Homework 6
ECON 8 5 : Advanced Macroeconomics
Svetlana Pashchenko

odel

Consider the following model:

Each period a continuum of agents is born. Agents live for T periods after which they die. The population growth rate is n per year (which is the model period length).

Newly born agents (i.e. $t = 1$) are endowed with no initial capital (i.e. $k_t = 0$) but can subsequently save in capital which they can rent to firms at rate r. A worker of age t supplies labor $l_t \in [0, 1]$ and pays proportional social security taxes τ on her labor income y_t until she retires at age $R < T$.

Labor income y_t is determined as follows: $y_t = w \exp(z_t)\lambda_t l_t$, where w is wage, z_t is idiosyncratic productivity shock, and λ_t is age-dependent deterministic productivity profile. The productivity shock z_t follows AR(1) process:

$$z_t = \rho z_{t\ \ 1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

Upon retirement, agent receives pension benefits b.

The instantaneous utility function of an agent of age t is given by:

$$u(c_t, l_t) = \frac{[c_t^{\mu}(1 \quad l_t)^{\ 1 \ \ \mu)}]^{\ 1 \ \ \sigma)}}{1 \quad \sigma}$$

with c_t denoting consumption and l_t denoting labor supply at age t. The weight on consumption is μ and the coefficient of the relative risk aversion is σ.

Preferences are then given by:

$$E_1 \sum_{t=1}^{T} \beta^{\ t \ \ 1)} u(c_t, l_t)$$

There is a constant returns to scale production technology $Y = F(K, L) = K \ \ L^{\ 1 \quad)}$ with denoting capital share, Y denoting aggregate output, K

denoting aggregate capital shock and L denoting aggregate effective labor supply. The capital depreciates at a rate δ. Capital and labor markets are perfectly competitive.

Parametrization

$$T = 65$$
$$R = 40$$
$$n = 0.011$$
$$k_1 = 0$$
$$\tau = 0.11$$
$$\mu = 0.5$$
$$\sigma = 3$$
$$\beta = 0.96$$
$$= 0.36$$
$$\delta = 0.06$$
$$\rho = 0.9$$
$$\sigma_\epsilon^2 = 0.03$$

Download lambda.in file from the course website to get λ_t .

1. Write a code to solve this model. Download the algorithm for the numerical procedure from the course website.

2. Evaluate the macroeconomic consequences of eliminating Social Security. You can use Table 1 to support your answers.

    1) Solve for the stationary competitive equilibrium of the benchmark model with Social Security. Is this economy dynamically efficient (compare the interest rate with the implicit return from Social Security, which is equal to the population growth rate)?

    2) Eliminate Social Security by setting $\tau = 0$ and solving for the new stationary equilibrium.

        a - Check how aggregate capital and labor supply change as a result of the tax reform.

b - Plot and compare the profiles of wealth and labor supply by age for the case with and without Social Security. Provide intuition for observed differences.

c - Will a newborn generation prefer to start in a steady state with or without Social Security? *int*: Compute the welfare of a newborn, $V^o = \sum_{ik} \sum_{iz} (ik, iz, 1) V(ik, iz, 1)$.

3. Repeat the analyses above when $\beta = 0.99$. How do your answers to question 2 change?

	Benchmark model	
	with SS	without SS
capital K		
labor L		
wage w		
interest r		
pension benefit b		
newborn welfare V^o		

Table 1: Results of the policy experiment