

Homework 5

Due: At the start of class on Monday, April 14. Please turn in a hard copy.

Question 1 Hansen 25.15, for this question, you need to code the probit estimator yourself (compute both estimates of the parameters and their standard errors); i.e., you cannot use built-in R functions such as `glm`. Hint: you can use R's optimization routines such as `optim`. In addition to what is asked in 25.15,

- Compare the estimates that you get to those coming from the `glm` function
- Calculate and report average partial effects for each regressor.
- Derive the asymptotic variance for the average partial effects.

Hint: It is helpful to notice that

$$\begin{aligned}\sqrt{n}(\widehat{APE} - APE) &= \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \phi(X'_i \hat{\beta}) \hat{\beta} - \mathbb{E}[\phi(X' \beta) \beta] \right) \\ &= \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \phi(X'_i \hat{\beta}) \hat{\beta} - \frac{1}{n} \sum_{i=1}^n \phi(X'_i \hat{\beta}) \beta \right) \\ &\quad + \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \phi(X'_i \hat{\beta}) \beta - \frac{1}{n} \sum_{i=1}^n \phi(X'_i \beta) \beta \right) \\ &\quad + \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \phi(X'_i \beta) \beta - \mathbb{E}[\phi(X' \beta) \beta] \right)\end{aligned}$$

which holds just by adding and subtracting some terms, and, just to be clear, the notation above is for the entire vector of average partial effects for all regressors. Figuring out the asymptotic distributions for the first and last lines is not too hard, but the middle expression requires using some kind of mean value theorem type of argument (as we have done before in the context of the delta method).

- Based on your result in part (c), compute the standard error of each average partial effect (again, use your own code to compute these).
- Use the bootstrap to calculate standard errors for the average partial effects that you calculated in part (b). Compare these standard errors to the analytical standard errors that you calculated in part (d).

Question 2: For this problem, we will be interested in computing an estimate of the *ATT* of a job training program. You can [download the data here](#) and [download a description here](#). For this problem, the outcome of interest is `re78`, the treatment is `train`, and suppose that unconfoundedness holds

after conditioning on `age`, `educ`, `black`, `hisp`, `married`, `re75`, and `unem75`.

- a. Given our discussion in class, we know that if we believe unconfoundedness and that untreated potential outcomes are linear in covariates, then we have that

$$ATT = \mathbb{E}[Y|D = 1] - \mathbb{E}[X'|D = 1]\beta_0$$

where β_0 can be estimated from the regression of Y on X using untreated observations.

Estimate ATT based on the above expression for it and report your result.

- b. Show that $\sqrt{n}(\widehat{ATT} - ATT) \xrightarrow{d} \mathcal{N}(0, V)$ and provide an expression for V . Based on this result, provide standard errors for your estimate of ATT .
- c. Use the bootstrap to compute standard errors for your estimate of ATT . How do these compare to the standard errors that you reported previously?
- d. Run a regression of Y on D and X (where X includes the same additional variables as above). Compare the coefficient on D to the estimate of ATT . How similar are they? What about their standard errors? Do you have any comment on these results?
- e. Calculate an estimate of ATT using propensity score re-weighting (as we discussed in class). You can estimate the propensity score model using logit (it is ok to use the `glm` function for this). Compute standard errors using the bootstrap and compare your results to the previous estimates.
- f. Calculate an estimate of ATT using the doubly robust AIPW estimator that we discussed in class. Again, you can estimate the propensity using logit from the `glm` function. Compute standard errors using the bootstrap and compare your results to the previous estimates.
- g. Finally, calculate an estimate of ATT using machine learning. For this problem, use the algorithm we discussed in class to estimate the ATT using machine learning. In particular, I'd like for you to use the `ranger` package to estimate the propensity score and the outcome regression model. Compute standard errors using the bootstrap and compare your results to the previous estimates.