Entanglement Entropy and Mutual Information of a Matrix Product State Circuit for a 4-qubit GHZ State Generated on an IBM Quantum Computer

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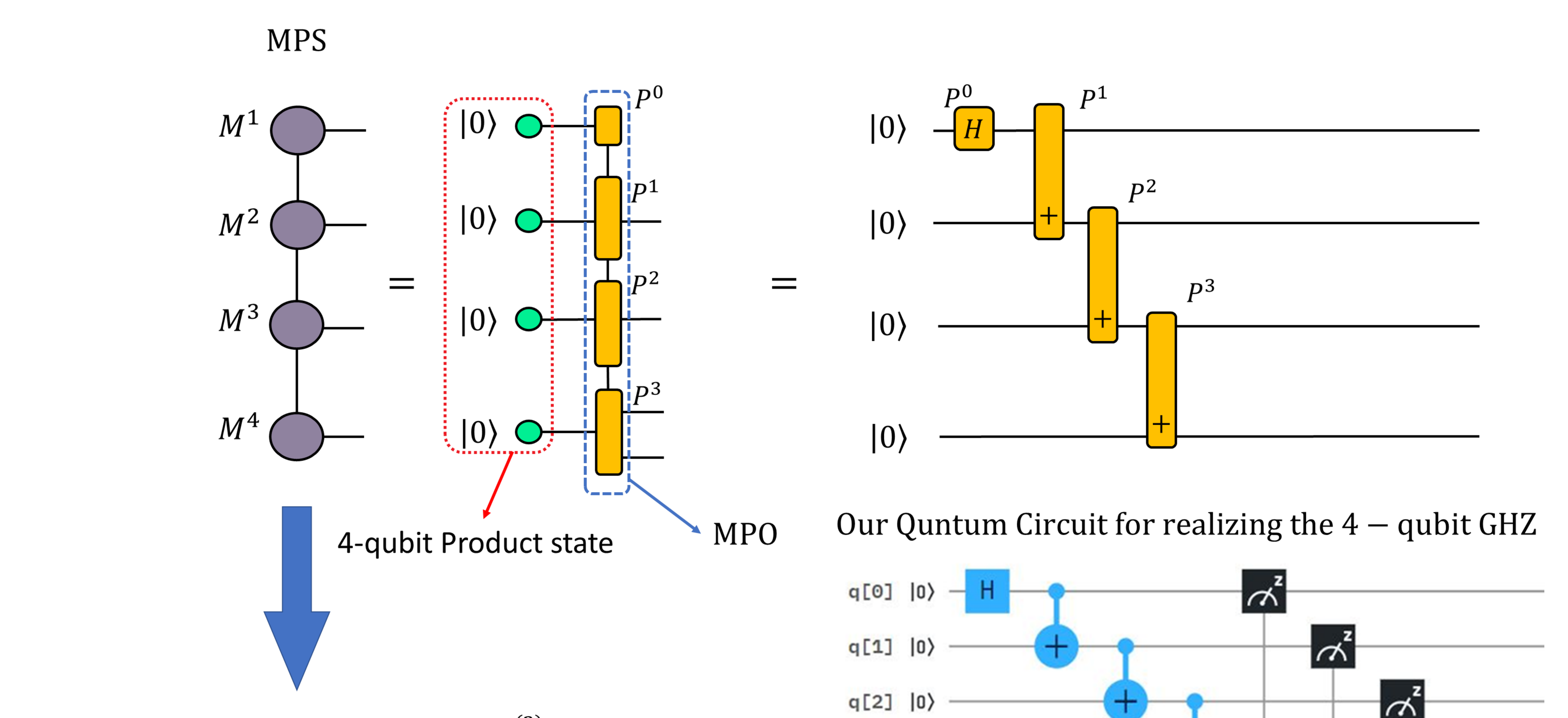
Abstract

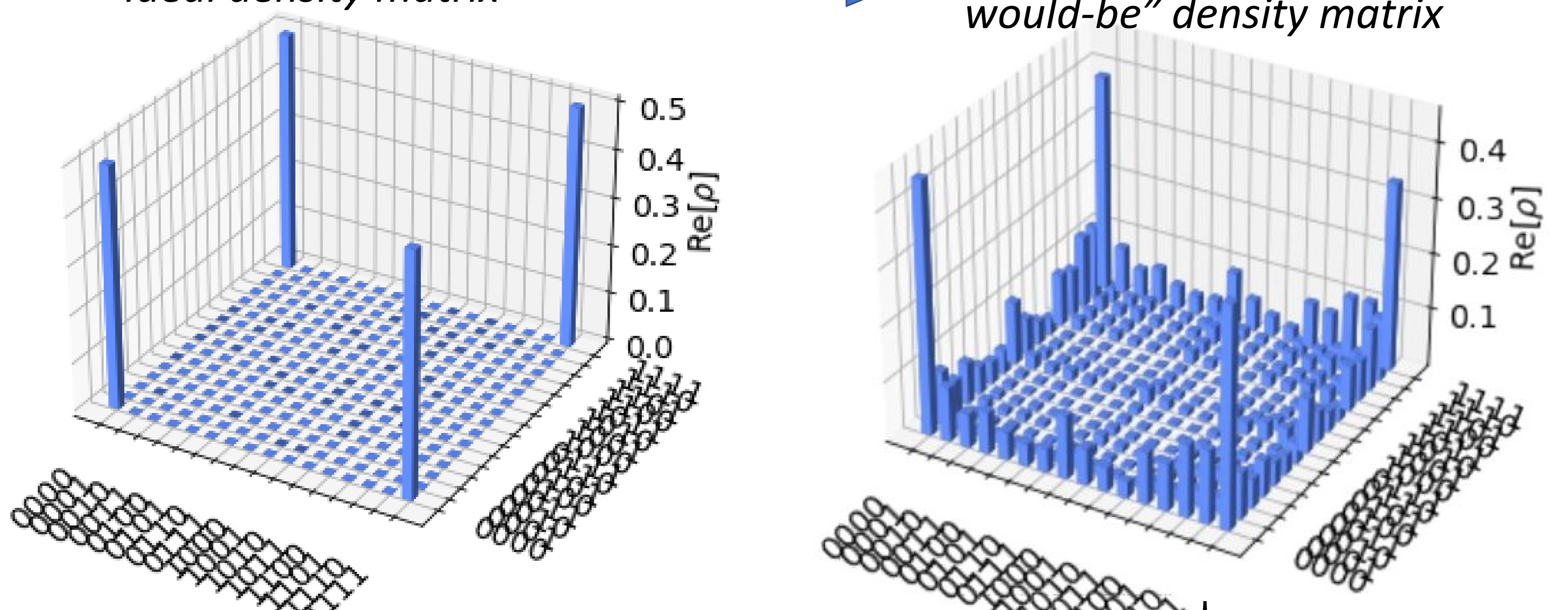
It is well established that the use of Matrix Product States (MPS's), given reasonable bond dimensions (χ), reduce the computational cost of running numerical operations on otherwise exponentially large multi-body quantum state vectors. There has been much focus recently on the one-to-one mapping between certain tensor networks and quantum circuits. This mapping perceives quantum circuits as quantum computer networks which manipulate multi-qubit product states using local unitary gates. Advantages of MPS's can extend to quantum circuit design, and since quantum computers also have memory requirements and computational runtimes, the cost of operation becomes a determining factor in the success of a circuit. Here we demonstrate this mapping, by running a trivial MPS quantum circuit on the "ibmq_london" quantum computer to reproduce the, MPS representable, 4-qubit GHZ state. 8192 runs of the MPS circuit with measurement were used to obtain a corresponding distribution. We proposed a non ideal way to infer a "would-be density matrix" for the pre-measurement state in an attempt to infer loss of entanglement due to decoherence. The entanglement, between the first qubit and the last three, for this hypothetical density matrix and ideal case were calculated using the von Neumann entropy for the ideal case and mutual information For the experimental results. In conclusion, keeping in mind our assumption, the trivial MPS circuit experienced a 23% reduction in total correlations.

Analogy of MPS's to Quantum Circuits



$$|GHZ\rangle^{(4)}_{MPS} = \frac{1}{\sqrt{2}}[\,|1\rangle\ |0\rangle\,]\begin{bmatrix}|1\rangle & 0\\ 0 & |0\rangle\end{bmatrix}^{(2)}\begin{bmatrix}|1\rangle\\ |0\rangle\end{bmatrix}$$

Analysis: Hypothetical Density Matrix



Reduced density matrix of partition **A**

1^{st} Qubit

$$\begin{pmatrix}0.5348 & 0.270896\\ 0.270896 & 0.46523\end{pmatrix}$$

Eigenvalues $(0.773135\,,0.226895)$

Reduced density matrix of partition **B**

$2^{nd}, 3^{rd}, 4^{th}$ Qubits

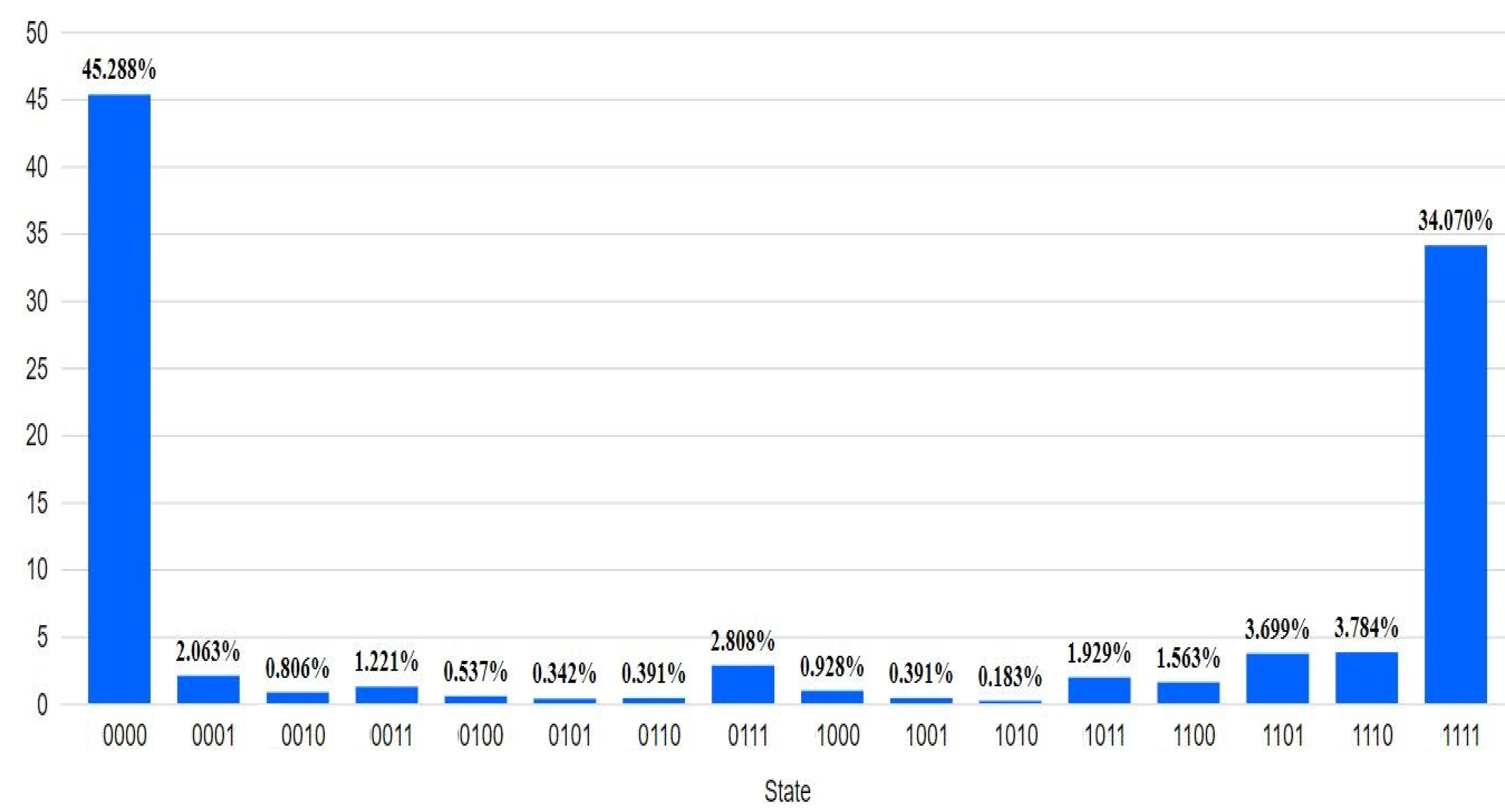
$$\begin{pmatrix}
0.47351 & 0.0762881 & 0.0577146 & 0.0661489 & 0.0738097 & 0.0487371 & 0.111758 & 0.214745\\
0.0762881 & 0.02027 & 0.013041 & 0.0241302 & 0.015558 & 0.0191876 & 0.032476 & 0.0819616\\
0.0577146 & 0.013041 & 0.00879 & 0.0143819 & 0.0107161 & 0.0112571 & 0.020409 & 0.0483898\\
0.0661489 & 0.0241302 & 0.0143819 & 0.03199 & 0.0165019 & 0.0259486 & 0.0400461 & 0.109974\\
0.0738097 & 0.015558 & 0.0107161 & 0.0165019 & 0.01319 & 0.0128057 & 0.0240698 & 0.0552376\\
0.0487371 & 0.0191876 & 0.0112571 & 0.0259486 & 0.0128057 & 0.02112 & 0.0320603 & 0.08939\\
0.111758 & 0.032476 & 0.020409 & 0.0400461 & 0.0240698 & 0.0320603 & 0.05262 & 0.13658\\
0.214745 & 0.0819616 & 0.0483898 & 0.109974 & 0.0552376 & 0.08939 & 0.13658 & 0.37854
\end{pmatrix}$$

Eigenvalues $(0.773135\,,0.226895\,,0\,,0\,,0\,,0,0\,,0)$

Qiskit's Quantum info Package was used to generate these reduced density

Results and Methodology Reservations

Probability distribution among the 16 different 4-qubit states prepared on IBM's "***ibmq_london***" quantum computer



- We could, in a sense, try and **infer a pre-measurement density matrix** for this result, although this is misleading, since we are using distributions in the classical register's totally mixed state, to infer some hypothetical pre-measurement pure state.
- This is not ideal, since we are ignoring the mixedness introduced by the gates and possible measurement errors and instead trying to cast it as some would-be pure state's matrix.
- However, we propose that this hypothetical inferred pure state would still **carry some sense of loss of entanglement information** from **experimental decoherence** by considering some "would be pure state" from our classical register, and we counted on that to **quantify the decoherence indirectly**.

Decoherence analysis & conclusion

Now to get mutual information $I(A;B)$:

$$I(A;B) = S\left(\rho^{(/A)}\right) + S\left(\rho^{(/B)}\right) - S\left(\rho^{(:AB)}\right)$$

Using Mathematica and Starting with

$$S\left(\rho^{(/A)}\right) = -tr\left(\rho^{(/A)}\log_2\rho^{(/A)}\right)$$

Whilst ignoring the eigenvalues of the order 0.001, We get

$$S\left(\rho^{(/A)}\right) = -\sum_{m_A}\lambda_{m_A}\log_2\,\lambda_{m_A} \sim 0.7725$$

$$S\left(\rho^{(/B)}\right) = -\sum_{m_B}\lambda_{m_B}\log_2\,\lambda_{m_B} \sim 0.7725$$

$$S\left(\rho^{(/AB)}\right) = -\sum_{m_{AB}}\lambda_{m_{AB}}\log_2\,\lambda_{m_{AB}} \sim 0$$

$$I(A;B)_{IBM} = 1.5451$$

$$I(A;B)_{Ideal} = 1 + 1 - 0 = 2$$

Conclusion:

The MPS quantum circuit, experienced an average of **23% reduction in correlation** which we attribute to decoherence.

REFERENCES

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