

Supplementary File for “Effects of Initialization Methods on the Performance of Multi-objective Evolutionary Algorithms”

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I. ADDITIONAL EXPERIMENTAL RESULTS ON THE EXPLORATION-ENHANCED STRATEGIES

In this section, we give complete experimental results to show the effects of the exploration-enhanced strategies by anytime performance curves of the average IGD value over 31 runs. We evaluate these initialization methods for MOEA/D, NSGA-II and NSGA-III on DTLZ1-4 and WFG4-9 test problems.

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A. Results on the Exploration-enhanced Strategies for MOEA/D

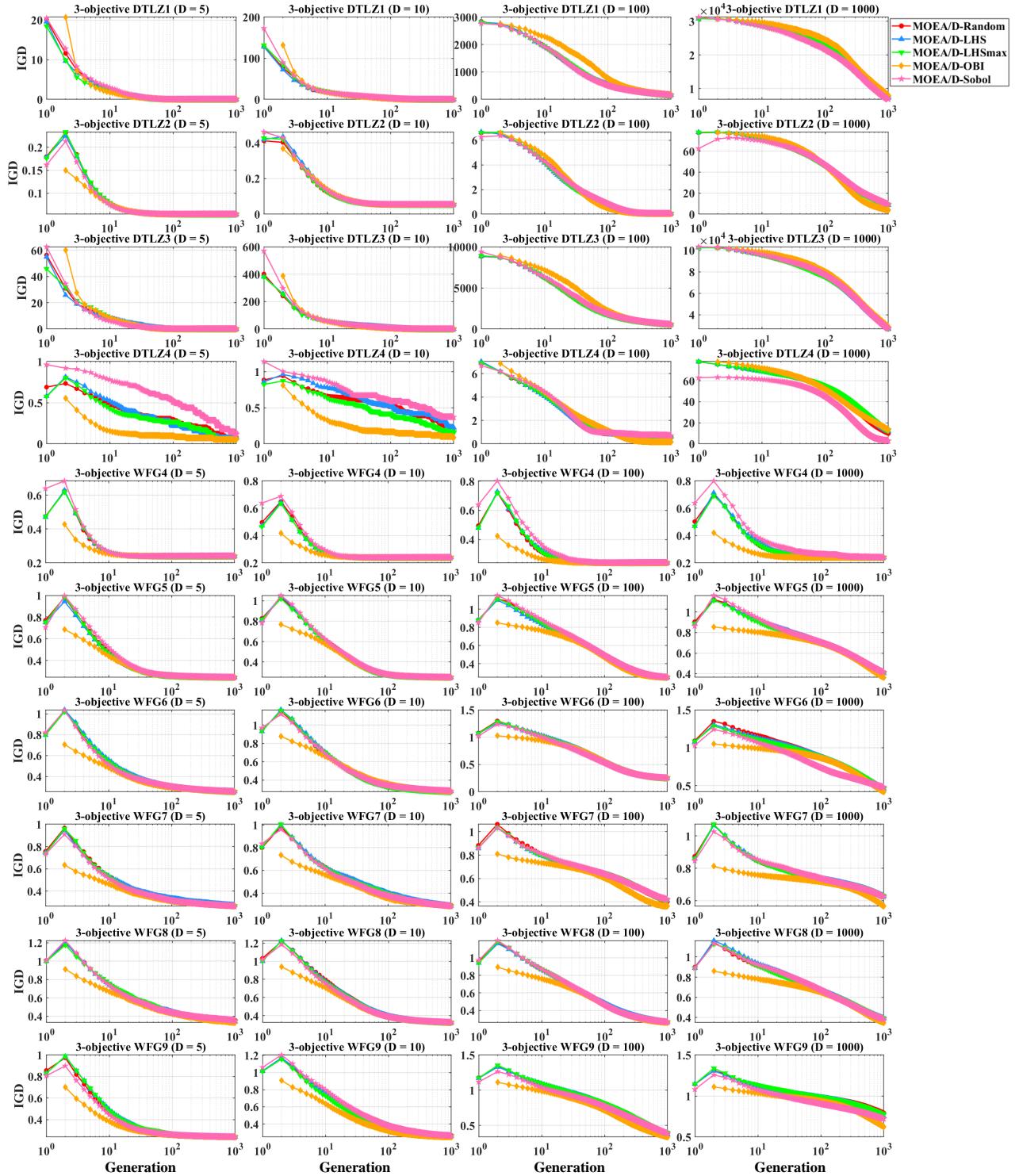


Figure S1. Anytime performance curves of the average IGD value over 31 runs that show the effects of various exploration-strategies for MOEA/D on different test problems.

B. Results on the Exploration-enhanced Strategies for NSGA-II

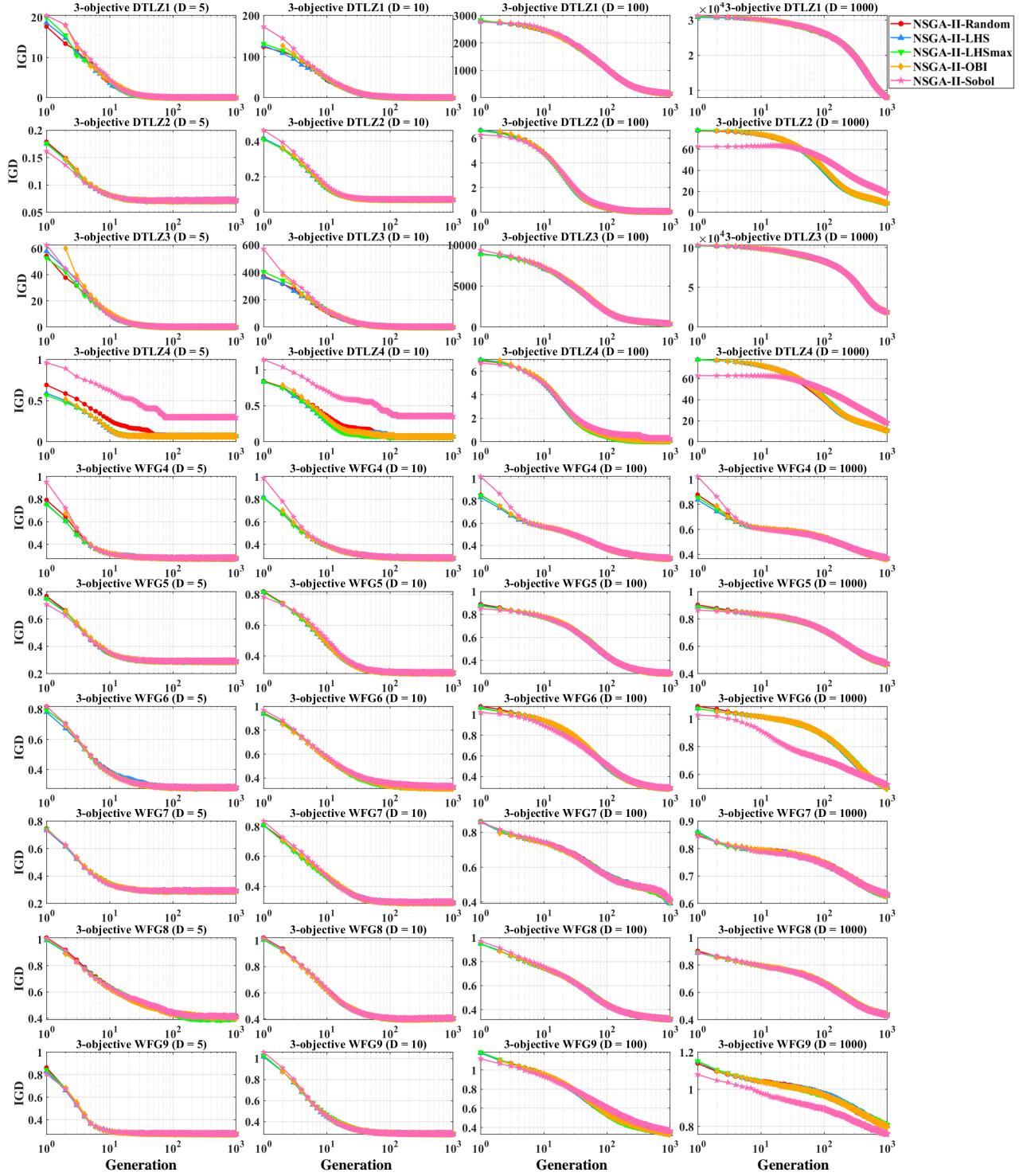


Figure S2. Anytime performance curves of the average IGD value over 31 runs that show the effects of various exploration-strategies for NSGA-II on different test problems.

C. Results on the Exploration-enhanced Strategies for NSGA-III

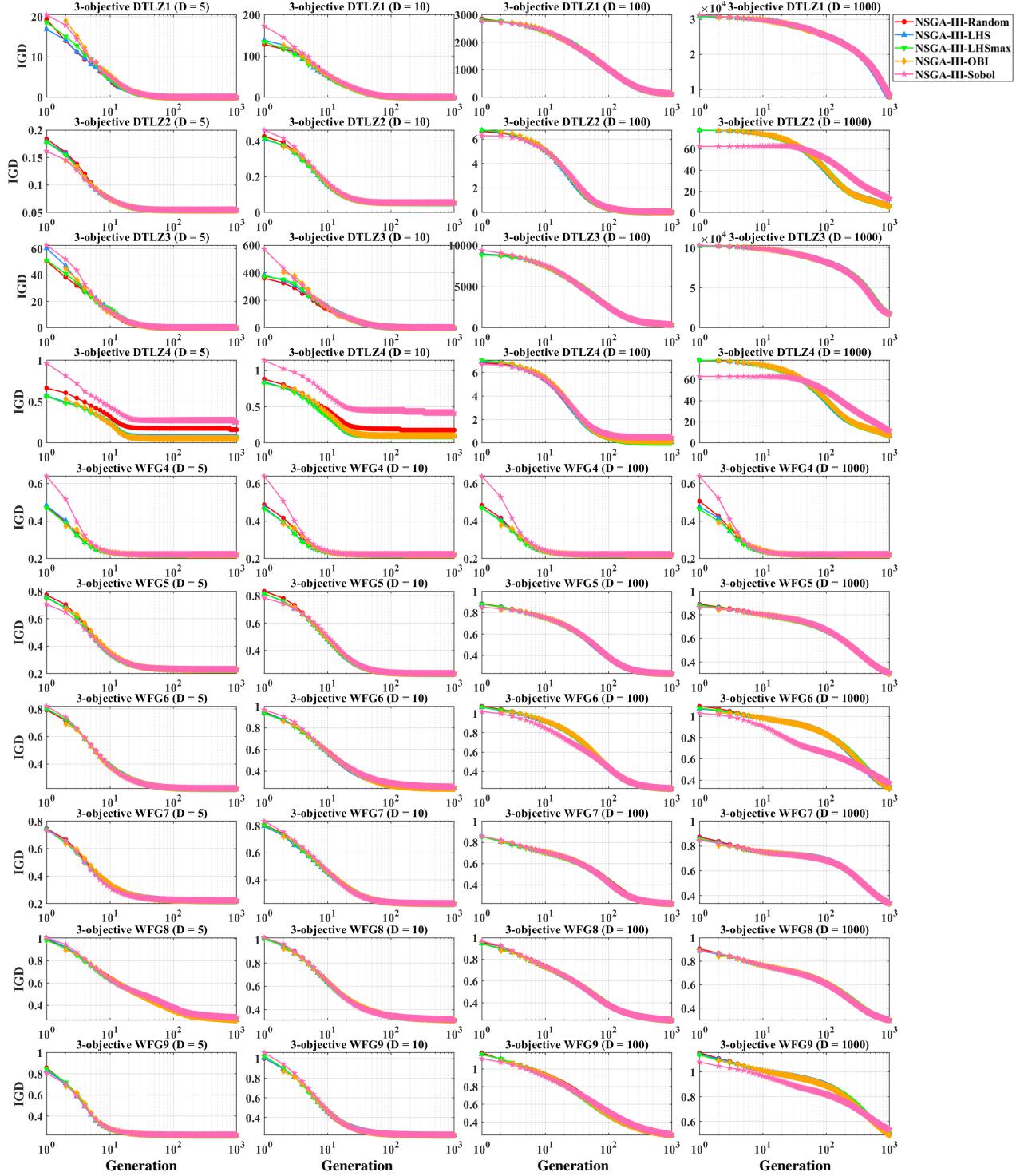


Figure S3. Anytime performance curves of the average IGD value over 31 runs that show the effects of various exploration-strategies for NSGA-III on different test problems.

II. ADDITIONAL EXPERIMENTAL RESULTS ON THE SUBSET SELECTION STRATEGIES

In this section, we give complete experimental results to show the effects of the subset selection strategies by anytime performance curves of the average IGD value over 31 runs. We evaluate these initialization methods for MOEA/D, NSGA-II and NSGA-III on DTLZ1-4 and WFG4-9 test problems.

A. Results on the Subset Selection Strategies for MOEA/D

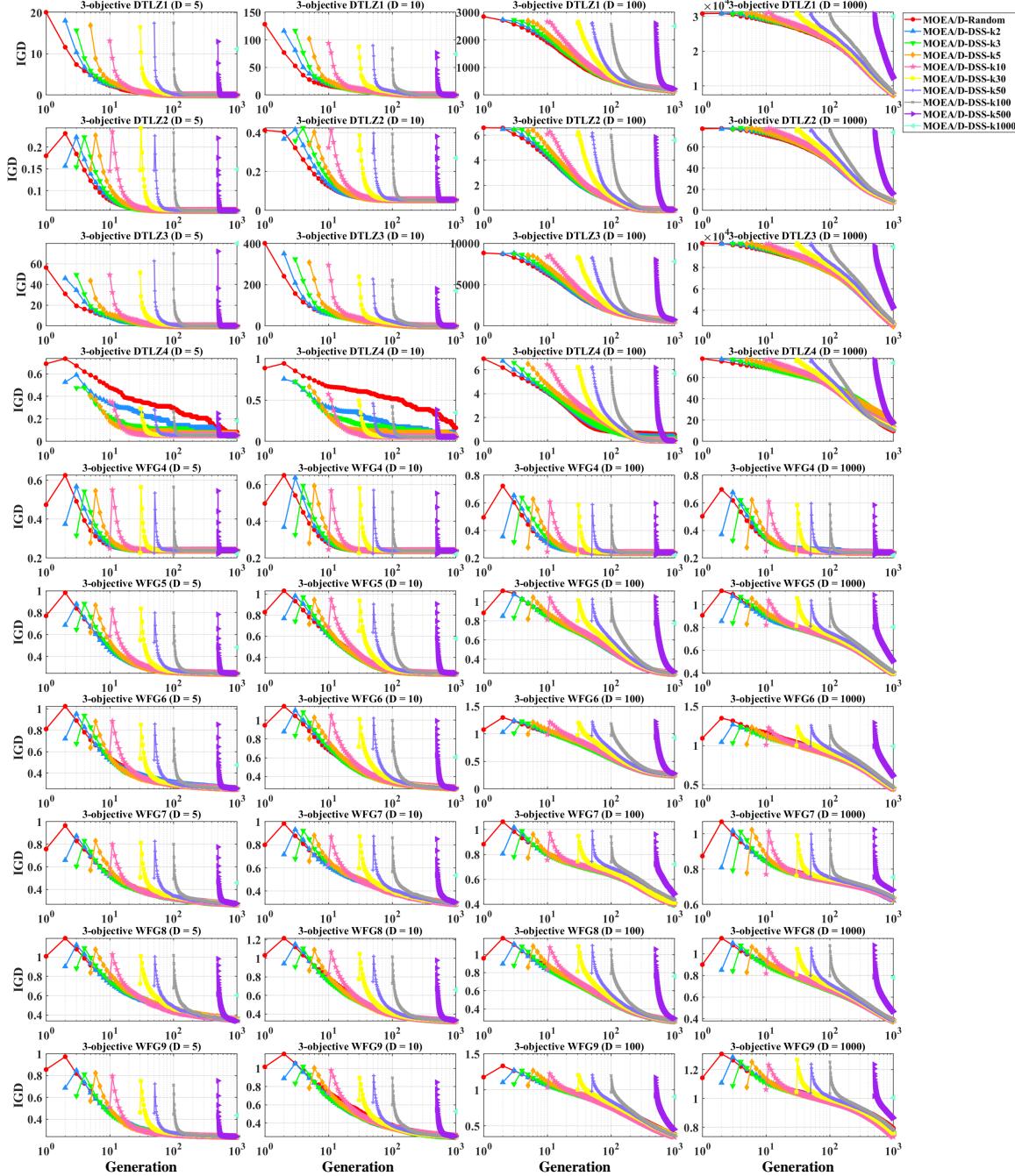


Figure S4. Anytime performance curves of the average IGD value over 31 runs that show the effects of DSS strategy for MOEA/D on different test problems.

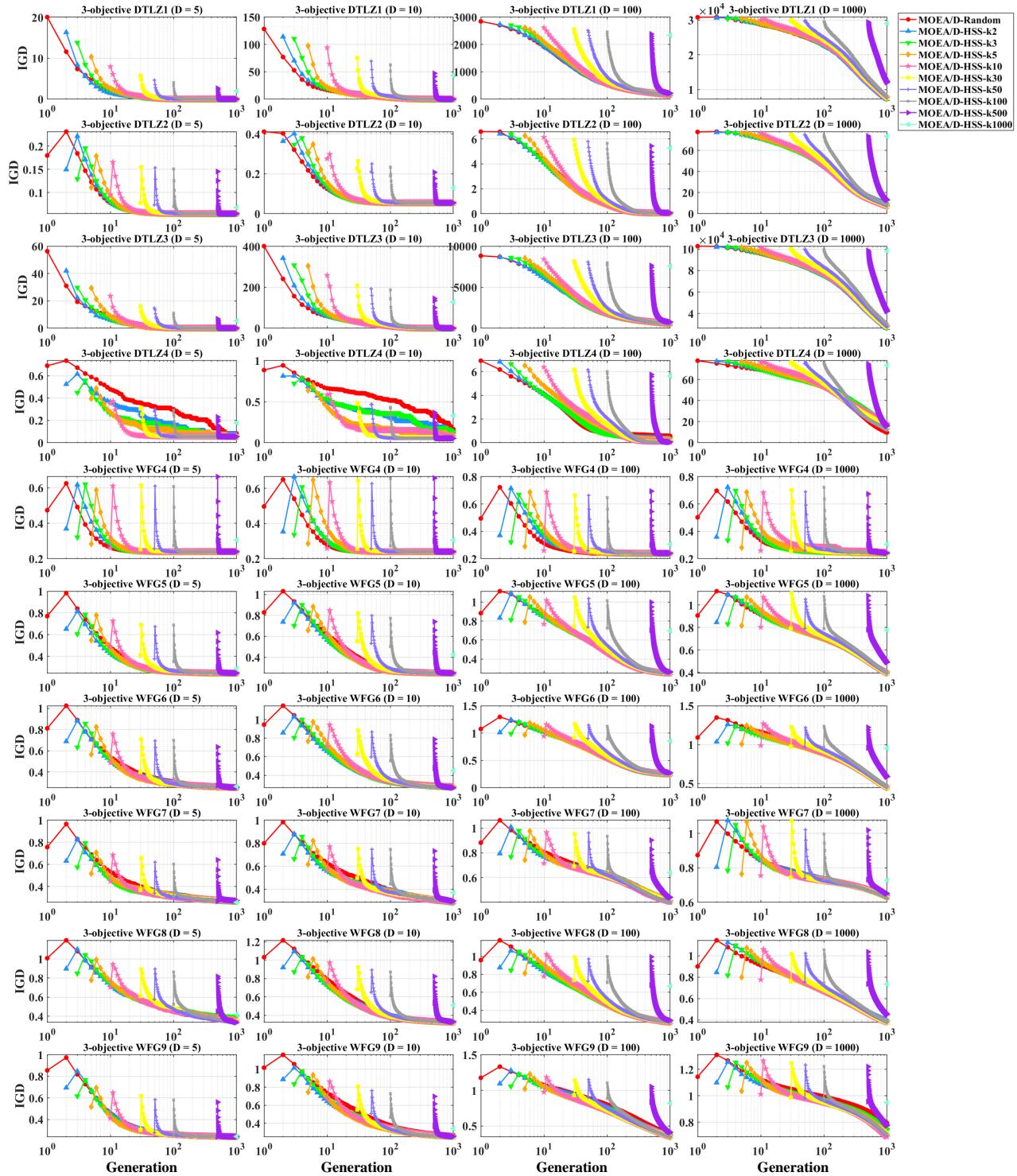


Figure S5. Anytime performance curves of the average IGD value over 31 runs that show the effects of HSS strategy for MOEA/D on different test problems.

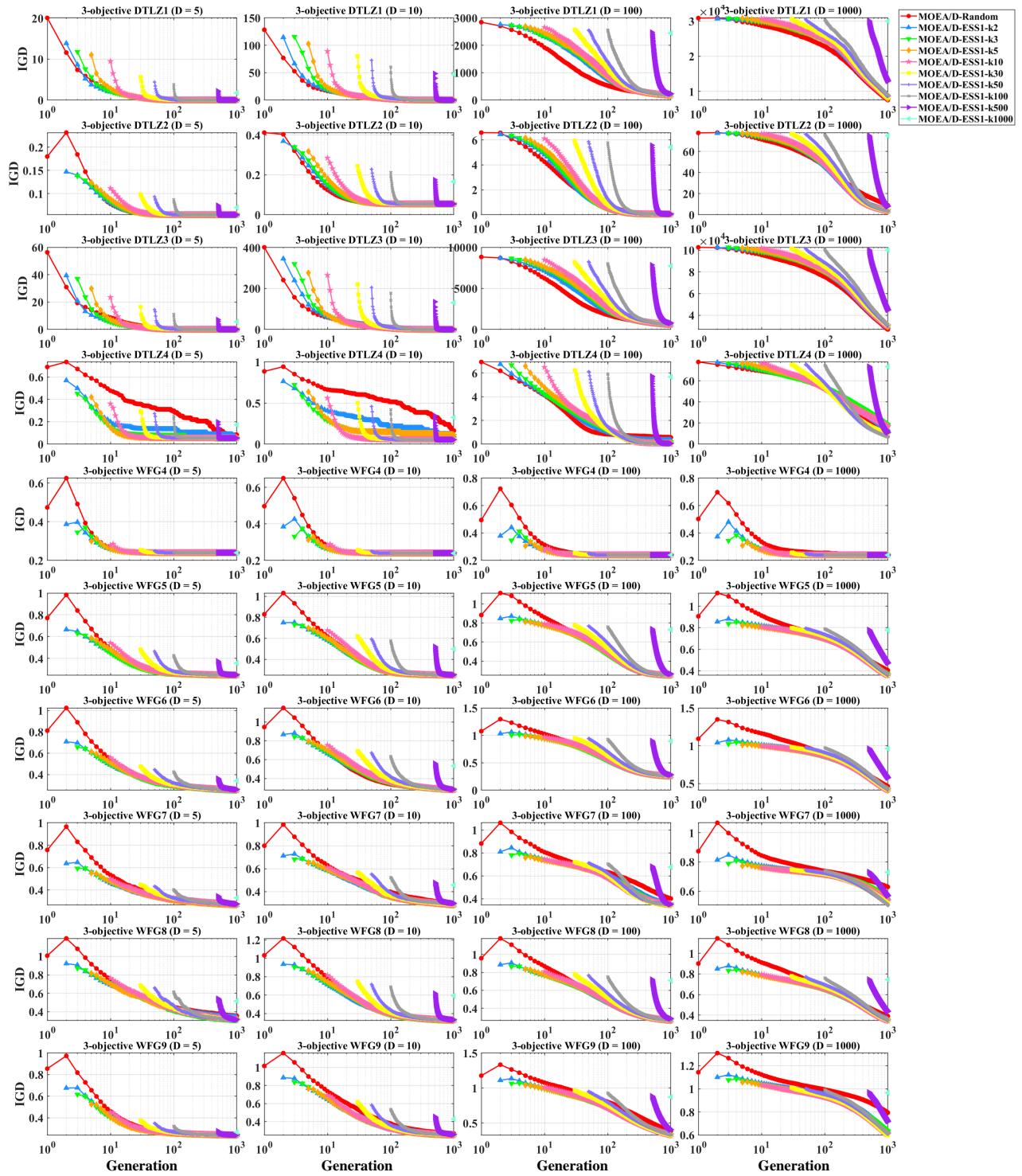


Figure S6. Anytime performance curves of the average IGD value over 31 runs that show the effects of ESS1 strategy for MOEA/D on different test problems.

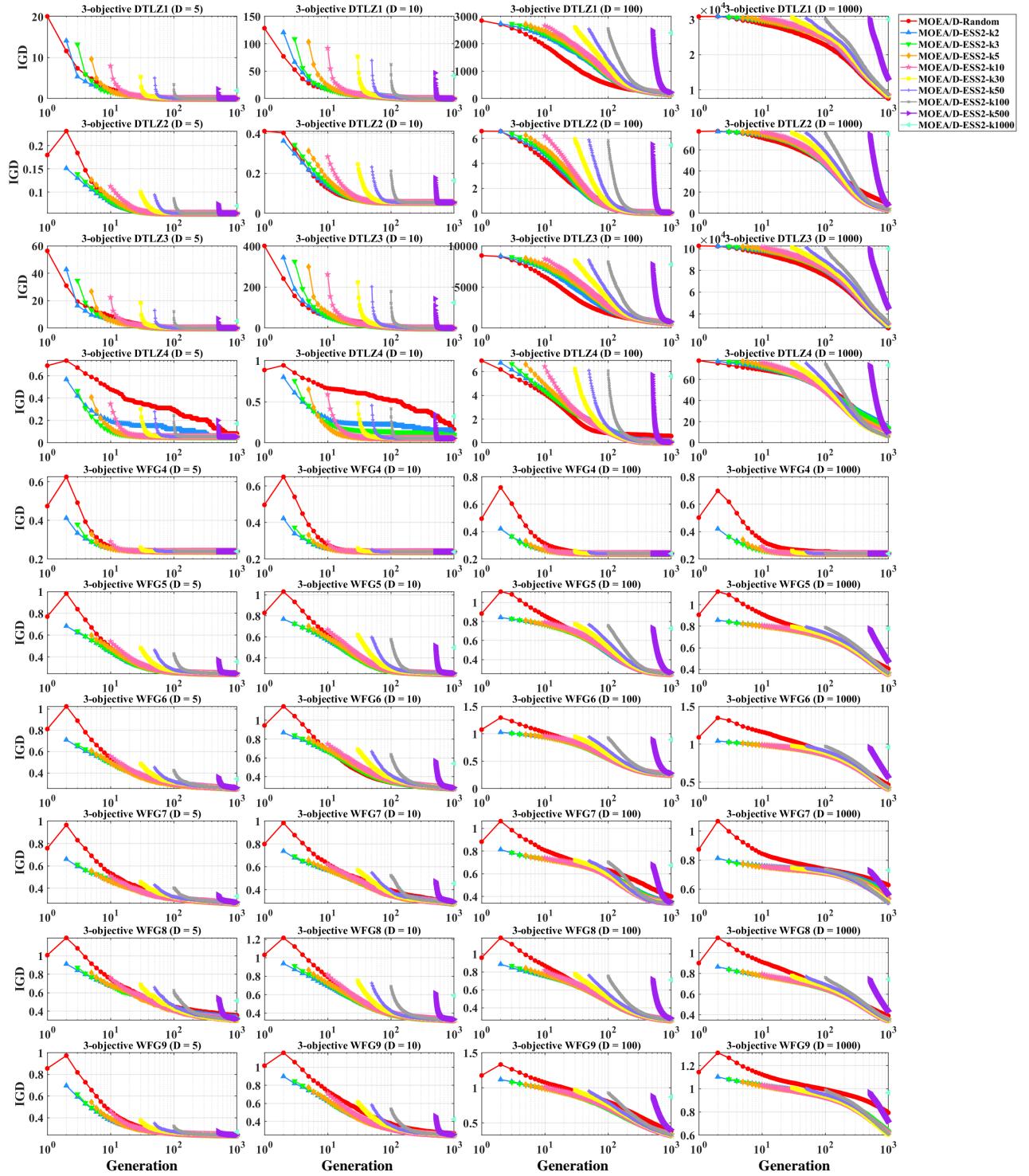


Figure S7. Anytime performance curves of the average IGD value over 31 runs that show the effects of ESS2 strategy for MOEA/D on different test problems.

B. Results on the Subset Selection Strategies for NSGA-II

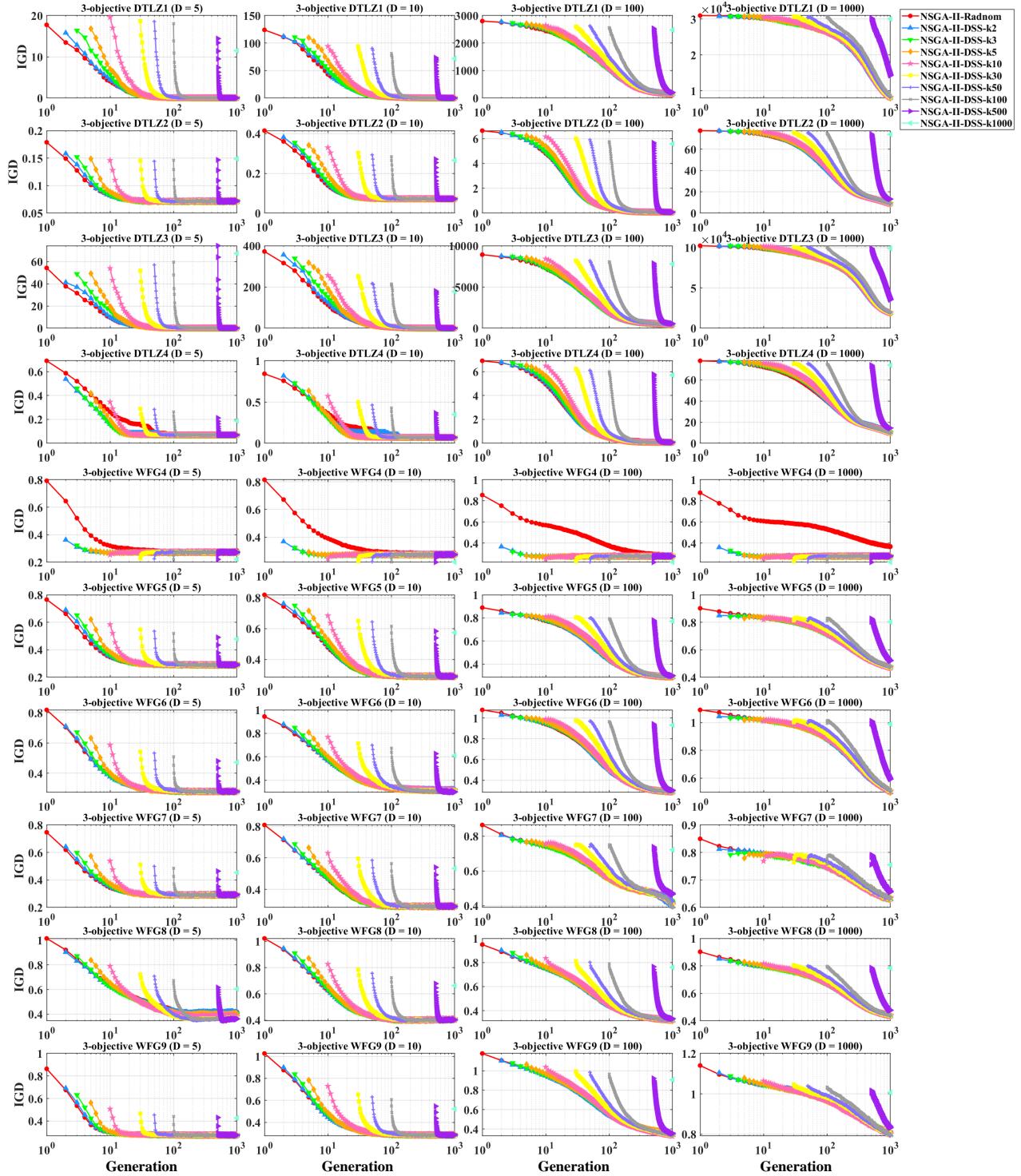


Figure S8. Anytime performance curves of the average IGD value over 31 runs that show the effects of DSS strategy for NSGA-II on different test problems.

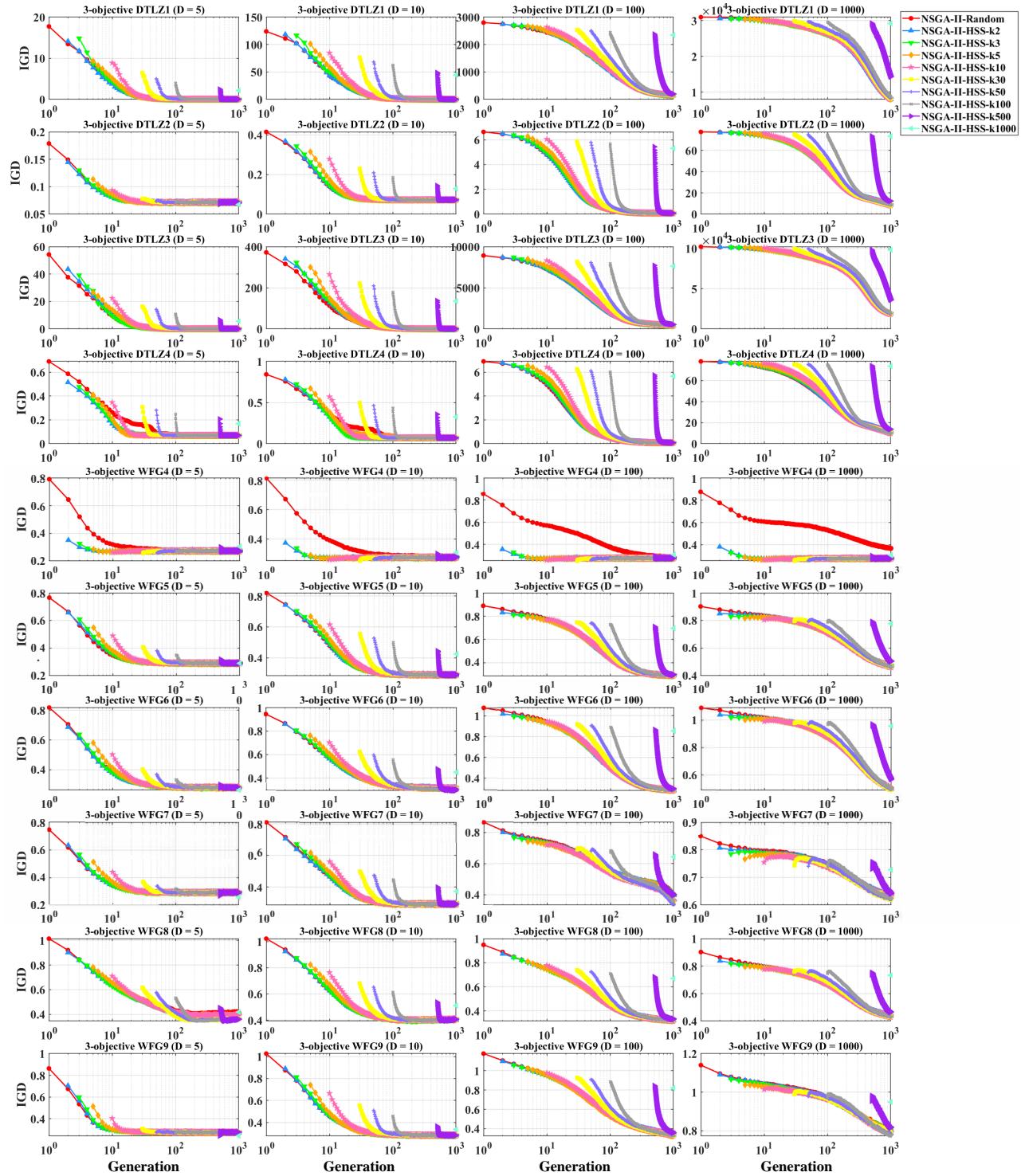


Figure S9. Anytime performance curves of the average IGD value over 31 runs that show the effects of HSS strategy for NSGA-II on different test problems.

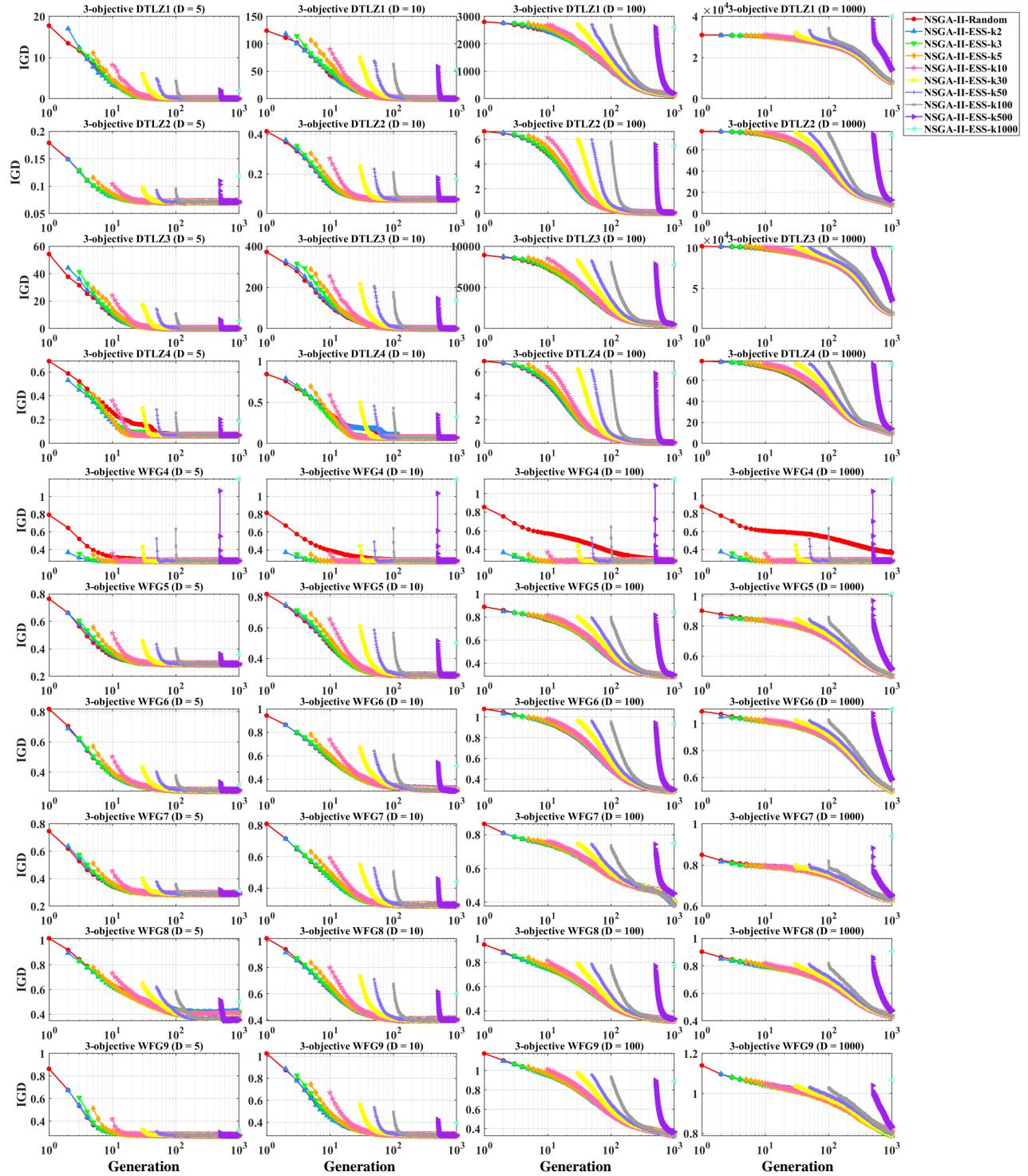


Figure S10. Anytime performance curves of the average IGD value over 31 runs that show the effects of ESS strategy for NSGA-II on different test problems.

C. Results on the Subset Selection Strategies for NSGA-III

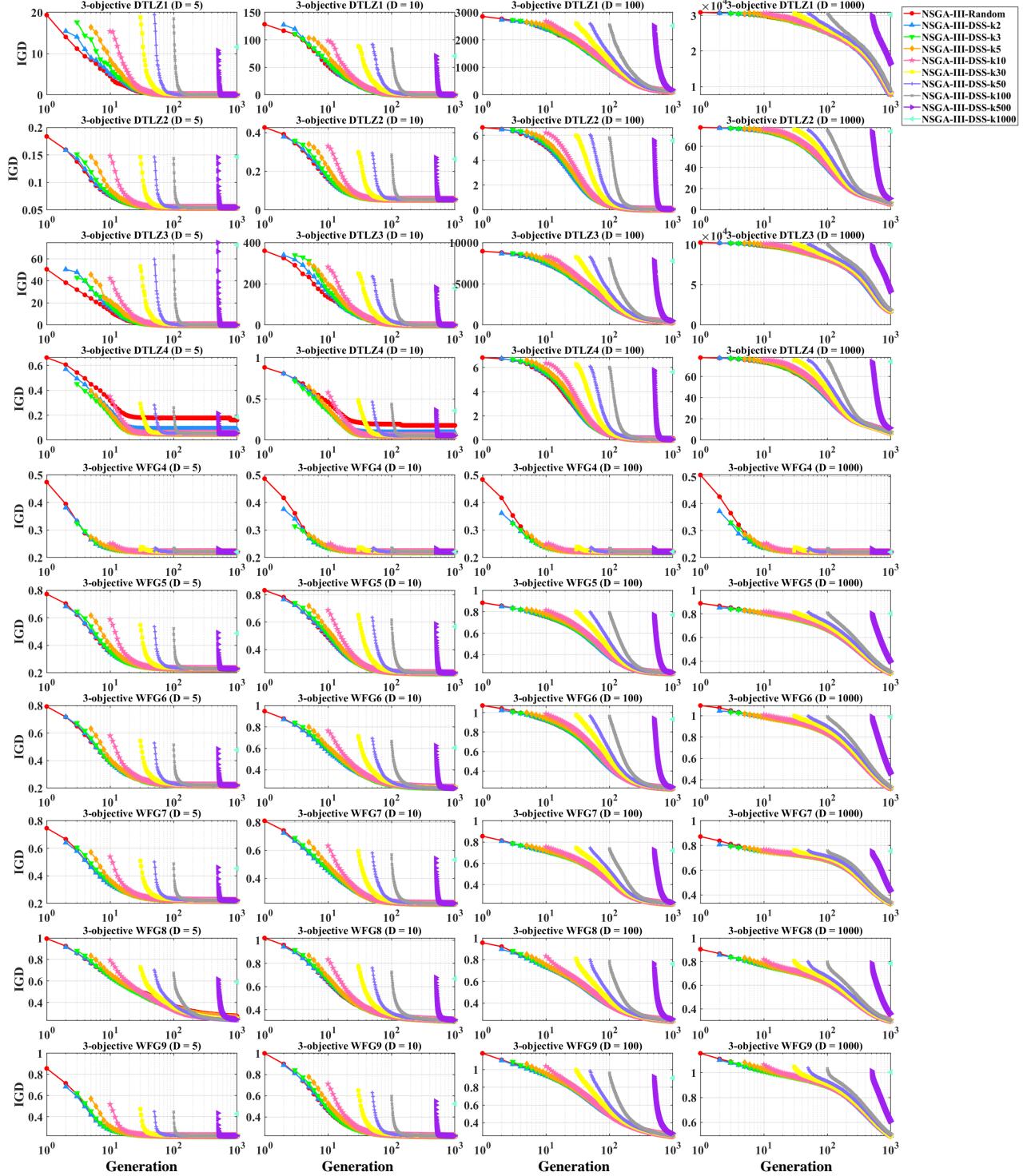


Figure S11. Anytime performance curves of the average IGD value over 31 runs that show the effects of DSS strategy for NSGA-III on different test problems.

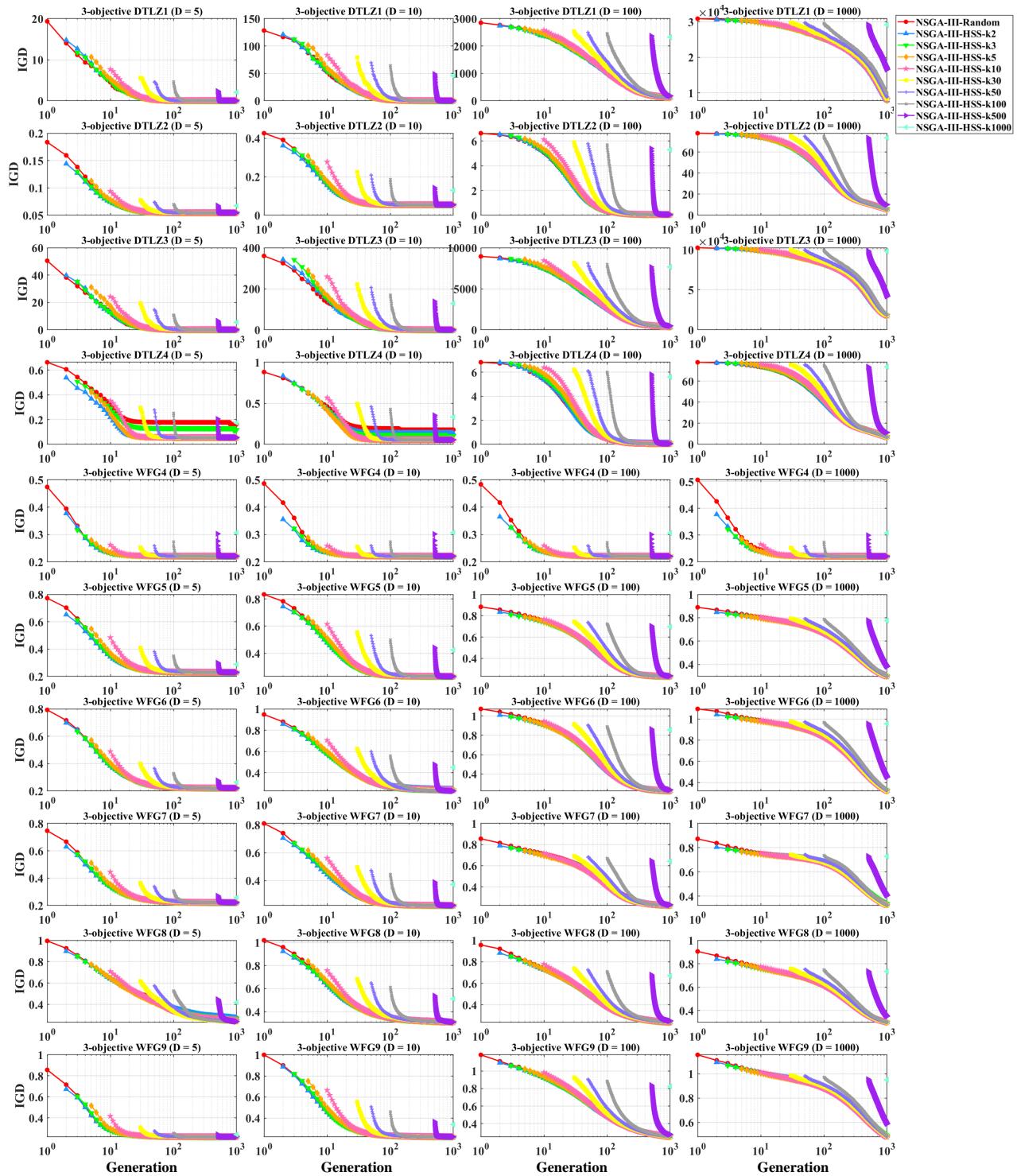


Figure S12. Anytime performance curves of the average IGD value over 31 runs that show the effects of HSS strategy for NSGA-III on different test problems.

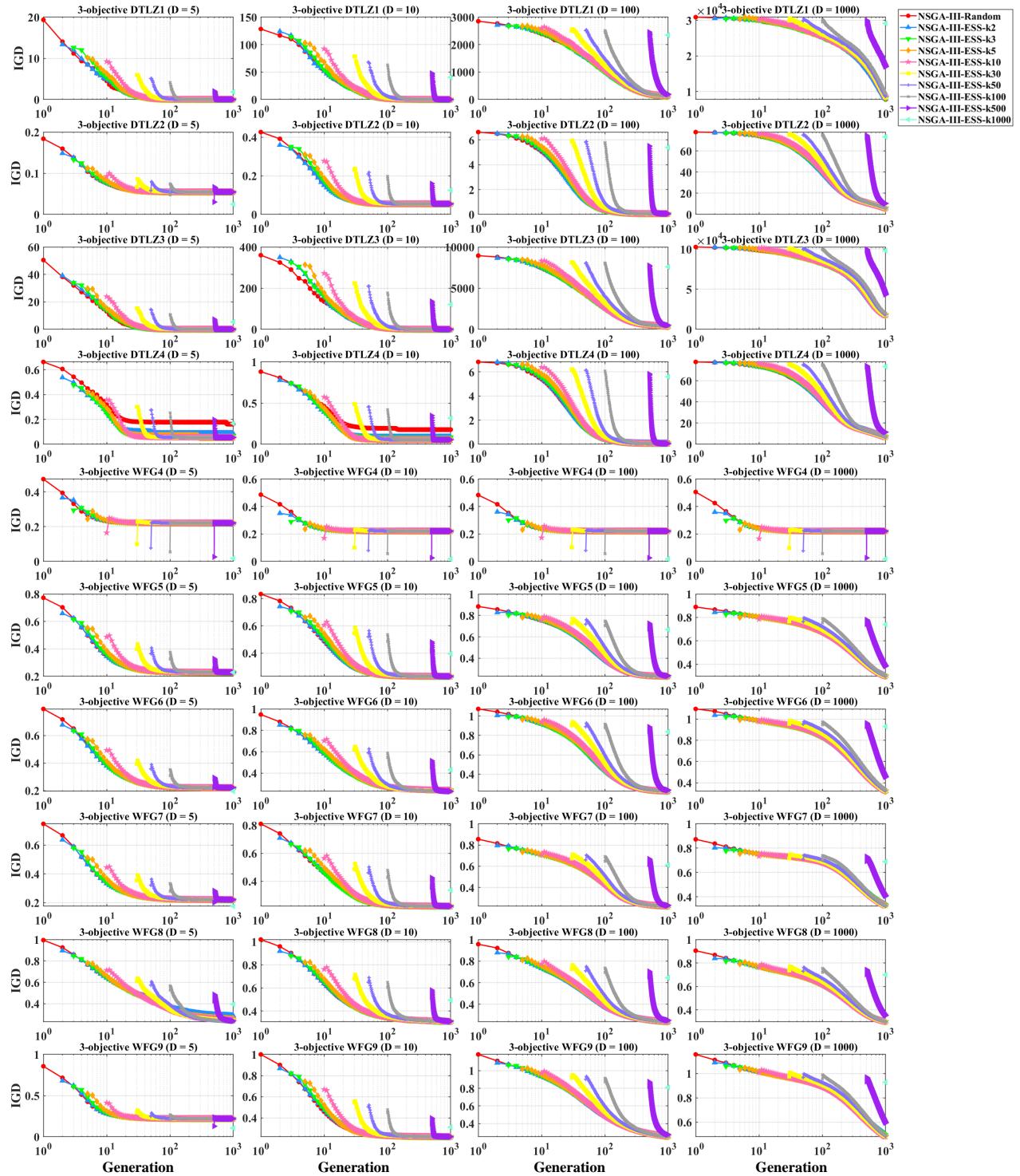


Figure S13. Anytime performance curves of the average IGD value over 31 runs that show the effects of ESS strategy for NSGA-III on different test problems.

III. SUMMARY OF STATISTIC TEST RESULTS FOR NSGA-II AND NSGA-III

In this section, we give additional statistic results on both exploration-enhanced strategies and subset selection strategies for NSGA-II and NSGA-III. The following tables summary their numerical results of significant test on all DTLZ1-4 and WFG4-9 test problems.

A. Summary of Statistic Test Results on Exploration-enhanced Strategies for NSGA-II and NSGA-III

TABLE S1: Summary of statistical test results (+/-/ \approx) on the IGD values for NSGA-II using exploration-enhanced strategies at different generations

Generation	NSGA-II-LHS	NSGA-II-LHSmax	NSGA-II-OBL	NSGA-II-Sobol
5	(3/0/37)	(1/1/38)	(0/2/38)	(8/11/21)
10	(3/0/37)	(1/0/39)	(1/4/35)	(7/6/27)
50	(4/1/35)	(1/1/38)	(2/3/35)	(5/4/31)
100	(3/0/37)	(2/0/38)	(0/2/38)	(4/9/27)
500	(3/2/35)	(0/1/39)	(2/1/37)	(4/10/26)
1000	(1/1/38)	(1/0/39)	(0/2/38)	(3/8/29)

TABLE S2: Summary of statistical test results (+/-/ \approx) on the IGD values for NSGA-III using exploration-enhanced strategies at different generations

Generation	NSGAIII-LHS	NSGAIII-LHSmax	NSGAIII-OBL	NSGAIII-Sobol
5	(1/0/39)	(4/0/36)	(0/3/37)	(10/7/23)
10	(4/1/35)	(5/3/32)	(2/2/36)	(9/8/23)
50	(1/1/38)	(4/3/33)	(0/2/38)	(5/12/23)
100	(3/0/37)	(2/0/38)	(3/2/35)	(4/7/29)
500	(2/1/37)	(2/1/37)	(2/0/38)	(3/8/29)
1000	(1/0/39)	(1/1/38)	(1/1/38)	(1/8/31)

B. Summary of Statistic Test Results on Subset Selection Strategies for MOEA/D, NSGA-II and NSGA-III

TABLE S3: Summary of statistical test results (+/-/ \approx) on the IGD values for MOEA/D using DSS strategy at different generations

Generation	MOEA/D-DSS-k2	MOEA/D-DSS-k5	MOEA/D-DSS-k10	MOEA/D-DSS-k50	MOEA/D-DSS-k100	MOEA/D-DSS-k500	MOEA/D-DSS-k1000
6	(1/11/28)	(2/37/1)	-	-	-	-	-
11	(4/4/32)	(2/24/14)	(1/38/1)	-	-	-	-
51	(4/2/34)	(4/9/27)	(4/18/18)	(1/38/1)	-	-	-
101	(3/4/33)	(4/8/28)	(5/14/21)	(2/33/5)	(1/38/1)	-	-
501	(5/5/30)	(2/3/35)	(6/3/31)	(4/13/23)	(2/23/15)	(0/39/1)	-
1000	(5/4/31)	(5/3/32)	(8/1/31)	(4/8/28)	(1/14/25)	(1/26/13)	(4/36/0)

TABLE S4: Summary of statistical test results (+/-/ \approx) on the IGD values for MOEA/D using HSS strategy at different generations

Generation	MOEA/D-HSS-k2	MOEA/D-HSS-k5	MOEA/D-HSS-k10	MOEA/D-HSS-k50	MOEA/D-HSS-k100	MOEA/D-HSS-k500	MOEA/D-HSS-k1000
6	(8/8/24)	(0/38/2)	-	-	-	-	-
11	(11/4/25)	(7/17/16)	(0/38/2)	-	-	-	-
51	(9/0/31)	(14/6/20)	(11/11/18)	(1/38/1)	-	-	-
101	(5/1/34)	(12/3/25)	(13/9/18)	(8/26/6)	(1/38/1)	-	-
501	(7/2/31)	(9/2/29)	(11/1/28)	(8/6/26)	(6/16/18)	(0/39/1)	-
1000	(5/1/34)	(9/2/29)	(8/3/29)	(11/3/26)	(7/6/27)	(3/22/15)	(1/38/1)

TABLE S5: Summary of statistical test results (+/-/≈) on the IGD values for MOEA/D using ESS2 strategy at different generations

Generation	MOEA/D-ESS2-k2	MOEA/D-ESS2-k5	MOEA/D-ESS2-k10	MOEA/D-ESS2-k50	MOEA/D-ESS2-k100	MOEA/D-ESS2-k500	MOEA/D-ESS2-k1000
6	(28/9/3)	(26/12/2)	-	-	-	-	-
11	(26/9/5)	(24/12/4)	(14/18/8)	-	-	-	-
51	(17/9/14)	(15/11/14)	(13/16/11)	(4/34/2)	-	-	-
101	(16/8/16)	(14/12/14)	(11/13/16)	(3/25/12)	(2/36/2)	-	-
501	(15/7/18)	(16/6/18)	(16/8/16)	(14/10/16)	(10/16/14)	(0/36/4)	-
1000	(14/5/21)	(14/8/18)	(16/6/18)	(14/7/19)	(15/11/14)	(4/27/9)	(3/36/1)

TABLE S6: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-II using DSS strategy at different generations

Generation	NSGA-II-DSS-k2	NSGA-II-DSS-k5	NSGA-II-DSS-k10	NSGA-II-DSS-k50	NSGA-II-DSS-k100	NSGA-II-DSS-k500	NSGA-II-DSS-k1000
6	(5/7/28)	(5/30/5)	-	-	-	-	-
11	(5/3/32)	(4/24/12)	(5/34/1)	-	-	-	-
51	(5/1/34)	(5/9/26)	(5/18/17)	(4/36/0)	-	-	-
101	(5/0/35)	(4/3/33)	(3/12/25)	(3/27/10)	(4/36/0)	-	-
501	(4/1/35)	(6/0/34)	(6/3/31)	(9/8/23)	(5/13/22)	(4/36/0)	-
1000	(5/0/35)	(5/2/33)	(5/2/33)	(5/3/32)	(7/7/26)	(6/20/14)	(4/36/0)

TABLE S7: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-II using HSS strategy at different generations

Generation	NSGA-II-HSS-k2	NSGA-II-HSS-k5	NSGA-II-HSS-k10	NSGA-II-HSS-k50	NSGA-II-HSS-k100	NSGA-II-HSS-k500	NSGA-II-HSS-k1000
6	(8/0/32)	(12/22/6)	-	-	-	-	-
11	(10/3/27)	(12/18/10)	(10/28/2)	-	-	-	-
51	(6/0/34)	(9/2/29)	(8/8/24)	(6/33/1)	-	-	-
101	(5/1/34)	(10/1/29)	(6/6/28)	(4/25/11)	(6/34/0)	-	-
501	(5/0/35)	(7/2/31)	(7/2/31)	(9/3/28)	(7/10/23)	(5/35/0)	-
1000	(4/1/35)	(6/0/34)	(8/0/32)	(10/1/29)	(8/3/29)	(4/18/18)	(5/33/2)

TABLE S8: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-II using ESS strategy at different generations

Generation	NSGA-II-ESS-k2	NSGA-II-ESS-k5	NSGA-II-ESS-k10	NSGA-II-ESS-k50	NSGA-II-ESS-k100	NSGA-II-ESS-k500	NSGA-II-ESS-k1000
6	(4/4/32)	(4/30/6)	-	-	-	-	-
11	(8/3/29)	(4/23/13)	(3/34/3)	-	-	-	-
51	(6/3/31)	(5/6/29)	(4/14/22)	(1/38/1)	-	-	-
101	(4/2/34)	(6/3/31)	(4/6/30)	(3/25/12)	(0/39/1)	-	-
501	(5/1/34)	(6/4/30)	(5/2/33)	(5/4/31)	(6/12/22)	(0/40/0)	-
1000	(4/1/35)	(5/0/35)	(5/0/35)	(9/2/29)	(6/4/30)	(8/20/12)	(0/40/0)

TABLE S9: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-III using DSS strategy at different generations

Generation	NSGA-III-DSS-k2	NSGA-III-DSS-k5	NSGA-III-DSS-k10	NSGA-III-DSS-k50	NSGA-III-DSS-k100	NSGA-III-DSS-k500	NSGA-III-DSS-k1000
6	(2/3/35)	(0/34/6)	-	-	-	-	-
11	(2/5/33)	(0/32/8)	(0/40/0)	-	-	-	-
51	(0/4/36)	(0/17/23)	(1/28/11)	(0/40/0)	-	-	-
101	(2/2/36)	(1/8/31)	(2/17/21)	(2/37/1)	(0/40/0)	-	-
501	(1/0/39)	(1/0/39)	(2/2/36)	(2/13/25)	(2/20/18)	(0/40/0)	-
1000	(1/2/37)	(2/0/38)	(2/2/36)	(2/9/29)	(3/12/25)	(3/28/9)	(0/36/4)

TABLE S10: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-III using HSS strategy at different generations

Generation	NSGA-III-HSS-k2	NSGA-III-HSS-k5	NSGA-III-HSS-k10	NSGA-III-HSS-k50	NSGA-III-HSS-k100	NSGA-III-HSS-k500	NSGA-III-HSS-k1000
6	(13/1/26)	(8/22/10)	-	-	-	-	-
11	(12/1/27)	(8/19/13)	(0/34/6)	-	-	-	-
51	(5/1/34)	(7/4/29)	(6/20/14)	(0/40/0)	-	-	-
101	(6/0/34)	(7/6/27)	(5/8/27)	(2/36/2)	(0/40/0)	-	-
501	(5/1/34)	(8/1/31)	(7/2/31)	(6/10/24)	(3/19/18)	(1/39/0)	-
1000	(3/1/36)	(8/1/31)	(5/1/34)	(6/4/30)	(4/8/28)	(3/27/10)	(0/40/0)

TABLE S11: Summary of statistical test results (+/-/≈) on the IGD values for NSGA-III using ESS strategy at different generations

Generation	NSGA-III-ESS-k2	NSGA-III-ESS-k5	NSGA-III-ESS-k10	NSGA-III-ESS-k50	NSGA-III-ESS-k100	NSGA-III-ESS-k500	NSGA-III-ESS-k1000
6	(0/2/38)	(1/35/4)	-	-	-	-	-
11	(1/3/36)	(1/31/8)	(0/40/0)	-	-	-	-
51	(2/1/37)	(0/13/27)	(2/25/13)	(0/36/4)	-	-	-
101	(3/0/37)	(2/4/34)	(1/14/25)	(2/37/1)	(4/36/0)	-	-
501	(4/0/36)	(9/2/29)	(8/2/30)	(4/12/24)	(2/17/21)	(4/36/0)	-
1000	(1/1/38)	(4/2/34)	(8/3/29)	(5/5/30)	(4/7/29)	(3/27/10)	(9/31/0)