

Models of Competition: Homogenous Goods

Peter W. Newberry

ECON 8210: Grad IO

Fall 2025



Terry College of Business

Department of Economics

UNIVERSITY OF GEORGIA

Is market power a problem?

Why do we care?

Active area of research: are mark-ups increasing over time?



Is market power a problem?

Why do we care?

- Mark-ups (e.g., debate on 'corporate greed' and inflation).

Active area of research: are mark-ups increasing over time?

Is market power a problem?

Why do we care?

- Mark-ups (e.g., debate on 'corporate greed' and inflation).
- Product quality.

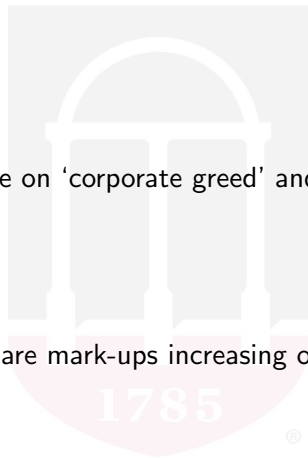
Active area of research: are mark-ups increasing over time?

Is market power a problem?

Why do we care?

- Mark-ups (e.g., debate on 'corporate greed' and inflation).
- Product quality.
- Product variety.

Active area of research: are mark-ups increasing over time?

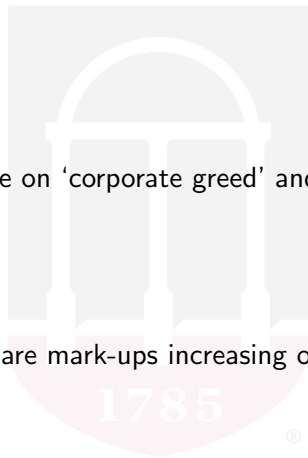


Is market power a problem?

Why do we care?

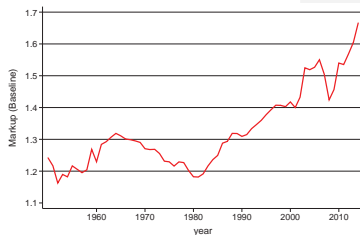
- Mark-ups (e.g., debate on 'corporate greed' and inflation).
- Product quality.
- Product variety.
- ...

Active area of research: are mark-ups increasing over time?

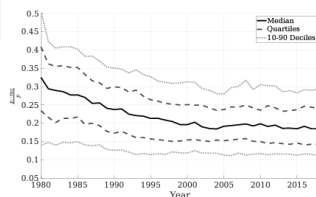


Are mark-ups increasing?

Figure: Mark-Ups Over Time



(a) DeLoecker and Eeckhout (2017),
Compustat Industries



(a) Distribution of Markups

(b) Grieco, Murry, & Yurukoglu (2024),
Autos

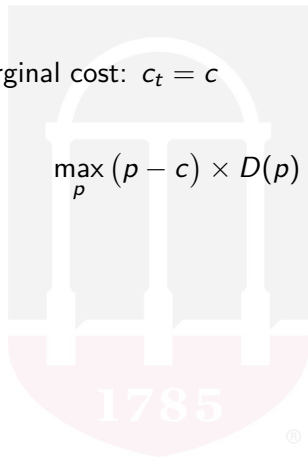
How do we measure mark-ups?

- If we have price and cost data, we're done.
- Cost data not often available.
- Even if available, it is accounting costs.

Where we're going: estimating mark-ups

- Demand: $D(p)$
- Supply: Constant marginal cost: $c_t = c$
- Profit-maximization:

$$\max_p (p - c) \times D(p)$$



Where we're going: estimating mark-ups

- Demand: $D(p)$
- Supply: Constant marginal cost: $c_t = c$
- Profit-maximization:

$$\max_p (p - c) \times D(p)$$

- FOC:

$$D(p) + (p - c) \times \frac{\partial D(p)}{\partial p} = 0$$

$$\Rightarrow p - c = - \underbrace{\left[\frac{\partial D(p)}{\partial p} \right]^{-1} D(p)}_{\text{Dollar Markup}}$$

Comment 1: We can estimate markups with a demand function $D(p)$ combined with data on prices (p) and quantities (Q).

Where we're going: estimating mark-ups

- Demand: $D(p)$
- Supply: Constant marginal cost: $c_t = c$
- Profit-maximization:

$$\max_p (p - c) \times D(p)$$

- FOC:

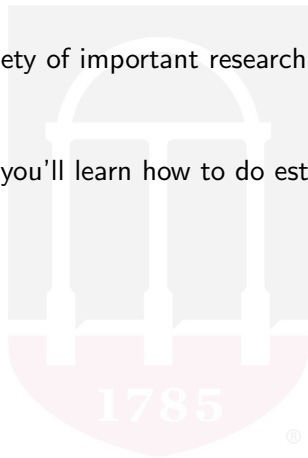
$$D(p) + (p - c) \times \frac{\partial D(p)}{\partial p} = 0$$

$$\Rightarrow p - c = - \underbrace{\left[\frac{\partial D(p)}{\partial p} \right]^{-1}}_{\text{Dollar Markup}} D(p)$$

Comment 2: We'll see that this will be equivalent to having estimated the demand elasticity.

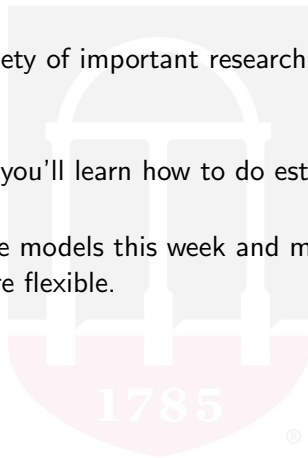
Estimating $D(p)$

- Answering a wide variety of important research questions requires estimation of $D(p)$.
- Over the next weeks, you'll learn how to do estimate demand.



Estimating $D(p)$

- Answering a wide variety of important research questions requires estimation of $D(p)$.
- Over the next weeks, you'll learn how to estimate demand.
- We'll start with simple models this week and move onto complicated models which are more flexible.



Estimating $D(p)$

- Answering a wide variety of important research questions requires estimation of $D(p)$.
- Over the next weeks, you'll learn how to estimate demand.
- We'll start with simple models this week and move onto complicated models which are more flexible.
- Don't confuse "complicated" with "better."
- You'll see that the big and deep ideas in the simple models are also contained in the complicated models.

A Basic Question

Q: How are prices & quantities determined when a **small number** of firms compete?

For now, we'll look at markets where firms produce a homogeneous (from the consumers' perspective) product?

Examples: crude oil, grain, lumber, cement, solar panels.

Key Issue

Monopoly and competitive firm behavior can be described by an optimization problem: e.g., firm chooses quantity to maximize profits given demand or price.

In oligopoly markets, when a firm chooses how much to produce or what price to charge, **its profits depend upon the quantities or prices chosen by its rivals and upon their subsequent reactions to its quantity or price choice.**

Open Questions:

- How to model behavior?
- What is an equilibrium?

Note: Play is inherently dynamic but we will initially consider static models.

Cournot Model



- Antoine Augustin Cournot (1801-1877)
- French philosopher and mathematician.
- In 1838 published
“Researches into the Mathematical Principles of the Theory of Wealth”.

1785

Cournot Model



- Antoine Augustin Cournot (1801-1877)
- French philosopher and mathematician.
- In 1838 published
“Researches into the Mathematical Principles of the Theory of Wealth”.
- Book not successful but was the first to use math to explain economic forces and theories.
- First to write supply and demand as a function of price as well as draw supply and demand curves on a graph;
i.e., not Alfred Marshall in 1890’s “Principles of Economics” as is commonly believed / taught.

Cournot Model

Consider the following *game*:

- Player Set: $i=1, 2$ (duopoly)
- Strategy for firm i : $y_i \in [0, \infty)$
 - *NB, this is the defining feature of Cournot models: Firms choose quantities.¹
- Payoffs for firm i depend upon not only its output choice (y_i) but also upon the output choice y_j of its rival, firm j :

$$\pi_i(y_i, y_j) = P(y_i + y_j) \times y_i - C_i(y_i)$$

¹When firms compete in quantities, we call the model “Cournot” but understand that Cournot developed his theory roughly 100 years before John Nash formalizes “Game Theory” via his 1950 dissertation.

Cournot Model

Consider the following *game*:

- Player Set: $i=1, 2$ (duopoly)
- Strategy for firm i : $y_i \in [0, \infty)$
 *NB, this is the defining feature of Cournot models: Firms choose quantities.¹
- Payoffs for firm i depend upon not only its output choice (y_i) but also upon the output choice y_j of its rival, firm j :

$$\pi_i(y_i, y_j) = P(y_i + y_j) \times y_i - C_i(y_i)$$

Interpretation: Two firms simultaneously decided how much to produce for the market. Given these amounts, price adjusts to equate demand to supply.

Makes sense in markets where it is hard to adjust production decisions before prices are determined (e.g., commodities, manufacturing).

¹When firms compete in quantities, we call the model “Cournot” but understand that Cournot developed his theory roughly 100 years before John Nash formalizes “Game Theory” via his 1950 dissertation.

What is an Equilibrium to this Game?

We have a non-cooperative game between two (or more) strategic agents so we'll use the equilibrium concept developed by John Nash.

Definition: A *Nash equilibrium* is a pair of quantities (y_1^*, y_2^*) satisfying

$$\pi_i(y_i^*, y_j^*) \geq \pi_i(y_i, y_j^*)$$

for any $y_i \in [0, \infty)$; $i \neq j$; $i, j = 1, 2$.

Interpretation: each firm is behaving optimally given its conjecture about its rival's choice of quantity and, in equilibrium, their conjectures are correct.

An Example

Suppose two firms

- produce a homogenous good at zero cost,
- sell to consumers with (inverse) demand $P(Y) = 120 - 20Y$, and
- each firm chooses between two outputs: low (1.5) and high (2).

Define payoffs $(\pi_1(y_1, y_2), \pi_2(y_1, y_2))$ then all possibilities are presented in the following table:

		Firm 2	
		$y_2 = 1.5$	$y_2 = 2$
Firm 1	$y_1 = 1.5$	\$90, \$90	\$75, \$100
	$y_1 = 2$	\$100, \$75	\$80, \$80

Equilibrium

		Firm 2	
		$y_2 = 1.5$	$y_2 = 2$
Firm 1	$y_1 = 1.5$	\$90, \$90	\$75, \$100
	$y_1 = 2$	\$100, \$75	\$80, \$80

- The Nash equilibrium is (2, 2).
- Notice that this Nash equilibrium is also the “dominant strategy” equilibrium: $y_i = 2$ yields higher payoffs for firm i no matter what firm j chooses.
- Notice that this is a prisoner’s dilemma: both firms could do better if they cooperate and choose $y_i = 1.5$

A More General Example

Environment:

- 2 firms indexed by $i = 1, 2$
- Demand: $P(Y) = a - bY$
- Costs: $C(y_i) = cy_i$; $i = 1, 2$

Recall the definition of a Nash equilibrium is that each firm is doing the best it can given the (expected) move of its opponent:

$$\pi_i(y_i^*, y_j^*) \geq \pi_i(y_i, y_j^*)$$

where the index j refers to other firm (i.e., if $i = 1$ then $j = 2$).

Important: Finding the Nash equilibrium requires figuring out how each firm will reply to the move of its opponent.

The “Best Response” (or “Best Reply”) Function

Suppose Firm 1 “believes” that Firm 2 produces y_2 . Its “best response” to that value of y_2 is to choose y_1 to maximize its profits; *i.e.*, it solves

$$\max_{y_1} \pi_1(y_1, y_2) \equiv \max_{y_1} \left\{ [a - b(y_1 + y_2) - c] \times y_1 \right\}$$

Differentiating and solving yields Firm 1’s “best response” function:

$$y_1 = (a - c - by_2)/2b$$

Similarly, Firm 2’s “best response” function is

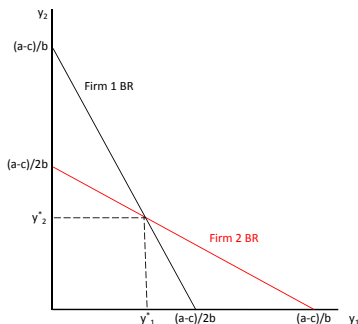
$$y_2 = (a - c - by_1)/2b$$

Important: Firm 1 can’t influence Firm 2’s choice (and vice versa) since the firms choose output simultaneously. When we solve for the best response functions, we therefore treat the other firm’s choice like it’s a constant (*i.e.*, $\frac{dy_2}{dy_1} = 0$).

The Nash Equilibrium

The Nash equilibrium is given by the intersection of the two best replies. Note that we can rewrite the best responses s.t. they resemble equations of lines:

$$BR_1(y_2) = \frac{a-c}{2b} - \frac{y_2}{2}; \quad BR_2(y_1) = \frac{a-c}{2b} - \frac{y_1}{2}$$



The Nash Equilibrium

Mathematically, this amounts to solving a simultaneous system of two equations and two unknowns:

$$\begin{cases} y_1 = \frac{a-c}{2b} - \frac{y_2}{2} \\ y_2 = \frac{a-c}{2b} - \frac{y_1}{2} \end{cases}$$

The solution is

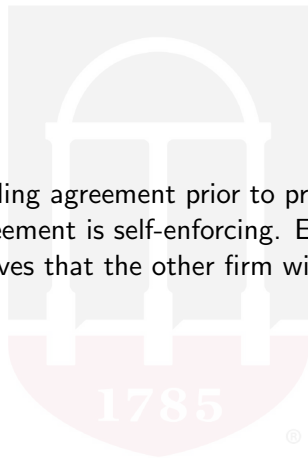
$$y_1^* = y_2^* = (a - c)/3b$$

Thus, equilibrium output, price and firm profits are

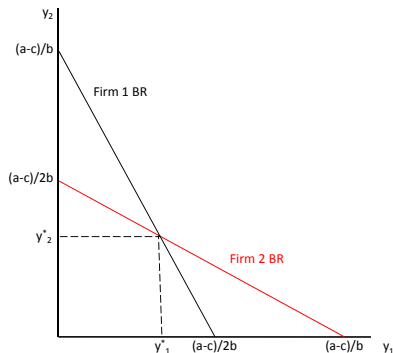
$$Y^* = \frac{2(a - c)}{3b}; P^* = \frac{a + 2c}{3}; \pi^* = \frac{(a - c)^2}{9b}$$

Comment 1

If firms reach a non-binding agreement prior to production to produce N.E. quantities, then the agreement is self-enforcing. Each firm cannot gain from deviating if it believes that the other firm will stick to the agreement.



Comment 2



Note: the best response for firm i is **decreasing** in y_j . This means that they are *strategic substitutes*.

Comment 3

Look at Firm 1's problem again:

$$\max_{y_1} \left(a - b(y_1 + y_2) \right) \times y_1 - C_1(y_1)$$

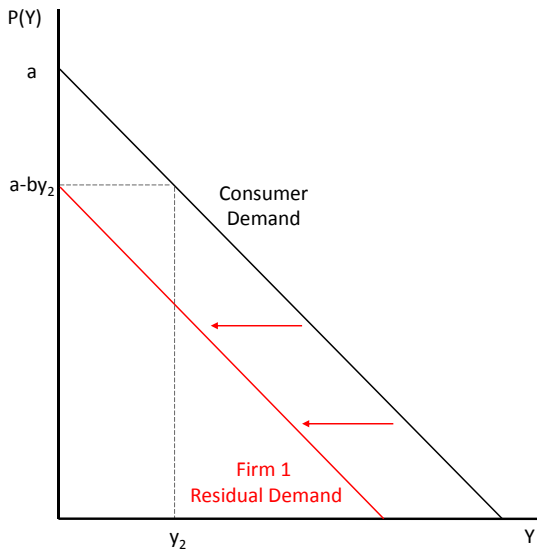
where I'm allowing for a general cost function $C_1(y_1)$ rather than our specific one.

Now move things around a bit:

$$\max_{y_1} \left(\underbrace{a - by_2}_{\substack{\text{Residual} \\ \text{(Inverse)} \\ \text{Demand}}} - by_1 \right) \times y_1 - C_1(y_1)$$

So the firm's best reply is the consequence of the firm maximizing profits given the “residual demand” curve which amounts to a downward-shift relative to the original demand curve.

Residual Demand



Extension to N Identical Firms

Let

$$Y_{-i} = \sum_{j \neq i}^n y_j$$

denote total output by firm i 's rivals. Substituting this quantity into firm i 's profit function and differentiating yields its best reply

$$y_i = (a - bY_{-i} - c)/2b$$

1785

®

Best Responses

In the symmetric equilibrium, firms produce the same amount,

$$y^* = [a - b(N - 1)y^* - c]/2b$$

Solving yields

$$y^* = (a - c)/[b(N + 1)]$$

Equilibrium output and price are

$$Y^* = Ny^* = \frac{N(a - c)}{b(N + 1)}; P^* = \frac{a}{N + 1} + \frac{N}{N + 1}c$$

Remark: price and output varies from monopoly to perfect competition as N goes from 1 to infinity.

Extension to Asymmetric Firms

Firms always face the same demand curve but their costs functions may vary.

In the above example, suppose $C_i(y_i) = c_i y_i$. Let $i = 1, 2$.

Firm i solves

$$\max_{y_i} (a - bY_{-i} - by_i) \times y_i - c_i y_i$$

Solve for the Best Responses and equilibrium outputs (y_1^*, y_2^*) .

Extension to Asymmetric Firms (cont'd)

Firm i solves

$$\max_{y_i} (a - bY_{-i} - by_i) \times y_i - c_i y_i$$

Differentiating, and assuming there are only two firms, the best reply functions are given by

$$y_1 = \frac{(a - c_1 - by_2)}{2b}$$

$$y_2 = \frac{(a - c_2 - by_1)}{2b}$$

Solving the pair of equations yields the N.E.:

$$y_1^* = \frac{(a + c_2 - 2c_1)}{3b}, \quad y_2^* = \frac{(a + c_1 - 2c_2)}{3b}$$

The remaining equilibrium variables (e.g., P^* , π_1^* , π_2^*) follow immediately.

Remarks:

- 1 In asymmetric case, you need to use all N equations to solve for the N unknowns.
- 2 Firm types (i.e., there are N_ℓ low cost firms and N_h high cost firms) to make problem easier: just have as many equations as types.
- 3 Lower cost firm produces more but the higher cost firm produces as long as its costs are not too much higher than the other firm's cost. Efficient?

The Inverse-elasticity Rule for Cournot Competition

Recall firm i 's FOC is

$$[a - bY_{-i}^* - 2by_i^*] - c_i = 0$$

This can be written as

$$P^* - c_i = by_i^*$$

or equivalently,

$$\frac{P^* - c_i}{P^*} = s_i^* \frac{bY^*}{P^*}$$

In the case of linear demand, the demand elasticity is:

$$\varepsilon(P) = \frac{-P}{bY(P)}$$

Thus,

$$\frac{P^* - c_i}{P^*} = \mu_i = -\frac{s_i^*}{\varepsilon(P^*)}$$

and we have an inverse-elasticity rule for Cournot with N firms (aka, the lerner index). Take the weighted average of both sides (where weights are s_i), then:

$$\bar{\mu} = \frac{HHI}{|\varepsilon(P^*)|}$$

Properties of the Cournot Solution

$$\frac{P^* - c_i}{P^*} = \frac{s_i^*}{|\varepsilon(P^*)|}$$

This is the fundamental markup equation for Cournot markets, but has insights that generalize to other (more complicated) models. Several results follow immediately from this equation:

- ① Firms exercise market power since price is above marginal cost $\forall i=1, \dots, N$.
- ② As with monopoly, markups are limited by elasticity of demand.
- ③ All else equal, firms with lower marginal costs have larger market shares.
- ④ All else equal, market share and markup are both decreasing in the number of firms.
- ⑤ If firms are symmetric, the oligopoly Cournot solution lies between the perfectly competitive equilibrium and monopoly in terms of quantities, price, profits, and surplus.

Does the Cournot model make sense for all industries?

Bertrand Model



- Joseph Louis François Bertrand (1822-1900)
- French mathematician.
- Worked in fields of number theory, differential geometry, probability theory, economics and thermodynamics.
- Developed theory underlying Bertrand model while writing a 1883 review of Cournot's book.
- “Theory” was words / intuition and was not formalized mathematically until Edgeworth in 1889.

Bertrand Model

- Player set: two firms, indexed by $i = 1, 2$.
- Strategy for firm i : $p_i \in [0, \infty)$;
 * This is the defining feature of Bertrand models: Firms choose price.
- Payoffs for firm i :

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c_i) \times D(p_i), & \text{if } p_i < p_j \\ (p_i - c_i) \times D(p_i)/2, & \text{if } p_i = p_j \\ 0, & \text{if } p_i > p_j \end{cases}$$

Interpretation: Two firms decide simultaneously what price to charge and produce to demand. Consumers know the prices and buy from the firm that offers the lowest price. Each firm has capacity to serve the entire market.

What is an Equilibrium to this Game?

Definition: A *Nash Equilibrium* is a pair of prices (p_1^*, p_2^*) satisfying

$$\pi_i(p_i^*, p_j^*) \geq \pi_i(p_i, p_j^*)$$

for any $p_i \in [0, \infty)$; $i \neq j$; $i, j = 1, 2$.

Question: What is the Nash Equilibrium?

What is an Equilibrium to this Game?

Definition: A *Nash Equilibrium* is a pair of prices (p_1^*, p_2^*) satisfying

$$\pi_i(p_i^*, p_j^*) \geq \pi_i(p_i, p_j^*)$$

for any $p_i \in [0, \infty)$; $i \neq j$; $i, j = 1, 2$.

Question: What is the Nash Equilibrium?

Answer: $p_1 = p_2 = c$.

Bertrand paradox: “one is monopoly, two is perfect competition.”

Extension to Asymmetric Firms

- Unit costs are $c_1 < c_2 < p^M(c_1)$ where $p^M(c_1)$ is monopoly price for firm 1.
- Assume prices are denominated in pennies.
- Undercutting logic implies the following Nash equilibrium:

$$p_1 = c_2 - .01$$

$$p_2 = c_2$$

- Firm 2 has a cost disadvantage and therefore does not participate (*i.e.*, it exits).
- Firm 1 is a monopoly, and exercise market power[®]
- BUT, can't set monopoly price. Why?

Key Assumptions

- 1 **Homogeneous Good:** Consumers care only about price and respond en masse to slightest difference in prices.
- 2 **Static Game:** Firms compete against each other repeatedly in the data
→ allows for the possibility of cooperation.
- 3 **Unlimited Capacity:** In the data firms generally do not have capacity to service the entire market.

If all customers try to buy from the low price firm, some customers are rationed and have to buy from the higher price firm - implies prices above marginal cost.

A Bertrand Model with Capacity Constraints

- Two ski resorts: Whistler and Blackcomb.
- Capacity constraints:
 - Blackcomb: 1000 skiers per day
 - Whistler: 1400 skiers per day
- Homogeneous Good: Quality of skiing is same on both mountains.
- Total Skiing Demand: $Q = 6000 - 60P$
- Marginal costs of lift services = \$10 per skier per day.

Important:

If $P = \$10$ (i.e., price equal marg. cost), total demand is 5400. **But** supply cannot exceed 2400.

Bertrand Nash Equilibrium with Capacity Constraints

Demand equals total capacity at $P = \$60$. Suppose both firms charge this price. Would either resort want to deviate?

There are two choices:

- 1 Decrease price. Clearly, the resort has no incentive to lower price since it cannot serve more skiers. **X**
- 2 Increase price. If it raises price above \$60, Whistler faces a residual demand curve:

$$Q_W = 5000 - 60P_W$$

Note: I have assumed that 1000 skiers with highest willingness to pay ski on Blackcomb.

Optimization

Question: Can Whistler Increase Profits by Increasing Price From \$60?

Whistler's optimization problem:

$$\max_{P_W} \pi(P_W) \equiv \max_{P_W} \left\{ (5000 - 60P_w)(P_W - 10) \right\}$$

Differentiating yields

$$\begin{aligned} \frac{d\pi(P_W)}{dP_W} &\equiv 5000 - 60P_W - 60(P_w - 10) \\ &\equiv 5600 - 120P_W \end{aligned}$$

Evaluating at $P_W = \$60$ yields

$$\frac{d\pi(P_W = 60)}{dP_W} = -1600$$

So the derivative is negative which means Whistler's profits fall if it increases price above \$60! **X**

Whistler's Best Response to $P_b = 60$

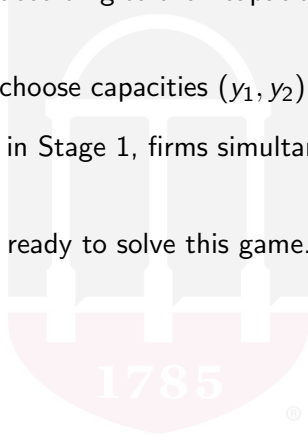
- This means that given Blackcomb sets a price of \$60, the best response for Whistler is to set a price of \$60.
- Solving for the best response of Blackcomb yields the same answer, so (60,60) is the Bertrand Nash Equilibrium and equilibrium prices exceed marginal cost.

More Generally

Firms divide the market according to their capacity. But capacity should be viewed as a choice: i.e.,

- 1 Firms simultaneously choose capacities (y_1, y_2) .
- 2 Given (y_1, y_2) choices in Stage 1, firms simultaneously choose prices (p_1, p_2) .

Unfortunately, we're not ready to solve this game.



More Generally

Firms divide the market according to their capacity. But capacity should be viewed as a choice: i.e.,

- 1 Firms simultaneously choose capacities (y_1, y_2) .
- 2 Given (y_1, y_2) choices in Stage 1, firms simultaneously choose prices (p_1, p_2) .

Unfortunately, we're not ready to solve this game.

How much capacity should the two resorts choose when they correctly anticipate the equilibrium prices that will follow?

- Kreps and Scheinkman (1983) show the capacities chosen will be the Cournot quantities!

Thus, the Cournot game is best understood as a capacity game.

Application: Genesove & Mullin (1998) “Testing Static Oligopoly Models: Conduct and Cost in the Sugar Industry, 1890-1914”

- Can we use consumer demand to infer market conduct (e.g., Cournot, Bertrand, Collusion) and firm cost?
- Recall firm FOC under Cournot

$$p + QP'(Q) = c$$

- Modify to include a conduct parameter θ :

$$p + \theta \times QP'(Q) = c$$

where

$$\theta = \begin{cases} 1, & \text{if collusion or monopoly} \\ 0, & \text{if perfect competition or } N \geq 2 \text{ Bertrand} \\ 1/N, & \text{if symmetric Cournot} \end{cases}$$

Empirical Approach

- Rewrite this new FOC as

$$\theta = \varepsilon(P) \times \frac{P - c}{P}$$

- If we know $\{P, c, \varepsilon(P)\}$ (and N), we can infer conduct θ .
- Authors use data on the sugar industry.
 - Reliable estimates of marginal cost
 - Industry underwent significant changes in competition during period.
 - We know firms colluded during certain periods and competed aggressively (price war) during others.
 - Can test whether theory and data are consistent.

Some Results

- Estimate $\{\theta, c\}$ assuming linear demand:

$$E[\{(1 + \theta)P - \alpha\theta - c_o - kP_{RAW}\}Z] = 0. \quad (12)$$

where Z are “instruments” for price (more on this later...)

- Suppose we know $\theta = 0.10$ from other data.
- Will our estimates generate a similar value?

Some Results

TABLE 7 **NLIV Estimates of Pricing Rule Parameters**

	Linear		Direct Measure
	(1)	(2)	(3)
$\hat{\theta}$.038 (.024)	.037 (.024)	.10
\hat{c}_o	.466 (.285)	.39 (.061)	.26
\hat{k}	1.052 (.085)		1.075

- $\hat{\theta}$ pretty close to the “Direct Measure”. Reject monopoly and Cournot ($N < 9$).

Some Results

TABLE 7 **NLIV Estimates of Pricing Rule Parameters**

	Linear		Direct Measure
	(1)	(2)	(3)
$\hat{\theta}$.038 (.024)	.037 (.024)	.10
\hat{c}_o	.466 (.285)	.39 (.061)	.26
\hat{k}	1.052 (.085)		1.075

- Can't reject perfect competition $\theta = 0$.

Some Results

TABLE 7 **NLIV Estimates of Pricing Rule Parameters**

	Linear		Direct Measure
	(1)	(2)	(3)
$\hat{\theta}$.038 (.024)	.037 (.024)	.10
\hat{c}_o	.466 (.285)	.39 (.061)	.26
\hat{k}	1.052 (.085)		1.075

- Costs c imprecisely estimated. Can't reject $k = 1.075$.

Some Results

TABLE 7 **NLIV Estimates of Pricing Rule Parameters**

	Linear		Direct Measure
	(1)	(2)	(3)
$\hat{\theta}$.038 (.024)	.037 (.024)	.10
\hat{c}_o	.466 (.285)	.39 (.061)	.26
\hat{k}	1.052 (.085)		1.075

- Including information about costs (k) does not help identify θ (column 2).

Next Time

- We now have a simple model to understand oligopoly competition.
- And we've learned some pretty deep insights into how competition influences equilibrium prices.
- Next time, we'll think harder about how firms compete and how to use the best responses to understand the dynamics and nuances of competition to develop intuition for underlying economic mechanisms.