

Supplementary Material of “Evolutionary Multi-Objective Optimization Framework with Three Solution Sets”

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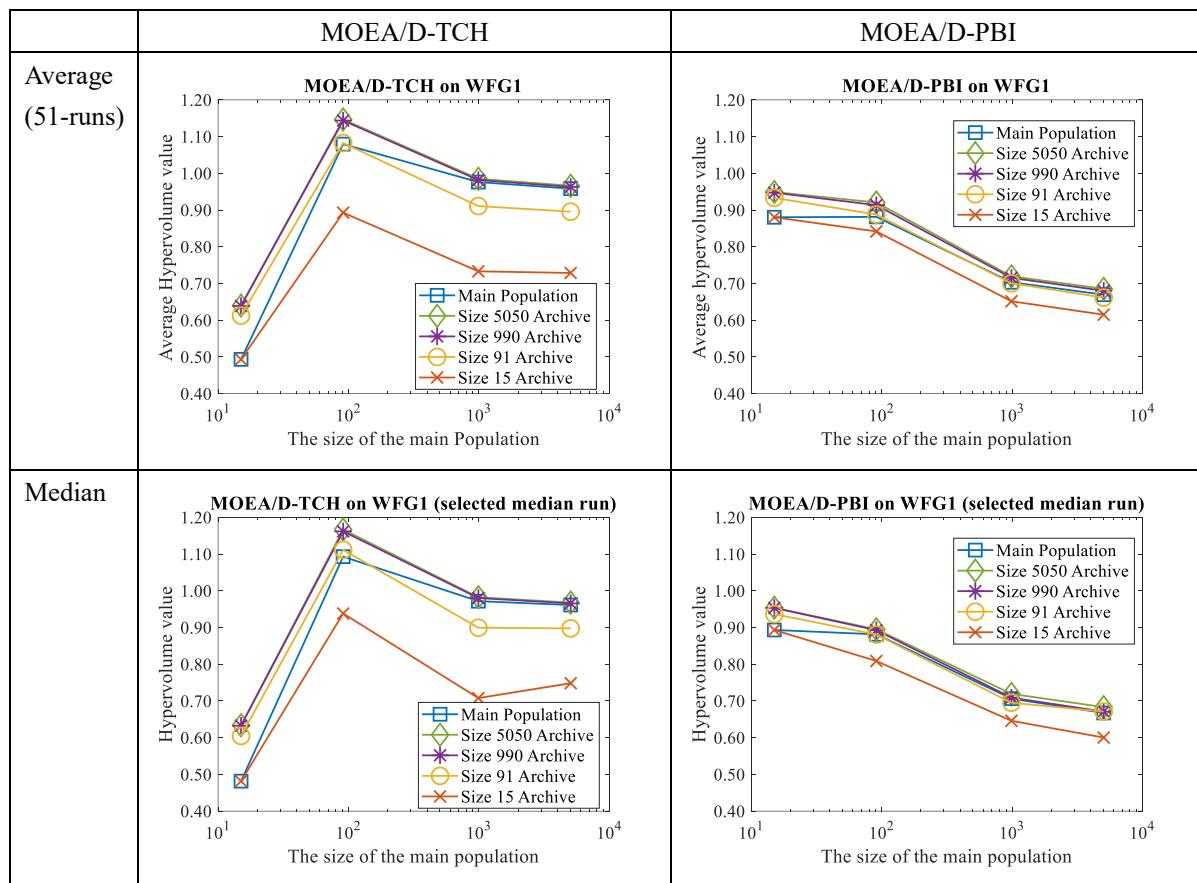
1. Experimental settings:

Number of objectives:	Three-objective ($M = 3$) and five-objective ($M = 5$) problems
EMO algorithm:	MOEA/D
Scalarizing functions:	<ul style="list-style-type: none"> - The weighted Tchebycheff function, g^{TCH} is defined as follows: $\text{Minimize } g^{\text{TCH}}(\mathbf{x} \mathbf{w}, \mathbf{z}^*) = \max_{i=1,2,\dots,M} \{w_i \cdot z_i^* - f_i(\mathbf{x}) \}, \quad (1)$ - The Penalty-based Boundary Intersection (PBI) function, g^{PBI} is defined as follows: $\text{Minimize } g^{\text{PBI}}(\mathbf{x} \mathbf{w}, \mathbf{z}^*) = d_1 + \theta d_2, \quad (2)$ $d_1 = (\mathbf{f}(\mathbf{x}) - \mathbf{z}^*)^T \mathbf{w} / \ \mathbf{w}\ , \quad (3)$ $d_2 = \ \mathbf{f}(\mathbf{x}) - \mathbf{z}^* - d_1(\mathbf{w}/\ \mathbf{w}\)\ , \quad (4)$ <p>where $\mathbf{z}^* = (z_1^*, z_2^*, \dots, z_M^*)$ is the reference point for the weighted Tchebycheff and PBI functions. Each element z_i^* ($i \in \{1, 2, \dots, M\}$) of \mathbf{z}^* is estimated by the minimum value of $f_i(\mathbf{x})$ among all solutions examined so far. For the PBI function in (2), the penalty parameter θ is set as 5.</p>
Population size:	<ul style="list-style-type: none"> - For three-objective problems, the population size 15 (Small), 91 (Standard), 990 (Large), and 5050 (Very Large) are used. - For five-objective problems, the population size 15 (Small), 210 (Standard), 1001 (Large), and 5985 (Very Large) are used.
Neighborhood size:	The neighborhood size 15, 20, 200, and 1000 are used for Small, Standard, Large, and Very Large population size, respectively.
Crossover operator:	SBX crossover with the probability 1.0 and the distribution index 30.
Mutation operator:	Polynomial mutation with the probability $1/D$ (D is the number of decision variables) and the distribution index 20.
Termination conditions:	<ul style="list-style-type: none"> - 50,000 solution evaluations for three-objective problems. - 200,000 solutions evaluations for five-objective problems.
Size of the external archives:	<p>Each MOEA/D has four external archives.</p> <ul style="list-style-type: none"> - For three-objective problems, the archive size 15 (Small), 91 (Standard), 990 (Large), and 5050 (Very Large) are used.

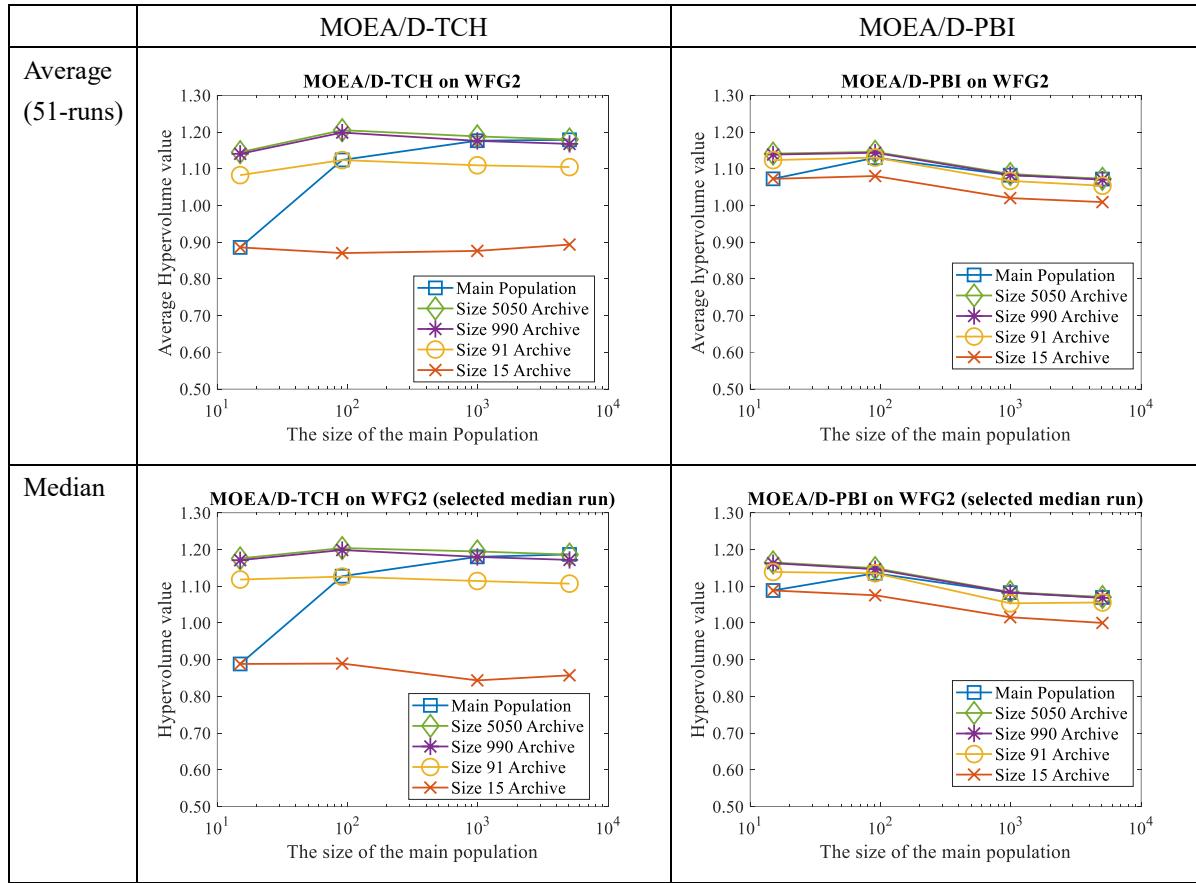
	<ul style="list-style-type: none"> - For five-objective problems, the archive size 15 (Small), 210 (Standard), 1001 (Large), and 5985 (Very Large) are used.
Test problems:	<p>DTLZ1-4 and WFG1-9.</p> <ul style="list-style-type: none"> - For the DTLZ test suite, the number of decision variable (i.e., D) for an M-objective DTLZ problem is set as $D = M + k - 1$. The parameter k is specified as $k = 5$ in DTLZ1, $k = 10$ in DTLZ2-4. - For the WFG test suite, an M-objective WFG problem has k position-related variables and l distance related variables. The number of decision variable for an M-objective WFG problem is $D = k + l$, where $k = M - 1$ and $l = 24 - (M - 1)$.
Number of independent runs:	51 runs.
Coding:	The coding implementation is modified from the MOEA/D code in PlatEMO [1].

2. Results of three-objective problems:

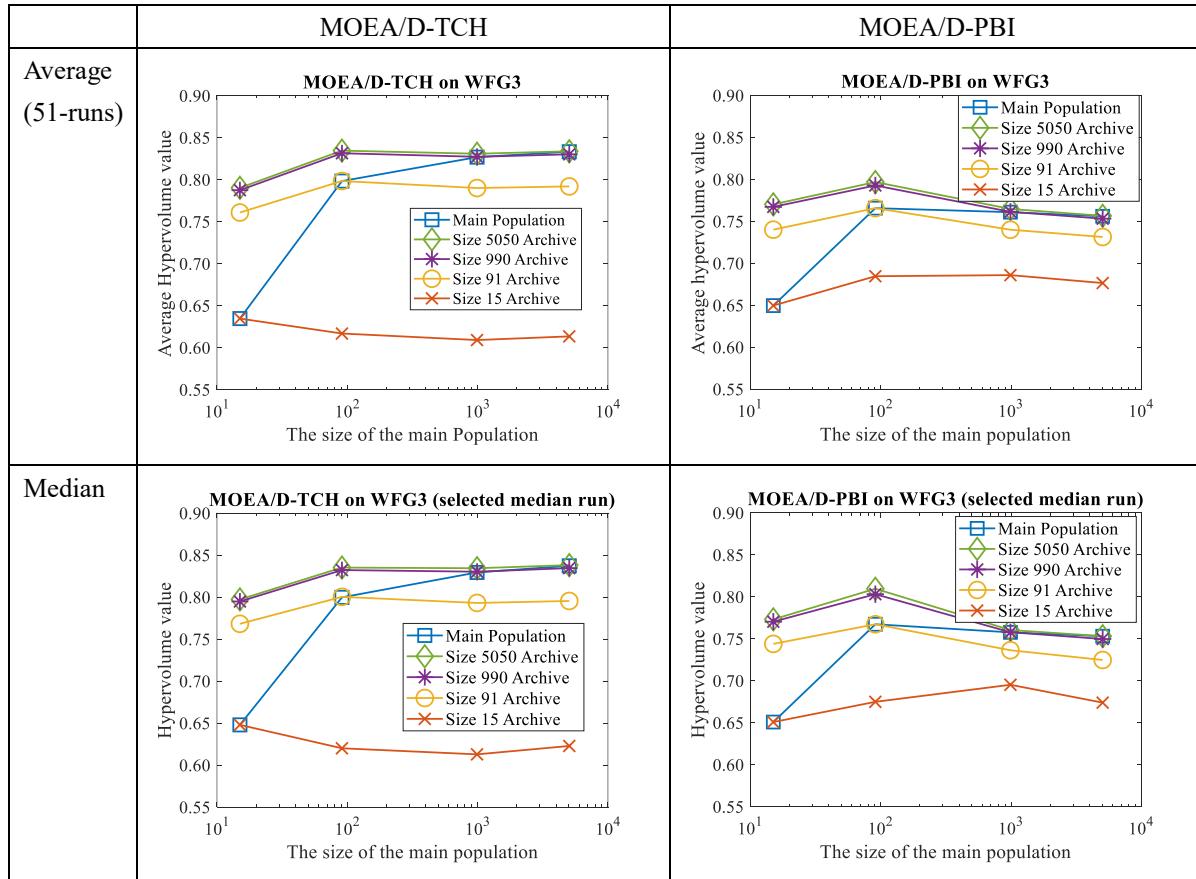
3-objective WFG1:



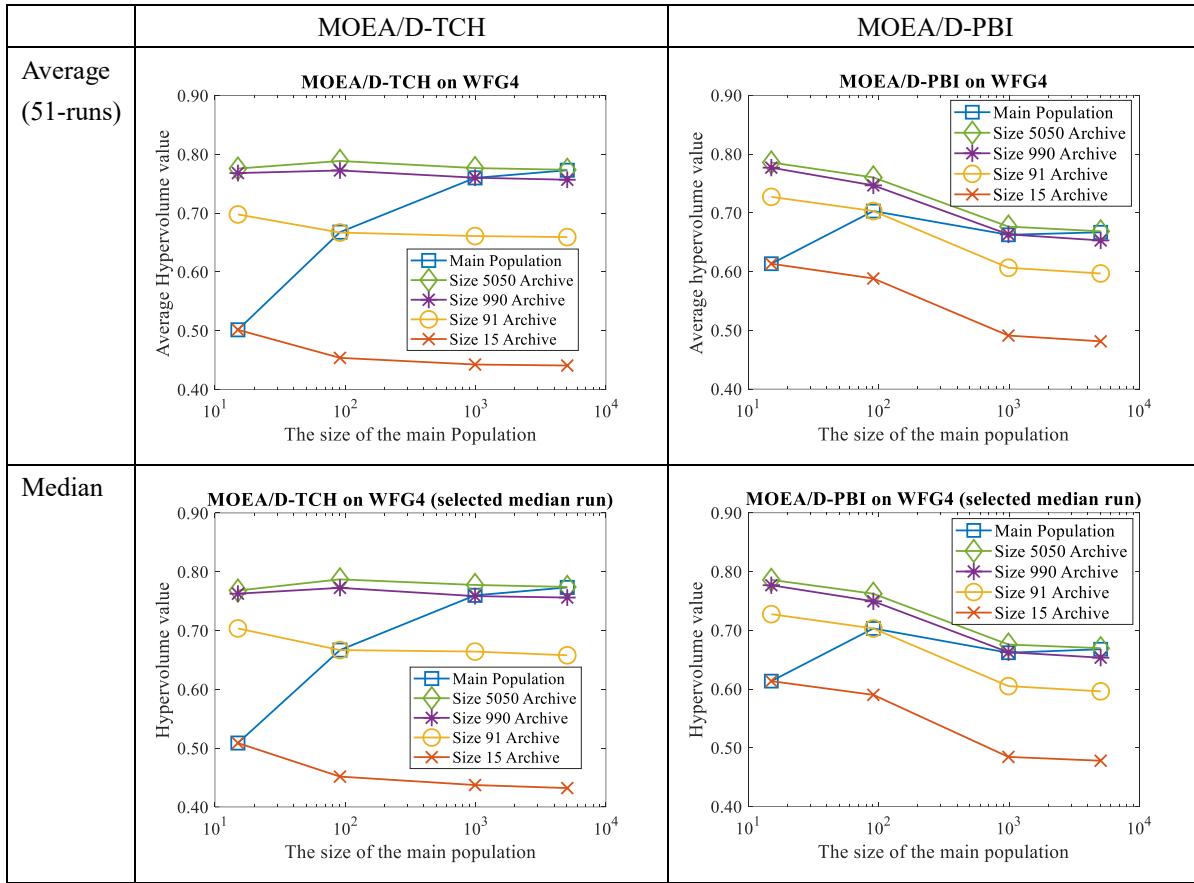
3-objective WFG2:



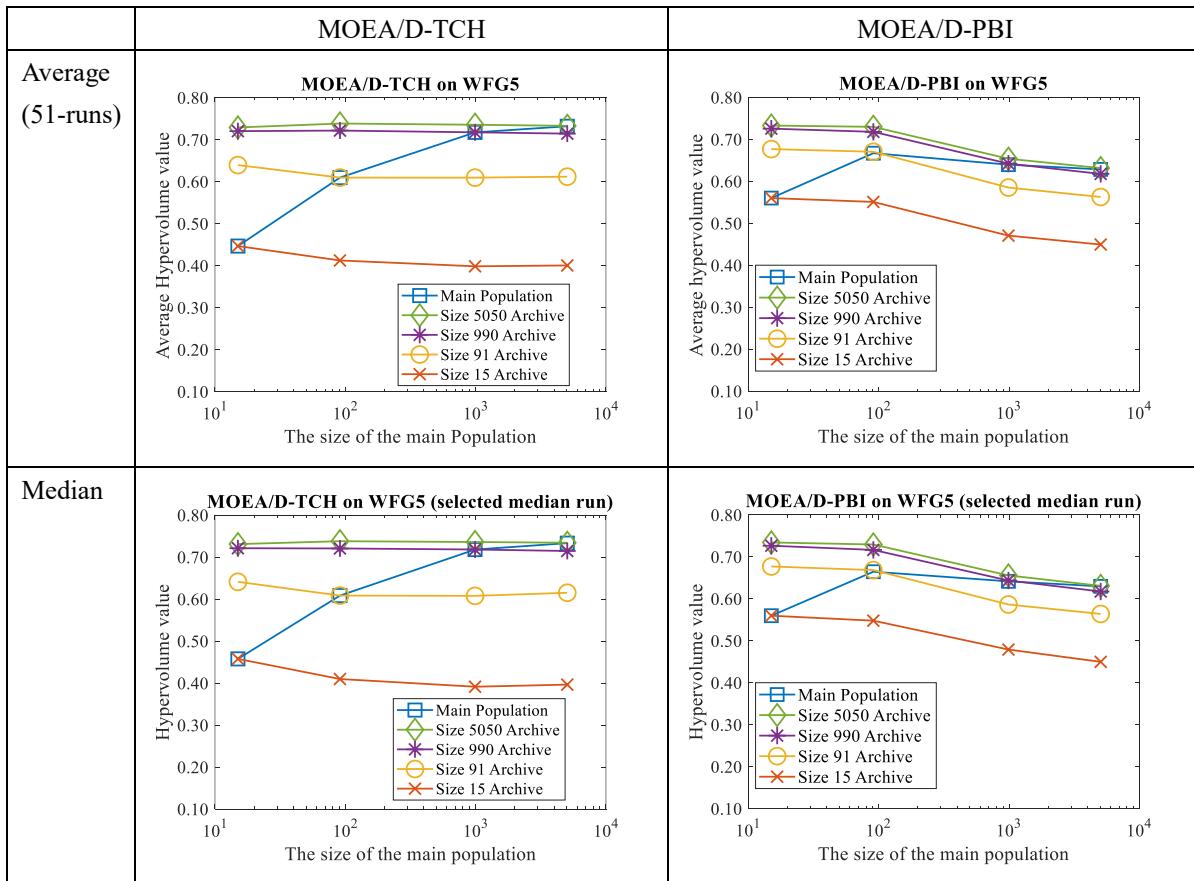
3-objective WFG3:



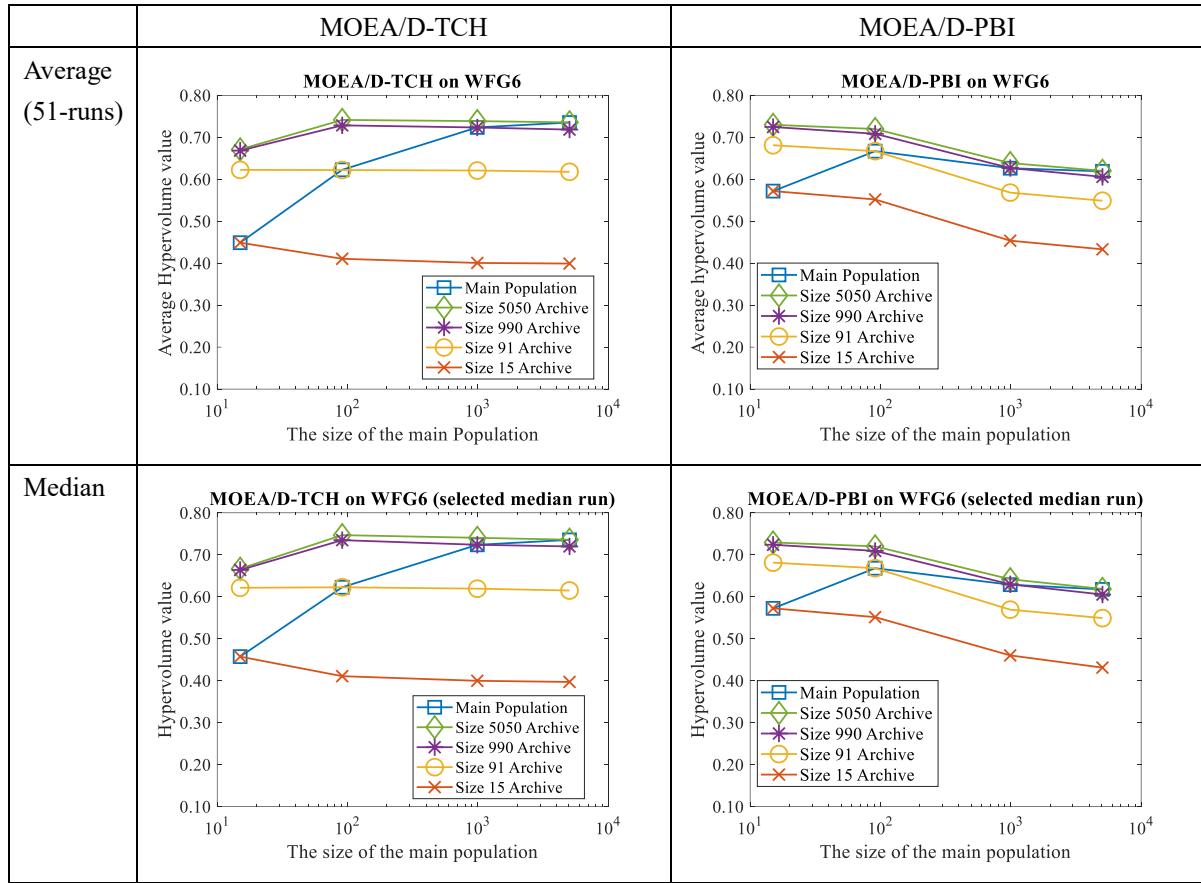
3-objective WFG4:



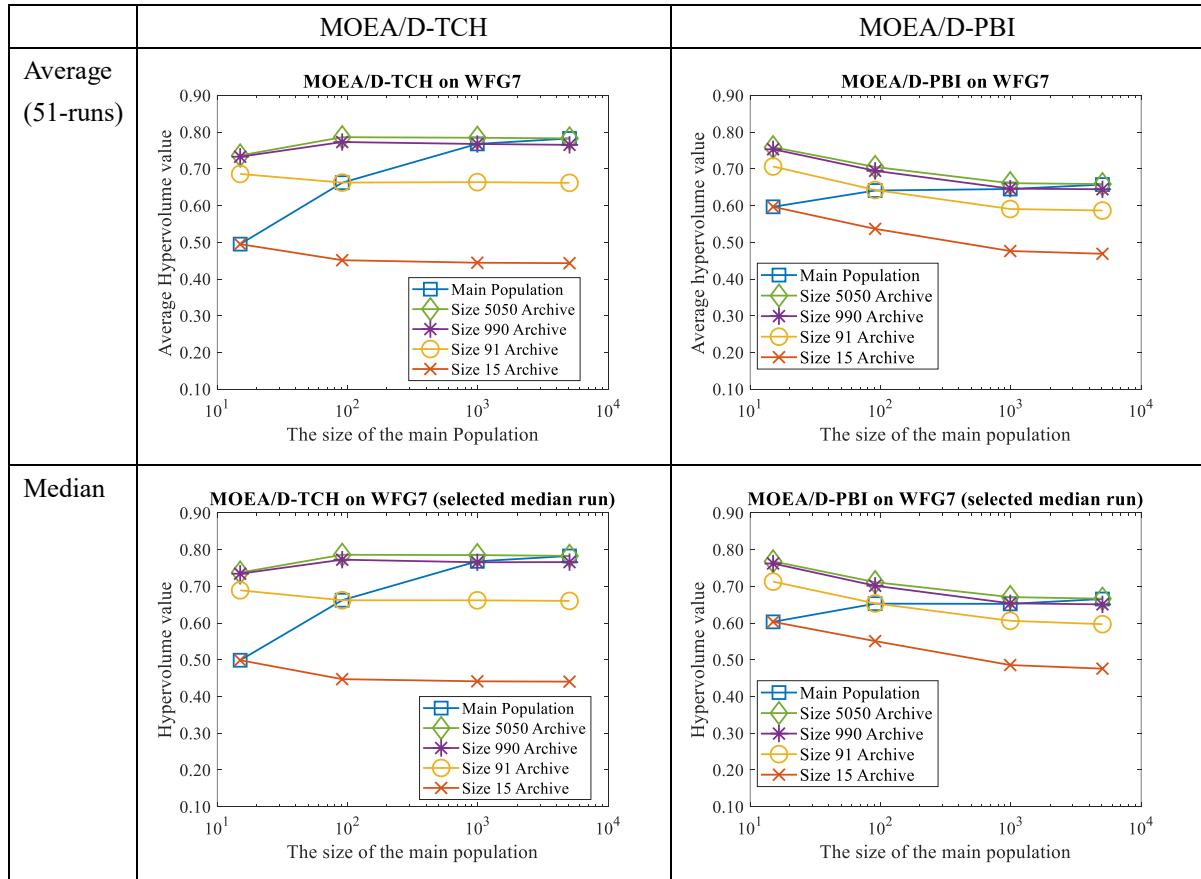
3-objective WFG5:



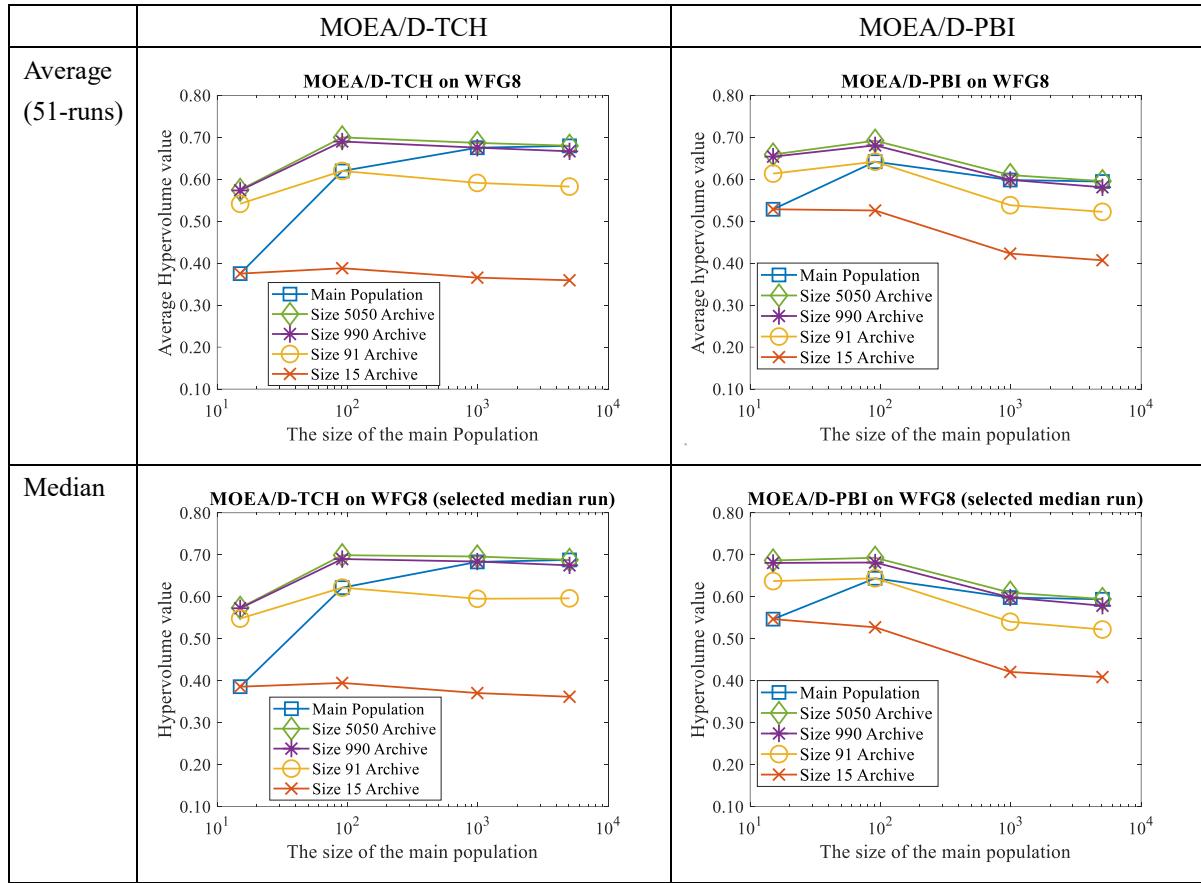
3-objective WFG6:



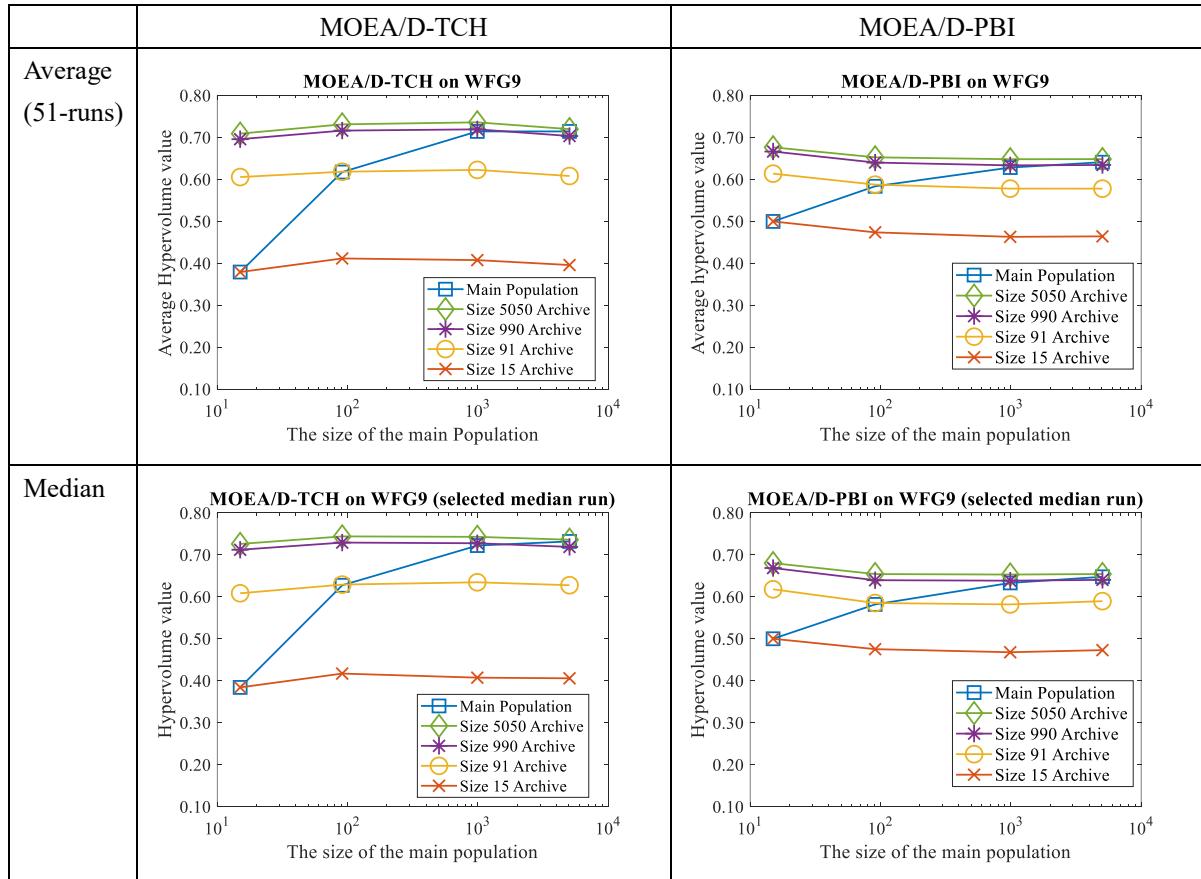
3-objective WFG7:



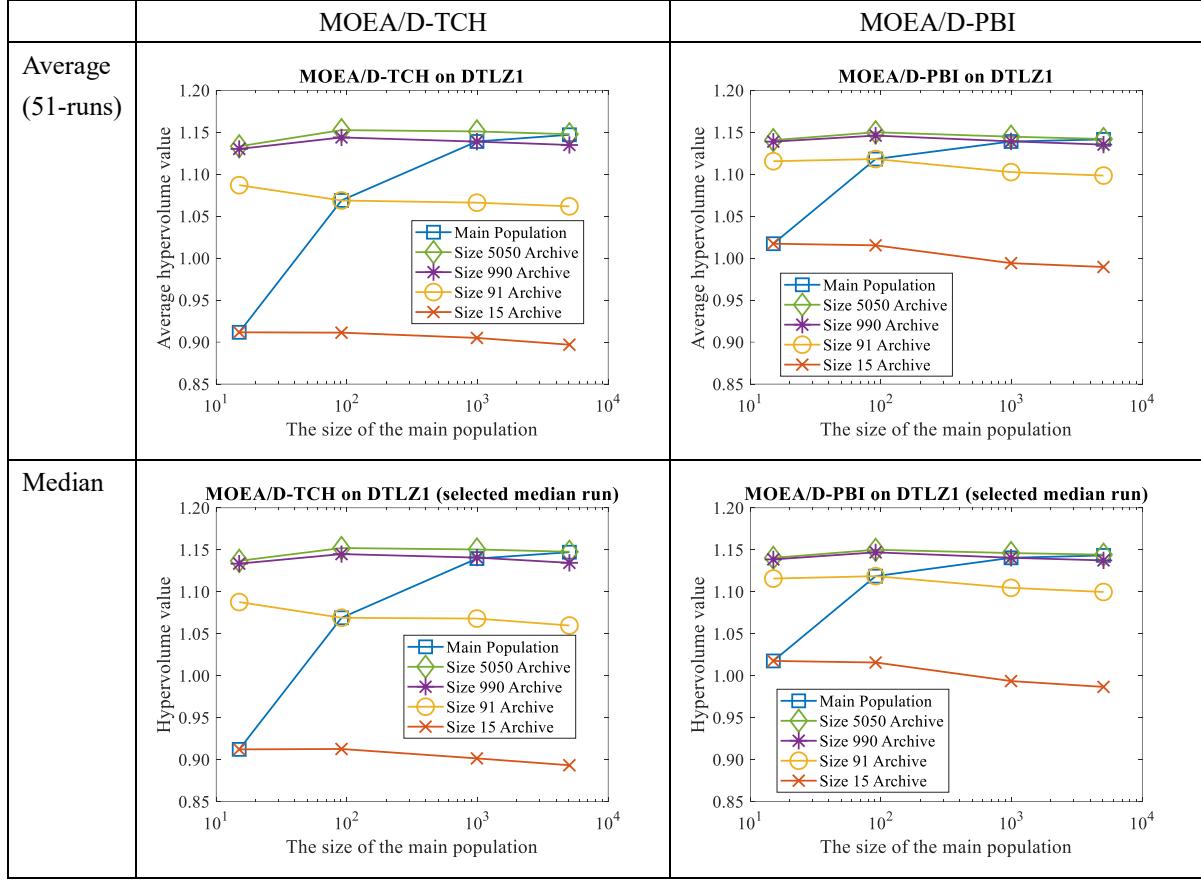
3-objective WFG8:



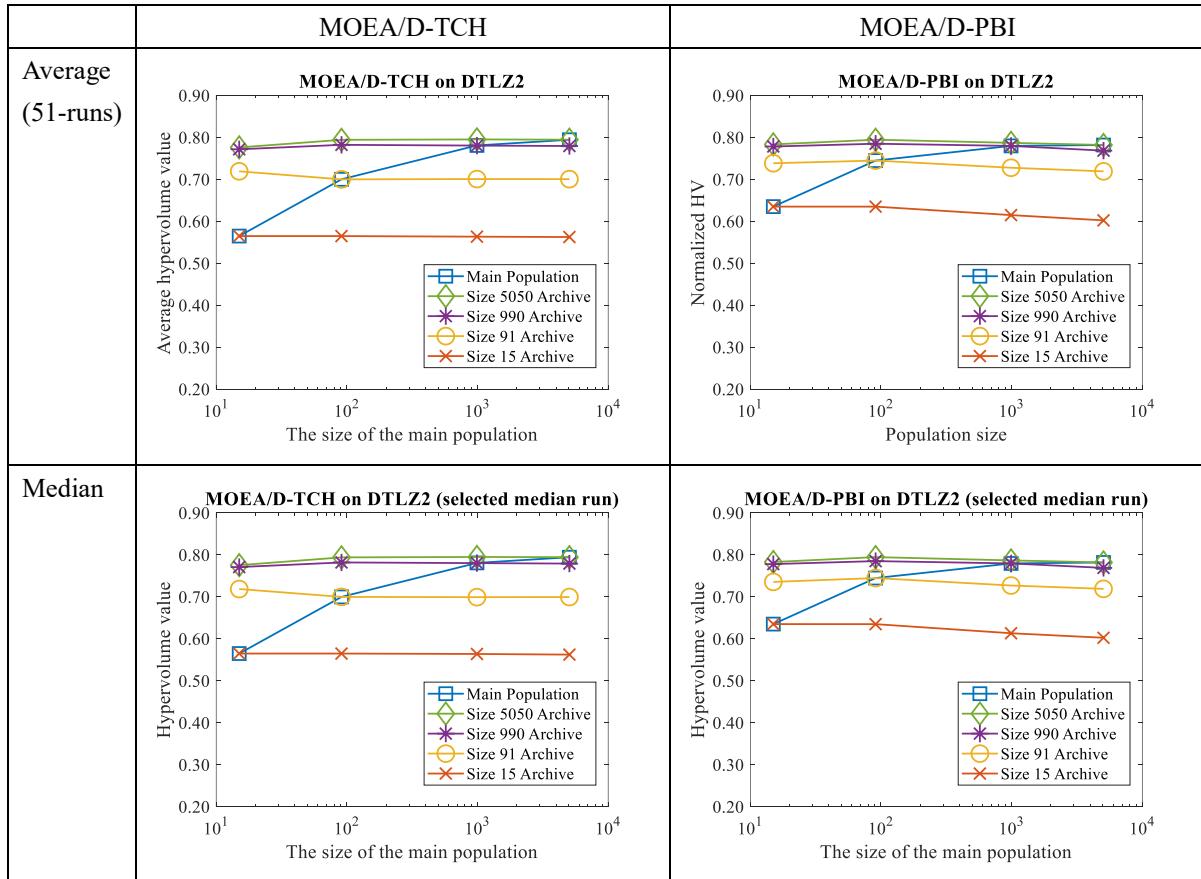
3-objective WFG9:



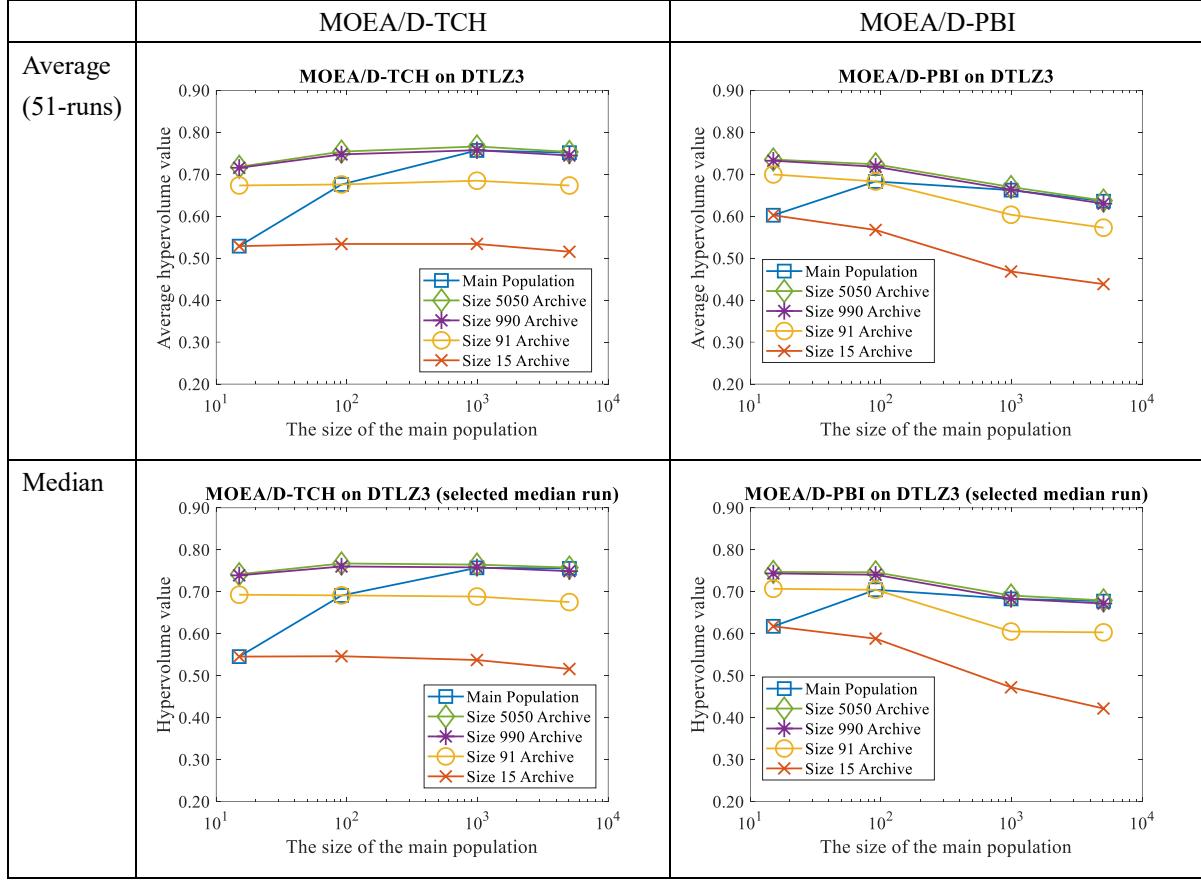
3-objective DTLZ1:



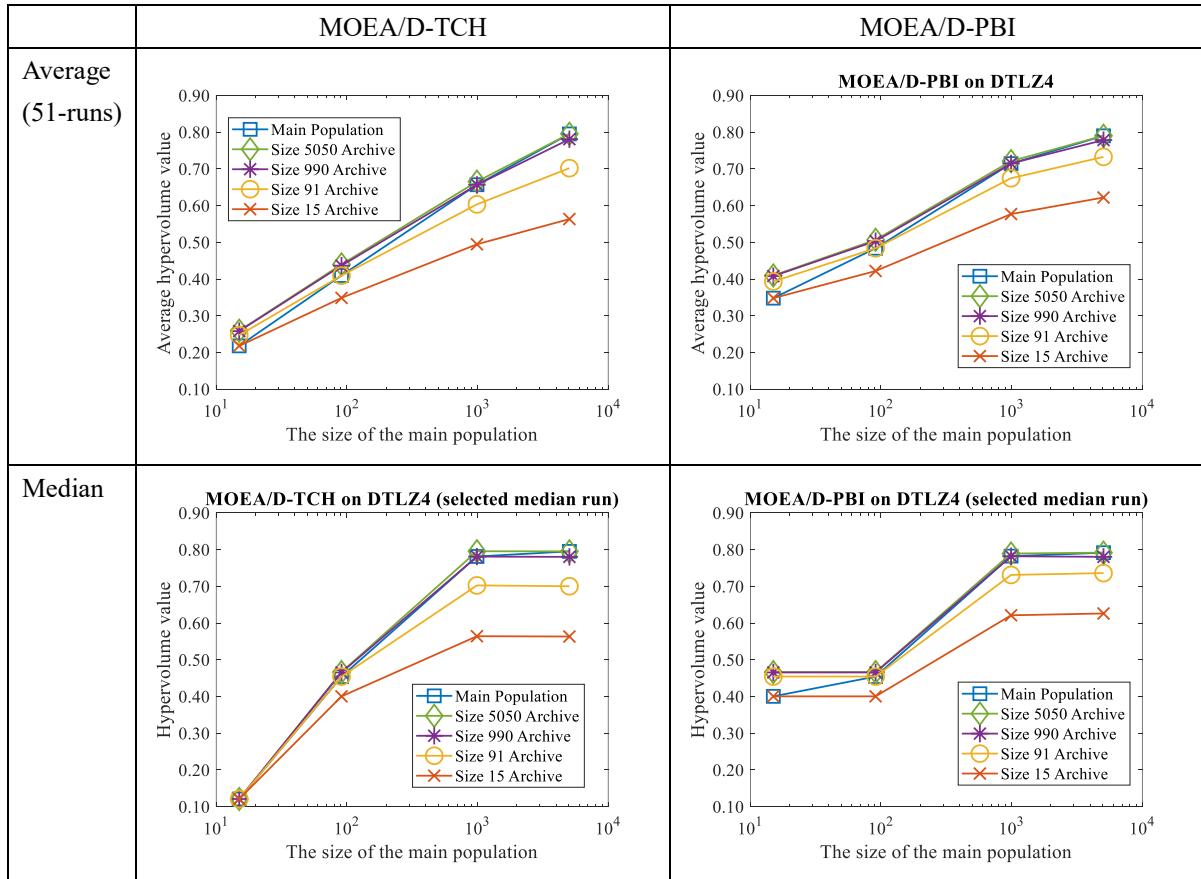
3-objective DTLZ2:



3-objective DTLZ3:

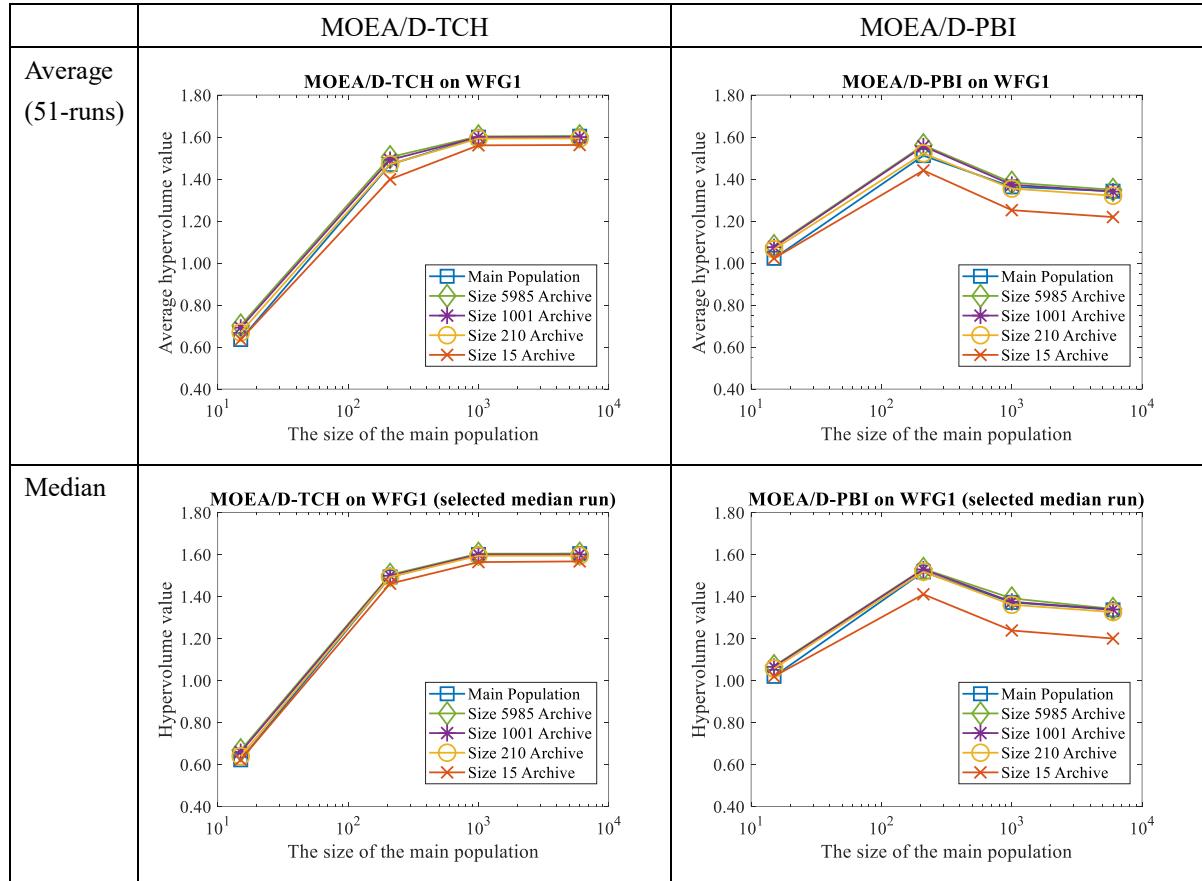


3-objective DTLZ4:

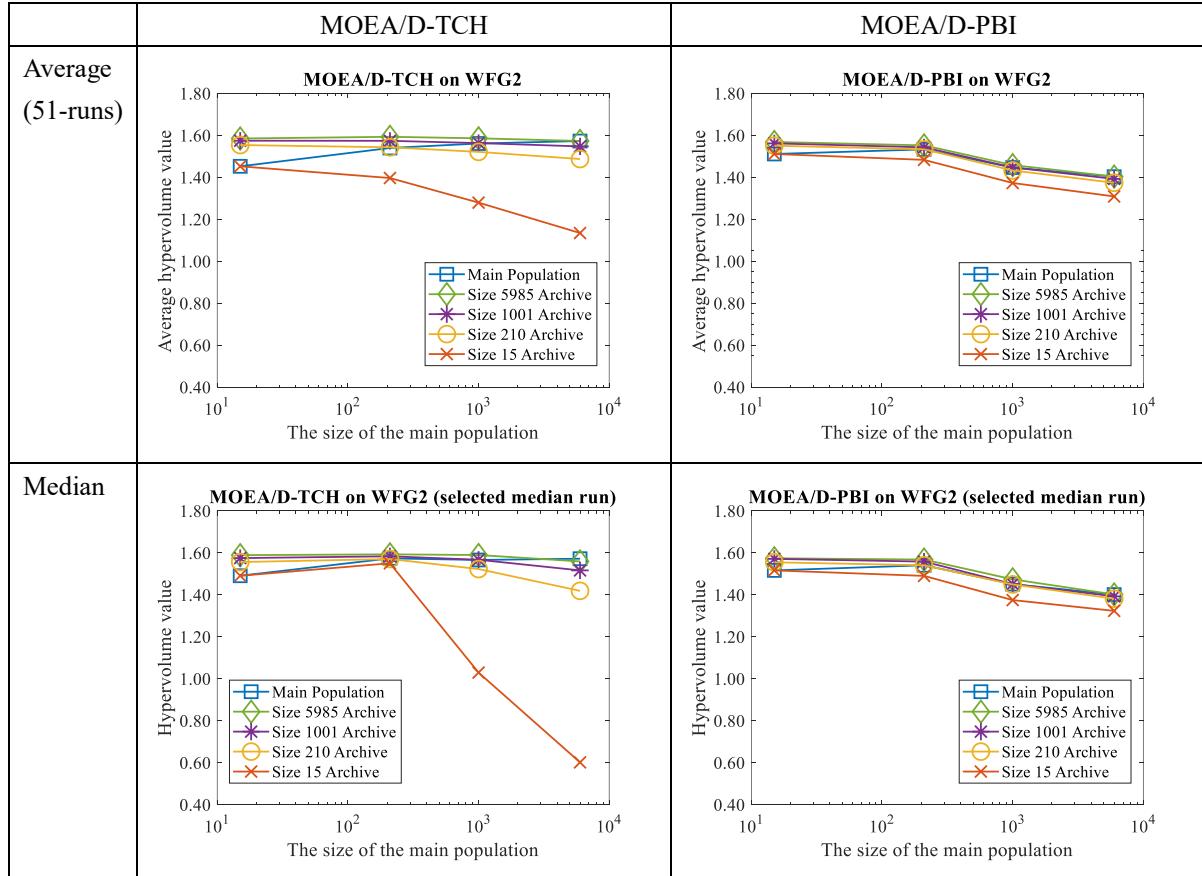


3. Results of five-objective problems:

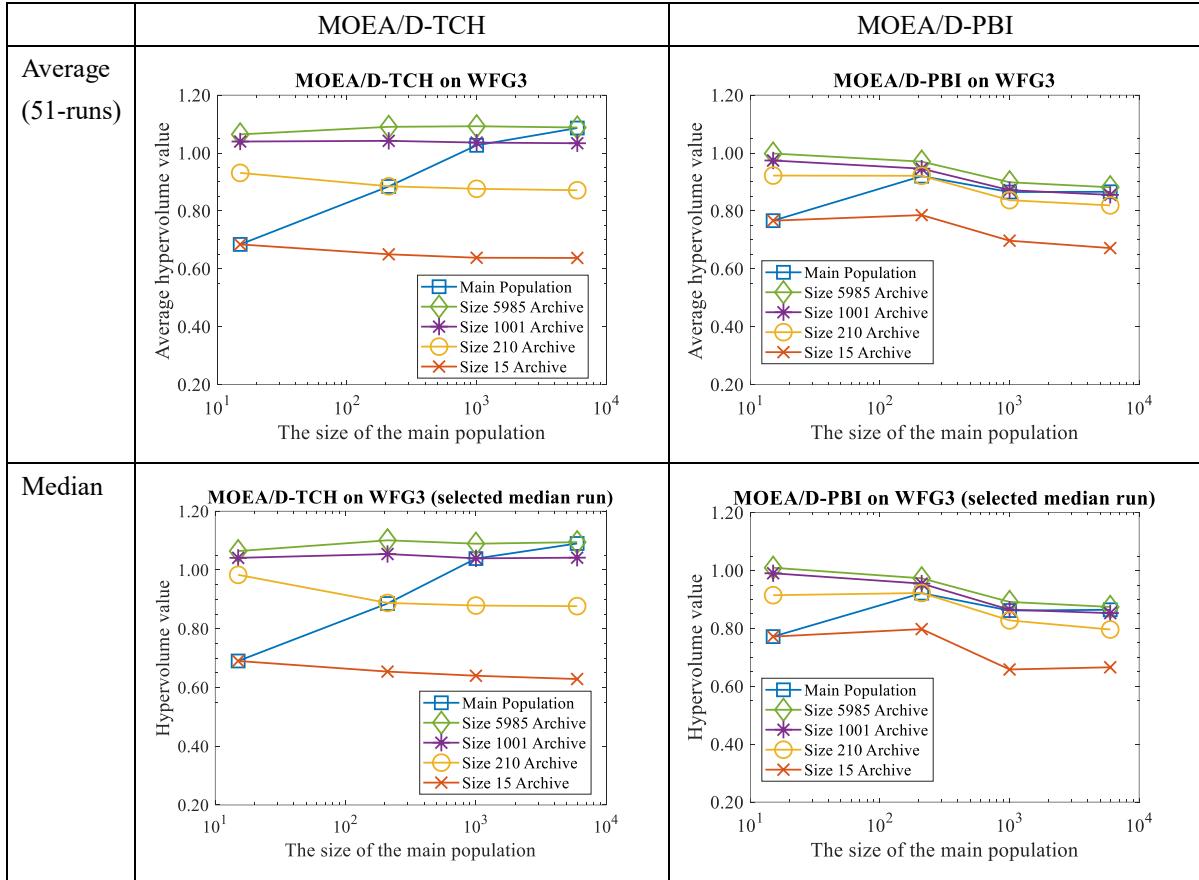
5-objective WFG1:



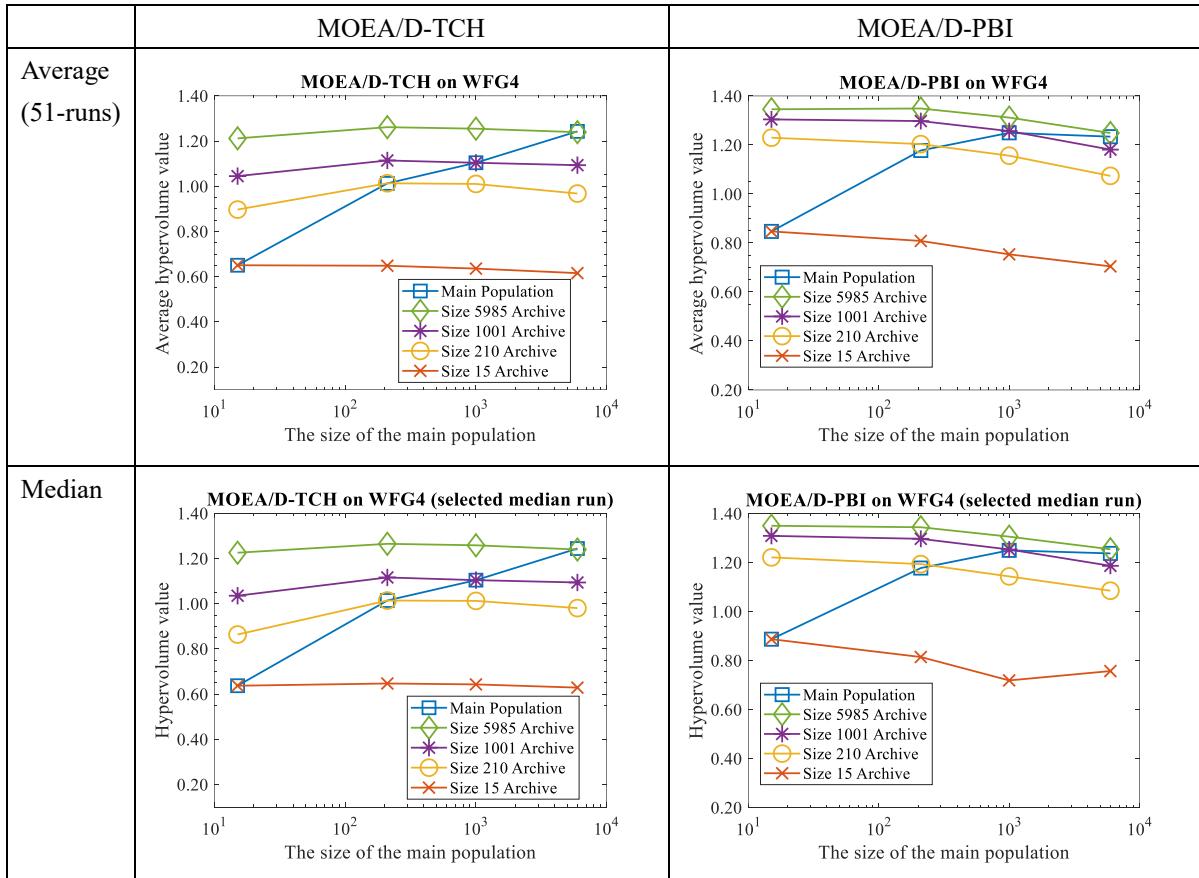
5-objective WFG2:



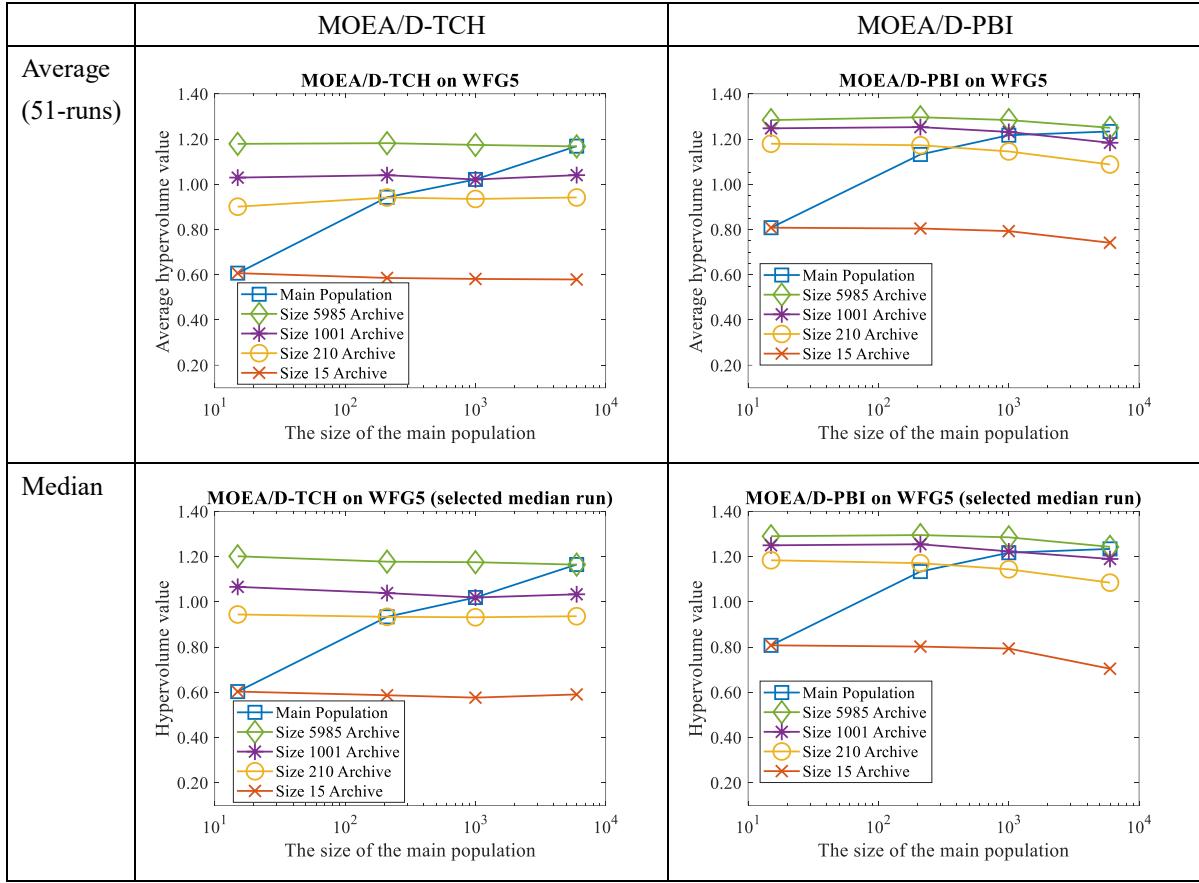
5-objective WFG3:



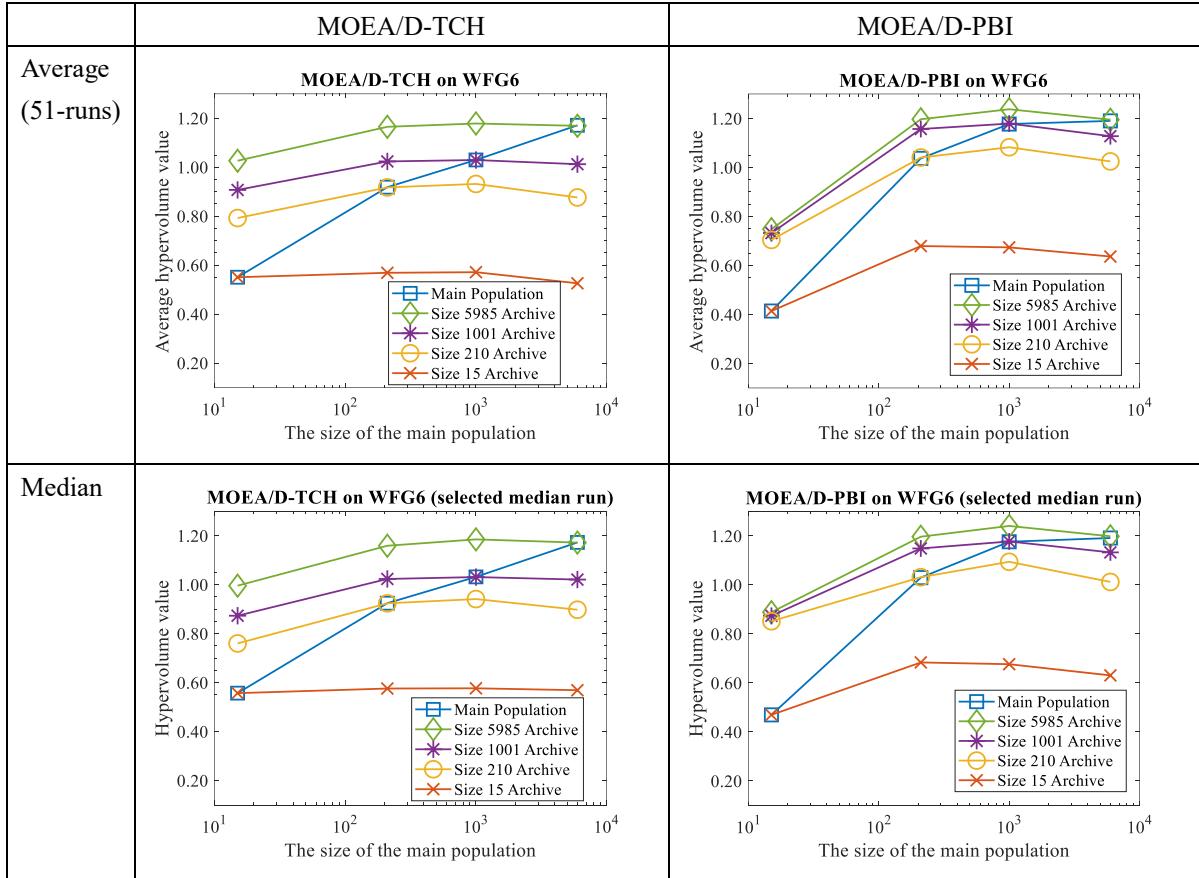
5-objective WFG4:



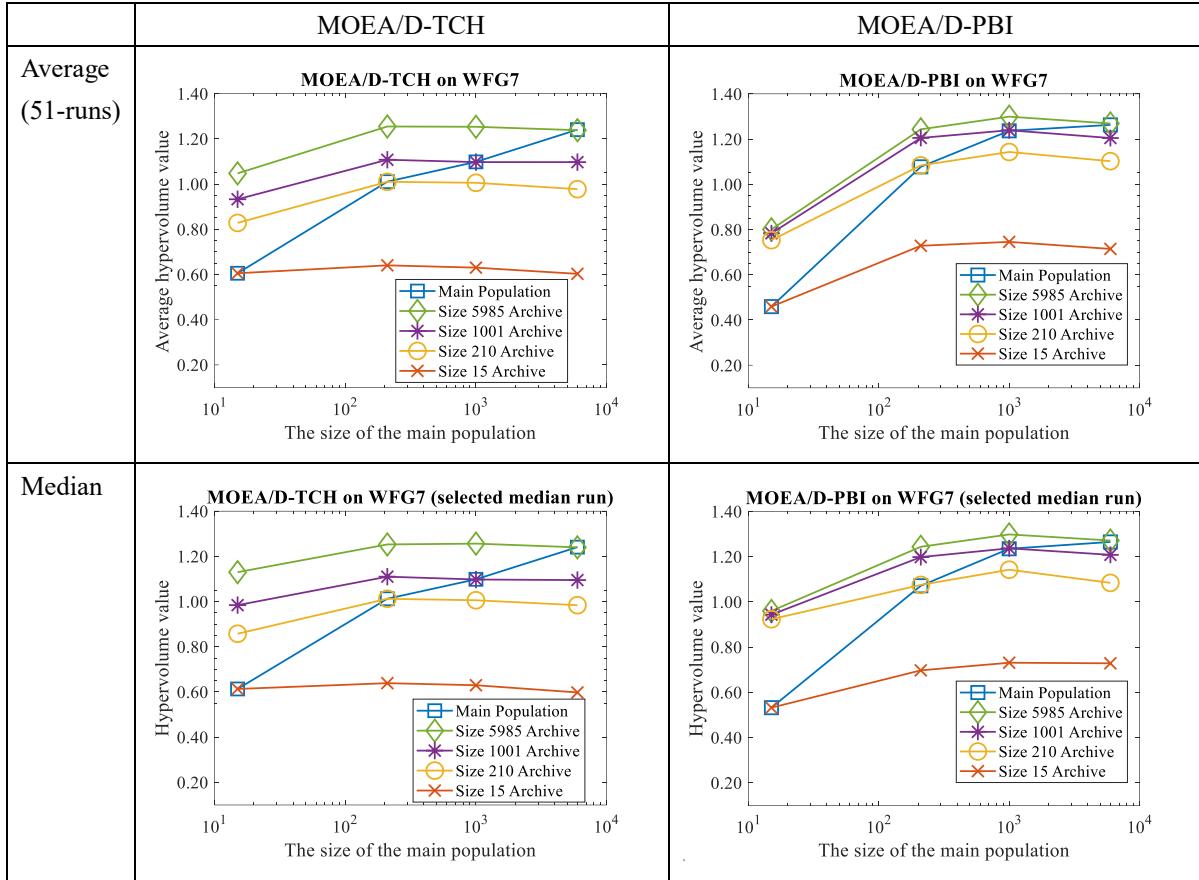
5-objective WFG5:



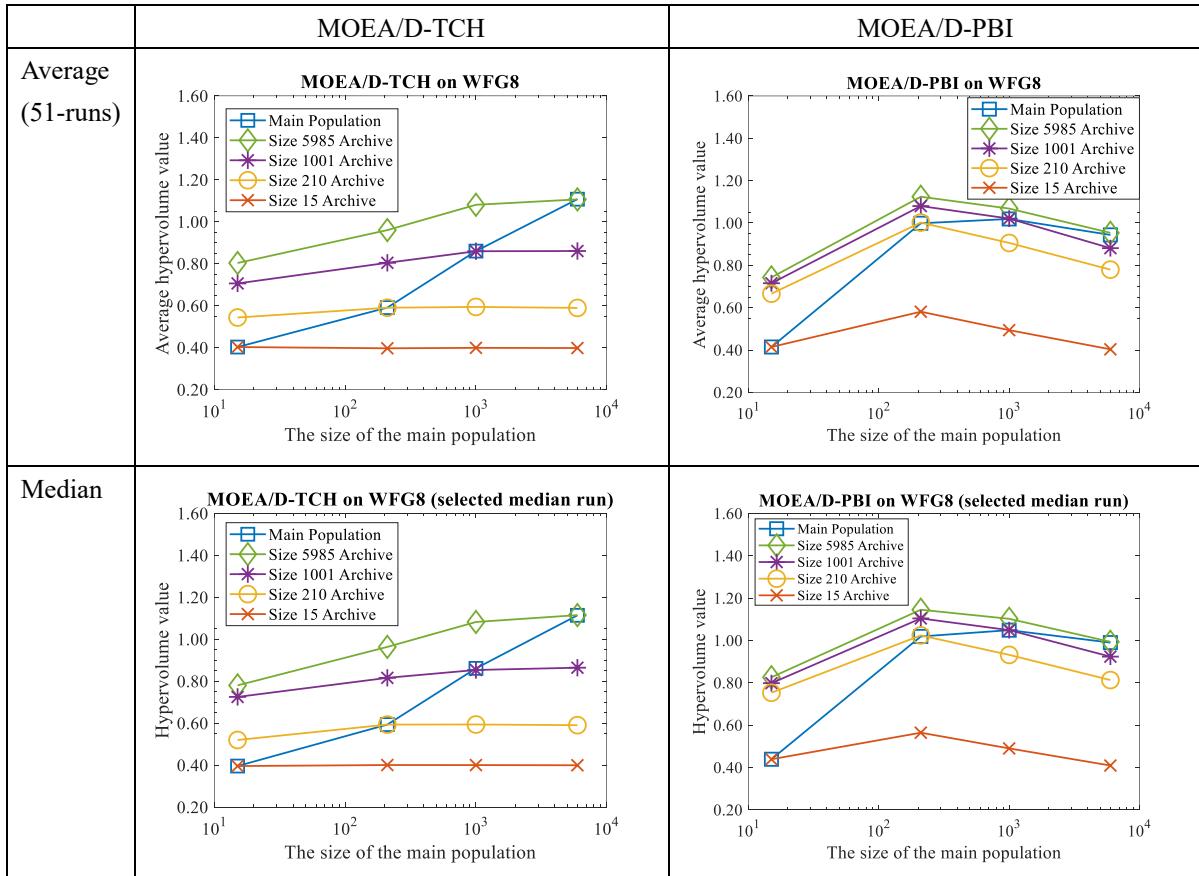
5-objective WFG6:



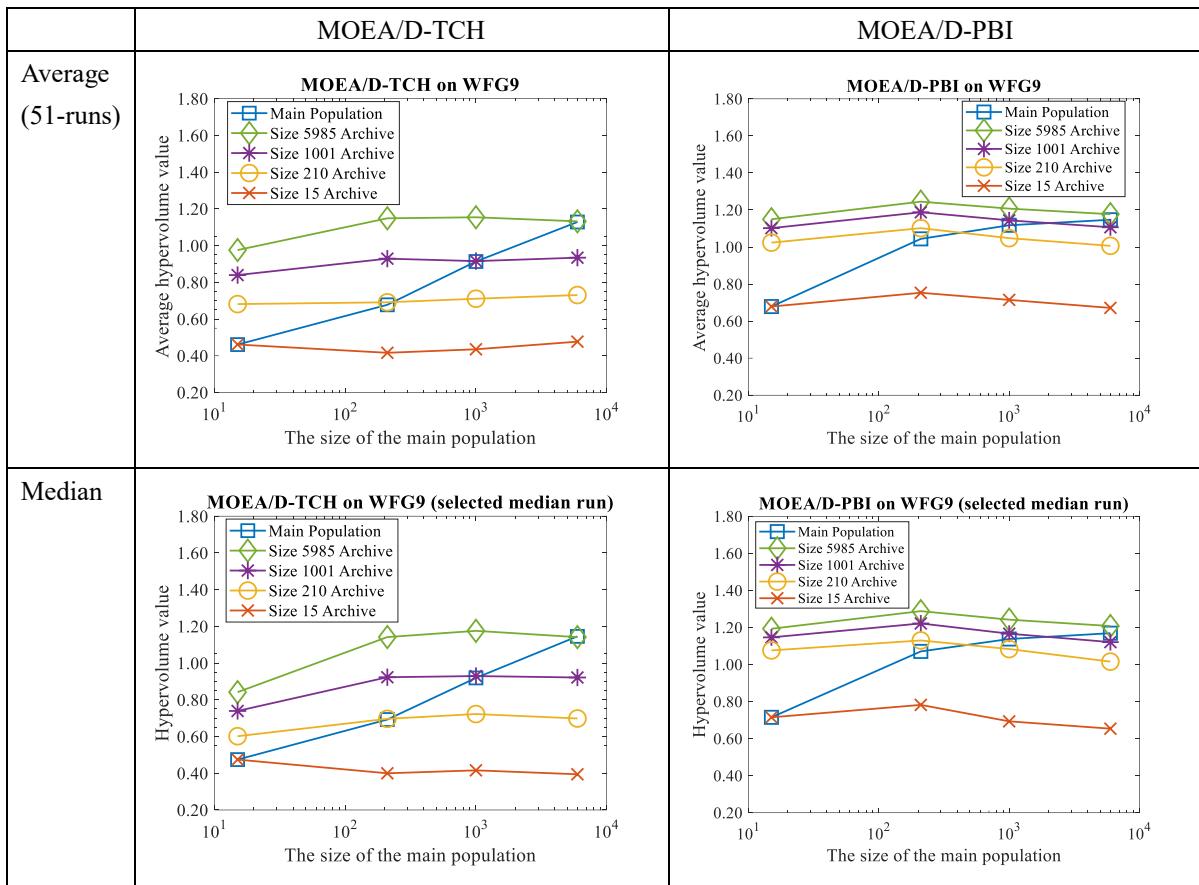
5-objective WFG7:



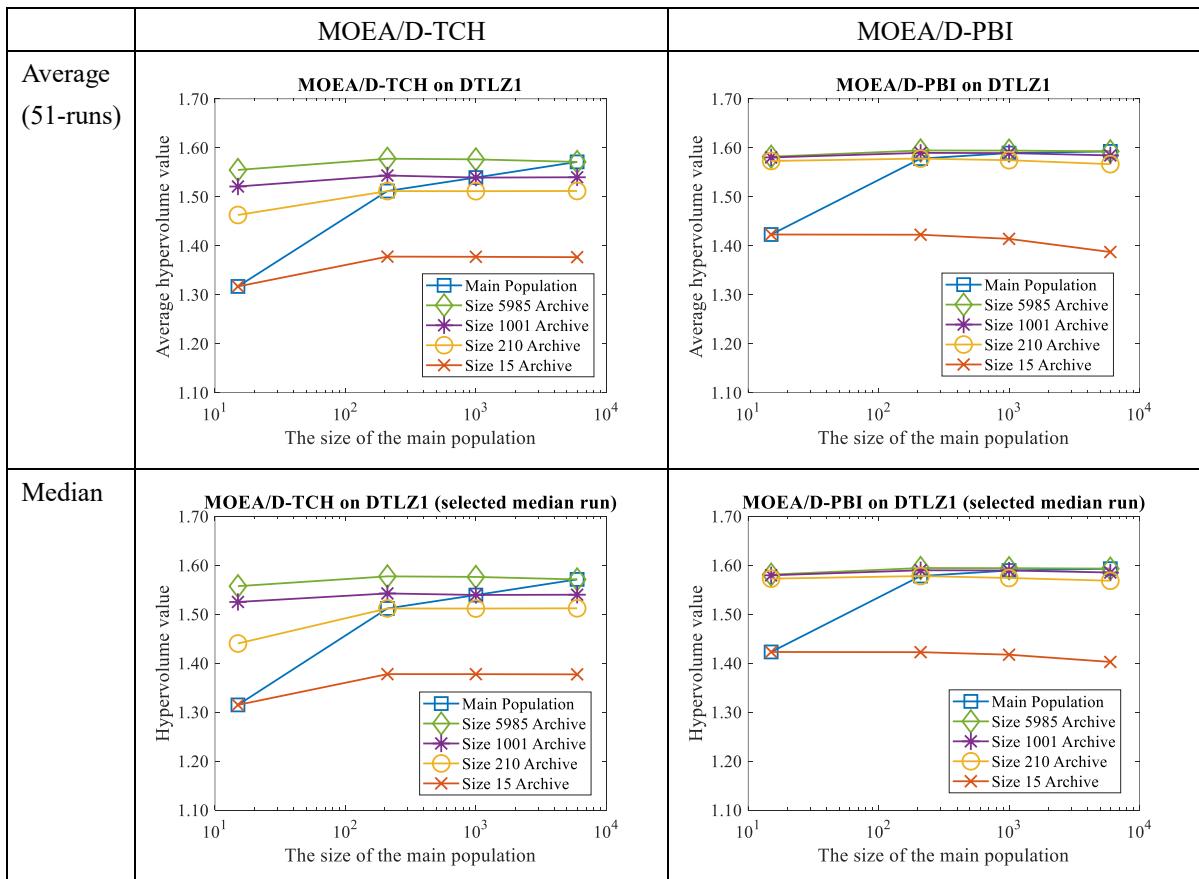
5-objective WFG8:



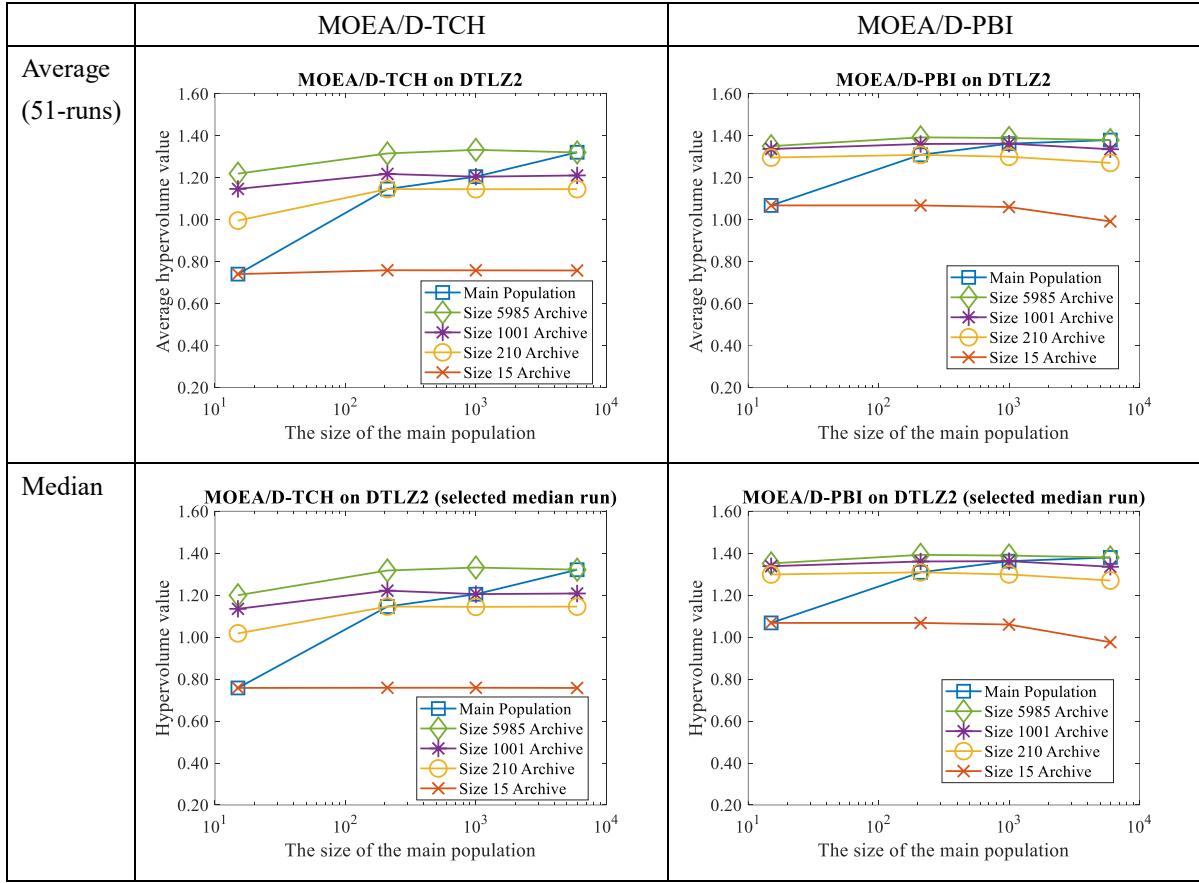
5-objective WFG9:



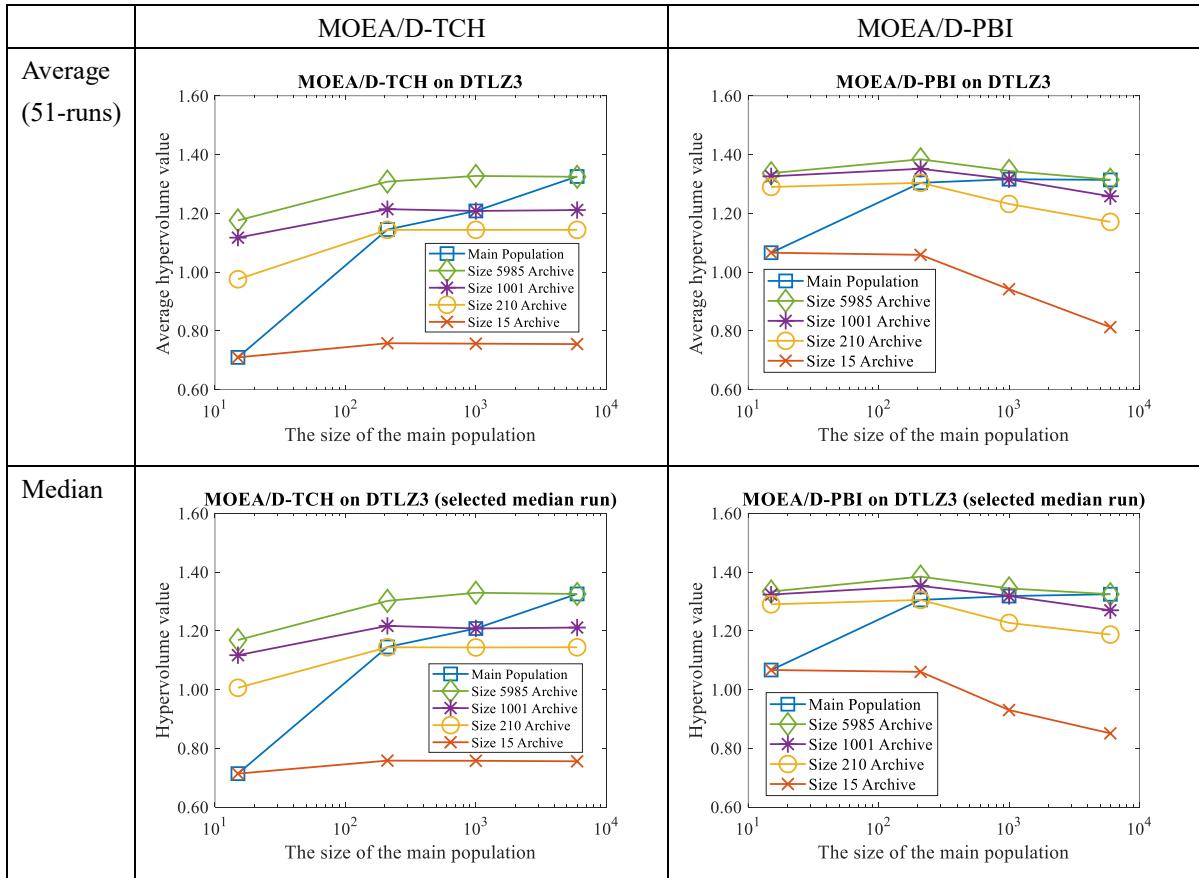
5-objective DTLZ1:



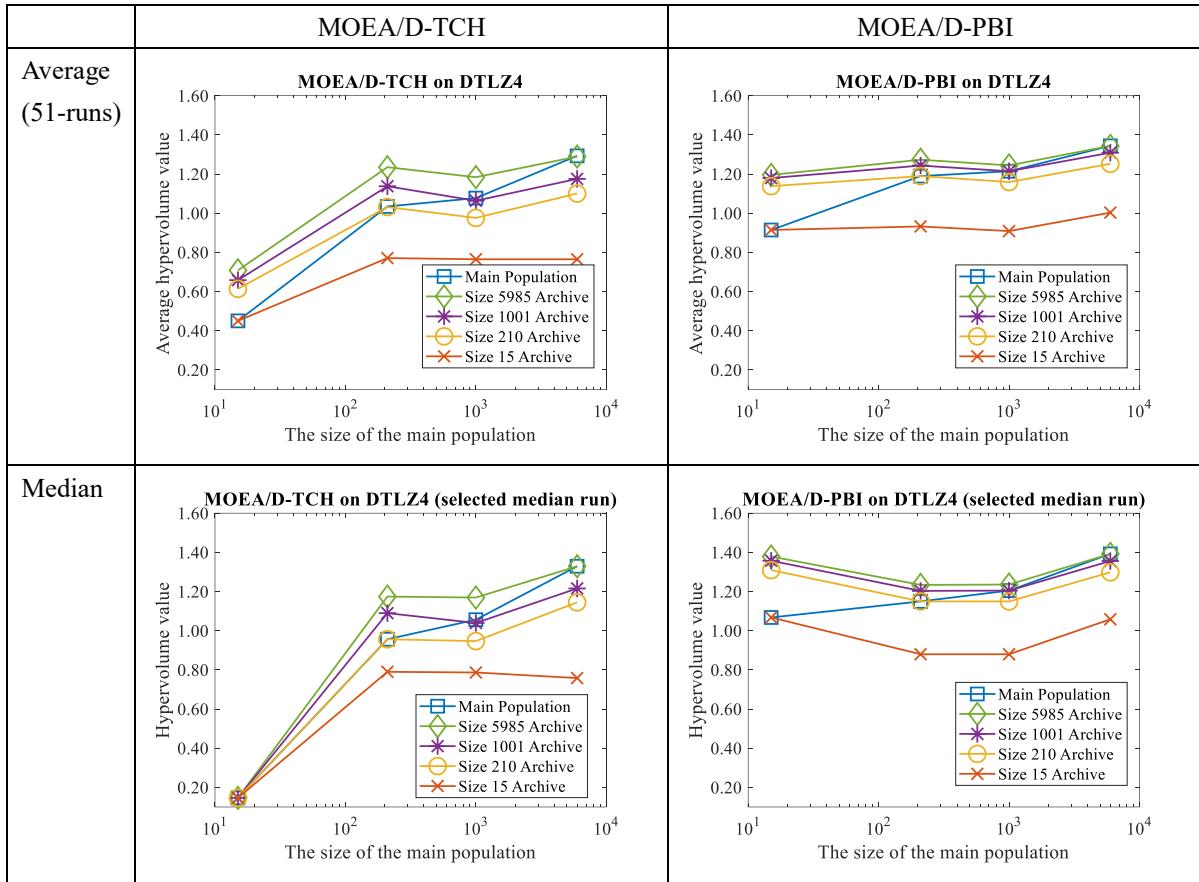
5-objective DTLZ2:



5-objective DTLZ3:



5-objective DTLZ4:



References

1. Tian, Y., Cheng, R., Zhang, X., and Jin, Y.: PlatEMO: A MATLAB platform for evolutionary multi-objective optimization [educational forum]. IEEE Computational Intelligence Magazine 12 (4), 73-87 (2017).