GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

ADDITONAL MATHEMATICS 2
0575

JUNE 2022   **ORDINARY LEVEL**

Subject Title	Additional Mathematics
Paper No.	2
Subject Code No.	0575

Two and a half hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS FROM EITHER SECTION B or SECTION C. IN SECTIONS B AND C, ALL QUESTIONS CARRY EQUAL MARKS.

Candidates are expected to answer a combination of Section A **and** Section B **OR** Section A and Section C but **NOT** a combination of all three

All necessary working must be shown. No marks will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic calculators are allowed.

Formulae booklets are allowed.

Where necessary take g as 10ms⁻².

_____ Turn Over

2022/0575/2/Q
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SECTION A: PURE MATHEMATICS
THIS SECTION IS COMPULSORY TO ALL CANDIDATES
(ANSWER ALL QUESTIONS)

1. (i)   Given that $(x + 1)$ is a factor of $f(x)$, where $f(x) = 2x^3 - 3x^2 + kx + 2$.
        (a) Find the value of k.   **(2 marks)**
            With this value of k,
        (b) factorise $f(x)$ completely   **(2 marks)**
    (ii) The quadratic equation $x^2 + 4x - 3 = 0$ has roots α and β.
        (a) Find the values $\alpha + \beta$ and $\alpha\beta$ **(1 mark)**
        (b) Find the value of $\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$.   **(3 marks)**

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2. (i)   Find the number of permutations of the letters of the word "**MAXIMUM**"   **(4 marks)**
    (ii) Find the coefficient of the term in x^4 in the binomial expansion of $\left(x^2 + \frac{1}{2x}\right)^8$ **(4 marks)**

3. The diagrams show a sequence of shapes.

        Diagram 1   Diagram 2   Diagram 3
(a) In your answer booklet, draw Diagram 4.   **(2 marks)**
(b) Copy and complete the table below showing the number of line segments in each diagram.

Diagram (n)	1	2	3	4	5
Number of lines	6	11			

                                                                                     (2 marks)
(c) Find the number of line segments in diagram 7.   **(2 marks)**
    Given that the number of line segments in diagram p is 66.
(d) Find the value of p.   **(2 marks)**

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4. (i) The operation $*$ is defined on the set $S = \{1, 4, 7, 13\}$ where $*$ is multiplication modulo 15.
        (a) Copy and complete the table below.   **(3 marks)**

$*$	1	4	7	13
1	1			
4				7
7		13		
13			1	

        (b) State the identity element.   **(1 mark)**
        (c) State the inverse of each element.   **(1 mark)**
    (ii) The linear transformation, T, is defined by $T: (x, y) \rightarrow (3x + 2y, x + 2y)$.
        Find,
        (a) the image of the point $(-3, -1)$ under the transformation T.   **(2 marks)**
        (b) the invariant line under the transformation T.   **(2 marks)**

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5. A woman has a maximum of 24,000 FCFA to buy x exercise books and y text books for her children.
   a) Given that an exercise book cost 600 FCFA and a text book cost 2400 FCFA. Show that $x + 4y \leq 40$.

   (2 marks)

   b) Given also that:
   - the number of text books is at most one quarter the number of exercise books,
   - the sum of an exercise book and ten text books is at least 40,

   write down two inequalities in terms of x and y that satisfy the conditions in (b) **(2 marks)**

   c) On a graph paper, taking 2 cm to represent 2 units on the y-axis and 5 units on the x-axis, shade, so as to leave unshaded, the region represented by these 3 inequalities. **(2 marks)**

   Given that she bought both exercise books and text books for the children.

   d) From your graph, find the maximum number of books she bought and the amount she spent. **(2 marks)**

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6. (i) Show that $\dfrac{1 - \cos 2x}{1 + \cos 2x} \equiv \tan^2 x$ **(3 marks)**

   (ii) The function $f(x)$ is defined as follows:
   $$f(x) = \cos x - 2\sin x, \; 0 \leq x \leq \pi.$$
   (a) Copy and complete the table below.

x	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π
$f(x)$	1			-2			

   (3 marks)

   Taking 2cm to represent $\dfrac{\pi}{6}$ radian on the x-axis and 4cm to represent 1 unit on the y-axis,

   (b) draw the graph of $y = f(x)$. **(2 marks)**

   (c) From your graph, find the minimum value of $f(x)$. **(1 mark)**

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7. The position vectors of the points A and B are $3i + j$ and i respectively.

   (a) Find the vector equation of the line L_1 passing through A and B in the form $p + tq$, where t is a scalar.

   (2 marks)

   Another line L_2 with vector equation $r = 2j + s(3i - j)$ where s is a scalar intersect with L_1 at point M,
   (b) Find the position vector of M. **(4 marks)**

   (c) Find the cosine of the angle between L_1 and L_2. **(2 marks)**

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8. (i) Differentiate with respect to x, the function $f(x) = x\sin x$. **(3 marks)**

   (ii) Find the area enclosed by the curve $y = 2 + 3x - x^2$, the x-axis and the ordinates $x = 0$ and $x = 2$. **(5 marks)**

Turn Over

SECTION B: MECHANICS

IF THIS SECTION IS CHOSEN, THEN SECTION C MAY NOT BE CHOSEN

(ANSWER ANY TWO QUESTIONS)

9. (i) Two particles A and B moving in the xy-plane such that at time t seconds, A has position vector
$[(2t^2 + 1)i + (2t)j]\, m$ and B has position vector $[t^2 i + (3t - 1)j]\, m$
    a) Calculate the distance between A and B when $t = 1$ **(3 marks)**
    b) Find the velocity of A relative to B when $t = 1$ **(2 marks)**
    c) Show that the acceleration of A is constant   **(1 mark)**

(ii) A ball of mass 2 kg moving of with a speed $6ms^{-1}$ collides with another ball
of mass 4 kg moving in opposite directions with a speed of $3ms^{-1}$.
Given that after impact, A moves with a speed of $3ms^{-1}$, find
    (a) The velocity of B after impact,   **(2 marks)**
    (b) The loss in kinetic energy of the system due to the impact.   **(3 marks)**

(iii)
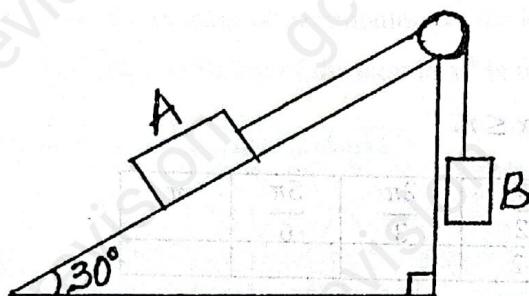
A particle A, of mass 4kg rest on a smooth surface inclined at an angle 30°to horizontal.
It is connected by light inextensible string that passes over a fixed pulley at the top of the plane to a particle,
of mass 3kg hanging freely as shown in the diagram above. Given that the system is released from rest with
the string taut, find,
    (a) the acceleration of the particles,   **(4 marks)**
    (b) the tension in the string.   **(2 marks)**

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10. (i) The rate of increase of the radius of a circle is $0.5cms^{-1}$ at the instant when the radius is 2 cm..
Find:
    (a) the rate at which area of the circle is increasing.   **(3 marks)**
    (b) the rate at which the circumference of the circle is increasing   **(3 marks)**

(ii) The area bounded by the curve $y^2 = 3x - 1$, the x-axis and the ordinates $x = 0$ and $x = 2$ is
rotated completely about the x-axis. Find the volume of the solid generated.   **(5 marks)**

(iii) Find the position vector of the centre of gravity of particles of the mass 3kg, 2kg and 4kgwhich
are at the points with position vectors $i + 2j, 3i - j \ and \ 4i + 5j$ respectively.   **(6 marks)**

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11. (i) The forces $F_1 = i - 3j$, $F_2 = pi + j$ and $F_3 = 5i + 2j$ act on a particle of mass $2kg$.
Given that the resultant of the three forces acting on the particle is $4i + qj$. Find
    (a) the values of p and q;   **(3 marks)**
    (b) the magnitude of the acceleration of the particle.   **(2 marks)**

(ii) A force acts vertically upwards on a particle of mass $2kg$. The particle rises from rest through
a distance of $20m$ at the instant when the speed of the particle is $8ms^{-1}$.
Calculate the work done by the force on the particle.   **(6 marks)**

(iii) A car of mass $1000kg$ starts from rest, accelerates uniformly on a horizontal plane at $0.5\ ms^{-2}$
and covers a distance of $100m$.
    (a) Find the speed of the car at this instant.   **(3 marks)**
        Given that the car develops a power of10KW,
    (b) Calculate the resistance the motion of the car.   **(3 marks)**

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SECTION C: STATISTICS AND PROBABILITY

(IF THIS SECTION IS CHOSEN, THEN SECTION B MAY NOT BE CHOSEN)

IF THIS SECTION IS CHOOSEN, THEN ANSWER ANY TWO QUESTIONS

12. The gain in mass in kilograms, of 100 pigs during a certain period were recorded as follows:

Gain (x kg)	5-9	10-14	15-19	20-24	25-29	30-34
f	2	29	37	16	14	2

   (a) (i) Draw a histogram to show the distribution.   **(5 marks)**
         From the histogram,

        (ii) estimate the mode.   **(3 marks)**

   (b) Find;

        (i) the mean of the distribution,   **(4 marks)**

        (ii) the standard deviation of this distribution   **(5 marks)**

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13.   (i) A discrete random variable X, has probability mass function, p, defined by

$$p(x) = \begin{cases} k(7-x), & x = 0, 1, 2, 3, 4 \\ 0, & otherwise \end{cases}$$

      a) Copy and complete the table below   **(2 marks)**

x	0	1	2	3	4
$P(X = x)$				$4k$	

      Find;
      b) the value of k,   **(2 marks)**
      c) the mean and the variance of X.   **(5 marks)**

   (ii) In a council election, the probability that a person supports party A is $\frac{1}{3}$.

      Given that 5 voters took part in the election.
      Using the binomial distribution, find the probability that,
      a) no voters supported party A,   **(2 marks)**
      b) more than 3 voters supported party A.   **(3 marks)**
      c) Find the mean and variance of the distribution   **(3 marks)**

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14. (i) Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{2}{3}$.

      Find;
      (a) $P(A \cap B)$,   **(2 marks)**
      (b) $P(A/B)$.   **(2 marks)**
      State, with reason, whether A and B are:
      (c) mutually exclusive   **(2 marks)**
      (d) independent   **(2 marks)**

   (ii) A bag contains 3 red and 5 black balls. A ball is drawn from the bag, its colour is noted and it is replaced by a ball of the other colour. A second ball is then drawn from the bag.
      (a) Draw a tree diagram showing all the possible outcomes.   **(3 marks)**
        Hence or otherwise, find the probability that
      (b) the second ball is red   **(3 marks)**
      (c) the balls are of the same colour.   **(3 marks)**