Homework 3

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An ECON - 8040 Homework Assignment

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1 Question 1

1.1 Problem

Consider the following two period planning problem

$$w(\bar{k}_1) = \max_{c_t, k_{t+1} \geq 0} \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma}$$

s.t.

$$\begin{aligned} c_1 + k_2 &= k_1^{\alpha} + (1-\delta)k_1 \\ c_2 &= k_2^{\alpha} + (1-\delta)k_2 \\ k_1 &= \bar{k}_1 \end{aligned}$$

The first order conditions for this problem is

$$c_1^{-\sigma} = \beta c_2^{-\sigma}(1 - \delta + \alpha k_2^{\alpha-1}).$$

Use the following parameters

β	σ	α	δ
0.95	2	0.4	0.1

Define

$$k_{ss} = \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha - 1}}$$

(a) Assume $\bar{k}_1 = k_{ss}$. Solve allocation of consumption and capital stock c_1, c_2, k_2. Note, you need to solve the following system of equations

$$\begin{aligned} c_1 + k_2 &= k_1^{\alpha} + (1-\delta)k_1 \\ c_2 &= k_2^{\alpha} + (1-\delta)k_2 \\ c_1^{-\sigma} &= c_2^{-\sigma}\beta(1 - \delta + \alpha k_2^{\alpha-1}) \end{aligned}$$

q using the Newton method.

(b) Now, make the following grid $\mathcal{K} = \{\frac{1}{2}k_{ss}, \frac{3}{4}k_{ss}, k_{ss}, \frac{3}{2}k_{ss}, 2k_{ss}\}$ for $\bar{k}_0$. Solve allocations c_1, c_2, k_2 for all points on the grid. Using your answers, find value of $w(\bar{k}_1)$ for every point on the grid and plot $w(\bar{k}_1)$.

1.2 Solution

2 Question 2

2.1 Problem