The connection between the cusp–to–core transformation and observational universalities of DM halos

Go Ogiya[1]⋆, Masao Mori[1,2], Tomoaki Ishiyama[2] and Andreas Burkert[3,4]

[1] *Graduate School of Pure and Applied Science, University of Tsukuba, 1-1-1, Tennodai, Tsukuba, Ibaraki, 305-8577, Japan*
[2] *Center for Computational Sciences, University of Tsukuba, 1-1-1, Tennodai, Tsukuba, Ibaraki, 305-8577, Japan*
[3] *Universitäts-Sternwarte München, Scheinerstraße 1, D-81679 München, Germany*
[4] *Max-Planck-Fellow, Max-Planck-Institut für extraterrestrische Physik, Postfach 1312, Giessenbachstraße, D-85741 Garching, Germany*

ABSTRACT

Observations have revealed interesting universal properties of dark matter (DM) halos especially around low-mass galaxies. Strigari et al. (2008) showed that DM halos have common enclosed masses within 300pc (Strigari relation). Kormendy & Freeman (2004) reported DM halos having almost identical central surface densities (the μ_{0D} relation). In addition, there exists a core–cusp problem, a discrepancy of the central density distribution between simulated halos and observations. We investigate whether a scenario where cuspy halos transform into cores by some dynamical processes can also explain their universal structural properties. It is shown that a cusp–to–core transformation model naturally reproduces the μ_{0D} relation and that Strigari relation follows from the μ_{0D} relation for dwarf galaxies. We also show that the central densities of cored dark halos provide valuable information about their formation redshifts.

Key words: cosmology: dark matter – galaxies: evolution – galaxies: formation – galaxies: dwarf – galaxies: Local Group

1 INTRODUCTION

The predictions of the present standard paradigm for structure formation in the universe, the cold dark matter (CDM) cosmology, excellently match observations on large scales ($\gtrsim$ 1Mpc). However, some serious discrepancies between CDM predictions and observations are being discussed on smaller scales ($\lesssim$ 1Mpc). One of them is the core–cusp problem, that is the mismatch of the observationally inferred central density structures of DM halos when compared with theoretical predictions. Cosmological CDM N–body simulations claim divergent density distributions, called cusps, in the centers of DM halos (e.g., Navarro et al. 1997, 2010; Fukushige & Makino 1997; Moore et al. 1999a; Ishiyama et al. 2013). The Navarro–Frenk–White (NFW) profile (Navarro et al. 1997) well fits the density structures of simulated CDM halos,

$$\rho(r) = \rho_s r_s^3 / [r(r + r_s)^2], \tag{1}$$

where r is the distance from the center and ρ_s and r_s are the scale density and scale length of the halo, respectively. On the other hand, DM halos of observed galaxies, especially for low-mass galaxies, indicate constant density profiles, called cores or at least shallower cusps (e.g., Moore 1994; de Blok et al. 2001; Swaters et al. 2003; Spekkens et al. 2005; Oh et al. 2011). The Burkert profile (Burkert 1995) reproduces well the density structures of DM halos obtained from observation data (see also Salucci & Burkert 2000),

⋆ E-mail:ogiya@ccs.tsukuba.ac.jp

$$\rho(r) = \rho_0 r_0^3 / [(r + r_0)(r^2 + r_0^2)], \tag{2}$$

where ρ_0 and r_0 are the central density and the core radius of the halo, respectively. If DM particles are only interacting gravitationally, flattening out a central DM cusp would require changes in the gravitational potential. Several mechanisms have been proposed and indeed succeed to reduce the central density. One of them are changes in the gravitational potential caused by stellar feedback redistributing gas or generating galactic winds (e.g., Navarro et al. 1996, Ogiya & Mori 2011, 2012; Pontzen & Governato 2012: Teyssier et al. 2013). Another example is the heating by dynamical friction of massive clumps (e.g., El–Zant et al. 2001; Goerdt et al. 2010; Inoue & Saitoh 2011).

In connection with this problem, some studies reported observational universalities of DM halos. Strigari et al. (2008) found there exists a common mass scale among DM halos of nearby dwarf galaxies. According to their analysis based on Jeans equation for spherical systems, the enclosed DM mass within 300pc from the halo centers, $M(< 300pc)$, is ∼ $10^7 M_\odot$ (hereafter, Strigari relation). Motivated by these observations, Walker et al. (2009) argued that all dwarfs are embedded in the same universal DM halo. Hayashi & Chiba (2012) evaluated $M(< 300pc)$ using non–spherical Jeans analysis. Independent of Strigari relation, Kormendy & Freeman (2004) revealed the quantity, $\mu_{0D} \equiv \rho_0 r_0$, which corresponds to the central surface density of DM halos for cored profiles, does not depend on blue magnitude of galaxies (hereafter, the μ_{0D} relation). Donato et al. (2009) confirmed this result for a larger sample.

The finding of Kormendy & Freeman (2004) is also consistent with the observed correlations of core parameters $\rho_0 \propto r_0^{-1}$ when fitting the observations by Burkert halos as confirmed by Salucci et al. (2012). They also obtained $\mu_{0D} \sim 140 M_\odot \mathrm{pc}^{-2}$. Using the recent observational data, Cardone & Tortora (2010), Boyarsky et al. (2010), and Cardone & Del Popolo (2012) argued for a weak dependence of a halo mass on the μ_{0D} relation. More recently, Del Popolo et al. (2013) proposed the theoretical interpretation.

The motivation of this work is to investigate the connection between processes transforming cusps to cores and the universalities of dark halo cores . The outline of this *letter* is as follows. In order to understand the meaning of the universalities in the cosmological context, we start in Section 2 with an investigation of the dependence of the dark halo concentration paremeter on halo mass and redshift. In Section 3, we argue the μ_{0D} relation and cusp–to–core transformation process are connected. The results derived in Section 3 provide information about halo properties, formation redshift and virial mass. Section 4 demonstrates Strigari and the μ_{0D} relation for nearby dwarf galaxies are linked that also shows both relations are inconsistent for larger and smaller galaxies. We summarize the results in Section 5.

2   CONCENTRATION PARAMETER

The concentration parameter of an NFW halo, c, is defined by $c \equiv r_{200}/r_\mathrm{s}$, where r_{200} is the radius inside of which the mean density of the DM halo is 200 times of the critical density of the universe and which is called the virial radius. This is related to the virial mass, M_{200}, by

$$M_{200} \equiv (4\pi/3)200\rho_\mathrm{crit}(1+z)^3 r_{200}^3, \qquad (3)$$

where ρ_crit and z are the critical density of the universe and redshift, respectively. If we know the dependence of c on M_{200} and z, we can determine the parameters of the NFW halo, ρ_s and r_s, for given M_{200} and z. Such dependence, $c(M_{200}, z)$, has been investigated by using cosmological N–body studies (e.g., Bullock et al. 2001; Macciò et al. 2008), most of which concluded that c decreases monotonically with increasing M_{200} and decreasing z. This is a result of the fact that c reflects the cosmic density at the formation (collapse) epoch of DM halos. In the standard CDM cosmology, the cosmic density decreases with time, and present massive objects are believed to have collapsed later than smaller ones.

Recently, Prada et al. (2012, hearafter P12) determined $c(M_{200}, z)$. According to their results, c is described by a nearly universal U–shaped function of M_{200}, and the dependence on z is more complex than proposed by Bullock et al. (2001). Ludlow et al. (2012) pointed out halos around the minimum of the U–shaped function are not relaxed dynamically and are in the early period of merger events. The analysis of P12 and Ludlow et al. (2012) was restricted to $M_{200} > 10^{11} M_\odot$. In order to determine $c(M_{200}, z)$ on less massive scales, we analysed the data of the cosmological simulation, *Cosmogrid*, performed by Ishiyama et al. (2013, hearafter I13). Though the covered volume is smaller than that of the Millenium simulations (Springel et al. 2005; Boylan-Kolchin et al. 2009), the Bolshoi (Klypin et al. 2011) and the MultiDark simulations (P12), the mass resolution is about 1000 times better (particle mass $\sim 10^5 M_\odot$). Therefore we could analyse c of DM halos down to $M \sim 10^8 M_\odot$ which is comparable to the virial mass of dSphs.

Along the lines of P12, we computed c of each halo from the velocity ratio V_max/V_{200}. V_max and V_{200} are the maximum circular velocity and the circular velocity at r_{200},
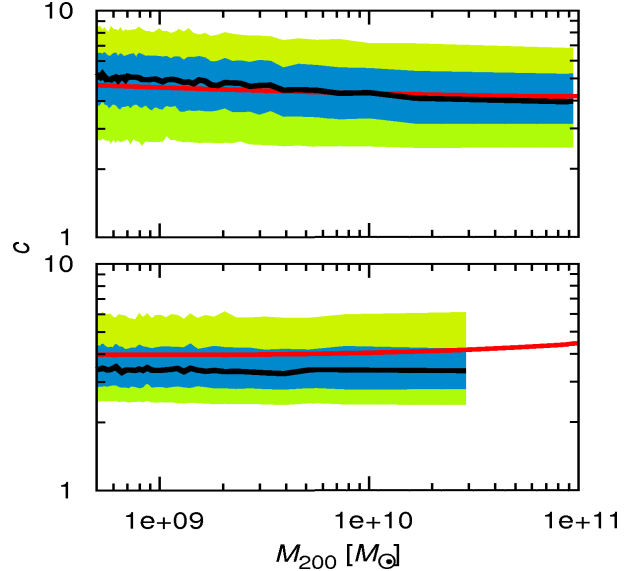

Figure 1. The concentration parameter, c, of DM halos as a function of the virial mass, M_{200}. Red and black lines indicate the fitting formula $c(M_{200}, z)$ proposed by Prada et al. (2012) and the median of our result, respectively. Blue and green areas cover halos distributed within the offsets of 25th and 45th percentiles from the median. Upper and lower panels correspond the results for $z = 3.0$ and $z = 5.4$, respectively.

respectively. Fig. 1 shows the concentration parameter of DM halos as a function of M_{200}. The fitting formula given by P12 perfectly predicts the result of our simulation at the redshift $z = 3$, and reasonably matches that of the redshift $z = 5.4$. The small offset seen in the lower panel arise from the difference of the adopted cosmological parameters, especially σ_8. A similar level of offset is observed at $z = 10.8$ (not shown). However, this is within a small error range and gives negligible effects on the following analysis.

3   THE ORIGIN OF THE μ_{0D} RELATION

Using the fitting formula $c(M, z)$, we study the connection between the cusp–to–core transformation and the μ_{0D} relation and estimate some characteristic properties of DM halos. The procedures and assumptions of the analysis are as follows:

(1) Halo formation: We assume that DM halos form following an NFW profile with the $c(M_{200}, z)$ of P12 for given M_{200} and z.

(2) Cusp–to–Core transformation: We then assume that NFW halos are transformed into Burkert halos by the dynamical process quoted in §1.

We then determine the parameters of the Burkert profile into which a given dark halo distribution will transform. For that we impose two conditions. First, we require mass conservation, that is, M_{200} is conserved during the cusp–to–core transformation. Secondly, we assume that the density distribution is affected only in the inner regions while the outer density distribution is preserved. This is a reasonable assumption if this transition is a result of variations in the gravitational potential caused by baryons and stellar feedback. From Equations (1) and (2), this condition requires

$$\rho_\mathrm{s} r_\mathrm{s}^3 = \rho_0 r_0^3. \qquad (4)$$

We show the results in Fig. 2 which plots r_0 vs. ρ_0. Though Spano et al. (2008) assumed an isothermal sphere to derive the core parameters, the difference between a Burkert profile and an isothermal sphere is small if the parameters (i.e., the central density and the core radius) are the same. Therefore we adopt the data from Spano et al. (2008) as parameters of the corresponding Burkert profile. The sample galaxies of Salucci et al. (2012, triangles) are nearby dSphs. The data from Donato et al. (2009, circles) and Spano et al. (2008, squares) covers more massive galaxies. In Fig. 2, the yellow and green regions indicate that the free-fall time is longer than the cooling time assuming the collisional equilibrium of the primordial composition of the gas with hydrogen molecules. Such a parameter range, the cooling is efficient so that these objects can cool and condense leading to the active star formation. Most of sample galaxies along the μ_{0D} relation lie in these regions. Therefore, we find that the scenario proposed in this study, the cusp–to–core transformation, naturally explains the μ_{0D} relation.

Figure 2 also shows that the central density, ρ_0, is nearly constant for each cusp–to–core transformation redshift, z_t, at which the central cusp has been transformed into core. We assume that DM halos have survived until now without significant change of the virial mass. Assuming that central cusps have been flattened out into cores by mechanisms which involve star formation activities (e.g., Ogiya & Mori 2011, 2012), z_t corresponds to the formation epoch of DM halos and galaxies, approximately. As shown in Fig. 1, the dependence of c on M_{200} is weak, especially at high redshifts (see also Fig. 12 in P12). Using this and the definition of the virial mass, Equation (3), $r_s^3 \propto M_{200}$ should be satisfied. We have defined the scale density, ρ_s, by the mass profile of NFW profile,

$$\rho_s = M_{200}\big/\big[4\pi r_s^3\big\{\ln(1+c) - c/(1+c)\big\}\big], \qquad (5)$$

and found ρ_s is almost independent of M_{200}. As one of the results of the anasysis, we know that the core radius of Burkert halos, r_0, is close to the scale length of corresponding NFW halos, r_s. This means that $\rho_0 \approx \rho_s$ since ρ_0 is determined by Equation (4). Combining everything, one can understand why the dependence of ρ_0 on r_0 is fairly weak. Fig. 2 allows us to estimate z_t from ρ_0. As expected, the higher the core density, the higher the transformation redshift of the corresponding halo. Halos with small core radii and correspondingly high core densities have small virial masses while large cores reside in massive dark halos.

Burkert (1995) obtained the density profiles of several DM halos. His sample halos had core radii of $10^3\mathrm{pc} \lesssim r_0 \lesssim 10^4\mathrm{pc}$. Fig. 1 in Burkert (1995) showed their outer density structures correspond to CDM halos formed at $z = 0.6$. He also proposed a scaling relation between r_0 and ρ_0, and Salucci & Burkert (2000) confirmed this relation for a larger sample. Our results match this relation (blue box). Salvadori et al. (2008) constructed a semi–analytical model to fit the observed features of the Sculptor dSph and found its transformation redshift to be about 7. Our core property estimate for this formation redshift matches nicely the observational measurements (large triangle). They also derived the virial mass of the Sculptor dSph which is about $10^8 M_{\odot}$. Using the observationally inferred r_0 and transformation redshift, z_t, a halo virial mass of $M_{200} \sim 1.2 \times 10^8 M_{\odot}$ which is a perfect match to their result is derived.

As demonstrated in this section, the cusp–to–core transformation naturally generates the μ_{0D} relation in the context of the CDM cosmology. We also find that the cusp–

to–core transformation redshift, z_t, is closely related to the central density of cored DM halos. Interestingly, the parameters of the Burkert profile (r_0 and ρ_0) are almost the same as the scaling parameters of corresponding NFW halos (r_s and ρ_s). Therefore we conclude that the observed universal correlation between the core parameters has been shaped by the physics of formation of these DM halos, in other words, the cores of dark halos retain a memory of their formation redshift. The $c(M_{200}, z)$ relation is essentially important for these results since it determines the parameters of NFW profile.

4 RELATION BETWEEN UNIVERSALITIES

The above mentioned $M(< 300\mathrm{pc})$ universality has been reported for DM halos surround nearby dwarf galaxies. These galaxies are also included in the samples which were used to derive the μ_{0D} relation. In this section, we show the consistency between both relationships among nearby dwarfs and the inconsistency for larger and smaller galaxies. We start by adopting a Burkert profile for the mass–density structures of DM halos. The mass enclosed within given radius, r, is then

$$M(r) = \pi\rho_0 r_0^3\big[-2\arctan(r/r_0) + 2\ln\big\{1 + (r/r_0)\big\} \\ + \ln\big\{1 + (r/r_0)^2\big\}\big] \qquad (6)$$

(Mori & Burkert 2000). We use Eqs. (5) and (6) in the analysis of this section.

The upper panel of Fig. 3 shows the DM mass enclosed within 300pc, $M(< 300\mathrm{pc})$, as a function of the core radius, r_0. As shown in this panel, the DM halos of dSphs have $\sim 10^7 M_{\odot}$ within 300pc. This is consistent with Strigari relation (black line) for lower core radii. Independent of this study, Faerman et al. (2013) show Burkert profile naturally produces Strigari relation for nearby dwarf galaxies. However, halos of larger galaxies deviate from it. Our theoretical prediction, shown by the solid red line fits the data very well and indicates that Strigari relation is just a coincidence resulting from the fact that dwarf galaxies like close to the maximum of $M(< 300\mathrm{pc})$ versus r_0 correlation. As a result we would predict that halos that are smaller than those typically found around dSphs should have smaller values of $M(< 300\mathrm{pc})$ as predicted by Strigari relation. We also show $M(< 300\mathrm{pc})$ of NFW halos supposing $\rho_s r_s = 140 M_{\odot}\mathrm{pc}^{-2}$ (red dashed line). For this line, the horizontal axis corresponds to r_s. Comparing the solid and dashed red lines one can see it is difficult to distinguish between NFW halos and Burkert halos for dwarfs, but that measurements of core properties for more massive halos should allow to distinguish between the different halo profiles. The lower panel of Fig. 3 shows the central density of the Burkert profile, ρ_0, as a function of r_0. This panel clearly demonstrates the two relations are consistent with each other among dSphs, but inconsistent for larger galaxies with scale radii of order 10 kpc or larger.

The consistency between Strigari and the μ_{0D} relation breaks down for larger halos surround massive galaxies, but these galaxies follow the μ_{0D} relation very well. From that we conclude that Strigari relation is just an indirect evidence of the μ_{0D} one for the case of dwarf galaxies, since the latter covers a broader range of halo masses and halo core radii. In addition, the NFW profile does not follow theses observations well, indicating that the cusp–to–core transformation is an important process for the evolution of the inner dark halo structure.
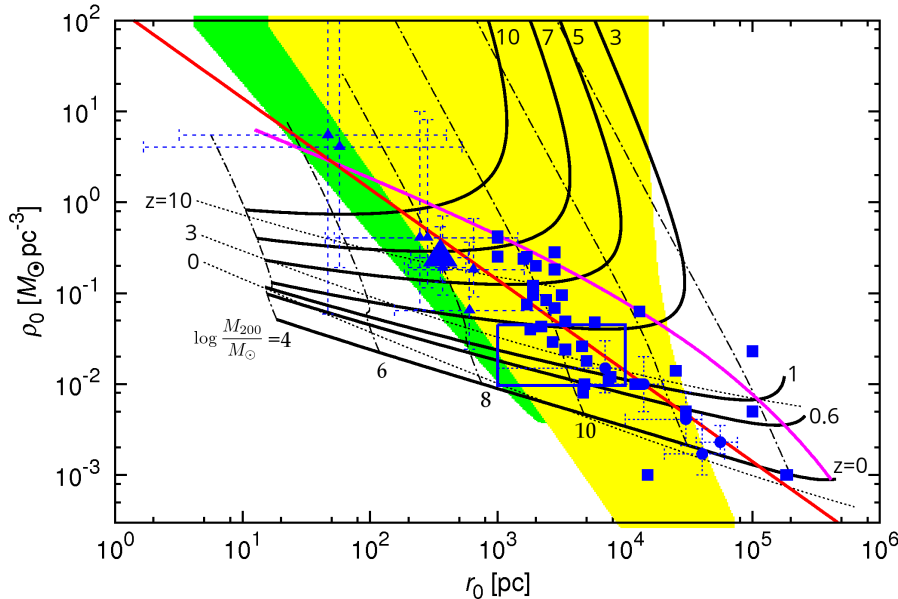
Figure 2. Diagram of r_0 vs. ρ_0. Respective points show the parameters of Burkert profile, ρ_0 and r_0, obtained by observations. Triangles, squares and circles represent the data from Salucci et al. (2012), Spano et al. (2008) and Donato et al. (2009), respectively. Large triangle denotes the Sculptor dSph. Red line is the scaling relation between ρ_0 and r_0 proposed by Salucci et al. (2012). The place embraced by blue box corresponds to the results of Burkert (1995). Solid and dotted black lines show the results of our analysis applying the $c(M, z)$ proposed by Prada et al. (2012) and Macciò et al. (2008), respectively. Subscripts denote the corresponding transformation redshift, z_t. Black dot–dashed lines are the contour of halo mass. Magenta line corresponds to the density peaks, $\delta = \delta_c = 3\sigma(M, z)$, where $\delta_c = 1.69$ and $\sigma(M, z)$ are the critical density peak to collapse and the linear rms fluctuation of the density field for given mass and redshift, $\sigma(M, z)$, respectively. Applying the Press–Schechter theory (Press & Schechter 1974), most of halos satisfy $\delta < 3\sigma(M, z)$ (below magenta line), stochastically. The yellow and green regions indicate the parameter range satisfying the condition, $t_{cool} \leqslant t_{ff}$, where t_{cool} and t_{ff} are the cooling time and the free–fall time of the gas, respectively. Aassuming a total gas mass equal to $0.1\,M_{200}$, and a molecular abundance equal to 0.1% of the neutral hydrogen, we estimate the cooling time using the cooling function of primordial gas given by Sutherland & Dopita (1993, yellow) including H_2 cooling (temperature below $\sim 10^4$ K, green) given by Galli & Palla (1998).

5   SUMMARY AND DISCUSSION

In this study, we have studied the relation between observational universalities of dark matter halos and the cusp–to–core transformation. We obtained three interesting results:

(1) We verified the dependence of the concentration parameter of NFW profiles, c, on virial mass of halo, M_{200}, and redshift, z, proposed by Prada et al. (2012) using the data of the cosmological simulation performed by Ishiyama et al. (2013). According to our analysis, the $c(M_{200}, z)$ proposed by Prada et al. (2012) is appropriate down to $M \sim 10^8 M_\odot$ which is the mass scale of dwarf galaxies.

(2) The cusp–to–core transformation shapes the μ_{0D} relation naturally. The central density of the Burkert profile, ρ_0, is almost independent of M_{200}. We therefore can estimate the formation epoch of the respective halos from ρ_0. Our estimations are consistent with previous studies. The μ_{0D} relation is a relict of the formation phase of DM halos and keeps a memory of their time of formation.

(3) We found that the μ_{0D} and Strigari relations are consistent with each other among DM halos surrounding dSphs. However, this consistency breaks down for halos of more massive galaxies that have larger cores. The μ_{0D} relation can be applied to a broader range of halo mass than Strigari relation. We conclude that Strigari relation is an indirect evidence of the μ_{0D} relation.

The cusp–to–core transformation represents a viable model in order to explain the origin of these universalities. In our analyses, we assumed a transformation from NFW profiles to Burkert profiles. This assumption requires some mechanisms to flatten the cusps and to create the cores with sizes comparable to the scale lengths of the cuspy halo. This requirement might allow to distinguish between different models of core formation and might provide important constraints for them. It is not only the flattening of a cusp but also the formation of a reasonably sized core that these models have to generate.

An interesting question is whether the cusp–to–core transformation also solves the missing satellite (e.g., Moore et al. 1999b) problem. This might be difficult to achieve as obviously the cores are observed in dwarf galaxies that were not disrupted by internal processes or that became invisible because of quenching of star formation but that still experienced major changes in the gravitational potential leading to cores (see also Peñarrubia et al. 2012). In addition, it is also intersting about a weak dependence of a halo mass on the μ_{0D} relation (Cardone & Tortora 2010; Boyarsky et al. 2010; Cardone & Del Popolo 2012). The subsequent studies will report on these issues.

It is noteworthy that the transformation redshifts of the Sextans and the Leo II dSphs, the leftmost points in Fig. 2, are beyond the redshift $z_t = 10$. In such an early epoch, the gas is almost metal–free, and H_2 acts as a prime coolant (e.g., Galli & Palla 1998; Omukai & Nishi 1998; Bromm &
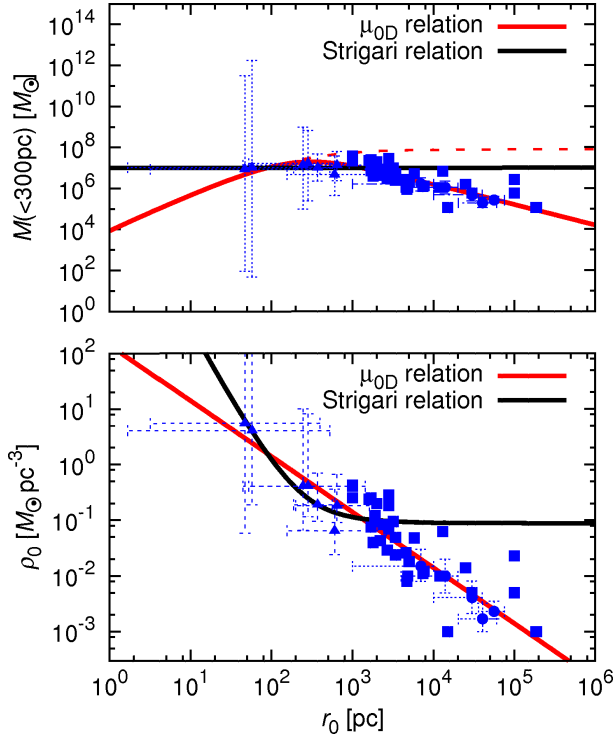
Figure 3. (*Upper*) DM mass enclosed within 300pc, $M(< 300\text{pc})$, as a function of core radius, r_0. Symbols with error bars show the observations. The red, solid line depicts the μ_{0D} relation. We show $M(< 300\text{pc})$ of NFW halos with the assumption of $\rho_s r_s = 140 M_\odot \text{pc}^{-2}$ (red dashed). The black line shows Strigari relation. (*Lower*) The central density of Burkert profiles, ρ_0, is shown as a function of r_0. The black and red lines show Strigari and the μ_{0D} relation, respectively. Each symbol represents same observed galaxy within Fig. 2.

Yoshida 2011 and references therein). Further studies about these galaxies may play a key role to unravel the formation of the first objects in the universe.

REFERENCES

Boyarsky A., Neronov A., Ruchayskiy O., Tkachev I., 2010, PhRvL, 104, 191301
Boylan-Kolchin M., Springel V., White S. D. M., Jenkins A., Lemson G., 2009, MNRAS, 398, 1150
Bromm V., Yoshida N., 2011, ARA&A, 49, 373
Bullock J. S., Kolatt T. S., Sigad Y., Somerville R. S., Kravtsov A. V., Klypin A. A., Primack J. R., Dekel A., 2001, MNRAS, 321, 559
Burkert A., 1995, ApJ, 447, L25
Cardone V. F., Tortora C., 2010, MNRAS, 409, 1570
Cardone V. F., Del Popolo A., 2012, MNRAS, 427, 3176
de Blok W. J. G., McGaugh S. S., Rubin V. C., 2001, AJ, 122, 2396
Del Popolo A., Cardone V. F., Belvedere G., 2013, MNRAS, 429, 1080
Donato F., et al., 2009, MNRAS, 397, 1169
El-Zant A., Shlosman I., Hoffman Y., 2001, ApJ, 560, 636
Faerman Y., Sternberg A., McKee C. F., 2013, ApJ, 777, 119
Fukushige T., Makino J., 1997, ApJ, 477, L9
Galli D., Palla F., 1998, A&A, 335, 403
Goerdt T., Moore B., Read J. I., Stadel J., 2010, ApJ, 725, 1707
Hayashi K., Chiba M., 2012, ApJ, 755, 145
Inoue S., Saitoh T. R., 2011, MNRAS, 418, 2527
Ishiyama T., et al., 2013, ApJ, 767, 146
Klypin A. A., Trujillo-Gomez S., Primack J., 2011, ApJ, 740, 102
Kormendy J., Freeman K. C., 2004, IAUS, 220, 377
Ludlow A. D., Navarro J. F., Li M., Angulo R. E., Boylan-Kolchin M., Bett P. E., 2012, MNRAS, 427, 1322
Macciò A. V., Dutton A. A., van den Bosch F. C., 2008, MNRAS, 391, 1940
Moore B., 1994, Natur, 370, 629
Moore B., Quinn T., Governato F., Stadel J., Lake G., 1999a, MNRAS, 310, 1147
Moore B., Ghigna S., Governato F., Lake G., Quinn T., Stadel J., Tozzi P., 1999b, ApJ, 524, L19
Mori M., Burkert A., 2000, ApJ, 538, 559
Navarro J. F., Eke V., Frenk C. S., 1996, MNRAS, 283, 72
Navarro J. F., Frenk C. S., White S. D. M., 1997, ApJ, 490, 493
Navarro J. F., et al., 2010, MNRAS, 402, 21
Ogiya G., Mori M., 2011, ApJ, 736, L2
Ogiya G., Mori M., 2012, arXiv, arXiv:1206.5412
Oh S.-H., de Blok W. J. G., Brinks E., Walter F., Kennicutt R. C., Jr., 2011, AJ, 141, 193
Omukai K., Nishi R., 1998, ApJ, 508, 141
Peñarrubia J., Pontzen A., Walker M. G., Koposov S. E., 2012, ApJ, 759, L42
Pontzen A., Governato F., 2012, MNRAS, 421, 3464
Prada F., Klypin A. A., Cuesta A. J., Betancort-Rijo J. E., Primack J., 2012, MNRAS, 423, 3018
Press W. H., Schechter P., 1974, ApJ, 187, 425
Salucci P., Burkert A., 2000, ApJ, 537, L9
Salucci P., Wilkinson M. I., Walker M. G., Gilmore G. F., Grebel E. K., Koch A., Frigerio Martins C., Wyse R. F. G., 2012, MNRAS, 420, 2034
Salvadori S., Ferrara A., Schneider R., 2008, MNRAS, 386, 348
Spano M., Marcelin M., Amram P., Carignan C., Epinat B., Hernandez O., 2008, MNRAS, 383, 297
Spekkens K., Giovanelli R., Haynes M. P., 2005, AJ, 129, 2119
Springel V., et al., 2005, Natur, 435, 629
Strigari L. E., Bullock J. S., Kaplinghat M., Simon J. D., Geha M., Willman B., Walker M. G., 2008, Natur, 454, 1096
Sutherland R. S., Dopita M. A., 1993, ApJS, 88, 253
Swaters R. A., Madore B. F., van den Bosch F. C., Balcells M., 2003, ApJ, 583, 732
Teyssier R., Pontzen A., Dubois Y., Justin I. R., 2013, MNRAS, 429, 3068
Walker M. G., Mateo M., Olszewski E. W., Peñarrubia J., Wyn Evans N., Gilmore G., 2009, ApJ, 704, 1274