Tutorial 7 : problem Solving 5

Reviewing Teleportation and Deutsch Algorithm

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QUESTION 1:

Given messages can only contain 0,1 or 2, if Alice wanted to send the message ‘220’ to Bob, can she use our superdense coding protocol to form an advantageous communication channel ? Why or How

Ans 1:

Use the encoding:

$0: 00$

$1: 01$

$2: 10$

So to send 220, Alice would need 3 qubits and so we need3 entangled states

And we would see them communicating:

$10;\ 10;\ 00$

QUESTION 2:

a) we can create multiples copies of the same qubit using quantum teleportation.

b) If we measure the second qubit, can we always decide the value of the first qubit?

$$\frac{1}{\sqrt{3}}(|010\rangle + |101\rangle + |011\rangle)$$

Can we always determine the state of the third qubit ?

Ans 2:

a) NO, since the original qubit is destroyed in the process

b) Yes , it’s anticorrelated

$$\frac{1}{\sqrt{3}}(|010\rangle + |101\rangle + |011\rangle)$$

Can we determine the state of the third qubit ? … NO, since if the second qubit was measured to be 1 it the state could have collapsed into either ($|010\rangle$ or $|011\rangle$) where it still has a 50/50 chance of being either 1 or 0

QUESTION 3:

Given 2 qubits, how many variables (real numbers) do we need to fully specify the combined state

Ans 3:

$$\psi^{AB} \ = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$a, b, c, d \in C^1$$

So without further constraints we would generally get $2^n \times 2 = 2^{n+1} = 8 \ for\ 2\ qubits$

But we still have 2 extra constraints, the general phase and the normalization:

General phase:

$$a, b, c, d = r_a e^{i\phi_a}, r_b e^{i\phi_b}, r_c e^{i\phi_c}, r_d e^{i\phi_d}$$

given $\psi = e^{i\phi}(\psi')$; ψ and ψ' are essentially the same for our purposes

$$\psi^{AB} \ = r_a e^{i\phi_a}|00\rangle + r_b e^{i\phi_b}|01\rangle + r_c e^{i\phi_c}|10\rangle + r_d e^{i\phi_d}|11\rangle$$

$$e^{\phi_a}\left(r_a|00\rangle + r_b e^{i(\phi_b - \phi_a)}|01\rangle + r_c e^{i(\phi_c - \phi_a)}|10\rangle + r_d e^{i(\phi_d - \phi_a)}|11\rangle\right)$$

Ans 3:

$$Normalization:$$

$$a^2 + b^2 + c^2 + d^2 = r_a^2 + r_b^2 + r_c^2 + r_d^2 = 1$$

$$r_a = \sqrt{1 - (r_b^2 + r_c^2 + r_d^2)}$$

So

$$\psi^{AB} = e^{i\phi_a}\left(\sqrt{1 - (r_b^2 + r_c^2 + r_d^2)}\,|00\rangle + r_b e^{i(\phi_b - \phi_a)}|01\rangle + r_c e^{i(\phi_c - \phi_a)}|10\rangle + r_d e^{i(\phi_d - \phi_a)}|11\rangle\right)$$

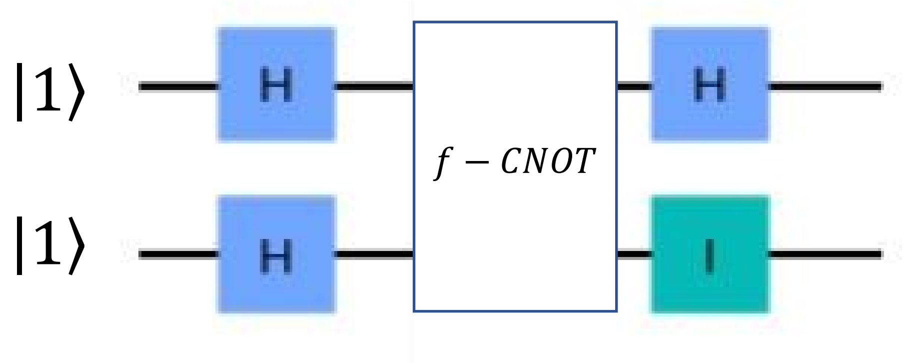
We get 6 variables

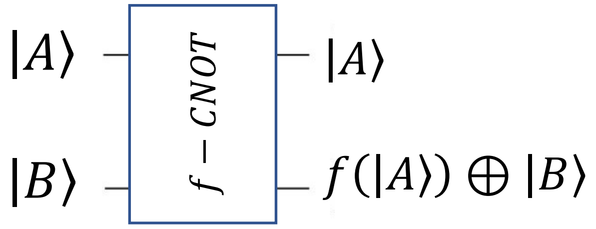
General equation for any n would be : $2^{n+1} - 2$

QUESTION 4:

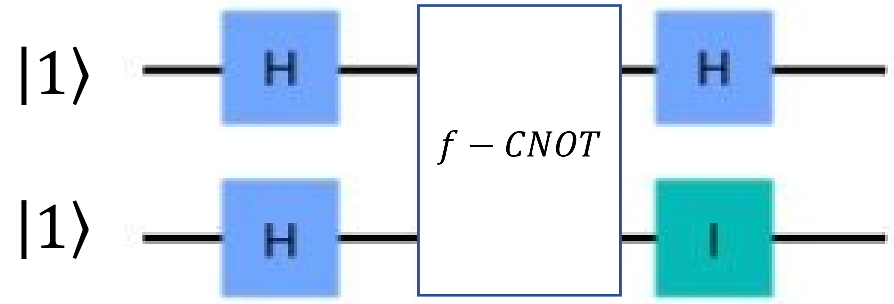
Could you classically solve the Deutsch problem by evaluating the function once ?

If on measuring the first qubit the output is 1 what does that say about our binary function f given the following definition of $f - CNOT$; (SHOW YOUR WORKING):





Ans 4:



$$\psi_1 = \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = \frac{1}{2}\left(|00\rangle - |01\rangle - |10\rangle + |11\rangle\right)$$

$$\psi_2 = \frac{1}{2}\left(|0\, f(0)\rangle - |0\ \overline{f(0)}\rangle - |1\, f(1)\rangle + |1\ \overline{f(1)}\rangle\right)$$

If $f(0) = f(1)$

2 options

$$\left.\begin{array}{l} \psi = \frac{1}{2}\left(|0\rangle - |1\rangle\right) \otimes \left(|f(0)\rangle - |\overline{f(0)}\rangle\right) \\ \psi = \frac{1}{2}\left(|0\rangle + |1\rangle\right) \otimes \left(|f(0)\rangle - |\overline{f(0)}\rangle\right) \end{array}\right\} \text{Apply } H^1$$

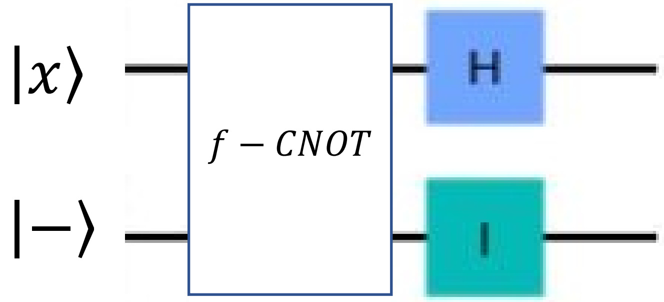
$$\psi = \frac{1}{2}|1\rangle \otimes \left(|f(0)\rangle - |\overline{f(0)}\rangle\right)$$

$$\psi = \frac{1}{2}|0\rangle \otimes \left(|f(0)\rangle - |\overline{f(0)}\rangle\right)$$

If $f(0) = \overline{f(1)}$

Ans 4:

GOAL: using phase kickback with quantum parallelism we want to act on both $f(0)$ and $f(1)$ in the first register.



By setting the target to a superposition state we get :

$$U_f(|x\rangle|-\rangle) = U_f\left(|x\rangle\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)\right) = \frac{U_f(|x\rangle|0\rangle) - U_f(|x\rangle|1\rangle)}{\sqrt{2}}$$

$$= \frac{|x\rangle|0\rangle \oplus f(x)\rangle - |x\rangle|f(x) \oplus 1\rangle}{\sqrt{2}}$$

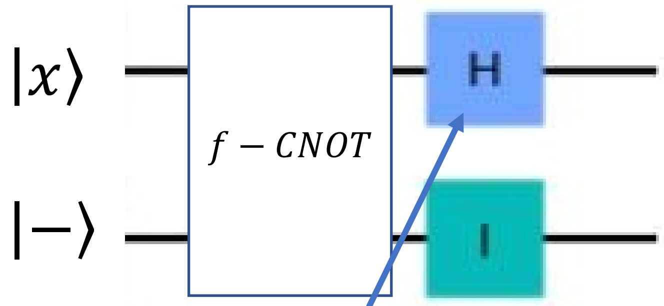
$$= \frac{|x\rangle|f(x)\rangle - |x\rangle|\overline{f(x)}\rangle}{\sqrt{2}} = |x\rangle\left(\frac{|f(x)\rangle - |\overline{f(x)}\rangle}{\sqrt{2}}\right)$$

$$U_f(|x\rangle|-\rangle) = |x\rangle \begin{cases} \dfrac{|0\rangle - |1\rangle}{\sqrt{2}}, & when\ f(x) = 0 \\ \dfrac{|1\rangle - |0\rangle}{\sqrt{2}}, & when\ f(x) = 1 \end{cases}$$

$$= |x\rangle(-1)^{f(x)}|-\rangle$$

Ans 4:

By setting the control to a superposition state we get :



$$U_f(|+\rangle|-\rangle) = U_f\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{U_f(|0\rangle|-\rangle) + U_f(|1\rangle|-\rangle)}{\sqrt{2}}$$

$$= \frac{(-1)^{f(0)}|0\rangle|-\rangle + (-1)^{f(1)}|1\rangle|-\rangle}{\sqrt{2}}$$

$$= \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}|-\rangle$$

So f is a balanced function

If $f(0) = \overline{\mathrm{f}(1)}$ we get a relative phase and the state $|-\rangle$
If $f(0) = f(1)$ we get a global phase and the state $|+\rangle$

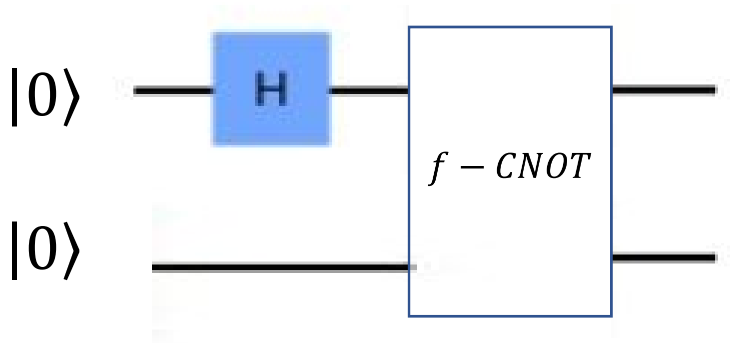
So f is a constant function

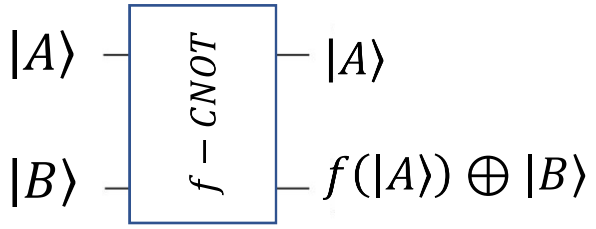
Apply H^1 to measure in the non standard basis $|+/-\rangle$

We can differentiate the 2 states and accordingly determine whether the function was balanced or constant by simply measuring the first qubit

QUESTION 5:

Given the circuit, if on measuring the first qubit the output is 0 what does that say about our binary function f given the following definition of $f - CNOT$; (SHOW YOUR WORKING):





Ans 5:

$$\psi_1 = \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

$$\psi_2 = \frac{1}{2}(|0\ f(0)\rangle + |1\ f(1)\rangle)$$

$$\text{IF}\ \ f(0) = 1\ ; f(1) = 0$$

$$\psi = \frac{1}{2}(|01\rangle + |10\rangle)$$

First Qubit can be in any state, no info inferred

QUESTION 6:

Describe phase kickback giving an example

Ans 5:

Phase kickback is the process where the eigenvalue added by a gate to a qubit is 'kicked back' into a different qubit via a controlled operation. Since our gates are unitary, that eigenvalue will be a phase.

Consider $X|-\rangle = -|-\rangle$ .. This adds a global phase to the state, but now add a control gate :

$$\mathrm{CNOT}|0-\rangle = |0-\rangle$$

$$\mathrm{CNOT}|1-\rangle = -|1-\rangle$$

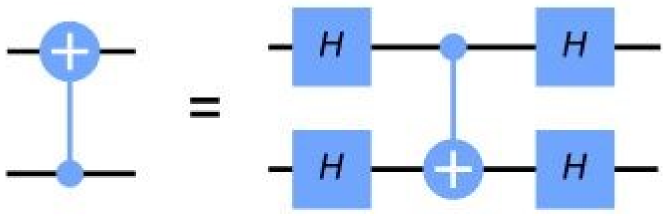
What if the control qubit is in superposition:

$$\mathrm{CNOT}|+-\rangle = \frac{1}{\sqrt{2}}(\mathrm{CNOT}|0-\rangle + \mathrm{CNOT}|1-\rangle)$$

$$= \frac{1}{\sqrt{2}}(|0-\rangle - |1-\rangle) = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes |-\rangle = |--\rangle$$

Ans 5:

This can be understood as the reverse $CNOT$:



What if our second qubit was 0 :

$$\mathrm{CNOT}|++\rangle = \frac{1}{\sqrt{2}}(\mathrm{CNOT}|0+\rangle + \mathrm{CNOT}|1+\rangle)$$

$$= \frac{1}{\sqrt{2}}(|0+\rangle + |1+\rangle) = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |+\rangle = |++\rangle \ldots \textit{nothing happens}$$