Midterm

Econ 8050: Macroeconomics II

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Problem 1

Assume that the consumer has CRRA utility over consumption given by

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

with $\gamma > 0$. The consumer discounts the future at the rate β and faces a gross interest rate R (note that the gross interest rate is equal to one plus the net interest rate). Assume that consumption is conditionally lognormal with mean $E_t\, ln\,(c_{t+1}) = \mu_t$ and variance v_t.

1) Find $E_t(\Delta\, ln\, c_{t+1}) = E_t(ln\, c_{t+1} - ln\, c_t)$.

Hint: it should depend only on γ, β, R and v_t.

2) Note that $E_t(\Delta\, ln\, c_{t+1})$ can be thought of (approximately) as expected consumption growth rate. Suppose $\beta R = 1$. Based on your result in 1), what the expected consumption growth rate will depend on? Provide intuition.

3) The expression you derived in 1) illustrates the effect of two saving motives on consumption choice. What are these saving motives?

Problem 2

Consider the following infinite-horizon model. An individual in the model is a male. Every period, he can be single or married, employed or unemployed. Single males can meet a wife with probability τ^m. A married male can divorce, in which case he will stay single for at least one period (agent cannot move straight from one marriage to another).

Unemployed individuals receive a job offer with probability τ^u. While unemployed, an individual receives unemployment benefits (b) that depend on the number of periods an individual has been unemployed (u), $b = b(u)$, $b'(u) < 0$.

Individuals are heterogeneous in their relationship skills (n), which are fixed and do not change over time. Relationship skills are useful at workplace and in marriage.

Individuals accumulate human capital (h) while working. The initial human capital (h_o) is the same for everyone. An individual who works has his human capital improving in the following way: $h' = h + \delta$, $\delta > 0$. An individual who does not work has his human capital declining: $h' = h - \delta$.

A job is characterized by a pair (c, v), where c is the level of job complexity and v is the relationship skill requirement. The discretized distribution of (c, v) can be represented by a matrix Π. An employed individual draws (c, v) pair from this distribution, $\Pi_{ij} = Prob(c = c_i, v = v_j)$.

The wage of an employed individual is determined as follows:

$$w(c, v, h, n) = \left(\gamma_0 c^{\gamma_1} + (1 - \gamma_0) h^{\gamma_1}\right)^{\frac{1}{\gamma_1}} \epsilon_w(n, v)$$

where γ_1 controls the degree of complementary between job complexity c and an individual's human capital h, and $\epsilon_w(n, v)$ is an iid stochastic match quality shock. The mean of the shock depends on n and v.

Note that while the pair (c, v) does not change as long as an individual keeps the job, ϵ_w is drawn every period. Denote the discretized distribution Ψ^w. If ϵ_w is too low, an individual may decide to quit. In this case, he will spend at least one period being unemployed (job-to-job transitions are not possible).

An individual who is single has utility $log(cons)$ where $cons$ is equal to his wage (if employed) or unemployment benefits (if unemployed). A married individual has utility $log(cons)\epsilon_m(n)$, where ϵ_m is iid shock that determines the quality of his marriage. This shock is drawn every period and its mean depends on the relationship skill n of an individual. Denote the discretized distribution Ψ^m. An individual who has bad realization of ϵ_m may choose to divorce.

Note that individuals in the model do not save. Agents discount the future at a rate β.

Write down the Bellman equations for this model. Use the following notations for the value functions:

$V^{s,u}$ for single unemployed,

$V^{s,e}$ for single employed,

$V^{m,u}$ for married unemployed,

$V^{m,e}$ for married employed.

Hints:

1. Think carefully through the state variables. Note that the state variables are different for employed and unemployed, married and single.

2. Note that some transitions between being employed/ being unemployed; being married/ being single are stochastic while some are voluntary. Make sure to take this into account.

3. Note that individuals can be in 4 states: (s, u), (s, e), (m, u), (m, e). For individuals in the first state, all transitions are exogenous. For individuals in states 2 and 3, there are

two options to choose from but some transitions can happen exogeneously. For individuals in state 4, there are four options and no transitions happen exogeneously. Think what are those options.

4. Assume that an agent who is not employed and receives a job offer always takes it. Also assume that an agent who is single and meet a partner always marries.

5. Be very explicit when writing expected value functions: use the information you have about the distribution of stochastic variables.

6. Note that for individuals who just become unemployed, $u = 0$. Make sure this is clear from your formulation of the problem.

7. In your notation, use ϵ'_m and ϵ'_w for future realizations of marriage quality and match quality shocks.

8. Two of the state variables in this model evolve non-stochastically (h and u). Make sure to state their dynamics explicitly for each type of agents (where it is relevant).

9. To simplify the notation, use Π instead of $\Pi(c, v)$.