SABARAGAMUWA UNIVERSITY OF SRI LANKA
FACULTY OF APPLIED SCIENCES

052

DEPARTMENT OF COMPUTING AND INFORMATION SYSTEMS

B.Sc. DEGREE PROGRAMME IN SOFTWARE ENGINEERING

SE 2019/ 2020 SEMESTER I EXAMINATION SEPT/ OCT 2022

FUNDAMENTALS OF MATHEMATICS – SE1106

Time allowed **Three (03) Hours**

INSTRUCTIONS TO CANDIDATES:

This paper consists of 05 questions. Answer ALL questions.

The marks given in brackets are indicative of the weight given to each part of the question.
Write your Index No. clearly in all places where appropriate.
Write clearly in English and use blue or black ink.
Non-programmable calculators are ALLOWED in this examination.
No clarifications will be provided on the given questions.
Strike a line through all unused pages in the answer booklet/sheets.
Cross out all scratch paper and hand in at the time of collection.

P.T.O

1) Consider the following system.

$$x + 3y - 2z = 5$$

$$3x + 5y + 6z = 7$$

$$2x + 4y + 3z = 8$$

I Use Cramer's rule to determine the solutions of above system if they exists. 10 Marks

II Use Gaussian elimination to solve above system of equations if they exist. 15 Marks

III Express the above system in the form $Ax = b$, where A is a 3×3 matrix and x and b are 3×1 vectors. 10 Marks

IV Find the determinant of A. 10 Marks

V Is A non-singular? Justify your answer. 05 Marks

VI Find the cofactor matrix of A. 15 Marks

VII Find the inverse of A if it exists. 10 Marks

VIII Hence find the solutions of the system if they exist. 10 Marks

IX Use Gaussian elimination to solve the above system of equations if they exist. 15 Marks

2) a) Determine whether the transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ $as, T(x, y, z) = (x, y)$ is a linear transformation. 20 Marks

b) Suppose that the set V is the set of positive real numbers (i.e. $V = \{x | x \in \mathbb{R}$ and $x > 0\}$) with addition and scalar multiplication defined as, $x + y = xy,\ cx = x^c$. Show that V is a vector space. 10 Marks

c) Show that $\{v_1, v_2, v_3\}$, where $v_1 = (1,-1,1), v_2 = (0,1,2)$ and $v_3 = (3,0,-1)$, is a basis for $\mathbb{R}^3$. 20 Marks

d) Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & 2 \end{pmatrix}$.

I Find the characteristic polynomial of A. 10 Marks

II Using part a, find the all eigenvalues of A. 10 Marks

III Determine eigenvectors corresponding to those eigenvalues. 10 Marks

P.T.O

IV Find an matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix whose diagonal elements are the eigenvalues of A. 20 Marks

3) a) Let U be the set of all integers without 0 (i.e. $U = \mathbb{Z} \setminus \{0\}$). Let A be the set of all even integers, B be the set of all odd integers and C be the set of all multiples of 3.

I Is C a subset of A? Justify your answer. 10 Marks

II Find $A \cup B$, $A \cap C$ and $A \cap B$. 15 Marks

III Find $B \setminus C$ and compliment of A. 10 Marks

IV Is B the compliment of A? Justify your answer. 10 Marks

b) Let " $\simeq$ " denote the relation on the set of symmetric matrices defined as $A \simeq B$ if only if $A = B^T$, where A and B are square matrices. Show that " $\simeq$ " is an equivalence relation. 35 Marks

c) Why is the relation "less than or equal to ("$\leq$") not an equivalence relation? 20 Marks

4) a) Let $f: \mathbb{R} \to \mathbb{R}$ define by $f(x) = 4x^3 - 5$ for all $x \in \mathbb{R}$.

I Is f one to one? Justify your answer. 10 Marks

II Is f onto? Justify your answer. 10 Marks

III Does inverse of f exist? Justify your answer. 10 Marks

IV Find the inverse of f if it exists. 10 Marks

b) Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x+1}$.

I Find $f \circ g(x)$ and state the domain of it. 30 Marks

II Find $g \circ f(x)$ and state the domain of it. 30 Marks

P.T.O

5) a) Complete the following truth table for the propositions p, q and r. 40 Marks

p	q	r	$\sim r$	$p \to q$	$q \leftrightarrow r$	$(p \to q) \vee r$	$p \wedge (q \leftrightarrow r)$

I Is $\{(p \to q) \vee r\}$ tautology? Justify your answer. 10 Marks

II Is $\{p \wedge (q \leftrightarrow r)\}$ Contradiction? Justify your answer. 10 Marks

b) Identify disjunctive normal form and conjunctive normal form among the following propositions.

I $(p \vee r) \wedge (q \wedge (p \vee \neg q))$ 05 Marks

II $(p \vee q \vee \neg r \vee s) \wedge (\neg q \vee s) \wedge \neg s$ 05 Marks

III $(p \wedge q \wedge \neg r \wedge s) \vee (\neg q \wedge s) \vee (p \wedge s)$ 05 Marks

c) Obtain the Karnaugh map expression of the following truth table of function F. 25 Marks

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1