

1  $\rightarrow$  f(x)

$$f(x) = \frac{x^{3-1} (1-x)^{\frac{1}{2}-1}}{B(3, \frac{1}{2})} \cdot \mathbb{1}_{[0 \leq x \leq 1]} \quad f \sim \text{Beta}(3, \frac{1}{2}) \quad \text{und } \mathbb{1}_{[0 \leq x \leq 1]}$$

$$g(x) = 1 \cdot \mathbb{1}_{[0 \leq x \leq 1]} \quad g \sim U(0, 1) \quad \text{und } \mathbb{1}_{[0 \leq x \leq 1]}$$

$0 \leq x \leq 1$        $\ell$        $\geq 1$        $\therefore$   $\text{Beta} \subset \text{U}(0, 1)$        $\rightarrow$   $\text{Beta}$

$$c = \max_x \{ f(x) / g(x) \} = \max_x \left\{ \frac{\frac{x^{3-1} (1-x)^{\frac{1}{2}-1}}{B(3, \frac{1}{2})}}{1} \right\} =$$

$$= \frac{1}{B(3, \frac{1}{2})} \max_x \left\{ x^2 \cdot (1-x)^{-\frac{1}{2}} \right\} = \frac{1}{B(3, \frac{1}{2})} \max_x h(x)$$

$$\begin{aligned} \frac{\partial h}{\partial x} &= 2x(1-x)^{-\frac{1}{2}} + \frac{1}{2}x^2(1-x)^{-3} = \frac{2x}{(1-x)^{\frac{1}{2}}} + \frac{x^2}{2(1-x)^{\frac{3}{2}}} = \frac{4x(1-x) + x^2(1-x)^{-1}}{2(1-x)^2} \\ &= \frac{(1-x)^{\frac{1}{2}} [4x(1-x) + x^2]}{2(1-x)^{\frac{3}{2}}} = \frac{4x + 3x^2}{2(1-x)^{\frac{3}{2}}} \end{aligned}$$

$$\frac{4x + 3x^2}{2(1-x)^{\frac{3}{2}}} = 0 \Rightarrow 4x + 3x^2 = 0 \Rightarrow x = \frac{4}{3} \quad \text{of } \text{all}$$

$0 \leq x \leq 1$        $1 < \frac{4}{3}$        $\ell$        $\geq 1$

$\forall x \in [0, 1]$   $\text{d}f(x) \text{ ist } \text{negativ}$   $\rightarrow$   $h(x)$   $\text{ist}$   $\text{monoton}$   $\downarrow$   $x=0$   $\text{und}$   $x=1$

$x=1$

$$\text{zu } \ell \quad c = \frac{f(1)}{g(1)} = f(1) = \frac{1}{B(3, \frac{1}{2}) \cdot \sqrt{1-1}} , \text{ of}$$

$\rightarrow$   $f(x)$   $\text{ist}$   $\text{negativ}$   $\leftarrow$   $\text{für } x = 1 = \frac{1}{2}$   $\text{ist}$   $f(1) = 0$

$h(0, 1) \sim U(0, 1)$   $\text{und}$   $B(3, \frac{1}{2}) \sim \text{Beta}(3, \frac{1}{2})$

(ii) . /c

$$f(x) = \frac{x^{3-1}(1-x)^{4-1}}{B(3,4)} \quad 0 \leq x \leq 1 \quad f - \text{Beta}(3,4) \text{ von } \underline{\underline{12/2}}$$

$$g(x) = 1 \cdot \mathbb{1}_{\{0 \leq x \leq 1\}} \quad g - U(0,1) \text{ von } \underline{\underline{12/2}}$$

$0 \leq x \leq 1$  der Fall  $\rightarrow$  nur  $x$  von  $0$  bis  $1$

$$c = \max_x \{ f(x)/g(x) \} = \max_x \left\{ \frac{\frac{x^{3-1} \cdot (1-x)^{4-1}}{B(3,4)}}{1} \right\} = \\ = \frac{1}{B(3,4)} \max_x \left\{ x^2 \cdot (1-x)^3 \right\} = \frac{1}{B(3,4)} \max_x h(x)$$

$$\frac{\partial h}{\partial x} = \frac{1}{B(3,4)} (2x(1-x)^3 - 3x^2(1-x)^2) = \frac{1}{B(3,4)} ((1-x)^2 [2x(1-x) - 3x^2]) =$$

$$= \frac{1}{B(3,4)} (1-x)^2 (2x - 5x^2) = x(1-x)^2 (2 - 5x) \cdot \frac{1}{B(3,4)}$$

$$x \quad 0 \quad 0.2 \quad 0.4 \quad 0.5 \quad 1$$

$$X=0, \quad X>1, \quad X=0.4 \quad \text{d.h. von } \underline{\underline{12/2}}$$

$h(x)$	$\nearrow$	$\max$	$\searrow$
$h(x)$	0	+	0

$$X=0.4 \quad \text{d.h. von } \underline{\underline{12/2}} \quad h(x) \quad [0,1] \quad \text{d.h. pr}$$

$$c = \frac{f(0)}{h(0)} = f(0.4) = \frac{0.4^2 \cdot 0.6^3}{B(3,4)} = \frac{0.16 \cdot 0.216}{B(3,4)} = \frac{0.03456}{B(3,4)} = 2.0736 \quad \text{pr}$$

$$B(3,4) = \frac{\Gamma(3) \cdot \Gamma(4)}{\Gamma(7)} = \frac{2! \cdot 3!}{6!} = \frac{2 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = \frac{1}{3 \cdot 4 \cdot 5} = \frac{1}{60}$$

(i) .2

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \quad 0 \leq x \leq 1 \quad f \sim \text{Beta}(\alpha, \beta)$$

$$g(x) = \alpha \beta x^{\alpha-1}(1-x^{\beta})^{\alpha-1} \quad g \sim \text{kumaraswamy}(\alpha, \beta)$$

$0 \leq x \leq 1 \quad \text{let } x^{\beta} = t \quad \text{then } x = t^{1/\beta}$

$$c = \max_x \left\{ f(x)/g(x) \right\} = \max_x \left\{ \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\frac{\alpha}{\beta} x^{\alpha-1}(1-x^{\beta})^{\alpha-1}} \right\} =$$

$$\max_x \frac{(1-x)^{-1/\beta}}{1.5 \cdot B(3, \beta)} = \max_x \frac{\frac{1}{(1-x)^{\beta}}}{\frac{1.5 \cdot B(3, \beta)}{(1-x^3)^{\beta}}} = \frac{(1-x^3)^{\beta}}{1.5 \cdot B(3, \beta) \cdot (1-x)^{\beta}} =$$

$$= \frac{1}{1.5 \cdot B(3, \beta)} \cdot \max_x \frac{(1-x^3)^{\beta}}{(1-x)^{\beta}} = \frac{1}{1.5 \cdot B(3, \beta)} \max_x h(x)$$

$$\frac{\partial h}{\partial x} = \frac{1/2 (1-x^3)^{-1/2} (-3x^2)(1-x)^{\beta-1} - 1/2 (1-x)^{-1/2} (1-x^3) \cdot (1-x^3)^{\beta-1}}{1-x} =$$

$$= \frac{-1.5x^2(1-x^3)^{-1/2}(1-x)^{\beta-1} + 1/2(1-x)^{-1/2}(1-x^3)^{\beta-1}}{1-x}$$

$$\frac{-1.5x^2(1-x^3)^{-1/2}(1-x)^{\beta-1} + 1/2(1-x)^{-1/2}(1-x^3)^{\beta-1}}{1-x} = 0 \Rightarrow$$

$$\Rightarrow \frac{-1.5x^2(1-x)^{\beta-1}}{(1-x^3)^{\beta-1}} + \frac{1/2(1-x^3)^{\beta-1}}{(1-x)^{\beta-1}} = 0 \Rightarrow \frac{-1.5x^2(1-x) + 1/2(1-x^3)}{(1-x^3)^{\beta-1} (1-x)^{\beta-1}} = 0$$

$$\Rightarrow -1.5x^2 + 1.5x^3 + 1/2 - 1/2x^3 = 0 \Rightarrow x^3 - 1.5x^2 + 1/2 = 0 \Rightarrow$$

$$\frac{x^3 - 0.5x^2 - 0.5}{x^3 - 1.5x^2 + 0.5} |(x-1) \quad \text{if } x=1 \text{ then } \text{not sol of eq}$$

$$\frac{x^3 - x^2}{-0.5x^2 + 0.5x} \Rightarrow (x-1)(x^2 - 0.5x - 0.5) = 0$$

$$\frac{-0.5x^2 + 0.5x}{-0.5x + 0.5} \quad X_{2,3} = \frac{0.5 \pm \sqrt{0.25 + 2}}{2} = \frac{0.5 \pm \sqrt{2.25}}{2} = \frac{0.5 \pm 1.5}{2} \quad \begin{array}{l} 1 \\ -1/2 \end{array}$$

$$\Rightarrow (x-1)^2 (x + 1/2) = 0$$

$$x \quad -1 \quad -\frac{1}{2} \quad 0 \quad 1$$

$$h(x) \quad \nearrow \quad \nearrow \max$$

$$h'(x) = 0 + 0$$

$$\text{at } x=1 \text{ do we have } h(x) \in [0,1] \text{ and } p \text{ is}$$

between 0 and 1  
 $C = \frac{f(x)}{g(x)} \Rightarrow x=1 \text{ is } 3 \text{ and } 1 < C < 2$

$$C = \frac{1}{1.5 D(3, \frac{1}{2})} \cdot \sqrt{\frac{1-x^3}{1-x}} = \frac{1}{1.5 D(3, \frac{1}{2})} \cdot \sqrt{x^2 x + 1}$$

$$\begin{aligned} & \frac{x^2 + x + 1}{x^3 + 0x^2 + 0x + 1} \Big|_{x=1} \\ & \frac{x^2 + x^2}{x^3 + x^2} \\ & \frac{-x^2 + 0x}{-x^3 - x^2} \\ & \frac{-x^2 + 1}{-x^3 - 1} \\ & \frac{0}{0} \end{aligned}$$

$$C = \frac{\sqrt{3}}{1.5 D(3, \frac{1}{2})}$$

l bsp 1 23)

$$D(3, \frac{1}{2}) = \int_0^1 x^2 (1-x)^{-\frac{1}{2}} dx = \left[ \begin{array}{l} u = x \\ u' = 1 \\ u = 2x \end{array} \quad \begin{array}{l} v = (1-x)^{\frac{1}{2}} \\ v' = \frac{(1-x)^{-\frac{1}{2}}}{-0.5} \\ v = -0.5 \end{array} \right] =$$

$$= -2x^2 (1-x)^{\frac{1}{2}} \Big|_0^1 - \int_0^1 -4x(1-x)^{\frac{1}{2}} dx = \int_0^1 x(1-x)^{\frac{1}{2}} dx = \left[ \begin{array}{l} u = x \\ u' = 1 \\ u = 1 \end{array} \quad \begin{array}{l} v = (1-x)^{\frac{1}{2}} \\ v' = \frac{(1-x)^{-\frac{1}{2}}}{-0.5} \\ v = -0.5 \end{array} \right] :$$

$$= -\frac{2}{3} x(1-x)^{\frac{3}{2}} \Big|_0^1 + \frac{4}{3} \int_0^1 (1-x)^{\frac{3}{2}} dx = \frac{2}{3} \int_0^1 (1-x)^{\frac{3}{2}} dx = \frac{2}{3} \cdot \frac{(1-x)^{\frac{5}{2}}}{-\frac{5}{2}} \Big|_0^1 :$$

$$\frac{8}{3 \cdot \frac{5}{2}} \cdot 1^{\frac{5}{2}} = \frac{16}{15}$$

$$C = \frac{\sqrt{3}}{1.5 \cdot \frac{16}{15}} = \frac{15 \cdot \sqrt{3}}{1.5 \cdot 16} = 1.082 \quad p \text{ p}$$

$$f(x) = \frac{x^{d-1}(1-x)^{p-1}}{B(d, p)} \quad 0 \leq x \leq 1 \quad f \sim \text{Beta}(3, 4)$$

$$g(x) = d p x^{d-1} (1-x^d)^{p-1} \quad g \sim \text{kumaraswamy}(3, 4)$$

$$0 \leq x \leq 1 \quad \text{let } h(x) = \frac{f(x)}{g(x)} = \frac{x^{3-1} (1-x)^{4-1}}{\frac{B(3, 4)}{x^3} (1-x^3)^{4-1}} = \frac{x^2 (1-x)^3}{\frac{1}{12 \cdot B(3, 4)} \cdot x^2 (x^3-1)^3}$$

$$\frac{\partial h}{\partial x} = \frac{-2x-1}{(x^2+x+1)^2} = \frac{-(2x+1)}{(x^2+x+1)^2}$$

$$x=0 \Rightarrow \frac{\partial h}{\partial x} = -1 \quad \text{at } x=0 \quad h'(0) = -1 \quad h(x) \text{ is decreasing}$$

$$C = \frac{1}{12 \cdot B(3, 4)} = \frac{1}{12 \cdot \frac{2! \cdot 3!}{7!}} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 5$$

2. Aufgabe

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$g(x) = \frac{1}{\pi s} \cdot \left( \frac{s^2}{(x-\mu)^2 + s^2} \right)$$

$N(\mu, \sigma)$

Cauchy( $\mu, s$ )

then 1/2

(i)

then 1/6

$$\text{so } h(x) = \frac{f(x)}{g(x)} \quad c = \max_x \left\{ \frac{f(x)}{g(x)} \right\} \quad \text{in case}$$

$c$  is the maximum value of  $h(x)$

we have to find  $x$  such that  $h(x) = c$  and  $h'(x) = 0$

$$c = \max_x \left\{ \frac{f(x)}{g(x)} \right\} = \max_x \left\{ \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}{\frac{1}{\pi s} \left( \frac{s^2}{x^2 + s^2} \right)} \right\} = \max_x \frac{e^{-\frac{x^2}{2}}}{\frac{1}{\sqrt{2\pi}}} \cdot \frac{\pi(x^2 + s^2)}{s} =$$

$$\frac{\sqrt{\pi}}{s\sqrt{2}} \cdot \max_x \left\{ e^{-\frac{x^2}{2}} \cdot (x^2 + s^2) \right\} = \frac{\sqrt{\pi}}{s\sqrt{2}} \cdot \max_x h(x)$$

$$\frac{\partial h}{\partial x} = -x e^{-\frac{x^2}{2}} (x^2 + s^2) + e^{-\frac{x^2}{2}} \cdot 2x = -x e^{-\frac{x^2}{2}} [x^2 + s^2 - 2]$$

$$-e^{-\frac{x^2}{2}} \cdot x \cdot [x^2 + s^2 - 2] = 0 \quad \text{or solve}$$

$$x_1 = 0 \quad x_2 = \sqrt{2-s^2} \quad x_3 = -\sqrt{2-s^2} \quad : \text{also zero values for}$$

$$x=0 \text{ is a local minimum and } x_2, x_3 \text{ are local maxima}$$

$$x = -1 \quad 0 \quad 1$$

$$h(x) \nearrow \searrow \quad 0 \text{ is a local minimum and } x=0 \text{ is a local maximum} \\ h(x) + 0 - \quad c = \frac{e^{-\frac{0^2}{2}}}{\sqrt{2\pi}} \cdot \frac{\pi(0^2 + s^2)}{s} = \frac{\sqrt{\pi}s}{\sqrt{2}}$$

$$0 < s < \sqrt{2} \quad \text{and}$$

$x$	$-2\sqrt{2-s^2}$	$-\sqrt{2-s^2}$	$-\frac{1}{2}\sqrt{2-s^2}$	$0$	$\frac{1}{2}\sqrt{2-s^2}$	$\sqrt{2-s^2}$	$2\sqrt{2-s^2}$
$h(x)$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
$h'(x)$	+	0	-	0	+	-0	-

$$\text{נול } C \text{ ב } \rho \theta \text{ ו } \theta = \pi/2 \quad x = \pm \sqrt{2-s^2} \quad \text{נול}$$

$$C = \frac{e^{\frac{s^2-2}{2}}}{\sqrt{2\pi}} \cdot \frac{\pi(2-s^2)}{s} = \frac{\sqrt{2\pi} \cdot e^{\frac{s^2-2}{2}}}{s}$$

$$C = \sqrt{\pi} \quad s = \sqrt{2} \quad \text{ונר. כ-} \quad \Leftrightarrow \quad C = \frac{s \cdot \sqrt{\pi}}{\sqrt{2}} \quad \Leftrightarrow \quad s \geq \sqrt{2}$$

$$C = \frac{\sqrt{2\pi} \cdot e^{\frac{s^2-2}{2}}}{s} \quad \Leftrightarrow \quad 0 < s < \sqrt{2}$$

הנימוק כפוי לערך  $s$

$$\frac{\partial C}{\partial s} = \frac{\sqrt{2\pi} \cdot e^{\frac{s^2-2}{2}} - \sqrt{2\pi} e^{\frac{s^2-2}{2}}}{s^2} = \frac{\sqrt{2\pi} e^{\frac{s^2-2}{2}} (s^2 - 1)}{s^2} = 0 \Rightarrow s = 1$$

$s$ $\downarrow$ $\zeta(s)$	$\frac{1}{2}$ $1$ $\nearrow$ $0$ $\searrow$	$C(s) \rightarrow$ $\zeta(s) \rightarrow$ $C(s) \rightarrow$ $\zeta(s) \rightarrow$ $C(s) \rightarrow$ $\zeta(s) \rightarrow$
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$$C(s) = \sqrt{2\pi} \cdot e^{-\frac{1}{2}}$$

$$1.77 \approx \sqrt{\pi} \quad \text{ונר. כ-} \quad s \geq \sqrt{2} \quad \text{נול}$$

$$1.52 \approx \sqrt{2\pi} \cdot e^{-\frac{1}{2}} \quad \text{ונר. כ-} \quad 0 < s < \sqrt{2} \quad \text{נול}$$

לפי גורם זה נובע כי  $\zeta(s)$  מוגדר רק עבור  $s > 1$ .

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$g(x) = \frac{1}{2s} e^{-\frac{|x-\mu|}{s}}$$

$N(0, 1)$   
double exp  $(0, s)$

(ii)

so  $h \approx g(x)$   $c = \max_x \left\{ \frac{f(x)}{g(x)} \right\}$

$c$  is a small number so we can

approximate by writing  $c$  as  $\frac{1}{c}$  and  $h(x) \approx c g(x)$

$$c = \max_x \left\{ \frac{f(x)}{g(x)} \right\} = \max_x \left\{ \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}{\frac{1}{2s} e^{-\frac{|x|}{s}}} \right\} = \max_x \left\{ \frac{s\sqrt{2}}{\sqrt{\pi}} e^{\frac{|x| - x^2}{2s}} \right\} =$$

$$\frac{s\sqrt{2}}{\sqrt{\pi}} \cdot \max_x \left\{ e^{\frac{|x| - x^2}{2s}} \right\} = \frac{s\sqrt{2}}{\sqrt{\pi}} \max_x h(x)$$

$$\frac{dh}{dx} = \left( \frac{\text{Sign}(x)}{s} - x \right) e^{\frac{|x| - x^2}{2s}} = \begin{cases} \left( \frac{1}{s} - x \right) \left( e^{\frac{|x| - x^2}{2s}} \right) & x > 0 \\ \left( -\frac{1}{s} - x \right) \left( e^{\frac{|x| - x^2}{2s}} \right) & x < 0 \end{cases}$$

			$x = \frac{1}{3}$				
			$x = -\frac{1}{3}$				
$x$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{0.5}{3}$	$\frac{0.5}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	
$h(x)$	$+$	$0$	$-$	$+$	$0$	$-$	
$h'(x)$	$+$	$0$	$-$	$+$	$0$	$-$	

$x = -\frac{1}{3}$  and  $x = \frac{1}{3}$  are roots of  $h'(x)$

$$c = \frac{s\sqrt{2}}{\sqrt{\pi}} \cdot e^{\frac{1}{s^2} - \frac{1}{2s^2}} = \frac{s\sqrt{2}}{\sqrt{\pi}} \cdot e^{\frac{1}{2s^2}}$$

$$\frac{\partial \zeta}{\partial s} = \frac{\sqrt{2}}{\sqrt{\pi}} \cdot e^{2s^2} - \frac{s\sqrt{2}}{\sqrt{\pi}} \cdot \sqrt{s} \cdot e^{2s^2} = \frac{\sqrt{2}}{\sqrt{\pi}} e^{2s^2} - \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \frac{1}{s} e^{2s^2} =$$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} e^{2s^2} \cdot \left[ 1 - \frac{1}{s^2} \right]$$

$s = \pm 1$  וריאנטים נוספים בפונקציית זטא

$s = 1$  וריאנט אחד בפונקציית זטא. נורמליזציה כפנית בפונקציית זטא

$s = \frac{1}{2}$  1 2 סדרה:

$\zeta(s) \rightarrow \infty \rightarrow$  פונקציית זטא שולחנית

$\zeta'(s) = 0 +$  סוליד

3. גורם

$$1 = C_i \leftarrow \text{הגדיל } C_i$$

$$y_{(0,1)} = \text{טב} \rightarrow y_1$$

$$y_i \cdot 1 \leq \frac{f(x_i)}{g(x_i)}$$

הגדיל  $\frac{f(x_i)}{g(x_i)} - x_i$   
הנ  $x_i \rightarrow \text{טב}$

$$y_i \cdot 1 > \frac{f(x_i)}{g(x_i)}$$

הגדיל  $x_i \rightarrow \text{טב}$

לפנ  $\frac{f(x_i)}{g(x_i)} > 1$  נזק  $x_i \rightarrow \text{טב}$  קבל  $y_i \rightarrow 1$   
 $\cdot \frac{f(x_i)}{g(x_i)} \rightarrow \text{טב} \leq \frac{f(x_i)}{g(x_i)}$  ולו  $\leq$

הגדיל  $y_i \rightarrow 1$  נזק  $x_i \rightarrow \text{טב}$  קבל  $y_i \rightarrow 1$  ולו  $\leq$

$$\frac{f(x_i)}{g(x_i)} \geq y_i \cdot C$$

$\downarrow$

ונזק  $y_i$  לפנ  $\frac{f(x_i)}{g(x_i)} \geq y_i \cdot C$   
 $x_i \rightarrow \text{טב}$

$$C \text{ נזק פולש נזק } C, \quad \frac{f(x_i)}{g(x_i)} \geq C \quad \{y_i\} \rightarrow \text{טב}$$

ונזק  $x_i \rightarrow \text{טב}$

ונזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק

ונזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק  $\rightarrow$  פולש נזק