Entanglement Entropy and Mutual Information of a Matrix Product State Circuit for a 4-qubit GHZ State Generated on an IBM Quantum Computer

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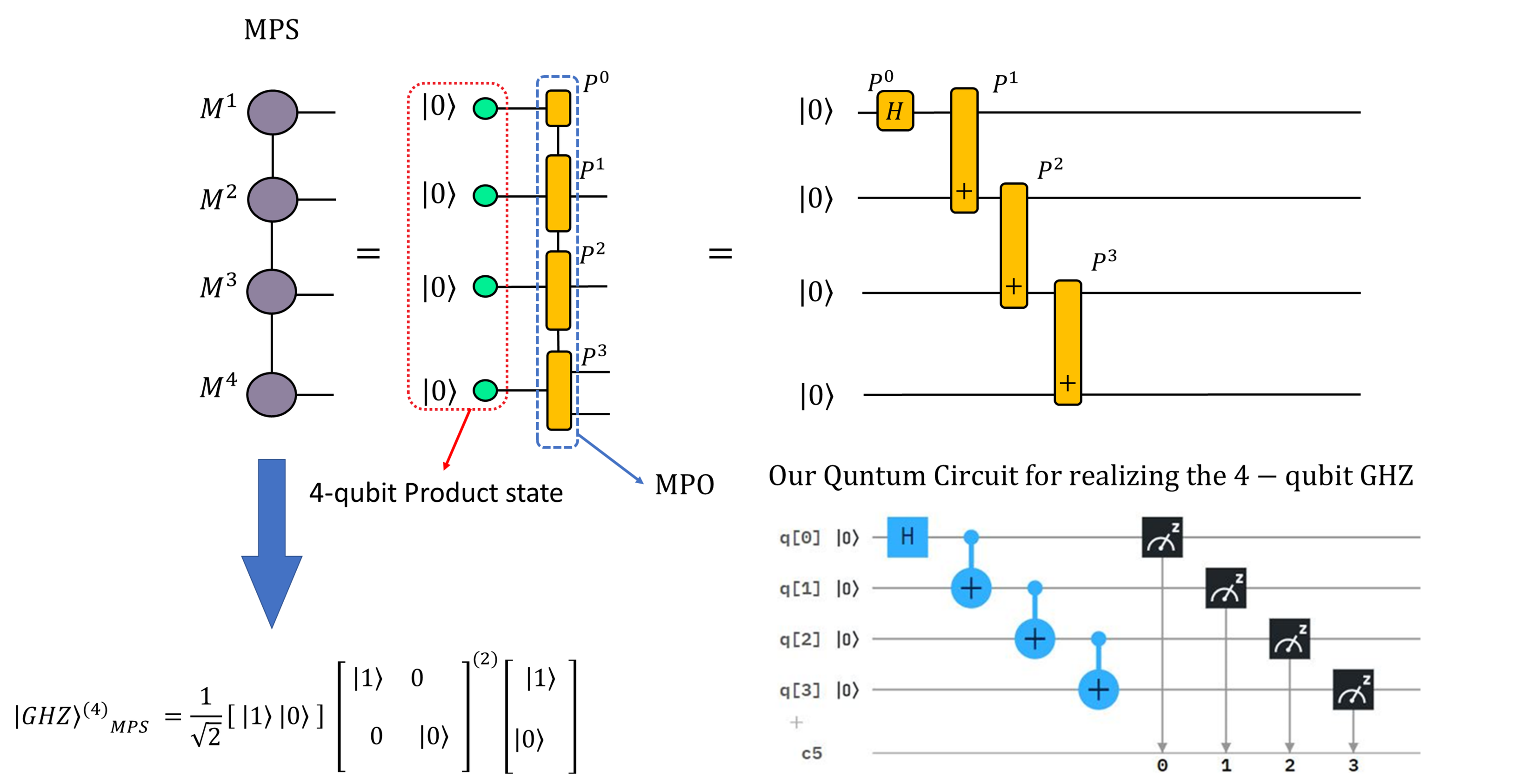
Abstract

It is well established that the use of Matrix Product States (MPS's), given reasonable bond dimensions (χ), reduce the computational cost of running numerical operations on otherwise exponentially large multi-body quantum state vectors. There has been much focus recently on the one-to-one mapping between certain tensor networks and quantum circuits. This mapping perceives quantum circuits as quantum computer networks which manipulate multi-qubit product states using local unitary gates. Advantages of MPS's can extend to quantum circuit design, and since quantum computers also have memory requirements and computational runtimes, the cost of operation becomes a determining factor in the success of a circuit. Here we demonstrate this mapping, by running a trivial MPS quantum circuit on the "ibmq_london" quantum computer to reproduce the, MPS representable, 4-qubit GHZ state. 8192 runs of the MPS circuit were used to obtain a corresponding distribution. We can infer a pre-measurement density matrix for such a distribution of classical bits through quantum state tomography, then use our matrix to indirectly quantify the decoherence relative to the ideal state. This was easier done on a simulator given job-size limitations with the cloud service, so we repeated the runs using the "ibmq_qasm_simulator", while applying a measurement noise model, and ran the circuit, once with, and another time without error correction methods. With error correction, the fidelity was ~0.166, while with error correction circuits, the fidelity rose to ~ 0.725. With our tomographic and ideal density matrices, quantified the loss of entanglement by referencing their mutual information. Accordingly, the trivial MPS circuit experienced an average of 5.7% reduction in correlation following error correction, indicating an equivalent loss of information we attribute to decoherence.

Analogy of MPS's to Quantum Circuits

Figure(1): Analogy between MPS's and MPO's and our 4-qubit GHZ state

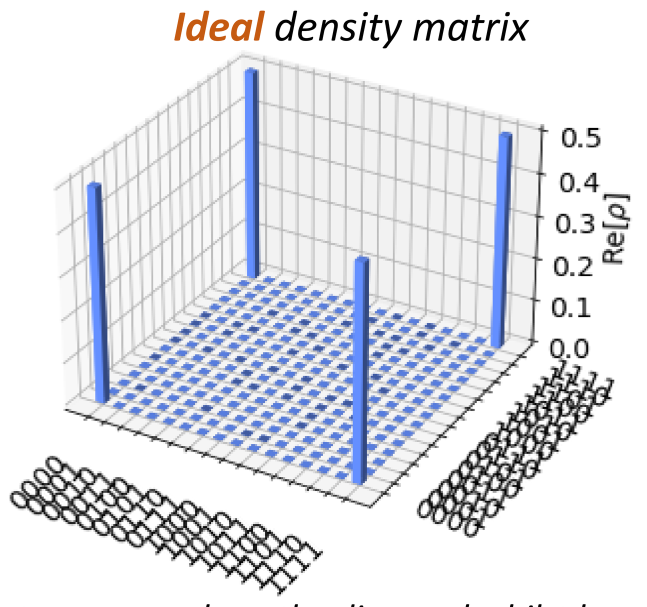


Analysis: Hypothetical Density Matrix

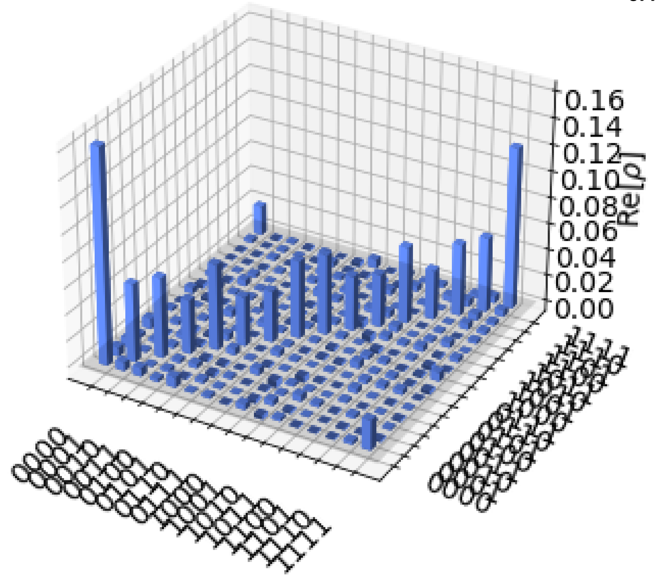
Figure(3): The evolution of the Tomographic ρ with error correction vs the Ideal ρ

Ideal *density matrix*

Qiskit's was used to generate these city plots

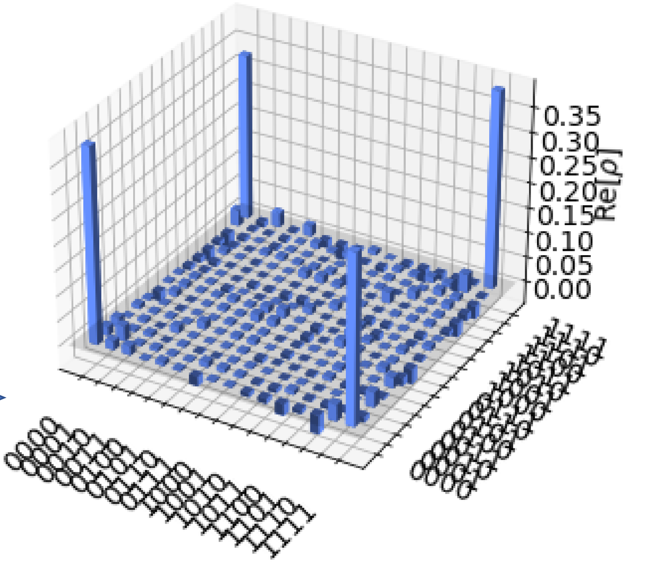


decoherence forces states to emerge along the diagonal whilst lowering the distributions amongst the off-diagonal elements



off-diagonal elements regain their values in favor of decreasing diagonal expression

error correction



Tomographic ρ ***before error correction***

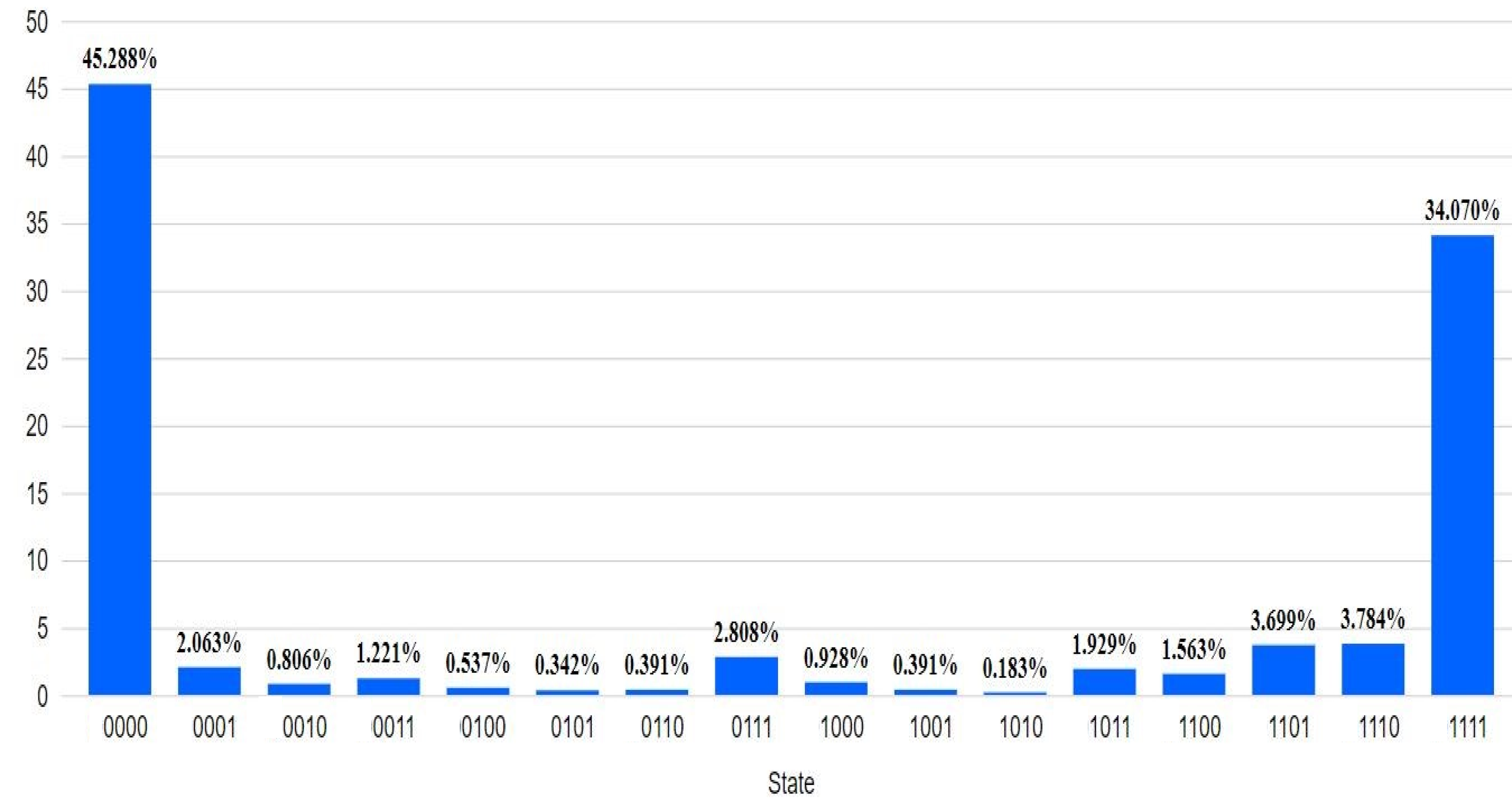
State Fidelity: 0.16564

Tomographic ρ ***after error correction***

State Fidelity: 0.72549

Results and Methodology

Figure(2):
Probability distribution among the 16 different 4-qubit states prepared on IBM's "***ibmq_london***" quantum computer



- We could **infer a pre-measurement density matrix** for such a measurement distribution of classical bits through **quantum state tomography**, then use our matrix to quantify the decoherence away from the ideal state indirectly.

- The tomographic density matrix would then be directly compared to the ideal density matrix. We expect noise models to induce a **loss of entanglement information** from **experimental** decoherence, one we could quantify using a quantifier of correlations that works even in mixed states. The obvious candidate would be **Quantum Mutual Information (QMI)**. By comparing QMI of the ideal and tomography density matrices, using the first qubit as subsystem A and qubits 2,3 and 4 as subsystem B, we were able to **quantify the decoherence indirectly**.

Decoherence Analysis & Conclusion

Now to get mutual information $I(A;B)$:

$$I(A;B) = S\big(\rho^{(/A)}\big) + S\big(\rho^{(/B)}\big) - S\big(\rho^{(:AB)}\big)$$

Using Mathematica and Starting with

$$S\big(\rho^{(/A)}\big) = -tr\big(\rho^{(/A)}\log_2 \rho^{(/A)}\big)$$

Whilst ignoring the eigenvalues of the order 0.001, We get

$$S\big(\rho^{(/A)}\big) = -\sum_{m_A} \lambda_{m_A}\log_2\ \lambda_{m_A} \sim 1$$

$$S\big(\rho^{(/B)}\big) = -\sum_{m_B} \lambda_{m_B}\log_2\ \lambda_{m_B} \sim 1.7889$$

$$S\big(\rho^{(/AB)}\big) = -\sum_{m_{AB}} \lambda_{m_{AB}}\log_2\ \lambda_{m_{AB}} \sim 1$$

$$I(A;B)_{IBM} = 1.7889$$

b)

$$I(A;B)_{Ideal} = 1 + 1 - 0 = 2$$

Conclusion:

The MPS quantum circuit, experienced an average of **10.6% reduction in correlation** which we attribute to decoherence.

REFERENCES

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