TDs 3 and 4

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Problem 1. (2.6.A)
Show that $(a_n) = \left(\frac{n\cos^n(n)}{\sqrt{n^2+2n}}\right)_{n=1}^{\infty}$ *has a convergent subsequence.*

Problem 2. (2.6.B)
Does the sequence $(b_n) = \left(n + \cos(n\pi)\sqrt{n^2+1}\right)_{n=1}^{\infty}$ *have a convergent subsequence?*

Problem 3. (2.6.D)
Show that every sequence has a monotone subsequence.

Problem 4. (2.6.H)
Let $(x_n)_{n=1}^{\infty}$ *be a sequence of real numbers. Suppose that there is a real number* L *with the property that every subsequence* $(x_{n_k})_{k=1}^{\infty}$ *has a subsubsequence* $\left(x_{n_{k(l)}}\right)_{l=1}^{\infty}$ *with*

$$\lim_{l\to\infty} x_{n_{k(l)}} = L.$$

Show that the whole sequence converges to L.
Hint. *If it were false, then you could find a subsequence bounded away from* L.

Problem 5. (2.6.J)
Suppose that $(x_n)_{n=1}^{\infty}$ *is a sequence of real numbers. If* $L = \liminf x_n$, *show that there is a subsequence* $(x_{n_k})_{k=1}^{\infty}$ *so that* $\lim_{k\to\infty} x_{n_k} = L$

Problem 6. (2.7.A)
Give an example of a sequence (a_n) *such that* $\lim_{n\to\infty} |a_n - a_{n+1}| = 0$, *but the sequence does not converge.*

Problem 7. (2.7.G)
Evaluate the continued fraction $1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\cdots}}}}}$.