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FLEXIBLE RETIREMENT AND OPTIMAL TAXATION*

Abdoulaye Ndiaye and Zhixiu Yu

May 21, 2025

Abstract

Raising the retirement age is a common policy response when social security schemes face fiscal pressures. We develop and estimate a dynamic life cycle model to study optimal retirement and tax policy when individuals face health shocks and income risk and make endogenous retirement decisions. The model incorporates key features of Social Security, Medicare, income taxation, and savings incentives and distinguishes three channels through which health affects retirement: nonconvexities in labor supply due to health-dependent fixed costs of working, earnings reductions, and mortality risk. We estimate our model to match US microdata and show that labor supply nonconvexities play a dominant role in driving early retirement, making rigid increases in the retirement age welfare reducing. In contrast, more flexible policies, such as increasing the dependence of Social Security benefits on the claiming age, can improve welfare and pay for themselves with a fiscal surplus. We map a range of policy reforms to their marginal values of public funds (MVPFs), showing that certain incentives to delay claiming offer MVPFs of infinity while broad-based retirement age increases have negative willingness-to-pay. These findings offer novel retirement policy prescriptions and challenge the prevailing emphasis on raising the retirement age.

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1 Introduction

Social security reform remains one of the most significant policy debates across developed economies, driven primarily by concerns about the fiscal sustainability of retirement benefits. Most policy discussions focus on increasing the full retirement age or making social security benefits less generous. However, such policy proposals are socially unpopular and rarely materialize in bipartisan reform[1] because they cut costs at the price of hurting recipients' welfare and retirement choices. In this paper, we posit that flexible retirement policies may better align with individuals' life cycle decisions and, in contrast to retirement age increases, improve overall welfare cost-effectively.

Central to this thesis is the recognition that retirement is an endogenous decision in which the deterioration of health as a person ages plays a determinant role. Poor health affects retirement decisions through several channels. It introduces labor supply nonconvexities by making work substantially less pleasant or feasible, leads to wage reductions due to productivity losses, and increases mortality risks (Blundell, French and Tetlow 2016). Each of these channels distinctly affects an individual's labor supply and retirement choices, such that any comprehensive retirement policy analysis must account for health, ability, and mortality shocks.

The paper aims to answer some key questions: How should retirement and taxation policies optimally account for health deterioration and endogenous retirement? How do health-induced distortions interact with policy-induced distortions to explain observed retirement behavior? What are the welfare implications of different retirement-related policy reforms, including reforms to Social Security, Medicare, and tax-advantaged retirement saving schemes?

To answer these questions, we develop a dynamic life cycle model featuring heterogeneous agents who face uncertain health transitions, wage risk, mortality risk, and medical expenditures. Agents choose their labor force participation, hours worked, consumption, and savings subject to realistic nonconvexities in their labor supply as in state-of-the-art retirement studies (French 2005; French and Jones 2011; Rogerson and Wallenius 2013). Our model explicitly integrates endogenous retirement choices with exogenous health shocks for the optimal tax analysis (Farhi and Werning 2013; Golosov, Troshkin and Tsyvinski 2016; Stantcheva 2017), and we derive how labor supply nonconvexities, productivity losses, and mortality risks from poor health shape the optimal policies. In short, we deliver the first dynamic taxation model that endogenizes the retirement decision, estimates heterogeneity in health, and maps reforms to estimates of their associated marginal values of public funds (MVPFs).

In our framework, wages from work vary because of health shocks and stochastic and

[1] For instance, as of April 2025, the retirement reform package France adopted in April 2023, whose showcase reform is an increase in the retirement age from 62 to 64, remains politically controversial, with multiple efforts to repeal it (Le Monde 2023).

persistent ability. We study optimal nonparametric policies targeting income histories and health but not ability. The government or planner maximizes social welfare subject to incentive compatibility constraints that reflect individuals' unobservable ability and participation constraints as in the standard Mirrlees (1971) income taxation model.

We characterize optimal policies through the use of labor and savings wedges—implicit distortions or markdowns in wages and returns on savings—that arise in equilibrium. We derive three key theoretical results.

First, for various widely used classes of preferences with nonconvexities in labor supply, the optimal labor wedge decreases in poor health. The intuition behind this result lies in the endogenous adjustment of labor supply elasticities to health. In contrast to the isoelastic preferences used in Farhi and Werning (2013) and Stantcheva (2017), preferences generating nonconvexities lead to labor supply elasticities that increase in poor health. Consequently, the planner finds it optimal to reduce labor distortions to provide utility compensation to unhealthy individuals. This result qualifies and generalizes Saez's (2002) finding—in a static setting—that if behavioral responses are concentrated on the extensive margin, the optimal tax is an earned income tax credit (EITC) with negative marginal tax rates. In our dynamic setting, the labor wedge is lower in poor health but remains *positive*, in contrast to what Saez (2002) finds. The intuition, in the spirit of Prescott, Rogerson and Wallenius's (2009) idea of *time averaging*, is that agents in a dynamic model can flexibly choose the *fraction of their lifetime* during which they work. The planner's tradeoff, per period, is therefore much milder than the "work forever vs. never work" knife-edge in Saez (2002) that necessitates subsidies for participation at the bottom of the income distribution. Instead, unhealthy workers work less, health workers work more, and extensive-margin incentives are provided predominantly by means of promised utility or consumption in retirement.

Second, the effect of poor health on wages—the productivity loss channel—alters the labor wedge primarily through its impact on the elasticity of wages with respect to ability rather than through its level effect on wages themselves. Specifically, a higher wage elasticity with respect to ability increases wage inequality among individuals with similar health status and thus enhances the insurance value of labor distortions. Empirical estimates, such as those from French and Jones (2011), typically assume a constant elasticity of wages with respect to ability across health states, implying that there is no direct impact of health-induced productivity declines on labor wedges in this class of models.

Third, by incorporating mortality risk from poor health in our model, we overturn standard results on the optimal savings wedge. While results from dynamic taxation models that abstract from mortality risk (Farhi and Werning 2013; Golosov et al. 2016) typically imply that positive distortions on savings enhance work incentives, we show that in the presence of mortality risk, a subsidy for savings can be optimal. As survival probabilities decrease due to poor health, the optimal savings wedge becomes negative, indicating a need for a subsidy. This occurs because mortality risk, when uninsured because of incomplete annuity markets, induces households to shift consumption disproportionately toward the present. Consequently, the government optimally intervenes by subsidizing savings, facilitating consumption smoothing, and insuring against mortality risk. This result provides a rationale for real-world policies such as preferential tax treatments for retirement accounts (e.g., traditional and Roth individual retirement accounts (IRAs) and 401(k)s), emphasizing that heterogeneous uninsurable mortality risk can justify such policies under time-consistent preferences.

However, these channels do not operate in isolation and can indirectly shape labor wedges through interactions with each other. When they interact, the implications for optimal labor distortions become richer and more nuanced. Specifically, an increase in the labor supply elasticity from nonconvexities reduces the optimal labor wedge, as labor distortions become costlier. Simultaneously, productivity declines associated with poor health reduce equilibrium work hours, amplifying labor supply elasticities among unhealthy workers (a level effect) and strengthening incentives for insurance-based redistribution (an elasticity effect). Moreover, the persistence of wedges increases with substantial consumption smoothing over time, which amplifies the long-term effects of both the productivity and nonconvexity channels.

To quantitatively assess the complex interactions among the three channels, we estimate our model to match key empirical features of retirement and Social Security claiming incentives in the US economy. We build on the established frameworks of French (2005) and French and Jones (2011) with a rich description of US status quo tax and retirement policies, which enables detailed positive and normative analysis. The model matches observed patterns in labor force participation, retirement, and asset accumulation, aligning closely with prior quantitative studies in the literature (e.g., De Nardi, French and Jones 2010; French and Jones 2011; De Nardi, Pashchenko and Porapakkarm 2024).

Our quantitative decompositions show that nonconvexities in labor supply, primarily attributable to participation costs, are the dominant preference and technology factors explaining the largest share of retirement behavior. These nonconvexities significantly shape retirement decisions, making it crucial to quantify their role in optimal policy design. Productivity losses due to poor health also play a notable role, whereas mortality risk differences have a more modest impact on participation, though they are important for welfare considerations.

The presence of nonconvexities in labor supply has long been recognized as critical to aggregate labor market outcomes (Holter, Ljungqvist, Sargent and Stepanchuk 2025). Lucas (1987), in his praise of Kydland and Prescott (1982), highlighted the value of incorporating microeconomic realism into macroeconomic models. However, earlier models relying on employment lotteries and complete markets, such as that of Rogerson (1988), faced criticism for being inconsistent with individual employment histories (Browning, Hansen and Heckman 1999). Prescott (2006a) later advocated for life cycle models with incomplete markets, emphasizing the importance of labor indivisibility as a determinant of lifetime labor supply and career length and bringing macroeconomic analysis into closer alignment with empirical reality and microeconomic evidence (Prescott 2006a,b). Recent studies, such as Graves, Gregory, Ljungqvist and Sargent (2023), further show that these nonconvexities significantly affect career length and labor supply responses to taxation.

Therefore, we quantify how the nonconvexity channel affects optimal retirement policies. We find that under the optimal retirement policies, higher retirement consumption is promised to mostly healthy individuals who delay retirement, and the optimal labor wedge is smaller for unhealthy individuals. In comparison with the status quo policies in our positive US economy, the dependence of Social Security on the claiming age through delayed retirement credits is not as strong as in the optimum. The status quo labor wedges differ from the optimal ones on many dimensions but retain the essential feature that labor distortions should be lower for unhealthy workers.

We then conduct our complete normative analysis by first assessing the *coarse* welfare contribution of each policy instrument—income taxes, Social Security, Medicare, disability insurance, and retirement savings subsidies—in the status quo US economy and then simulating *granular* reforms of each policy.

First, we quantitatively assess the welfare contributions of key retirement policy instruments by separately eliminating each and measuring the associated welfare changes in consumption- and leisure-equivalent terms. Eliminating Social Security benefits would lead to the largest welfare reduction, which highlights this program's critical role in retirement well-being. Extending Medicare coverage to eliminate out-of-pocket medical expenditures after age 65 would provide substantial welfare gains, but these would still be smaller than the benefits provided by Social Security. Savings subsidies, income taxes, and disability benefits follow in descending order of their importance to welfare. Notably, removing disability benefits would substantially increase labor force participation among individuals with disabilities over 50, though the overall welfare impact would remain modest because of this group's smaller size. These findings emphasize the centrality of Social Security over Medicare in shaping welfare and retirement decisions, refining previous insights from French and Jones (2011).

Second, we study detailed reforms to Social Security, Medicare, income tax, and savings subsidy policies to evaluate their welfare impacts, fiscal implications, and effects on labor force participation. Within Social Security, reforms that increase its dependence on the claiming age offer the highest MVPF. Increasing the delayed retirement credit (DRC) would increase welfare, promote later retirement, and even generate government savings relative to the US status quo, achieving an infinite MVPF—that is, the credits would pay for themselves. The Pareto-improving nature of these reforms can be understood through their effect on participation over the life cycle. They would increase participation at a young age—representing a positive substitution effect—and after the normal retirement age (NRA)—representing a positive substitution effect for healthier individuals who opt to retire later and claim larger benefits. In addition, the reforms could reduce the participation of unhealthy early retirees, thus improving self-selection into retirement.[2] Conversely, reducing the government deficit by raising the NRA would reduce welfare as it would worsen outcomes absent any behavioral change and restrict retirement flexibility. Policymakers should therefore tread cautiously in formulating policies aimed solely at increasing participation, such as raising the NRA, because of their potential adverse welfare effects.[3]

Among reforms that would affect the level or progressivity of Social Security benefits, increasing the replacement rate of the lowest earnings bracket would offer the highest MVPF, at approximately 0.52 to 1.14. However, linear reforms to the Social Security benefit replacement rate yield lower MVPFs compared to progressive reforms. This result contrasts with findings in the dynamic taxation literature(e.g. Farhi and Werning 2013), which typically concludes that *linear*, history-independent but age-dependent policies are nearly optimal. In contrast, we show that when retirement decisions are endogenous, Social Security–an income-history-dependent policy–becomes the most important policy instrument for welfare. Moreover, the cumulative impact of ability and health shocks on lifetime earnings implies that progressive reforms to Social Security benefits yield substantially larger MVPFs than linear reforms.

Medicare Part A and Part B expansions, although welfare improving, entail substantial direct costs because of old-age medical expenses, with MVPFs of approximately 0.15, at the low end of our estimates. Reductions in income taxes at older ages[4] would boost participation and welfare but would incur significant net fiscal costs, yielding modest MVPFs of 0.23–0.26. Finally, savings subsidies would substantially increase welfare and can have high MVPFs, particularly if higher income tax revenue from taxation of capital gains made possible by the additional savings materializes in the long term. However, such subsidies may lower old-age participation by facilitating retirement savings.

[2] This does not mean, however, that the DRC can be indefinitely increased. The effects of a large DRC and an increase in the early retirement penalty are similar to those of uniform increases in the NRA.

[3] The 1983 Social Security reform gradually increased the NRA to 67 for individuals born in 1960 and later. This policy was motivated by substantial gains in average life expectancy throughout the 20th century; however, recent trends—including the impact of the COVID-19 pandemic—have reversed some of these gains (Arias et al. 2022). A contemporary reform proposal suggests gradually raising the NRA to 70 for individuals born in 1981 and later (Congressional Budget Office 2024).

[4] An example of such a policy is the income tax cliff for senior citizens proposed by Barack Obama during his presidential campaign in 2008.

Overall, our results suggest that Social Security reforms that increase the claiming-age dependence of benefits, reforms that marginally improve benefit progressivity, and retirement saving incentives can achieve substantial welfare gains at favorable fiscal tradeoffs.

We extend our analysis along several key dimensions. We develop a tractable continuous-time model to derive analytical insights into optimal retirement choices under labor supply nonconvexities. Unhealthy workers optimally retire earlier than their healthy counterparts. An option value of working under uncertainty, similar to what arises in real-options models (Dixit 1989; Leland 1994), naturally emerges and decreases with age, increasing endogenous labor supply elasticities. We also propose a novel, simplified implementation of optimal Pareto-improving policies for consumption-risk-neutral agents as studied by Golosov et al. (2016). This implementation features retirement benefits similar to Social Security benefits that depend only on lifetime income and the retirement age, complemented by history-independent taxes and lump-sum transfers. It thus overcomes the complexity that typically characterizes full-history-dependent schemes (Farhi and Werning 2013; Werning 2011). Finally, we explore robustness using alternative preference structures and discuss quantitative tradeoffs of our model.

1.1 Literature Review

This paper builds on studies on the empirics of health shocks and retirement incentives. It contributes to the literature on dynamic taxation with and without endogenous retirement and on quantitative life cycle models of retirement and to the literature that estimates the fiscal tradeoffs of policy reforms.

Empirics of Health Shocks and Retirement Incentives. An extensive empirical literature documents the relationship between retirement behavior, health status, and social security systems around the world (McGarry 2004; Blau and Goodstein 2010; Maestas and Zissimopoulos 2010; Coile 2019). Gruber and Wise (1998, 2004), with accompanying volumes of comparative studies documenting that various disincentives to continue working created a trend toward early retirement in OECD countries. Our main result that tightly links Social Security *claiming* incentives with *retirement* is consistent with Shoven, Slavov and Wise's (2017) survey evidence showing that two-thirds of respondents claim Social Security retirement benefits upon stopping work. Liebman and Luttmer's (2015) experiment, which provided information on the benefits of delay, found no significant impact on actual claiming behavior. This suggests that a lack of awareness or behavioral biases may not be the primary determinants of individuals' claiming decisions. Blundell et al. (2016) review the empirical evidence on factors affecting retirement and labor supply. These studies motivate our modeling framework and the incorporation of health as an essential driver of retirement behavior. Prior and concurrent macroeconomic studies have analyzed the impacts of different dimensions of health, such as medical treatments Papageorge (2016), frailty Hosseini, Kopecky and Zhao (2022), and mental health Abramson, Boerma and Tsyvinski (2024), without systematically studying their policy implications.

Dynamic Income Taxation without Retirement. Our paper builds on the insights of the early literature on nonlinear income taxation. Mirrlees (1971) develops the theory and optimal tax formulas that Saez (2001) links to estimated elasticities. Albanesi and Sleet (2006) develop a dynamic Mirrlees model and focus on implementing the optimal allocations with a restricted set of instruments. The subsequent literature develops the dynamic Mirrlees model with persistent productivity shocks (Farhi and Werning 2013) and focuses on the evolution of the labor wedge. Golosov et al. (2016) disentangle the motives of insurance and redistribution. Stantcheva (2017) and Makris and Pavan (2021), respectively, incorporate endogenous human capital acquisition and learning-by-doing. Comprehensive surveys of the dynamic taxation literature can be found in Golosov and Tsyvinski (2015),Stantcheva (2020), and Kaplow (2024).

These papers assume that the retirement age is exogenous and abstract from health shocks. They find that the labor wedge should increase with age and that linear history-independent but age-dependent taxes are close to optimal. We qualify the results of this literature with our assessment that, when retirement is endogenous, Social Security—a history-dependent and progressive instrument—is the most important policy instrument.

Dynamic Income Taxation with Endogenous Retirement. The first analysis of retirement and optimal taxation came from Diamond and Mirrlees (1978). In their framework, workers are subject to disability shocks (as they subsequently were in Golosov and Tsyvinski 2006). All able workers choose the same retirement age and share the same productivity at any given age. Hence, their retirement decisions do not interact with the income distribution. In addition, Diamond and Mirrlees (1978) do not allow for an intensive margin of labor supply. Other papers study optimal taxation with an extensive margin of labor supply in a static framework (Jacquet, Lehmann and Van der Linden 2013; Gomes, Lozachmeur and Pavan 2017; Rothschild and Scheuer 2013; Saez 2002).

Recent literature has analyzed optimal tax and retirement benefits and the timing of retirement. Choné and Laroque (2014), Cremer, Lozachmeur and Pestieau (2004), Michau (2014), and Shourideh and Troshkin (2015) introduce the retirement margin in the analysis of optimal tax and retirement benefit systems. In these papers, a permanent shock deterministically pins down the whole history of productivity, as in a static setting. Shourideh and Troshkin (2015) find that when the fixed cost of work increases in wages, the static retirement wedge incentivizes delayed retirement. Other papers study aspects of retirement, taxation, and social security design with fundamental differences from the current paper. Nishiyama and Smetters (2007) and Hosseini and Shourideh (2019) study the privatization and funding of social security in overlapping-generation economies.

Our theoretical results contribute to this literature and provide new insights that require modeling of lifetime persistent income shocks and health shocks. Our result that the non-convexities in labor supply imply the optimal labor wedge in poor health is smaller qualifies Saez's (2002) EITC result. It also highlights the role of promised utilities and retirement consumption in a dynamic setting. On retirement savings subsidies in the presence of mortality risk, we show that such subsidies can arise optimally under incomplete markets and rational preferences (Bastani and Ring 2024). In contrast, Moser and Olea de Souza e Silva (2019) rationalize such policies with present bias.

Quantitative Life Cycle Models of Retirement. Our analysis of the Mirrlees optimal policies sheds new light on the quantitative literature that positively or parametrically optimizes social insurance policies (Karabarbounis 2016; Jones and Li 2023; Bairoliya and McKiernan 2023; Borella, De Nardi and Yang 2023; Yu 2024; Fan, Seshadri and Taber 2024). Our results emphasize the importance of addressing the negative welfare effects of policies aimed at increasing participation in old age, such as NRA increases as in French (2005) and French and Jones (2011). The paper closest to ours that studies welfare-increasing policies is Huggett and Parra (2010), which quantifies the level of insurance provided by the US Social Security and tax system in a model with a fixed retirement age. The authors quantitatively find that Social Security benefits that are linear or progressive in lifetime income are equally desirable under the status quo tax system. However, as the authors acknowledge, their analysis cannot identify which policies come close to achieving larger welfare gains, and they abstract from the endogeneity of the retirement choice. Accounting for endogenous retirement and health shocks, we find, among other differences, that progressive reforms are more effective than linear reforms of US Social Security and that reforms of the claiming-age dependence are the most effective among the menu of common reforms we consider.

Estimation of Fiscal Tradeoffs of Policy Reforms. Our paper is related to the growing literature that estimates the MVPF of policy reforms (Mayshar 1990; Slemrod and Yitzhaki 1996; Kleven and Kreiner 2006; Hendren 2016; Hendren and Sprung-Keyser 2020). Empirical estimates of the MVPF of life cycle policies such as Social Security, Medicare, permanent tax reforms, and retirement subsidies are hard to come by because (i) exogenous variation in and reforms of these large-scale, often national programs are rare and (ii) their effects materialize in the long run over the time frame of cohorts, such that a dynamic analysis is needed. For instance, even though Finkelstein and McKnight (2008) find a large MVPF for the *introduction* of Medicare in the US, our model predicts a much smaller MVPF of the *expansion* of Medicare with respect to the US status quo because Medicare expenditures in old age are so large. To our knowledge, our structural estimates of the life cycle MVPFs in Section 7 are novel in this literature. More research is needed to empirically test the quantitative prediction of our model on the MVPFs of retirement and health policies.

Outline. The paper is structured as follows. Section 2 sets up the life cycle model of endogenous retirement with health and wage shocks. Section 3 develops the planning problem and its recursive formulation. Section 4 determines the optimal retirement policies and discusses our three theoretical results. Section 5 presents our full quantitative model estimation, calibration, and decomposition of how the three channels of our interest affect retirement. Section 6 quantifies the effect of nonconvexities on the optimal policies and presents the labor and savings distortions induced by status quo US policies. Section 7 decomposes the welfare contribution of each policy instrument and studies granular reforms of Social Security, Medicare, income taxes, and savings subsidies in the US. Section 8 presents extensions. Section 9 concludes. All proofs are presented in Theory Appendix A. Data Appendix B and Quantitative Appendix C contain our data, some additional figures, and the methods we use in our quantification analysis.

2 A Model of Endogenous Retirement Choice

2.1 Environment

Technology: The economy is populated with workers who live up to T years. Agents work $n_t \geq 0$ hours at time t at a wage rate w_t. Technology is linear, as in Mirrlees (1971).

$$y_t = w_t n_t \tag{1}$$

Wages are a function of two shocks that affect workers: ability θ_t and health h_t.

$$w_t = w_t(\theta_t, h_t) \tag{2}$$

where $w_t(\cdot, h_t)$ is strictly increasing and concave in ability (for each health status h_t, $\partial w_t(\theta, h_t)/\partial\theta > 0$, $\partial^2 w_t(\theta, h_t)/\partial\theta^2 \leq 0$). This wage determination equation, as in the similar frameworks of Erosa et al. (2016), French (2005), and French and Jones (2011), allows health, age, and idiosyncratic ability shocks to affect wages. The notation $\varepsilon_{w\theta} \equiv d\ln\left(w_t(\theta, h_t)\right)/d\left(\ln\theta_t\right)$ represents the elasticity of wages to ability.

Preferences: Workers have preferences in consumption and labor that are impacted by their health status $u(c_t, n_t; h_t)$. This specification captures many preferences, such as those described in French and Jones (2011) and Rogerson and Wallenius (2013), that can lead, with fixed costs of working, to nonconvexities in labor supply. For instance, in French and Jones

(2011), the utility from consumption and leisure is

$$u(c_t, n_t; h_t) = \frac{1}{1-\nu} \left(c_t^{\gamma} l_t \left(n_t; h_t \right)^{1-\gamma} \right)^{1-\nu} \tag{3}$$

where $\gamma \in [0, 1]$ is the weight on consumption and ν is the coefficient of relative risk aversion (CRRA). Leisure in period t is given by:

$$l_t = L - n_t - \phi_n I\{n_t > 0\} - \phi_{RE} RE_t - \phi_h(h_t) \tag{4}$$

where L is total leisure endowment, ϕ_n is the fixed time cost of work incurred when hours worked are positive, ϕ_{RE} is a reentry cost incurred by individuals who reenter the labor market $RE_t = 1$, and $\phi_h(h_t)$ is time lost because of poor health.

Health, Disability, and Mortality: In each period, individuals face uncertainty in health, $h_t \in \mathcal{H} \equiv \{0, 1, 2\}$, which takes three values: 0 for good health, 1 for poor health, and 2 for a disability. The initial health status in period 1 has probability $\pi_{j,t=1} = Pr(h_1 = j)$. Health in the next period, h_{t+1}, depends on the individual's current health and age and evolves according to a Markov chain across the three states, with an age-dependent Markov transition matrix π. A typical element of the health transition matrix at age t is given by:

$$\pi_{j,i,t+1} = Pr(h_{t+1} = j | h_t = i, t+1), \qquad i, j \in \mathcal{H}^2 \tag{5}$$

Blundell et al. (2016) conclude in their study of retirement incentives that *"falling health is an important determinant of retirement and it is a feature worth capturing in a retirement model."*. Our model captures the three properties of health that they find to be important for retirement decisions, with some nuances.

1. **Labor supply nonconvexity channel:** *"Declining health makes work less pleasant"*. We consider several forms of nonconvexities in labor supply: a time cost of work as in French and Jones (2011), setup time costs as in Prescott et al. (2009), and fixed utility costs of work. In particular, these nonconvexities can depend on health and feature larger fixed costs in poor health and disability.

2. **Productivity channel:** Declining health *"can reduce an individual's productivity and thus the individual's wage."* For each ability θ, $w(\theta_t, 0) \geq w(\theta_t, 1) \geq w(\theta_t, 2)$.

3. **Mortality channel:** *"Health shocks might reduce life expectancy."* The parameter s_{t+1} denotes the probability that an individual is alive at age $t + 1$ conditional on their being alive at age t. The survival probability depends on age and previous health status, as $s_{t+1}(h_t) = S(h_t, t + 1)$. Because individuals live up to a maximum age T, $s_{T+1} = 0$ for any health status h.

Ability and Its Evolution: Following Farhi and Werning (2013), agents are born in period 1 with heterogeneous earning ability θ_1 with distribution $f_1(\theta_1)$ and at time t with distribution $f^t(\theta_t|\theta_{t-1})$ over a fixed support $\Theta = [\underline{\theta}, \overline{\theta}]$. Let the probability of a history $\{\theta^t\} \in \Theta^t$ be $P(\theta^t) = f^t(\theta_t|\theta_{t-1}) \cdots f^2(\theta_2|\theta_1) f_1(\theta_1)$.

We introduce notation for the novel elements of health and mortality shocks. Denote by $\{h^t\} \equiv (h_1, \ldots, h_t)$ the history of health shocks up to period t,[5] and by $\Pi(h^t)$ the probability of a history $\{h^t\}$, $\Pi(h^t) \equiv \pi_{h_t,h_{t-1},t} \cdots \pi_{h_2,h_1,2}\pi_{h_1,t=1}$. Similarly, let $S(h^0) = 1$ and $S(h^{t-1}) \equiv s_t(h_{t-1}) \cdots s_2(h_1)$ be the probability of survival until t. The expected utility, with discount factor β, from an allocation $\mathbf{x}$ that specifies consumption and hours as a function of the history of ability $\{\theta^t\}$ and health $\{h^t\}$ is

$$U(\mathbf{x}) = \sum_{t=1}^{T} \beta^{t-1} \sum_{h^t \in \mathcal{H}^t} S(h^{t-1})\Pi(h^t) \int_{\Theta^t} u\big(c(\theta^t, h^t),\, n(\theta^t, h^t); h_t\big)\, P(\theta^t) d\theta^t \tag{6}$$

3 Planning Problem

In every period, the planner observes consumption c_t, work status, health status, and income from work y_t. However, the planner does not observe θ_t and therefore does not observe labor $n_t = y_t/w_t(\theta_t, h_t)$, either. This private information friction reflects the fact that the planner cannot set policies that depend on unobservable ability but can set policies that depend on income, such as taxes, and health status, such as disability insurance and Medicare.

This section first sets up the planning problem and explains the structure of the incentive constraints. Second, we relax the incentive problem using the first-order approach (FOA) procedure developed in Farhi and Werning (2013), and we incorporate health status and the retirement decision. Third, through a redefinition of the state space, we write a recursive program for the planner's relaxed problem.

3.1 Incentive Compatibility

In constrained efficient allocations, we focus on direct revelation mechanisms and suppose, based on the revelation principle, that agents can make a reporting strategy that specifies their reported ability after each history. Formally, a direct revelation mechanism $\mathcal{M}$ is defined as a tuple $\mathcal{M} = (M, (c, y))$ where the message space $M = \Theta^t$ and agents report their ability history $\hat{\theta}^t = (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_t) \in M$ potentially as a function of the observed health history h^t, $\hat{\theta}^t = r(\theta^t, h^t)$, where $r : \Theta^t \times \mathcal{H}^t \to \Theta^t$ is the reporting strategy. Allocations $c : \Theta^t \times \mathcal{H}^t \to \mathbb{R}_+, y : \Theta^t \times \mathcal{H}^t \to \mathbb{R}_+$ determine consumption c and output y based on reported ability $\hat{\theta}^t$ and observed health h^t. Let the continuation utility after history (θ^t, h^t) under a reporting

[5] We use the underscore notation h_t for realizations at time t and the superscript h^t for the history of realizations $h_1, h_2, \ldots$ until time h_t.

strategy r, denoted by $\omega^r(\theta^t, h^t)$, solve

$$\omega^r(\theta^t, h^t) = u\left(c(\hat{\theta}^t, h^t), \frac{y(\hat{\theta}^t, h^t)}{w_t(\theta_t, h_t)}; h_t\right) + \beta s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega^r(\theta^{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1},$$

with $\omega^r(\theta^{T+1}, h^{T+1}) = 0$. The continuation utility after truthful revelation $\omega(\theta^t, h^t)$ solves

$$\omega(\theta^t, h^t) = u\left(c(\theta^t, h^t), \frac{y(\theta^t, h^t)}{w_t(\theta_t, h_t)}; h_t\right) + \beta s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega(\theta^{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

An allocation $c(\theta^t, h^t), y(\theta^t, h^t)$ is incentive compatible if and only if truth telling yields a weakly higher continuation utility than any reporting strategy r:

$$\text{[IC]: } \omega(\theta_1, h_1) \geq \omega^r(\theta_1, h_1) \quad \forall\ \theta_1, h_1, r \tag{7}$$

Denote by X^{IC} the set of incentive-compatible allocations. To solve this dynamic problem, we now relax the planner's incentive constraints using the FOA, which requires the following assumptions:

Assumption 1. *1.A The utility function $u(c, y; h, \theta)$ is bounded and twice continuously differentiable. The partial derivative $u_\theta(c, y; h, \theta)$ is bounded.*

1.B The density $f^t(\theta_t|\theta_{t-1})$ has full support and has a continuously differentiable derivative with respect to its second argument $\partial f^t(\theta_t|\theta_{t-1})/\partial\theta_{t-1}$ that is also bounded.

Suppose the agent has reported ability truthfully until time $t-1$ and deviates to $\hat{\theta}$ at time t; that is, the reporting strategy $r_s(\theta^{s-1}) = \theta_s$ for all $s \leq t-1$ and $r_t(\theta^t) = \hat{\theta} \neq \theta_t$. The continuation utility under this reporting strategy solves

$$\omega^r(\theta^t, h^t) = u\left(c(\theta^{t-1}, \hat{\theta}, h^t), \frac{y(\theta^{t-1}, \hat{\theta}, h^t)}{w_t(\theta_t, h_t)}; h_t\right) + \beta s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega^r(\theta^{t-1}, \hat{\theta}, \theta_{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

Incentive compatibility in (7) implies that for almost all θ^t, h^t, temporal incentive compatibility holds:

$$\omega(\theta^t, h^t) = \max_r \omega^r(\theta^t, h^t) \tag{8}$$

Inversely, if equality (8) holds after all θ^{t-1}, h^t and almost all θ_t, then (7) also holds (Kapička 2013). Applying the envelope theorem of Milgrom and Segal (2002) under Assumption 1 to the agent's reporting problem, we obtain the marginal variation in continuation utility with respect to reports:

$$\dot{\omega}(\theta^t, h^t) \equiv \frac{\partial \omega(\theta^t, h^t)}{\partial \theta_t} = -\frac{w_{\theta,t}(\theta_t, h_t)}{w(\theta_t, h_t)} n(\theta^t, h^t) u_n\left(c(\theta^t, h^t), n(\theta^t, h^t); h_t\right) + \beta s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega(\theta^{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} \frac{f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1} \tag{9}$$

Let X^{FOA} be the set of allocations that satisfy the necessary condition for incentive compatibility (9). This expression characterizes the slope of promised utility with respect to ability types at incentive-compatible allocations. The first term is the static rent as in Mirrlees (1971), while the second term is a dynamic rent (Pavan et al. 2014) that summarizes the agent's advance information about his future ability (this second term vanishes when ability is not persistent).

3.2 Government's Objective

The analysis is in partial equilibrium, and intergenerational transfers are exogenous. The planner can save aggregate resources in a small open economy at a gross return $R \equiv 1+r$. Her objective is to minimize the net present value of the cost of providing an allocation, subject to incentive compatibility (7) and the expected lifetime utility of each initial type (θ, h) being above a utility floor $\underline{U}(\theta, h)$. Let $U(\{c, y\}; \theta, h)$ be lifetime utility as defined in (6) for an agent of initial type (θ, h). The relaxed planning problem, denoted by P^{FOA}, replaces the incentive constraint with the envelope condition (9) and is given by

$$\begin{aligned} K(\underline{U}(\theta, h)_\Theta) \equiv \min_{\{c,y\}} & \sum_{t=1}^{T} \left(\frac{1}{R}\right)^{t-1} \sum_{h^t \in \mathcal{H}^t} S(h^{t-1}) \Pi(h^t) \int_{\Theta^t} \left[c(\theta^t, h^t) - y(\theta^t, h^t)\right] P(\theta^t) d\theta^t \\ & \text{subject to } U(\{c, y\}; \theta, h) \geq \underline{U}(\theta, h), \\ & y(\theta^t, h^t) \geq 0, c(\theta^t, h^t) \geq 0, \\ & \{c, y\} \in X^{\text{FOA}} \end{aligned}$$

3.3 Recursive Problem

To write the problem recursively, we introduce the following notation for the expected future continuation utility and the future marginal rent:

$$v(\theta^t, h^t) \equiv s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega(\theta^{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

$$\Delta(\theta^t, h^t) \equiv s_{t+1}(h_t) \int \sum_{h_{t+1} \in \mathcal{H}} \omega(\theta^{t+1}, h^{t+1}) \pi_{h_{t+1}, h_t, t+1} \frac{f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1}$$

The continuation utility $\omega(\theta^t, h^t)$ and envelope condition can be rewritten as:

$$\omega(\theta^t, h^t) = u\left(c(\theta^t, h^t), n(\theta^t, h^t); h_t\right) + \beta v(\theta^t, h^t)$$
$$\dot{\omega}(\theta^t, h^t) = -\frac{w_{\theta,t}(\theta_t, h_t)}{w(\theta_t, h_t)} n(\theta^t, h^t) u_n\left(c(\theta^t, h^t), n(\theta^t, h^t); h_t\right) + \beta\Delta(\theta^t, h^t)$$

We can then write a recursive formulation of the relaxed planning problem for the states of promised utility (to keep track of the participation constraint), marginal future rents (to account for dynamic incentives), current ability, health, and time $(v, \Delta, \theta_-, h_-, t)$:

$$K(v, \Delta, \theta_-, h_-, t) = \min_{X^{\text{FOA}}} s_t(h_-) \sum_{h\in\mathcal{H}} \pi_{h,h_-,t} \int_\Theta \Big[c(\theta, h) - w_t(\theta, h) n(\theta, h) + \frac{1}{R} K(v(\theta, h), \Delta(\theta, h), \theta, h, t+1) \Big] f^t(\theta|\theta_-) d\theta$$

subject to

$$\omega(\theta, h) = u\left(c(\theta, h), n(\theta, h); h\right) + \beta v(\theta, h)$$
$$\dot{\omega}(\theta, h) = -\frac{w_{\theta,t}(\theta, h)}{w(\theta, h)} n(\theta, h) u_n\left(c(\theta, h), n(\theta, h); h\right) + \beta\Delta(\theta, h),$$
$$v = s_t(h_-) \int \sum_{h\in\mathcal{H}} \omega(\theta, h) \pi_{h,h_-,t} f^t(\theta|\theta_-) d\theta,$$
$$\Delta = s_t(h_-) \int \sum_{h\in\mathcal{H}} \omega(\theta, h) \pi_{h,h_-,t} \frac{f^t(\theta|\theta_-)}{\partial\theta_-} d\theta,$$
$$\omega(\theta, h) \geq \underline{U}(\theta, h) \text{ if } t = 1$$

where the minimization is over the functions $c(\theta, h), n(\theta, h), \omega(\theta, h), v(\theta, h), \Delta(\theta, h)$. We obtain the solution of the relaxed problem by minimizing the problem at time 1, treating Δ as a free variable:

$$K\left(\underline{U}(\theta, h)_\Theta\right) = \min_\Delta K\left(\underline{U}(\theta, h)_\Theta, \Delta, 1, 0, 1\right)$$

This recursive formulation allows the dimensionality of the problem to be reduced.

The solution to the relaxed planning problem might not be a solution to the full program because the envelope condition is only a necessary condition. In the static Mirrlees (1971) nonlinear income taxation model, the FOA is valid if the utility function satisfies the Spence–Mirrlees single-crossing property and a monotonicity condition on the allocation of income. Boerma et al. (2022) show that bunching is a natural feature of multidimensional mechanisms with general correlations across types. Surprisingly, Golosov and Krasikov (2023) find that the FOA is likelier to hold in the multidimensional setting than in unidimensional settings of optimal income taxation where there are no correlations across types. Our problem with serial correlation across types, but with uncorrelated innovations in our calibration in Section

6.1, lies between these two extremes. Hence, for our quantification, we numerically check the incentive compatibility of the candidate allocation, any omitted nonnegativity constraints, and the interiority conditions, using a procedure in the spirit of Farhi and Werning (2013).

4 Optimal Policies

In this section, we characterize the optimal allocations, obtained as solutions to the relaxed program above, using wedges, or implicit taxes, and subsidies, distortions, or markups. To simplify the exposition, from here on, the dependence on the full history is often left implicit; for example, $c_t = c(\theta^t, h^t)$ and $\tau_{L,t} = \tau_{Lt}(\theta^t, h^t)$. Similarly, function arguments are sometimes left out; for example, $w_{\theta,t} = (\partial/\partial\theta) w_t(\theta_t, h_t)$. The term $\mathbb{E}_t$ denotes the expectation as of time t, conditional on (θ_t, h_t).

Definition 1. *(Wedges) The labor wedge is the gap between the marginal rate of substitution of consumption for leisure and the marginal product of labor (or the wage).*

$$\tau_L(\theta^t, h^t) = 1 + \frac{1}{w_t} \frac{u_n\left(c_t, n_t; h_t\right)}{u_c\left(c_t, n_t; h_t\right)} \tag{10}$$

The savings wedge is the gap between the marginal rate of substitution in consumption across $t+1$ and t and the real return on savings (or the interest rate).

$$\tau_K(\theta^t, h^t) = 1 - \frac{1}{R\beta} \frac{u_c\left(c_t, n_t; h_t\right)}{\mathbb{E}_t u_c\left(c_{t+1}, n_{t+1}; h_{t+1}\right)} \tag{11}$$

Wedges are akin to locally linear subsidies and taxes and would all be zero in the first-best economy. In this section, we characterize these wedges for allocations that solve the relaxed planning problem.

We now consider the effects of the channels through which health affects retirement in our life cycle model by examining each in isolation and show how they affect the optimal wedges. Then, we study their interactions.

4.1 Labor Supply Nonconvexity Channel

In this section, we study the effect of labor supply nonconvexities on the optimal policies. We isolate this channel with the nonconvexity of utility from leisure but assume no wage difference due to differences in health or survival probability differences due to health, $w_t(\theta_t, h_t) = w_t(\theta_t, 0) = \theta_t$, $s(h_t) = s_t(0) = 1$, $\forall h_t \in \mathcal{H}$. We call this setting Economy 1.

Assumption 2. *2.A Utility is separable in consumption and labor:*

$$u\left(c_t, n_t; h_t\right) = \tilde{u}(c_t) - \psi\left(n_t; h_t\right)$$

where $\tilde{u}$ is concave and $\psi(\cdot, h_t)$ is bounded and twice continuously differentiable in n_t.

2.B Ability is a log autoregressive process with persistence $\rho \leq 1$:

$$\ln(\theta_t) = \rho \ln(\theta_{t-1}) + \epsilon_t$$

where ϵ_t has density $f^t(\epsilon|\theta_{t-1})$ with $\mathbb{E}[\epsilon|\theta_{t-1}] = 0$.

Note that the disutility from labor ψ in Assumption 2.A need not be convex in labor or separable in health. Let $\varepsilon_t^u(h)$ and $\varepsilon_t^c(h)$ be the uncompensated and compensated labor supply elasticities to the net wage, with savings held fixed.[6]

Lemma 2. *Suppose that Assumptions 1 and 2 hold in Economy 1. The optimal labor wedge evolves according to*

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{\varepsilon_t^c(h_t)}{1+\varepsilon_t^u(h_t)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h_t\Big] =$$

$$Cov\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right) + \rho\frac{\tau_{L,t-1}^b}{1-\tau_{L,t-1}^b}\frac{\varepsilon_{t-1}^c(h_{t-1})}{1+\varepsilon_{t-1}^u(h_{t-1})} \tag{12}$$

for all $h_t \in \mathcal{H}$, where ε^u and ε^c are the endogenous uncompensated and compensated labor supply elasticities to the net wage with savings held fixed.

The proof and expressions of the elasticities are in Appendix A.1. The expectation is taken for the history up until $t-1$ *and* for each health status $h_t \in \mathcal{H}$, i.e., for given $\{\theta^{t-1}, h^t\}$ and conditional on survival. Like the labor wedge in Farhi and Werning (2013), equation (12) has a drift term and an autoregressive term. Long-run incentives cause a positive covariance between consumption growth and productivity. The government promises higher consumption growth to induce higher-ability agents to reveal their true types. This, however, makes insurance valuable and is captured by the drift term. In the autoregressive term, the persistence of the labor wedge equals the persistence of ability ρ multiplied by the elasticity ratio $\frac{\varepsilon_{t-1}^c(h_{t-1})}{1+\varepsilon_{t-1}^u(h_{t-1})}$.[7] With high persistence, the optimal labor wedge increases over time as age increases, until the elasticity ratio $\frac{\varepsilon_t^c(h_t)}{1+\varepsilon_t^u(h_t)}$ that modulates the left-hand side increases endogenously. At that point, this higher elasticity forces a reduction in the optimal labor wedge. In addition, the results from the static Mirrlees model on the standard zero labor distortions at the bottom and top continue to apply when retirement is endogenous.

Proposition 3 below shows that the optimal labor wedge trends downward under a variety of preferences that lead to nonconvexity of labor supply. Even though our autoregressive expres-

[6] We define the uncompensated and compensated elasticities as in Saez (2001) at constant savings:

$$\varepsilon^u = \frac{\frac{\psi_n(n;h)}{n} + \frac{\psi_n(n;h)^2}{u'(c)^2}u''(c)}{\psi_{nn}(n;h) - \frac{\psi_n(n;h)^2}{u'(c)^2}u''(c)}, \quad \varepsilon^c = \frac{\frac{\psi_n(n;h)}{n}}{\psi_{nn}(n;h) - \frac{\psi_n(n;h)^2}{u'(c)^2}u''(c)}$$

[7] The endogenous elasticities depend on the full history $\{\theta^t, h^t\}$. For ease of notation, we emphasize the dependence on health, as this will be the focus of our analysis.

sion for the optimal labor wedge (12) applies for preferences that are separable in consumption and labor, it is useful for analyzing the optimal policies in the limiting case of prevalent forms of labor supply nonconvexities. First, it allows us to analyze nonconvexities in the form of lost time out of leisure that can worsen due to poor health in equation (4) of French and Jones (2011) at the separable limit where $\nu \to 1$ or generally under preferences of the form:

$$u\left(c_t, n_t; h_t\right) = \tilde{u}\left(c_t\right) + \frac{1}{1-\gamma}\left(L - n_t - \phi_n I\{n_t > 0\} - \phi_{RE} RE_t - \phi_h(h_t)\right)^{1-\gamma} \tag{13}$$

for $\gamma > 0$ and CRRA utility of consumption $\tilde{u}$. Second, it can capture nonconvexities in the form of a smooth version of a production setup time cost $g(n) = \max\{0, n - n_h\}$ as in Prescott et al. (2009) and Rogerson and Wallenius (2013).[8] Third, it allows us to analyze nonconvexities in the form of a health-dependent utility cost of participation, in the spirit of Cho and Rogerson (1988) and Ndiaye (2018):

$$u\left(c_t, n_t; h_t\right) = \tilde{u}\left(c_t\right) - \psi(n_t) - \phi(h_t) I\{n_t > 0\}, \quad \phi(0) \le \phi(1) < \phi(2) \tag{14}$$

where ψ is convex in labor and $\phi(h_t)I\{n_t > 0\}$ is a utility cost of participation. This formulation turns out to be very tractable while maintaining the nonconvexity of labor supply.[9]

Proposition 3. *Under the assumptions of Lemma 2, then, for all these preferences,*

(i) the separable limit of French and Jones (2011) fixed time cost preferences (3) and (4) at $\nu \to 1$,

(ii) general separable and nonconvex labor supply preferences with time fixed costs of participation defined by (13),

(iii) preferences with fixed setup costs of production as in Prescott et al. (2009),

(iv) nonconvex labor supply preferences with a utility cost of participation defined by (14),

the labor wedge ultimately decreases in poor health, that is, $\exists t' \ge 1$ such that

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}} \frac{1}{\beta R} \frac{u'(c_{t-1})}{u'(c_t)} | h_t\Big] \le \frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}$$

for all $t \ge t'$ and (almost) all $\{\theta^{t-1}, h^{t-1}, h_t\}$ with $h_t > h_{t-1}$.

[8] Our strict analog in the spirit of Prescott et al. (2009) and Rogerson and Wallenius (2013) considers health-dependent setup costs $n_2 \ge n_1 \ge n_0$ so that production from labor is $g(n_t) = \max\{0, n_t - n_h\}$. This function, and preferences with a fixed utility cost at $\phi_h I\{n > n_h\}$, are not differentiable at n_h. However, a smooth family of functions that approximates g is the tanh function $g_{\tanh}(n_t) = \frac{1}{k}\tanh\big(k\,(n - n_h)\big)$, where $k > 0$ is the smoothing parameter and $g_{\tanh}(n_t) \to_{k \to 0} g(n_t)$. This smoothing satisfies the properties of g in Prescott et al. (2009) (see Fig. 4 in Prescott et al. 2009, p. 27, for a graphical discussion of g). Our result applies to general smoothed nonconvexities that satisfy two simple conditions presented in Appendix A.2.

[9] This nonconvexity is different from a relabeling of the utility of consumption in Farhi and Werning (2013) as fixed costs are paid only when working.

The proof is in Appendix A.2. The intuition lies in analyzing the endogenous response of the elasticity ratio $\frac{\varepsilon_t^c(h)}{1+\varepsilon_t^u(h)}$. For isoelastic disutility of labor $\psi(n_t) = \frac{n_t^{1+1/\varepsilon}}{1+1/\varepsilon}$, then the elasticity ratio is constant, equal to $\frac{\varepsilon}{1+\varepsilon}$. This is the case for Farhi and Werning (2013) and the numerical exercise of Stantcheva (2017). However, for preferences that lead to nonconvexities in labor supply, compensated and uncompensated elasticities are endogenous and increase in poor health—even when ψ in (14) is isoelastic.

Intuitively, for general preferences (13), the separable limit of French and Jones (2011), and setup time costs as in Prescott et al. (2009), time lost out of leisure, especially in poor health, increases the disutility of labor supply. For utility cost preferences (14), participation and poor health lead to a drop in flow utility. Since, as age increases, the government distorts consumption of workers less, it delivers promised utility to the unhealthy workers by lowering labor distortions. This is consistent with Saez's (2002) result in a static setting that if behavioral responses are concentrated on the extensive margin (what we characterize as nonconvexities), the optimal schedule switches from a negative income tax (NIT) with high positive marginal tax rates to an earned income tax credit (EITC) with negative or small marginal tax rates. However, in our dynamic setting, the labor wedge remains positive, in contrast to Saez (2002). The intuition is that households in our dynamic model and Prescott et al. (2009)–type models can optimally choose a fraction of periods to work. The planner's tradeoff is therefore, per period, much milder than the "work forever vs. never work" knife-edge in Saez (2002). Because individuals eventually supply labor, the planner does not need to make a large unconditional transfer to lifelong nonworkers. Hence, the intensive-margin distortion can remain positive without anyone being permanently pushed out of the labor force, and extensive-margin incentives are provided by means of retirement consumption.

4.2 Productivity Loss Channel

In this section, we study the effect of the fact that declining health lowers wages. We isolate this channel with a wage that decreases in poor health but assume no labor supply nonconvexity or differences in survival probability due to poor health, $w_t(\theta_t, h_t) \leq w_t(\theta_t, h')$, $s(h_t) = s_t(0)\ \forall h_t \in \mathcal{H}, h' \leq h_t$ and $u(c_t, n_t) = \tilde{u}(c_t) - \psi(n_t)$, where ψ is isoelastic with Frisch elasticity ε. We call this setting Economy 2.

Lemma 4. *Suppose that Assumptions 1 and 2 hold in Economy 2; then, the labor wedge evolves according to*

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\varepsilon_{w\theta,t}(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] =$$
$$\rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}\frac{1}{\varepsilon_{w,\theta_-,t-1}(h_-)} + \left(1+\frac{1}{\varepsilon}\right) Cov\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right)$$

The proof is in Appendix A.1. This expression is similar to the one derived in Stantcheva (2017) for the effect of human capital on the optimal labor wedge. The lemma expresses that the wage effect of poor health affects the labor wedge not through changes in the level of wages but through the wage elasticity with respect to ability. A higher elasticity $\varepsilon_{w\theta,t}(h)$ increases the magnitude of the labor wedge, as it increases the value of insurance and redistribution. The intuition is that if a health status has a higher wage elasticity with respect to ability, then agents with that health status have higher wage inequality among themselves after ability realizations.

Estimates of the variation in the wage elasticity with respect to ability are uncommon since ability is often empirically treated as a residual. French and Jones (2011) estimate a specification where the logarithm of hourly wages at age t, $\ln w_t$, is a function of health status and age, $W(h_t, t)$, and an autoregressive component of wages, θ_t, as follows: $\ln\left(w_t(\theta_t, h_t)\right) = W(h_t, t) + \ln(\theta_t)$. For such specifications, the wage elasticity to ability is constant and independent of health status $\varepsilon_{w\theta,t}(h) = 1$.[10] Therefore, the adverse productivity effect of poor health, in isolation, does not dampen the labor wedge. However, we see in Section 4.4 how it affects the labor wedge through interactions with other channels.

4.3 Mortality Channel

In this section, we study the effect of the fact that health shocks might reduce mortality. Our results here on the savings wedge apply to our full baseline economy, without assumptions that isolate a specific channel.

Proposition 5. *Suppose that Assumptions 1 and 2 hold. Then, the savings wedge satisfies*

$$1 - \frac{1}{s_{t+1}(h_t)}\left(1 - \frac{\sigma^2_{c,t+1} u_c(c_t)}{\beta R u_c(\underline{c_{t+1}})^3}\right) \leq \tau_K \leq 1 - \frac{1}{s_{t+1}(h_t)}\left(1 - \frac{\sigma^2_{c,t+1} u_c(c_t)}{\beta R u_c(\overline{c_{t+1}})^3}\right)$$

where $\sigma^2_{c,t+1}$ is the variance of the marginal utility of consumption in the next period $\sigma^2_{c,t+1} \equiv Var_t\left[\tilde{u}_c\left(c_{t+1}\right)|\theta_t, h_t\right]$ and, respectively, $\underline{c_{t+1}}$ and $\overline{c_{t+1}}$ are the minimum and maximum values of future consumption $\underline{c_{t+1}} \equiv \min_{\{\theta\in\Theta, h\in\mathcal{H}\}} c_{t+1}(\theta^t, \theta, h^t, h)$ and $\overline{c_{t+1}} \equiv \max_{\{\theta\in\Theta, h\in\mathcal{H}\}} c_{t+1}(\theta^t, \theta, h^t, h)$.

The proof is in Appendix A.3. This result generalizes the standard results on the savings wedge in the dynamic taxation literature and qualifies it with new economic intuition.

In the left-hand inequality, if survival is certain, $s_{t+1}(h_t) = 1$, then the inequality yields $0 \leq \frac{\sigma^2_{c,t+1} u_c(c_t)}{\beta R u_c(\underline{c_{t+1}})^3} \leq \tau_K$. This is a generalization of the standard result (Farhi and Werning 2013; Golosov et al. 2016) that when there is no mortality risk, the savings wedge is positive. Methodologically, this result is well known and stems from an application of Jensen's

[10] Similarly to Stantcheva (2017), with a constant elasticity of substitution (CES) wage as a function of ability and health $\varepsilon_{w,\theta} = (\frac{w(\theta_t, h_b)}{\theta_t})^{\eta-1}$, where η is the Hicksian complementarity coefficient between health and ability in wages.

inequality to Rogerson's (1985) inverse Euler equation (IEE). Our result stems from a modern improvement of Jensen's inequality by Liao and Berg (2019).

The right-hand inequality, however, gives an upper bound on the savings wedge. Importantly, this upper bound has nontrivial implications under mortality risk. As the probability of survival decreases, the savings wedge decreases and will turn negative. In this case, optimal allocations subsidize savings. The intuition is that the IEE (Rogerson 1985) rationale for savings distortions without mortality risk is to facilitate work incentives. With mortality risk and observable health, the government will still want to insure against that risk. Since, in the absence of government intervention or annuities in the baseline market structure,[11] households cannot insure against mortality risk and will tend to tilt consumption more toward the present, it is optimal for the government to subsidize savings under high mortality risk.

A broad interpretation of our novel result is that subsidies for savings, such as preferential treatment of 401(k) retirement plans, can be motivated by uninsurable mortality risk despite the existence of the standard savings distortions on incentives to work. Given that the savings distortion from the IEE is small (Farhi and Werning 2013), we should expect savings subsidies to be the norm, and heterogeneous mortality should be part of a quantitative model of retirement (e.g., Jones and Li 2023).

4.4 Interactions

The previous sections considered separately three distinct channels through which health affects the optimal policies: nonconvexities in labor supply, productivity losses, and mortality risk. Here, we explicitly characterize their combined effects.

Lemma 6. *Suppose that Assumptions 1 and 2 hold.*

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\varepsilon_{w\theta,t}}\frac{\varepsilon^c_t(h)}{1+\varepsilon^u_t(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] =$$
$$\rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}\frac{1}{\varepsilon_{w,\theta_-}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}+Cov\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)},\ln(\theta_t)\right)$$

The proof is in Appendix A.1. This lemma emphasizes how the interaction between the nonconvexity and productivity channels jointly shapes the dynamics of the optimal labor wedge. Specifically, an increase in the labor supply elasticity—particularly through nonconvex preferences—shrinks the labor wedge by making distortions of labor choices costlier. Productivity losses in poor health further reduce equilibrium hours worked by unhealthy workers as a share of disposable leisure hours. This leads to an endogenous increase in the labor supply elasticities of unhealthy workers, as the proof of Proposition 3 makes clear. A higher wage elasticity with respect to ability amplifies the insurance motive of the government and thus

[11]See Lockwood's (2018) evidence that most retirees do not buy annuities or long-term care insurance.

increases the magnitude of labor wedges. Finally, the optimal allocations tend to smooth consumption significantly over time, thereby dampening the intertemporal variation in the marginal utility of consumption. As a result, the autoregressive structure in the labor wedge equation gains relative importance. The consumption-smoothing effect increases the persistence of wedges, reinforcing the impact of the productivity and nonconvexity channels in subsequent periods.

5 Quantitative Model

In this section, we develop a comprehensive quantitative model that matches key empirical patterns of retirement behavior and allows us to analyze how Social Security (SS), Medicare, taxes, and savings incentives shape retirement decisions. In Section 5.4, we use this framework to quantify the impact of health on retirement through the channels we identify. Section 6 presents the wedges in the positive economy and a calibrated numerical example of optimal wedges. Section 7 evaluates the welfare contributions of each policy in the positive economy and explores granular policy reforms.

5.1 Setup

We now extend our model to incorporate key institutional features of the status quo US economy. Our approach to modeling retirement—specifically labor force participation and SS claiming decisions—closely follows empirical insights from the retirement literature and builds on the established frameworks of French (2005) and French and Jones (2011). We estimate the model parameters based on US data. This enriched setup will allow us to conduct the detailed policy analyses presented in Sections 6 and 7.

In each period t, individuals face shocks to health (good, poor, or disability), mortality risk, wages, and medical expenditure. They make decisions on how much to consume, how much to work (including both participation and hours worked), and whether to claim SS retirement benefits (if eligible). With respect to our normative economy, here, we specify and introduce several additional elements to better reflect the US economy:

Preferences: Individuals enter the model at age 30 and live up to age 85, with nonseparable preferences over consumption and leisure as specified in equation (3).[12] They also incur health-dependent time costs for labor force participation and reentry,[13] so that

$$l_t = L - n_t - \phi_n(h_t)I\{n_t > 0\} - \phi_{RE}RE_t$$

[12] Grochulski and Kocherlakota (2010) study preferences that are separable in consumption and leisure and nonseparable in time. Our analysis in not directly comparable to theirs.

[13] French and Jones (2011) features a time cost of being unhealthy that does not depend on participation and a time cost of participation that does not depend on health. We consider a time cost of participation that depends on health to allow for nonconvexities in labor supply that more directly depend on health.

Health Transitions and Mortality: Individuals face uncertainty in their health status, $h_t \in \{0, 1, 2\}$, which evolves according to a Markov process with probabilities of transitioning from state i to j of $\pi_{j,i,t+1}$. Mortality risk at age $t+1$ is captured by the probability $s_{t+1} = S(h_t, t+1)$, which depends on both the individual's current health status and age.

Medical Expenses: Out-of-pocket medical expenditures m_t vary by age and health status and are given by

$$m_t = M(h_t, t, P_t) \tag{15}$$

defined as the individual's total medical expenditure net of coverage provided by insurance prior to age 65 and Medicare thereafter.[14] These medical expenditures are modeled to reflect the fact that out-of-pocket expenses increase as individuals age or their health worsens and depend on participation.

Hourly Wages: Hourly wages have two components: a deterministic function of health and age, and an autoregressive process for ability that evolves according to:

$$\begin{aligned} \ln w_t(\theta_t, h_t) &= W(h_t, t) + \theta_t, \\ \ln(\theta_t) &= \rho \ln(\theta_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\rho^2) \end{aligned} \tag{16}$$

SS Retirement Benefits: At a high level, SS retirement benefits depend on a measure of lifetime earnings and are adjusted actuarially based on the age at which benefits are claimed. In addition, benefits recipients under age 70 are subject to an earnings test according to which retirement benefits can be reduced if the recipients continue to earn labor income.

We model SS benefits in detail to match the current US system closely. Individuals become eligible for benefits by contributing payroll taxes during their working years. Benefit levels are determined by the primary insurance amount (PIA), which is based on a worker's average indexed monthly earnings (AIME)— approximately the average of a worker's highest 35 years of earnings. Eligibility to claim benefits begins at the early retirement age (ERA), and claiming is an irreversible, one-time decision. Once retirement benefits are claimed, individuals receive SS payments ss_t until death, subject to actuarial adjustments and the earnings test. Three key factors determine retirement benefit levels: lifetime career earnings, the claiming age, and labor income after benefit collection.

First, lifetime career earnings, which refer to the AIME, are calculated based on a worker's highest 35 years of earnings. To reflect this in the model, we construct an annualized version

[14] Blau and Gilleskie (2008), Imrohoroğlu and Kitao (2012) and French and Jones (2011) explicitly model employer-provided health insurance, while Ozkan (2023) models self-insurance in the form of preventative medical expenditures. Our model abstracts from heterogeneity in health insurance to incorporate other dimensions of heterogeneity.

of AIME, $aime_t$, that evolves according to:

$$aime_{t+1} = \begin{cases} \min\{aime_t + \frac{w_t(\theta_t,h_t)n_t}{35}, aime_{max}\} & \text{if age} < 60 \\ \min\{aime_t + \max\{0, \frac{w_t(\theta_t,h_t)n_t - aime_t}{35}\}, aime_{max}\} & \text{if age} \geq 60 \end{cases}$$

which is capped at a maximum threshold $aime_{max}$.[15] This limits the incentives to continue working after a worker has accumulated 35 years of earnings, unless the income from an additional year exceeds that in earlier lower-earnings years.

PIA_t, the base benefit amount, is a piecewise linear function of $aime_t$:

$$\begin{aligned} PIA_t = & \; 0.9 * \min\{aime_t, aime_0\} \\ & + 0.32 * \min\{\max\{aime_t - aime_0, 0\}, aime_1 - aime_0\} \\ & + 0.15 * \max\{aime_t - aime_1, 0\} \end{aligned}$$

where $aime_0$ and $aime_1$ denote the bend points in the formula. This structure results in a more generous replacement rate for low earners.

Second, the age at which benefits are claimed affects the benefit level through actuarial adjustments. Benefits claimed before the NRA are reduced because of an actuarial penalty for early claiming, while delaying benefit claims beyond the NRA, up to age 70, increases the benefit level through the DRC. These adjustments are designed to provide actuarial fairness, approximately equalizing lifetime benefits across different claiming ages, based on average life expectancy and interest rates at the time the rules were established. Table 1 summarizes the benefit adjustments for claiming ages between 62 and 70, expressed as the percentage of unreduced benefits at the NRA, under scenarios with an NRA of 65 and a DRC of 4.5%.

Table 1: Effects of Early or Delayed Social Security Claiming

		Benefit, as a percentage of PIA, payable at age								
NRA	DRC	62	63	64	65	66	67	68	69	70
65	4.5	80	86.67	93.33	100	104.5	109	113.5	118	122.5

Notes: Social Security retirement benefits, expressed as a percentage of the primary insurance amount, that an individual can receive if he claims at ages 62–70. Abbreviations: NRA = normal retirement age; DRC = delayed retirement credit (percentage points); PIA = primary insurance amount.

Third, individuals under age 70 who work while receiving retirement benefits are subject to the retirement earnings test (RET). If their labor income exceeds a specified threshold y_{ret}, each dollar earned above that threshold reduces their benefit payments by τ_{ret} dollars, until

[15] Individuals are assumed to enter the labor market at age 25. That is, by age 30, the starting age of the model, they have accumulated 5 years of labor earnings. This is a common assumption (French 2005). In addition, according to the Social Security Administration (SSA), an individual's earnings up to two years before retirement eligibility age (currently age 60) are indexed to average wage growth to account for increases in living standards over time. In our model, to capture the general rise in the standard of living during working periods, we adjust $AIME$ for inflation by indexing wages to the personal consumption expenditure price index.

the entire benefit is withheld. For example, before 2000, individuals under age 65 had their benefits reduced by \$1 for every \$2 of earnings, while those aged 65 and older faced a \$1 reduction for every \$3 of earnings. Thus, the actual benefit received at age t, ss_t, reflects this reduction.

$$ss_t = \max\{0, ssb_t - \tau_{ret} * \max\{0, (w_t(\theta_t, h_t)n_t - y_{ret})\}\} \quad \text{if age} < 70$$

Here, ssb_t is the PIA_t amount adjusted by early or delayed claiming. Benefits withheld due to the RET are credited through increased future payments.[16]

Disability Insurance and Pension Benefits: Disability insurance (DI) benefits db_t are modeled as the standard SS benefits for individuals who meet disability criteria, adjusted by a probability factor reflecting typical DI qualification rates. Pension income pb_t is modeled as a function of SS benefits to reflect historical earnings and age-specific pension accruals. Appendix C.1 presents more details on the disability insurance and pension benefits.

Taxes: We model income taxation following Bénabou (2002), Heathcote et al. (2020), and Borella et al. (2023), reflecting the US system. Individuals face effective tax rates dependent on age (and thus time). Taxes paid on annual total income y_t are given by $T_t(y_t, \tau_t) = y_t - \lambda_t y_t^{1-\tau_t}$, where τ_t denotes the degree of progressivity and λ_t captures the average level of taxation. In addition, the government uses a proportional payroll tax τ_t^p on labor income to help finance SS and Medicare. This tax applies up to a threshold $\bar{y}_t^{ss}$, with the payroll tax paid at age t given by $T_t^p(w_t(\theta_t, h_t)n_t, \tau_t^p) = \tau_t^p \min[w_t(\theta_t, h_t)n_t,\ \bar{y}_t^{ss}]$.

Assets, Borrowing Constraints, and Government Transfers: In each period, individuals receive from several sources: interest on assets ra_t, labor income $w_t(\theta_t, h_t)n_t$, pension benefits pb_t, SS retirement benefits net of the RET ss_t (if applicable), DI benefits db_t (if applicable), and government transfers tr_t (if applicable). The budget constraint is given by:

$$\begin{aligned} a_{t+1} =& a_t + Y_t(y_t, \tau_t, \tau_t^p) + (b_t * ss_t) + db_t I\{h_t = 2\} + tr_t - m_t - c_t \qquad (17) \\ y_t =& ra_t + w_t(\theta_t, h_t)n_t + pb_t \\ Y_t(y_t, \tau_t, \tau_t^p) =& y_t - T_t(y_t, \tau_t) - T_t^p(w_t(\theta_t, h_t)n_t, \tau_t^p) \\ a_{t+1} \geq& \underline{a} \end{aligned}$$

The variable $b_t \in \{0, 1\}$ is an indicator that equals one if the individual has claimed retirement benefits and zero otherwise. The term y_t is annual taxable income in period t, with r denoting the net return on assets.[17] Income taxes $T_t(\cdot)$ depend on total taxable income y_t, and payroll

[16]Following French and Jones (2011), if a year's worth of benefits is withheld because of the RET before the NRA, future benefits are increased by the actuarial reduction factor for early claiming. If benefits are withheld after the NRA, future benefits are increased by the DRC.

[17]That is, $r = (1 - \tau_k)r_f$ with the pretax interest rate r_f, and τ_k is an unmodeled corporate tax and other direct capital taxes as in Stantcheva (2017).

taxes T_t^p depend on labor income $w_t(\theta_t, h_t)n_t$. After-tax income is represented by $Y_t(\cdot)$. Since individuals cannot borrow against future SS benefits and can rarely borrow against most forms of pension wealth, we set the borrowing limit $\underline{a} = 0$.[18] In addition, to ease comparison with the literature on life cycle taxation with exogenous retirement, we allow for accidental bequests but assume that there are no planned bequests at the terminal age $a_{T+1} = 0$.[19] Government transfers, tr_t, ensure individuals have access to a minimum level of consumption, $\underline{c}$, as in De Nardi et al. (2010) and Hubbard et al. (1995). This term captures safety net programs in the United States, such as the Supplemental Nutritional Assistance Program (SNAP) and Supplemental Security Income.[20]

Choices: In each state, individuals choose their policy functions of consumption rules $c_t(\cdot)$, labor force participation rules $p_t(\cdot)$, hours worked $n_t(\cdot)$, savings $a_{t+1}(\cdot)$ and SS benefit application rules $b_t(\cdot)$ to maximize their discount expected utility (discount rate β), subject to budget and borrowing constraints (17). The recursive program is detailed in Appendix C.1.1.

Summary: Overall, our framework captures essential features from French (2005) and French and Jones (2011) but differs with important innovations. First, we allow people to choose to work until death, rather than restricting endogenous retirement decisions to end at age 70. Second, in contrast to prior studies that focus on male household heads, our model targets aggregate variables and outcomes. Third, we explicitly model the disability state and DI benefits, which are important components of retirement policies.

5.2 Estimation

We estimate the model using a two-step strategy, as is standard in the literature (e.g., Cagetti 2003; Gourinchas and Parker 2002). In the first step, we estimate or calibrate the parameters that can be cleanly identified without using the model. These parameters are obtained directly from the data, taken from values used in the existing literature, or obtained from program rules. In the second step, we estimate the remaining preference parameters using the method of simulated moments (MSM), taking the parameters estimated in the first step as given. The goal is to find a vector of parameters that generates simulated life cycle profiles that best match the observed profiles from the data.

[18] This no-borrowing assumption is standard in the literature (e.g., Hubbard et al. 1995; Zeldes 1989). In our setting, since the lower bound of the support of the distribution of wages is zero, the natural borrowing limit is zero.

[19] We target the time discount factor to match mean and median assets over the life cycle.

[20] That is, $tr_t = \max\{0, \underline{c} + m_t - (a_t + Y_t + ss_t + db_t)\}$. Papers such as Braun et al. (2017) explicitly model these safety net programs.

5.2.1 First-Step Estimation

The first-step estimates include parameters for health transitions, mortality rates, out-of-pocket medical spending, and wage profiles, all estimated directly from raw data. We also calibrate parameters based on values used in the existing literature: The gross interest rate is set to 4% (French 2005),[21] and the consumption floor is set to $3,500 (French and Jones 2011). SS policy parameters, such as retirement ages and benefit formulas, are taken from official SSA rules.

Data: We primarily use the Panel Study of Income Dynamics (PSID) for the first-step estimation and supplement it with data from the Medical Expenditure Panel Survey (MEPS) to estimate medical expenditure profiles.

The PSID is a longitudinal study of a representative sample of the US population. It provides high-quality information on, among other things, labor market behaviors, income, health, and wealth. It began in 1968 with interviews of approximately 4,800 families and has since tracked these families and their descendants. Participants have been interviewed annually (biennially since 1997), yielding rich panel data on labor market activity, income, health, and wealth. For our analysis, we utilize data from the 1968–2015 waves, focusing on individuals born between 1920 and 1940. This cohort was subject to similar SS rules, and we observe outcomes for it throughout the life cycle.

The MEPS is a nationally representative survey of families, individuals, their medical providers, and employers across the United States, providing detailed information on medical expenditures, sources of payment, insurance coverage, health status, and demographics. We use data from the 1999–2012 waves, dropping observations with missing values of age, medical spending, health insurance information during the working periods, or health status. Appendix B.1 presents details on the data and sample selection.

We use these data to estimate various parameters, including health transitions, survival probabilities, out-of-pocket medical expenses, and hourly wages. Health and disability are measured with self-reported work limitation questions from the PSID and self-perceived health rankings from the MEPS. We estimate health transitions by running probit regressions on age dummies and survival probabilities through logistic regressions that include age polynomials and lagged health status, using the data from the PSID. We construct out-of-pocket medical expenses from the MEPS by combining information on total medical expenditures and insurance coverage, controlling for age and health status. Hourly wages are derived from PSID data on annual earnings and hours worked, adjusted for selection bias by means of the Heckman

[21] We set a reduction from the gross to the net interest rate of 20% as in Stantcheva (2017). Even though capital gains are taxed as part of income in our Bénabou (2002) tax function, this distinction between gross and net interest rate captures unmodeled corporate and direct capital taxes. Our analysis is equivalent to considering a baseline 3.2% interest rate.

selection model. Appendix B.3 describes the estimation procedures in detail.

Estimated Profiles by Health Status: Based on the health measures described above, we estimate age- and health-dependent inputs from the data. Figure 1 displays the resulting life cycle profiles of health transitions, mortality rates, out-of-pocket medical expenditures, and hourly wages by health status. Panel A presents the probability of a transition out of good health, $\Pr(h_t|h_{t-1} = 0)$, highlighting individuals' declining probability of staying healthy with age. The decline is primarily absorbed by the increasing probabilities of a transition into poor health, which rise with age, especially at older ages. Panel B reports mortality rates by health type, which rise with age for all health categories. The increase is rapid for people with a disability at older ages, despite the similar starting points across health types. Panel C shows out-of-pocket medical expenses, calculated as total medical expenditure net of insurance coverage from MEPS data. These expenses vary significantly by health status, with individuals in poorer health facing considerably higher costs. Panel D presents deterministic wage profiles. Healthy individuals earn consistently higher hourly wages than workers in poor health, who, in turn, outearn individuals with a disability throughout the life cycle.

5.2.2 Second-Step Estimation

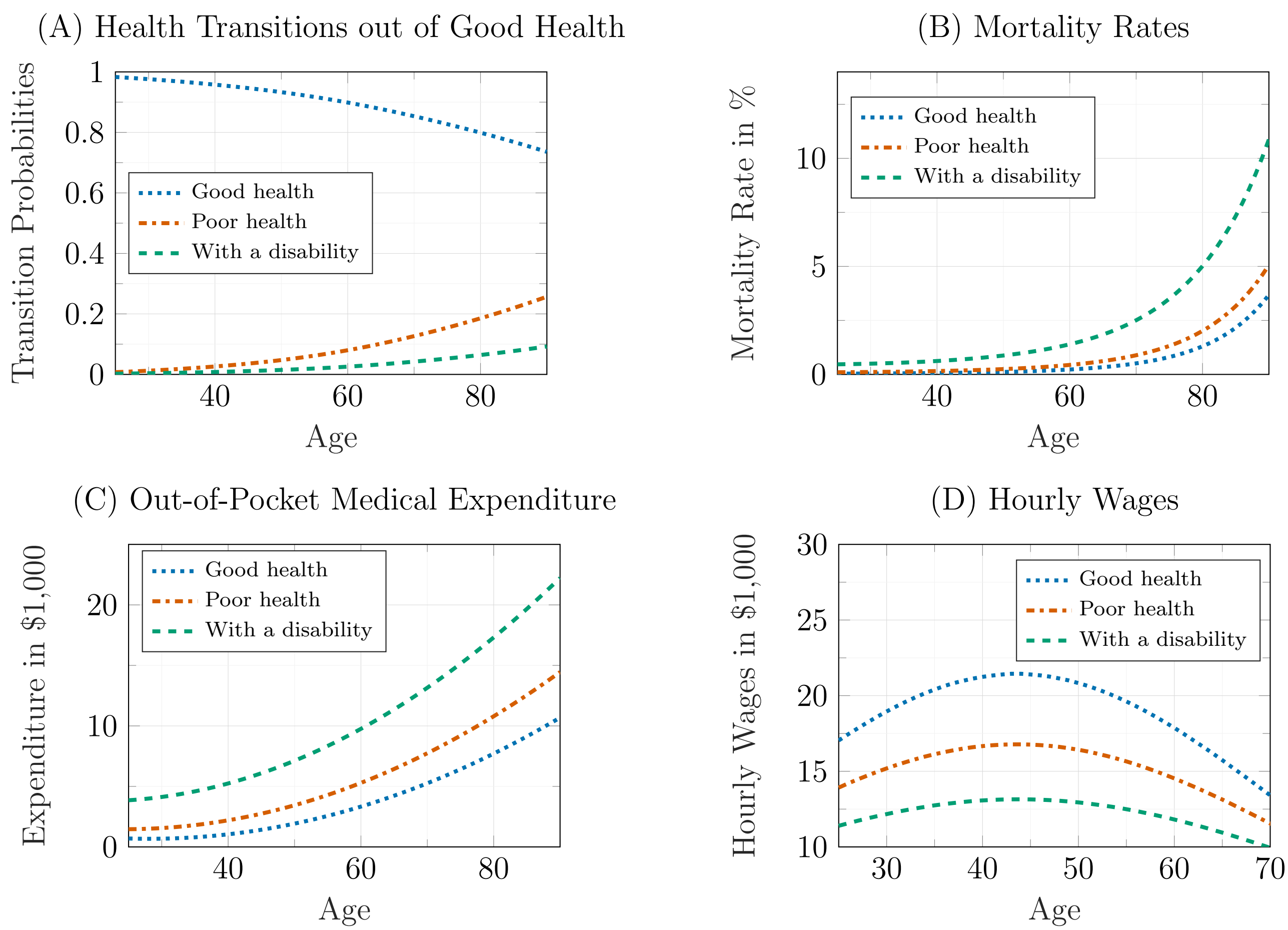
In the second step of the estimation, we use the MSM to estimate the preference parameters: $\mathcal{P} = (\gamma, \nu, \phi_n(h = 0), \phi_n(h = 1, 2), \phi_{RE}, L, \beta)$. Our MSM estimator $\hat{\mathcal{P}}$ minimizes the weighted distance between the estimated target profiles from the PSID and the simulated profiles generated by the model, as measured by a generalized method of moments (GMM) criterion function. We require our model to match a set of life cycle moments observed in the data for individuals from age 30 to 69, including average labor force participation by health status, hours worked conditional on participation by health status, mean and median nonpension assets, and mean labor market reentry rates, resulting in a total of 280 moment conditions. Because of the limited number of observations of individuals with a disability in the PSID, individuals in poor health and those with disabilities ($h = 1, 2$) are assigned the same fixed cost of working, $\phi_n(h = 1, 2)$, to simplify the model. The MSM procedure involves estimating empirical profiles from the data, generating simulated profiles based on the model with initial conditions and simulated shocks, and using them to construct moment conditions. We evaluate the match between the simulated and empirical moments using a GMM criterion. We then search over the preference parameter space to find the values that minimize the criterion. Further details are provided in Appendix 5.2.2.

5.3 Match of Moments

Table 2 reports the estimated structural parameters, and Figure 2 compares the simulated life cycle profiles generated from the estimated model to the empirical profiles observed in the

PSID, with 95% confidence intervals.[22] The model successfully reproduces the key patterns of decision-making observed in the data over the life cycle.

Figure 1: First-Step Estimation, Profiles by Health Status



Notes: Probabilities of transitioning out of good health as a function of age by health status (panel A), mortality rates as a function of age by health status (panel B), out-of-pocket expenditure as a function of age by health status (panel C), and hourly wages as a function by health status (panel D). The blue/red/green (dotted/dash-dotted/dashed) lines represent people in good health/people in poor health/people with a disability. Monetary values are expressed in 2016 dollars. Profiles highlight the declining probability of an individual's staying healthy with age, which is primarily absorbed by increasing probabilities of transition into poor health, especially at older ages. Mortality rates and out-of-pocket medical expenses increase with age, with sizable differences by health. Healthy individuals consistently earn higher hourly wages than people in poor health, who, in turn, earn more than individuals with a disability throughout the life cycle.
Data Source: Panel Study of Income Dynamics, Medical Expenditure Panel Survey.

The estimated discount factor is 0.98, identified by individuals' intertemporal substitution of consumption and leisure and thus by labor supply and asset profiles, especially for the age group accumulating wealth for retirement. The annual time endowment is estimated at 4,491 hours, with fixed costs of working equal to 791 hours for healthy individuals and 868 hours for

[22] For a more in-depth discussion of identification of preference parameters, see French (2005) and French and Jones (2011). As in previous studies, the overidentification test statistics reported in Table 2 formally reject the model. Nonetheless, the model successfully reproduces life cycle profiles that closely match those observed in the data.

those in poor health or with a disability. These values are estimated to match the employment rates and hours worked by health status over the life cycle. These estimates fall within the range of estimates from earlier studies, such as French and Jones (2011) and Borella et al. (2023). Furthermore, the cost of reentering the labor market is estimated at 3,312 hours and to match the low reentry rates observed in the data. The estimated CRRAs for flow utility and consumption weight are 1.70 and 0.54, respectively, implying a CRRA for consumption, $-(\partial^2 u/\partial c_t^2)c_t/(\partial u/\partial c_t)$, of approximately 1.38, identified by the asset and labor supply profiles. Higher risk aversion contributes to greater precautionary savings and encourages more work earlier in life.

Our model effectively replicates the observed labor force participation patterns over the life cycle. It captures the high and stable participation rates among individuals in their 30s and 40s, the earlier declines beginning in the 50s for those in poor health or with a disability, and sharp drops during the 60s across all health categories. These patterns coincide with falling wages and the growing retirement incentives through pensions and SS. As highlighted by Yu (2024), modeling disability status and incorporating DI benefits are essential for explaining the observed participation declines among unhealthy individuals starting in their 50s, as DI provides an alternative pathway to retirement for people with disabilities (Maestas et al. 2013; French and Song 2014; Low and Pistaferri 2015; Li 2018). This feature allows the model to more accurately replicate the labor supply patterns of this group, providing a closer fit than in earlier work, such as French (2005).

Additionally, our model matches mean and median assets well until the mid-50s. The divergence in asset holdings beyond the 50s likely arises because, to make valid comparisons with the literature on optimal taxation with exogenous retirement over the life cycle (Farhi and Werning 2013; Golosov et al. 2016; Stantcheva 2017), we model accidental bequests but abstract from incidental bequests, which typically increase savings in older age groups (e.g., De Nardi et al. 2010; Lockwood 2018). Labor market reentry rates are already low in the data, below 5%, and our model generates no reentry in equilibrium. Appendix C.2.1 presents these additional matched profiles. The large estimated reentry costs justify our assumption that retirement is irreversible in the quantitative analysis of our optimal taxation model.

Overall, the estimation results show that the model captures key patterns of labor supply and savings decisions over the life cycle, indicating that it offers a useful laboratory in which to study the impact of policy reforms.

5.4 Decomposition of Allocations

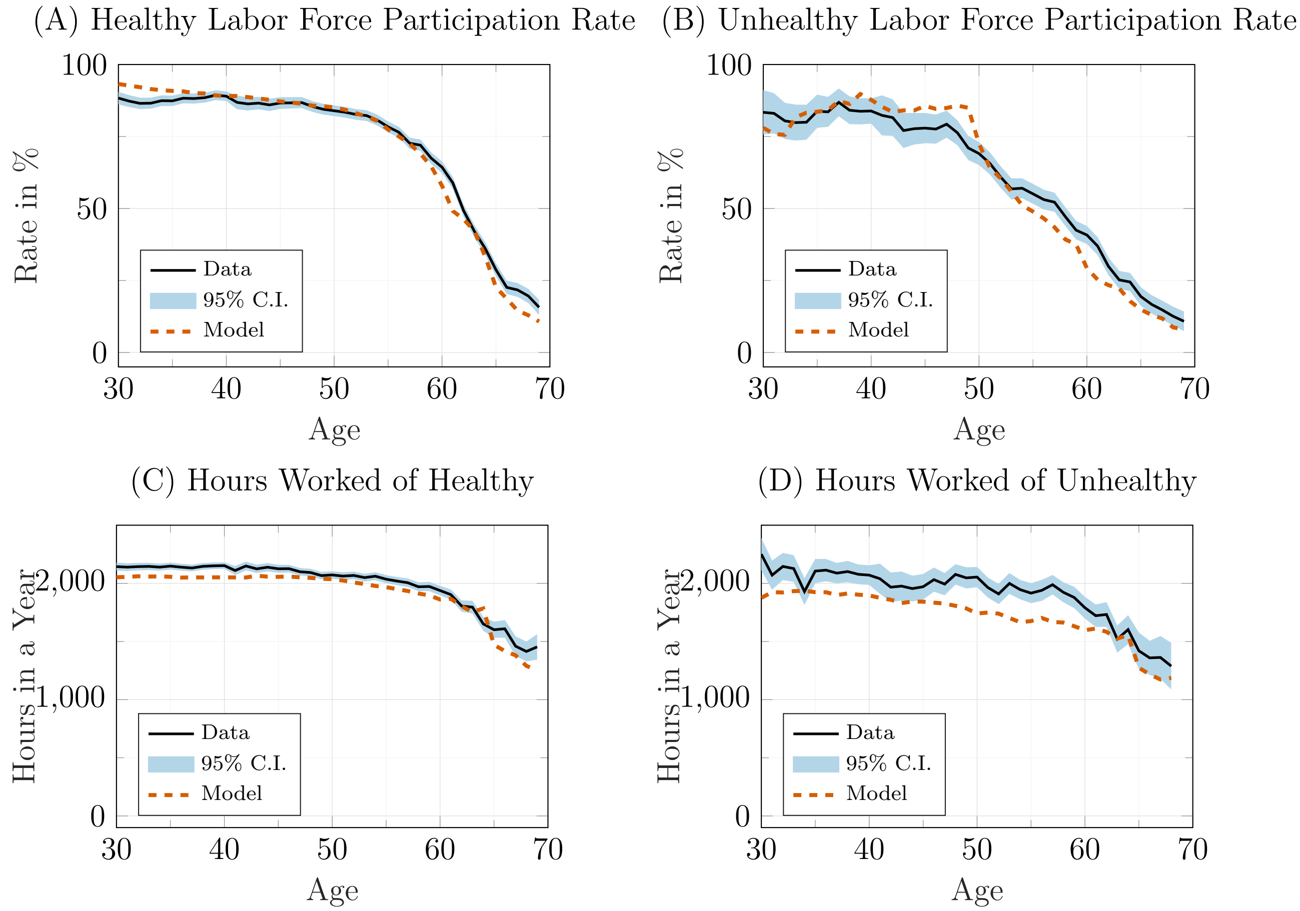
In this section, we quantify the extent to which the nonconvexity, productivity loss, and mortality channels affect labor force participation and retirement. To do so, we run simulations while eliminating each model feature corresponding to the respective channels.

Table 2: Estimated Structural Parameters

Parameter	Definition	Estimates	S.E.
γ	Consumption weight	0.5354	0.0042
ν	CRRA for flow utility	1.7029	0.0587
β	Time discount factor	0.9797	0.0020
L	Leisure endowment	4490.5	24.71
ϕ_{RE}	Labor market reentry cost	3311.5	5.1666
$\phi_n(h=0)$	Fixed cost of work, healthy	791.07	20.003
$\phi_n(h=1,2)$	Fixed cost of work, unhealthy	867.54	30.613
χ^2 statistic (degrees of freedom = 273)		1386.8	

Notes: Estimated structural parameters and standard errors from the method of simulated moments procedures.

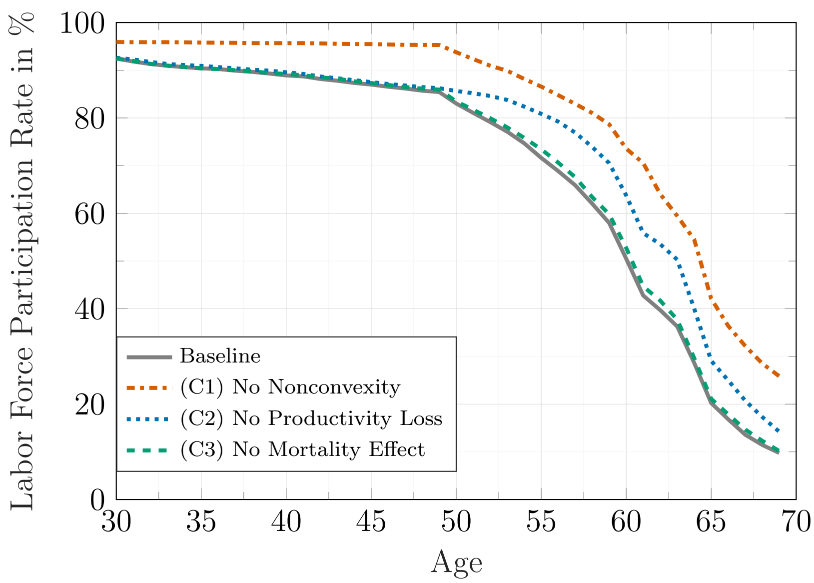
Figure 2: Data and Estimated Profiles



Notes: Panels A and B show the model fit for labor force participation by health status. Panels C and D show the model fit for hours worked conditional on participation by health status. Data profiles are estimated with the Panel Study of Income Dynamics. The shaded region represents the 95% confidence intervals. The orange dashed lines represent model profiles. Monetary values are expressed in 2016 dollars. The model fits these targeted data profiles very well and reproduces the observed key patterns of labor supply decisions over the life cycle.

We first consider our *No Nonconvexity* benchmark (C1), where we set the fixed costs of labor market participation equal to zero. Second, in our *No Productivity Loss* benchmark (C2), we equalize wages across all health categories to the wage level of workers in good health. Third, in our *No Mortality* benchmark (C3), we remove health disparities in mortality by setting the survival probabilities to those of individuals in good health. In each of these exercises, we eliminate one channel at a time. The excess labor force participation rates relative to the estimated model (Baseline) participation rates capture the extent to which each of our channels explains retirement.

Figure 3: Effects of Preferences and Technological Factors on Labor Force Participation



Notes: Labor force participation rates as a function of age for the estimated quantitative model and counterfactual simulations in which the different channels are individually eliminated. The *No Nonconvexity* benchmark (C1) sets the fixed costs of participation to zero. The *No Productivity Loss* benchmark (C2) sets wages equal to the wages of healthy individuals. The *No Mortality Effect* benchmark (C3) sets the survival probabilities equal to those of healthy individuals. The *No Nonconvexity* benchmark features the largest increase in labor force participation, followed by the *No Productivity Loss* benchmark. Health differences in mortality affect labor force participation only slightly.

Figure 3 illustrates the results of these decompositions. We find that nonconvexities in labor supply are the primary preference and technological factors that explain labor force nonparticipation in old age, followed by the productivity differences in health. Absent the nonconvexities, labor force participation would be larger at each working age. Productivity differences related to health start affecting labor force participation at age 50 onward. Mortality differences related to health, however, hardly affect labor force participation.

When we keep the nonconvexities in labor supply through time costs of participation but eliminate the differences in these time costs in health, $\phi_n(h) = \phi_n(0)$ for all h, we find that labor force participation changes only slightly. This qualifies the Blundell et al. (2016) statements, in that it is the presence of nonconvexities in labor supply that mainly explains retirement behavior rather than variation in these convexities in health.

Table 3: Effects on Labor Force Participation Through the Three Channels, by Age Group and Health Status

		Good Health			Poor Health			With a Disability		
	Age Group	30-49	62-64	65-69	30-49	62-64	65-69	30-49	62-64	65-69
(C1)	No Nonconvexity	6.52	27.62	21.40	7.77	20.78	15.00	10.98	11.66	10.86
(C2)	No Productivity Loss	0.41	11.73	6.04	1.65	15.11	8.67	3.15	17.88	8.49
(C3)	No Mortality Effect	0.19	1.41	0.79	0.30	1.97	0.86	0.47	1.15	1.12

Notes: Percentage-point increases relative to model-estimated labor force participation under the three benchmarks, broken down by health status and for age categories 30–49, 62–64, and 65–69. Nonconvexities are the primary preference and technological factor explaining labor force participation across ages and health states, while heterogeneous mortality has small effects on participation. Wage differences by health status have modest differences but dominate for the category of individuals aged 62–64 with a disability.

To further elucidate what drives these results, Table 3 provides a breakdown of the excess percentage-point increases in labor force participation for each of our benchmarks, by health status and for age categories young/prime age 30–49, early retirement 62–64, and late retirement 65–69, relative to the estimated model. We find that nonconvexities consistently explain labor participation across ages and health categories while health-dependent mortality rates have much lower effects. Health-related differences in wages, despite affecting overall participation less than nonconvexities do, dominate for individuals in the 62–64 age category with a disability, when DI benefits are available.

6 Quantitative Optimal Policies

We now turn to quantifying the optimal policies. Because we showed that the nonconvexity channel is the primary preference and technological factor explaining the retirement decision, we focus on this channel in our calibrated Mirrlees economy. This simplification and numerical example has the advantage of keeping the Mirrlees model (a sequence of partial differential equations) quantitatively solvable while preserving the main insights for optimal policies. We next contrast these policies with the labor and savings wedges that arose in our status quo model of the US economy in Section 5.

6.1 Calibrated Quantification of Effects on Optimal Policies Through the Labor Supply Nonconvexity Channel

6.1.1 Calibration

Assumptions: We illustrate the properties of optimal allocations and wedges in an economy where agents live for $T = 55$ years, from ages 30 to 84. Consistent with the large reentry cost ϕ_{RE} estimated in Section 5.2, we assume that retirement, $n_t = 0$, is an irreversible decision, i.e., that there is no labor force reentry. We consider two health states, healthy and unhealthy.[23]

[23] The unhealthy category combines poor health and disability.

Consistent with the low transition rates from unhealthy to healthy, we assume that the unhealthy state is absorbing. Assumptions 1 and 2 hold, with log preferences over consumption $\tilde{u}(c_t) = \ln(c_t)$ and disutility of labor equal to $\psi(n_t; h_t) = \frac{1}{1+\frac{1}{\varepsilon}} \left(\frac{y_t}{\theta_t}\right)^{1+\frac{1}{\varepsilon}} + \phi(h_t) I\{n_t > 0\}$ with $\varepsilon = 0.5$ as in of Chetty (2012). We capture the labor supply nonconvexities but leave the heterogeneous effects of health on wages and mortality to our full quantitative model, i.e., $s_t(h_t) = 1, w(\theta_t, h_t) = w(\theta_t)\ \forall t \leq T, h_t \in \mathcal{H}$. We set the gross interest rate to 4% and the autoregressive parameter of ability $\rho = 1$ and endogenously calibrate the other functions and variables.

Endogenously Estimated Parameters: We reestimate the wage function and health transition probabilities along the assumptions of this numerical example and following the procedure described in Section 5. Then, in Appendix C.5, we jointly estimate the fixed costs of participation for the healthy and unhealthy $\phi(0), \phi(1)$, the variance of normally distributed innovations to log ability σ, the discount rate β, and the other parameters to match empirical profiles as in Section 5.

Table 4: Calibration

Parameter	Definition	value	S.E.	Source/Target
Exogenous parameters				
T	Duration	55		ages 30–84
ε	Disutility elasticity	0.5		Chetty (2012)
r_f	Gross interest rate	4%		
ρ	Autoregressivity of ability	1		
Endogenously estimated in baseline US economy				
$\phi(0)$	Fixed cost healthy	0.53	0.0082	Employment, 30–69, healthy
$\phi(1)$	Fixed cost unhealthy	0.59	0.0152	Employment, 30–69, unhealthy
σ	Variance of ability	0.019		Panel Study of Income Dynamics
β	Discount rate	0.97	0.0015	Assets ages 30–69

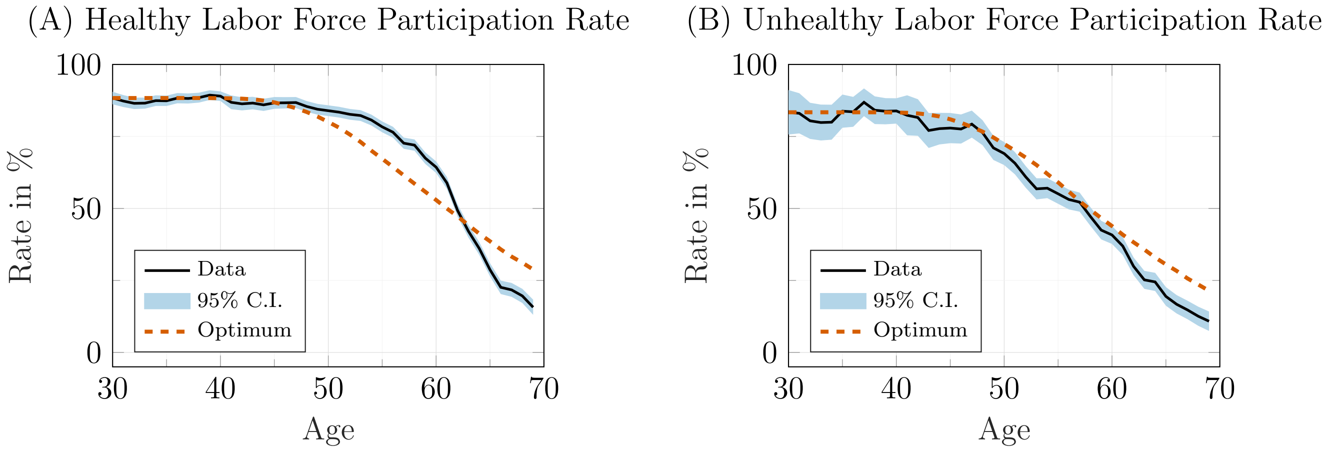
Notes: Calibration of dynamic Mirrlees model with nonconvex labor supply. The last column shows the source of the parameter for exogenously calibrated parameters and the target moment (and its source) for parameters endogenously estimated in the baseline economy. If the source/target is blank, the parameter is normalized.

Table 4 summarizes the estimated values. These fixed cost estimates are within the range of estimates in Chang et al. (2019). The variance of log wages is in the medium to high range of the estimates in Heathcote et al. (2010). We now use these parameters to simulate the effects of labor supply nonconvexities on optimal policies in the Mirrlees model.

For each simulation, we compute the policy functions for the calibrated values above. From these policy functions, we perform a Monte Carlo simulation with $N = 1,000,000$ draws. Ex ante welfare is set to result in an aggregate consumption equal to aggregate output so that we can study optimal reforms that do not involve intergenerational transfers.

Figure 4 illustrates labor force participation rates with age for the healthy (panel A) and the

Figure 4: Optimal Labor Force Participation Rates



Notes: Labor force participation rates as a function of age for healthy (panel A) and unhealthy individuals (panel B). Data profiles are estimated with the Panel Study of Income Dynamics. The shaded region represents the 95% confidence intervals. Model profiles are represented by the orange dashed lines. Optimal labor participation rates for the unhealthy match well most of the data profiles before age 62, and the model generates a less steep slope of the optimal labor force participation curve. Healthy individuals optimally exit the labor force later than unhealthy individuals.

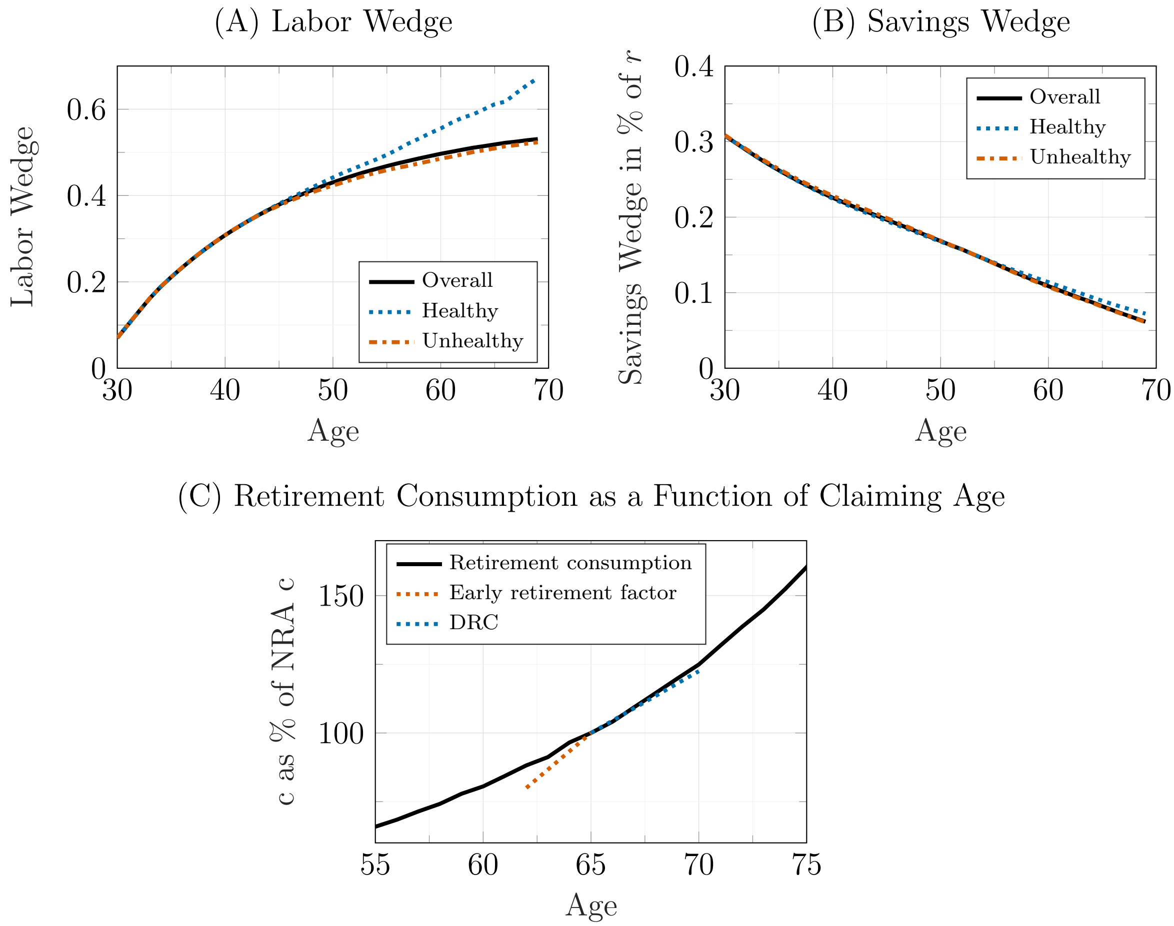
unhealthy (panel B). Healthy individuals optimally exit the labor force later than unhealthy individuals. In comparison to the data, the model generates a less sloped optimal labor force participation curve and therefore more gradual exits from the labor market. In the data, healthy individuals participate more than optimal at ages 45–62, while they participate less than optimal at ages 63–70. In most of the life cycle before age 62, the optimal labor participation rates for the unhealthy are within the error term of those in the data, except after age 50, which corresponds to early retirement for individuals with a disability. After age 62, unhealthy individuals still participate less than what is optimal.

Panels A and B of Figure 5 show the optimal allocations summarized by the labor and savings wedges defined in Section 4. For our numerical example, the analysis of the variation in wedges across health status is more instructive than that of their variation over time, which was the main result of Farhi and Werning (2013).[24]

Labor and Savings Wedges: The optimal labor wedges in panel A are larger for healthy than for unhealthy workers. This is consistent with our result on the effect through the nonconvexity channel for the labor wedge across health. The savings wedge in panel B is small in absolute terms and is indistinguishable between the healthy and unhealthy individuals. This echoes the fact that the main driver for large changes in savings distortion is heterogeneity in mortality rates that correlate with health, which we abstract from here.

[24]For the time series analysis, the optimal labor wedge remains increasing in the 30–70 age range in our calibration, as in Farhi and Werning (2013). As workers further exit the labor market, selection of the remaining workers toward high-earning and healthy individuals counters the endogenous labor supply elasticity increases in old age. The savings wedge is decreasing over time.

Figure 5: Optimal Allocations with Labor Supply Nonconvexity



Notes: Panel A shows the optimal labor wedge by age and health status. Panel B plots the savings wedge by health status and age. The black solid lines represent people on average, and the blue/red dashed lines represent healthy/unhealthy individuals. Panel C plots the average consumption of new retirees by age, normalized to 100% at the normal retirement age and superimposed on the early retirement factor and delayed retirement credit curves. The optimal labor wedges are lower for unhealthy than for healthy workers. The savings wedge is small in absolute terms and indistinguishable between the healthy and unhealthy individuals. The optimal consumption for new retirees is convex in retirement age.

Retirement Consumption: To illustrate allocations in retirement, Panel C plots the consumption of new retirees at ages ±10 years from the NRA. The graph is normalized at 100% at the NRA, and we contrast these new-retiree consumption patterns with the early retirement factor curve and the DRC.[25] Interestingly, the optimal new-retiree consumption is convex in the retirement age in our calibration and all noncalibrated economies we analyzed. This result is particularly robust to several alternative modeling choices (Ndiaye 2018). Policies that implement the constrained optimum would promise higher retirement consumption to those who retire later, whether through higher retirement benefits, subsidized retirement savings, or government-provided health insurance. In addition, the flatter pattern of labor force participation in old age in the Mirrlees economy suggests that policies that make retirement more flexible by expanding rewards for delayed retirement will be closer to the optimum in our full quantitative model.

6.2 Wedges in the Status Quo US Economy

We now turn to the labor and savings distortions that arise from the lack of complete markets and full insurance, taxes, retirement benefits, medical expenditures, and other policies in our quantitative model. These distortions can be summarized formally by the labor and savings wedges in equations (10) and (11) computed for each state and whose averages over our estimated model are reported in Figure 6.

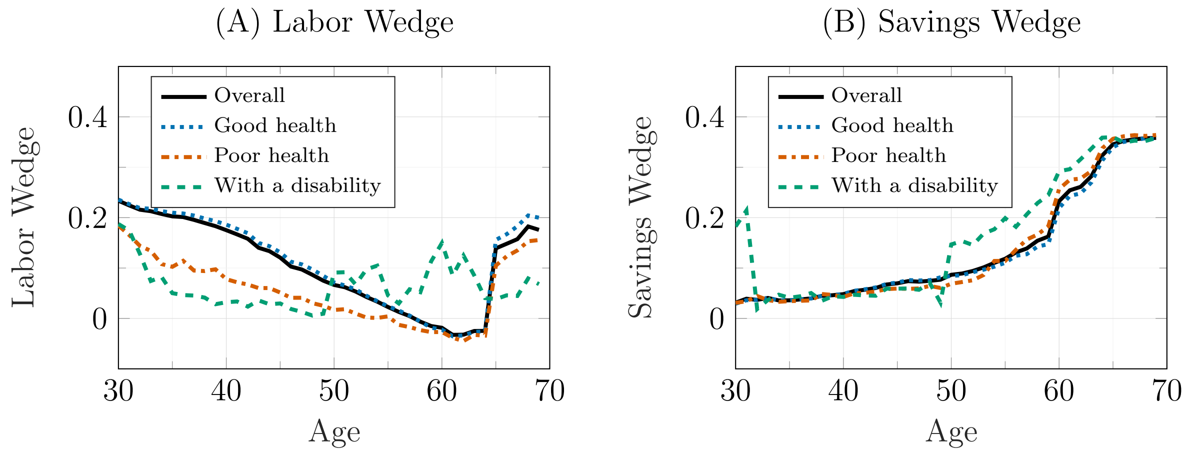
Labor Wedge: The labor wedge (panel A) is larger for those in good health, followed by that for those in poor health, and then the one for individuals with a disability. This ranking by health status aligns with the properties of the optimal distortions. However, the introduction of DI at age 50 generates a substantially larger labor wedge, creating a significant disincentive to work for individuals with a disability relative to people in the other health categories. This DI distortion gradually diminishes and nearly disappears by age 65, with the relative ranking of the wedges across health states remaining the same.

Additionally, from ages 30 to 60, the labor wedge shows a decreasing time trend, which contrasts with existing optimal taxation literature that typically suggests an increasing wedge profile. This invites further consideration of the specific factors that a nonlinear taxation model might incorporate in future work to explain the difference from the status quo.

Savings Wedge: The savings wedges (panel B) for healthy and unhealthy individuals do not differ much. This aligns with our previous finding on the variations in the savings wedge by health. Individuals with a disability, however, and the newly retired face larger savings

[25] Although Social Security claiming is implicit in our normative analysis, which derives optimal allocations, we anticipate a close link between retirement and benefit claiming decisions. Shoven et al. (2017) showed that approximately two-thirds of individuals claim Social Security retirement benefits immediately upon stopping work.

Figure 6: Labor and Savings Wedges in Status Quo US economy



Notes: Panel A shows the optimal labor wedge by age and health status. Panel B plots the savings wedge by health status and age. The black solid lines represent people on average, and the blue/red/green nonsolid lines represent people in good health/in poor health/with a disability. The labor wedge is larger for healthy individuals, consistent with the optimal distortions. Disability insurance at age 50 sharply increases the wedge for people with a disability, creating a strong work disincentive that diminishes by age 65, while the health ranking is preserved. The savings wedge is similar for individuals in good and poor health but larger for individuals with a disability and retirees, which might be due to the nonseparability of consumption and leisure.

distortions. This is explained by the nonseparability of consumption and leisure. As individuals with a disability and retirees have more leisure, they gain more value from present consumption, which increases the savings distortions.

The savings distortions in our positive economy (in rates akin to wealth taxes) significantly exceed those found to be optimal in our numerical example and prior literature. The larger savings wedge can be explained by the limited opportunities for individuals to smooth consumption due to borrowing constraints in our positive economy and larger incentives to consume in retirement due to complementarities.

Summary: These preliminary findings highlight that labor distortions should decrease in poor health and that optimal policies should promise higher retirement consumption to those who retire beyond the NRA and warn of the complexity of the labor and savings distortions present in the status quo US economy. To effectively evaluate refined policy reform proposals, we leverage our comprehensive quantitative model in the next section and study how various policy instruments direct affect welfare and retirement choices.

7 Welfare and Fiscal Analysis

7.1 Welfare Decomposition

We start by decomposing the welfare contribution of each of our policy instruments by running simulations where we eliminate, one by one, the role of these instruments. Table 5 presents the consumption-equivalent (CE) and leisure-equivalent (LE) welfare change as a

Table 5: Welfare Changes from Counterfactual Elimination of Each Policy Instrument

		CE Welfare Δ	LE Welfare Δ
(D1)	No Social Security Benefits	-5.89	-6.79
(D2)	Loss From Incomplete Coverage in Medicare (A-B)	-4.28	-4.93
(D3)	Full Savings Subsidy	3.60	4.15
(D4)	No Income Tax	3.16	3.64
(D5)	No Disability Benefits	-0.34	-0.40

Notes: Consumption-equivalent (CE) and leisure-equivalent (LE) welfare changes in percentage for each counterfactual simulation that eliminates a policy instrument. The *No Social Security* benchmark (D1) sets Social Security benefits to zero. The *Loss from Incomplete Coverage in Medicare* benchmark (D2) sets out-of-pocket medical expenses after age 65 to zero for nonparticipants and participants. The *Full Savings Subsidy* benchmark (D3) sets the net return to assets r to the gross interest rate r_f. The *No Income Tax* benchmark (D4) sets income tax to zero. The *No Disability Benefit* benchmark (D5) sets disability benefits to zero. The *No Social Security* experiment features the highest welfare change. The blue gradient indicates larger absolute values.

result of these experiments. Our first decomposition (D1) eliminates all Social Security benefits and shows the highest welfare change in absolute value of our experiments. This suggests that Social Security is the most important policy instrument to provide welfare over the life cycle.[26] D2 quantifies the difference between status quo welfare and welfare under coverage of all medical expenditures after age 65 for nonparticipants and participants. D2, therefore, shows that all potential gains from expanding Medicare Parts A to D are less than current welfare gains from Social Security. Then, in decreasing order of importance in their effect on lifetime welfare, come savings subsidies up to the return on savings (D3), elimination of income taxes (D4), and elimination of disability benefits (D5).

Appendix C.3.2 shows the labor force participation rates by health status that result from experiments D1–D5. The labor force participation changes are largest for D1. In particular, eliminating disability benefits would substantially increase the labor force participation of people with disabilities after age 50, even though it would not impact welfare as much because of the smaller size of the population with a disability.

These experiments provide an order of prioritization of the policy instruments and qualify the results of French and Jones (2011) that emphasize the role of Medicare for retirement.[27] Here, we find that even though Medicare is important, Social Security is the most important for retirement from a welfare and participation perspective. In addition, we will show that policy instruments can, however, have differential effects on welfare, participation, and the government budget. For instance, in our simulations, increasing the NRA as studied by French and Jones (2011) hurts welfare, so we warn against policies that aim simply to increase

[26] When we eliminate only the retirement benefits portion of OASDI benefits of Social Security, we find CE (respectively LE) welfare reduction of 5.34% (respectively 6.15%), giving a lower bound on the role of Social Security retirement benefits for welfare.

[27] In an earlier working paper version, French and Jones (2004) highlights Social Security as having a larger effect on labor supply than Medicare, while their subsequent published paper places greater emphasis on Medicare, finding its impact nearly as significant as that of Social Security.

participation. We next study more granular reforms to derive a better sense of the most impactful reforms of our policy instruments.

7.2 Retirement Policy Reforms

7.2.1 Social Security Reforms

From our description of policies in the setup of our quantitative model in Section 5.1, we see that Social Security benefits feature progressivity, a level, and age dependence. Here, we study reforms of these different aspects.

Social Security Progressivity and Level Reforms. Table 6 presents the welfare, fiscal, and labor force participation effects of reforms that increase Social Security benefit progressivity (P reforms) and levels (L reforms).

Progressivity reforms P1.1–1.9 progressively increase, by 1 percentage points (pp), the first bracket of the Social Security PIA as a function of the AIME from 0.91 to 0.99. Progressivity reforms P2.0–2.9 (resp., P3.0–3.9) increase the second (resp., third) bracket by 0.32 pp (resp., 0.15 pp), that is, by 1% increments of the initial bracket level. Level or linear reforms L0–9 increase all PIA brackets proportionally by 1% at a time.

For each reform, we present the CE welfare change from the status quo in percentage, which is individuals' willingness-to-pay (WTP) for the policy; the direct fiscal effects of the policy (here, the percentage change in retirement benefits paid by Social Security);[28] the indirect fiscal effects of the policy (here, the percentage change in all other taxes and transfers); and the net fiscal effect, which combines the direct and indirect impacts and reflects changes in government spending G. From these, we calculate the MVPF, which measures the amount of welfare that can be delivered per dollar of government spending, taking into account both the direct and indirect effects of the policy. As is common with any welfare analysis, "the creation of MVPFs requires various judgment calls" (Hendren and Sprung-Keyser 2020). When the indirect effects of welfare-increasing policy strongly offset its direct costs, up to making the net cost turn negative, leading to an MVPF of infinity (policies that pay for themselves in equilibrium), we also provide, for robustness, a lower-bound MVPF that accounts only for the direct fiscal effects. Finally, we present changes in labor force participation at ages 30–49 (young or prime ages), 62–64 (early retirement ages), and 65–69 (late retirement ages) in percentage points. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. Appendix C.4 presents the detailed quantitative implementation of these reforms and calculations of the WTP, budget, MVPF, and participation effects of all the reforms.

We study these reforms that increase benefit progressivity and levels by starting with a

[28]We calculate total retirement benefits as their NPV at age 62, the early eligibility age.

Table 6: Social Security Retirement Benefit Progressivity and Level Reforms

	Policy Reforms	%Δ Welfare		%Δ Budget			Δ Participation		
	PIA Brackets	MVPF	WTP	Net	Direct	Indir.	30–49	62–64	65–69
(P1.1)	(0.91, 0.32, 0.15)	[0.52, +∞)	0.10	-0.03	0.57	-0.10	0.08	0.20	0.06
(P1.2)	(0.92, 0.32, 0.15)	0.97	0.22	0.09	1.23	-0.07	0.14	-0.07	0.00
(P1.3)	(0.93, 0.32, 0.15)	0.92	0.31	0.13	1.66	-0.09	0.14	0.15	0.01
(P1.4)	(0.94, 0.32, 0.15)	0.84	0.40	0.19	2.16	-0.10	0.14	-0.17	-0.23
(P1.5)	(0.95, 0.32, 0.15)	0.91	0.50	0.21	2.32	-0.09	0.16	-0.34	-0.31
(P1.6)	(0.96, 0.32, 0.15)	1.14	0.59	0.20	2.80	-0.16	0.19	-0.27	-0.15
(P1.7)	(0.97, 0.32, 0.15)	1.01	0.67	0.26	3.22	-0.16	0.21	-0.57	-0.50
(P1.8)	(0.98, 0.32, 0.15)	0.86	0.75	0.34	3.72	-0.15	0.24	-0.71	-0.49
(P1.9)	(0.99, 0.32, 0.15)	0.69	0.80	0.44	4.12	-0.11	0.25	-0.85	-0.60
(P2.0)	(0.99, 0.3232, 0.15)	0.56	0.83	0.57	4.75	-0.07	0.25	-1.10	-0.83
(P2.1)	(0.99, 0.3264, 0.15)	0.50	0.86	0.67	5.28	-0.05	0.24	-1.29	-1.03
(P2.2)	(0.99, 0.3296, 0.15)	0.49	0.88	0.70	5.99	-0.11	0.25	-1.05	-1.01
(P2.3)	(0.99, 0.3328, 0.15)	0.44	0.90	0.78	6.56	-0.10	0.24	-1.37	-1.15
(P2.4)	(0.99, 0.336, 0.15)	0.38	0.92	0.93	7.21	-0.05	0.24	-1.60	-1.63
(P2.5)	(0.99, 0.3392, 0.15)	0.37	0.93	0.98	7.95	-0.10	0.27	-1.63	-1.58
(P2.6)	(0.99, 0.3424, 0.15)	0.36	0.95	1.03	8.46	-0.11	0.28	-1.47	-1.68
(P2.7)	(0.99, 0.3456, 0.15)	0.33	0.94	1.11	8.99	-0.10	0.27	-1.74	-1.99
(P2.8)	(0.99, 0.3488, 0.15)	0.31	0.95	1.19	9.49	-0.10	0.28	-2.29	-2.1
(P2.9)	(0.99, 0.352, 0.15)	0.29	0.95	1.27	9.97	-0.09	0.28	-2.26	-2.05
(P3.0)	(0.99, 0.352, 0.1515)	0.29	0.95	1.26	10.04	-0.10	0.28	-2.22	-2.05
(P3.1)	(0.99, 0.352, 0.153)	0.29	0.96	1.27	10.13	-0.10	0.28	-2.30	-2.07
(P3.2)	(0.99, 0.352, 0.1545)	0.29	0.96	1.28	10.19	-0.10	0.28	-2.30	-2.07
(P3.3)	(0.99, 0.352, 0.156)	0.29	0.96	1.26	10.26	-0.13	0.28	-2.25	-2.03
(P3.4)	(0.99, 0.352, 0.1575)	0.29	0.96	1.27	10.34	-0.13	0.28	-2.23	-2.03
(P3.5)	(0.99, 0.352, 0.159)	0.29	0.97	1.29	10.41	-0.12	0.28	-2.41	-2.02
(P3.6)	(0.99, 0.352, 0.1605)	0.29	0.97	1.27	10.48	-0.14	0.28	-2.39	-2.02
(P3.7)	(0.99, 0.352, 0.162)	0.30	0.97	1.27	10.55	-0.15	0.28	-2.39	-2.02
(P3.8)	(0.99, 0.352, 0.1635)	0.30	0.97	1.25	10.63	-0.18	0.28	-2.37	-2.01
(P3.9)	(0.99, 0.352, 0.165)	0.30	0.98	1.24	10.70	-0.20	0.28	-2.37	-1.99
(L0)	(0.909, 0.3232, 0.1515)	0.51	0.16	0.12	1.34	-0.05	0.11	-0.05	0.01
(L1)	(0.918, 0.3264, 0.153)	0.59	0.32	0.21	2.47	-0.11	0.14	-0.15	-0.13
(L2)	(0.927, 0.3296, 0.1545)	0.49	0.46	0.36	3.48	-0.10	0.13	-0.49	-0.08
(L3)	(0.936, 0.3328, 0.156)	0.61	0.60	0.38	4.25	-0.18	0.18	-0.29	-0.3
(L4)	(0.945, 0.336, 0.1575)	0.59	0.74	0.49	5.28	-0.21	0.21	-0.48	-0.43
(L5)	(0.954, 0.3392, 0.159)	0.52	0.87	0.64	6.31	-0.19	0.23	-0.72	-0.68
(L6)	(0.963, 0.3424, 0.1605)	0.47	0.95	0.79	7.41	-0.20	0.24	-1.00	-1.22
(L7)	(0.972, 0.3456, 0.162)	0.42	1.00	0.92	8.66	-0.23	0.26	-1.40	-1.43
(L8)	(0.981, 0.3488, 0.1635)	0.35	1.02	1.14	9.80	-0.18	0.27	-1.76	-1.86
(L9)	(0.99, 0.352, 0.165)	0.30	0.98	1.24	10.70	-0.20	0.28	-2.37	-1.99

Notes: Progressivity reforms P1.1–1.9 progressively increase, by 1 percentage point (pp), the first bracket of the Social Security primary insurance account (PIA) as a function of the average indexed monthly earnings from 0.91 to 0.99. Progressivity reforms P2.0–2.9 (resp., P3.0–3.9) increase the second (resp., third) bracket by 0.32 pp (resp., 0.15 pp), that is, by 1% increments of the initial bracket level. Level reforms L0–9 increase all PIA brackets proportionally by 1% at a time. For each reform, we present the consumption-equivalent welfare change from the status quo in percentage or willingness-to-pay (WTP), the percentage change in total Social Security payments (the direct fiscal effect), the indirect and net fiscal effects accounting for equilibrium changes in all parts of government spending, the marginal value of public finds (MVPF), and changes in labor force participation at ages 30–49, 62–64, and 65–69 in pp. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. Reforms of the first bracket of Social Security progressivity offer the highest MVPF among the progressivity and level reforms.

bias toward those that increase welfare. Table 6 offers several takeaways. All the welfare gains from increasing the progressivity and level of Social Security benefits come with sizable budget costs in terms of extra Social Security payments. An expansion of Social Security generosity increases participation slightly at young ages 30–49 (a substitution effect for young households, which work more at younger ages to guarantee a large AIME) but can lead to large drops in participation in old age (an income effect for old workers, who can now claim higher benefits). The indirect fiscal effects lead to less taxes collected from old workers and more government spending, except for small progressivity reforms in the first bracket, which can lead to a net government surplus and an MVPF of infinity driven mainly by an increase in income taxes collected from additional participants (57% of the indirect effects). Overall, if one were to reform Social Security progressivity, increasing the first bracket would be the most efficient way to do so, as it has the highest MVPF of approximately 0.52–1.14. A uniform increase in the level of retirement benefits (L reforms) or an increase in the top or 3rd bracket of PIAs (P3 reforms) would not be as effective.

In summary, reforms that aim to increase the generosity of retirement benefits increase welfare but come at a fiscal cost. Among them, reforms that increase the progressivity of the lowest bracket have the highest MVPF. This result contrasts with findings in the dynamic taxation literature(e.g. Farhi and Werning 2013), which typically concludes that *linear*, age-dependent and history-independent policies are nearly optimal. In contrast, we show that when retirement decisions are endogenous, Social Security–an income-history-dependent policy–becomes the most important policy instrument for welfare. Moreover, the cumulative impact of ability and health shocks on lifetime earnings implies that progressive reforms to Social Security benefits yield substantially larger MVPFs than linear reforms.

Social Security Age-Dependence Reforms. We now study reforms of Social Security that change the claiming-age dependence of retirement benefits (A reforms).

Table 7 presents how our reforms shift or alter the benefits paid at different ages as a percentage of the PIA. The blue gradient indicates high values, while the red gradient indicates low values, and white represents 100% of the PIA.

The first line (AN0) describes claiming-age adjustments in retirement benefits at the US status quo. The NRA for our analysis cohort is age 65. Benefits claimed after the NRA increase by 4.5 pp with the DRC, and benefits claimed before the NRA decrease by 6.67 pp. Reforms that change the NRA (AN1–3) increase its value to ages 66, 67, and 68. These reforms gradually shift the Social Security benefit replacement rates to the right and have the effect of lowering all retirement benefits *without any behavioral change*, which can be seen in the shift of the red-white-blue color gradient.

Reforms of the age dependence of Social Security retirement benefits that increase the DRC

(AD reforms) or linearly change age adjustments of retirement benefits (both the DRC and early retirement penalties at $\pm x\%$) do not, however, shift the benefit replacement rate away from the NRA but rather tilt and steepen the benefits adjustment rate around the NRA. These reforms are therefore expected to affect benefits insofar as *individuals change their retirement behavior in response to the reforms.*

Table 7: Effects of Reforms to Social Security Claiming-Age Dependence on Retirement Benefit Replacement Rate

Claiming Ages		62	63	64	65	66	67	68	69	70
Reforms		Benefit as a Percentage of PIA at Different Claiming Ages								
(AN0)	NRA at 65	80	86.67	93.33	100	104.5	109	113.5	118	122.5
(AN1)	NRA at 66	75	80	86.67	93.33	100	104.5	109	113.5	118
(AN2)	NRA at 67	70	75	80	86.67	93.33	100	104.5	109	113.5
(AN3)	NRA at 68	65	70	75	80	86.67	93.33	100	104.5	109
(AD1)	DRC at 8%	80	86.67	93.33	100	108	116	124	132	140
(AD2)	DRC at 8.5%	80	86.67	93.33	100	108.5	117	125.5	134	142.5
(AD3)	DRC at 9%	80	86.67	93.33	100	109	118	127	136	145
(AD4)	DRC at 9.5%	80	86.67	93.33	100	109.5	119	128.5	138	147.5
(AD5)	DRC at 10%	80	86.67	93.33	100	110	120	130	140	150
(AL1)	Linear ±7%	79	86	93	100	107	114	121	128	135
(AL2)	Linear ±8%	76	84	92	100	108	116	124	132	140
(AL3)	Linear ±9%	73	82	91	100	109	118	127	136	145
(AL4)	Linear ±10%	70	80	90	100	110	120	130	140	150
(AL5)	Linear ±11%	67	78	89	100	111	122	133	144	155
(AL6)	Linear ±12%	64	76	88	100	112	124	136	148	160
(AL7)	Linear ±13%	61	74	87	100	113	126	139	152	165
(AL8)	Linear ±14%	58	72	86	100	114	128	142	156	170
(AL9)	Linear ±15%	55	70	85	100	115	130	145	160	175

Notes: Social Security retirement benefits, expressed as the percentage of the primary insurance amount (PIA), that an individual can receive upon claiming at ages 62–70 under each reform to Social Security claiming-age dependence, following rules from the Social Security Administration. AN1–AN3 increase the normal retirement age to 66, 67, and 68. AD1–AD5 increase, by 0.5%, the delayed retirement credit (DRC) from 8% to 10%. AL1–AL9 equalize the DRC and early claiming penalties in absolute values and increase them from ±7% to ±15%. The blue gradient captures larger values, while the red gradient represents lower values, and blank means 100% of the PIA.

Reforms AD1–5 incentivize late claiming by increasing the DRC (thus affecting benefits claimed after the NRA) from 8% to 10% without changing the early retirement penalty. Reforms AL1–9 incentivize late claiming and penalize early claiming by equalizing the DRC and early retirement penalties in absolute values and increasing them from ±7% to ±15%.

Table 8 presents the results from these reforms of the claiming-age dependence of SS retirement benefits. Increasing the NRA is welfare decreasing but reduces retirement benefit payments and direct costs. Participation decreases slightly at ages 30–49 (representing a negative substitution effect) and 62–64 and increases substantially at ages 65–69 (representing an adverse income effect). The indirect fiscal effects of these reforms lower government spending further, as more taxes are collected from the older participants. The MVPF of these welfare-decreasing reforms is approximately 0.16–0.18, lower than the values for the SS reforms we have calculated. These MVPFs can be interpreted as the "bang-for-buck" of an NRA decrease from 66, 67, or 68 to age 65.[29] Once we account for endogenous and optimal retirement choices by individuals, it is intuitive that increasing the NRA would be welfare decreasing. Doing so would equally penalize all retirees who optimally choose when to claim retirement.

Table 8: Reforms of Social Security Claiming-Age Dependence

Policy Reforms		%Δ Welfare		%Δ Budget			Δ Participation		
		MVPF	WTP	Net	Direct	Indir.	30–49	62–64	65–69
(AN1)	NRA at 66	0.17	-0.38	-0.87	-6.13	-0.02	-0.04	0.38	0.60
(AN2)	NRA at 67	0.16	-0.74	-1.75	-11.82	-0.12	-0.07	-0.04	1.98
(AN3)	NRA at 68	0.18	-1.09	-2.38	-16.75	-0.08	-0.14	-0.43	3.27
(AD1)	DRC at 8%	∞	0.16	-0.45	-2.46	-0.10	0.11	0.15	0.74
(AD2)	DRC at 8.5%	∞	0.19	-0.44	-2.87	-0.04	0.10	0.18	0.57
(AD3)	DRC at 9%	∞	0.23	-0.66	-4.22	-0.07	0.09	-0.36	1.20
(AD4)	DRC at 9.5%	∞	0.26	-0.73	-4.56	-0.09	0.12	-0.49	1.37
(AD5)	DRC at 10%	∞	0.3	-0.78	-5.05	-0.08	0.14	-0.97	1.39
(AL1)	Linear ±7%	∞	0.10	-0.12	-0.74	-0.01	0.05	0.24	0.22
(AL2)	Linear ±8%	∞	0.15	-0.46	-2.81	-0.07	0.11	0.05	0.61
(AL3)	Linear ±9%	∞	0.21	-0.72	-4.74	-0.07	0.11	-0.31	1.21
(AL4)	Linear ±10%	∞	0.29	-0.86	-5.54	-0.09	0.14	-0.93	1.42
(AL5)	Linear ±11%	∞	0.37	-1.17	-7.02	-0.19	0.19	-1.15	2.01
(AL6)	Linear ±12%	∞	0.47	-1.37	-7.72	-0.28	0.30	-1.46	2.61
(AL7)	Linear ±13%	∞	0.57	-1.38	-8.21	-0.23	0.32	-2.90	2.50
(AL8)	Linear ±14%	∞	0.69	-1.47	-8.22	-0.31	0.34	-3.30	2.42
(AL9)	Linear ±15%	∞	0.80	-1.57	-8.45	-0.37	0.36	-3.49	2.26

Notes: Social Security age-dependence reforms AN1–AN3 increase the normal retirement age to 66, 67, and 68. Reforms AD1–AD5 increase, by 0.5%, the delayed retirement credit (DRC) from 8% to 10%. Reforms AL1–AL9 equalize the DRC and early claiming penalties in absolute values and increase them from ±7% to ±15%. For each reform, we present the consumption-equivalent welfare change from the status quo in percentage or willingness-to-pay (WTP), the percentage change in total Social Security payments (the direct fiscal effect), the indirect and net fiscal effects accounting for equilibrium changes in all parts of government spending, the marginal value of public finds (MVPF), and the changes in labor force participation at ages 30–49, 62–64, and 65–69 in percentage points. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. Reforms that increase the retirement age lower welfare when endogenous retirement is accounted for. Preserving retirement flexibility by increasing the DRC pays for itself, increasing welfare at a negative cost (MVPF of infinity).

Reforms of the age dependence of Social Security retirement benefits that increase the DRC or linearly change the age adjustments of the benefits would increase welfare and reduce benefit payments directly. Given the US status quo, these reforms would therefore pay for themselves and have an MVPF of infinity. The Pareto-improving nature of these reforms can be understood through their effect on participation. These reforms would increase participation among young individuals aged 30–49 (representing a positive substitution effect) and people above the NRA, aged 65–69 (representing positive substitution among healthier individuals who opt to retire later and claim larger benefits), but would have heterogeneous effects on early retirees. An increase in the DRC up to 8.5% and linear adjustments of SS benefits by age up to ±8% would increase participation in the 62–64 age group (the positive substitution effect

[29] As Hendren and Sprung-Keyser (2020) put it, "Equivalently, the MVPF measures the shadow price of raising revenue from the beneficiaries of the policy by reducing spending on the policy."

would still dominate). However, further increases in the DRC would decrease participation for early retirees aged 62–64—and do so even more markedly at very steep linear adjustment rates, as unhealthier workers self-select into earlier retirement.

Through selection, the indirect fiscal effects of the reforms would lower government expenses, as more taxes would be collected from the larger number of healthier workers who participate longer. Our measurement of the MVPF of these reforms is infinity, whether we account for the net or direct fiscal effect alone, as they lead to a net decrease in direct and indirect government expenses. The intuition behind the welfare gains from the age-dependency reforms of Social Security retirement benefits that increase the DRC or linearly change the age adjustments of retirement benefits is that they do not constitute a strict worsening of the individuals' benefits without any behavioral change, in contrast to changes to the NRA (which induce a negative substitution effect for the young and a negative income effect for the old). Instead, they increase the reward for those who delay retirement and the penalty on those who retire early (particularly the L reforms) and thus preserve more retirement flexibility than increases to the NRA. It turns out, given current US retirement benefit adjustment rates, that increasing the DRC to 8% can increase welfare, lower Social Security payments, and increase participation in all the age groups considered in our simulations.

In summary, reforms that aim to increase the retirement age lower welfare when one accounts for endogenous retirement. In contrast, preserving retirement flexibility by increasing the DRC would pay for itself, increasing welfare at a negative cost (an MVPF of infinity), and could increase participation in all the age groups considered.

7.2.2 Medicare and Tax Reforms

Medicare Reforms. We now study reforms that change Medicare coverage and affect medical expenditures depending on participation. While the conversation around Medicare reform in the US tends to focus on Medicare Advantage, the privately managed alternative to traditional Medicare, and adjustments to insurer payment models to reduce overpayments and waste, we abstract from issues around *efficiency of government program administration* (Skinner and Wennberg 1998; Skinner et al. 2001) and focus on policies that expand Medicare coverage, which are welfare-improving in our framework. Because most beneficiaries receive premium-free Medicare Part A (mostly covering hospitalizations and inpatient services) if they or their spouse have paid Medicare taxes for at least 10 years, the reform of Medicare Part A that we consider is an increase in health coverage of all individuals over age 65 by 20%. Medicare Part B requires a monthly premium[30] and covers physician and outpatient services that most employer-provided health insurance plans already cover. Therefore, we assume that

[30] Medicare Part B premiums are currently $174/month and increase with delays in election after age 65 to avoid adverse selection in participation.

only nonparticipants over age 65 receive Medicare Part B. Our reform of Medicare Part B would increase coverage of nonparticipants by 20%.

Table 9: Medicare Part A and Part B Reforms

Policy Reforms		%Δ Welfare		%Δ Budget			Δ Participation		
		MVPF	WTP	Net	Direct	Indir.	30–49	62–64	65–69
Medicare Part A	+ 20 pp	0.146	1.36	3.53	45.17	0.10	0.22	-1.70	-0.38
Medicare Part B	+ 20 pp	0.155	1.30	3.19	41.61	0.03	0.21	-1.95	-3.44

Notes: The reform of Medicare Part A increases health coverage of all individuals over age 65 by 20%. The reform of Medicare Part B increases coverage of nonparticipants by 20%. For each reform, we present the consumption-equivalent welfare change from the status quo in percentage or willingness-to-pay (WTP), the changes in the medical payment by the program to the targeted beneficiaries as a share of payroll tax collection at baseline (the direct fiscal effect), the indirect and net fiscal effects accounting for equilibrium changes in all parts of government spending, the marginal value of public funds (MVPF), and the changes in labor force participation at ages 30–49, 62–64, and 65–69 in percentage points. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. The Medicare expansions have MVPFs within the low range of our estimates and would lead to large direct costs and indirect costs.

Table 9 presents the results for these proposed Medicare reforms.[31] An expansion of Medicare Part A would increase welfare but would come at a sizable direct fiscal cost, with a 45% increase in the payroll taxes T^p that fund Medicare. The costs are driven mainly by large medical expenditures over age 70. In addition, it would reduce participation at ages 62–64 and 65–69, as older individuals would need to work less to save for medical expenses. This drop in participation would lead to an increase in indirect costs, as less income tax would be collected. Therefore, its MVPF of 0.146 is in the low range among the estimates for the reforms studied.

An expansion of Medicare Part B, which primarily benefits nonparticipants, would increase welfare slightly less than the Medicare Part A expansion in coverage by an equivalent number of percentage points because it would cover a smaller group of individuals. It would also, therefore, be less fiscally costly. Expansion of Medicare among nonparticipants, however, would strongly decrease participation after 65 by 3.44 pp within the group of workers attached to their jobs primarily for employer-provided benefits. It would yield only a slightly better MVPF of 0.155 than our Medicare Part A reform because it is better targeted toward nonparticipants who need it most.

Our MVPF estimates, in particular those for Medicare expansion, are useful because their empirical counterparts and random variation in Medicare expansion are rare. Using variation in the preexisting fraction of the population that was insured, Finkelstein and McKnight (2008) and Hendren and Sprung-Keyser (2020) find that the MVPF of the *introduction* of Medicare falls in the range $[0.52, 3.83]$.[32] It is therefore intuitive that, with diminishing marginal returns,

[31] To implement Medicare reforms targeting individuals aged 65 and older, we reestimate the average Medicare coverage rates for this age group by health and working status using the MEPS data. The model is applied with these updated estimates as the new baseline.

[32] Finkelstein, Hendren and Shepard (2019) finds a willingness-to-pay for health insurance of approximately 30% of the cost imposed on the insurer by low-income adults, but 70% of care to these individuals goes uncom-

Medicare *expansions* would have a lower MVPF than the introduction of Medicare. Our MVPF estimate for Medicare expansion of 0.15 aligns with the MVPFs of reforms of the NRA (AN1–3). This result is consistent with French and Jones's (2011) conclusion that Medicare is almost, if not equally, as important as Social Security for retirement. Nonetheless, we find more efficient SS reforms, such as increasing the DRC, that can pay for themselves and increase participation given the US status quo.

Income Tax Reforms. We now study reforms of income taxes. Income tax reforms I1–5 would decrease income taxes after age 62, so that statutory after-tax incomes $y - T(y)$ increase by 1% up to 5%. Table 10 summarizes the results from these reforms.

Table 10: Income Tax Reforms

Policy Reforms		%Δ Welfare		%Δ Budget			Δ Participation		
		MVPF	WTP	Net	Direct	Indir.	30–49	62–64	65–69
(I1)	$y^{\text{post-tax}}_{t\geq 62} + 1$ %	0.25	0.44	0.69	0.70	-0.81	0.06	0.76	1.98
(I2)	$y^{\text{post-tax}}_{t\geq 62} + 2$ %	0.26	0.87	1.28	1.31	-1.51	0.15	1.58	4.12
(I3)	$y^{\text{post-tax}}_{t\geq 62} + 3$ %	0.25	1.29	2.01	2.03	-2.18	0.18	2.76	6.07
(I4)	$y^{\text{post-tax}}_{t\geq 62} + 4$ %	0.23	1.69	2.80	2.80	-2.78	0.2	3.19	7.71
(I5)	$y^{\text{post-tax}}_{t\geq 62} + 5$ %	0.23	2.09	3.46	3.47	-3.58	0.26	4.11	8.92

Notes: Income tax reforms I1–I5 decrease income taxes after age 62, so that statutory after-tax incomes $y - T(y)$ increase by 1% up to 5%. For each reform, we present the consumption-equivalent welfare change from the status quo in percentage or willingness-to-pay (WTP), the percentage change in negative total income tax collection (the direct fiscal effect), the indirect and net fiscal effects accounting for equilibrium changes in all parts of government spending, the marginal value of public finds (MVPF), and the changes in labor force participation at ages 30–49, 62–64, and 65–69 in percentage points. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. Reductions in income taxes at old age would increase participation, welfare, and direct costs that are not fully offset by indirect government savings. Their MVPFs are in the lower range of our estimates.

Lowering income taxes in old age would increase welfare at a relatively significant cost to the government's budget. These policies would incentivize participation at all ages. However, the induced savings in government spending through lower indirect costs would not offset the income tax losses. Their MVPFs of 0.23–0.26 are at the low end of our range of estimates for the other reforms. In summary, tax reforms that reduce income taxes in old age would increase participation and welfare at a relatively important net fiscal cost and yield only modest MVPFs.

7.2.3 Savings Subsidies

We now study reforms that subsidize savings. Savings subsidy reforms K1–3 increase the net interest rate on savings by, respectively, 0.2 pp, 0.4 pp, and 0.8 pp. Table 11 summarizes the results of these reforms.

pensated. Hendren and Sprung-Keyser (2020) assumes that uncompensated care is paid by the government and finds an upper bound for the MVPF of health insurance subsidies for low-income adults of 0.8. These numbers, though instructive, are not comparable to our estimates, as these studies focus on the expansion of Medicaid.

Table 11: Savings Subsidy Reforms

Policy Reforms		%Δ Welfare		%Δ Budget			Δ Participation		
		MVPF	WTP	Net	Direct	Indir.	30–49	62–64	65–69
(K1)	$\Delta r = +0.2$ pp	$[0.96, 20.37]$	0.87	0.08	1.77	-1.69	0.15	-1.27	-0.85
(K2)	$\Delta r = +0.4$ pp	$[0.92, 19.38]$	1.75	0.18	3.77	-3.59	0.36	-2.58	-1.75
(K3)	$\Delta r = +0.8$ pp	$[0.84, 5.29]$	3.6	1.34	8.44	-7.10	0.38	-5.98	-4.11

Notes: Savings subsidy reforms K1–3 increase net returns to savings by 0.20 percentage points (pp), 0.4 pp, and 0.8 pp. For each reform, we present the consumption-equivalent welfare change from the status quo in percentage or willingness-to-pay (WTP), the total amount of the subsidy as a share of government spending at baseline (the direct fiscal effect), the indirect and net fiscal effects accounting for equilibrium changes in all parts of government spending, the marginal value of public finds (MVPF), and the changes in labor force participation at ages 30–49, 62–64, and 65–69 in pp. The blue gradient captures desirability, while the red gradient represents undesirability, and blank is neutral. Savings subsidies can have large MVPFs, but whether they pay for themselves depends on their size and whether they significantly decrease indirect costs through higher capital gains tax revenue from extra savings.

Savings subsidies would increase welfare but would do so at the cost of a direct loss in government revenue (as a share of total government spending). Accounting for these direct costs alone gives us a lower bound of the MVPF for these reforms of 0.84–0.96. Reductions in indirect costs would offset the direct revenue losses. These indirect cost reductions are driven by additional income taxes from younger households that save more (representing a positive substitution effect). Since savings subsidies, particularly those for retirement accounts, are geared toward long-term investments, if we assume that all capital gains are realized and taxed in each period, we can set an upper bound on the indirect effects. Upon accounting for the net fiscal effects, we see that small savings subsidies of 0.2 pp can have an MVPF greater than 1. As a conservative measure, the lower bound of 0.84 is among the highest among the policies we have studied, except for the age-dependency increases in the DRC, which would pay for themselves directly. This suggests that policies aimed at further subsidizing retirement saving—such as tax exemptions for Roth IRAs and Roth 401(k) plans—or facilitating retirement saving—such as tax deferral for traditional IRAs and 401(k) plans—can have a return on the dollar close to or above 1. However, savings subsidies can significantly trigger an income effect in old age and lower participation, as individuals find it easier to save for retirement. Therefore, reforms aimed at increasing retirement savings can be more effective at increasing welfare but optimally ease retirement in old age.[33]

8 Extensions and Robustness

Optimal Retirement: To tractably characterize the optimal retirement decisions, we develop a continuous-time analog of our model (e.g., Sannikov 2008; Grochulski and Zhang 2017) in Appendix A.4, where wages evolve according to a geometric Brownian motion (GBM). Retirement is modeled as a stopping-time problem under nonconvexities in labor supply. With

[33]Our MVPF calculations in Appendix C.4 for income tax reductions after age 62 and savings subsidies at all ages are comparable, so the differences in effectiveness from the age-dependent income tax reforms and savings subsidies do not come from the fact that income taxes would be reduced only at old ages.

full information, highly productive workers retire later than less productive ones. Similarly to Dixit (1989) and Leland (1994), we highlight an option value of continuing work in the presence of uncertain future incomes. This option value decreases as workers age, causing the nonconvexities in labor supply to increase the retirement threshold over time. This result holds under general risk-averse preferences that are separable in consumption and labor and for Greenwood et al.'s (1988) GHH preferences that are not separable.

We next analyze optimal retirement under asymmetric information. For risk-neutral individuals, we provide a novel application to the problem of Pareto-optimal taxation of Bergemann and Strack's (2015) restriction to "consistent deviations" as a necessary condition for incentive compatibility. In this setting, incentive compatibility simplifies to a single condition involving an impulse response function similar to that of Pavan et al. (2014), enabling an FOA. The constrained-efficient optimal retirement can be characterized as a modification of the first-best threshold by a "virtual fixed cost," where we adjust the fixed utility cost of labor by incorporating informational rents and the elasticity of labor supply. Unhealthy workers who face larger fixed costs of work retire earlier in the constrained optimum than in the first-best optimum.

Implementation with a Simple Social Security Program: The usual implementation of second-best policies in Farhi and Werning (2013) and Werning (2011), which applies for separable preferences, requires full-history dependence of both the retirement benefits and the labor income tax and is somewhat different from status quo policies. We provide in Appendix A.6 a novel and simpler implementation of the optimal policies in our continuous-time setup with risk neutrality in consumption. Our simplified system of instruments that resembles the US Social Security program consists of retirement benefits dependent only on lifetime income and retirement age, a history-independent labor income tax, and a lump-sum transfer. The key idea involves introducing post-retirement transfers that align agents' private retirement decisions with the optimal second-best solution. We achieve this alignment by setting transfers equal to the expected discounted value of future earnings and fixed costs, evaluated along a "reflected" productivity process that always remains above the optimal retirement threshold (Kruse and Strack 2015). This construction provides a tractable way of implementing optimal retirement policies without complex dependencies.

Alternative Preferences: In our quantification of the effects through the nonconvexity channel on optimal policies in Section 6.1, we considered a calibration based on log preferences in consumption for reasons of quantitative solvability. Appendix C.5 presents the reestimation of our full quantitative model with these alternative preferences. In particular, this model does not match empirical labor force participation rates as well as our model with nonseparable consumption and leisure. However, it matches the mean assets in old age better than our baseline quantitative model.

9 Conclusion

In this paper, we have integrated endogenous retirement choices and health shocks into a dynamic life cycle income taxation model to study the interplay between optimal taxation, Social Security, Medicare, and disability insurance. Our theoretical analysis highlights that optimal policy design must account for several key channels through which health impacts retirement decisions: labor supply nonconvexities, productivity loss, and mortality risk. Specifically, we find that unhealthy workers optimally face lower labor wedges because of their higher cost of continued labor while healthy workers benefit from policies incentivizing delayed retirement.

Quantitatively, our estimation shows that labor supply nonconvexities significantly drive retirement behavior in the US, more so than productivity or mortality risks alone. Among policy instruments, retirement benefits through the US Social Security system have the largest impact on retirement choices and on welfare, followed by Medicare, retirement savings subsidies (such as tax exemptions for Roth plans and tax deferrals for traditional IRAs and 401(k) plans), then income taxes, and finally disability insurance. Our policy experiments provide estimates of the MVPFs of feasible reforms of these policies. Our results suggest that increasing the claiming-age dependence of Social Security benefits relative to the status quo would yield substantial welfare gains and could pay for itself with an MVPF of infinity while uniform increases to the NRA would be welfare decreasing once one accounts for endogenous retirement.

Overall, our results suggest that flexible retirement policies, rather than rigid increases in the retirement age alone, are optimal from both welfare and fiscal perspectives. Future research and policy discussions should, therefore, focus on nuanced yet simple adjustments to social insurance programs that align worker and government incentives.

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Part I

Appendix

A Theory Appendix

A.1 Proof of Lemmas 2, 4, and 6

The Hamiltonian of the planner's problem is

$$\begin{aligned}
&[C^t(y(\theta,h),\omega(\theta,h)-\beta v(\theta,h),\theta,h)-y(\theta,h)]s(h_-)\pi_h f^t(\theta|\theta_-) \\
&+\frac{1}{R}[K(v(\theta,h),\Delta(\theta,h),\theta,h,t+1)]s(h_-)\pi_h f^t(\theta|\theta_-) \\
&+\lambda[v-\omega(\theta,h)s(h_-)\pi_h f^t(\theta|\theta_-)]+\gamma[\Delta-\omega(\theta,h)s(h_-)\pi_h \tilde{f}^t(\theta,\theta_-)] \\
&+p(\theta,h)[u^t_\theta(C^t(y(\theta,h),\omega(\theta,h)-\beta v(\theta,h),\theta,h),y(\theta,h),\theta,h)+\beta\Delta(\theta,h)]
\end{aligned}$$

with boundary conditions of the costate p

$$\lim_{\theta\to\bar{\theta}} p(\theta,h)=\lim_{\theta\to\underline{\theta}} p(\theta,h)=0 \quad \forall h\in\mathcal{H}$$

Taking the first-order condition (FOC) with respect to y,

$$\frac{\tau_L(\theta,h)}{1-\tau_L(\theta,h)}=\frac{p(\theta,h)}{s(h_-)\pi_h f^t(\theta|\theta_-)}\frac{w_{\theta,t}}{w_t}u'\left(c\left(\theta,h\right)\right)\left(1+\frac{n(\theta,h)\psi_{nn}\left(n(\theta,h);h\right)}{\psi_n\left(n(\theta,h);h\right)}\right)$$

Define the uncompensated and compensated elasticities as in Saez (2001) at constant savings:

$$\varepsilon^u=\frac{\frac{\psi_n(n)}{n}+\frac{\psi_n(n)^2}{u'(c)^2}u''(c)}{\psi_{nn}(n)-\frac{\psi_n(n)^2}{u'(c)^2}u''(c)},\quad \varepsilon^c=\frac{\frac{\psi_n(n)}{n}}{\psi_{nn}(n)-\frac{\psi_n(n)^2}{u'(c)^2}u''(c)}$$

With these elasticity definitions, we have

$$\frac{\tau_L(\theta,h)}{1-\tau_L(\theta,h)}=\frac{p(\theta,h)}{s(h_-)\pi_h f^t(\theta|\theta_-)}\frac{\varepsilon_{w,\theta}}{\theta}u'\left(c\left(\theta,h\right)\right)\left(\frac{1+\varepsilon^u_t(h)}{\varepsilon^c_t(h)}\right). \tag{18}$$

For the costate, the FOC with respect to $\omega(\theta,h)$ yields:

$$\frac{dp(\theta,h)}{d\theta}=-H^*_\omega$$

where H^* is the Hamiltonian, that is,

$$\frac{dp(\theta,h)}{d\theta}=\left[-\frac{1}{u'\left(c\left(\theta,h\right)\right)}+\lambda+\gamma\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]s(h_-)\pi_h f^t(\theta|\theta_-)$$

Integrating over θ and using both boundary conditions, we have for any h

$$\lambda=\int_\Theta\frac{1}{u'\left(c\left(\theta,h\right)\right)}f^t(\theta|\theta_-)d\theta \tag{19}$$

The FOCs for $v(\theta,h)$ and $\Delta(\theta,h)$ are, using the envelope theorem to replace K_v and K_Δ with $\lambda.\gamma$,

$$\frac{1}{u'(c(\theta,h))}=\frac{\lambda(\theta,h)}{\beta R} \tag{20}$$

and

$$-\frac{\gamma(\theta,h)}{\beta R}=\frac{p(\theta,h)}{s(h_-)\pi_h f^t(\theta|\theta_-)}$$

Multiplying (18) with an arbitrary weighting function $x(\theta)$, and letting $\mathcal{X}$ be the integral of x, we have

$$\begin{aligned}
&\int \frac{\tau_L(\theta,h)}{1-\tau_L(\theta,h)}s(h_-)\pi_h f^t(\theta|\theta_-)\frac{\theta}{\varepsilon_{w,\theta}}\frac{\varepsilon_t^c(h)}{1+\varepsilon_t^u(h)}\frac{1}{u'\left(c\left(\theta,h\right)\right)}x(\theta)d\theta=\int_\Theta p(\theta,h)x(\theta)d\theta\\
&=-\int\frac{dp(\theta,h)}{d\theta}\mathcal{X}(\theta)d\theta\\
&=\int\left[\frac{1}{u'\left(c\left(\theta,h\right)\right)}-\frac{\beta R}{u'_{t-1}}-\gamma\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]s(h_-)\pi_h f^t(\theta|\theta_-)\mathcal{X}(\theta)d\theta\\
&=\int\left[\frac{1}{u'\left(c\left(\theta,h\right)\right)}-\frac{\beta R}{u'_{t-1}}+\frac{\beta Rp(\theta_-,h_-)}{s(h_{-2})\pi_{h_-}f^t(\theta_-|\theta_{-2})}\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]s(h_-)\pi_h f^t(\theta|\theta_-)\mathcal{X}(\theta)d\theta\\
&=\int\left[\frac{1}{u'}-\frac{\beta R}{u'_{t-1}}+\frac{\beta R\tau_L(\theta_-,h_-)}{1-\tau_L(\theta_-,h_-)}\frac{\theta_-}{\varepsilon_{w,\theta_-}}\frac{1}{u'_{t-1}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]s(h_-)\pi_h f^t(\theta|\theta_-)\mathcal{X}(\theta)d\theta\\
&=\int\left[\frac{1}{u'}-\frac{\beta R}{u'_{t-1}}+\frac{\beta R\tau_L(\theta_-,h_-)}{1-\tau_L(\theta_-,h_-)}\frac{\theta_-}{\varepsilon_{w,\theta_-}}\frac{1}{u'_{t-1}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]s(h_-)\pi_h f^t(\theta|\theta_-)\mathcal{X}(\theta)d\theta
\end{aligned}$$

so with $\theta x(\theta)=1$, $\mathcal{X}(\theta)=\ln(\theta)$

$$\begin{aligned}
&\int\frac{\tau_L}{1-\tau_L}\frac{1}{\varepsilon_{w,\theta}}\frac{\varepsilon_t^c(h)}{1+\varepsilon_t^u(h)}\frac{u'_{t-1}}{\beta Ru'}f^t(\theta|\theta_-)d\theta\\
&=\int\left[\frac{u'_{t-1}}{\beta Ru'}-1+\frac{\tau_L(\theta_-,h_-)}{1-\tau_L(\theta_-,h_-)}\frac{\theta_-}{\varepsilon_{w,\theta_-}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}\frac{\tilde{f}^t(\theta,\theta_-)}{f^t(\theta|\theta_-)}\right]\ln(\theta)f^t(\theta|\theta_-)d\theta\\
&=\mathrm{Cov}\left(\frac{u'_{t-1}}{\beta Ru'},\ln(\theta_t)\right)+\frac{\tau_L(\theta_-,h_-)}{1-\tau_L(\theta_-,h_-)}\frac{\theta_-}{\varepsilon_{w,\theta_-}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}\int_\Theta\tilde{f}^t(\theta,\theta_-)\ln(\theta)d\theta\\
&=\mathrm{Cov}\left(\frac{u'_{t-1}}{\beta Ru'},\ln(\theta_t)\right)+\rho\frac{\tau_L(\theta_-,h_-)}{1-\tau_L(\theta_-,h_-)}\frac{1}{\varepsilon_{w,\theta_-}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)}
\end{aligned}$$

with AR(1) with persistence ρ

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\varepsilon_{w\theta,t}}\frac{\varepsilon^c_t(h)}{1+\varepsilon^u_t(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] =$$
$$\rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}\frac{1}{\varepsilon_{w,\theta_-}}\frac{\varepsilon^c_{t-1}(h_-)}{1+\varepsilon^u_{t-1}(h_-)} + \text{Cov}\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right) \tag{21}$$

This proves the general Lemma 6 with interactions. We can then focus on the nonconvexity channel in Lemma 2, which corresponds to $\varepsilon_{w,\theta_-} = \varepsilon_{w,\theta_t}$ and yields

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{\varepsilon^c_t(h)}{1+\varepsilon^u_t(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] =$$
$$\text{Cov}\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right) + \rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}\frac{\varepsilon^c_{t-1}(h_{-1})}{1+\varepsilon^u_{t-1}(h_{-1})}$$

and the productive loss channel in Lemma 4, which corresponds to $\frac{\varepsilon^c_t(h)}{1+\varepsilon^u_t(h)} = \frac{\varepsilon^c_{t-1}(h_{-1})}{1+\varepsilon^u_{t-1}(h_{-1})} = \frac{\varepsilon}{1+\varepsilon}$, where ε is the constant Frisch elasticity of the isoelastic disulity of labor $\psi(n)$ in Economy 2. Equalizing the labor supply elasticities in (21) yields

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\varepsilon_{w\theta,t}(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] =$$
$$\rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}\frac{1}{\varepsilon_{w,\theta_-,t-1}(h_-)} + \left(1+\frac{1}{\varepsilon}\right)\text{Cov}\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right)$$

A.2 Proof of Proposition 3

The proof proceeds as follows. After showing the direction of movement of these elasticities, we obtain an *epsilon-of-a-room* for our inequality. For the innovation covariance term, we invoke the reduction in the savings wedge/dynamic incentives over time to fit it inside the room.

We start by observing that close to retirement, the intratemporal insurance term in covariance grows invariably small, so for any $\zeta > 0$, there exists t' so that for any $t \geq t'$, $\frac{1+\varepsilon^u_{t-1}(h_{-1})}{\varepsilon^c_{t-1}(h_{-1})}\text{Cov}\left(\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}, \ln(\theta_t)\right) \leq \zeta$ and

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{\varepsilon^c_t(h)}{1+\varepsilon^u_t(h)}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] \leq \rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}(1+\zeta)\frac{\varepsilon^c_{t-1}(h_{-1})}{1+\varepsilon^u_{t-1}(h_{-1})}$$

We then characterize how the endogenous elasticities evolve with health.

1. For general separable and nonconvex labor supply preferences defined by (13): We calculate the compensated and uncompensated elasticities. First, $u'(c) = c^{-\nu}$, $u''(c) = -\nu c^{-\nu-1}$, where $u(c) = \frac{c^{1-\nu}}{1-\nu}$ is the CRRA utility function. Second, $\psi_n(n) = -(L-n)^{-\gamma}$ and $\psi_{nn}(n) = \gamma(L-n)^{-\gamma-1}$, where $\psi(n) = -\frac{(L-n)^{1-\gamma}}{1-\gamma}$ represents the disutility of labor n and L is the available leisure time.

Substituting into the definitions of elasticities, we have:

$$\varepsilon^u = \frac{\frac{(L-n)^{-\gamma}}{n} - \nu(L-n)^{-2\gamma}c^{\nu-1}}{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1}},$$

$$\varepsilon^c = \frac{\frac{(L-n)^{-\gamma}}{n}}{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1}}.$$

The ratio of the elasticities $\frac{\varepsilon^c}{1+\varepsilon^u}$ is calculated as follows, on the denominator $1+\varepsilon^u$:

$$1+\varepsilon^u = 1 + \frac{\frac{(L-n)^{-\gamma}}{n} - \nu(L-n)^{-2\gamma}c^{\nu-1}}{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1}}$$

Combine terms over the common denominator:

$$1+\varepsilon^u = \frac{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1} + \frac{(L-n)^{-\gamma}}{n} - \nu(L-n)^{-2\gamma}c^{\nu-1}}{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1}}$$

Simplify the numerator:

$$1+\varepsilon^u = \frac{-\gamma(L-n)^{-\gamma-1} + \frac{(L-n)^{-\gamma}}{n}}{-\gamma(L-n)^{-\gamma-1} + \nu(L-n)^{-2\gamma}c^{\nu-1}}$$

The ratio becomes:

$$\frac{\varepsilon^c}{1+\varepsilon^u} = \frac{\frac{(L-n)^{-\gamma}}{n}}{-\gamma(L-n)^{-\gamma-1} + \frac{(L-n)^{-\gamma}}{n}}$$

Simplify further:

$$\frac{\varepsilon^c}{1+\varepsilon^u} = \frac{1}{\frac{n}{L-n} - \gamma}$$

The available leisure time depends on health, so $L = L(h)$. We evaluate the elasticities at the endogenous hours worked, which depend on the whole history, but we emphasize its dependence on health and write $n^* = n^*(h)$ for ease of notation. Then,

$$\frac{\varepsilon^c}{1+\varepsilon^u} = \frac{1}{\frac{n^*(h)}{L(h)-n^*(h)} - \gamma}$$

Take the derivative with respect to h:

$$\frac{d}{dh}\left(\frac{\varepsilon^c}{1+\varepsilon^u}\right) = -\frac{\frac{d}{dh}\left(\frac{n^*(h)}{L(h)-n^*(h)}\right)}{\left(\frac{n^*(h)}{L(h)-n^*(h)} - \gamma\right)^2}$$

The derivative of $\frac{n^*(h)}{L(h)-n^*(h)}$ using the quotient rule is:

$$\frac{d}{dh}\left(\frac{n^*(h)}{L(h)-n^*(h)}\right) = \frac{\frac{dn^*(h)}{dh}(L(h)-n^*(h)) - n^*(h)\left(\frac{dL(h)}{dh} - \frac{dn^*(h)}{dh}\right)}{(L(h)-n^*(h))^2}$$

If $\frac{dL(h)}{dh} = 0$ (total available time does not depend on health), this simplifies to:

$$\frac{d}{dh}\left(\frac{n^*(h)}{L(h)-n^*(h)}\right) = \frac{\frac{dn^*(h)}{dh}L(h)}{(L(h)-n^*(h))^2}$$

Thus, the derivative becomes:

$$\frac{d}{dh}\left(\frac{\varepsilon^c}{1+\varepsilon^u}\right) = -\frac{\frac{dn^*(h)}{dh}L(h)}{\left(\frac{n^*(h)}{L(h)-n^*(h)} - \gamma\right)^2 \cdot (L(h)-n^*(h))^2}$$

The intuition is that if health worsens (poor h), $n^*(h)$ decreases ($\frac{dn^*(h)}{dh} < 0$). The numerator $\frac{dn^*(h)}{dh}L(h)$ is negative. The denominator is positive because $\left(\frac{n^*(h)}{L(h)-n^*(h)} - \gamma\right)^2$ and $(L(h)-n^*(h))^2$ are both positive. Thus, $\frac{d}{dh}\left(\frac{\varepsilon^c}{1+\varepsilon^u}\right) < 0$, meaning the elasticity ratio increases as health worsens. As health worsens, labor supply $n^*(h)$ decreases, making labor more sensitive to incentives relative to consumption. This raises the elasticity ratio $\frac{\varepsilon^c}{1+\varepsilon^u}$.

If $\frac{dL(h)}{dh} \neq 0$,

$$\frac{\varepsilon^c}{1+\varepsilon^u} = \frac{1}{\frac{n^*(h)/L(h)}{1-n^*(h)/L(h)} - \gamma} \tag{22}$$

and the function $x \mapsto x/(1-x)$ is increasing in $[0,1)$. Hence, the elasticity ratio increases in poor health if the ratio of hours to available time $n^*(h)/L(h)$ decreases in poor health. All the terms in the expectations are positive such that $E(XY) \leq E(X)E(Y)$, so

$$\begin{aligned} E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] &\leq E_{t-1}\Big[\left(\frac{\varepsilon^c_t}{1+\varepsilon^u_t}\right)^{-1}|h\Big]E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{\varepsilon^c_t}{1+\varepsilon^u_t}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] \\ &\leq \rho\frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}(1+\zeta)\underbrace{\frac{\varepsilon^c_{t-1}}{1+\varepsilon^u_{t-1}}E_{t-1}\Big[\left(\frac{\varepsilon^c_t}{1+\varepsilon^u_t}\right)^{-1}|h\Big]}_{<1} \end{aligned}$$

where the expectation around $\left(\frac{\varepsilon^c_t}{1+\varepsilon^u_t}\right)^{-1}$ is because the endogenous elasticities can depend on $\{\theta^t\}$. Given that the elasticity ratio is decreasing in poor health, $\phi \leq 1$, and taking ζ small enough, we have the result for $t > t'$

$$E_{t-1}\Big[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{\beta R}\frac{u'(c_{t-1})}{u'(c_t)}|h\Big] \leq \frac{\tau^b_{L,t-1}}{1-\tau^b_{L,t-1}}$$

2. For the separable limit of French and Jones (2011) preferences: At the separable limit of French and Jones (2011) fixed time cost preferences (3) and (4) at $\nu \to 1$, we have

$$u(c_t, n_t; h_t) = \gamma \ln(c_t) + (1-\gamma) \ln\left(L - n_t - \phi_n I\{n_t > 0\} - \phi_{RE} RE_t - \phi_h(h_t)\right)$$

The elasticity ratio is the same as in (22), where $\gamma = 1$ and the available leisure time is $L(h) = L - \phi_n - \phi_{RE} - \phi_h(h)$, thus the result.

3. For preferences with a fixed setup cost of work as in Prescott et al. (2009): Hours devoted to market work are denoted by $n \in [n_h, 1]$, where

$$g(n) = \max\{\, 0, n - n_h \,\}$$

maps hours into *labor services*.[34]

For analytical traceability, it helps to work with a smoothed version of the max function as in Prescott et al. (2009). That is, to differentiate at the kink, we replace g by the shifted hyperbolic tangent:

$$\tilde{g}(n) = \frac{1}{k} \tanh\big(k\,(n - n_h)\big), \quad k > 0.$$

Its inverse

$$\varphi(n) = \frac{1}{k} \operatorname{artanh}\big(k\,(n - n_h)\big) \tag{23}$$

is *strictly increasing and strictly convex* on $(n_h, 1/k + n_h)$.

The disutility of work is isoelastic in φ, $\psi(x) = \frac{x^{1+1/\epsilon}}{1+1/\epsilon}$. For ease of notation, let d denote the disutility of hours and R the endogenous elasticity ratio $\frac{\varepsilon^c}{1+\varepsilon^u}$

$$d(n) \equiv \psi\big(\varphi(n)\big), \quad R(n) \equiv \frac{\varepsilon^c}{1+\varepsilon^u} = \frac{d'(n)/n}{d'(n)/n + d''(n)}$$

From (23),

$$\varphi'(n) = \frac{1}{1 - k^2(n-n_h)^2}, \quad \varphi''(n) = \frac{2k^2(n-n_h)}{\big[1 - k^2(n-n_h)^2\big]^2} > 0.$$

Hence

$$\begin{aligned}
d'(n) &= \psi'\big(\varphi(n)\big)\,\varphi'(n) = \varphi(n)^{1/\epsilon}\varphi'(n),\\
d''(n) &= \psi''\big(\varphi(n)\big)\,[\varphi'(n)]^2 + \psi'\big(\varphi(n)\big)\,\varphi''(n)\\
&= \frac{1}{\epsilon}\varphi(n)^{1/\epsilon-1}[\varphi'(n)]^2 + \varphi(n)^{1/\epsilon}\varphi''(n)
\end{aligned}$$

We can now do Taylor approximations around the kink n_h. Let $\tilde{n} \equiv n - n_h \downarrow 0^+$. Using

[34] See Fig. 4, p. 27, of Prescott et al. (2009) for a graphical discussion of g.

$\operatorname{artanh}(x) = x + \frac{x^3}{3} + O(x^5)$,

$$\varphi(n) = \tilde{n} + \frac{k^2\tilde{n}^3}{3} + O(\tilde{n}^5), \quad \varphi'(n) = 1 + k^2\tilde{n}^2 + O(\tilde{n}^4), \quad \varphi''(n) = 2k^2\tilde{n} + O(\tilde{n}^3)$$

Substituting into $d'(n)$ and $d''(n)$ and retaining the lowest nonzero orders gives

$$\frac{d'(n)}{n} = \frac{\tilde{n}^{1/\epsilon}}{\tilde{n}} + O(\tilde{n}^{1/\epsilon+1}) = \tilde{n}^{1/\epsilon-1} + O(\tilde{n}^{1/\epsilon+1})$$

$$d''(n) = \frac{1}{\epsilon}\tilde{n}^{1/\epsilon-1} + 2k^2\tilde{n}^{1/\epsilon+1} + O(\tilde{n}^{1/\epsilon+2})$$

Insert into the elasticity ratio $R(n)$:

$$R(n) = \frac{1}{1 + \dfrac{d''(n)\,n}{d'(n)}} = \frac{1}{1 + \dfrac{1}{\epsilon} + 2k^2\tilde{n}^2 + O(\tilde{n}^3)}$$

The final Taylor approximation yields

$$R(n) = \frac{1}{1+\epsilon}\Big[1 - \underbrace{\Big(\frac{2k^2\epsilon}{1+\epsilon}\Big)}_{>0}\tilde{n}^2 + O(\tilde{n}^3)\Big] \tag{24}$$

Equation (24) implies that

$$R'(n) = -\frac{4k^2\epsilon}{1+\epsilon}\,\tilde{n} \; + \; O(\tilde{n}^2) < 0 \quad \text{for } 0 < \tilde{n} \ll 1/k$$

As hours supplied n fall toward the fixed-cost kink n_h, $R(n)$ *rises*. A deterioration in health that reduces the optimal n therefore increases the effective Frisch elasticity, thus the result.

Our result goes beyond the tanh smoothing function. The proof relied only on two facts:

1. *Strict convexity* of the smoothing immediately right of n_h ($\varphi''(n) > 0$);

2. A *finite, positive* first derivative $\varphi'(n)\big|_{n\downarrow n_h} = 1$.

Any other smoothing (e.g., soft-plus, shifted logistic, spline) that shares these local properties generates

$$R(n) = \frac{1}{1+\epsilon} - C\left(n - n_h\right)^2 + o\big((n-n_h)^2\big), \quad C > 0$$

and thus the same comparative static conclusion.

4. For preferences with a utility cost of participation: For nonconvex labor supply preferences with a utility cost of participation defined by (14) with a minimum part

$$u\left(c_t, n_t; h_t\right) = \tilde{u}\left(c_t\right) - \psi(n_t) - \phi(h_t)I\{n_t > 0\}$$

We can ignore dependencies of the fixed cost on health and focus on the presence of nonconvexities with a fixed cost $\phi I\{n_t > 0\}$. We will work with a smoothed approximation of the nonconvexity term.

To differentiate at the kink at 0, we adopt a logistic switch that starts at zero and asymptotes to $\phi > 0$:

$$\phi_S(n) \equiv \phi\left(\tfrac{2}{1+e^{-kn}} - 1\right) = \phi \tanh\left(\tfrac{k}{2}\, n\right), \quad k, \phi > 0 \tag{25}$$

We can now add this fixed cost proxy directly to an isoelastic disutility of hours $d(n) \equiv \psi(n) + \phi_S(n)$, $\psi(n) = \frac{n^{1+1/\epsilon}}{1+1/\epsilon}$. Hence, $d'(n) = n^{1/\epsilon} + A + O(n^2)$, $d''(n) = \frac{1}{\epsilon}\, n^{1/\epsilon - 1} + O(n)$, with $A \equiv \phi k/2 > 0$.

The elasticity ratio $R(n)$ can be then approximated near $n = 0$, using

$$\frac{d'(n)}{n} = A\, n^{-1} + n^{1/\epsilon - 1} + O(n), \quad d''(n) = \frac{1}{\epsilon}\, n^{1/\epsilon - 1} + O(n)$$

Write for ease of notation $X \equiv A\, n^{-1}$ and $Y \equiv n^{1/\epsilon - 1}$. Then,

$$R(n) = \frac{X + Y}{X + Y + \frac{1}{\epsilon} Y} = 1 - \frac{\frac{1}{\epsilon} Y}{X + Y + \frac{1}{\epsilon} Y} = 1 - \frac{2}{\phi k \epsilon}\, n^{1/\epsilon} + O\left(n^{\min\{2, 1+1/\epsilon\}}\right)$$

Because the coefficient $2/(\phi k \epsilon)$ is positive, the first derivative satisfies

$$R'(n) = -\frac{2}{\phi k \epsilon^2}\, n^{1/\epsilon - 1} + o\left(n^{1/\epsilon - 1}\right) < 0 \quad \text{for all } 0 < n \ll 1$$

Therefore, for the disutility $d(n) = \dfrac{n^{1+1/\epsilon}}{1 + 1/\epsilon} + \phi_S(n)$ with ϕ_S in (25), the elasticity ratio $R(n)$ is strictly decreasing in the neighborhood of the kink $n = 0$. Consequently, poorer health that lowers the optimal hours supplied raises the labor supply elasticity ratio.

Our result is more general than the smoothing function considered. Only two local properties of the smoothing were used:

1. *Finite, positive slope:* $\phi'_S(0) = \phi k/2$.

2. *Curvature not exploding:* $\phi''_S(n) = O(n)$ as $n \downarrow 0$.

Any alternative smoothing, switch—logistic, shifted exponential, cubic spline, etc.—that shares these properties produces the expansion

$$R(n) = 1 - C\, n^{1/\epsilon} + o\left(n^{1/\epsilon}\right), \quad C > 0$$

and therefore the same comparative static sign. The result is thus robust to the exact choice of smooth approximation. Therefore, this elasticity ratio increases if n decreases in poor health. Thus the result.

This highlights the importance of fixed costs in shaping labor supply decisions when work effort is near the participation threshold.

A.3 Proof of Proposition 5

Replacing (20) for λ in the previous period in (19), we obtain an inverse Euler equation (IEE) that applies by future health state and conditional on survival, that is,

$$\frac{\beta R}{u'(c(\theta_-,h_-))}=\int_\Theta \frac{1}{u'\left(c\left(\theta,h\right)\right)}f^t(\theta|\theta_-)d\theta=\mathbb{E}_{t-1}\left[\frac{1}{u'\left(c\left(\theta,h\right)\right)}|h\right]\,h\in\mathcal{H}_t$$

That is, the IEE (Rogerson 1985) that applies in a setting with dynamic moral hazard applies here not in expectation of future $\{\theta,h,\text{survival}\}$ but, pathwise, conditional on each future health realization h and conditional on survival.

By definition, the savings wedge is constructed with the unconditional expectations at time $t-1$:

$$\tau_{K,t-1}(\theta_-,h_-)=1-\frac{1}{R\beta}\frac{u'\left(c\left(\theta_-,h_-\right)\right)}{s_t(h_-)\int_\Theta\sum_{h\in\mathcal{H}}\pi_{h,h_-,t}u'\left(c\left(\theta,h\right)\right)f^t(\theta|\theta_-)d\theta}$$

We can apply Liao and Berg's (2019) sharpening of Jensen's inequality to the scalar random variable $X=u'(c(\theta,h))$ and $\varphi(x)=1/x$ to obtain, taking expectations on (θ,h), and conditional on survival,

$$\frac{\sigma^2_{c,t}}{u_c(\underline{c_t})^3}\leq\int_\Theta\sum_{h\in\mathcal{H}}\pi_{h,h_-,t}\frac{1}{u'\left(c\left(\theta,h\right)\right)}f^t(\theta|\theta_-)d\theta-$$

$$\frac{1}{\int_\Theta\sum_{h\in\mathcal{H}}\pi_{h,h_-,t}u'\left(c\left(\theta,h\right)\right)f^t(\theta|\theta_-)d\theta}\leq\frac{\sigma^2_{c,t}}{u_c(\overline{c_t})^3}$$

where $\sigma^2_{c,t}$ is the variance of of marginal utility of consumption in the next period $\sigma^2_{c,t}\equiv\mathrm{Var}_{t-1}\left[\tilde{u}_c\left(c_t\right)|\theta_-,h_-\right]$ and, respectively, $\underline{c_t},\overline{c_t}$ are the minimum and maximum values of future consumption: $\underline{c_t}\equiv\min_{\{\theta\in\Theta,h\in\mathcal{H}_t\}}c_t(\theta^{t-1},\theta,h^{t-1},h)$, $\overline{c_t}\equiv\max_{\{\theta\in\Theta,h\in\mathcal{H}_t\}}c_t(\theta^{t-1},\theta,h^{t-1},h)$. Multiplying by $u'\left(c\left(\theta_-,h_-\right)\right)/(\beta R)$, the first integral will equal one due to the IEEs:

$$\frac{u'\left(c\left(\theta_-,h_-\right)\right)\sigma^2_{c,t+1}}{\beta Ru_c(\underline{c_{t+1}})^3}\leq 1-\frac{u'\left(c\left(\theta_-,h_-\right)\right)}{\beta R\int_\Theta\sum_{h\in\mathcal{H}}\pi_{h,h_-,t}u'\left(c\left(\theta,h\right)\right)f^t(\theta|\theta_-)d\theta}\leq\frac{u'\left(c\left(\theta_-,h_-\right)\right)\sigma^2_{c,t+1}}{\beta Ru_c(\overline{c_{t+1}})^3}$$

That is, subtracting 1, dividing by the survival probability $s_t(h_-)$, and adding 1, we obtain

$$1-\frac{1}{s_t(h_-)}\left(1-\frac{u'\left(c\left(\theta_-,h_-\right)\right)\sigma^2_{c,t+1}}{\beta Ru_c(\underline{c_{t+1}})^3}\right)\leq\underbrace{1-\frac{u'\left(c\left(\theta_-,h_-\right)\right)}{\beta Rs_t(h_-)\int_\Theta\sum_{h\in\mathcal{H}}\pi_{h,h_-,t}u'\left(c\left(\theta,h\right)\right)f^t(\theta|\theta_-)d\theta}}_{\tau_{K,t-1}}$$

$$\leq 1-\frac{1}{s_t(h_-)}\left(1-\frac{u'\left(c\left(\theta_-,h_-\right)\right)\sigma^2_{c,t+1}}{\beta Ru_c(\overline{c_{t+1}})^3}\right)$$

The middle term of the inequality corresponds to the savings wedge. Thus the result.

A.4 Optimal Retirement in a Continuous-Time Model

In this section, we describe a continuous-time economy in which workers are ex ante heterogeneous in productivity, experience idiosyncratic productivity shocks over their lifetime, and adjust their labor supply through flexible working hours and the timing of their retirement.

Productivity, Technology, and Preferences: Consider a continuous-time economy populated by a continuum of agents who live until age T. At each time t, each agent privately observes the realization of his current labor productivity $\theta_t \in (0, +\infty)$. Agents provide $n_t \geq 0$ units of labor at time t at a wage rate equal to their productivity and earn gross income $y_t = \theta_t n_t$.

At time $t = 0$, initial productivity $\theta_0 \in (0, +\infty)$ is drawn from a distribution F with density f. A standard Brownian motion $B = \{B_t, \mathcal{F}_t; 0 \leq t \leq T\}$ on $(\Omega, \mathcal{F}, \mathcal{P})$ drives the productivity shocks in future periods. A history of productivities $(\theta^t) = \{\theta_s\}_{s\in[0,t]}$ is a sequence of realizations of the productivity process that evolves according to the law of motion

$$\frac{d\theta_t}{\theta_t} = \mu_t dt + \sigma dB_t \tag{26}$$

The real constants $\mu_t - \frac{1}{2}\sigma^2$ and σ are, respectively, the drift and volatility of log productivity. When the drift and volatility are independent of time, productivity is a geometric Brownian motion (GBM), and log productivity is the continuous-time limit of a random walk. Health quality $h_t \in \mathbb{R}$ is a diffusion process $dh_t = \mu_t^h dt + \sigma_h dW_t$, where W is in general a standard Brownian motion with its filtration. The correlation structure of B_t, W_t matters for a general treatment of our problem and would lead to a two-dimensional Hamilton–Jacobi–Bellman (HJB) equation that is not tractable with analytical methods. Instead, we assume a perfect correlation between high productivity and good health, $W_t = -B_t$. The interpretation of this assumption within the lens of our discrete-time model in the main text is that the wage (here, $w_t = \theta_t$) deterministically decreases in poor health[35] and all available information on wages and health is the same.

Agents have time-separable preferences over consumption $\{c_t\}_{0\leq t\leq T}$ and labor $\{n_t\}_{0\leq t\leq T}$ processes that are progressively measurable with respect to the filtration $\mathcal{F}_t$.[36] When an agent is working, ($n_t > 0$), he incurs a health-dependent utility cost of staying in the labor market $\phi_t(h_t)$, and his current-period utility is $u(c_t, n_t; h_t) - \phi_t(h_t)$, where u is increasing in consumption, decreasing in labor, twice continuously differentiable, and concave and ϕ_t is decreasing. Given our perfect correlation assumption between wages and health, the fixed cost of participation can be written directly in terms of productivity $\phi_t(\theta_t)$. Utility along the

[35] Applying Ito's lemma, we have $h_t = h_0 + \int_0^t \mu_s^h ds - \frac{\sigma_h}{\sigma}\left(\log\left(\frac{\theta_t}{\theta_0}\right) - \int_0^t \left(\mu_s - \frac{1}{2}\sigma^2\right) ds\right)$.

[36] Consumption $c_t(\theta^t)$ and labor $n_t(\theta^t)$ depend on the whole history of productivities until time t. To simplify the notation, we drop the realizations θ^t when referring to $\mathcal{F}_t$-measurable processes $\{c_t, y_t\}$.

intensive margin is separable in consumption and labor and isoelastic in labor:

$$u(c_t, n_t) = u(c_t) - h(n_t) = u(c_t) - \kappa \frac{n_t^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$

where $\varepsilon > 0$ is the intensive Frisch elasticity of labor supply.

Retirement, $n_t = 0$, is an irreversible decision. Define a stopping time $\mathscr{T}_R \in \mathcal{T}$,[37] the age after which a retired agent provides zero labor effort and does not incur the fixed utility cost. After retirement, an agent's utility in each period is $u(c_t, 0)$. We define the retirement age as the age at which an individual chooses to exit the labor force forever[38]—which the model allows to differ from the age at which an individual chooses to start claiming Old-Age, Survivors, and Disability Insurance (OASDI) benefits.[39]

Planning Problem: Preferences over consumption and labor $\{c_t, n_t\}$ and retirement decisions $\{\mathscr{T}_R\}$ are summarized by an agent's expected lifetime utility:

$$v_0(\{c_t, n_t, \mathscr{T}_R\}) \equiv \mathrm{E}\Big\{ \int_0^{\mathscr{T}_R} e^{-\rho t}[u(c_t, n_t) - \phi_t(\theta_t)]dt + \int_{\mathscr{T}_R}^{T} e^{-\rho t} u(c_t, 0) dt \Big\} \tag{27}$$

in which ρ is the rate of time preference. A utilitarian planner chooses incentive-compatible (IC) allocations to maximize social welfare:

$$\max_{\{c_t, n_t, \mathscr{T}_R\}} v_0(\{c_t, n_t, \mathscr{T}_R\})$$

subject to the law of motion of productivity (26), the definition of indirect utility (27), and an intertemporal resource constraint. For simplicity, we work in partial equilibrium, and the planner can save aggregate resources in a small open economy and borrow at a net rate of return r. We study the planner's problem for a single cohort in isolation and abstract from intergenerational redistribution.[40] The planner's resource constraint is therefore

$$\mathrm{E}\Big\{ \int_0^{T} e^{-rt} c_t dt \Big\} + G \le \mathrm{E}\Big\{ \int_0^{\mathscr{T}_R} e^{-rt} \theta_t n_t dt \Big\} \tag{28}$$

[37] A random variable $\mathscr{T}_R$ is a stopping time if $\{\mathscr{T}_R \le t\} \in \mathcal{F}_t, \forall t \ge 0$. Intuitively, this definition means that at any time t, one must know whether retirement has occurred.

[38] The irreversible retirement assumption is motivated by empirical and theoretical considerations. Rogerson and Wallenius (2013) find empirical evidence in Current Population Survey data that retirement occurs as an abrupt transition from full-time to little or no work in the US. By age 70, the age at which individuals should start claiming Social Security benefits, 75% of men report working zero hours. In addition, this assumption is without loss of generality and can be relaxed. The main predictions of the model remain unchanged if we allow retirees to return to the labor market at a lower wage.

[39] In a decentralized economy, workers can actually claim SS benefits whenever they want, and their optimal retirement benefits are computed according to the history of their earnings. Because we work directly with allocations in this primal approach, the Social Security benefits are implicit in the model.

[40] Given that we study insurance and redistribution across one cohort, time is equivalent to age for the cohort.

The left-hand side includes exogenous government spending G[41] and the cost of providing lifetime consumption to agents. The right-hand side is the sum of the net present value (NPV) of income y_t generated by workers until they retire. Because of the law of large numbers, the aggregate resource constraint is the expectation over the histories of productivities (θ^t).

A.4.1 Optimal First-Best Retirement Choice

This section solves the planning problem with complete information. We highlight features of the optimal retirement decision that are absent in existing models with no endogenous retirement choice but that have important implications for optimal policy.

Let the rate of time preference equal the rate of return of government savings, $\rho = r$. From the IEE, productivity shocks are fully insured, and consumption is the same across different histories: $u'(c_t(\theta^t)) = \lambda$, where λ is the marginal social cost of public funds.[42] When it is optimal to work, the marginal rate of transformation of labor into consumption is the wage rate, θ_t. Therefore, labor supply satisfies $\kappa n_t^{\frac{1}{\varepsilon}} = \lambda\theta_t$. With full information, the planner maximizes social welfare by maximizing total resources available in the economy. Consumption is smoothed, and more productive agents work more hours and produce more output. It is therefore only natural that, as long as the fixed cost of staying in the labor market for highly productive workers is not too high compared with that of low-productivity workers (Technical Assumption 3), the planner makes highly productive workers retire later than low-productivity workers.

Assumption 3. *For some constant ψ, $\phi'(\theta) \leq \psi\theta^{\varepsilon}$, $\forall\theta$.*

Proposition 7. *(First-best retirement decision) Suppose that Assumption 1 holds. Then, there exists a time-dependent productivity threshold $\theta_R^{fb}(t)$ such that retirement occurs if and only if productivity falls below it: $\mathscr{T}_R^{fb} = \inf\{t; \theta_t \leq \theta_R^{fb}(t)\}$.*

Proof. The planner's problem is

$$\max_{\{\lambda, c_t, n_t, \mathscr{T}_R\}} \mathrm{E}\Big\{\int_0^T e^{-\rho s}[u(c_t) - \lambda c_t]dt + \int_0^{\mathscr{T}_R} e^{-\rho s}[\lambda\theta_t n_t - \kappa\frac{(n_t)^{1+\frac{1}{\varepsilon}}}{1+\varepsilon} - \phi_t(\theta_t)]dt\Big\}$$

subject to the law of motion of productivity (26). From the optimal allocations $u'(c) = \lambda$ and $\kappa n_t^{\frac{1}{\varepsilon}} = \lambda\theta_t$, denote $\mathrm{E}\Big\{\int_0^T e^{-\rho s}[u(c_t) - \lambda c_t]dt\Big\} = h(\lambda)$. Then, the above objective is rewritten as

$$\max_{\{\lambda, \mathscr{T}_R\}} h(\lambda) + \mathrm{E}\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}[\lambda^{1+\varepsilon}\frac{(\theta_t)^{1+\varepsilon}}{\kappa^{\varepsilon}(1+\varepsilon)} - \phi_t(\theta_t)]dt\Big\}$$

[41] G can capture many sources of exogenous government revenues and expenses as well as intergenerational transfers to or from another cohort and so on.

[42] λ is the multiplier on the planner's resource constraint (28).

Denote a maximizer by λ^*. By an envelope condition, the expected change in the payoff if retirement is delayed by an infinitesimally short time is $\lambda^{*1+\varepsilon}\frac{(\theta_t)^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)} - \phi_t(\theta_t)$. Taking $\psi < \frac{\lambda^{*1+\varepsilon}}{\kappa^\varepsilon}$ in the condition of growth bounded from above of $\phi_t(\theta)$ in Proposition 7 or assuming that G is high enough that the marginal utility of consumption $\lambda^{*1+\varepsilon}$ is high and the inequality holds, then the expected change in payoff is increasing in productivity. The dynamic single crossing condition in Kruse and Strack (2015) holds, and Theorem 4.3 of Jacka and Lynn (1992) implies that a time-varying threshold determines the shape of the stopping region (retirement rule).

Note that when ϕ_t is independent of productivity, or nonincreasing in productivity, the "bounded growth from above" condition in the proposition holds, implying Proposition 7. $\square$

This proposition means that the planner balances the need to induce the highly productive (high-wage) agents to continue working with the need to avoid the fixed utility cost for less productive (low-earning) workers. In the first-best case, it is therefore optimal to set productivity cutoffs below which retirement occurs.

To understand the determinants and lifetime evolution of these retirement cutoffs, we consider the case in which agents are risk neutral. In this tractable case, we analytically show that there is an option value of waiting for higher productivity shocks before retirement. In addition, this option value decreases over time. Therefore, the implicit labor supply elasticity over the retirement margin increases over time. The following corollary summarizes this result in terms of the retirement thresholds $\theta_R^{fb}(t)$.

Corollary 8. *(Option value of continued work vs. retirement) Suppose that Assumption 3 holds and productivity is a GBM. Denote by θ_S the static participation threshold.*

1. *For all $t < T$, $\theta_R^{fb}(t) \leq \theta_S$, and the marginal social value of continued work is negative at retirement, that is, $\theta_R^{fb}(t)n^{fb}(\theta_R^{fb}(t)) - h(n^{fb}(\theta_R^{fb}(t))) - \phi(\theta_R^{fb}(t)) \leq 0$.*
2. *The retirement thresholds $\theta_R^{fb}(t)$ are increasing in t. In addition, $\lim_{t\to T} \theta_R^{fb}(t) = \theta_S$.*

Point 1 of the corollary states that retirement occurs below a level of productivity at which it would be inefficient to work in a static environment. This creates an option value of waiting for higher productivity shocks and higher earnings before retirement, which is not present in models with permanent productivity shocks such as those in Michau (2014) and Shourideh and Troshkin (2015). Working today instead of retiring preserves the option of retiring later at a higher wage, hence the term "option value of work." Indeed, when there is no uncertainty on future earnings, the marginal value of labor is equal to the fixed utility cost of work at retirement, and the option value is zero. In practice, this option value is negative at retirement. Lazear and Moore (1988), Rust (1989) and Stock and Wise (1990) estimate structural models of retirement with uncertain earnings and find that people continue to work at any age, as long as the expected present utility value of continuing work is greater than or equal to the expected present value of immediate retirement.

Point 2 of the corollary states that the option value of continued work decreases over time as the horizon shortens. The option value of continued work vanishes at the end of the horizon, and only then is the irreversible retirement decision similar to a static participation decision, and the marginal value of labor is equal to the fixed utility cost of work.

Proof. To qualify results further, we now consider agents who are risk-neutral in consumption, so that $u(c_t) = c_t$. Consumption is not pinned down by the Euler equation. We eliminate consumption from the planner's problem by replacing the resource constraint in the planner's social welfare function:

$$w \equiv \max_{\mathscr{T}_R} \mathrm{E}\Big\{ \int_0^{\mathscr{T}_\mathscr{R}} e^{-\rho t}[\theta_t n_t^{fb} - \kappa \frac{(n_t^{fb})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} - \phi(\theta_t)]dt \Big\} - G$$

subject to the law of motion of productivity (26). Normalizing government spending to zero, $G = 0$, and replacing the first-best labor allocations using the optimality condition $\kappa(n_t^{fb})^{\frac{1}{\varepsilon}} = \theta_t$, the social welfare function $w(\theta_t, t)$ satisfies the following HJB equation:

$$0 = \max \Big\{ -w(\theta,t),\ -\rho w(\theta,t) + \frac{\theta^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)} - \phi(\theta_t) + (\mu_t\theta)\partial_\theta w(\theta,t) + \frac{\sigma^2\theta^2}{2}\partial_{\theta\theta} w(\theta,t) + \partial_t w(\theta,t) \Big\}$$

The terms to the right of $-\rho w(\theta, t)$ consist of the marginal social value of labor minus the fixed cost and derivatives of social welfare with respect to time and productivity.

Now, consider the case of productivity that evolves according to a GBM; that is, μ_t and σ are, respectively, constants μ and σ. We show that even when the fixed cost is a constant $\phi(t) = \phi$, there is an option value of waiting for higher productivity shocks before retirement. In addition, this option value decreases over time. Therefore, even when the fixed cost is constant over time, the elasticity of the retirement margin increases over time. Hence, the extensive margin elasticity of labor supply increases over time, despite the intensive Frisch elasticity and the fixed cost being time independent.

Consider the infinite horizon model, $T = +\infty$. To ensure convergence of social welfare, we assume

$$\rho > (1+\varepsilon)(\mu + \frac{1}{2}\sigma^2\varepsilon). \tag{29}$$

Social welfare is now time independent, and replacing the HJB equation in this setting is

$$\max\{0 - w(\theta), -\rho w(\theta) + \mu\theta w_\theta + \frac{\sigma^2\theta^2}{2} w_{\theta\theta} + \frac{\theta^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)} - \phi(\theta)\} \tag{30}$$

We conjecture that the solution is of the following form: There is a threshold θ_R^{fb} such that an agent retires if and only if his productivity falls below the threshold $\theta_t \leq \theta_R^{fb}$. This implies that $w(\theta) = 0$ for all $\theta \leq \theta_R^{fb}$ and for $\theta > \theta_R^{fb}$, w is a nonnegative solution to the equation

$$-\rho w(\theta) + \mu\theta w_\theta + \frac{\sigma^2\theta^2}{2} w_{\theta\theta} = -\frac{\theta^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)} + \phi(\theta)$$

Moreover, w must be C^1 on its entire domain. This implies $w(\theta_R^{fb}) = 0$, a value-matching condition, and $w_\theta(\theta_R^{fb}) = 0$, a smooth-pasting condition. Finally, observe that, for $\theta \leq \theta_R^{fb}$, the second term jon the right-hand side of (30) implies that $\dfrac{\theta^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)} \leq \phi(\theta)$; that is, at retirement and afterward, the marginal social value of continued work is negative. In particular, $\hat{\theta}_R^{fb} \leq \theta_S$.

Set $\phi_1\theta^{1+\varepsilon_\phi} + \phi_0$ with $\varepsilon_\phi < \varepsilon$. Define the quadratic polynomial $P(x) = -\rho + \mu x + \frac{\sigma^2}{2}x(x-1)$. The homogeneous equation

$$-\rho w(\theta) + \mu\theta w_\theta + \frac{\sigma^2\theta^2}{2} w_{\theta\theta} = 0$$

admits the general solution

$$w(\theta) = C_-\theta^{x_-} + C_+\theta^{x_+}$$

in which x_- and x_+ are the negative and positive roots of P. We find a particular solution for each nonhomogeneous term, denoted $A\theta^{1+\varepsilon}$, $A'\theta^{1+\varepsilon_\phi}$, and B in which $A = -\dfrac{1}{\kappa^\varepsilon(1+\varepsilon)P(1+\varepsilon)}$, $A' = \dfrac{\phi_1}{P(1+\varepsilon_\phi)}$ and $B = -\dfrac{\phi}{\rho}$. By the assumption in (29), $P(1+\varepsilon) < 0$. The sum of these particular solutions $A\theta^{1+\varepsilon} + A'\theta^{1+\varepsilon_\phi} + B$ is the value of social welfare if agents never retire.

By the superposition principle of linear homogeneous ordinary differential equations (ODEs), the solution takes the form

$$w(\theta) = A\theta^{1+\varepsilon} + A'\theta^{1+\varepsilon_\phi} + B + C_-\theta^{x_-} + C_+\theta^{x_+}$$

for $\theta > \theta_R^{fb}$ and $w(\theta) = 0$ for $\theta \leq \theta_R^{fb}$. From (29), we ensure that $x_+ > 1+\varepsilon$. Since $n^{fb} - \kappa\frac{(n^{fb})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} = \dfrac{\theta^{1+\varepsilon}}{\kappa^\varepsilon(1+\varepsilon)}$, we can conjecture that $w(\theta) =_{\theta\to+\infty} \mathcal{O}(\theta^{1+\varepsilon})$. Therefore, $C_+ = 0$.

By the value-matching and smooth-pasting conditions:

$$A(\theta_R^{fb})^{1+\varepsilon} + A'(\theta_R^{fb})^{1+\varepsilon_\phi} + B + C_-(\theta_R^{fb})^{x_-} = 0 \tag{31}$$

$$(1+\varepsilon)A\frac{(\theta_R^{fb})^{1+\varepsilon_\phi}}{\theta_R^{fb}} + (1+\varepsilon_\phi)A'\frac{(\theta_R^{fb})^{1+\varepsilon_\phi}}{\theta_R^{fb}} + x_-C_-\frac{(\theta_R^{fb})^{x_-}}{\theta_R^{fb}} = 0 \tag{32}$$

Multiplying (31) by x_- and (32) by θ_R^{fb} and subtracting the two yields

$$(1+\varepsilon-x_-)A(\theta_R^{fb})^{1+\varepsilon} + (1+\varepsilon_\phi-x_-)A'(\theta_R^{fb})^{1+\varepsilon_\phi} = x_-B$$

When $\varepsilon_\phi = \varepsilon$, the equation becomes simply

$$(1 + \varepsilon - x_-)(A + A')(\theta_R^{fb})^{1+\varepsilon} = x_- B$$

Setting $A(\phi_1) = A + A'$, the expression of θ_R^{fb} and w in Corollary 8 follows by replacing the values of $A(\phi_1)$ and B.

$$\theta_R^{fb} = \Big(\frac{\phi_0}{\rho} \frac{x}{A(\phi_1)(1 + \varepsilon + x)}\Big)^{\frac{1}{\varepsilon}}$$

and the static participation threshold is

$$\theta_S = (\frac{\phi_0}{[\kappa^\varepsilon(1 + \varepsilon)]^{-1} - \phi_1})^{\frac{1}{\varepsilon}}$$

Both θ_R^{fb} and θ_S are increasing in ϕ_0 and in ϕ_1. In addition, since $\frac{\rho-(1+\varepsilon)(\mu+\frac{\sigma^2}{2}\varepsilon)}{\rho} < 1$ and $\frac{(x)}{(1+\varepsilon+x)} < 1$, we obtain $\theta_R^{fb} < \theta_S$. Now, in a finite horizon, the problem is time dependent, and the thresholds are time dependent. When time goes to T, the value of waiting for productivity to improve decreases, and the thresholds converge to θ^*. Only the dynamic single-crossing property of the derivative operator is needed in the finite horizon for this to hold. This is again an application of Jacka and Lynn (1992). □

To develop some intuition for the proof, when we set $\phi(\theta) = \phi_0 + \phi_1\theta^{1+\varepsilon}$ and consider the infinite horizon limit $T \to \infty$, the retirement threshold is independent of time, θ_R^{fb}. We proceed similarly to Leland (1994) by decomposing the value of social welfare into two terms:

$$w(\theta) = \underbrace{A(\phi_1)\theta^{1+\varepsilon} - \frac{\phi_0}{\rho}}_{\text{social value of working forever (SVWF)}} - \underbrace{(\frac{\theta_R^{fb}}{\theta})^x}_{\text{discounting at retirement } \mathrm{E}[e^{-\rho\mathcal{T}_R^{fb}}|\theta]} \underbrace{[A(\phi_1)(\theta_R^{fb})^{1+\varepsilon} - \frac{\phi_0}{\rho}]}_{\text{SVWF starting at retirement threshold}}$$

with positive constant x and nonincreasing function $A(\phi_1)$. The value of social welfare $w(\theta)$ is the value of the lifetime utility of output if the agent were to work forever, minus the value of lifetime utility of output if he were to work forever at the optimal retirement threshold, discounted by the expected value of the discount factor at retirement. This value is zero at retirement. From a smooth-pasting argument as in Dixit (1993), the value of his marginal social welfare is also zero at retirement. This gives an explicit value of the retirement threshold

$$\theta_R^{fb} = \Big(\frac{\phi_0}{\rho} \frac{x}{A(\phi_1)(1 + \varepsilon + x)}\Big)^{\frac{1}{\varepsilon}}$$

and the static participation threshold is

$$\theta_S = (\frac{\phi_0}{[\kappa^\varepsilon(1 + \varepsilon)]^{-1} - \phi_1})^{\frac{1}{\varepsilon}}$$

Note that both θ_R^{fb} and θ_S are increasing in ϕ_0 and in ϕ_1, meaning that workers retire earlier when their fixed costs are large. In addition, the marginal social value of continued work is negative at retirement $\theta_R^{fb} < \theta_S$.

Figure 7: First-Best Retirement Decision

Notes: Example of productivity history. Horizontal axis t, vertical axis θ_t. Retirement region shaded. θ_s: static participation cutoff. The retirement region expands with age.

In summary, the solution to the first-best planning problem generates the following insights about the implications of optimal retirement: First, low-productivity agents retire earlier than highly productive agents. Second, there is an option value of waiting for higher earnings before retiring. Therefore, the implicit labor supply elasticity increases over time.

Nonseparable preferences. The following lemma characterizes the first-best retirement decision with preferences that are nonseparable in consumption and labor.

Lemma 9. *Suppose u is a Greenwood et al. (1988)–type utility function. The optimal retirement rule in the first-best is a cutoff rule $\mathscr{T}_R^{fb} = \inf\{t; \theta_t \leq \theta_R^{fb}(t)\}$.*

Proof. Denote as λ the Lagrangian on the government's resource constraint. The FOC on c_t when an agent works is $\left(c_t - \frac{n_t^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\right)^{-\nu} = \lambda$ and $c_t^{-\nu} = \lambda$ when an agent is retired. The FOC for the labor supply of workers is $n_t^{\frac{1}{\varepsilon}}\lambda = \lambda\theta_t$ so that $n_t = \theta_t^{\varepsilon}$. After rearranging and simplifying, the terms in λ cancel out, and the planner's retirement problem is rewritten as:

$$\max_{\{\lambda, \mathscr{T}_R\}} \mathrm{E}\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}[\lambda \frac{(\theta_t)^{1+\varepsilon}}{(1+\varepsilon)} - \phi_t(\theta_t)]dt\Big\}$$

The proof ends as in the proof of Proposition 7 applying Theorem 4.3 in Jacka and Lynn (1992). □

When the planner cannot observe productivity, first-best allocations with constant consumption are not achievable, as any agent would be better off retiring immediately. Nevertheless, history-dependent versions of these intuitions carry through in the second-best retirement policies.

A.5 Optimal Second-Best Retirement

A.5.1 Second-Best Problem Under Risk Neutrality

We introduce the first-order approach (FOA) in the simpler setting in which agents are risk neutral in consumption and productivity is a GBM. We relax incentive compatibility by considering a family of deviations that Bergemann and Strack (2015) call *consistent deviations*. The effect of these deviations on promised utility can be summarized by what Pavan et al. (2014) call the *impulse response function*. This FOA is standard in the dynamic contracting literature with persistent shocks.

The value of the agent's productivity if he reports his productivity truthfully is

$$\theta_t = \theta_0 \exp((\mu - \frac{\sigma^2}{2})t + \sigma B_t)$$

We define Φ by $\theta_t \equiv \Phi(t, \theta_0, B_t)$ and set the following definition, which is motivated by Bergemann and Strack (2015).

Definition 10. *(Consistent deviations). A deviation is called consistent if an agent, with real productivity $\theta_t = \Phi(t, \theta_0, B_t)$ and associated initial shock θ_0, misreports his initial shock by announcing $\tilde{\theta}_0 \in \Theta_0$ at $t = 0$ and continues to misreport $\tilde{\theta}_t = \Phi(t, \tilde{\theta}_0, B_t)$ instead of his true productivity θ_t at all future dates $t \leq T$.*

With this definition, an agent who follows a consistent deviation misreports his true type in all future periods. An agent's reported productivity $\tilde{\theta}_t = \Phi(t, \tilde{\theta}_0, B_t)$ would be equal to the productivity he would have had if his initial shock had been $\tilde{\theta}_0$ instead of θ_0. From these misreports, the planner can infer the true realized path of Brownian shocks B_t. However, since the allocations depend on the history of productivities instead of the Brownian shocks, the inference on the Brownian shocks is not of immediate use for the principal. Bergemann and Strack (2015) show that incentive compatibility with respect to consistent deviations—a one-dimensional class of deviations—is sufficient for full incentive compatibility in the risk-neutral and GBM case. This result allows us to derive the IC optimal allocations and shadow fixed costs.

Consider the ex ante utility at time 0 of an agent with initial productivity θ_0 who announces $\tilde{\theta}_0$ and follows consistent deviations; denote it $v(\theta_0, \tilde{\theta}_0)$. Then,

$$v(\theta_0, \tilde{\theta}_0) = \mathrm{E}^{\{\tilde{\theta}\}}\Big\{\int_0^T e^{-\rho t} c_t(\tilde{\theta}_0)dt - \int_0^{\mathscr{T}_R(\tilde{\theta}_0)} e^{-\rho t}[\kappa \frac{\left(\frac{y_t(\tilde{\theta}_0)}{\Phi(t,\theta_0,B_t)}\right)^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} + \phi_t\Big(\Phi(t, \theta_0, B_t)\Big)]dt\Big|\tilde{\theta}_0\Big\}$$

Restricting attention to consistent deviations alone, the incentive problem turns into a static one. Truthful reports at time zero are necessary for incentive compatibility; that is, $v(\theta_0) = \max_{\tilde{\theta}_0} v(\theta_0, \tilde{\theta}_0)$, and an envelope condition allows us to obtain the derivative of ex ante utility. The sensitivity of ex ante utility with respect to initial reports satisfies:

$$v_\theta(\theta_0) = \mathrm{E}\Big\{ \int_0^{\mathscr{T}_R} e^{-\rho t}[(1+\frac{1}{\varepsilon})(\frac{\Phi_\theta(t,\theta_0,B_t)}{\theta_t})\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} - \Phi_\theta(t,\theta_0,B_t)\phi_t'(\theta_t)]dt\Big|\theta_0\Big\}$$

$\Phi_\theta(t,\theta_0,B_t)$ is what Pavan et al. (2014) call the *impulse response function* and Bergemann and Strack (2015) call the *stochastic flow* in continuous time. Here, with GBM productivity, the stochastic flow is the ratio of current productivity to initial productivity, that is,

$$\Phi_\theta(t,\theta_0,B_t) = \exp((\mu - \frac{\sigma^2}{2})t + \sigma B_t) = \theta_t/\theta_0$$

Then, the IC constraint simplifies to

$$v_\theta(\theta_0) = \frac{1}{\theta_0}\mathrm{E}\Big\{ \int_0^{\mathscr{T}_R} e^{-\rho t}[(1+\frac{1}{\varepsilon})\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} - \theta_t\phi_t'(\theta_t)]dt\Big|\theta_0\Big\} \tag{33}$$

A.5.2 Optimal Second-Best Retirement Choice

Consider the case of agents who are risk neutral in consumption and productivity that evolves according to a GBM. Risk neutrality in consumption implies that consumption need not be distorted. Because of the strict concavity of $u(c)$ in the case of risk-averse agents with a utilitarian planner, the equivalent generalized social marginal welfare weights (Saez and Stantcheva 2016) reflect decreasing marginal utility of consumption. Low-productivity agents have lower consumption and higher marginal utility and therefore higher social welfare weights. To ensure comparability between the risk-averse utilitarian and the risk-neutral cases, we assume that the planner puts Pareto welfare weights $\alpha(\theta_0)$ on each agent with initial type θ_0. Since, with concave utility, the marginal utility of consumption is nonincreasing, we assume the function $\alpha : \Theta_0 \mapsto (0; +\infty)$ is nonincreasing. We normalize the sum of Pareto weights to one $\int_0^\infty \alpha(\theta_0)dF(\theta_0) = 1$ and call the summand of weights $\mathit{\Lambda}(\theta) = \int_0^\theta \alpha(\theta_0)dF(\theta_0)$.

The following lemma formulates the second-best retirement decision problem by substituting the optimal allocations in the planner's problem.

Lemma 11. *(Allocations and wedges) The labor wedges are time invariant and depend only on the initial heterogeneity and the welfare weights:*

$$\frac{\tau_t^L}{1-\tau_t^L} = \frac{\tau(\theta_0)}{1-\tau(\theta_0)} = (1+\frac{1}{\varepsilon})\frac{1}{\theta_0}\frac{\mathit{\Lambda}(\theta_0) - F(\theta_0)}{f(\theta_0)}$$

In addition, the planner's problem is to choose the retirement rule so as to solve:

$$\max_{\mathscr{T}_R} \int_0^\infty \mathrm{E}\Big\{ \int_0^{\mathscr{T}_R} e^{-\rho t}\Big[(1-\tau(\theta_0))^\varepsilon[y_t^{fb} - \kappa\frac{(\frac{y_t^{fb}}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}] - [\phi_t - \frac{\tau(\theta_0)}{1-\tau(\theta_0)}\frac{\varepsilon}{1+\varepsilon}\theta_t\phi_t'(\theta_t)]dt\Big\}dF(\theta_0)$$

Proof. The problem of the planner is to choose allocations $\{c, y\}$ and a retirement rule $\mathscr{T}_R$ to maximize social welfare subject to the definition of ex ante utility, the resource constraint (28), the relaxed IC constraint (33) and the law of motion of productivity (26). We rewrite the problem from the FOA with risk neutrality below for reading convenience.

$$\begin{aligned}
&\max_{\{c,y,v,\mathscr{T}_R\}} \int_0^\infty \alpha(\theta_0)v(\theta_0)dF(\theta_0)\\
&\text{s.to } \frac{d\theta_t}{\theta_t} = \mu dt + \sigma dB_t\\
&\quad v(\theta_0) = \mathrm{E}_0\Big\{\int_0^T e^{-\rho t}c_t dt - \int_0^{\mathscr{T}_R} e^{-\rho t}[\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} + \phi_t]dt\Big|\theta_0\Big\}\\
&\quad 0 \leq \mathrm{E}\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}y_t dt\Big\} - \mathrm{E}\Big\{\int_0^T e^{-\rho t}c_t dt\Big\}\\
&\quad v_\theta(\theta_0) = \frac{1}{\theta_0}\mathrm{E}_0\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}[(1+\frac{1}{\varepsilon})\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} - \theta_t\phi_t'(\theta_t)]dt\Big|\theta_0\Big\} \qquad \text{(FOA)}
\end{aligned}$$

Eliminate consumption from the problem by plugging the definition of ex ante utility at time zero into the feasibility constraint (28). The feasibility constraint then becomes:

$$\int_0^\infty \Big(v(\theta_0) + \mathrm{E}_0\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}\Big[\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} + \phi_t\Big]dt\Big|\theta_0\Big\}\Big)dF(\theta_0) \leq \int_0^\infty \mathrm{E}_0\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}y_t dt\Big|\theta_0\Big\}dF(\theta_0) \tag{34}$$

Denote by λ the multiplier on the new feasibility constraint (34). If $v(\theta_0)$ is interior, the FOC on v: $\alpha(\theta_0)f(\theta_0) - \lambda f(\theta_0) = 0$ integrated over Θ_0 yields $\lambda = 1$. The problem is then to maximize the Lagrangian

$$\begin{aligned}
\int_0^\infty \alpha(\theta_0)v(\theta_0)dF(\theta_0) - \Big[\int_0^\infty \Big(v(\theta_0) + \mathrm{E}_0\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}[\kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} + \phi_t]dt\Big|\theta_0\Big\}\Big)dF(\theta_0)\\
- \int_0^\infty \mathrm{E}_0\Big\{\int_0^\nu e^{-\rho t}y_t dt\Big|\theta_0\Big\}dF(\theta_0)\Big]
\end{aligned}$$

subject to the incentive constraints from the FOA (33) and the law of motion of productivity (26). By partial integration,

$$\begin{aligned}
\int_0^\infty v(\theta_0)dF(\theta_0) &= \int_0^\infty \frac{1-F(\theta_0)}{f(\theta_0)}v_\theta(\theta_0)dF(\theta_0) + \lim_{\theta\to 0} v(\theta)\\
\int_0^\infty \alpha(\theta_0)v(\theta_0)dF(\theta_0) &= \int_0^\infty \frac{1-\mathit{\Lambda}(\theta_0)}{f(\theta_0)}v_\theta(\theta_0)dF(\theta_0) + \lim_{\theta\to 0} v(\theta)
\end{aligned}$$

Eliminating v from the Lagrangian using partial integration and the expression of v_θ from the IC constraint, the planner's problem becomes

$$\int_0^\infty \mathrm{E}_0\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}\Big[y_t - \kappa\frac{(\frac{y_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}[1+(1+\frac{1}{\varepsilon})\frac{\mathit{\Lambda}(\theta_0)-F(\theta_0)}{f(\theta_0)}\frac{1}{\theta_0}] - [\phi_t - \frac{\mathit{\Lambda}(\theta_0)-F(\theta_0)}{f(\theta_0)}\frac{\theta_t}{\theta_0}\phi_t'(\theta_t)]\Big]dt\Big|\theta_0\Big\}\Big\}dF(\theta_0) \tag{35}$$

The FOC for y_t implies that the labor wedge is time invariant and depends only on initial heterogeneity and the welfare weights.

$$\frac{\tau^L_t}{1-\tau^L_t} = \frac{\tau(\theta_0)}{1-\tau(\theta_0)} = (1+\frac{1}{\varepsilon})\frac{1}{\theta_0}\frac{\Lambda(\theta_0)-F(\theta_0)}{f(\theta_0)}$$

Since $y^{fb}_t - \kappa\frac{(\frac{y^{fb}_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} = \frac{\theta_t^{1+\varepsilon}}{\kappa^{\varepsilon}(1+\varepsilon)}$ and $y^{sb}_t - \kappa\frac{(\frac{y^{sb}_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\left[1+(1+\frac{1}{\varepsilon})\frac{\Lambda(\theta_0)-F(\theta_0)}{f(\theta_0)}\frac{1}{\theta_0}\right] = (1-\tau(\theta_0))^{\varepsilon}\frac{\theta_t^{1+\varepsilon}}{\kappa^{\varepsilon}(1+\varepsilon)}$, we can replace y^{sb} in the planner's objective (35) to obtain

$$\max_{\nu}\int_{\underline{\theta}}^{\infty}\mathrm{E}\Big\{\int_0^{\mathscr{T}_R} e^{-\rho t}\Big[(1-\tau(\theta_0))^{\varepsilon}[y^{fb}_t - \kappa\frac{(\frac{y^{fb}_t}{\theta_t})^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}] - [\phi_t - \frac{\tau(\theta_0)}{1-\tau(\theta_0)}\frac{\varepsilon}{1+\varepsilon}\theta_t\phi'_t(\theta_t)]dt\Big\}dF(\theta_0)$$

□

The normalization of Pareto weights and the assumption of nonincreasing weights implies that $\Lambda(\theta_0) - F(\theta_0)$ is always nonnegative. The labor wedges are therefore nonnegative. In the risk-neutral case, with GBM productivity, the labor wedges depend only on the inverse intensive Frisch elasticity of labor supply, initial heterogeneity, and the welfare weights of the planner. Because there is no income effect, consumption can be allocated freely over time without distorting the labor margin.

In the context of private information, labor distortions are such that the flow utility of consumption and disutility of labor are lower than in the first best. This is captured by the factor $(1-\tau(\theta_0))^{\varepsilon} < 1$ in front of $[y^{fb}_t - \kappa\frac{(y^{fb}_t/\theta_t)^{1+1/\varepsilon}}{1+1/\varepsilon}]$ in the planner's objective. These labor distortions create incentives for the agents to retire early. However, the virtual fixed cost either increases or decreases depending on the sign of $\phi'_t(\theta_t)$.

If ϕ'_t is negative, the virtual fixed cost increases compared to the first best. Its effect goes in the same direction as the decrease in output y, and agents retire earlier than in the first best. Therefore, if ϕ'_t is negative, all agents retire earlier in the second best than in the first best. In addition, retirement is a cutoff rule. If ϕ'_t is positive, the virtual fixed cost decreases compared to the first best and depends negatively on the intensive Frisch elasticity of labor and the labor wedge. Its effect goes in the opposite direction as the decrease in y. Once we have solved the retirement decision problem in the first-best case, the derivation of the analogous rule for the second-best scenario is relatively simple. Dividing the planner's objective by $(1-\tau(\theta_0))^{\varepsilon}$, one can observe that the choice of the retirement rule in the second best is equivalent to the choice of the retirement rule in the first best when the fixed utility cost is replaced by a virtual cost $\tilde{\phi}$, defined as $\tilde{\phi}(t,\theta_t) = \dfrac{\phi(t,\theta_t)}{(1-\tau(\theta_0))^{\varepsilon}}(1-\dfrac{\tau(\theta_0)}{1-\tau(\theta_0)}\dfrac{\varepsilon}{1+\varepsilon}\varepsilon_{\phi,\theta}(\theta_t,t))$. In contrast to the first-best case, the retirement rule depends on initial productivity. Define $S(\tau(\theta_0),t) \equiv \tilde{\phi}(t,\theta_t)/\phi(t,\theta_t)$, and $S(\tau(\theta_0)) \equiv \tilde{\phi}(\theta_t)/\phi(\theta_t)$ when ϕ is time-invariant. The following proposition summarizes the results on the second-best retirement decision.

Proposition 12. *(Second-best retirement decision)*

1. *There exists a time-dependent and initial productivity–dependent deterministic retirement threshold* $\theta_R^{sb}(t,\theta_0)$ *such that* $\mathscr{T}_R^{sb} = \inf\{t; \theta_t \leq \theta_R^{sb}(t,\theta_0)\}$.

2. *Set* $\phi(\theta) = \phi_1\theta^{1+1/\varepsilon_\phi} + \phi_0$. *At the infinite horizon limit,* $T = +\infty$, *the retirement thresholds are time invariant* $\hat{\theta}_R^{sb} : \Theta_0 \mapsto \mathbb{R}^{+*}$, $\mathscr{T}_R^{sb} = \inf\{t; \theta_t \leq \theta_R^{sb}(\theta_0)\}$ *and*
$$\theta_R^{sb}(\theta_0) = \theta_R^{fb} S(\tau(\theta_0))^{\frac{1}{\varepsilon}}$$

3. *If* $\phi_1 \leq 0$, *retirement occurs earlier in the second best than in the first best for all agents* $\theta_R^{sb}(t,\theta_0) \geq \theta_R^{fb}(t)$. *If* $\phi_1 > 0$, *a criterion for whether retirement happens early or is delayed compared to the first best is*
$$S(\theta_0) = \frac{1}{(1-\tau(\theta_0))^{\varepsilon}}(1 - \frac{\tau(\theta_0)}{1-\tau(\theta_0)}\frac{1+1/\varepsilon_\phi}{1+1/\varepsilon})$$
For a given $T < +\infty$, *retirement occurs earlier in the second best than in the first best:* $\theta_R^{sb}(t,\theta_0) \geq \theta_R^{fb}(t)$ *for all* $t \leq T$ *if and only if* $S(\theta_0) \geq 1$.

Point 1 of the proposition highlights that the retirement thresholds depend on the initial productivity of the agents. Again, the option of continued work compared to retiring is negative at retirement. The second point gives an explicit formula for the optimal retirement threshold at the infinite horizon as in the discussion after Corollary 8.[43] Point 2 gives an explicit expression for the retirement thresholds at the infinite horizon.

Point 3 of the proposition states that if the fixed utility cost is increasing in productivity, there is a force that pushes for delayed retirement. High types have a high fixed cost and lower information rents than in the case when the fixed cost is independent of productivity. This creates an effect that goes in the opposite direction of the income tax. Depending on the strength of this effect, retirement may occur early or be delayed with respect to the first best. The proposition shows that the relative weight of the two forces depends on the criterion S that, in turn, depends on the intensive Frisch elasticity of labor supply, the elasticity of the fixed cost with respect to the wage and the welfare weights of the planner. This criterion allows one to determine which productivity types should be induced to retire before $S(\theta_0) \geq 1$ or after $S(\theta_0) < 1$ the first best.

A.6 Implementation with a Simple Social Security Program

When can one reduce the history dependence of the optimal policies? In this subsection, we show that in the limit case of workers who are risk neutral in consumption, the optimal

[43] There is no concern over immiseration at the infinite horizon here since, with risk neutrality in consumption, consumption is not pinned down by FOCs.

policies can be implemented by a retirement benefit system that looks similar to the US Social Security system (in that it depends on lifetime income and retirement age) and by a history-independent labor income tax. To construct this implementation, we proceed in two steps. First, we construct retirement-age-dependent post-retirement transfers that replicate the effects of the shadow fixed cost. Given optimal hours and said transfers, the agent's private retirement decision would coincide with the optimal retirement decision. Second, using these post-retirement transfers and a labor wedge, we construct a social security system and a history-independent income tax that implement the optimum.

A.6.1 Shadow Fixed Costs

Definition 13. *We define the shadow fixed cost as the change in fixed cost $\tilde{\phi}_t = (1+\tau_t^{\phi})\phi(\theta_t)$ that makes the agent privately choose the second-best retirement decision $\mathscr{T}_R^*$ given $\{c_t^*, y_t^*, \tilde{\phi}_t\}$, that is,*

$$\mathscr{T}_R^* = \arg\max_{\mathscr{T}_R} v_t^{lf}(\mathscr{T}_R; \{c_t^*, y_t^*, (1+\tau_t^{\phi})\phi(\theta_t)\})$$

The change in fixed utility cost that would make the agent privately choose the second-best retirement decision given $\{y_t^*\}$ is:

$$\tau_t^{\phi}\phi(\theta_t) = \underbrace{\tau_0^L \frac{\varepsilon}{1+\varepsilon} y_t^*}_{\substack{\text{increased shadow fixed cost} \\ \text{from labor wedge}}} \quad \underbrace{-\frac{\tau_0^L}{1-\tau_0^L}\frac{\varepsilon}{1+\varepsilon}\varepsilon_{\phi,\theta}(\theta_t)}_{\text{net shadow fixed cost}} \ \phi(\theta_t), \tag{36}$$

where $\varepsilon_{\phi,\theta}(\theta_t)$ is the elasticity of the fixed utility cost with respect to productivity. The first term is a positive fixed cost and comes from the fact that the labor wedge distorts retirement downward. The net shadow fixed cost τ_t^R corrects for this effect; $(\tau_t^R - \tau_t^{\phi})\phi(\theta_t) = -\tau_0^L \frac{\varepsilon}{1+\varepsilon} y_t^*$ and is equal to the second term of (36) in equilibrium.

A.6.2 Post-Retirement Transfers

Recall from A.5.2 that if agents are risk neutral in consumption, then consumption is undistorted and the labor wedge at age t is equal to the time-zero labor wedge $\tau_L^t(\{\theta^t\}) = \tau_L^0(\theta_0)$, where $\tau_L^0(\theta_0)$ is determined by the government's redistributive motive in the initial period. Lemma 14 below gives general conditions on the distribution of initial heterogeneity such that there exist government Pareto weights that rationalize a constant optimal labor wedge, $\tau_L^t(\{\theta^t\}) = \tau_L$. More specifically, these conditions are satisfied if initial productivity is Pareto distributed for a range of social welfare functions, from utilitarian (labor wedge equal to zero) to Rawlsian (largest labor wedge) and a Rawlsian-utilitarian mixture (intermediate levels of labor wedge).[44]

Lemma 14. *For any smooth distribution f such that $\theta_0 f(\theta_0) \to_{\theta_0\to\infty} 0$ (which is satisfied by all densities that have a mean), for all τ, there exist Pareto weights that are smooth on the support of f but put weight on the min of the support of f, such that the optimal tax is constant, $\tau(\theta_0) = \tau$.*

Proof. $\Lambda(\theta) = \int_0^\theta \alpha(\theta_0)dF(\theta_0)$. We want in the interior $\Lambda(\theta) - F(\theta) = \frac{\tau}{1-\tau}\theta f(\theta)$. The limit condition in the lemma comes from the fact that $\Lambda(\infty) - F(\infty) = 0$. Now, the derivative is a necessary condition in the interior:

$$[\alpha(\theta) - 1] = \frac{\tau}{1-\tau}[1 + \underline{\theta}].$$

Thus,

$$\alpha(\theta) = 1 + \frac{\tau}{1-\tau}[1 + \frac{\theta f'(\theta)}{f(\theta)}]$$

with a mass at the bottom support $\underline{\theta}$ such that the sum of weights adds up to 1, that is,

$$\alpha_{\underline{\theta}} = 1 - \int_{\underline{\theta}}^{\theta} \alpha(\theta_0)dF(\theta_0) = -\frac{\tau}{1-\tau}(1 + \int_{\underline{\theta}}^{\infty} \theta f'(\theta))d\theta = \frac{\tau}{1-\tau}\underline{\theta} f(\underline{\theta})$$

and, of course, the condition that the weights are positive everywhere, that is,

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{1}{\tau}$$

In particular, if the support of the distribution starts at zero, there is no mass at zero. If the distribution is Pareto $(\underline{\theta}, \alpha)$ such that $f(\theta) = \frac{a\underline{\theta}^a}{\theta^{a+1}}$, then $\frac{\theta f'(\theta)}{f(\theta)} = -1 - a$, and the weights are constant:

$$\alpha(\theta) = 1 - \frac{\tau}{1-\tau}a$$

and

$$\alpha_{\underline{\theta}} = \frac{\tau}{1-\tau}a$$

One can also invert this to show that with a Pareto distribution and the types of Pareto weights that we call Rawlsian-utilitarian, the tax is given by:

$$\tau = \frac{1}{a}\frac{\alpha_{\underline{\theta}}}{1+\alpha_{\underline{\theta}}}$$

which is a zero marginal tax $\tau = 0$ if $\alpha_{\underline{\theta}} = 0$ and utilitarian and is $\tau = \frac{1}{2a}$ if Rawlsian. □

If the government sets a flat labor income tax equal to τ and a post-retirement transfer π

[44] If the distribution is Pareto with shape parameter a on $[\underline{\theta}, \infty)$ and the government puts weight $\alpha_{\underline{\theta}}$ at $\underline{\theta}$ and equal weights on $(\underline{\theta}, \infty)$, then the labor wedge is $\tau_L = \frac{1}{a}\frac{\alpha_{\underline{\theta}}}{1+\alpha_{\underline{\theta}}}$. The labor wedge is $\tau_L = 0$ if $\alpha_{\underline{\theta}} = 0$ (utilitarian) and $\tau = \frac{1}{2a}$ if $\alpha_{\underline{\theta}} = 1$ (Rawlsian) and is increasing in $\alpha_{\underline{\theta}}$.

is a function of retirement age, then the agent chooses hours conditional on work optimally, $y_t = y_t^*$, and his private retirement decision satisfies

$$\max_{\nu} \mathrm{E}\Big\{ \int_0^{\nu} e^{-\rho t}\Big[(1-\tau)y_t^* - h(\frac{y_t^*}{\theta_t}) - \phi(\theta_t)\Big]dt + e^{-\rho\nu}\pi(\nu)\Big\} \tag{37}$$

The planner's choice of the optimal retirement decision is different from the agent's private choice in two respects. First, because of labor income taxes, the government values output relative to the fixed cost more than the agent does. Second, the government wants to distort the fixed cost faced by the agent because of the redistributive value of the net shadow fixed cost. The transfer π implements the optimal retirement decision if $\mathscr{T}_R^*$ is a solution to the agent's private retirement decision problem (37).

Under Assumption 3, we construct π by evaluating the agent's expected utility at the productivity process reflected at the second-best retirement cutoff $\theta_R^*(t)$. Intuitively, the reflected productivity is a process that equals productivity as long as it stays above the cutoff. Once productivity falls below the cutoff and the planner would want the agent to retire, the reflected process follows its own dynamics and is defined so that it stays above the cutoff at all times.

We started with the definition of a reflected process above the second-best retirement boundary.

Definition 15. *(Reflected process) Let $\theta_R^* : [0,T] \to \mathbb{R}$ be a càdlàg function. A process $\{\tilde{\theta}_t\}_t$ on $(\Omega, \mathcal{F}, \mathcal{P})$ is called a reflected version of $\{\theta_t\}_t$ with barrier θ_R^* if it satisfies the following conditions:*

1. *$\tilde{\theta}_t$ is constrained to stay above θ_R^*: For every $t \in [0,T]$, we have $\tilde{\theta}_t \geq \theta_R^*(t)$ a.s.*
2. *Until $\{\theta_t\}$ hits the barrier, both processes coincide: For every $0 \leq t < \mathscr{T}_R^*$, we have $\theta_t = \tilde{\theta}_t$ a.s.*
3. *$\tilde{\theta}$ is always higher than θ: For every $t \in [0,T]$, we have $\tilde{\theta}_t \geq \theta_t$ a.s.*
4. *When $\{\theta_t\}$ hits the barrier, $\{\tilde{\theta}_t\}_t$ is at θ_R^*: $\tilde{\theta}_{\mathscr{T}_R^*} = \theta_R^*(\mathscr{T}_R^*)$ a.s.*

The next proposition ensures the existence of a reflected version of the productivity process at the second-best retirement boundary for our GBM diffusion process.

Proposition 16. *Let $\theta_R^* : [0,T] \to \mathbb{R}$ be a càdlàg function. If $\{\theta_t\}_t$ has no jumps, then there exists a version of $\{\theta_t\}_t$ reflected at θ_R^*.*

The conditions of this proposition are just the necessary ones to obtain our result. In general, there exist reflected versions at càdlàg boundaries for an extensive class of processes, including those with downward jumps.

Proposition 17. *Suppose Assumption 3 holds. Define as* $\{\tilde{\theta}_t\}_t$ *the reflected process above* $\theta_R^*(t)$*; then,*

$$\pi(t) = \mathrm{E}_t\Big\{ \int_t^T e^{-\rho s}\Big[(1-\tau)\tilde{y}_s^* - h(\frac{\tilde{y}_s^*}{\tilde{\theta}_s}) - \phi(\tilde{\theta}_s)\Big] ds\Big\}$$

implements the second-best retirement decision, where $\tilde{y}_t^* = (1-\tau)^{\varepsilon}\frac{\tilde{\theta}_t^{1+\varepsilon}}{\kappa^{\varepsilon}(1+\varepsilon)}$.

Proof. If the government sets a flat labor income tax equal to τ from Lemma 14 and a post-retirement transfer π is a function of the retirement age, then the agent chooses his hours conditional on work optimally $y_t = y_t^*$, and his private retirement decision satisfies:

$$\max_{\nu} \mathrm{E}\Big\{ \int_0^{\nu} e^{-\rho t}\Big[(1-\tau)y_t^* - h(\frac{y_t^*}{\theta_t}) - \phi(\theta_t)\Big] dt + e^{-\rho\nu}\pi(\nu)\Big\}$$

The occurrence of π is here to implement the wedges τ_ϕ with post-retirement transfers. From Proposition 12, we know that the second-best retirement decision is a cutoff rule, θ_R^*. Then, Theorem 5 of Kruse and Strack (2015) applies and implies that

$$\pi(t) = \mathrm{E}_t\Big\{ \int_t^T e^{-\rho s}\Big[(1-\tau)\tilde{y}_s^* - h(\frac{\tilde{y}_s^*}{\tilde{\theta}_s}) - \phi(\tilde{\theta}_s)\Big] ds\Big\}$$

implements the second-best retirement decision, where $\tilde{y}_t^* = (1-\tau)^{\varepsilon}\frac{\tilde{\theta}_t^{1+\varepsilon}}{\kappa^{\varepsilon}(1+\varepsilon)}$. □

The transfer achieves the implementation of the second-best retirement decision by doing the following. First, when the net shadow fixed cost and labor wedge result in distortions for delayed (resp., early) retirement, the planner provides a marginal change in the transfer that increases (resp., decreases) the option value of continued work of the agent until (resp., after) productivity falls to $\theta_R^*(t)$. Proposition 17 states that the marginal change in the optimal transfer is the agent's private value of work at a level of labor income that is constrained to stay above the level of labor income that triggers retirement in the second best. In particular, if π implements $\mathscr{T}_R^*$, then a lump-sum transfer added to π implements $\mathscr{T}_R^*$. This will allow us to complement any smooth history-independent labor income tax with a history-dependent retirement benefit and a lump-sum transfer to implement the optimum.

Proposition 18. *Let* $T(y_t)$ *be a differentiable history-independent labor income tax. There exist retirement benefits* b *and a lump-sum transfer* t_0 *such that* (T, b, t_0) *implements the optimum. In addition,*

$$b(\nu, \{y_t\}) = \quad \delta(\nu)\underbrace{\mathrm{E}\Big\{\int_0^{\mathscr{T}_R^*} e^{-\rho t}\tau y_t^*)\Big\}}_{\textit{level around second best}} + \underbrace{\pi(\nu) - \delta(\nu)\mathrm{E}[e^{-\rho\mathscr{T}_R^*}\pi(\mathscr{T}_R^*)]}_{\textit{deferral rate}} + \underbrace{f(\{y_s\})}_{\textit{function of past earnings}}$$

for any retirement age, where $e^{-\rho \mathscr{T}_R^*} f(\{y_s\}) = \int_0^{\mathscr{T}_R^*} e^{-\rho t}[T(y_t) - \tau y_t]dt$ *and* $\delta(t) \equiv \frac{1-e^{-\rho(T-t)}}{1-e^{-\rho T}}$ *is the lifetime equivalent of a stream of unit of consumption from time* t *until death.*

Proof. We start by setting the savings tax such that the agents are not willing to save privately. Because of risk neutrality in consumption, the savings tax can be set to zero. Given a history-independent income tax $T(y_t)$ and a history-dependent lifetime retirement benefit $b(\{y_t\}, \mathscr{T}_R)$, the agent's consumption before retirement is $c_t = y_t - T(y_t)$, and after retirement, the NPV of consumption is $c_{\mathscr{T}_R} g(\mathscr{T}_R) = b(\{y_t\}, \mathscr{T}_R)$. The agent's private optimization problem is:

$$\max_{y_t, \nu} \int_{\underline{\theta}}^{\infty} \mathrm{E}\Big\{ \int_0^{\nu} e^{-\rho t}[y_t - T(y_t) - h(\frac{y_t}{\theta_t})] - \phi_t(\theta)\Big] dt + e^{-\rho \nu} b(\{y_s\}, \nu)\Big\} dF(\theta_0)$$

We search for affordable benefits of the form

$$b(\{y_t\}, \nu) = -T(0)g(\nu) + \pi(\nu) + f(\{y_t\}, \mathscr{T}_R^*)$$

that guarantees that $y_t = y_t^*$, after which we will know that $\nu = \mathscr{T}_R^*$ by construction of the transfer π. The necessary condition for optimal hours is, given $\mathscr{T}_R^*$,

$$(1 - T'(y_t) + e^{-\rho(\mathscr{T}_R^* - t)} \frac{\partial b(\{y_s\}, \mathscr{T}_R^*)}{y_t} = h(\frac{y_t^*}{\theta_t}) = 1 - \tau$$

Setting

$$e^{-\rho(\mathscr{T}_R^* - t)} \frac{\partial b(\{y_s\}, \mathscr{T}_R^*)}{y_t} = T'(y) - \tau$$

and integrating pathwise over y,

$$e^{-\rho \mathscr{T}_R^*} f(\{y_s\}, \mathscr{T}_R^*) = \int_0^{\mathscr{T}_R^*} e^{-\rho t}[T(y_t) - \tau y_t]dt$$

guarantees that $y_t = y_t^*$. The transfer $\pi(\mathscr{T}_R)$ guarantees that $\nu = \mathscr{T}_R^*$ as long as it is affordable by the aggregate resource constraint. Now observe that if π implements the second-best retirement decision, any lump-sum transfer added to π also implements the second-best retirement decision. This will allow us to adjust the lump-sum transfer $-T(0)$ until the aggregate resource constraint is satisfied in equilibrium.

$$T(0)g(0) + \mathrm{E}\Big\{ \int_0^{\mathscr{T}_R^*} e^{-\rho t} T(y_t^*) \Big\} = \mathrm{E}[e^{-\rho \mathscr{T}_R^*} \pi(\mathscr{T}_R^*)] + E[e^{-\rho \mathscr{T}_R^*} f(\{y_s^*\}, \mathscr{T}_R^*)]$$

Replacing the expression of $-T(0)$ in b yields that, for any $\mathscr{T}_R$,

$$b(\{y_t\}, \nu) = \underbrace{\frac{g(\nu)}{g(0)} \Big(\mathrm{E}\Big\{ \int_0^{\mathscr{T}_R^*} e^{-\rho t} T(y_t^*) \Big\} - \mathrm{E}[e^{-\rho \mathscr{T}_R^*} f(\{y_s^*\}, \mathscr{T}_R^*)] \Big)}_{\text{level around second best corrected for tax distortion}}$$

$$+\underbrace{\pi(\nu)-\frac{g(\nu)\mathrm{E}[e^{-\rho\mathscr{T}_R^*}\pi(\mathscr{T}_R^*)]}{g(0)}}_{\text{actuarial adjustment}}$$

$$+\quad\underbrace{f(\{y_s\},\mathscr{T}_R^*)}_{\text{function of past earnings}}$$

which simplifies from the expression of f to:

$$b(\{y_t\},\nu)=\qquad\underbrace{\frac{g(\nu)}{g(0)}\Big(\mathrm{E}\Big\{\int_0^{\mathscr{T}_R^*}e^{-\rho t}\tau y_t^*)\Big\}}_{\text{level around second best corrected for tax distortion}}$$

$$+\underbrace{\pi(\nu)-\frac{g(\nu)\mathrm{E}[e^{-\rho\mathscr{T}_R^*}\pi(\mathscr{T}_R^*)]}{g(0)}}_{\text{actuarial adjustment}}$$

$$+\quad\underbrace{f(\{y_s\},\mathscr{T}_R^*)}_{\text{function of past earnings}}$$

We rename without loss of generality the income tax T to a tax on labor income without the lump-sum transfer. The benefits $b(\{y_t\},\nu)$, combined with labor income tax and the lump-sum transfer, implement the planner's optimum. $\square$

B Data Appendix

B.1 Data

Our analysis primarily relies on data from the Panel Study of Income Dynamics (PSID), supplemented with data from the Medical Expenditure Panel Survey (MEPS) for analyses related to medical spending. This section describes the construction of the sample from these sources and the methods used to define health and disability status across the two datasets.

Panel Study of Income Dynamics: The PSID is a longitudinal survey of a representative sample of the US population, collecting detailed information on labor market outcomes, income, health, and wealth. Initiated in 1968 with interviews of approximately 4,800 families, the PSID has continuously followed the original respondents and their descendants, with interviews conducted annually until 1997 and biennially thereafter. The original sample comprises two components: 1,872 low-income families drawn from the Survey of Economic Opportunity (SEO) and 2,930 families selected by the Survey Research Center (SRC) at the University of Michigan to represent the general US population.

In constructing our sample, we restrict to individuals from the SRC component who participated in interviews between 1968 and 2015. To make the sample more representative of the US population, we exclude the SEO sample, as most of these families were dropped from the survey after 1997. Our analysis focuses on individuals born between 1920 and 1940, a cohort that experienced a similar set of Social Security rules and for whom the PSID provides sufficient data observations across the life cycle.

Table 12: PSID Sample Selection

Selection	Individuals	Observations
Initial sample	76,880	3,075,200
Non-SEO sample	36,430	1,457,200
Household heads and spouse if present	21,225	355,314
Age 20 to 90	21,031	350,617
No missing labor or health information	19,433	222,319
Born between 1920 and 1940	2,248	44,404

Notes: Description of our Panel Study of Income Dynamics subsample selection. We focus on the 1920 to 1940 birth cohort over the age of 20.

Table 12 reports the sample selection and resulting sample size. The final sample includes 2,248 individuals and 44,404 person–year observations. We use data from the PSID to estimate health status transitions, wage profiles and dynamics, survival probabilities, the initial distribution over state variables, and the moment conditions that the model is estimated to match.

Medical Expenditure Panel Survey: The MEPS is a nationally representative survey of families, individuals, medical providers, and employers across the United States. The survey covers individuals of all ages, although age is top-coded at 85. Medical spending is reported at the individual level and cross-validated with information from medical providers and insurance companies, which enhances data accuracy.

We use data from the 1999–2012 waves of the MEPS, restricting the sample to household heads aged 20 and older. We exclude observations with missing information on age, medical spending, health insurance coverage during working years, or health status. The final sample includes 120,731 individuals and 211,709 person–year observations, as summarized in Table 13. We use the MEPS data to estimate out-of-pocket medical expenditures for a representative population.

Table 13: MEPS Sample Selection

Selection	Individuals	Observations
Initial sample	259,263	491,795
Household heads	130,978	231,866
Age above 20	125,587	222,495
With insurance information at 25–65	121,385	213,005
Nonmissing health status	120,731	211,709

Notes: Description of our Medical Expenditure Panel Survey subsample section. We keep household heads aged 20 and older and exclude observations with missing information.

B.2 Health and Disability Measurement

In the PSID, health and disability status are constructed based on responses to a series of self-reported work limitation questions. In each wave, respondents are asked the following: (1) *Do you have any physical or nervous condition that limits the type of work or the amount of work that you can do?* (2) *Does this condition keep you from doing some types of work?* (3) *For work you can do, how much does it limit the amount of work you can do—a lot, somewhat, or just a little?*

The first question allows *Yes* or *No* responses. The second question, available beginning in 1986, offers *Yes*, *No*, or *Can do nothing* as response options. The third question has been coded differently over time: Since 1976, the response options have been *Not at all*, *Somewhat*, *Just a little*, or *A lot*; before 1976, they were *We can't work*, *It limits me a lot*, *Some, not much*, or *Limitation but not on work*. Since the second question became available in 1986, for observations from before that year, health status is defined based only on the responses to the first and third questions.

Following Low and Pistaferri (2015), we classify individuals into three health categories: good health ($h_t = 0$) if they respond "No" to the first question or "Not at all" to the third question; poor health ($h_t = 1$) if they answer "Yes" to the first question and "Somewhat" or "Just a little" to the third question; and with a disability ($h_t = 2$) if they answer "Yes" to the first question, "Can do nothing" to the second question, and "A lot" to the third question, in accordance with the Social Security Administration (SSA) criterion for disability insurance qualification. Alternatively, Hosseini et al. (2022) construct a frailty index to measure health status using PSID survey questions available since 2003, while the PSID has also included self-reported health rankings since 1984. However, these measures are not suitable for our analysis, as we require consistent information on health dynamics over the entire life cycle. The restricted availability of these measures precludes their use in our analysis.

In the MEPS, health status is based on self-reported health rankings, following Pashchenko and Porapakkarm (2019). Each year t includes three survey waves, and in each wave, respondents rate their health on a five-point scale: (1) *excellent*, (2) *very good*, (3) *good*, (4) *fair*, or (5) *poor*. We define individuals as being in good health ($h_t = 0$) if their health rank falls in categories 1–3 across all three waves; as with a disability ($h_t = 2$) if they report "poor" in any wave; and as being in poor health ($h_t = 1$) otherwise. Table 14 below compares the health measures across the two data sources and demonstrates their consistency.

Table 14: Health Measures Over Ages – Comparison of Two Datasets

	Ages 50–60		Ages 60–70		Ages 70 and older	
Health Status	MEPS	PSID	MEPS	PSID	MEPS	PSID
With a disability (%)	10	9.3	11.7	12.9	15	19.7
Poor health (%)	21	10.6	23.6	17.0	28.7	19.3
Good health (%)	69	80.1	64.7	70.1	56.3	60.0

Data Source: Panel Study of Income Dynamics and Medical Expenditure Panel Survey.

B.3 First Step of Estimation

This section describes the procedures used to estimate the first-step inputs, including health transitions, survival probabilities, hourly wages, and out-of-pocket medical expenses directly from raw data.

Health Transitions: We estimate the evolution of health status, $Pr(h_t = j|h_{t-1} = i)$ for $i, j \in \{0, 1, 2\}$, by running probit regressions of the indicator variable $\mathrm{I}(h_t = j|h_{t-1} = i)$ on a set of age dummies, using the sample with ($h_{t-1} = i$). The predicted values from these regressions provide estimates of the transition probabilities. We then linearly interpolate the estimated transition probabilities across ages so that the transition matrices change smoothly over the life cycle.

Survival Probabilities: Using data from the PSID, we estimate age- and health-dependent survival probabilities by running a logistic regression of an indicator for survival on a polynomial in age, previous health status, and their interactions whenever they are statistically significant. The estimated coefficients are reported in Table 15.

Table 15: Estimated Coefficients for the Survival Probabilities

	Coefficients	S.E.
Age^2	-.0006***	(0.0000)
Health$_{t-1} = 1$	-1.3198***	(0.2288)
Health$_{t-1} = 2$	-3.1419***	(0.4189)
Health$_{t-1}\times$Age^2	0.0109***	(0.0032)
Constant	7.8801***	(0.1373)
Observations	132,070	

Notes: Table shows estimated coefficients of logistic regression using the Panel Study of Income Dynamics. Dependent Variable: indicator of survival. Robust standard errors in parentheses, clustered at the individual level.

Out-of-Pocket Medical Expenses: Out-of-pocket medical expenses are computed as total medical expenditures net of the amount covered by health insurance, based on data from the MEPS. Following the procedure of Pashchenko and Porapakkarm (2019), we estimate the profiles of total medical expenditures by running weighted regressions on age and year dummies separately for each health group, applying both the cross-sectional and longitudinal weights. We similarly estimate the profiles of insurance coverage by regressing them on age and year dummies, separately by health status. For working-age individuals, the medical coverage is computed as the amount paid by insurance programs, while for individuals aged 65 and older, coverage reflects expenses paid by Medicare. Out-of-pocket expenditures are then computed as the product of estimated coinsurance rates and total medical expenses at each age and health status. Following De Nardi et al. (2024), we adjust the estimated medical expenditures by multiplying them by 1.60 for individuals younger than age 65 and by 1.90 for those aged 65 and older, to make medical spending consistent with the aggregate spending patterns reported in the National Health Expenditure Account. Finally, we smooth the resulting profiles by regressing them on a quadratic function of age.

Hourly Wages: Hourly wages are calculated as annual earnings divided by annual hours worked, based on data from the PSID. Respondents report their annual earnings and hours worked for the previous year. To account for extreme wage values, observations with hourly wages below \$6.50 or above \$250 (in 2016 dollars) are excluded. Since wages are observed only for labor market participants, we adjust for selection bias in the observed wages by estimating age- and health-specific hourly wage profiles using the two-stage Heckman selection model (Heckman 1976), as adapted in recent studies, such as Guner et al. (2012). Some previous studies address this selection bias by matching profiles of labor market participants using the model to estimate the parameters of wage equations (French 2005; French and Jones 2011). We choose the Heckman selection model as it provides a good fit to the targeted moments while also being computationally efficient.

Specifically, in the first stage, we estimate the selection equation (labor force participation) using a probit regression on all PSID observations, generating an inverse Mill's ratio. In the second stage, we estimate hourly wages by regressing them on age polynomials, the interactions of health and age polynomials, and the inverse Mill's ratio obtained in the first stage. The coefficients from this two-step procedure are reported in Table 16.

To estimate the stochastic components of hourly wages (ρ, σ_ρ^2), we use the wage residuals from the previous steps. We restrict the age range to between 25 and 65 and drop the top 0.5% of the residuals to mitigate the effect of extreme outliers on the variance. The estimation is performed with maximum likelihood and standard Kalman filter recursions. The estimated values for (ρ, σ_ρ^2) are (0.981, 0.016), which are consistent with the estimates from the standard literature.

Table 16: Estimated Coefficients for the Logarithm of Hourly Wages

	Coefficients	S.E.
	Wage Equation	
Age	0.0582***	(0.00500)
Age^2	-0.00067***	(0.00006)
Health×Age	-0.0113***	(0.00127)
Health×Age^2	0.00013***	(0.00002)
Constant	1.447***	(0.104)
	Selection Equation	
Age	0.0977***	(0.00453)
Age^2	-0.00162***	(0.00004)
Health	-1.371***	(0.0682)
Health×Age	0.0122***	(0.00119)
Family Size	0.0225***	(0.00369)
Birth Year	-0.0123***	(0.00080)
Constant	23.79***	(1.581)
Inverse Mill's Ratio	0.182***	(0.0482)
Observations	64,941	

Notes: Table shows estimated coefficients of two-stage Heckman selection model based on the Panel Study of Income Dynamics. Dependent variable of wage equation: logarithm of wages. Dependent variable of selection equation: indicator of labor force participation. Robust standard errors in parentheses, clustered at the individual level. $*p < 0.10, **p < 0.05, ***p < 0.01$.

Initial Distribution: To construct the initial distribution of the relevant state variables at age 30, we draw randomly from the empirical joint distribution of wages, health, assets, and labor force participation for individuals aged 28–32 in the target cohort, using data from the PSID. We adjust the mean of log wages to match the estimated wage profiles within each health group. For the initial Social Security wealth, we assume that all individuals have worked for five years, working 2,000 hours per year at the hourly wage rate at age 30, following the procedure used by French (2005), to impute initial values of average indexed monthly earnings. Table 17 summarizes the initial distribution of assets, wages, previous labor force participation, and health status. It shows that individuals in good health have higher wages and assets, on average, than those in poor health and with disabilities.

Table 17: Summary Statistics for the Initial Conditions

		Health Status		
	Overall	Good health	Poor health	With a disability
Assets (in 2016 dollars)				
Mean	60,241	61,388	42,022	30,289
Standard Deviation	146,975	149,670	91,894	19,310
Wages (in 2016 dollars)				
Mean	18.95	19.19	15.13	12.50
Standard Deviation	9.32	9.32	8.29	6.98
Participation				
Mean	0.96	0.97	0.89	0.45
Standard Deviation	0.20	0.18	0.31	0.50
Percentage		94.74	4.17	1.09
Observations	4,849	4,594	202	53

Data Source: Panel Study of Income Dynamics.

B.4 Second Step of Estimation

Target Profiles: The target profiles include labor force participation by health status, hours worked conditional on participation by health status, mean and median assets, and mean rates of labor market reentry over the life cycle. We construct these profiles with the data from the PSID by estimating fixed-effects regressions, following the procedure outlined in French (2005).

Labor force participation is measured as a dummy variable, taking the value of one if an individual's reported annual hours worked exceed 800 hours, thereby capturing both full-time and part-time employment. Hours worked are based on self-reports from the PSID, with respondents in survey year t reporting their hours worked during year $t-1$.

Asset measurement includes real estate, farm or business equity, vehicles, stocks, mutual funds, individual retirement accounts (IRAs), Keogh plans, liquid assets, bonds, and investment trusts, net of mortgages and other debts. Pension wealth and Social Security wealth are excluded. We define assets as the sum of all these categories plus home equity net of any outstanding debts. Following Borella et al. (2018), we assume equal division of asset ownership between partners for aggregation purposes.

Asset data are available only for selected survey years starting in 1984. To impute asset values for years without direct observations, we estimate a fixed-effects regression of assets on age polynomials, log wages, family size, education, unemployment rates, health status indicators, interactions between log wages and the age polynomials, and interactions between health status and the age polynomials, education, family size, unemployment rates, and wages. We use both imputed and observed values to construct asset profiles.

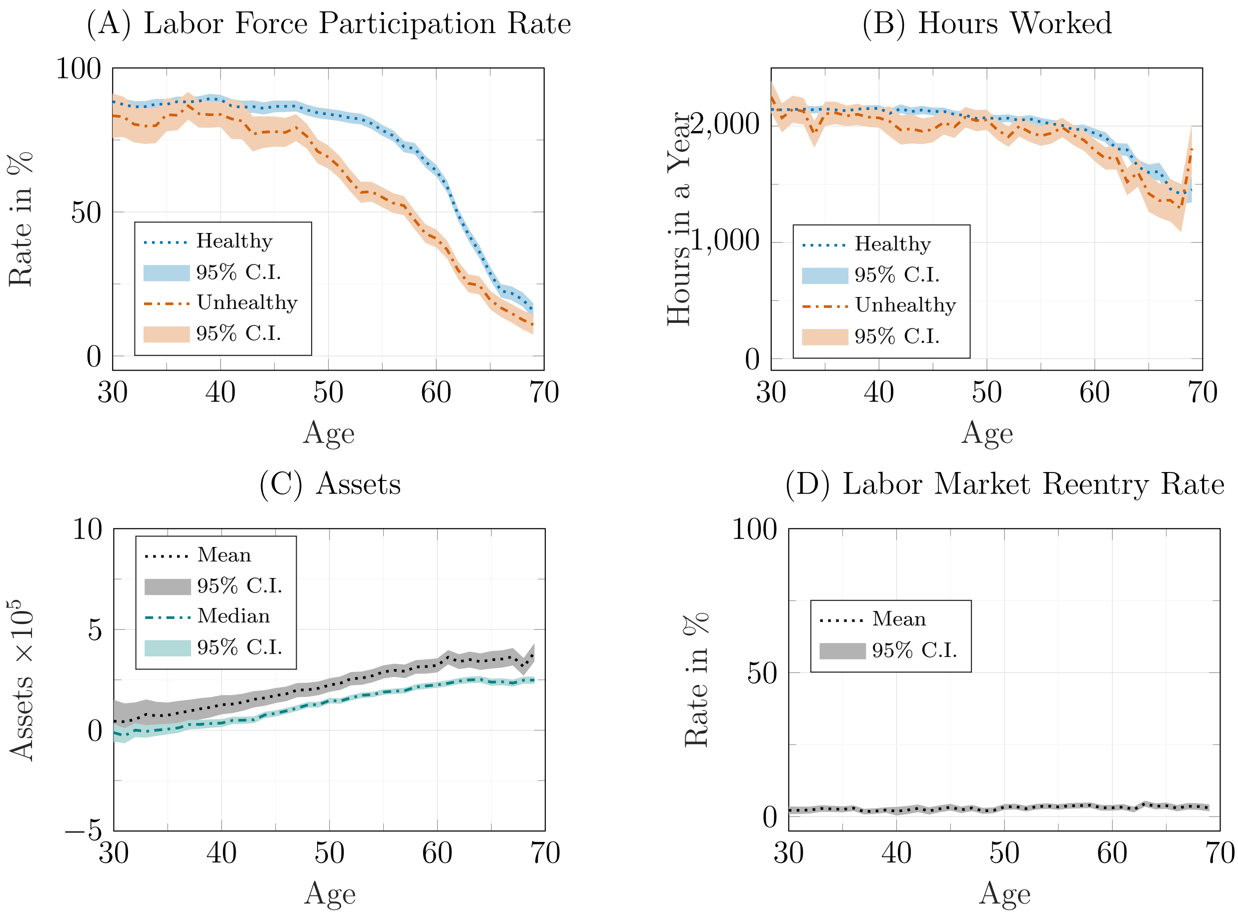
We estimate the target profiles by running the following fixed-effects regression model:

$$
\begin{aligned}
Z_{it} = f_i + \sum_{k=1}^{T} B_{gk} I\{age_{it} = k\} \times I\{h_{it} = 0\} + \sum_{k=1}^{T} B_{bk} I\{age_{it} = k\} \times I\{h_{it} \neq 0\} \\
+ \sum_{f=1}^{F} B_f \text{familysize}_{it} + B_u U_t + u_{it}
\end{aligned}
$$

where Z_{it} represents the outcome of interest, either hours worked or labor force participation, for individual i at age t, f_i denotes an individual-specific effect, familysize_{it} is family size dummies, U_t is the unemployment rate, and $\{\{B_{gk}\}_{k=1}^{T}, \{B_{bk}\}_{k=1}^{T}, \{B_f\}_{f=1}^{F}, B_u\}$ are parameters. To ensure the target profiles are not contaminated by family size, year (business cycle) effects, or individual-specific effects, we set family size to 3 and the unemployment rate to 6.5%. To control for birth year (cohort) and the correlation between the person-specific effect and health status, we use the mean individual-specific effect for individuals who are aged 50, in average health at age 50, and born in 1935. For the asset and labor market reentry profiles, we do not condition on health status.

The resulting estimated profiles are displayed in Figure 8. Healthy individuals work significantly more than unhealthy individuals over the life cycle, particularly after age 50. Labor market reentry rates are low in the data, remaining below 5% throughout the observed period.

Figure 8: Data Profiles



Notes: Panel A shows the labor force participation by health status. Panel b shows the hours worked conditional on participation by health status. The blue and red lines represent healthy and unhealthy individuals. Panel C shows the mean and median assets. Panel D shows the mean labor market reentry rate. Data profiles are estimated with the Panel Study of Income Dynamics. The shaded region represents the 95% confidence intervals.

C Quantitative Appendix

C.1 Additional Specifications of the Quantitative Model

Social Security Policy Rules: According to the Social Security Administration (SSA), individuals born between 1920 and 1940 faced an early retirement age (ERA) of 62 and, on average, a normal retirement age (NRA) of 65. Their delayed retirement credit (DRC) ranged from 3% to 6%, depending on their year of birth. In our analysis, we use the cohort-average DRC of 4.5%.

The primary insurance amount (PIA) formula included bend points at $3,720 and $22,392, with maximum average indexed monthly earnings (AIME) capped at $43,800. The retirement earnings test imposed different tax rates and thresholds based on age: Beneficiaries under 65 lost $1 of benefits for every $2 of earnings above $6,000, while those aged 65 and older lost $1 for every $3 above $8,186. These parameters, summarized in Table 18, are incorporated into the model.

Table 18: Social Security Parameters Summary

Parameter	Description	Value		Source
τ_t^{ss}	Payroll tax rate	3%–7.65%		SSA
$\bar{y}_t^{ss}$	Threshold, payroll tax	$14,459–$64,974		SSA
y_{db}	Disability benefits income threshold	$3,600		SSA
$aime_0$	1st bend point in PIA formula	$3,720		SSA
$aime_1$	2nd bend point in PIA formula	$22,392		SSA
$aime_{max}$	Maximum Social Security wealth	$43,800		SSA
ERA	Early retirement age	62		SSA
NRA	Normal retirement age	65		SSA
DRC	Delayed retirement credit	4.5%		SSA
RET	Retirement earnings test	Under NRA	NRA-69	
τ_{ret}	Tax rate	50%	33%	SSA
y_{ret}	Threshold	$6,000	$8,186	SSA

Notes: Monetary values are expressed in 1987 dollars.
Source: Social Security Administration.

Disability Insurance: Individuals with a disability are eligible to receive disability insurance (DI) benefits prior to the NRA if their labor income falls below a specified threshold. In our model, the DI application is not treated as an endogenous decision because it is not the primary focus of the paper and obtaining relevant data is challenging. In studies that explicitly model DI applications, such as Low and Pistaferri (2015), the probability of a successful application for DI depends on age and health status. To keep our model tractable, we assume that DI benefits are granted exclusively to individuals who have a disability and meet the income requirement. Specifically, we assume that individuals with a disability are eligible for

DI benefits if they are older than 50 and their labor income is below \$3,600, corresponding to the monthly substantial gainful activity threshold of \$300, as defined by the SSA for nonblind individuals with a disability.

$$db_t = \pi^{db} PIA_t I_{\{h_t=2\}} I_{\{w_t(\theta_t,h_t)n_t \leq y_{db}\}} \quad \text{if age} < NRA$$

The DI benefit amount db_t is set equal to the individual's PIA, without the early claiming penalty that would normally reduce retirement benefits received before the NRA. To reflect the fact that not all individuals with a disability receive benefits successfully, the benefit amount is discounted by π^{db}, the probability of receiving DI benefits for old-age groups with severe work limitations, as estimated by Low and Pistaferri (2015). This approach captures the average DI benefit amount received by individuals with a disability.

Pension Benefits: Pension benefits are modeled to capture the basic features of defined benefit (DB) plans. Similarly to Social Security benefits, pensions depend on an individual's work history and are illiquid before a specified age, which we assume to be 62. In contrast to Social Security, DB pension plans often create strong incentives for workers to exit the labor force at particular ages through the structure of accrual rates (Gustman et al. 2000; Ippolito 1997). To reflect these features, we impute pb_t as a function of Social Security benefits, following the approach of French (2005), and model it as a regressive function of $pb(PIA_t)$, adjusted by a term ϵ_t to reflect age-specific pension accrual.

$$pb_t = pb(PIA_t) + \epsilon_t$$

The parameters used in $pb(PIA_t)$ are based on estimates by Gustman and Steinmeier (1999), and the age-specific pension accrual rates are taken from Gustman et al. (2000). The adjustment ϵ_t is made to account for the difference in accrual between Social Security and pension wealth over the life cycle, which tends to be positive at older ages. French (2005) provides a detailed description of this procedure.

Taxes: Individuals in our model face the effective time-varying tax rates on their total income, estimated from PSID data as in Borella et al. (2023). For tractability, we assume that individuals anticipate changes in the tax rates on total income. Payroll tax rates and thresholds are based on historical data from the SSA for Old-Age, Survivors, and Disability Insurance (OASDI) and Medicare's Hospital Insurance (HI) programs. Since 1960, the combined OASDI and HI tax rates for employees and employers have ranged from 3% to 7.65%.

C.1.1 Dynamic Program of Positive Economy

The life cycle in the model is divided into three distinct stages. The first stage, from ages 30 to 61, represents the working years during which individuals are not eligible for pension or Social Security retirement benefits. In this period, they make decisions only about consumption and labor supply. The second stage, from ages 62 to 69, is a transitional period when individuals continue to choose consumption and labor supply but also decide whether to claim Social Security retirement benefits. The third stage, from ages 70 to 85, reflects a post-benefits-claiming period during which individuals have already claimed retirement benefits and continue to make decisions about consumption and labor supply.

The individual's problem at age t is summarized by a value function in state (X_t):

$$\begin{aligned} V_t(X_t) = \max_{c_t,n_t,b_t} \{ &\frac{1}{1-\nu}\left(c_t^{\gamma} \left[L - n_t - \phi_n(h_t)I\{n_t > 0\} - \phi_{RE}RE_t\right]_t^{1-\gamma} \right)^{1-\nu} \\ &+ \beta s_{t+1} \int V_{t+1}(X_{t+1})dF(X_{t+1}|X_t, t, c_t, n_t, b_t)\} \\ X_t = (&a_t, w_t, h_t, b_{t-1}, P_{t-1}, aime_t) \end{aligned} \tag{38}$$

subject to equations (1)–(2), (5), (15)–(16), and (17). The function $F(\cdot|\cdot)$ defines the conditional distribution of states, subject to the laws of motion of shocks and the budget constraint. Variables such as m_t and pb_t are not explicitly included in X_t since they depend on other state variables.

The model is solved by backward induction with value function iteration. An individual's decisions in period t depend on the state X_t, preference parameters $\mathcal{P} = (\gamma, \nu, \phi_n(h = 0)$, $\phi_n(h = 1, 2), \phi_{RE}, L, \beta)$, and structural parameters that determine the data-generating process for the state variables $\chi = (r, \sigma_\eta^2, \rho, \{W(h_t, t)\}_{t=1}^{T}, \{\pi_{h_{t+1},h_t,t}\}_{t=1}^{T-1}, \{s_{t+1}\}_{t=1}^{T-1}, \{m_t\}_{t=1}^{T}$, $\{pb_t\}_{t=1}^{T}, \{ss_t\}_{t=1}^{T}, \{db_t\}_{t=1}^{T}, Y_t(\cdot))$. At time T, the consumption decision c_T is made by solving the above problem $V_T(X_T)$ with the terminal value. Our model accounts for savings used for self-insurance against retirement and risks such as medical expenses and income fluctuations, and accidental bequests, while abstracting from any legacy bequests after age 85. Given the decision rules and value function at time T, we then solve the decision rules at time $T - 1$, $T - 2$, ..., 0. The solution to the problem (38) consists of sequences of consumption rules $\{c_t(X_t, \mathcal{P}, \chi)\}_{1\leq t\leq T}$, hours worked rules $\{n_t(X_t, \mathcal{P}, \chi)\}_{1\leq t\leq T}$, and Social Security benefit application rules $\{b_t(X_t, \mathcal{P}, \chi)\}_{1\leq t\leq T}$. The labor force participation rules at t, $p_t(X_t, \mathcal{P}, \chi)$, are determined by the hours worked decisions. Assets in the next period, $a_{t+1}(X_t, \mathcal{P}, \chi)$, are obtained by applying these decision rules to the asset accumulation equation.

To numerically solve these decision rules, we discretize the state variables into a finite number of points within a grid that includes 30 asset states, 10 wage states, 10 AIME states, 2 Social Security application states, three health states, and two previous labor force participation states. We directly compute the value function at these points and integrate the value function with respect to the innovation of wages using five-node Gauss–Hermite quadrature (Judd 1998). We use linear interpolation within the grid and linear extrapolation outside of the grid to evaluate the value function at points that cannot be directly computed.

After solving the model, we use the decision rules to simulate individual life cycle patterns using forward induction. Starting from an initial realized state variables X_0, decisions are made with the decision rules at $t = 0$. Then, given time-zero decisions, the state X_1 is generated by means of transition probabilities and shock realizations. This process is repeated through age T, generating complete simulated histories.

C.2 Second Step of Estimation Strategy

In the second step, we estimate the remaining preference parameters, $\mathcal{P} = (\gamma, \nu, \phi_n(h = 0), \phi_n(h = 1, 2), \phi_{RE}, L, \beta)$, using the method of simulated moments (MSM). Because of the limited number of observations on individuals with a disability in the PSID, individuals in poor health and those with disabilities are assigned the same fixed cost of working to simplify the model. The objective is to find a vector of preference parameters $\hat{\mathcal{P}}$ that minimizes the weighted distance between the estimated target profiles from the PSID and the corresponding simulated profiles generated by the model. The MSM estimator $\hat{\mathcal{P}}$ minimizes the GMM criterion function, which evaluates the distance between the model and data moments:

$$\hat{\mathcal{P}} = \arg\min_{\mathcal{P}} \frac{I}{1+\tau} \hat{\varphi}(\mathcal{P};\chi)' \hat{\mathbf{W}}_I \hat{\varphi}(\mathcal{P};\chi)$$

The parameter τ denotes the ratio of the number of empirical observations to the number of simulated observations, and $\hat{\varphi}(\mathcal{P};\chi)$ is a $7\mathbb{T}$-element vector of moment conditions, such that

$$\hat{\varphi}(\mathcal{P};\chi) = \begin{bmatrix} E[p_{iht}|h,t] - \int p_t(X,\mathcal{P},\chi)dF_{h,t}(X|h,t) \\ E[n_{iht}|h,t] - \int n_t(X,\mathcal{P},\chi)dF_{h,t}(X|h,t) \\ E[a_{iht}|h,t] - \int a_t(X,\mathcal{P},\chi)dF_{h,t-1}(X|h,t) \\ E[re_{it}|t] - \int re_t(X,\mathcal{P},\chi)dF_t(X|t) \end{bmatrix}_{t\in\{30,\ldots,69\},h\in\{healthy,unhealthy\}}$$

where $E[p_{iht}|\cdot]$, $E[n_{iht}|\cdot]$, $E[a_{iht}|\cdot]$, and $E[re_{iht}|\cdot]$ are the moments that our model is estimated to match:

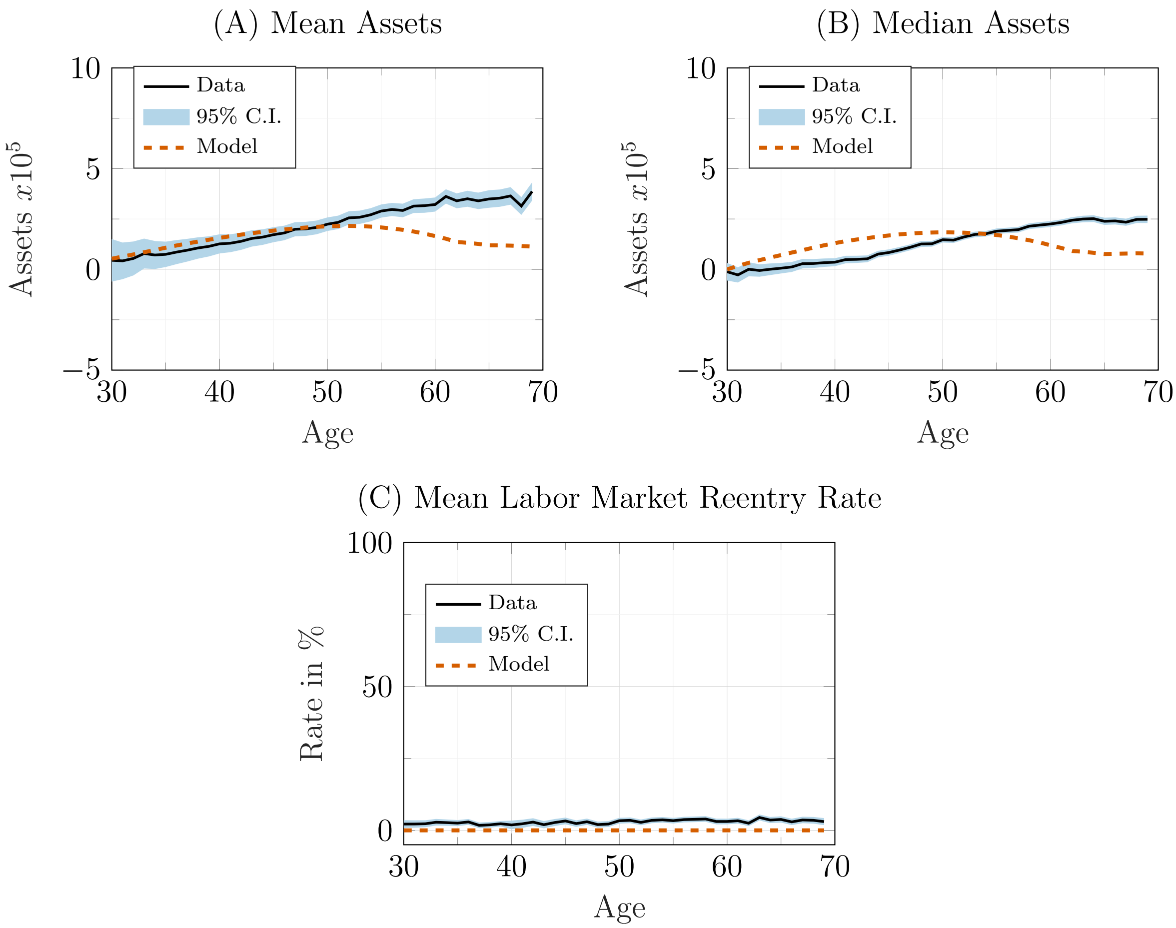
1. Labor force participation by health status (healthy, unhealthy) over ages 30–69 ($2\mathbb{T}$ moments),
2. Hours worked conditional on participation by health status (healthy and unhealthy) over ages 30–69 ($2 \times \mathbb{T}$ moments),
3. Mean and median assets over ages 30–69 ($2 \times \mathbb{T}$ moments),
4. Mean labor market reentry rates over ages 30–69 ($\mathbb{T}$ moment).

This gives a total of $7 \times \mathbb{T} = 280$ moment conditions. The model generates $\int p_t(X,\mathcal{P},\chi)dF(\cdot)$, $\int n_t(X,\mathcal{P},\chi)dF(\cdot)$, $\int a_t(X,\mathcal{P},\chi)dF(\cdot)$, and $\int re_t(X,\mathcal{P},\chi)dF(\cdot)$, where$F_t(\cdot)$ is the cumulative distribution function (CDF) of the state variables at time t and $F_{ht}(\cdot)$ indicates the CDF of the state variables at period t given health status h. The weighting matrix, $\hat{\mathbf{W}}_I$, is set to the inverse of a diagonal $7\mathbb{T} \times 7\mathbb{T}$ matrix, which consists of the elements of the variance–covariance matrix from the data along its main diagonal. To identify the preference parameters in $\mathcal{P}$, we follow the approach in French (2005) and impose two assumptions: that individuals' work decisions do not affect their health status or wage and that age changes work and savings incentives but not preferences.

The estimation proceeds in the following steps: First, we estimate the empirical life cycle profiles for labor force participation, hours worked, and assets by health status from the data. Second, given the initial distribution of relevant state variables and a set of parameters χ that determine the data-generating process for the state variables, we generate matrices of health and wage shocks. Third, given the values of $\mathcal{P}$, we solve the model numerically using value function iteration and generate decision rules that allow us to simulate life cycle profiles for participation, hours worked, wealth accumulation, and labor market reentry. Fourth, we construct moment conditions and calculate the distance between the simulated profiles and data profiles using the GMM criterion. Fifth, we pick a new vector of preference parameters, $\mathcal{P}_{new}$, and iterate the above two steps until the parameter vector that minimizes the criterion function is found.

C.2.1 Additional Estimation Profiles

Figure 9: Data and Estimated Profiles

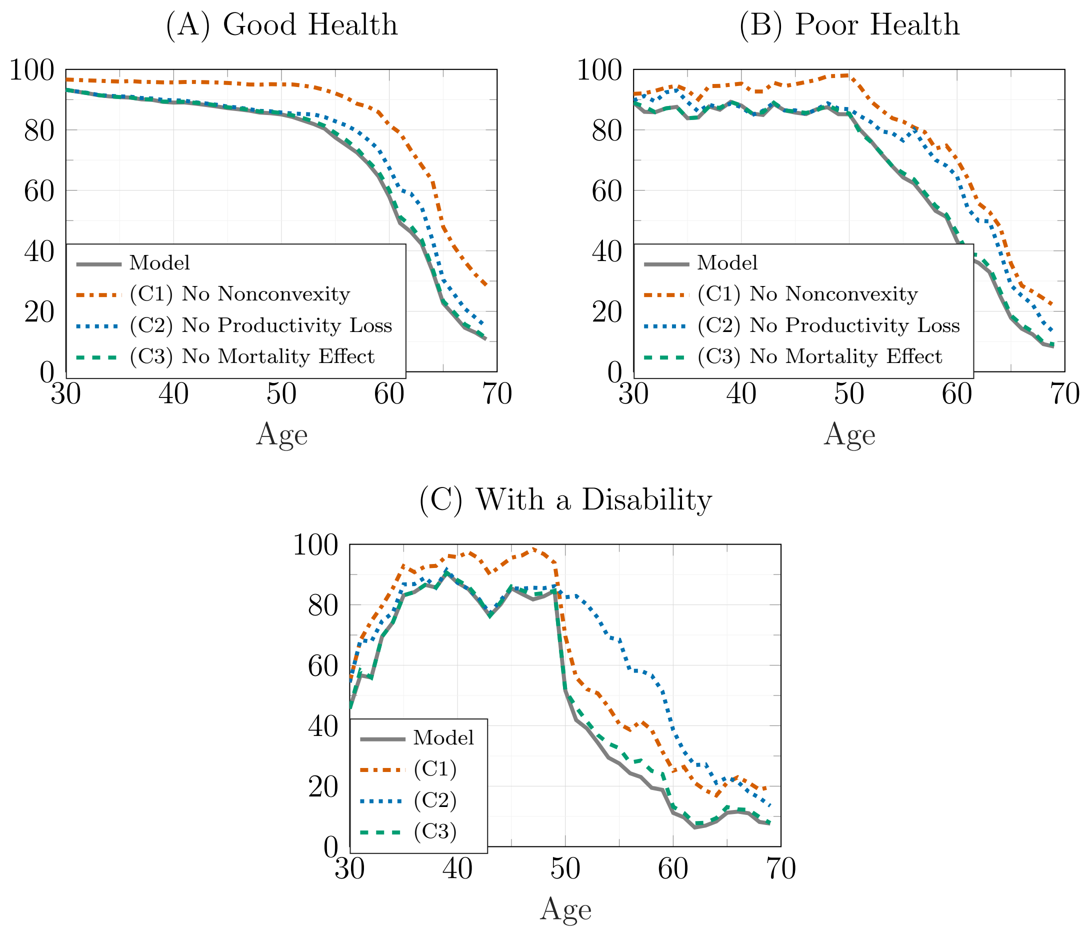


Notes: Panels A and B show the model fit for mean and median assets, and panel C shows mean reentry. Data profiles are estimated with the Panel Study of Income Dynamics. The shaded region represents the 95% confidence intervals. The orange lines represent model profiles. Monetary values are expressed in 2016 dollars. The model matches well the mean and median assets until age 55. Reentry is already low in the data (below 5%), and our model generates no reentry in equilibrium.

C.3 Decomposition

C.3.1 Additional Figures: Decomposition of Allocations

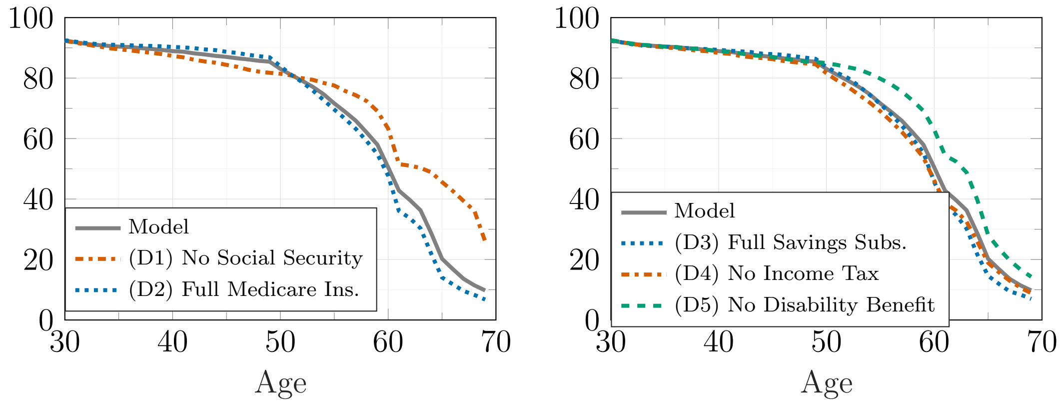
Figure 10: Effects of Preferences and Technological Factors on Labor Force Participation by Health Status



Notes: Labor force participation rates as a function of age by health status for the estimated quantitative model and counterfactual simulations that shut down different channels. Panels A/B/C represent people in good health/people in poor health/people with a disability. The *No Nonconvexity* benchmark (C1) sets fixed costs of participation to zero. The *No Productivity Loss* benchmark (C2) sets wages equal to the wages of healthy individuals. The *No Mortality Effect* benchmark (C3) sets survival probabilities equal to those of healthy individuals. The *No Nonconvexity* benchmark features the largest increase in labor force participation for people in good and poor health, followed by the *No Productivity Loss* benchmark. For people with a disability, however, the *No Productivity Loss* benchmark features the largest increase in labor force participation. That could be explained by the availability of disability benefits, so a large increase in wages could provide work incentives to forgo what they could obtain from disability insurance.

C.3.2 Additional Figures: Effects of Policies on Participation

Figure 11: Effects of Policies on Labor Force Participation



Notes: Labor force participation rates as a function of age for the estimated quantitative model and counterfactual simulations that eliminate different policy instruments. The *No Social Security* benchmark (D1) sets Social Security benefits to zero. The *Full Medicare Insurance* benchmark (D2) sets medical expenses after age 65 to zero. The *Full Savings Subsidy* benchmark (D3) sets the net return on assets to the risk-free rate. The *No Income Tax* benchmark (D4) sets income tax to zero. The *No Disability Benefit* benchmark (D5) sets disability benefits to zero. The *No Social Security* benchmark features the largest increase in labor force participation, followed by the *No Disability Benefit* benchmark.

C.4 Reforms

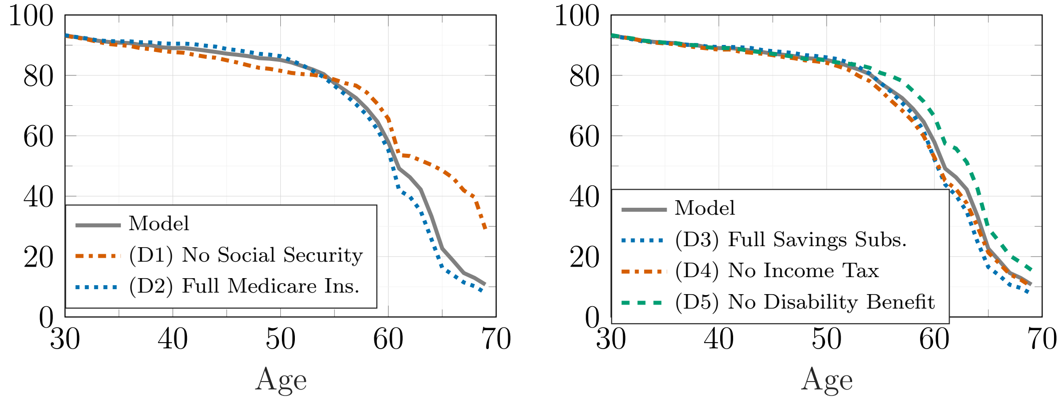
This section describes the quantitative implementation and evaluation of the policy reforms analyzed in the paper. We study a range of policy scenarios, including reforms to Social Security, income taxation, savings subsidies, and Medicare, and assess their impacts on welfare, government budgets, labor force participation, and the marginal value of public funds (MVPF). The subsections below provide details on how the reforms are constructed and how their effects are evaluated.

C.4.1 Construction of Reforms

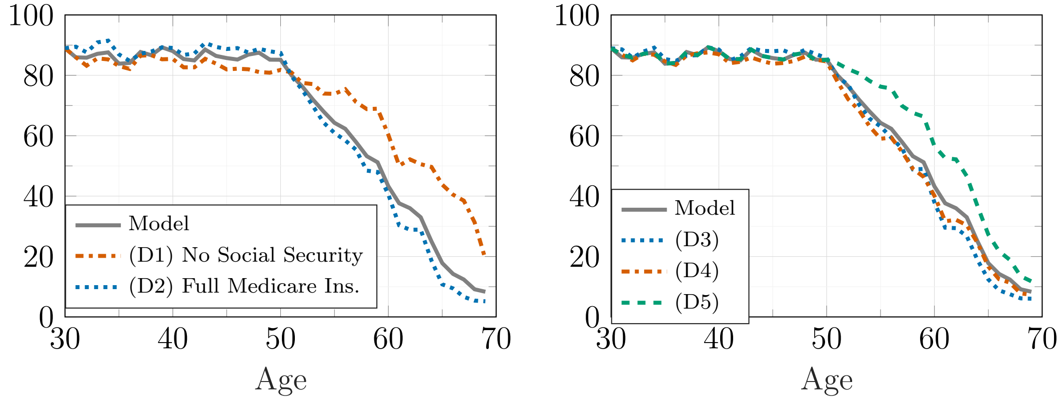
Social Security Progressivity and Levels Reforms: Social Security progressivity reforms P1.1–P1.9 modify the first bracket of the primary insurance amount (PIA) formula by gradually increasing its replacement rate. Specifically, the initial rate of 0.90 is raised by one percentage point (pp) in each step, from 0.91 under P1.1 up to 0.99 under P1.9. Reforms P2.0–P2.9 and P3.0–P3.9 target the second and third brackets, respectively: P2.0–P2.9 increase the second bracket's replacement rate by 0.32 pp per step, while P3.0–P3.9 raise the third bracket's replacement rate by 0.15 pp per step. Each set of reforms implements changes in 1% increments relative to the initial bracket levels. In addition, Social Security level reforms L0–L9 uniformly increase the replacement rates across all PIA brackets by 1% in each step.

Figure 12: Effects of Policies on Labor Force Participation by Health Status

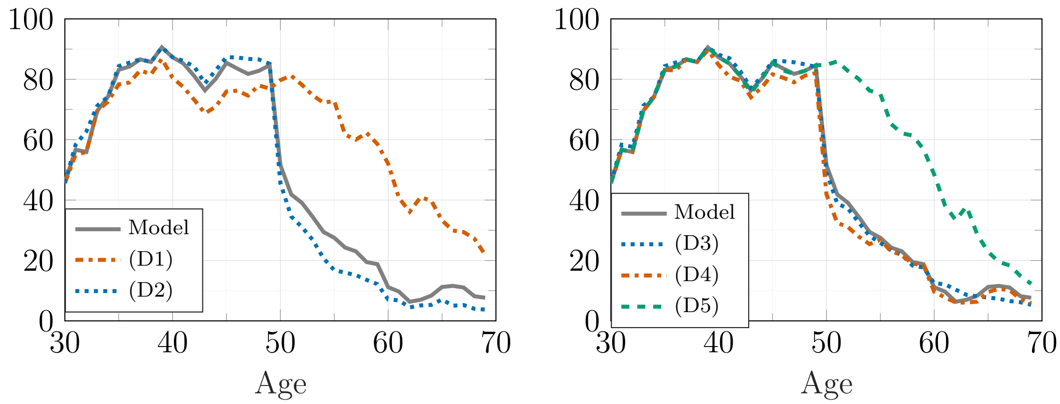
(a) Good health



(b) Poor health



(c) With a disability



Notes: Labor force participation rates as a function of age by health status for the estimated quantitative model and counterfactual simulations that eliminate different policy instruments. Panels A/B/C represent people in good health/people in poor health/people with a disability. The *No Social Security* benchmark (D1) sets Social Security benefits to zero. The *Full Medicare Insurance* benchmark (D2) sets medical expenses after age 65 to zero. The *Full Savings Subsidy* benchmark (D3) sets the net return on assets to the risk-free rate. The *No Income Tax* benchmark (D4) sets income tax to zero. The *No Disability Benefit* benchmark (D5) sets disability benefits to zero. The *No Social Security* benchmark features the largest increase in labor force participation. Eliminating disability benefits substantially increases the labor force participation of people with a disability after age 50.

Reforms of Social Security Claiming-Age Dependence: Reforms that modify the NRA, denoted as AN1–3, increase the NRA to ages 66, 67, and 68, respectively. Reforms that adjust the DRC, referred to as AD1–5, increase the DRC to 8%, 8.5%, 9%, 9.5%, and 10%, respectively. Reforms that change the linear adjustment of retirement benefits (AL1–9) modify both the DRC and early retirement penalties, increasing both rates from 7% to 15%, with a 1 pp increase for each reform step. These reforms effectively alter the percentage of the PIA that workers receive from Social Security based on their claiming age.

Medicare Reforms: To implement Medicare reforms targeting individuals aged 65 and older, we first compute the average Medicare coverage rates for this age group by health and working status using MEPS data. We then apply these updated coverage rates to the estimated model and use the newly simulated model as the new baseline. Building on it, we introduce two Medicare reforms: Medicare Part A reform increases coverage for all individuals over age 65 by 20%, reflecting expanded access to inpatient services; Medicare Part B reform increases coverage among nonparticipants by 20%, improving access to outpatient care for those not previously enrolled.

Tax Reforms: Income tax reforms (I1–I5) reduce income taxes after age 62, leading to increases in after-tax income $y - T(y) = \lambda_t y^{1-\tau_t}$. In particular, $\tilde{\lambda}_t = \lambda_t + x(t-61)$, x increases from 1% to 5%, in 1 pp increments.

Savings Subsidy Reforms: Savings subsidy reforms K1–3 increase the net returns to savings by 0.20 pp, 0.4 pp, and 0.8 pp.

C.4.2 Construction of Outcomes

Welfare Measurements: We calculate consumption-equivalent (CE) and leisure-equivalent (LE) welfare measures using the prereform value V_{before}, which is the average of $V_{t=30}$, as the reference point. Following a policy reform, the post-reform value function is linked to the consumption-equivalent welfare change x by $V_{after} = e^{\gamma x(1-\nu)} V_{before}$. Based on this equation, the CE welfare change from the status quo, which represents the individuals' willingness-to-pay for the policy, is calculated as $\frac{\ln(V_{before}) - \ln(V_{after})}{\gamma(1-\nu)}$, and, equivalently, the LE welfare change is computed as $\frac{\ln(V_{after}) - \ln(V_{before})}{(1-\gamma)(1-\nu)}$.

Fiscal Effects: For the Social Security reforms, the direct fiscal effect is calculated as the percentage change in the net present value (NPV) of total retirement benefits paid out by the program, discounted to age 62 by a factor of $(1+r)^{-1}$. The indirect fiscal effects are computed as the percentage change in the NPV of all other taxes and transfers, including payments to disability benefits and government transfers net of income and payroll tax collections. The net fiscal effect combines the direct and indirect effects.

For the Medicare reforms, the direct fiscal effect is calculated as the changes in the NPV of medical payments made by the program to the targeted beneficiaries after age 65 (Part A for everyone; Part B for labor market nonparticipants), expressed as a share of payroll tax collections at the baseline. The indirect fiscal effects are computed as the percentage change in the NPV of all taxes and transfers, including payments to all Social Security benefits and government transfers net of income and payroll tax collections. The net fiscal effect is measured as the sum of the NPV of the change in government spending and the total medical payments to the targeted beneficiaries, divided by the NPV of government spending at the baseline.

For the income tax reforms, the direct fiscal effect is calculated as the percentage change in the NPV of negative total income tax collection. The indirect fiscal effect is computed as the percentage change in the NPV of all other taxes and transfers, including payments to all Social Security benefits and government transfers net of payroll tax collections. The net fiscal effect combines the direct and indirect effects.

For the savings subsidy reforms, the direct fiscal effect is calculated as the NPV of the subsidy as a share of government spending at the baseline. The indirect fiscal effect is computed as the percentage change in the NPV of government spending, excluding subsidies. The net fiscal effect is measure as the percentage change in the NPV of the government spending, including subsidies.

Marginal Value of Public Funds Construction: For reforms related to Social Security, Medicare, and income tax, the MVPF is defined as the ratio of the average dollar value of CE welfare gains to the dollar change in the NPV of the government spending per capita, taking into account both direct and indirect effects. The dollar value of CE welfare gain is calculated as the product of the CE welfare change and the NPV of per capita consumption in the baseline (starting at age 62 and discounted to age 62). For reforms that generate an infinite MVPF, we report a lower bound as the average dollar value of CE welfare gain, divided by the dollar change in the NPV of the direct cost.

For savings subsidy reforms, the MVPF is defined as the average dollar value of CE welfare gains (starting at age 30 and discounted to age 62), divided by the average dollar change in the NPV of government spending, including the subsidy. The lower bound uses only the subsidy in the denominator.

C.5 Log Preferences Estimation

As a robustness check, we estimate the model using an alternative utility specification based on log preferences.

C.5.1 Setup

We adopt a utility function that is separable in consumption and labor to calibrate preferences for our quantitative illustration of the effects of labor supply nonconvexities on the optimal policies. Specifically, it incorporates logarithmic utility over consumption, a disutility of labor with a Frisch elasticity, and fixed utility costs associated with labor market participation and reentry:

$$u(c,n) = \ln(c) - \kappa \frac{n^{1+1/\varepsilon}}{1+1/\varepsilon} - \phi(h)\mathbf{1}\{P=1\} - \phi_{RE}\mathbf{1}\{RE=1\}$$

where c is consumption, n is working hours, $\phi(h)$ captures the utility loss associated with labor market participation P, and ϕ_{RE} reflects a fixed utility cost of reentering the labor market. In addition, κ is the scaling factor for the disutility of work, and ϵ is the Frisch elasticity of labor supply. We set $\epsilon = 0.5$ following Chetty (2012) and endogenously estimate the other parameters.

C.5.2 Targets and Matched Profiles

We reestimate the model under this specification using the same method of simulated moments (MSM) approach as in the baseline analysis. All other components of the model, including the health and income processes, policy rules, and estimation targets, remain unchanged.

Table 19 summarizes the key estimated parameters and moment matching performance under the log utility specification. Figure 13 compares the simulated and empirical life cycle profiles for labor supply and savings under the log preferences.

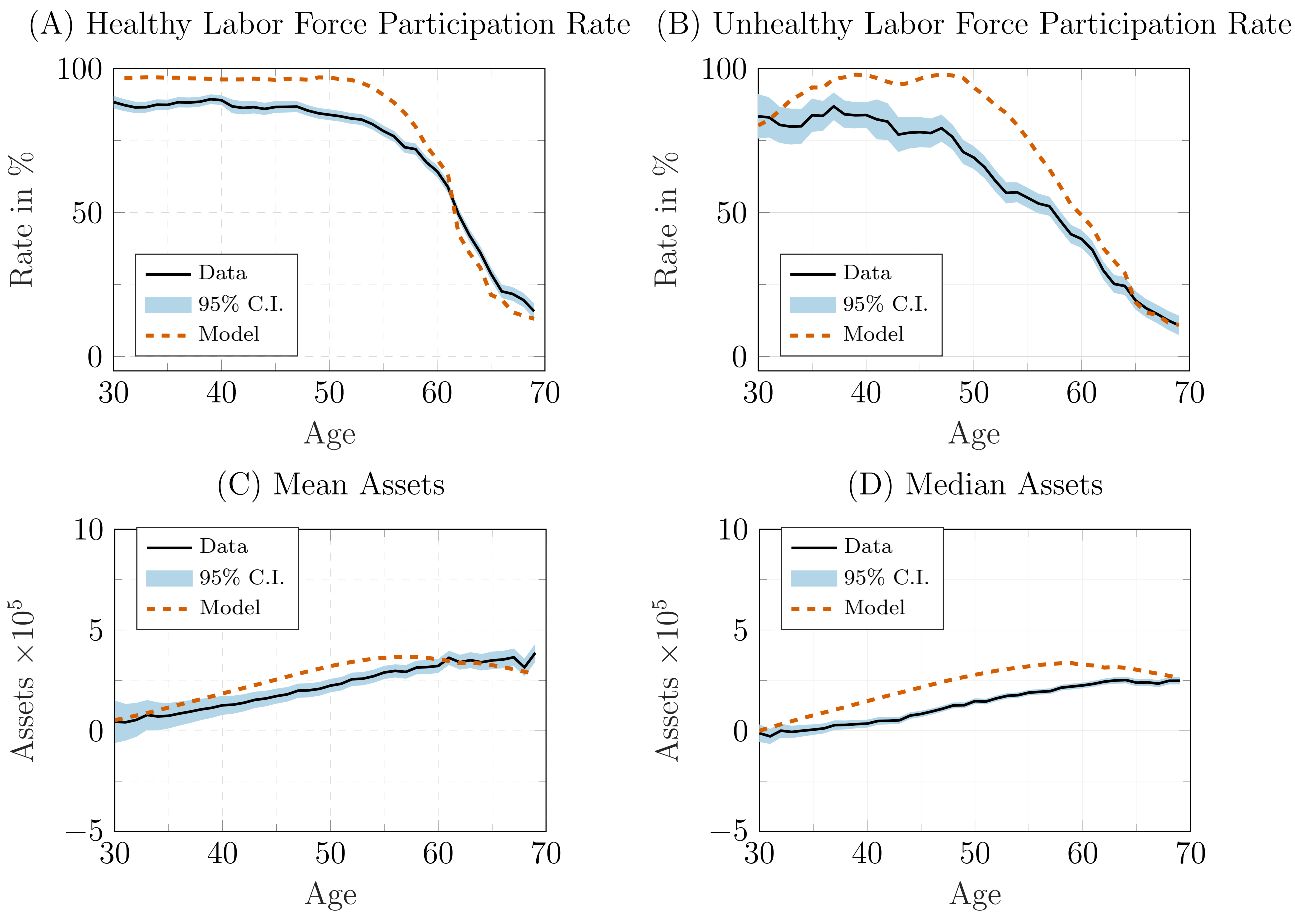
The results under the log preference specification are broadly consistent with those from the baseline model. In particular, the utility cost of working is higher for unhealthy individuals than for healthy ones. While the log preference specification fits assets at older ages better, it matches labor force participation rates less accurately than the baseline model with nonseparable preferences.

Table 19: Estimated Parameters

Parameter	Definition	Estimates	S.E.
β	Time discount factor	0.97	0.0015
ϕ_{RE}	Labor market reentry cost	36.15	1.2686
$\phi(h=0)$	Fixed cost of work, healthy	0.53	0.0082
$\phi(h=1,2)$	Fixed cost of work, unhealthy	0.59	0.0152
κ	Scaling factor for work disutility	1.37×10^{-10}	0.14×10^{-10}

Notes: Estimated structural parameters from the method of simulated moments procedures.

Figure 13: Data and Estimated Profiles



Notes: Panels A and B show the model fit for labor force participation by health status. Panels C and D show the model fit for mean and median assets. Data profiles are estimated with the Panel Study of Income Dynamics. The shaded region represents the 95% confidence intervals. The orange lines represent model profiles. Monetary values are expressed in 2016 dollars. Compared to the baseline model with nonseparable preferences, the log preference model fit assets at older ages better but matches labor force participation rates less accurately.