each cluster as a function of its mean radius in figure 4. Here, the cluster mean radius is defined as $r \equiv (1/s)\sum_i^s |\boldsymbol{r}_i - \boldsymbol{r}_{\rm CM}|$ with $\boldsymbol{r}_{\rm CM} = (1/s)\sum_i^s \boldsymbol{r}_i$ for each cluster with a size of $s\,(>1)$, where $\boldsymbol{r}_i$ is the coordinates of the i-th site belonging to the cluster. From the fitting, we find roughly $s \propto r^D$ with $D = 1.9$.

3. Discussions

We now address two key questions: what relates the fractal structure to the temporal fluctuation in the simulated MHD disk? How can such a fractal distribution arise? Turbulence is generally known to exhibit a fractal behavior (Procaccia 1984); although this fact may be related to our present finding, the predicted fractal dimension is $D \sim 2.6$, differing from that of the present case. Alternatively, we note the notion of self-organized criticality (SOC: Bak 1996; Jensen 1998), one of the most attractive concepts developed in the study of complex systems.

Bak, Tang, and Wiesenfeld (1988) proposed a sand-pile model to describe a system exhibiting $1/f^{\alpha}$ fluctuation. Suppose that we drop sand particles one after another on a table. The fallen sand particles will form a pile, onto which other sand particles will be added. When the slope of the pile in either direction exceeds a critical value, an avalanche occurs and sand particles slide down in that direction. Then, the system spontaneously evolves to, and stays at, SOC. In our case, the addition of a sand particle corresponds to energy input to magnetic fields, while the critical slope corresponds to the critical j/ρ over which energy dissipation occurs via reconnection (Mineshige et al. 1994).

For systems in an SOC state, long-distance spatial communication among different sites is naturally built up, which yields a long-term time correlation. If each flare light curve is expressed by a time-symmetric profile with exponential grow and decay of about $t = 0$, $L(t) \propto \exp(-|t|/\tau)$ with τ ($\le \tau_{\max}$) being constant, its PSD is $P_\tau(f) \propto \epsilon^2/(1+4\pi^2 f^2\tau^2)^2$. If the energy ($\epsilon$) of each flare is distributed as $N(\epsilon) \propto \epsilon^{-p}$ and each flare duration is related to the energy as $\epsilon \propto \tau^D$, the total PSD becomes

$$\begin{aligned} P(f) &= \sum_{\tau}^{\tau_{\max}} P_\tau(f) N(\tau)\Delta\tau \\ &\propto \sum_{\tau}^{\tau_{\max}} \frac{\tau^{2D}}{(1+4\pi^2 f^2\tau^2)^2}(\tau^D)^{-p}\frac{d\epsilon}{d\tau}\Delta\tau, \\ &\longrightarrow P(f) \propto \left(\frac{1}{f}\right)^{(3-p)D} F(f) \quad (\Delta\tau \to 0). \qquad (1) \end{aligned}$$

Here, $F(f)$ $\{= \int_0^{2\pi\tau_{\max} f} dx[x^{(3-p)D-1}]/(1+x^2)^2\}$ is a slowly varying function of f for $f > 1/(2\pi\tau_{\max})$. We may regard $\epsilon \propto s$ (volume of a clump) and $\tau \propto r$ (mean radius of a clump). Then, we found from a simulation that $p \sim 2$ and $D \simeq 2$ (see figure 4). Hence, equation (1) leads to $P(f) \propto f^{-2}$, in agreement with the numerical result (see figure 1). That is the reason why the fractal magnetic field produces $1/f^\alpha$ fluctuations (Takeuchi et al. 1995; Kawaguchi et al. 1998).

One of the most conspicuous natures of the SOC is its ubiquity; namely, it is supposed to describe various non-equilibrium open systems, such as earthquakes, forest fires, the evolution of biological species, and traffic flow (Bak 1996). In an astrophysical context, it is important to note that coronal magnetic fields in the Sun are suggested to be in a SOC state (Lu, Hamilton 1991; Vassiliadis et al. 1998) as well, thus exhibiting the power-law occurrence rate of flares and $1/f^\alpha$ fluctuations in solar flare curves (UeNo et al. 1997).

Gamma-ray bursts (GRBs) also occasionally exhibit $1/f^\alpha$ fluctuations (Beloborodov et al. 1998). In some models of GRBs which involve the merger of two compact objects (white dwarf, neutron star, and black hole), a sort of accretion disk is thought to be formed by debris of one component around another (Mészáros 1999). This situation could be similar. We thus expect frequent reconnection events with a smooth size (amplitude and duration) distribution to occur in GRBs, which would give rise to $1/f^\alpha$ fluctuations (Panaitescu et al. 1999). Likewise, any other magnetic systems, regardless of the system size, may show similar effects.

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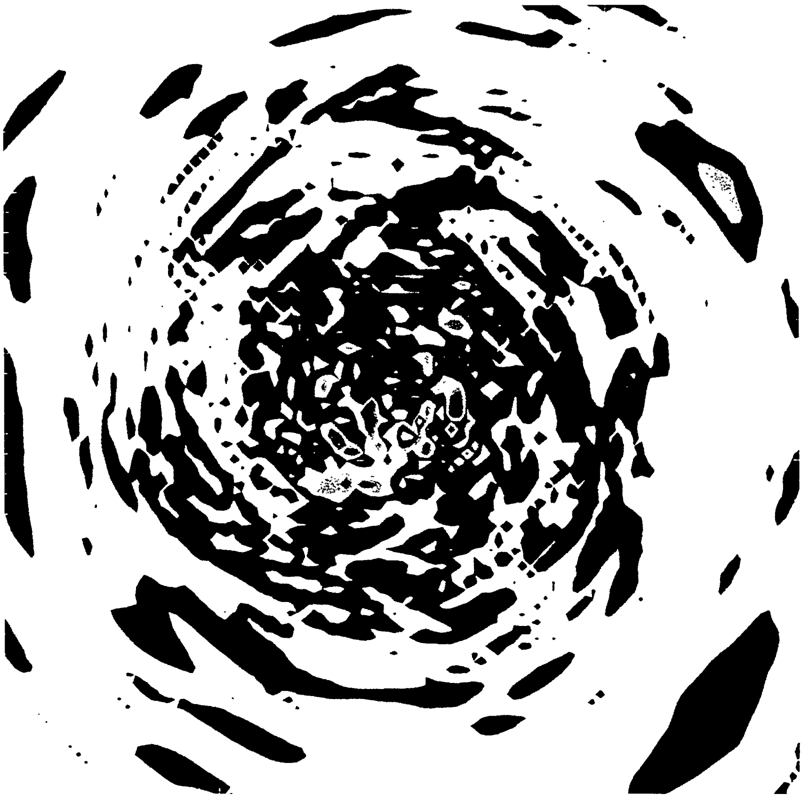

Fig. 2. Color contour map of the (j/ρ) distribution on a horizontal plane slightly above the equatorial plane. Here, the values of (j/ρ) where the colors change are as follows: 70 from white to blue, 120 to green, 200 to yellow, and 300 to red, respectively.

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