Homework 3

ECON 8050: Macroeconomics II
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Problem 1: Dynamic Programming

Consider the following model with a disability shock. There are three sources of uncertainty:

- Out-of-pocket medical shock evolving according to transition matrix $\Psi(x_t|x_{t-1})$.
- Productivity evolving according to $T(z_t|z_{t-1})$.
- Disability shock.

The timing of events is as follows: At the beginning of the period, an individual with savings k_t learns their productivity z_t and medical shock x_t. Then they decide whether to work ($l_t = 0$ or $l_t > 0$). If working, labor income is wz_tl_t. Then, they decide about consumption c_t and savings k_{t+1}.

At the end of the period, the disability shock is realized with probability d. Disabled individuals stay permanently disabled, do not work, receive constant benefits DI, and make only consumption/savings decisions. Medical spending for disabled individuals is fully covered by public insurance.

(1) Write down the dynamic programming problem of a non-disabled individual, denoting the value function as V_t.

(2) Write down the dynamic programming problem of a disabled individual, denoting the value function as V_t^d.

(3) Modify the problem assuming disabled individuals can recover with probability f. Recovered individuals draw new productivity realizations from the invariant distribution.

(4) Extend the model to allow non-disabled individuals to falsely claim disability benefits, introducing the value function for falsely disabled V_t^{fd}.

Problem 2: Consumption-Savings Model

A consumer with infinite life maximizes quadratic utility:

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$$

where future utility is discounted at rate β and borrowing/savings occur at interest rate r with $\beta(1+r) = 1$.

The consumer's endowment y_t is i.i.d. with values y_H and y_L occurring with probabilities p_H and p_L respectively. The budget constraint is:

$$c_t = a_t(1+r) + y_t - a_{t+1}.$$

(1) Solve for the consumption and saving functions. Provide intuition on when savings are positive or negative.

(2) Introduce a borrowing constraint $a_{t+1} \geq 0$. Solve the consumer's problem in recursive form numerically using given parameters.

(3) Plot policy functions a_{t+1} and c_t as functions of current assets a_t for cases with and without borrowing constraints.

(4) Simulate the income process and optimal decision rules over $T = 100$ periods. Compare results with and without borrowing constraints.

Solution 1

Part (i)

The Bellman for an able bodied person with probability of becoming disabled d is as follows:

$$V_t(k_t, x_t, z_t) = \max_{c_t, l_t, k_{t+1}} \{u(c_t, l_t) + \beta(1-d)\sum_{x_t}\sum_{z_t} \Psi(x_t|x_{t-1})\mathrm{T}(z_t|z_{t-1})V_{t+1}(k_{t+1}, x_{t+1}, z_{t+1})$$
$$+\beta d \sum_{x_t} \Psi(x_t|x_{t-1})V_{t+1}(k_{t+1}, x_{t+1}, z_{t+1})\}$$

s.t.

$$c_t + k_{t+1} + x_t = wz_tl_t + k_t(1+r)$$

Part (ii)

The Bellman for an individual who is disabled and has no probability of recovery can be represented as follows:

$$V_t^d(k_t, x_t) = \max_{c_t, k_{t+1}} \{u(c_t, 0) + \beta \sum_{x_t} \Psi(x_t|x_{t-1})V_{t+1}^d(k_{t+1}, x_{t+1})\}$$

s.t.

$$c_t + k_{t+1} = DI + k_t(1+r)$$

Part (iii)

The Bellman equation for an individual who is disabled but has a probability of recovery is as follows:

$$V_t^{df}(k_t, x_t) = \max_{c_t, k_{t+1}} \{u(c_t, l_t) + \beta f \sum_{x_t|x_{t-1}} \sum_{z_t|z_{t-1}} \Psi(x_t|x_{t-1})\mathrm{T}(z_t|z_{t-1})V_{t+1}^d(k_{t+1}, x_{t+1}, z_{t+1})$$
$$+\beta(1-f)\sum_{x_t} \Psi(x_t|x_{t-1})V_{t+1}^d(k_{t+1}, x_{t+1})\}$$

s.t.

$$c_t + k_{t+1} + x_t = DI + k_t(1+r)$$

Part (iv)

Finally, the Bellman for someone who has the option to fake disability is as follows:

$$V_t^{df}(k_t, x_t, z_t) = \max_{c_t, k_{t+1}, l_t} \{u(c_t, l_t) + \beta f \sum_{x_t|x_{t-1}} \sum_{z_t|z_{t-1}} \Psi(x_t|x_{t-1})\mathrm{T}(z_t|z_{t-1})V_{t+1}^{d}(k_{t+1}, x_{t+1}, z_{t+1})$$
$$+\beta(1-f)\sum_{x_t} \Psi(x_t|x_{t-1})V_{t+1}^{d}(k_{t+1}, x_{t+1})$$
$$+\beta f \mathbb{1}_{fake=1} \sum_{x_t}(x_t|x_{t-1})V_{t+1}^{fd}(k_{t+1}, x_{t+1})\}$$

s.t.

$$c_t + k_{t+1} + x_t = wz_t l_t + DI\mathbb{1}_{D=1orfake=1} + k_t(1+r)$$