

# FORMATION OF MASSIVE BLACK HOLES IN GLOBULAR CLUSTERS

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## ABSTRACT

The hydrodynamic formation of massive black holes (BHs) in globular clusters is considered. In particular, we examine the possibility of BH formation induced by the radiation drag that is exerted on the interstellar matter by stellar radiation in globular clusters. The radiation drag extracts angular momentum from interstellar gas and thus allows the gas to accrete onto the cluster center. By incorporating the realistic chemical evolution of globular clusters, we scrutinize the efficiency of the radiation drag to assess the total accreted mass. As a result, we find that if a globular cluster is more massive than  $\approx 6 \times 10^6 M_\odot$  in the present-day stellar component, a massive BH with  $>260 M_\odot$  can form within it. But the BH-to-cluster mass ratio is considerably smaller than the BH-to-bulge mass ratio ( $\approx 10^{-3}$ ) found in galactic bulges. The results are not sensitive to the assumed stellar initial mass function and star formation rate in the cluster, as long as the resulting color-magnitude relation, metallicity, and mass-to-luminosity ratio satisfy those observed in globular clusters. Hence, the putative linear relation between BH mass and bulge mass ( $M_{\text{BH}}-M_{\text{bulge}}$  relation) cannot be extrapolated to globular cluster systems. In the present regime, we discuss the BH formation in M15, G1,  $\omega$  Cen, the M33 nucleus, and the compact X-ray sources in M82. Finally, we discuss observational indications of the formation process of massive BHs in globular clusters. We find that the final phase of BH growth due to the radiation drag can be observed as ultraluminous X-ray sources (ULXs) with  $\sim 10^{41}$  ergs s $^{-1}$ .

*Subject headings:* black hole physics — globular clusters: general — hydrodynamics — radiation mechanisms: general

## 1. INTRODUCTION

The recent compilation of the kinematical data on galactic centers has revealed that a central “massive dark object” (MDO), which is a candidate for a supermassive black hole (BH), does correlate with the mass of the galactic bulge; the BH-to-bulge mass ratio is  $\approx 0.002$  as a median value (e.g., Kormendy & Richstone 1995). This correlation suggests that the formation of a supermassive BH is physically connected with the formation of a galactic bulge.

If the BH-to-bulge relation can be extrapolated to small spheroidal systems like globular clusters, massive BHs with  $10^3$ – $10^4 M_\odot$  may inhabit globular clusters. To date, some candidates of massive BHs in globular clusters have been reported; the estimated mass of a possible BH in M15 is  $M_{\text{BH}} = (1.7^{+2.7}_{-1.7}) \times 10^3 M_\odot$  (Gerssen et al. 2003), or  $M_{\text{BH}} \approx 2 \times 10^4 M_\odot$  in G1 (Gebhardt et al. 2002). On the contrary, recent sophisticated numerical simulations (Baumgardt et al. 2003a, 2003b) show that a massive BH with  $>500$ – $1000 M_\odot$  is not mandatory to account for the observational data in M15 and that the luminosity profile of G1 can be well fitted without a massive BH. On the other hand, the M33 nucleus does not appear to possess a massive BH with an upper limit of  $\approx 10^3 M_\odot$  (Gebhardt et al. 2001; Merritt et al. 2001). In addition, the latest radio observations have suggested that the BH mass in  $\omega$  Cen should be less than about  $100 M_\odot$  for the spherical Bondi-Hoyle accretion rate, but the data are marginally consistent with a BH of about  $1000 M_\odot$  for a more plausible accretion rate (Maccarone et al. 2005). Thus, the existence of massive BHs in globular clusters is under debate.

Some theoretical models have been proposed for the formation of massive BHs. One of them is the merger of multiple small BHs or stars (e.g., Lee 1987; Quinlan & Shapiro 1990). However, this scenario predicts that even in a very compact star cluster like a globular cluster, the core collapse is halted by the binary heating before the stellar density becomes high enough for stars to merge in a runaway fashion (Hut et al. 1992). However, it has been pointed out recently that the mass difference of the constituents plays an important role. Stars of different masses are not always able to reach energy equipartition. This causes the heaviest stars to undergo core collapse on a timescale that is much shorter than the core-collapse time for the cluster as a whole. Portegies Zwart & McMillan (2002) show by  $N$ -body simulations that a runaway merger among these massive stars leads to the formation of a massive BH. On the other hand, Miller & Hamilton (2002) examine a model in which a single BH ( $>50 M_\odot$ ) located at the center of the cluster grows in mass through merging with stellar mass BHs.

Another possibility is the hydrodynamic formation of massive BHs through supermassive stars. Recently, as a potential hydrodynamic mechanism in a bulge, Umemura (2001) has considered the effects of radiation drag, which is equivalent to the well-known Poynting-Robertson effect in the solar system. The radiation drag extracts angular momentum from interstellar gas and thus allows the gas to accrete onto the center. This is a parallel model to the formation of massive BHs by Compton drag in the early universe (Umemura et al. 1993). The mass accretion by the radiation drag has been explored in detail (Umemura et al. 1997; Fukue et al. 1997). The angular momentum loss rate by the radiation drag is given by  $d \ln J / dt \simeq -\chi E / c$ , where  $J$  is the total

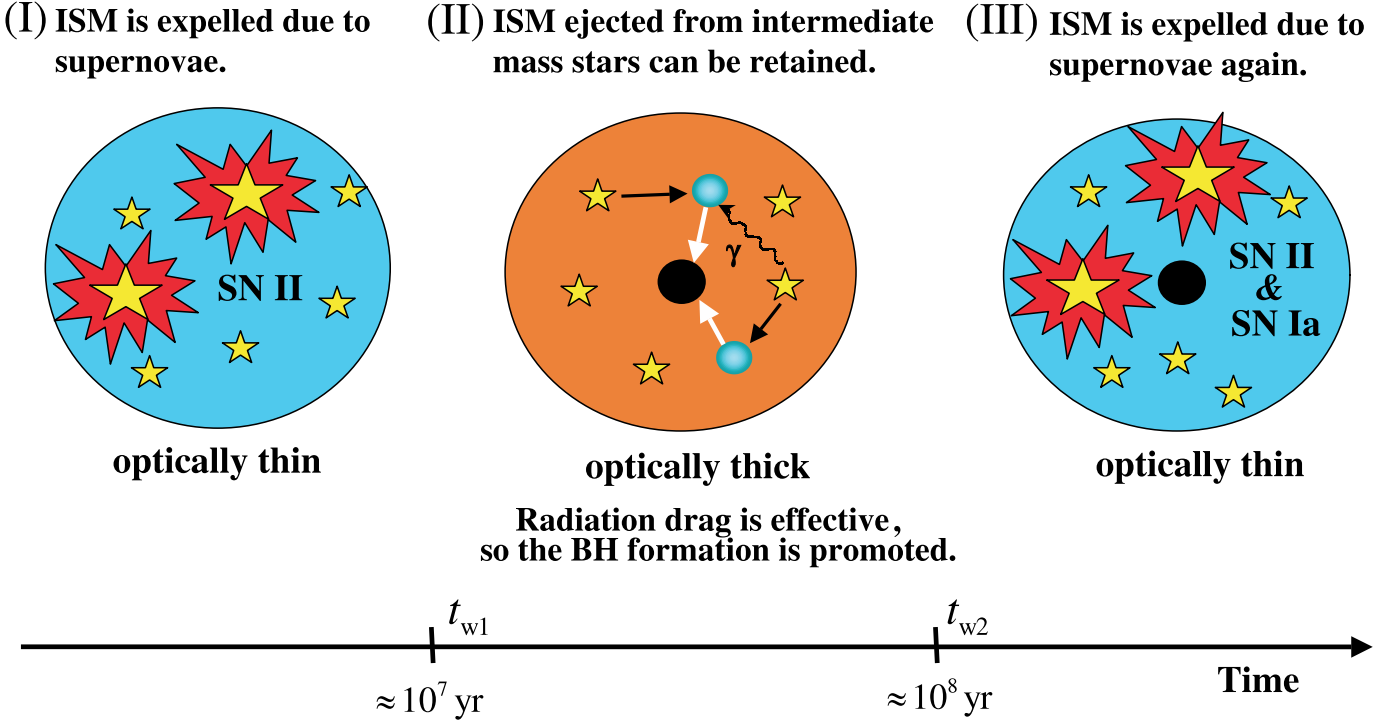


FIG. 1.—Outline of the present scenario for the formation of a massive BH in a globular cluster.

angular momentum of the gaseous component,  $E$  is the radiation energy density, and  $\chi$  is the mass extinction coefficient. Therefore, in an optically thin regime,  $d \ln J/dt \simeq -(\tau L_*/c^2 M_{\text{gas}})$ , where  $\tau$  is the total optical depth of the system,  $L_*$  is the total luminosity of the spheroidal system, and  $M_{\text{gas}}$  is the total mass of gas. In an optically thick regime, the radiation drag efficiency is saturated due to the conservation of photon number (Tsuribe & Umemura 1997). Thus, an expression of the angular momentum loss rate that includes both regimes is given by  $d \ln J/dt \simeq -(L_*/c^2 M_{\text{gas}})(1 - e^{-\tau})$ . Then the mass accretion rate is estimated to be  $\dot{M} = -M_{\text{gas}} d \ln J/dt = L_*/c^2 (1 - e^{-\tau})$ . In an optically thick regime, this gives simply  $\dot{M} = L_*/c^2$  (Umemura 2001). Then the total accreted mass onto the MDO,  $M_{\text{MDO}}$ , is maximally  $M_{\text{MDO}} \simeq \int L_*/c^2 dt$ . Although this is a simple estimation in a one-zone interstellar medium (ISM), the ISM is observed to be highly inhomogeneous in an active star-forming galaxy (Sanders et al. 1988; Gordon et al. 1997). In such an inhomogeneous ISM, optically thin surface layers of optically thick, clumpy clouds are stripped by the radiation drag, and the stripped gas loses angular momentum, eventually accreting onto the center (Sato et al. 2004). Kawakatu & Umemura (2002) have shown that the inhomogeneity of the ISM plays an important role for the radiation drag to attain its maximal efficiency. Then the final mass of the MDO is proportional to the total radiation energy from stars. Taking realistic chemical evolution into consideration, the radiation drag model predicts a mass ratio  $M_{\text{BH}}/M_{\text{bulge}} \simeq 0.001$  in a galactic bulge (Kawakatu et al. 2003). The theoretical upper limit of the BH-to-bulge mass ratio is determined by the energy conversion efficiency of nuclear fusion from hydrogen to helium, i.e., 0.007 (Umemura 2001). Here the question is whether the radiation drag mechanism can work also in small spheroidal systems like globular clusters.

In the present work, we examine the possibility of radiation drag-induced formation of massive BHs in globular clusters. Here we assume that the formation of a globular cluster begins with the coeval starburst within it. The outline of the physical processes

that we consider is shown in Figure 1: (1) First, the energy input from Type II supernovae (SNe) drives the outflow of ISM from the system (*left*). Therefore, the system becomes optically thin in an early evolutionary phase. (2) Afterward, intermediate-mass ( $2-8 M_{\odot}$ ) stars shed the gas envelope into the interstellar space, accumulating the ISM, and thus the optical depth of the system increases (*middle*). In this phase, the radiation drag can work effectively, and the mass accretion onto the center is induced. (3) Finally, Type Ia SNe, with Type II SNe in newly formed stars, expel the ISM from the system again (*right*). As a result, the efficiency of the radiation drag descends abruptly. Through these physical processes, it is expected that the BH formation in a globular cluster is strongly related to the star formation history.

In this paper, we attempt to elucidate whether the formation of massive BHs can be prompted by a radiation-hydrodynamic mechanism in globular clusters. Also, based on the present model, we reconsider the formation of massive BHs in M15, G1,  $\omega$  Cen, the M33 nucleus, and the compact X-ray sources in M82. The paper is organized as follows. In § 2, we build up the model for the chemical evolution and the radiation-hydrodynamic process. In § 3, we investigate the relation between the star formation history and the final BH mass in a cluster. Then we derive the condition under which globular clusters can possess massive BHs. In § 4, we compare our predictions with the observational results for M15, G1,  $\omega$  Cen, the M33 nucleus, and the compact X-ray sources in M82 and discuss whether massive BHs can form through the radiation-hydrodynamic process in these globular clusters. In § 5, based on the present regime, we discuss observational indications of the formation process of massive BHs in globular clusters. Section 6 is devoted to the conclusions.

## 2. MODELS

### 2.1. Chemical Evolution of Globular Cluster

We suppose a two-component system that consists of a spheroidal stellar cluster and dusty interstellar matter within it. First,

we construct the model for the chemical evolution of globular clusters, using the evolutionary spectral synthesis code PEGASE (Projet d'Etude des Galaxies par Synthèse Evolutive; Fioc & Rocca-Volmerange 1997). At the initial epoch, we set a spherical system composed of gas with mass given in the range  $M_0 = 10^5 - 10^9 M_\odot$ . The radius,  $r_s$ , is determined to follow a mass-radius relation,  $[r_s/(10 \text{ pc})] = [M_0/(10^6 M_\odot)]^{1/2}$ , found by Chiosi & Carraro (2002). It is assumed that the star formation starts with an overall starburst with a duration of  $\approx 10^7$  yr, which reproduces the color-magnitude relation of globular clusters at the present epoch (Yoshii & Arimoto 1987). We employ a stellar initial mass function (IMF) in the form of  $\phi = dn/d \log m_* = A(m_*/M_\odot)^{-\alpha}$ , where  $m_*$  is the stellar mass. The lower mass in the IMF is assumed to be  $m_l = 0.1 M_\odot$  and the upper mass to be  $m_u = 60 M_\odot$ . The index  $\alpha$  is changed from 0.2 to 2.0 in order to see the effects of the slope of the IMF on the final results. Also, stars with masses greater than  $8 M_\odot$  are postulated to undergo Type II SN explosions. As for the Type Ia SN, the evolutionary spectral synthesis code PEGASE adopts the single-degenerate scenario (Nomoto et al. 1994) for the Type Ia SN progenitor, in which a white dwarf (WD) in a close binary undergoes a thermonuclear explosion when the companion star evolves off the main sequence and transfers a large enough amount of mass to the WD.<sup>1</sup> From the evolution model of progenitor stars, Hachisu et al. (1996, 1999) obtained the mass range of companion stars of the WDs that become Type Ia SNe as  $[0.9, 3.5] M_\odot$ . The star formation rate (SFR) per unit mass,  $C(t)$ , is assumed to be in proportion to the gaseous mass fraction,  $f_{\text{gas}}(t) \equiv M_{\text{gas}}(t)/M_0$ , where  $M_{\text{gas}}(t)$  is the total gas mass at time  $t$ . The SFR is given by

$$C(t) = \begin{cases} \beta f_{\text{gas}}, & t < t_{w1}, \\ k f_{\text{gas}}, & t_{w1} \leq t < t_{w2}, \\ 0, & t \geq t_{w2}. \end{cases} \quad (1)$$

The epoch  $t_{w1}$  is determined by the age of the minimal star ( $8 M_\odot$ ) that undergoes a Type II SN, that is,  $t_{w1} = 6 \times 10^7$  yr. Before  $t_{w1}$ , the interstellar gas is assumed to blow out from the system due to the energy input from successive Type II SNe. The epoch  $t_{w2}$  is the second blowout epoch when the interstellar gas is expelled by Type Ia SNe as well as Type II SNe in newly formed stars. The second blowout epoch is estimated by the epoch when the kinetic energy ( $E_{\text{kin}}$ ) equals the binding energy ( $E_{\text{bin}}$ ). The kinetic energy is evaluated by  $E_{\text{kin}} = \epsilon 10^{51} N_{\text{SN}}$  ergs, where  $N_{\text{SN}}$  is the number of SNe, which is calculated by PEGASE, and  $\epsilon$  is the efficiency at which the total SN energy is converted to the kinetic energy of the blast wave. The Sedov solution gives  $\epsilon = 0.28$  for the adiabatic exponent of  $\gamma = 5/3$ . After  $t_{w2}$ , the system is assumed to be ISM-deficient and thus optically thin. The coefficient  $\beta$  is assumed to be  $\beta \approx (10^7 \text{ yr})^{-1}$  (Yoshii & Arimoto 1987), and  $k$  is changed in order to see the dependence on the SFR.

As for the stellar component, we assume star clusters with a specific stellar initial mass function (see above). Stars are distributed uniformly inside the spheroidal system. Actually,  $N_*(=300)$  small star clusters are distributed randomly. In addition, the angular momentum is smeared, assuming rigid rotation, according

<sup>1</sup> In present-day galaxies, it is well known that more than half of main-sequence stars are observed in binaries, while it is not observationally clear if Population III stars are able to form in binary systems. But Saigo et al. (2004) have recently shown by numerical simulations that a significant fraction of Population III stars are expected to form in binary systems, and thus Type Ia SNe would occur even in metal-deficient environments.

to the spin parameter of  $\lambda = (J_T |E_T|^{1/2}) / (GM_T^{5/2}) = 0.05$ , where  $J_T$ ,  $E_T$ , and  $M_T$  are, respectively, the total angular momentum, energy, and mass (Barnes & Efstathiou 1987; Heavens & Peacock 1988; Steinmetz & Bartelmann 1995).

As for the ISM, we consider clumpy matter, since the ISM is observed to be highly inhomogeneous in active star-forming regions (Sanders et al. 1988; Gordon et al. 1997). Here  $N_c (=10^4)$  identical clouds are distributed randomly. (It is noted that simulations with a 3 times larger number of clouds did not lead to any fundamental difference in the final BH mass, although at least  $10^4$  clouds are necessary to treat the radiative transfer effect properly in a clumpy ISM.) The distribution and motion of the ISM is the same as those of the stellar component. We assume that the cloud-covering factor is unity according to the previous analysis (Kawakatu & Umemura 2002). The internal density in a cloud is assumed to be uniform, and the optical depth is determined by a dust-to-gas ratio that is calculated by PEGASE. Thus, the total optical depth of the globular cluster,  $\tau_T$ , is a function of time. In addition, the optical depth of a cloud, and therefore the overall optical depth of the globular cluster, depends on the cloud size  $r_c$ . Here  $r_c = 0.01 r_s$  is assumed as a fiducial case. But we have confirmed that no essential difference in the final BH mass is found by changing  $r_c$  so as to enhance  $\tau_T$  by an order of magnitude.

## 2.2. Mass Accretion Due to Radiation Drag

Next we model the formation of a BH in a globular cluster, based on the radiation drag-driven mass accretion. The radiation drag, which drives the mass accretion, originates in the relativistic effect in absorption and subsequent reemission of the radiation. This effect is naturally involved in relativistic radiation-hydrodynamic equations (Umemura et al. 1997; Fukue et al. 1997). The angular momentum transfer in radiation-hydrodynamics is given by the azimuthal equation of motion in cylindrical coordinates,

$$\frac{1}{r} \frac{d(rv_\phi)}{dt} = \frac{\chi_d}{c} [F^\phi - (E + P^{\phi\phi})v_\phi], \quad (2)$$

where  $E$  is the radiation energy density,  $F^\phi$  is the radiation flux,  $P^{\phi\phi}$  is the radiation stress tensor, and  $\chi_d$  is the mass extinction coefficient, which is given by  $\chi_d = n_d \sigma_d / \rho_{\text{gas}}$ , with the number density of dust grains  $n_d$ , the dust cross section  $\sigma_d$ , and the gas density  $\rho_{\text{gas}}$ . By solving radiative transfer including dust opacity, we evaluate the radiative quantities,  $E$ ,  $F^\phi$ , and  $P^{\phi\phi}$ , and thereby obtain the total angular momentum loss rate. Then we can estimate the total mass of the dusty ISM accreted onto a central massive dark object,  $M_{\text{MDO}}$ . Using the relation  $\dot{M}_{\text{gas}}/M_{\text{gas}} = -\dot{J}/J$ , where  $J$  and  $M_{\text{gas}}$  are the total angular momentum and gas of the ISM, the MDO mass is assessed as

$$M_{\text{MDO}}(t) = \int_0^t \dot{M}_{\text{gas}} dt = - \int_0^t M_{\text{gas}} \frac{\dot{J}}{J} dt. \quad (3)$$

In the optically thick regime of the radiation drag,  $-\dot{M}_{\text{gas}} \dot{J}/J = \int_0^\infty [L_{\text{star}, \nu}(t)/c^2] d\nu$ , where  $L_{\text{star}, \nu}$  is the total luminosity of the globular cluster at frequency  $\nu$ . The radiation drag efficiency depends on the optical depth  $\tau$  in proportion to  $(1 - e^{-\tau})$  (Umemura 2001). Thus, the total mass of the MDO can be expressed by

$$M_{\text{MDO}}(t) = \eta_{\text{drag}} \int_0^t \int_0^\infty \frac{L_{\text{star}, \nu}(t)}{c^2} [1 - e^{-\tau_\nu(t)}] d\nu dt, \quad (4)$$

where  $\tau_\nu$  is the optical depth of the globular cluster measured from the center. Here we estimate the evolution of  $\tau_\nu$  using PEGASE. The efficiency  $\eta_{\text{drag}}$  is found to be maximally 0.34 (Kawakatu & Umemura 2002). After  $t_{w2}$ , the radiation drag is inefficient, because the system becomes ISM-deficient and optically thin. In this paper, at  $t < t_{w1}$  and  $t > t_{w2}$ ,  $\tau_\nu$  is assumed to drop abruptly to  $\tau_\nu \ll 1$ . Hence, the final mass of an MDO is given by

$$M_{\text{MDO}} = \eta_{\text{drag}} \int_{t_{w1}}^{t_{w2}} \int_0^\infty \frac{L_{\text{star}, \nu}(t)}{c^2} [1 - e^{-\tau_\nu(t)}] d\nu dt. \quad (5)$$

### 2.3. The Fate of the MDOs

In § 2.2, we estimated the MDO mass in the context of the radiation drag-induced mass accretion. However, the MDO itself does not evolve into the massive BH directly, because the radiation drag is not likely to remove the angular momentum thoroughly (Sato et al. 2004). Hence, some residual angular momentum will terminate the radial contraction of the accreted gas. Thus, the MDO is likely to be a compact rotating disk. We should consider the further collapse of the MDO through other physical mechanisms. In the MDO, the viscosity is expected to work effectively, because the timescale for viscous accretion is shortened by the radiation drag (Mineshige et al. 1998). Thus, the MDO would be a massive self-gravitating viscous disk. For a massive self-gravitating viscous disk, some self-similar solutions are known to give an inside-out disk collapse in a flat temperature distribution of a disk (Mineshige & Umemura 1996, 1997; Tsuribe 1999). A flat temperature profile is plausible as long as the viscous heating is balanced with the radiative cooling in the geometrically thin disk (see Mineshige & Umemura [1996] for details). In fact, Tsuribe (1999) derived a series of self-similar solutions for a rotating isothermal disk, taking into account the growth of the central point mass, and then provided a convenient formula for the inside-out mass accretion rate for a variety of outer flows,

$$\dot{M}_\alpha = \frac{3\alpha c_s^3}{Q\dot{G}}, \quad (6)$$

where  $\alpha$  is the viscous parameter,  $c_s$  is the sound speed, and  $\dot{Q}$  is Toomre's  $\dot{Q}$ , which is  $\kappa c_s / \pi G \Sigma$  for epicycle frequency  $\kappa$  and surface density  $\Sigma$ . Tsuribe (1999) has found that an accretion with  $\dot{Q} \approx 2$  is stable for a wide range of  $\alpha$ . The critical accretion rate is given by

$$\dot{M}_\alpha \simeq \frac{c_s^3}{G} = (0.24 M_\odot \text{ yr}^{-1}) \left( \frac{T_{\text{disk}}}{10^4 \text{ K}} \right)^{3/2}. \quad (7)$$

It must be noted that the viscous accretion rate depends sensitively on the disk temperature. Under an intense starburst, the disk is exposed to strong ultraviolet radiation. Then the disk could be heated up to  $10^4$  K, although the details depend on the dust extinction and radiative cooling. Here we employ  $10^4$  K as the disk temperature. Through this inside-out collapse, almost all the MDOs can accrete onto a central core. Finally, the core is likely to be a rigidly rotating very massive star (VMS), because the angular momentum transfer via viscosity works to smear out any differential rotation in a self-gravitating system. Such VMSs possess large helium cores that reach carbon ignition. The fate of VMSs sensitively depends on the initial mass (Bond et al. 1984; Heger & Woosley 2002). A VMS below  $260 M_\odot$  (in the range  $140\text{--}260 M_\odot$ ) results in a pair-instability SN, leaving no remnant,

because the core of such a star undergoes the electron-positron pair creation instability after the helium burning (Barkat et al. 1967; Bond et al. 1984; Kippenhahn & Weigert 1990; Fryer et al. 2001). Above  $\sim 260 M_\odot$ , a VMS collapses directly into a BH because the nuclear burning cannot halt the core collapse (Fryer et al. 2001; Heger et al. 2003). There must be the effect of rotation, because the progenitor stars are likely to be rotating (Bond et al. 1984; Fryer et al. 2001). Recently, Shibata (2004) found that a rigidly rotating VMS with several hundreds of solar masses can be unstable in general relativity for softer equations of state, eventually forming a BH.

Here we should pay attention to the metallicity dependence of the VMS formation. In the solar abundance, any stars more massive than  $100 M_\odot$  are not expected to form (Fryer et al. 2001; Heger et al. 2003). On the other hand, VMSs can preferentially form in metal-free gas (e.g., Nakamura & Umemura 2001 and references therein). The problem is the critical metallicity, under which VMSs can form. Recently, Omukai & Palla (2003) have explored this problem in detail and found the critical metallicity to be around  $0.01 Z_\odot$ , under which dust opacity is not effective in hindering the mass accretion onto a protostar. Hence, in the light of the critical metallicity, VMSs are expected to form in globular clusters. Taking such theoretical advances into account, we assume  $260 M_\odot$  as the minimum mass for the formation of massive BHs. If  $M_{\text{MDO}}$  exceeds this threshold mass ( $260 M_\odot$ ), a massive BH whose mass is equal to an MDO mass would form in a globular cluster. However, it is not clear enough whether the self-similar solutions employed in this paper are feasible in realistic situations for the MDO formed via the radiation drag. The problem should be explored with sophisticated numerical simulations, which are left for future study. In this paper, we regard the BH mass estimated here as the maximal mass of a BH within a globular cluster.

## 3. RESULTS

By changing the physical parameters  $\alpha$  and  $k$ , which control the star formation, we investigate the relation between the formation of massive BHs and the star formation history. First, we examine the dependence on the stellar IMF and then on the star formation rate. Finally, we derive the relation between the mass of the globular cluster and the BH mass.

### 3.1. Dependence on Initial Mass Function

Here we investigate the effects of the IMF slope  $\alpha$  on the final BH mass, setting  $M_0 = 10^8 M_\odot$  and  $k = 8.6 \text{ Gyr}^{-1}$ . In Figure 2, the resulting mass of the BH,  $M_{\text{BH}}$ , is shown for the range  $\alpha = 0.2\text{--}2.0$ . The details of the results are summarized in Table 1. As seen in Figure 2, we find that the mass of the central massive objects exceeds the threshold mass ( $260 M_\odot$ ) for the BH formation in the range  $\alpha = 0.2\text{--}2.0$ . In addition, the BH mass shows a peak around  $\alpha \simeq 1$ . The reason can be understood as follows. First, let us repeat that the final BH mass is determined by the optical depth of the system and the duration between  $t_{w1}$  and  $t_{w2}$ , as shown in equation (5). In the present scenario, the optical depth is controlled by the mass loss from stars with  $< 8 M_\odot$ . If  $\alpha$  is smaller, a larger number of massive stars form, and therefore the optical depth can be increased in a shorter timescale by the mass loss from intermediate-mass stars. But, simultaneously, an increasing number of SN explosions shorten the second blowout time  $t_{w2}$ . Table 1 shows that  $t_{w2}$  reduces vastly with decreasing  $\alpha$ , so that  $M_{\text{BH}}$  lessens. On the other hand, if  $\alpha$  is larger, the fraction of massive stars decreases. As a result, fewer SN explosions lengthen  $t_{w2}$ . But the optical depth cannot be augmented because of a reduced number of intermediate-mass

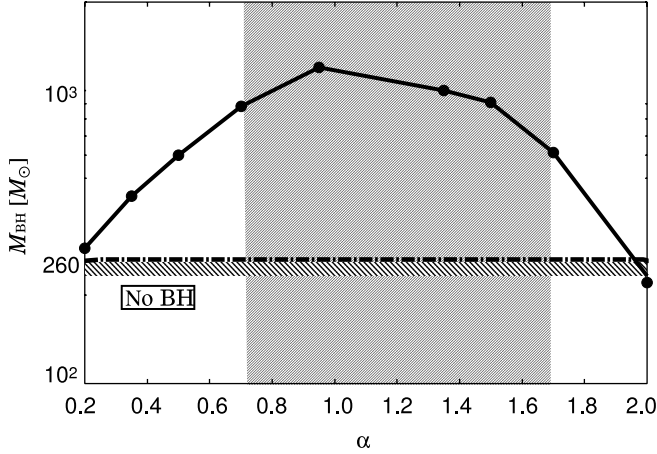


FIG. 2.—Dependence of the final BH mass ( $M_{\text{BH}}$ , in units of solar masses) on the IMF slope ( $\alpha$ ). Here  $M_0 = 10^8 M_\odot$  and  $k = 8.6 \text{ Gyr}^{-1}$  are assumed. The solid line shows our prediction. The shaded region satisfies to the observed mass-to-luminosity ratio and metallicity in globular clusters. The BH mass is maximal around  $\alpha \simeq 1$ .

stars. As a consequence,  $M_{\text{BH}}$  lessens with increasing  $\alpha$ . By these two effects,  $\alpha \simeq 1$  leads to the maximum efficiency of the radiation drag.

The change of  $\alpha$  brings a different mass-to-luminosity ratio and metallicity of a globular cluster. The shaded region satisfies the observed mass-to-luminosity ratio of globular clusters, 1.0–3.0 in solar units (e.g., Mandushev et al. 1991; Djorgovski et al. 2003), and also the metallicity,  $\approx 0.01\text{--}0.1 Z_\odot$  (Lee 1990; Chaboyer et al. 1996), where  $Z_\odot$  is the solar metallicity. If the slope of the IMF is in the range  $\alpha = 0.7\text{--}1.7$ , the properties of globular clusters are consistent with the observations. In this range of  $\alpha$ , massive BHs with  $\approx 10^3 M_\odot$  are expected to form.

In this calculation, we have assumed  $m_l = 0.1 M_\odot$  and  $m_u = 60 M_\odot$ . To see the effect of a different choice of the lower and upper mass of the IMF, we also investigated the cases of  $(m_l, m_u) = (0.5, 60)$ ,  $(0.1, 30)$ , and  $(0.1, 120) M_\odot$ , all of which reproduce the observed mass-to-luminosity ratio. As a result, we found that the final BH mass is not strongly dependent on  $m_l$  and  $m_u$ . To conclude, as long as the mass-to-luminosity ratio and metallicity satisfy those observed, the final BH mass is not sensitive to the stellar initial mass function.

### 3.2. Dependence on Star Formation Rate

In this section, to examine the dependence of the BH mass on the star formation history in globular clusters, we alter the co-

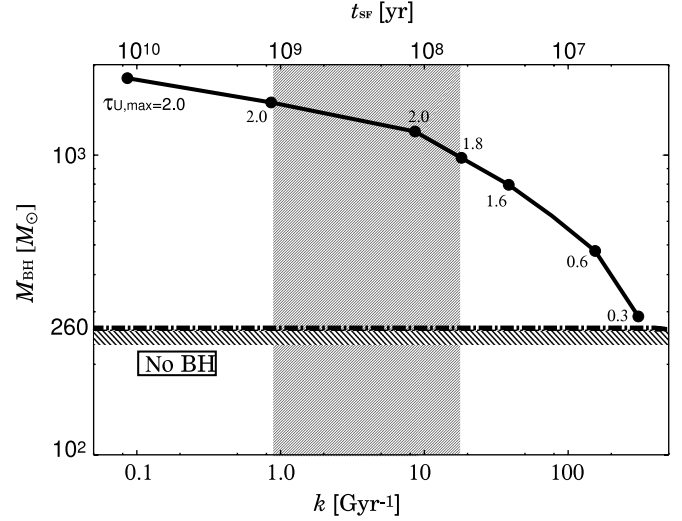


FIG. 3.—Final BH mass ( $M_{\text{BH}}$ ) against the star formation rate coefficient ( $k$ ), assuming  $M_0 = 10^8 M_\odot$  and  $\alpha = 0.95$ . The abscissa is the coefficient in units of  $\text{Gyr}^{-1}$ , and the ordinate is the final BH mass in units of solar masses. The top horizontal axis is the star formation timescale ( $t_{\text{SF}}$ , in units of years). The attached values are the maximal optical depth at the  $U$  band. For  $t_{\text{SF}} > 10^8 \text{ yr}$ , the BH mass is almost saturated. For  $t_{\text{SF}} < 10^8 \text{ yr}$ , the BH mass decreases rapidly, because the system becomes optically thin. The shaded region satisfies the observed mass-to-luminosity ratio and metallicity in globular clusters ( $Z_* = 0.01\text{--}0.1 Z_\odot$ ).

efficient  $k$  from 0.1 to  $300 \text{ Gyr}^{-1}$ . Here we assume  $M_0 = 10^8 M_\odot$  and set  $\alpha = 0.95$ , which gives the maximal radiation drag efficiency for the BH formation, as shown in § 3.1. In Figure 3, the final BH mass is shown against  $k$ . The corresponding star formation timescale, which is defined by  $t_{\text{SF}} = k^{-1}$ , is also shown on the top horizontal axis. The maximal optical depth at the  $U$  band is given for each point simulated. The shaded region satisfies the observed mass-to-luminosity ratio and metallicity of globular clusters. As seen in Figure 3, if  $t_{\text{SF}}$  is shorter than  $\sim 10^8 \text{ yr}$ , the BH mass becomes lower. This is because  $t_{\text{SF}}$  is shorter than the typical age of intermediate-mass stars,  $t_* \approx 10^8 \text{ yr}$ ; the optical depth cannot increase to a high level before  $t_{w2}$ . In contrast, if  $t_{\text{SF}}$  is longer than  $t_*$ , massive BHs with  $\gtrsim 10^3 M_\odot$  can form, because the optical depth of the system can reach a high level. However, if  $t_{\text{SF}} \approx 10^{10} \text{ yr}$ , the number of intermediate-mass stars increases slowly with time. Hence, the BH cannot grow rapidly and its mass is almost saturated for longer  $t_{\text{SF}}$ . As a result, we find that as long as the mass-to-luminosity ratio and metallicity satisfy those observed, the final BH mass is almost independent of the star formation rate, and massive BHs with  $\approx 10^3 M_\odot$  are expected to form.

### 3.3. Globular Cluster-to-BH Mass Relation

Finally, we derive the relation between the final BH mass,  $M_{\text{BH}}$ , and the final stellar mass of globular clusters,  $M_{\text{star, final}}$ . Here we set  $k$  to be constant as  $k = 8.6 \text{ Gyr}^{-1}$ , but  $\alpha$  is changed. We consider the range of initial mass as  $M_0 = 10^6\text{--}10^9 M_\odot$ , because we focus on the final mass range  $M_{\text{star, final}} = 10^5\text{--}10^8 M_\odot$ . The results are shown in Figure 4. As shown in §§ 3.1 and 3.2, the final BH mass is almost independent of the stellar initial mass function and star formation rate, as long as the resulting properties of globular clusters are consistent with the observations. In Figure 4, the shaded area is the region that satisfies the observed mass-to-luminosity ratio and metallicity of globular clusters. The region of no BH means that the mass of the MDO is less than  $260 M_\odot$ , which is the present criterion for a massive BH. Figure 4 shows that massive BHs can form only for a final globular cluster

TABLE 1  
BH MASS-TO-IMF SLOPE IN GLOBULAR CLUSTER

$\alpha$	$M_{\text{BH}}$ ( $M_\odot$ )	$M_{\text{star, final}}$ ( $M_\odot$ ) <sup>a</sup>	$\tau_{U, \text{max}}$ <sup>b</sup>	$t_{w2}$ (yr)	$M_\odot/L_\odot$
0.2.....	290	$3.4 \times 10^5$	1.3	$9 \times 10^7$	0.1
0.35.....	436	$3.0 \times 10^6$	1.4	$10^8$	0.47
0.5.....	600	$6.9 \times 10^6$	1.5	$1.1 \times 10^8$	0.7
0.7.....	880	$1.5 \times 10^7$	1.5	$1.2 \times 10^8$	1.0
0.95.....	1200	$3.0 \times 10^7$	2.0	$1.4 \times 10^8$	1.3
1.35.....	1000	$6.0 \times 10^7$	1.2	$2.0 \times 10^8$	2.0
1.5.....	909	$7.0 \times 10^7$	0.9	$2.5 \times 10^8$	2.3
1.7.....	613	$8.2 \times 10^7$	0.4	$4.5 \times 10^8$	2.9
2.0.....	(221)	$9.2 \times 10^7$	0.2	$1.6 \times 10^9$	4.0

<sup>a</sup> Mass of globular cluster in the present day.

<sup>b</sup> Maximal optical depth at the  $U$  band.

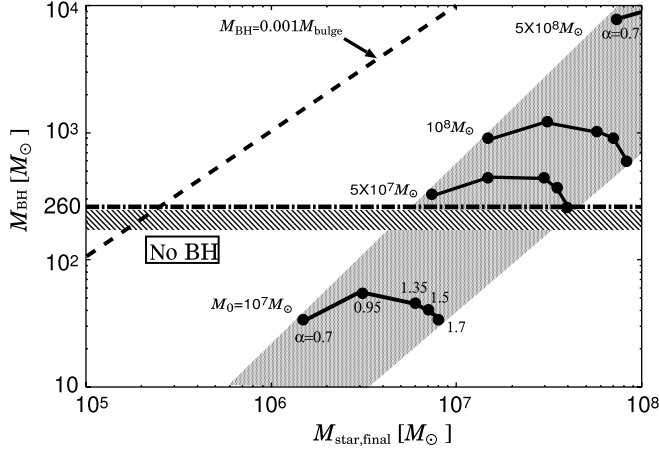


FIG. 4.—Relation between the final BH mass ( $M_{\text{BH}}$ ) and the final stellar mass of the globular cluster ( $M_{\text{star, final}}$ ). Here we assume  $k = 8.6 \text{ Gyr}^{-1}$ . The ordinate is the BH mass in units of solar masses, and the abscissa is the final mass of the globular cluster in units of solar masses. The thick solid lines show the results for different initial masses of the system ( $M_0$ ). The attached values denote the IMF index ( $\alpha$ ). The shaded area shows the region that satisfies the observed mass-to-luminosity ratio and metallicity. The BH-bulge mass relation,  $M_{\text{BH}} = 0.001 M_{\text{bulge}}$ , is shown by a dashed line. The predicted BH fraction is considerably smaller than  $10^{-3}$ .

mass of  $\geq 6 \times 10^6 M_{\odot}$ . The putative BH-to-bulge mass relation,  $M_{\text{BH}} = 0.001 M_{\text{bulge}}$ , is shown by a dashed line. We can see that the predicted BH-to-globular cluster mass ratio ( $M_{\text{BH}}/M_{\text{star, final}}$ ) is considerably lower than the BH-to-bulge mass ratio. The main reason is that the duration when the system is optically thick is shorter in globular clusters than that in galactic bulges, because the supernova explosions are more devastating for smaller spheroidal systems like globular clusters. It is also noted that the relation between the BH mass and the globular cluster mass is not linear, in contrast to the BH-to-bulge mass relation.

#### 4. COMPARISON WITH OBSERVATIONS

To compare the present prediction with the observations of the massive BHs in globular clusters, we translate the obtained  $M_{\text{BH}}-M_{\text{star, final}}$  relation into a  $M_{\text{BH}}-\sigma$  relation, based on the virial theorem, where  $\sigma$  is the stellar velocity dispersion in globular clusters. The resulting  $M_{\text{BH}}-\sigma$  relation is shown in Figure 5. Here we compare the prediction with BH candidates in M15, G1,  $\omega$  Cen, the M33 nucleus, and the compact X-ray sources in M82.

In the globular cluster M15 in the Milky Way, the BH mass is estimated to be  $M_{\text{BH}} = (1.7^{+2.7}_{-1.7}) \times 10^3 M_{\odot}$  by Gerssen et al. (2003), which includes the possibility of no BH. By detailed comparison with numerical simulations, Baumgardt et al. (2003a) claim that a massive BH with  $>500\text{--}1000 M_{\odot}$  is not indispensable to account for the observational data in M15. The total stellar mass of M15 is roughly  $10^6 M_{\odot}$ , and the velocity dispersion is  $\sigma \approx 14 \text{ km s}^{-1}$ . As seen in Figure 5, the present model predicts that no massive BH forms for the velocity dispersion in M15. Hence, the prediction of the present model supports the case of no BH.

In the globular cluster G1 in M31, a candidate for a massive BH with  $M_{\text{BH}} \approx 2 \times 10^4 M_{\odot}$  is reported (Gebhardt et al. 2002). But Baumgardt et al. (2003b) argue that the observational data in G1 can be well fitted even without a massive BH. Hence, the above mass of the BH in G1 should be considered as an upper limit. The total mass of G1 is  $(0.7\text{--}2) \times 10^7 M_{\odot}$ , and the velocity dispersion is estimated as  $\sigma \approx 25 \text{ km s}^{-1}$  (Meylan et al. 2001). It is predicted for this velocity dispersion that no massive BH forms if the system mass is initially lower than  $5 \times 10^7 M_{\odot}$ ,

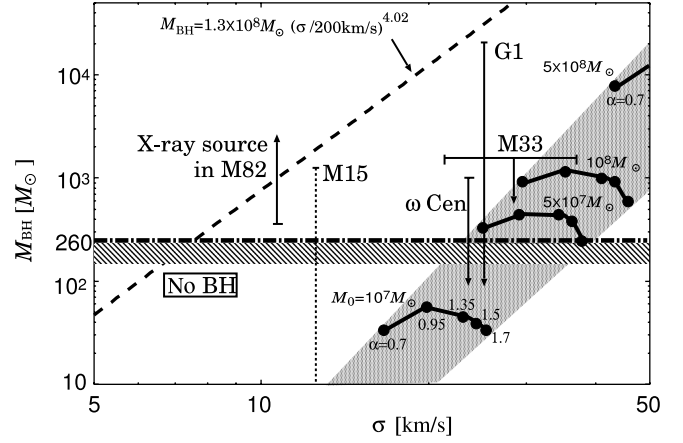


FIG. 5.—Comparison of the theoretical prediction with the observational data on BH mass in globular clusters. The ordinate is the BH mass ( $M_{\text{BH}}$ , in units of solar masses), and the abscissa is the velocity dispersion of globular cluster ( $\sigma$ , in units of kilometers per second). The parameters are the same as in Fig. 4. The shaded region is the prediction of the present analysis. The relation between the BH mass and the velocity dispersion of the galactic bulges is shown by a dotted line,  $M_{\text{BH}} = (1.3 \times 10^8 M_{\odot}) [\sigma / (200 \text{ km s}^{-1})]^{4.02}$  (Tremaine et al. 2002). The observational data of M15, G1,  $\omega$  Cen, M33 nucleus, and M82 are plotted.

while a BH with  $\approx 500 M_{\odot}$  can form if the system mass is initially higher than that mass.

In addition,  $\omega$  Cen possibly harbors a massive BH. The object  $\omega$  Cen is a globular cluster in the Milky Way in which member stars show a wide range of metallicity. The total stellar mass of  $\omega$  Cen is  $5 \times 10^6 M_{\odot}$ , and the velocity dispersion is  $\sigma \approx 22 \text{ km s}^{-1}$  (Meylan et al. 1995). As seen in Figure 5, whether  $\omega$  Cen can have a massive BH with  $\approx 300 M_{\odot}$  is marginal. The latest radio observations have suggested that the black hole mass in  $\omega$  Cen should be less than about  $100 M_{\odot}$  for the spherical Bondi-Hoyle accretion rate, but the data are marginally consistent with a black hole of about  $1000 M_{\odot}$  for a more plausible accretion rate (Maccarone et al. 2005).

On the other hand, no strong evidence of a massive BH is found in a stellar cluster at the center of M33 (Merritt et al. 2001; Gebhardt et al. 2001). The M33 nucleus has a velocity dispersion of  $\sigma = 21\text{--}34 \text{ km s}^{-1}$  (Merritt et al. 2001). For this velocity dispersion, our model predicts that a massive BH with  $\approx 10^3 M_{\odot}$  can form only if the initial system mass is higher than  $5 \times 10^7 M_{\odot}$ .

Finally, Matsumoto et al. (2001) have identified nine bright compact X-ray sources in M82 by the *Chandra X-Ray Observatory*. The brightest source (No. 7 in their Table 1) has a luminosity of  $9 \times 10^{40} \text{ ergs s}^{-1}$ . If this luminosity is assumed to be the Eddington luminosity, the derived mass of the black hole is  $350 M_{\odot}$ . From the infrared luminosity, the total mass of the cluster is estimated to be  $\sim 5 \times 10^5 M_{\odot}$ , and the velocity dispersion is estimated to be  $11 \text{ km s}^{-1}$  (McCrady et al. 2003). As seen in Figure 5, the present model predicts that no BH forms for this velocity dispersion, although the possibility of BH formation via runaway stellar collision is proposed (Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004).

#### 5. DISCUSSION

In §§ 3 and 4, we have shown that massive BHs are expected to be hosted by globular clusters. Here we consider the detectability of the BH formation in globular clusters. The present scenario of BH formation is summarized as follows. (1) First, a massive self-gravitating viscous disk (MDO) forms because of the radiation drag in a timescale of  $\approx 10^8\text{--}10^9 \text{ yr}$ , and the core evolves into a VMS. (2) Second, the VMS collapses directly

into a BH if its mass is higher than  $260 M_{\odot}$ . (3) Finally, the BH grows to a massive BH by mass accretion from the ambient gas.

A VMS more massive than  $260 M_{\odot}$  evolves in a short lifetime of  $\leq 10^6$  yr (e.g., Schaerer 2002). Therefore, stages 1 and 2 correspond to a high redshift, if we consider the age of globular clusters. Hence, it would be hard to detect a VMS itself, because the apparent magnitude is too faint to observe. However, a VMS may result in a gamma-ray burst (GRB), if it is Population II and a forming BH entails the sufficient angular momentum (Heger et al. 2003). GRBs are detectable even at redshifts higher than 10 (Lamb & Reichart 2000). The present scenario predicts that GRB events would take place at the centers of forming globular clusters at early stages of  $\approx 10^8$ – $10^9$  yr.

In stage 3, the luminosity of BH accretion is expected to be  $L = \eta \dot{M} c^2$ , where  $\dot{M}$  is the mass accretion rate and  $\eta$  is the efficiency. The temperature in the innermost regions of the accretion disk and the efficiency  $\eta$  depend on whether  $\dot{M}$  is higher or lower than the Eddington accretion rate  $\dot{M}_E = L_E/c^2$  (e.g., Kato et al. 1998), where  $L_E$  is the Eddington luminosity, given by

$$L_E = (1.25 \times 10^{41} \text{ ergs s}^{-1}) \left( \frac{M_{\text{BH}}}{10^3 M_{\odot}} \right). \quad (8)$$

If  $\dot{M} \approx \dot{M}_E$ , a so-called standard disk forms. Then the efficiency is  $\eta \approx 0.1$ , and the temperature is given by

$$T_{\text{disk}} = (5.8 \times 10^6 \text{ K}) \left( \frac{M_{\text{BH}}}{10^3 M_{\odot}} \right)^{-1/4} \left( \frac{r}{3r_{\text{sch}}} \right)^{-3/8}, \quad (9)$$

where  $r_{\text{sch}}$  is the Schwarzschild radius. If  $\dot{M} \ll \dot{M}_E$ , the solution of accretion is optically thin and called a radiatively inefficient accretion flow (RIAF). Then  $\eta \approx 0.1(\dot{M}/\dot{M}_E)$ , and the temperature is on the order of  $10^9$  K for electrons. If the accretion rate is super-Eddington ( $\dot{M} > \dot{M}_E$ ), the solution is optically thick and called a “slim disk,” in which the temperature is higher than that in the standard disk. In this type of disk, the photon-trapping effect in the accretion flow plays an important role, and then the efficiency becomes  $\eta \approx 0.1(\dot{M}/\dot{M}_E)^{-1/2}$ . Actually, the accretion luminosity can achieve up to  $\approx 7L_E$  (Ohsuga et al. 2002, 2003).

Thus, if  $\dot{M} \gtrsim \dot{M}_E$ , it is expected that an accreting BH in a globular cluster would be a far-UV and X-ray source with  $L \gtrsim L_E$ . If a slim disk is considered,  $L \approx 10^{41}$  ergs s $^{-1}$  is realized even for a BH with several hundreds of solar masses. Such a source may be observed as an ultraluminous X-ray source (ULX), the luminosity of which is  $10^{39}$ – $10^{41}$  ergs s $^{-1}$ . The lifetime of the accretion phase is pretty shorter than the present age of globular clusters. But in some globular clusters, the final accretion phase may be detected. In fact, Liu & Bregman (2005) have recently reported that some ULXs are located at old globular clusters.

These ULXs can be the candidates for accreting BHs with  $10^3 M_{\odot}$  formed by the radiation-hydrodynamic process.

## 6. CONCLUSIONS

Based on the radiation-hydrodynamic model, we have examined the possibility for the formation of massive BHs in globular clusters, incorporating realistic chemical evolution. In the present model, the mass of a central massive dark object in a globular cluster is determined by the duration of the optically thick phase. We have found that massive BHs can form only in globular clusters with a final mass of  $\gtrsim 6 \times 10^6 M_{\odot}$  or a velocity dispersion of  $\gtrsim 20$  km s $^{-1}$ . The BH mass is almost independent of the stellar initial mass function and star formation rate. The results show that massive BHs are not likely to form in typical globular clusters with  $\approx 10^6 M_{\odot}$ . The predicted BH-to-globular cluster mass ratio ( $M_{\text{BH}}/M_{\text{star, final}}$ ) is considerably lower than the BH-to-bulge mass ratio.

We have applied the present model to some globular clusters, in which the possibilities of massive BHs are reported. In M15, no massive BH is predicted to form, although the evidence of a massive BH is under debate. The globular cluster G1 can harbor a massive BH with  $\approx 500 M_{\odot}$ , which is smaller by an order of magnitude than an estimate,  $M_{\text{BH}} \approx 2 \times 10^4 M_{\odot}$ , by Gebhardt et al. (2002). In  $\omega$  Cen, the formation of a massive BH with  $\approx 300 M_{\odot}$  is marginal. As for M82, it is suggested that massive BHs with  $350 M_{\odot}$  can reside in bright compact X-ray sources. But no BH is expected to form in the present radiation-hydrodynamic model. Finally, the present scenario predicts that a final phase of BH growth in a globular cluster can be observed as a ULX with  $\sim 10^{41}$  ergs s $^{-1}$ . Also, GRBs may arise in early phases ( $10^8$ – $10^9$  yr) of globular cluster formation.

In this paper, we have investigated the possibility of BH formation from a hydrodynamic point of view. There may be a different path to form massive BHs in globular clusters. To elucidate the mechanism for the formation of massive BHs, high-resolution multiwavelength observations, especially optical, far-UV, X-ray, and gamma-ray, of the central regions of globular clusters would be greatly important in the future.

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