enumitem

Linear Model:

- Assumptions:

a) $\mathbb{E}(u) = 0, cov(x_j, u) = 0$ zero mean

b) $\mathbb{E}(u|x) = 0$ uncorrelated mean

c) $Rank[\mathbb{E}(xx')] = K$ full rank

d) $\mathbb{E}(u^2xx') = \sigma^2\mathbb{E}(xx')$ s.t. $\mathbb{E}(u^2) \equiv u^2$: errors constant in variance

e) $cov(x_k, e_k) = 0$ or $cov(x_k^*, e_k) = 0$: classical errors in variables

f) for some $Lx1$ vector z, $\mathbb{E}(zu) = 0$: any exog elements of x, are included in z

g) $Rank[\mathbb{E}(zz')] = L$ and $Rank[\mathbb{E}(zx')] = K$: must have at least as many instruments as explanatory variables

h) $\mathbb{E}(u^2zz') = \sigma^2\mathbb{E}(zz')$: homoskedasticity

- OLS estimator: $\hat{\beta} = (\sum_{i=1}^{n} x_i x_i')^{-1} \sum_{i=1}^{n} x_i y_i$

- 2SLS estimator: $\hat{\beta} = [(\sum_{i=1}^{n} \mathrm{x}_i z_i')(\sum_{i=1}^{n} \mathrm{z}_i z_i')^{-1}(\sum_{i=1}^{n} \mathrm{z}_i x_i')]^{-1}(\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i x_i')(\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i z_i')^{-1}(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} \mathrm{z}_i u_i)$

- IV estimator: $\hat{\beta}[\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i x_i']^{-1}\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i y_i$. z is the instrument which satisfies criteria uncorrelated with u and slightly correlated with x.

- Omitted Variables (IV): put omitted in error term and find instruments for any elements in the explanatory in the omitted

Nonlinear Model:

- Probit: $f(y|x_i) = [\Phi(x_i'\theta)]^y[1 - \Phi(x_i'\theta)]^{1-y}$

- Logit: $G(z) = \frac{e^z}{1+e^z}$

- Partial effect of probit: $\frac{\partial f}{\partial x}\phi$

- log-likelihood function: $l_i = y_i \log[G(x_i'\beta)] + (1 - y_i)\log[1 - G(x_i'\beta)]$

- likelihood function: $\mathcal{L} = \sum_{i=1}^{n} \mathrm{l}_i$

- multinomial logit: $P(y = j|x) = \frac{e^{x'\beta_j}}{1+\sum_{h=1}^{J} e^{x'\beta_h}}$; $P(y = j \ or \ y = h|x) = \frac{p_j(x,\beta)}{p_j(x,\beta)+p_h(x,\beta)}$

Roy:

$$y_1 = \begin{cases} 1 \ if \ y_1^* > 0 \\ 0 \ if \ y_1^* \leq 0 \end{cases}$$

$$y_2 = \begin{cases} y_2^* \ if \ y_1^* > 0 \\ y_3^* \ if \ y_1^* \leq 0 \end{cases}$$

$y_1^* = x_1'\beta_1 + \epsilon_1$

$y_2^* = x_2'\beta_2 + \epsilon_2$

$y_3^* = x_3'\beta_3 + \epsilon_3$

Heckman 2-Step:

$- y_{2i} = x_{2i}'\beta_2 + \sigma_{12}\lambda(x_{1i}'\hat{\beta}_1) + v_i$

$-$ v is the error, beta hat is the first step probit reg of y_1 on x_1

since $Pr[y_1^* > 0] = \lambda(x_1'\beta_1)$ and $\lambda_1(x_1'\hat{\beta}_1) = \phi(x_1'\beta_1)/\Phi(x_1'\beta_1)$ is the inverse Mills ratio

- Likelihood ratio test: $2(\mathcal{L}_{UR} - \mathcal{L}_r)$