ECON 8050
Advanced Macroeconomics II
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Homework 2

Problem 1 (Costs of business cycle)

Let utility be given by:

$$E_{-1}\sum_{t=0}^{\infty}\beta^t U(c_t)$$

where the utility function is CRRA:

$$U(c_t)=\frac{c_t^{1-\gamma}}{1-\gamma}$$

The consumption process is

$$c_t=c_{t-1}^{\alpha}\,\varepsilon_t\,exp(\mu)$$

where

$$\mu=-\frac{\sigma_{\varepsilon}^2\,(1-\alpha)}{2\,(1-\alpha^2)}\ ,\quad log\,\varepsilon_t\sim N(0,\sigma_{\varepsilon}^2)\ \text{ and }\ \text{i.i.d.}$$

so the *log* of consumption follows an AR(1) with persistence parameter α :

$$log\,c_t=\mu+\alpha\,log\,c_{t-1}+log\,\varepsilon_t$$

A) Find the unconditional mean of c_t, $E(c_t)$. (Hint: recall the properties of the lognormal distribution).

B) Define $V_0=E_{-1}\sum\limits_{t=0}^{\infty}\beta^t U(c_t)$. This is ex-ante lifetime utility before any uncertainty is realized, i.e., even c_0 is not realized. Assume c_0 is drawn from the invariant (unconditional) distribution of c.

Define $V(\lambda) = E_{-1} \sum_{t=0}^{\infty} \beta^t U[(c_t(1+\lambda)]$. This is lifetime utility when every period consumption is increased by $(1+\lambda)$. Express $V(\lambda)$ as a function of μ, σ_ε^2, α, γ, β.

C) Denote by V_0 the lifetime utility for the case when c_t is deterministic and is equal to its unconditional mean you found in part A). Find how much compensation the consumer has to be given in order to be indifferent between the stochastic and deterministic cases, i.e find λ s.t. $V(\lambda) = V_0$.
Provide economic intuition.

D) Denote the interest rate as r. Find consumption c_t.

Problem 2

This problem uses non-expected utility framework (as in Kreps and Porteus, 1978; Epstein and Zin, 1991; Weil, 1990).

Let remaining lifetime utility at time t once c_t is known (uncertainty regarding current consumption is realized) be given by v_t. Assume v_t satisfies:

$$v_t = \left[(1-\beta)\, c_t^\rho + \beta \left(E_t v_{t+1}^\alpha\right)^{\frac{\rho}{\alpha}}\right]^{\frac{1}{\rho}} \tag{1}$$

Here, $1-\alpha$ represents the risk-aversion and $1-\rho$ - the inverse of the intertemporal elasticity of substitution. In the standard expected utility framework, $\alpha = \rho$.

Denote lifetime utility at time t before the uncertainty regarding current consumption is realized as U_t, $U_t = (E_t v_t^\alpha)^{\frac{1}{\alpha}}$.

A) Suppose we multiply c_t by λ for all $t = 0 \ldots \infty$. Prove that this is equivalent to multiplying v_t by λ.

(Hint: start with a guess that this is true. Plug this guess back into equation (1). Show that the function you get back (denote it $\tilde{v}_t$) is equal to λv_t, i.e., $\tilde{v}_t = \lambda v_t$.

B) Suppose for all $t = 0 \ldots \infty$ we substitute c_t with a deterministic constant $\bar{c}$, $\bar{c} = Ec_t$. Compare individuals' welfare in this situation without uncertainty with the situation with uncertain c_t. Specifically, find η such that when you multiply c_t (for $t = 0 \ldots \infty$) by $(1+\eta)$, the ex-ante welfare U_0

in the case with uncertainty is the same as in the deterministic case. Express η in terms of U_0 and $\bar{c}$.

C) Suppose the consumption process is as follows:

With probability $\frac{1}{2}$, consumption sequence is such that $c_t = c_l$ for all t, with probability $\frac{1}{2}$ the consumption sequence is such that $c_t = c_h$ for all t. The uncertainty regarding which sequence consumption will follow is revealed at $t = 0$. Find the value of η in this case. Does it depend on ρ? Does it depend on α? Provide economic intuition.

D) Suppose the consumption stream is i.i.d.. Every period with probability $\frac{1}{2}$, $c_t = c_l$ and with probability $\frac{1}{2}$, $c_t = c_h$.

a) Derive expression that implicitely define U_0.

(Hint: The following facts will be useful. First, because consumption is i.i.d., $[E_0 v_1^\alpha]^{\frac{1}{\alpha}} = [E_1 v_1^\alpha]^{\frac{1}{\alpha}}$. Second, because we have infinite horizon and i.i.d. process, $U_0 = U_1$.)

b) Will η in this case depend on α? What about ρ? Explain the difference from part C).

E) Use the equation that implicitely defines U_0 that you found in part D) to solve for U_0 in Matlab. Use the following parameters: $\beta = 0.95$, $c_l = e^{0.98}$, $c_h = e^{1.02}$. Find η for the following 9 combinations of parameters:

$\alpha = 1\,,\, 0.5\,,\, -1$

$\rho = 1\,,\, 0.5\,,\, -1$

Report as a table. Provide intuition.

(Hint: Your equation should have a form:

$$U_0 = f(U_0) \text{ where } f(\cdot) \text{ is some function}$$

Start with a guess, for example, $U_0 = 1$. Compute $U_0^{(1)} = f(U_0 = 1)$. Plug in $U_0^{(1)}$ back, find $U_0^{(2)} = f(U_0^{(1)})$. Continue until $|U_0^{(n+1)} - U_0^{(n)}| < tol$ where $tol = 10^{-8}$.)