Supplementary file for "Partially Degenerate Multi-Objective Test Problems"

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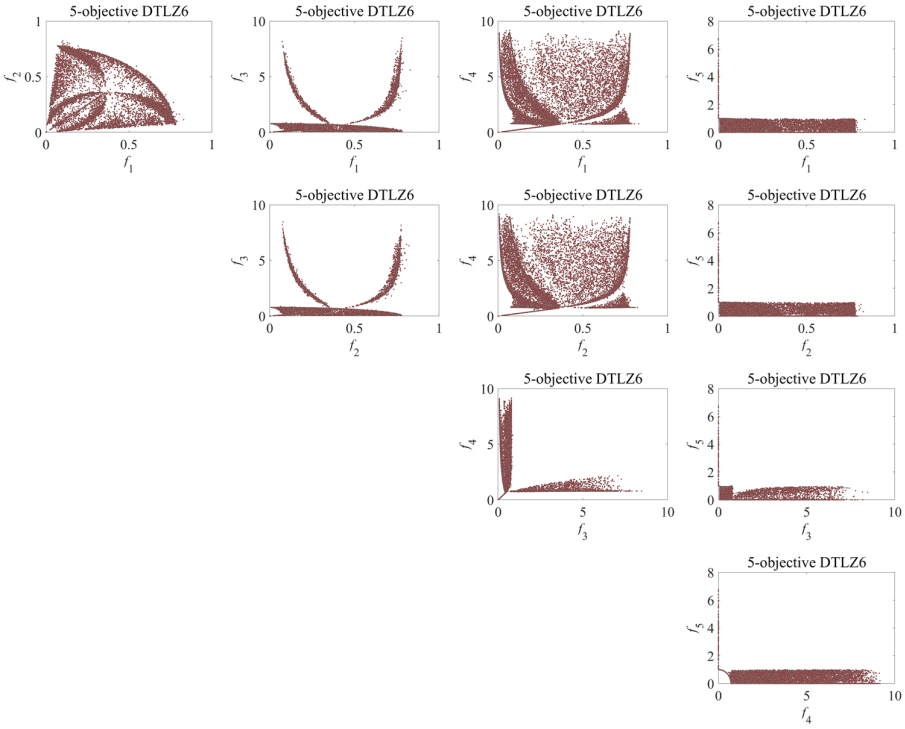

Fig. S1. The approximated Pareto front for the five-objective DTLZ6 test problem. Solutions are projected to the two-dimensional subspace.

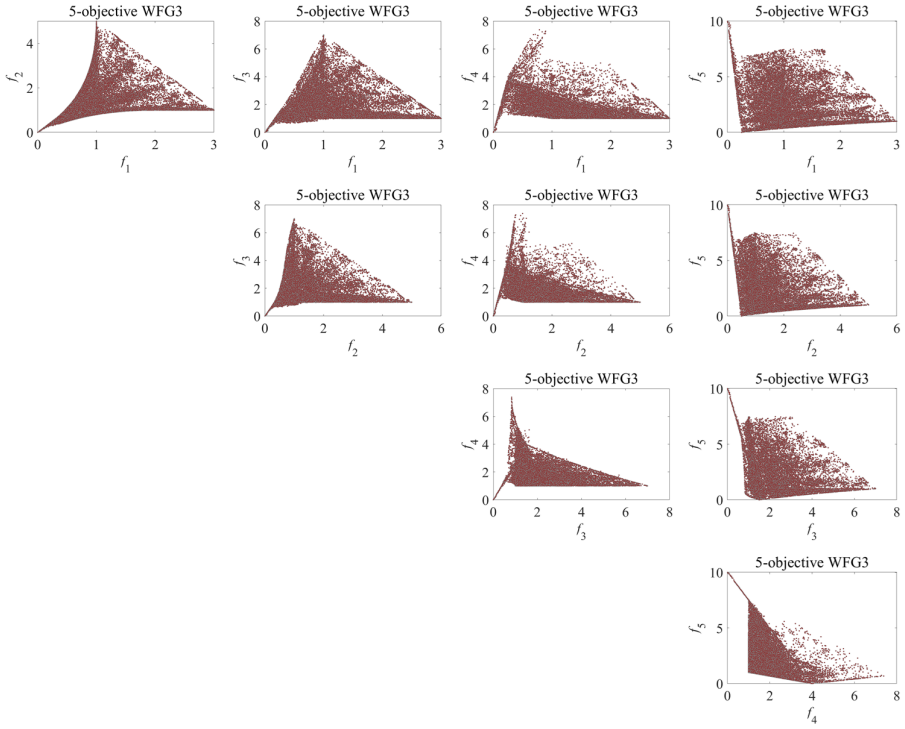

Fig. S2. The approximated Pareto front for the five-objective WFG3 test problem. Solutions are projected to the two-dimensional subspace.

Table S1. IGD values calculated using the reference point set provided on PlatEMO. The best value is highlighted in bold. The Wilcoxon's rank sum test at a significant level of 0.05 is used. The symbols "+", "-", and "=" are used to indicate whether the compared algorithm is statistically better than, worse than, or comparable to PREA.

Problem	M	MOEA/D	NSGA-III	θ-DEA	NSGA-II/SDR	PREA
DTLZ5	3	3.3902E-2 (1.39E-5) -	1.2402E-2 (1.69E-3) -	3.1531E-2 (2.75E-3) -	3.1546E-2 (5.08E-3) -	**5.1260E-3 (1.63E-4)**
	5	**2.2729E-2 (2.77E-6)** +	9.6933E-2 (2.91E-2) =	3.2915E-1 (5.89E-2) -	6.3068E-2 (1.18E-2) +	9.8630E-2 (1.44E-2)
	10	**1.9882E-2 (5.87E-6)** +	3.2004E-1 (9.01E-2) =	1.6041E-1 (4.48E-2) +	1.5781E-1 (2.74E-2) +	3.4764E-1 (5.81E-2)
DTLZ6	3	3.3923E-2 (3.25E-6) -	1.8897E-2 (2.67E-3) -	4.0363E-2 (9.41E-3) -	6.3470E-2 (1.55E-2) -	**5.1240E-3 (1.28E-4)**
	5	**2.2693E-2 (6.08E-5)** +	1.9964E-1 (8.67E-2) =	2.0084E-1 (6.27E-2) =	9.1514E-2 (2.13E-2) +	2.1224E-1 (4.81E-2)
	10	**1.8991E-2 (4.32E-4)** +	6.2197E-1 (2.87E-1) -	2.7028E-1 (5.98E-2) +	2.8766E-1 (6.80E-2) +	3.9775E-1 (1.92E-1)
WFG3	3	1.5717E-1 (1.01E-3) -	1.1232E-1 (1.38E-2) -	1.3274E-1 (1.81E-2) -	1.0916E-1 (3.04E-2) -	**8.7952E-2 (1.06E-2)**
	5	5.7262E-1 (6.09E-2) -	4.5529E-1 (4.71E-2) -	4.6917E-1 (1.61E-2) -	**3.4658E-1 (3.85E-2)** +	3.9269E-1 (3.72E-2)
	10	5.0598E+0 (1.27E-1) -	1.9893E+0 (1.60E-1) -	**1.2203E+0 (2.70E-1)** +	1.7405E+0 (3.02E-1) -	1.5719E+0 (1.41E-1)
+/-/=		4/5/0	0/6/3	3/5/1	5/4/0	

Table S2. IGD value calculated using the reference point set obtained by the five algorithms with the termination condition of 1,000 generations. The best value is highlighted in bold. The Wilcoxon's rank sum test at a significant level of 0.05 is used. The symbols "+", "-", and "=" are used to indicate whether the compared algorithm is statistically better than, worse than, or comparable to PREA.

Problem	M	MOEA/D	NSGA-III	θ-DEA	NSGA-II/SDR	PREA
DTLZ5	3	2.4689E-2 (9.17E-6) -	9.9390E-3 (1.29E-3) -	2.1964E-2 (2.92E-3) -	3.2404E-2 (5.49E-3) -	**5.0678E-3 (4.66E-4)**
	5	4.6147E-1 (1.14E-3) -	1.7525E-1 (1.50E-2) =	3.0037E-1 (1.86E-2) -	4.4753E-1 (4.55E-2) -	**1.7023E-1 (2.45E-2)**
	10	8.3070E-1 (1.52E-2) -	4.3085E-1 (3.27E-2) -	4.8048E-1 (7.05E-2) -	5.1220E-1 (2.68E-2) -	**4.1471E-1 (1.91E-2)**
DTLZ6	3	2.2166E-2 (2.61E-6) -	1.0802E-2 (2.47E-3) -	2.5765E-2 (1.21E-2) -	5.4907E-2 (1.29E-2) -	**4.6754E-3 (3.90E-4)**
	5	1.4720E+0 (1.48E-1) -	3.0741E-1 (4.10E-2) -	4.6734E-1 (1.16E-1) -	1.8105E+0 (2.09E-1) -	**2.6672E-1 (1.36E-2)**
	10	1.8476E+0 (8.12E-2) -	1.0309E+0 (2.91E-1) -	8.9413E-1 (5.77E-2) -	1.1568E+0 (3.70E-2) -	**8.1311E-1 (1.51E-1)**
WFG3	3	1.2939E-1 (1.06E-3) =	1.3477E-1 (7.21E-3) -	2.5842E-1 (8.65E-2) -	2.5427E-1 (4.31E-2) -	**1.2732E-1 (1.05E-2)**
	5	4.8206E-1 (4.62E-3) -	5.4445E-1 (7.84E-3) -	6.0760E-1 (9.31E-3) -	6.5687E-1 (4.03E-2) -	**4.6495E-1 (5.76E-3)**
	10	2.1308E+0 (2.46E-2) -	1.8235E+0 (3.43E-2) -	3.6931E+0 (6.33E-1) -	2.0530E+0 (8.51E-2) -	**1.7219E+0 (4.19E-2)**
+/-/=		0/8/1	0/8/1	0/9/0	0/9/0	

Table S3. IGD value calculated using the reference point set obtained by the five algorithms with the termination condition of 10,000 generations. The best value is highlighted in bold. The Wilcoxon's rank sum test at a significant level of 0.05 is used. The symbols "+", "-", and "=" are used to indicate whether the compared algorithm is statistically better than, worse than, or comparable to PREA.

Problem	M	MOEA/D	NSGA-III	θ-DEA	NSGA-II/SDR	PREA
DTLZ5	3	2.3686E-2 (9.13E-6) -	9.4189E-3 (1.26E-3) -	2.0539E-2 (2.78E-3) -	3.1167E-2 (5.30E-3) -	**4.9274E-3 (4.77E-4)**
	5	4.3954E-1 (1.17E-3) -	1.7181E-1 (1.50E-2) =	3.1099E-1 (2.01E-2) -	4.3021E-1 (4.34E-2) -	**1.7011E-1 (2.66E-2)**
	10	8.5248E-1 (1.47E-2) -	4.3625E-1 (3.38E-2) -	4.8805E-1 (8.33E-2) -	5.2790E-1 (2.79E-2) -	**4.1156E-1 (2.10E-2)**
DTLZ6	3	2.2350E-2 (2.54E-6) -	1.1504E-2 (2.42E-3) -	2.6490E-2 (1.09E-2) -	5.3830E-2 (1.32E-2) -	**4.6394E-3 (3.29E-4)**
	5	1.7710E+0 (1.63E-1) -	3.5092E-1 (4.96E-2) -	5.4234E-1 (1.18E-1) -	2.1432E+0 (2.57E-1) -	**2.8858E-1 (1.28E-2)**
	10	1.8997E+0 (7.89E-2) -	1.0398E+0 (2.87E-1) -	9.2309E-1 (6.19E-2) -	1.1750E+0 (3.61E-2) -	**8.4386E-1 (1.43E-1)**
WFG3	3	1.3277E-1 (9.95E-4) =	1.3443E-1 (8.14E-3) =	2.6323E-1 (9.06E-2) -	2.6262E-1 (4.47E-2) -	**1.3060E-1 (1.12E-2)**
	5	4.7849E-1 (1.67E-2) -	5.4505E-1 (7.95E-3) -	6.1015E-1 (9.44E-3) -	6.6678E-1 (4.06E-2) -	**4.6718E-1 (5.64E-3)**
	10	1.9293E+0 (2.31E-2) -	1.7063E+0 (2.90E-2) -	4.3026E+0 (8.04E-1) -	2.1410E+0 (9.08E-2) -	**1.6368E+0 (3.20E-2)**
+/-/=		0/8/1	0/7/2	0/9/0	0/9/0	

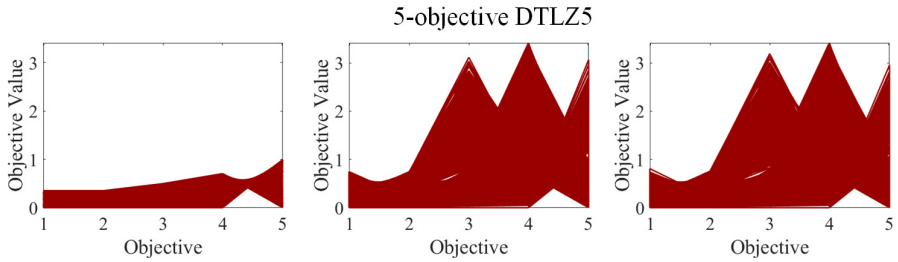

Fig. S3. The reference point sets used for IGD calculation: (left) provided by PlatEMO, (middle) all the non-dominated solutions obtained by the five algorithms with the termination condition of 1,000 generations, (right) all the non-dominated solutions obtained by the five algorithms with the termination condition of 10,000 generations.

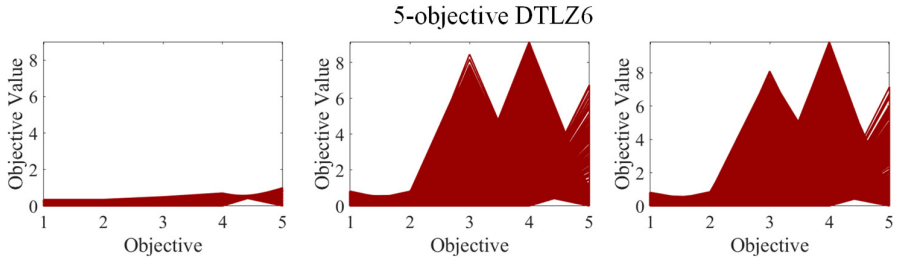

Fig. S4. The reference point sets used for IGD calculation: (left) provided by PlatEMO, (middle) all the non-dominated solutions obtained by the five algorithms with the termination condition of 1,000 generations, (right) all the non-dominated solutions obtained by the five algorithms with the termination condition of 10,000 generations.

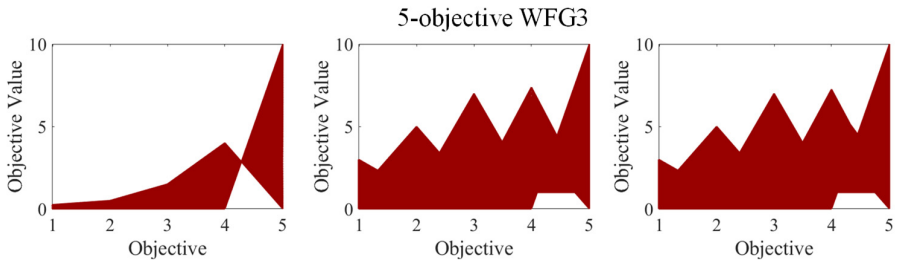

Fig. S5. The reference point sets used for IGD calculation: (left) provided by PlatEMO, (middle) all the non-dominated solutions obtained by the five algorithms with the termination condition of 1,000 generations, (right) all the non-dominated solutions obtained by the five algorithms with the termination condition of 10,000 generations.

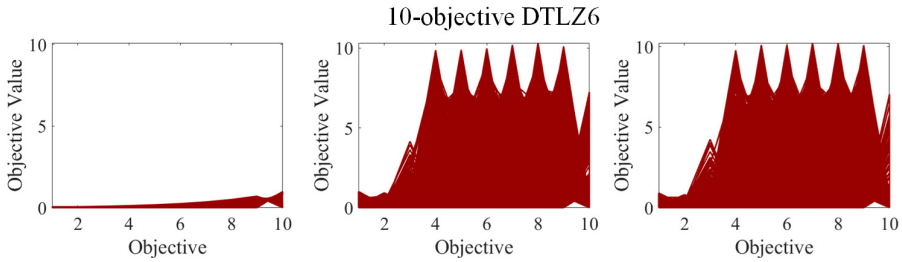

Fig. S6. The reference point sets used for IGD calculation: (left) provided by PlatEMO, (middle) all the non-dominated solutions obtained by the five algorithms with the termination condition of 1,000 generations, (right) all the non-dominated solutions obtained by the five algorithms with the termination condition of 10,000 generations.