Homework # 5
ECON 8050: Advanced Macroeconomics II
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Problem 1 (50pts)

Consider the following overlapping generations model. Time is indexed by $t = 0, 1, 2, ..., \infty$. In period t, L_t two-period-lived consumers are born, where

$$L_t = L_{t-1}(1+n)$$

with $n > 0$. Every young consumer is endowed with one unit of labor, old consumers cannot work. The preferences are given by

$$u(c_t^y, c_{t+1}^o) = \min(c_t^y, \beta c_{t+1}^o)$$

The representative firm has a production technology given by

$$Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$$

where $0 < \alpha < 1$, K_t is the aggregate capital and L_t is the aggregate labor.

1. Determine consumption of the young, consumption of the old, and the capital/labor ratio in the optimal steady-state (i.e. solve the Social Planner problem).

2. Determine consumption of the young, consumption of the old, and the capital/labor ratio in the competitive equilibrium steady-state. How does the capital/labor ratio differ from part 1?

3. Now suppose that the government issues B_{t+1} bonds in period t, where $B_{t+1} = bL_t$ for all t, with b a constant. Each young agent is taxed lump-sum (denote the individual tax τ_t and total taxes collected as T_t) so that the government can finance any interest payments on the debt that cannot be financed with the current bond issue. Determine the value for b that implies that an optimal steady state is achieved as a competitive equilibrium steady state. (You only need to compare the capital/labor ratio).

4. Assume instead that the government runs pay-as-you-go (or unfunded) pension system: it levies the tax τ on the young which is used to finance benefits P for the old. Determine the value for τ that implies that an optimal steady state is achieved as a competitive equilibrium steady state. (You only need to compare the capital/labor ratio).

Problem 2 (50pts)

The effect of state-contingent savings on aggregate outcomes and welfare

Consider the following overlapping generations model. Time is indexed by $t = 0, 1, 2, ..., \infty$. In period t, L_t two-period-lived consumers are born, where

$$L_t = L_{t-1}(1+n).$$

Every young consumer works and receives wage w_t, saves s_t; old consumers cannot work and live from their savings: $s_t(1+r_t)$. There is no pension system. Every old individual can receive medical shock x with probability π.

The lifetime utility is given by

$$\frac{(c_t^y)^{1-\sigma}}{1-\sigma} + \beta E \frac{\left(c_{t+1}^o\right)^{1-\sigma}}{1-\sigma}$$

The representative firm has a production technology given by

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$$

where $0 < \alpha < 1$, K_t is the aggregate capital and L_t is the aggregate labor.

Consider two cases. Case 1: no insurance against medical shock is available. Case 2: young individuals can buy actuarially fair insurance by paying premium $p = \dfrac{\pi x}{1+r}$, which fully covers medical shock x.

For each case, solve for the steady-state competitive equilibrium for this economy for a set of values of x from 0 to 0.3 with a step 0.001. Use the following parameter values: $\alpha = 0.3, \sigma = 3, \beta = 0.99, \pi = 0.1, n = 0.01$.

(Hint: you have to add premiums collected by insurance firms to the market clearing condition for the capital market)

a) Plot equilibrium capital per worker as a function of x for the two cases on the same graph.

b) Plot equilibrium wage as a function of x for the two cases on the same graph.

c) Plot welfare of a newborn individual as a function of x for the two cases on the same graph.

d) Discuss which case brings higher welfare and why.

Repeat a)-c) for the case when young individuals survive to the second period with probability $surv = 0.8$. Savings of young individuals who do not survive are allocated in a lump sum fashion to the newborns, i.e. each newborn receives $(1 - surv)s_t L_t / L_{t+1}$. Does your answer to d) changes? Why or why not?

(Hint: you need to adjust the actuarial fair premium, market clearing condition for the capital market, and consumers optimization problem for the presence of survival uncertainty).

Coding hint: To compute equilibrium capital per worker you should start with a guess, e.g., k_0. Given this guess, you can compute factor prices and insurance premiums and solve individual optimization problem. Then plug savings s and factor prices in the market clearing condition for capital and find new capital k_{new}. In the next iteration, set capital as the weighted average between

k_0 and k_{new}. Continue until the difference between old and new capital is below some tolerance level (10^{-5}). In case of survival uncertainty, you should also iterate on transfers received by newborns from accidental bequests.