GENERAL CERTIFICATE OF EDUCATION (GCE) BOARD

General Certificate of Education Examination

JUNE 2021   **ORDINARY LEVEL**

Subject Title	Additional Mathematics
Paper No.	2
Subject Code No.	0575

Two and a half hours

Answer ALL QUESTIONS IN SECTION A and ANY TWO QUESTIONS FROM EITHER SECTION B or SECTION C. IN SECTIONS B AND C, ALL QUESTIONS CARRY EQUAL MARKS.

Candidates are expected to answer a combination of Section A **and** Section B **OR** Section A **and** Section C but **NOT** a combination of all three

All necessary working must be shown. No marks will be awarded for answers without brief statements showing how the answers have been obtained.

Electronic calculators may be used.

Where necessary take g as 10ms⁻².

Formulae Booklets may be used.

Turn Over

SECTION A: PURE MATHEMATICS
THIS SECTION IS COMPULSORY TO ALL CANDIDATES
(ANSWER ALL QUESTIONS)

1. (i) Given that $(x + 2)$ is a factor of $f(x)$, where $f(x) = x^3 + 3x^2 + kx + 6$.
    (a) Find the value of k. (2 marks)
    With this value of k,
    (b) Show that $f(x) = (x + 2)(x^2 + x + 3)$. (2 marks)

    (ii) Given that α and β are the roots of $2x^2 + x - 3$,
    (a) find the values of $\alpha + \beta$ and $\alpha\beta$. (2 marks)
    Hence,
    (b) write down another quadratic equation with integral coefficients whose roots
    are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. (2 marks)

2. (i) Find the number of ways of selecting a class prefect boy and a class prefect girl from a class of 4 boys and 5 girls. (4 marks)

    (ii) Write down the first three terms of the binomial expansion of $(1 - 2x)^{-1}$ in ascending powers of x, simplifying your answer as far as possible. (4 marks)

3. The basic salary of a teacher in a school is 100,000 FCFA and his salary is increased monthly by 10%. The table below shows his salary for the first 3 months.

Month	Monthly Increment (I)	Monthly salary (M)
1	$I_1 = 0$	$M_1 = 100,000$
2	$I_2 = 1,000$	$M_2 = 110,000$
3	$I_3 = 1,100$	$M_3 = 121,000$

    (a) Show that $\frac{M_3}{M_2} = \frac{M_2}{M_1}$. (2 marks)

    Hence or otherwise, find:
    b) the salary for the sixth month, (3 marks)
    c) the total salary earned by the teacher within the first 6 months. (3 marks)

4. (i) The binary operation $*$ is defined over the set, $S = \{1, 2, 3, 4\}$ as $x * y = x^y$ modulo 5.
    (a) Copy and complete the operation table below for $(S, *)$. (3 marks)

*	1	2	3	4
1	1		1	1
2	2	4		1
3	3	4	2	
4	4		3	1

    (b) Give a reason why $(S, *)$ does not form a group. [Assume associativity] (1 mark)

    (ii) Given a matrix, $M = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.
    (a) Find the inverse of M. (3 marks)
    Hence or otherwise,
    (b) Find the point whose image is $(5, -1)$ under the transformation represented by the matrix, M. (2 marks)

5. A woman has x cups of beans and y cups of corn to prepare corn chaff. Given that:
    - She must use at least 2 cups of beans and at least 4 cups of corn
    - The quantity of corn she uses must be greater than or equal to the quantity of beans.
    If the quantity of bean and corn that she has to prepare the corn chaff is 20 cups,
    (a) Write down four inequalities in terms of x and y that satisfy these conditions. **(3 marks)**
    On a graph paper, taking 1cm for 2 units on both axes,
    (b) Shade so as to leave unshaded, the region represented by these inequalities. **(3 marks)**
    Given that a cup of beans costs 100 francs and a cup of corn cost 50 francs,
    (c) Find the maximum expenditure that can be incurred by the woman. **(2 marks)**

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6. (i) Solve for θ, in the range, $0° \le \theta \le 360°$, the equation $\sin 2\theta - \cos \theta = 0$. **(3 marks)**
    (ii) The function $f(x) = \sqrt{3} \cos x + \sin x$, where $0 \le x \le 2\pi$.
    (a) Copy and complete the table. **(3 marks)**

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$f(x)$	1.7		1.7	1		-1	-1.7		-1.7	-1	0		1.7

Taking 1cm to represent $\frac{\pi}{6}$ radian units on the x-axis and 2cm to represent 1 unit on the y-axis,
    (b) draw the graph of $y = f(x)$. **(2 marks)**
    From the graph or otherwise,
    (c) write down the maximum value of $f(x)$. **(1 mark)**

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7. The points P and Q have position vectors $3i - 2j$ and $2i + 3j$ respectively.
    (a) Write down the vector equation of the line l_1 passing through P and Q in the form $r = a + tb$, where t is a scalar. **(2 marks)**
    Another line, l_2 has vector equation, $l_2 : r = 2i + 3j + s(5i + j)$, where, s is also a scalar.
    Find:
    (b) the position vector of the point of intersection of the lines l_1 and l_2. **(4 marks)**
    (c) the angle between the lines l_1 and l_2. **(2 marks)**

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8. (i) $f(x) = \begin{cases} -x^2, x < 0 \\ x^2, \ x \ge 0 \end{cases}$
    (a) Define in a similar manner, the gradient function of $f'(x)$ **(2 marks)**
    (b) Evaluate $f'(-2)$. **(2 marks)**
    (ii) Evaluate $\int_0^{\frac{\pi}{4}} (2x + \sin 2x) dx$. **(4 marks)**

SECTION B: MECHANICS
IF THIS SECTION IS CHOSEN, THEN SECTION C MAY NOT BE CHOSEN
(ANSWER ANY TWO QUESTIONS)

9. (i) A particle moves from rest in a straight line from a fixed point O, and after t seconds of its motion, the displacement from O is $x = (t^3 + 4t^2 + 6)m$.
    Find:
    (a) the velocity of the particle when $t = 2$. **(3 marks)**
    (b) the acceleration of the particle when $t = 2$. **(3 marks)**

(ii) A particle P, of mass $3kg$ lies at rest on a smooth plane, inclined at $30°$ to the horizontal. Given that the particle, P is attached to an inelastic string which passes over a smooth fixed pulley at the top of the plane, to another particle, Q of mass 6kg hanging freely. Given that the system is released from rest with the string taut, find:

    (a)   the common acceleration of the particles,   (4 marks)

    (b)   the tension in the string.   (2 marks)

(iii) Two particles S and T of masses $2kg$ and $3kg$ respectively are moving in a straight line with velocities $12\ ms^{-1}$ and $6\ ms^{-1}$ respectively. S collides with T and after collision S moves with a velocity of $6\ ms^{-1}$.

Find:

    (a)   the velocity of T after collision.   (2 marks)

    (b)   the loss in kinetic energy after collision.   (3 marks)

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10. (i) The rate of change of the radius of a circle is $2 cms^{-1}$ at the instant when the radius is $2\ cm$.
Find:

    (a)   the rate of change of the area of the circle,   (3 marks)

    (b)   the rate of change of the circumference of the circle.   (2 marks)

(ii) The area bounded by the curve $y^2 = x^2 + 1$, the x-axis and the ordinates $x = 1$ and $x = 3$ is rotated completely about the x-axis. Find the volume of the solid generated.   (6 marks)

(iii) The position vectors of three particles of masses 2kg, 5kg and 9kg are $(-2i + 3j)$, $(4i - 3j)$ and $(3i + 5j)$ respectively.

Find the position vector of the centre of gravity of these three particles.   (6 marks)

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11. (i) The forces $F_1 = (pi + j)N$, $F_2 = (2qi + 3pj)N$ and $F_3 = (i + qj)N$ act on a particle of mass $2kg$.

    (a)   Find the resultant of the three forces in terms of p and q.   (3 marks)

Given that a fourth force, $F_4 = (6i)N$ is added to the system and the system is at equilibrium.

    (b)   Find, the values of p and q.   (5 marks)

(ii) A car of mass $2000kg$ has a maximum speed of $10\ ms^{-1}$ up a slope inclined at an angle θ to the horizontal, where, $\sin \theta = 0.4$, against a constant resistance of $2000\ N$.

    (a)   Find the power generated by the engine of the car.   (4 marks)

Given that the power generated and the resistance remain unchanged,

    (b)   Find the acceleration of the car on a level road at the instant when the speed is $20\ ms^{-1}$.   (5 marks)

SECTION C: STATISTICS AND PROBABILITY
IF THIS SECTION IS CHOSEN, THEN SECTION B MAY NOT BE CHOSEN
(ANSWER ANY TWO QUESTIONS)

12. The marks scored by 100 students in an examination are distributed as follows:

Marks (x)	1 - 5	6 -10	11-15	16-20	21-25	26-30	31-35	36 - 40
Number of students (f)	8	10	12	20	23	12	9	6

    (a) Draw a cumulative frequency graph of the distribution.   (5 marks)

    From your graph, estimate:

    (b) the median,   (3 marks)

    (c) the semi-interquartile range.   (4 marks)

    (d) Find the mean of the distribution.   (5 marks)

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13. (i) A discrete random variable, X has probability mass function, p defined by

$$p(x) = \begin{cases} \frac{(x+1)}{k}, & for\ x = 0,1,2,3,4 \\ 0, & elsewhere \end{cases}$$

where, k, is a constant.

    (a) Copy and complete the distribution table.   (2 marks)

x	0	1	2	3	4
P(X = x)	$\frac{1}{k}$		$\frac{3}{k}$		

Find :

    (b) the value of the constant k.   (2 marks)

    (c) the mean and variance of X.   (5 marks)

  (ii) A random variable, X is such that $X \sim Bin(10, \frac{1}{2})$.

Find:

    (a) The mean and the standard deviation of the distribution.   (3 marks)

    (b) $P(X < 2)$ (3 marks)

    (c) $P(X \geq 2)$ (2 marks)

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14. (i) Two events A and B are independent such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{3}{20}$.

Find:

    (a) $P(A \cap B)$,   (2 marks)

    (b) $P(A \cup B)$,   (3 marks)

    (c) $P(A' \cap B)$.   (3 marks)

  (ii) In a class of 200 students 80 are boys and 120 are girls. Given that $\frac{3}{4}$ of the boys and $\frac{2}{5}$ of the girls offer the sciences. A student X is chosen at random from the class. By drawing a tree diagram or otherwise, find the probability that:

    (a) X offers the sciences.   (3 marks)

    (b) X is a girl or offers the sciences.   (3 marks)

    (c) X is a boy given that he offers the sciences.   (3 marks)