

**A WELFARE ANALYSIS OF SOCIAL SECURITY
IN A DYNASTIC FRAMEWORK***

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In this article, we study the welfare effects of unfunded social security in a general equilibrium model populated with overlapping generations of altruistic individuals that differ in lifetime expectancy and earnings ability. Contrary to previous research, our results indicate that steady-state welfare increases with social security for most households, although by very different amounts. This result is mainly due to two factors. First, the presence of two-sided altruism significantly mitigates the crowding out effect of unfunded social security. Second, ability shocks and uncertain lifetimes generate significant heterogeneity among households to yield different induced preferences for social security.

1. INTRODUCTION

Since the work of Auerbach and Kotlikoff (1987) it is well understood that social security can have large negative effects on capital accumulation and labor supply decisions. Yet, the current unfunded social security system appears to have considerable political support in the U.S. economy and is, perhaps, the most popular redistributive mechanism used by the U.S. government. The distributional implications of social security are far from trivial. Social security redistributes income across generations since it transfers income from workers to retired individuals. It also redistributes income within generations since pension benefits increase less than proportionately with income. The social security system also has nontrivial insurance effects against different types of risk.

In this article, we study the welfare effects of unfunded social security in a two-sided altruistic framework in which social security provides insurance against individual income and lifespan uncertainty.² We develop an economy populated

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² Unfunded social security may also serve to distribute aggregate risks across generations. In this article, we abstract from this potential benefit of social security.

by overlapping generations of individuals that differ in income and lifetime expectancy. Following Laitner (1992), individuals derive utility from their own lifetime consumption and from the felicity of their predecessors and descendants. As a result, we can study the impact of social security on *intervivos* transfers and bequests within the family (see Barro, 1974). This economy consists of a rich set of households. First, there are households in which the father and the children are alive and can possess different abilities. Second, there are households in which the father or the children have died. This framework allows us to compare the annuity role of social security for single individuals versus for households in which families can also provide annuity insurance to their members. Our goal is to quantitatively assess the effects of social security for aggregate allocations and its impact on the distribution of welfare across these households that differ in earnings ability and lifetime expectancy of their members.

Our main findings can be summarized as follows. First, similar to the findings in Fuster (1999), an unfunded social security system with a 44% replacement rate crowds out only 6% of the capital stock, which is much smaller than the one obtained in pure life-cycle models.³ This finding owes much to our assumption of two-sided altruism. In models populated by overlapping generations of pure life-cycle consumers, social security lowers the capital stock since it redistributes income away from younger agents with higher marginal propensities to save to older agents with lower marginal propensities to save. In a framework with altruism, in addition to the insurance and life-cycle motives, individuals save for bequest motives, and therefore old individuals do not necessarily have a lower marginal propensity to save than young individuals. As a result, social security has a small effect on the aggregate saving rate and on capital accumulation.⁴ Second, contrary to previous research, our results indicate that steady-state welfare increases with social security for *most* household types. In the vast majority of the literature that abstracts from altruism, the large negative effect of social security on capital accumulation leads to an important reduction in steady-state welfare. In these papers, the benefits of social security are never large enough to compensate its large negative effect on capital accumulation, consequently giving rise to a reduction in steady-state welfare as the economy moves further away from the golden rule steady state.⁵ In our model, the benefits of social security outweigh the relatively small cost in terms of the crowding out of the capital stock for a

³ For example, Auerbach and Kotlikoff (1987) find that a social security system with a 60% replacement rate reduces the steady-state capital stock by 24%. In İmrohoroglu et al. (1999), capital stock decreases by 26% with a 40% social security replacement rate. Conesa and Krueger (1999) find the reduction in capital stock to be around 11% to 17%. Some other examples of papers supporting the reduction in individual savings due to social security are Blinder et al. (1981), Feldstein and Pellechio (1979), Kotlikoff (1979), and Hubbard (1986).

⁴ As in Barro (1974), the transfer induced by social security is partially undone by altruistic transfers from parent to children and thus social security has a small impact on saving.

⁵ Auerbach and Kotlikoff (1987), İmrohoroglu et al. (1999), Conesa and Krueger (1999), Hubbard and Judd (1987), Cubeddu (1996), and Storesletten et al. (1999) all find significant reductions in welfare due to an unfunded social security system. İmrohoroglu et al. (1995) find that steady-state welfare increases with social security. However, they study an economy that is dynamically inefficient in the absence of social security.

majority of the households.⁶ Third, we also find that the distributional impact of social security varies significantly across household types. In this framework, households differ in terms of the value they assign to the insurance role of social security due to differential age earnings profiles and mortality risk. The household type where the children pay the highest taxes and the father has the shortest lifetime is the one that likes social security the least. Households that like social security in this framework are those for which family insurance plays an important role in undoing the borrowing constraints due to government policy. These are mainly households where the father is alive and transfers income to his children. Our results also indicate that, in addition to altruism, incorporating mortality risk and ability shocks are very important in order to obtain differences in induced preferences over the desirability of social security. Abstracting from either one of these features or simply examining the average utility in an economy with heterogeneous agents may give the false impression that the majority of the population prefers to be born into economies without social security. Overall, these features are likely to be especially important for models building a positive political economy theory of social security.

The main results of this article owe much to our assumption of two-sided altruism. Whether models with two-sided altruism are more useful abstractions for the U.S. economy is an empirical question. Our findings suggest that the answer has very important policy implications on the issues related to social security.

The article is organized as follows. Section 2 describes the model and Section 3 describes the calibration of the benchmark economy. Section 4 presents the results and the sensitivity analysis, and Section 5 concludes.

2. THE MODEL

2.1. Demographics and Endowments. Time is discrete and starts in the infinite past. The setup is an overlapping generations model with two-sided altruistic bequests, uninsurable “labor ability” shocks realized at birth, and borrowing constraints. At each date t a new generation of individuals is born. Individuals face random lives and some live through age $2T$, the maximum possible life span. Conditional on survival, an individual’s lifetime support overlaps with the lifetime support of his father during the first T periods and during the last T periods with the lifetime support of his children. At any point in time, the economy is populated by $2T$ overlapping generations of individuals with total measure one.

Each period, individuals are endowed with one unit of time that they supply inelastically to firms as labor services. At birth, an individual receives the realization of a random variable $z \in Z = \{H, L\}$ that determines his lifetime labor ability; z is a two-state, first-order Markov process with the transition probability matrix

$$\Pi(z', z) = [\pi_{ij}], \quad i, j \in \{H, L\}$$

where $\pi_{ij} = \Pr\{z' = j \mid z = i\}$, z' is the labor ability of the newborn in the dynasty, and z is the labor ability of his father. The transition probabilities are consistent

⁶ Fuster (1999) gets comparable results on capital accumulation; however, since she abstracts from lifetime uncertainty, there are no welfare benefits of social security in her model.

with the existence of a unique stationary measure of abilities $\lambda(z)$. It is important to note that there are no insurance markets in the economy to diversify the risk of being born as a low-ability-type individual.

Labor ability affects two features of an individual's lifetime opportunities. First, z determines the individual's age-efficiency profile $\{\varepsilon_j(z)\}_{j=1}^{2T}$. If $z = H$, an individual has a higher labor productivity throughout his lifespan than an individual with $z = L$.⁷ Second, labor ability determines an individual's life expectancy. Let $\psi_j(z)$ denote the probability of surviving to age $j + 1$ conditional on having survived to age j for an individual with ability z for age $j = 1, 2, \dots, 2T$, where $\psi_{2T}(z) = 0$ and $z \in \{H, L\}$.

The demographic structure of population is assumed to be stationary in the sense that the population shares of cohorts are time invariant. The size of cohort 1 (newborns), with ability z , relative to that of cohort $(T + 1)$ (parents) is $\mu_1(z) = \lambda(z)(1 + n)^T$ where $(1 + n)^T$ is the number of children per parent and $\lambda(z)$ is the measure of newborn individuals with ability z . The relative sizes of the other generations are obtained recursively as follows:

$$\mu_{i+1}(z) = \frac{\psi_i(z)\mu_i(z)}{(1 + n)}, \quad i = 1, \dots, 2T - 1$$

Since the population growth rate, n , and conditional survival probabilities, $\psi_i(z)$, are constant, the cohort shares are time-invariant.

2.2. Technology. There are firms in this economy that use capital, K , and labor, N , to produce a single good according to a constant returns to scale Cobb–Douglas production function:

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}$$

where $\alpha \in (0, 1)$ is the output share of capital, Y_t is output at time t , K_t is aggregate capital input at time t , N_t is aggregate labor input at time t , and A_t denotes a technology index that grows at a constant rate γ . Capital depreciates at a constant rate $\delta \in (0, 1)$. Firms maximize profits renting capital and hiring labor from the households so that marginal products equal factor prices

$$(1) \quad \tilde{r}_t = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha}$$

$$(2) \quad \omega_t = (1 - \alpha) K_t^\alpha (A_t N_t)^{-\alpha}$$

where \tilde{r}_t is the rental price of capital and ω_t is the wage per effective labor.

2.3. Social Security and Fiscal Policy. There is a social security system that provides pension benefits to retired individuals. Pensions are financed by taxing

⁷ At the mandatory retirement age R and thereafter, the endowment of efficiency units of labor is assumed to be zero, so that no time is supplied to market activities.

earnings of current workers. The payroll tax, τ , is set so that the budget of the social security system is balanced each period. We assume that the benefits that individuals receive are related to their average lifetime earnings according to a piecewise linear function.

There is a government that taxes labor income, capital income, and consumption in order to finance exogenously given government purchases. We assume that the government's budget is balanced each period. Since tax rates and government expenditure are exogenous, the budget is balanced by an endogenous lump-sum transfer to the individuals.

The government also collects the asset holdings and capital income of individuals who die without descendants. These resources are transferred in a lump-sum fashion to all survivors.

2.4. Altruistic Preferences and the Households' Decision Problem. The formalization of preferences follows Laitner (1992). Individuals derive utility from their own lifetime consumption and from the felicity of their predecessors and descendants. This two-sided altruism framework allows us to evaluate the annuity role of social security when families can also provide annuity insurance to their members. In this setup, strategic behavior between the father and the children does not arise because their decisions maximize the same objective function. Because of this commonality of interests during the periods when their lives overlap, the father and the children constitute a single decision unit by pooling their resources. This decision unit is called a household and is constituted by an adult male, the "father," of generation j and age $T + 1$, and his $m = (1 + n)^T$ adult children of generation $j + T$ and age 1. A household lasts T periods or until the father and the children have died.⁸ A dynasty is a sequence of households that belong to the same family line. If the children survive to age $T + 1$, each of them becomes a father in the next-generation household of the dynasty. Otherwise, the family line is broken, and this particular dynasty is over. We assume that such dynasties are replaced by new dynasties to maintain our assumption of a stationary demographic structure. Since mortality rates are higher for low-ability individuals, the number of new dynasties of low-ability is higher than the number of dynasties of high ability. A new dynasty begins with an individual of age 1 that holds zero assets.

Households are heterogeneous regarding their asset holdings, age, abilities, and their composition. The composition of a household changes when either the father or his m children die. Since the lifespan shocks that affect each of the children are perfectly correlated, there are three types of households. Households of type 1 are those in which the father has died. Households of type 2 are those in which the m children have died. Households of type 3 are those in which both the father and the children are still alive.

The budget constraint facing an age- j household, where $j = 1, 2, \dots, T - 1$ is the age of the youngest member(s), is given by

⁸ In a given household, all children are born at the same period and all of them die at the same period. They are also identical regarding their labor abilities and vector of conditional survival probabilities. These assumptions allow us to restrict the number of different household types.

$$\begin{aligned}
 (3) \quad & [\phi_s(h) + \phi_f(h)](1 + \tau_c)c_j + (1 + \gamma)a_j \\
 & = [1 + r(1 - \tau_k)]a_{j-1} + e_j(h, z, z') \\
 & \quad + [\phi_s(h) + \phi_f(h)](\xi_1 + \xi_2)
 \end{aligned}$$

where ϕ_s is an indicator function that takes the value m if the children are alive and 0 otherwise, and ϕ_f is an indicator function that takes the value 1 if the father is alive and 0 otherwise; $h \in \{1, 2, 3\}$ is an indicator of household composition, $r = \tilde{r} - \delta$, $e_j(h, z, z')$ are the aftertax earnings, c_j is the consumption of each household member, a_j denotes the asset holdings to be carried over to age $j + 1$, ξ_1 is a lump-sum redistribution of accidental bequests left behind by fathers without sons and confiscated by the government, ξ_2 is a lump-sum transfer to balance the government's budget, and τ_c and τ_k denote the consumption and capital income tax rates, respectively. Consumption, asset holdings, lump-sum transfers, and earnings are transformed to eliminate the effects of labor augmenting, exogenous productivity growth. In particular, we have divided those variables by the level of the technology, A_t , at each period t .⁹

The function $e_j(h, z, z')$ gives the net-of-tax earnings of an age- j household:

$$\begin{aligned}
 (4) \quad & e_j(h, z, z') \\
 & = \begin{cases} \phi_s(h)\omega(1 - \tau - \tau_\ell)\varepsilon_j(z') + \phi_f(h)B_{j+T}(z) & \text{if } j \geq R - T \\ \phi_s(h)\omega(1 - \tau - \tau_\ell)\varepsilon_j(z') + \phi_f(h)\omega(1 - \tau - \tau_\ell)\varepsilon_{j+T}(z), & \text{otherwise} \end{cases}
 \end{aligned}$$

where τ is the social security tax rate and τ_ℓ is the tax rate on labor income. $B_{j+T}(z)$ denotes the pension at age $j + T$, which depends on average lifetime earnings calculated using the wage at the retirement period, that is, $\omega/(1 + \gamma)^{j+T-R}$. An individual's pension remains constant during retirement whereas technology grows at the rate γ . Thus the pension per effective labor decreases during retirement at rate γ , that is, $B_{j+T}(z) = B_R(z)/(1 + \gamma)^{j+T-R}$. Put differently, the retirement benefits of successive cohorts increase at the rate γ .

For $j = T$, the budget constraint of the household is given by

$$\begin{aligned}
 (5) \quad & [\phi_s(h) + \phi_f(h)](1 + \tau_c)c_T + (1 + n)^T(1 + \gamma)a_T \\
 & = [1 + r(1 - \tau_k)]a_{T-1} + e_T(h, z, z') \\
 & \quad + [\phi_s(h) + \phi_f(h)](\xi_1 + \xi_2)
 \end{aligned}$$

If the children survive to age T , $(1 + n)^T$ new households are constituted in the dynasty and each of them will hold a_T assets. If the children do not survive to age T , the family line is broken.

⁹ For the sake of clarity, we will drop the remaining time subscripts from now on.

It is assumed that households face borrowing constraints and cannot hold negative assets at any age:

$$(6) \quad a_j \geq 0, \quad \forall j$$

Individuals obtain utility from their consumption and from their predecessors' and descendants' consumption.¹⁰ We restrict the utility function to the CRRA class because we assume a balanced growth path for our economy. We will use the language of recursive economic theory to describe the household's decision problem.

Let $V_j(a, h, z, z')$ denote the maximized value of expected, discounted lifetime utility of an age- j household with the state vector (a, h, z, z') . For a household of age $j \leq T$,

$$(7) \quad V_j(a, h, z, z') = \max_{\{c, a'\}} \left\{ \frac{[\phi_s(h) + \phi_f(h)]^{\frac{1-\sigma}{1-\sigma}}}{+\beta(1+\gamma)^{1-\sigma}} \tilde{V}_{j+1}(a', h', z, z') \right\}$$

subject to (3)–(6)

where σ is the coefficient of relative risk aversion and

$$\begin{aligned} & \tilde{V}_{j+1}(a', h', z, z') \\ &= \begin{cases} \sum_{h'=1}^3 \chi_j(h, h'; z, z') V_{j+1}(a', h', z, z') & \text{for } j = 1, 2, \dots, T-1 \\ \psi_T(z')(1+n)^T \sum_{z'' \in \{H, L\}} \pi_{z'z''} V_1(a', 3, z', z'') & \text{for } j = T \end{cases} \end{aligned}$$

$\chi_j(h, h'; z, z')$ is the probability that a household of age j and type h becomes type h' the next period given that the father is of ability z and the children are of ability z' .¹¹ Note that a household of age T faces two shocks. One is the lifespan shock that affects the youngest members of the household, the other is the ability shock that affects the new generation of the dynasty. The youngest members will survive with probability $\psi_T(z')$ and constitute $(1+n)^T$ new households. The ability of the new generation of the dynasty is denoted by z'' and is correlated with the ability z' of the father. Note that the household members pool their resources and act as a single decision-making unit.

2.5. Steady-State Equilibrium. Given a fiscal policy $\{G, B, \tau_\ell, \tau_k, \tau_c, \tau\}$, a stationary recursive competitive equilibrium is a set of value functions $\{V_j(a, h,$

¹⁰ We describe the utility function of individuals in detail in the Appendix. For a description of the solution method, see Fuster (1999).

¹¹ This transition probability matrix is a function of the age of the household and of the abilities of the father and the son, and is given by

$$[\chi_j(h, h'; z, z')]_{h, h' \in \{1, 2, 3\}} = \begin{bmatrix} \psi_j(z') & 0 & 0 \\ 0 & \psi_{j+T}(z) & 0 \\ \psi_j(z')(1 - \psi_{j+T}(z)) & (1 - \psi_j(z'))\psi_{j+T}(z) & \psi_j(z')\psi_{j+T}(z) \end{bmatrix}$$

$z, z')\}_{j=1}^T$, households' decision rules $\{c_j(\cdot), a_j(\cdot)\}_{j=1}^T$, time-invariant measures of households $\{X_j(a, h, z, z')\}_{j=1}^T$, relative prices of labor and capital $\{\omega, r\}$, a lump-sum distribution of unintended bequests ξ_1 , and a lump-sum government transfer ξ_2 such that the following conditions are satisfied:

1. given policy, prices, and lump-sum transfers, households' decision rules solve households' decision problem (7);
2. factor prices are competitive, i.e., (1) and (2) hold;
3. aggregation holds,

$$\begin{aligned}\tilde{K} &= \sum_{j,a,h,z,z'} a_{j-1}(a, h, z, z') X_j(a, h, z, z') (1+n)^{1-j} \\ \tilde{N} &= \sum_{j=1}^{R-1} \sum_{z \in \{H, L\}} \varepsilon_j(z) \mu_j(z) \\ C &= \sum_{j,a,h,z,z'} [\phi_s(h) + \phi_f(h)] c_j(a, h, z, z') X_j(a, h, z, z') (1+n)^{1-j}\end{aligned}$$

4. the set of age-dependent measures of households satisfies

$$(8) \quad X_{j+1}(a', h', z, z') = \sum_{\{a,h,z:a'=a_j(a,h,z,z')\}} X_j(a, h, z, z') \chi_j(h, h'; z, z'),$$

for $j = 1, \dots, T-1$

the invariant distribution of age-1 households is given by conditions

$$(9) \quad X_1(a', 3, z', z'') = \pi_{z'z''} \sum_{\{a,h,z:a'=a_T(a,h,z,z')\}} X_T(a, h, z, z') \chi_T(h, 3; z, z')$$

and

$$(10) \quad X_1(0, 1, z', z'') = \lambda(z') \pi_{z'z''} - \sum_{a'} X_1(a', 3, z', z'')$$

that is, new dynasties, holding zero assets, substitute for the family lines broken during the last period;

5. the lump-sum redistribution of unintended bequests satisfies

$$\xi_1 = (1+r) \sum_{j=1}^T a_j(a, h, z, z') X_j(a, h, z, z') \left[1 - \sum_{h'=1}^3 \chi_j(h, h'; z, z') \right] (1+n)^{1-j}$$

6. the government's budget is balanced

$$\xi_2 = \tau_k r \left[\tilde{K} - \frac{\xi_1}{1+r} \right] + \tau_\ell \omega \tilde{N} + \tau_c C - G$$

7. the social security tax is such that the budget of the social security system is balanced

$$\sum_{j=R}^{2T} \sum_{z=H,L} B_j(z) \mu_j(z) = \tau \omega \tilde{N}$$

8. the goods market clears

$$C + [(1+n)(1+\gamma)\tilde{K} - (1-\delta)\tilde{K}] + G = \tilde{K}^\alpha \tilde{N}^{1-\alpha}$$

3. CALIBRATION OF THE BENCHMARK ECONOMY

3.1. Demographic and Labor Market Parameters. We assume that individuals are born when they are 20 years old and live to be at most 90 years old. If they survive, they retire from the labor market at the age of 65. Also conditional on surviving, individuals' fertile lifetimes conclude when they are 35 years old. At this time they have m children. If individuals reach the age of 55, they form a household with their m children. For computational reasons the model period is five years. These assumptions imply the following parameter values for the model: $T = 7$ and $R = 10$. These imply that when the father reaches the model age of 8 (real time age 55) his children reach the model age 1 (real time age 20) and this household starts making joint decisions.¹² The father retires at the model age of 10 (real time age 65) when the son is 3 periods old (real time age 30).

Labor efficiency profiles for high- and low-ability individuals, $\varepsilon_j(\cdot)$, are calibrated to the profiles of efficiency units of labor of college and non-college graduate males, respectively. We construct these indices using data on earnings from the U.S. Bureau of the Census (1991).

We choose the values for the transition probabilities so that our benchmark economy matches two observations. First, the proportion of full-time male workers that were college graduates in 1991 was 28% (see U.S. Bureau of the Census, 1991, p. 145). Second, the correlation between the permanent component of income of parents and children is 0.4 according to the estimates by Zimmerman (1992) and Solon (1992). These observations imply that $\pi_{HH} = 0.57$ and $\pi_{LL} = 0.83$.

Labor ability determines the vector of conditional survival probabilities. We obtain these probabilities from Elo and Preston (1996), who present data for college and non-college graduate males in the U.S. economy. They show that lifetime expectancy at the real age of 20 is 5 years longer for a college graduate than for a non-college graduate.

Although the model period is 5 years, in what follows we express flow variables as rates per year. The population growth rate is 1.2%, which equals the average annual population growth rate of the U.S. economy over the last 50 years. This implies for the model that $n = 0.012$ and $m = 1.52$.

¹² Note that the children are born when the father is 35 years old, but the joint decision making only starts after the children reach the age of 20 and start working.

3.2. *Preferences and Technology.* The utility function displays a constant elasticity of intertemporal substitution of consumption ($1/\sigma$), which is consistent with a balanced growth path. There is an ample range of estimated values for the intertemporal elasticity of substitution. Auerbach and Kotlikoff (1987) cite empirical studies that provide estimates of σ in the range of 1 to 10. For the benchmark economy we assume $\sigma = 2$.

The exogenous productivity growth rate is taken as $\gamma = 1.65\%$, which is the postwar annual average growth rate of per capita real GDP. Following İmrohoroglu et al. (1999), the income share of capital, α , and the depreciation rate, δ , are set at 0.31 and 4.4%, respectively. The subjective discount factor, β , is chosen so that the benchmark economy produces a capital–output ratio of 2.5, which is close to the postwar average. This procedure yields a value of 0.988 for β .

3.3. *Social Security and Taxation.* The model's treatment of the social security arrangements follows the existing benefit formulas used by the Social Security Administration. Retirement benefits depend on individuals' average lifetime earnings via a concave, piecewise linear function. The marginal replacement rate decreases with average lifetime earnings indexed to productivity growth. It is equal to 0.9 for earnings lower than 20% of the economy's average earnings. Above this limit and below 125% of the economy's average earnings the marginal replacement rate decreases to 0.33. For income within 125% and 246% of the economy's average earnings the marginal replacement rate is 0.15. Additional income above 246% of the economy's average earnings does not provide any additional pension payment. In order to capture the progressivity of social security, we use different benefit formulas for individuals of low labor ability and of high labor ability.

In the benchmark economy, individuals without college education have an average lifetime earnings between 20% and 125% of the economy's average earnings. The average lifetime earnings of individuals with college education is between 125% and 246% of the economy's average earnings. Therefore, the pension payment for each ability group is calculated as follows:

$$B_{nc}(M_{nc}) = 0.9(0.2M) + 0.33(M_{nc} - 0.2M)$$

$$B_c(M_c) = 0.9(0.2M) + 0.33(1.25M - 0.2M) + 0.15(M_c - 1.25M)$$

where M_{nc} and M_c are the average lifetime earnings of a non-college and a college graduate, respectively, and M denotes the economy's average earnings. The average replacement rate is $\theta = 0.44$.¹³

¹³ The benefit formula used in the benchmark economy can be generalized for any average replacement rate. This general benefit formula is

$$B_{nc}(M_{nc}) = \theta \left[\frac{0.9}{0.44}(0.2M) + \frac{0.33}{0.44}(M_{nc} - 0.2M) \right]$$

$$B_c(M_c) = \theta \left[\frac{0.9}{0.44}(0.2M) + \frac{0.33}{0.44}(1.25M - 0.2M) + \frac{0.15}{0.44}(M_c - 1.25M) \right]$$

where θ is the average replacement rate which equals 0.44 in the benchmark economy.

TABLE 1
LIST OF PARAMETERS

Population	
$2T = 14$	Maximum lifetime (70 years)
$R = 10$	Retirement age (45 years)
$n = 0.012$	Population growth rate (annual)
Utility	
$\sigma = 2$	Coefficient of relative risk aversion
$\beta = 0.988$	Annual discount factor
Production	
$\gamma = 0.0165$	Annual rate of growth of technology
$\alpha = 0.31$	Capital share of GNP
$\delta = 0.044$	Annual depreciation rate
$\lambda(H) = 0.28$	Measure of individuals with high ability
$\pi_{LL} = 0.83$ $\pi_{HH} = 0.57$	Transition probability matrix of abilities
Fiscal Policy	
$\tau_l = 0.2$	Labor income tax rate
$\tau_k = 0.4$	Capital income tax rate
$\tau_c = 0.055$	Consumption tax rate
$G/Y = 18\%$	Government purchases to GDP ratio

We set the government purchases equal to 18% of output in all experiments. The capital income tax rate is 40%. This tax rate and the technology assumptions imply that the aftertax interest rate is 4.6% in the benchmark economy. We assume a consumption tax rate of 5.5% and a labor income tax rate, exclusive of social insurance taxes, of 20%, which is consistent with the U.S. data (see İmrohoroglu, 1998). Table 1 summarizes all the parameters used in the benchmark economy.

4. RESULTS

Before we discuss the steady-state findings, it will be useful to summarize the benefits and costs of an unfunded social security system in our environment. On the benefit side, there are two insurance roles played by the unfunded social security system. First, it partially substitutes for missing private annuity markets by insuring pensioners against outliving their resources. Second, the system insures against the “ability” shock by redistributing across ability types. On the cost side, the main adverse impact of social security is the distortion introduced on the consumption–saving choice. Furthermore, labor income taxes that are required to pay for the social security system have an additional adverse impact on liquidity-constrained agents. Social security also impacts decisions on the timing, direction, and the size of inter vivos transfers and bequests. In order to examine the combined effects of these various features of social security, which allows us to document who benefits from having an unfunded social security system, we conduct several experiments that consist of comparing the steady-state properties of economies indexed by

TABLE 2
WELFARE OF NEWBORNS

θ	τ	Type 3				Type 1		Average
		HH	HL	LH	LL	H	L	
1.00	0.21	-48.65	-55.27	-56.92	-67.12	-50.34	-63.72	-61.28
0.80	0.17	-48.62	-55.55	-56.43	-66.94	-49.01	-61.85	-60.98
0.60	0.12	-48.63	-55.90	-55.97	-66.87	-47.67	-60.05	-60.75
0.44	0.09	-48.66	-56.13	-55.72	-66.85	-46.89	-59.03	-60.65
0.30	0.06	-48.72	-56.42	-55.45	-66.88	-46.04	-57.92	-60.57
0.20	0.04	-48.76	-56.63	-55.28	-66.89	-45.48	-57.18	-60.51
0.10	0.02	-48.82	-56.84	-55.11	-66.91	-44.94	-56.43	-60.47
0.05	0.01	-48.85	-56.96	-55.02	-66.92	-44.67	-56.05	-60.44
0.00	0.00	-48.88	-57.07	-54.94	-66.93	-44.39	-55.66	-60.42
CV		-0.49%	-1.71%	1.39%	-0.13%	5.62%	6.06%	0.45%
Measure of types		0.147	0.11	0.107	0.53	0.025	0.08	

different social security replacement rates.¹⁴ For each artificial economy, we report the summary statistics and compare the steady-state equilibria on the basis of these quantities as well as the welfare of different types of households.¹⁵

4.1. Social Security and Welfare across Household Types. Households in our economy differ in terms of their demographic composition and labor ability. Because of lifetime uncertainty, households can be classified into three categories according to their demographic composition. A household in which only the children are alive is denoted as type 1. When the father is the only member alive, the household is labeled as type 2. Households where both the father and the children are alive are denoted as type 3. A very small fraction of the population is of type 2 and none of the newborns can be of this type (children live at least one period). At a given point in time, type 1, type 2, and type 3 households constitute 29%, 2%, and 69% of the population. Since individuals can be of high or low labor ability, type 3 households can be subdivided into four categories according to the abilities of the father and his children. We thus denote a type 3 household where both the father and children are of high human capital by HH. The remaining type 3 households are denoted as HL, LL, and LH, where the first letter indicates the ability of the father and the second the ability of the children.

Table 2 provides information on newborn household preferences over social security replacement rates. Each row in this table corresponds to a steady state with a given (average) replacement rate. The first column reports the average

¹⁴Fuster et al. (2002) examine the welfare effects of eliminating social security under different schemes to finance the transitions.

¹⁵In all the computations reported below, the capital income, consumption and labor income tax rates, and government purchases are held constant across steady states. Although the lump-sum transfer that balances the government budget differs across steady states, its magnitude is very small and unlikely to have any significant quantitative impact.

replacement rate which varies between 0% and 100%. The corresponding social security payroll tax rates are reported in the second column. Each of the remaining columns corresponds to a household type. These columns report the steady-state expected lifetime utility of newborn households.¹⁶ For each household type, maximum steady-state welfare is indicated in bold.

Table 2 indicates that most type 3 households would like to be born in an economy with social security (positive replacement rate) and the preferred replacement rate is quite different across household types. For example, HH household prefers an 80% replacement rate, HL household prefers 100%, and LL household prefers to be born into an economy with 44% replacement rate. On the other hand, a type 3 LH household and both type 1 households favor a 0% replacement rate. Notice that even though the majority of households prefer to be born into economies with social security, the average utility measure is not able to capture that feature primarily because the preferences of households who prefer social security are much flatter than those of others who dislike it. Another way to see this result is by examining the consumption compensation that would be required in order to make a household indifferent between two economies. In the row labeled CV, we present the compensating variation in consumption that is necessary to make a household indifferent between eliminating social security and a 44% replacement rate. If the number is negative, it means that the household is worse off when social security is eliminated. For example, the HH household would require a 0.49% increase in consumption forever in order to agree to live in an economy with no social security. However, notice that if we compute the average consumption compensation using the measure of households given in the last row, we would conclude that the average household would be willing to give up 0.45% of consumption forever in order to be born into an economy without social security.

There are several reasons that contribute to the differences in induced preferences over social security. First, households differ in terms of the value they assign to the annuity role of social security due to differential expected life spans. Second, due to the progressivity of the benefits formula, social security may redistribute income to low-ability households. In addition, since some households may be more borrowing constrained than others, the “utility cost” of social security taxes on these households may be stronger.

One measure that can be used to partially understand the results in Table 2 is the rate of return of social security. This return can be computed for a newborn individual or for a household. This return for a newborn depends on the type of the individual, and is computed by taking into account the contributions made and benefits received by that individual during his entire lifespan. We find that

¹⁶ Since there are significant differences in the distribution of assets across newborn households at each steady state, we use the distribution at the 44% social security replacement rate in order to aggregate the utility across asset levels for different steady states. An alternative method is to aggregate the utility across asset levels using the invariant distribution of initial assets that corresponds to each steady state. With this method, differences in preferences toward social security will also reflect preferences due to differences in the invariant distribution of assets across newborn households in each steady state. Our results for this case indicate optimal social security replacement rates of 60%, 80%, 0%, and 100% for households HH, HL, LH, and LL, respectively.

TABLE 3
RATES OF RETURN

Father's Age at Death (d_f)	Ability Type			
	HH	HL	LH	LL
10	<0	<0	<0	<0
11	<0	<0	<0	<0
12	<0	8.4	<0	6.3
13	5.7	10.2	<0	9.1
14	7.4	10.9	4.1	10.0
Expected returns	3.2	8.6	<0	5.7

the expected rate of return for a newborn individual in our benchmark economy is 2.96% for a low human capital type and 2.72% for a high human capital type. Both of these returns are lower than the return to capital in this economy, which is 4.62%. However, to understand the results in Table 2, it is useful to compute the realized return for a household. This return depends on the abilities and the mortality history of the members of the household. For example, type 1 households do not receive benefits because they are constituted by children only.¹⁷ As a result, they face a negative return for all mortality histories. Moreover, the burden of social security taxes is particularly heavy for them because they are likely to be borrowing constrained when young. Thus, it should not be surprising that type 1 households are better off in a steady state with no social security.

Table 3 presents rates of return of social security for a type 3 household conditional on the father's lifespan, d_f , and the household's ability type (zz'). If the return of social security is positive, it is higher the longer the father lives.¹⁸ For a given mortality history, these returns are higher the lower are the taxes paid and the higher is the pension received. This is why returns are higher when the ability of the taxpayer (children) is low and when the ability of the father is high. In particular, Table 3 shows the relation $r_{ss}^{LH}(d_f) < r_{ss}^{HH}(d_f) < r_{ss}^{LL}(d_f) < r_{ss}^{HL}(d_f)$ for all $d_f \geq R$. The ability type LH is the one that pays highest taxes because the children are of high ability, and gets the lowest pension because the father is of low ability. If we compare the returns of social security with the aftertax return on capital, 4.62%, we see that the LH household is the only household that faces a return of social security lower than the aftertax interest rate for all possible mortality histories. This observation explains why the preferred replacement rate is zero for LH households. For the other types of households, the return of

¹⁷ The son will retire as a member of the next household in the dynasty line.

¹⁸ If the father only survives to model age 10 (real time age 65) or 11 (real time age 70), the return on social security is negative, regardless of ability type, because the sum of taxes paid is higher than the sum of pension benefits. The same is true if the father survives to age 12 and the son has high ability and if the father survives to age 13 and the abilities are LH.

TABLE 4
AGGREGATE EFFECTS OF SOCIAL SECURITY

θ	τ	K	Y	K/Y	$r(1 - \tau_k)$	C
1.00	0.210	98.3	97.8	2.49	0.0469	99.6
0.80	0.170	98.5	99.5	2.46	0.0468	99.7
0.60	0.122	99.2	99.7	2.47	0.0465	99.8
0.44	0.094	100.0	100.0	2.48	0.0462	100.0
0.30	0.063	101.2	100.4	2.50	0.0456	100.3
0.20	0.042	102.4	100.7	2.53	0.0451	100.5
0.10	0.021	104.0	101.2	2.55	0.0445	100.8
0.05	0.011	105.0	101.5	2.57	0.0441	101.0
0.00	0.00	106.1	101.8	2.59	0.0437	101.2

social security is higher than the return of capital provided the father lives long enough.¹⁹

The last row in Table 3 reports the expected returns for these different households. Notice that the expected return on social security is lower for an LL household compared to an HL household. This is precisely because the high human capital father is able to collect benefits for a longer amount of time compared to a low human capital father. In addition, the expected return of social security for the HH household, 3.2%, is lower than the after tax return on capital (4.6%). However, in the event that the father survives to age 13, the return increases to 5.7%. The median return for type HH is close to 7.4% since there are 47% of households of type HH that get this return.

4.1.1. Aggregate properties of the economies. In this subsection, we examine some of the statistical properties of the economies discussed above. In particular, we document the impact of social security on the aggregate capital stock and on the redistribution of assets across households.

Table 4 shows the levels of capital stock, output, and consumption relative to the corresponding levels at the 44% replacement rate economy. Note that the introduction of social security gives rise to a small crowding-out effect. For example, an economy with a 0% replacement rate generates 6.1% more capital and 1.8% more output than an economy with a 44% replacement rate.²⁰ In line with the findings of previous research, aggregate steady-state capital stock, output, and consumption monotonically increase as the social security replacement rate is reduced.

¹⁹ We compute the return of social security for a newborn household, that is, a household composed by a father of age $T + 1$ (8) and m children of age 1 (type 3 household). Since father and children face uncertain lifetimes, the return of social security depends on the length of their lifetime in this household. Also, since the father is older than 8 and about to retire, the taxes paid by the father do not play as important a role as the pensions received by the father in calculating the returns from social security. See the Appendix for the computation of these returns.

²⁰ It is possible that the crowding out of the capital stock could be larger if our model did not abstract from endogenous labor choice. However, Fuster (1999) uses a model very similar to ours together with an endogenous labor choice and finds the reduction in capital stock to be 8%.

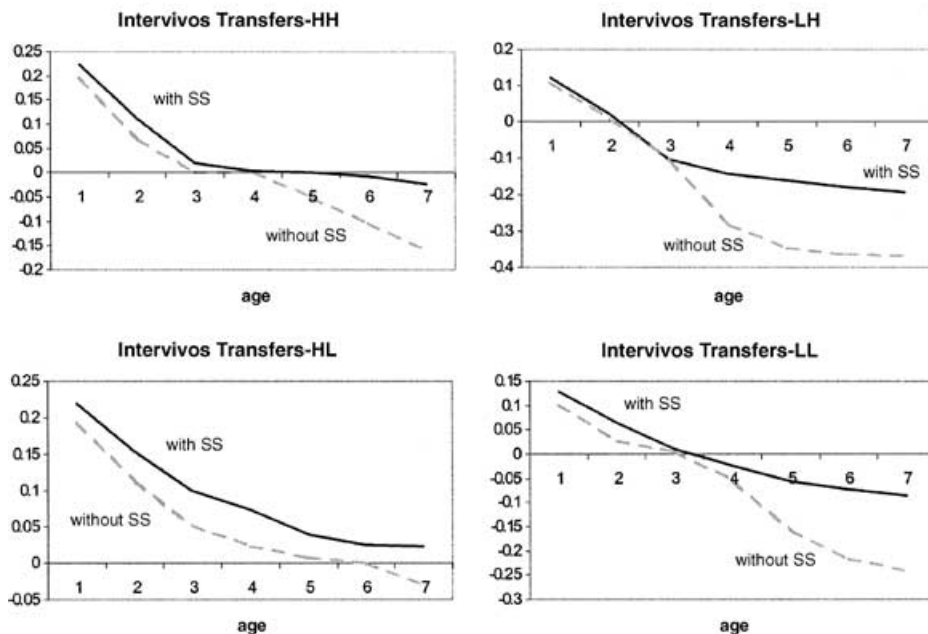


FIGURE 1

INTERVIVOS TRANSFERS

Social security leads to changes in private transfers that (partially) undo social security transfers. Hence, a pay-as-you-go system has a small impact on private household saving. In our economy, private transfers can go from parent to children and from children to parents. Since the purpose of the household is to smooth consumption not only across time and states of nature, but also within the household, in the economy with social security there are more transfers from the father to the son. This takes place even in a household where both members are of high human capital type because by the time the son is born the father will be enjoying a high income level due to being at a later stage of his life-cycle earnings profile.

Figure 1 displays the amount of intervivos transfers for four categories of type 3 households in two steady states, one with a social security replacement rate of 44% and another one without social security. In these frames, the vertical axis measures the average amount of transfers where negative numbers indicate a net transfer from the children to the father and positive numbers indicate a net transfer from the father to the children. The horizontal axis measures the age of the children; when the children are 1 period old (real time age 20) the father will be 8 periods old (real time age 55). The father retires when the children are 3 periods old (real time age 30), and dies when the children are 7 periods old. We observe that there are net transfers from the father to the children when the children are young, and from the children to the father when the father is retired. In general transfers from the father to the children increase with social security. Because of the hump shape of life-cycle earnings profile, the father faces higher wages compared to the

children. As a result, the father increases his transfers to compensate the increase in labor income taxes faced by the children due to the social security program. In addition, the transfers from the children to the father are lower in an economy with social security compared to an economy without social security. These transfers are substantial in LH and LL households, where the father has low ability.

The effects of social security on private transfers lead to a significant increase in the Gini coefficient of the distribution of assets. Indeed, the Gini coefficient in the benchmark economy is 0.65 whereas in an economy without social security it is only 0.51. In an earlier paper, Fuster (1999) also finds that wealth becomes more concentrated with social security due to the increase in transfer wealth.²¹ Since social security distributes income from younger generations to older generations, altruistic old individuals increase their transfers to compensate the tax burden of children. On the other hand, social security substitutes for life-cycle saving. The former effect is relatively more important than the latter effect for rich households whereas the opposite is true for poor households. As a consequence, higher replacement rates induce rich households to accumulate more assets and induce poor households to accumulate fewer assets, which explains the increase in the concentration of wealth.

It is interesting to compare the data generated by our model on private transfers with the available empirical evidence. There is substantial disagreement about the size of private transfers in the data and their relative importance for explaining wealth accumulation in the U.S. economy. The debate about the size of transfer wealth between Kotlikoff and Summers (1981) and Modigliani (1988) illustrates this disagreement. It has become clear from this debate that the size of transfer wealth depends on how it is measured. Without taking a stand in the aforementioned debate, we borrow the methodology used by Kotlikoff and Summers (1981) to measure transfer wealth in our economy and compare our findings with theirs.²² These authors report that transfer wealth in the U.S. economy is about 130% of the capital stock (with a lower bound of 80%). This statistic is about 180% in our benchmark economy. We emphasize that this statistic is particularly sensitive to the interest rate and the growth rate of the economy. In the last section of the article we discuss the sensitivity of our results to key parameters and to the importance of transfer wealth.

Using data from The Survey of Consumer Finances for 1983–85, Gale and Scholz (1994) find that in the United States about 75% of transfers involve parents giving to children. In our model, we find that 72% of inter vivos transfers are from the parents to children (that is, 28% from children to parents), which is consistent with the findings of these authors. Gale and Scholz (1994) report that inter vivos transfers plus bequests constitute at least 51% of net worth. When college expenditures are added, this estimate goes up to 64%. They conclude that, in evaluating the effects of fiscal policies, it is important to understand the degree to which transfers respond to economic incentives. Our findings do indicate that this response is important.

²¹ De Nardi (1999) shows that voluntary bequests are important in generating a concentrated wealth distribution.

²² We describe the methodology to compute the ratio of transfer wealth to capital used in Kotlikoff and Summers (1981) in the Appendix.

TABLE 5
WELFARE OF NEWBORNS

θ	τ	HH	HL	LH	LL	Average
1.00	0.21	-55.73	-65.15	-64.12	-76.99	-70.62
0.80	0.17	-55.30	-64.97	-63.28	-76.37	-70.05
0.60	0.12	-54.84	-64.77	-62.43	-75.80	-69.51
0.44	0.10	-54.57	-64.65	-61.95	-75.48	-69.21
0.30	0.06	-54.26	-64.51	-61.41	-75.12	-68.86
0.20	0.04	-54.07	-64.42	-61.07	-74.92	-68.66
0.10	0.02	-53.87	-64.33	-60.73	-74.69	-68.44
0.05	0.01	-53.78	-64.29	-60.57	-74.59	-68.34
0.00	0.00	-53.68	-64.24	-60.40	-74.48	-68.23
Measure of types		0.16	0.12	0.12	0.60	

4.2. *Sensitivity Analysis.* In this section, we present the sensitivity of our results to some of the modeling choices that were carried out and to the choice of our calibrated parameters. We start by examining the contribution of some of the features of the model to the findings obtained in this environment. In particular, we report the welfare effects of social security in four different cases in order to investigate the impact of lifespan uncertainty, differential mortality, differential fertility, and random ability shocks on the desirability of social security.

4.2.1. *Certain lifetimes and random ability differentials.* In this first experiment, all individuals face certain lifetimes and live for 12 periods, approximately equal to the expected lifespan (at birth) in the uncertain lifetime version of the model, and they face random ability shocks.²³ With certain lifetimes and random ability shocks, the economy consists entirely of type 3 households in which both the father and m children are alive. Table 5 summarizes the steady-state welfare implications of various values of the replacement rate θ . The last column presents the aggregate welfare measures and indicates that a replacement rate of 0% maximizes steady-state welfare for an “average” household. Conditioning our welfare measure on household types produces the same result. Households of all types prefer to live in an economy with a replacement rate of 0%.

Since the only potential benefit of social security in this economy is its insurance role against the ability shock, the results indicate that this role is not important enough to compensate for the reduction in the capital stock caused by the displacement of private saving. In other words, since there is no annuity role of social security in this framework, the welfare loss due to the reduction in the capital stock and due to borrowing constraints outweighs the small benefit of social security.

4.2.2. *Average mortality and average earnings.* We now consider a modified version of the previous economy in which all agents face the same but uncertain

²³ β in this case is chosen so that in the steady state with 44% replacement rate the capital–output ratio is equal to 2.5.

TABLE 6
WELFARE OF NEWBORNS

θ	τ	Type 3	Type 1	Average
1.00	0.22	-65.67	-65.09	-65.61
0.80	0.17	-65.64	-63.42	-65.41
0.60	0.13	-65.62	-61.84	-65.22
0.44	0.10	-65.59	-60.63	-65.07
0.30	0.07	-65.58	-59.60	-64.96
0.20	0.04	-65.56	-58.85	-64.85
0.10	0.02	-65.55	-58.13	-64.77
0.05	0.01	-65.52	-57.73	-64.71
0.00	0.00	-65.54	-57.40	-64.69
Measure of types		0.896	0.104	

lifespan, and the same age-efficiency profile. Hence, this version abstracts from heterogeneity and uncertainty with respect to labor abilities, allowing us to try to isolate the annuity role of social security. The efficiency profile and the vector of conditional survival probabilities are calibrated to the average efficiency units and conditional survival probabilities of college and non-college individuals, respectively, using their fractions as weights.²⁴ This is an economy with very limited heterogeneity: All members of the same cohort and same wealth level are identical. However, since individuals face random survival, some measure of households will die before the children get to child-bearing age, and hence a fraction of dynasties will disappear. We replace these with new dynasties consisting of type 1 households. As a result, we have two types of households; type 1, in which only the children are alive, and, type 3, in which both the father and the children are alive.²⁵

As can be seen from Table 6, the steady-state welfare of an average newborn household is maximized with a 0% replacement rate. Conditioning the welfare on the newborn household types shows that type 3 household prefers to be born in an economy with a small amount of social security, demonstrating that the annuity role of social security alone may not yield much support for a replacement rate similar to the one observed in the United States.

4.2.3. Average mortality with ability shocks. In this economy, all agents face the same but uncertain lifespan and face the random ability shocks. Hence, this version abstracts from differential mortality that is present in the benchmark economy.²⁶ In Table 7, we present the welfare results for type 3 individuals in

²⁴ The other parameters are set to the values reported in Table 1 with the exception of β , which is chosen so that in the steady state with 44% replacement rate, the capital-output ratio is equal to 2.5.

²⁵ Note that this economy is essentially the İmrohoroglu et al. (1995) setup with Laitner (1992) preferences.

²⁶ The elimination of mortality differences induces a decrease in the social security tax that balances the social security budget at each steady state.

TABLE 7
WELFARE OF NEWBORNS

θ	τ	Type 3			
		HH	HL	LH	LL
1.00	0.21	-47.14	-55.96	-55.42	-67.88
0.80	0.17	-47.04	-56.19	-54.94	-67.77
0.60	0.12	-46.96	-56.49	-54.5	-67.78
0.44	0.10	-46.97	-56.70	-54.26	-67.82
0.30	0.06	-46.95	-56.96	-54.0	-67.86
0.20	0.04	-46.96	-57.15	-53.85	-67.89
0.10	0.02	-46.98	-57.34	-53.70	-67.94
0.05	0.01	-46.99	-57.45	-53.64	-67.971
0.00	0.00	-47.0	-57.55	-53.56	-67.973
Measure of types		0.1425	0.108	0.108	0.537

this economy. As can be observed, the qualitative results in this case are similar to the results obtained in the benchmark economy in which most households prefer to be born into economies with social security. We also observe that optimal social security replacement rates vary among households. While the LH household prefers to be born into an economy without social security, the rest of type 3 households prefer rates between 30% and 100%. A quantitative comparison of Tables 7 and 2 illustrates the effect of eliminating mortality differences on the preferences for social security for different household types. In Table 7 the HH (LL) household prefers a lower (higher) replacement rate when we eliminate mortality differences because mortality rates of the H (L) type are lower (higher) than the average. Thus the annuity role of social security is relatively less for the HH household in Table 7 in which average mortality rates are used.

4.2.4. Differential fertility. In the following experiment, we examine the sensitivity of our results to the benchmark assumption of identical fertility across different human capital types. In what follows, the ability type also determines the number of children. We calibrate the number of children of high and low types to match the average number of children $m = (1 + n)^T = 1.52$ and the ratio of the number of children of college graduates to the average number of children, which was equal to 0.81 in 1994.²⁷ The fertility rate of non-college graduates is chosen such that $m = 0.72m_L + 0.28m_H$ where m_L and m_H stand for the average number of children by low and high ability types, respectively. The transition probability matrix of abilities is calibrated to match the intergenerational correlation of earnings, 0.4, and the invariant distribution of ability types $\lambda_H = 0.28$. This matrix is characterized by $\pi_{HH} = 0.536$ and $\pi_{LL} = 0.795$.²⁸

In this economy, there are four ability categories even within the type 1 household since the size of the household depends on the ability of the father. We can

²⁷ U.S. Department of Health and Human Services, Monthly Vital Statistics Report vol. 45, no. 10, April 1997.

²⁸ Transition probabilities do not coincide with the values at the benchmark economy because of differential fertility across ability types.

TABLE 8
WELFARE OF NEWBORNS

θ	τ	Type 3			
		HH	HL	LH	LL
1.00	0.20	− 37.05	− 49.22	−56.06	−77.03
0.80	0.16	−37.24	−49.61	−55.67	−76.78
0.60	0.12	−37.47	−50.08	−55.31	−76.63
0.44	0.09	−37.64	−50.39	−55.10	−76.55
0.30	0.06	−37.85	−50.76	−54.87	−76.43
0.20	0.04	−38.00	−51.01	−54.74	−76.36
0.10	0.02	−38.17	−51.28	−54.58	−76.26
0.05	0.01	−38.25	−51.43	−54.52	−76.22
0.00	0.00	−38.33	−51.57	− 54.45	− 76.17
Measure of types		0.113	0.097	0.14	0.543

observe from Table 8, which reports the welfare numbers for type 3 households, that the optimal replacement rate is 100% for HH and HL households, whereas it is 0 for LH and LL households. Although the numbers for type 1 households are not provided in the table, they all prefer to be born into economies without social security. In this framework, the return to social security is higher for households with few children since they pay lower taxes.

Although the results of this experiment suggest that only a minority of households prefer to be born into an economy with social security, perhaps a better treatment of this issue would involve endogenous fertility choice in which households are allowed to change their choice for the number of children depending on the availability of social security.

The results in this subsection demonstrate that, in addition to altruism, incorporating mortality risk and random ability shocks are very important in order to describe the differences in preferences over to the desirability of social security. Abstracting from either one of these features or simply examining the average utility in an economy with heterogeneous agents may give the false impression that the majority of the population prefers to be born into economies without social security. On the other hand, differences in mortality rates are not necessary for the qualitative results obtained in the benchmark economy.

4.2.5. Calibration issues. In addition to understanding the impact of certain features of the model on our results, we are also interested in examining the robustness of our findings to the parameter values used. For example, our benchmark economy generates a relatively high ratio of transfer wealth to capital stock. Since transfer wealth could play an important role in issues regarding social security, we would also like to investigate the sensitivity of our results to the transfer wealth implied by our calibration. It is important to note that the size of the transfer wealth is particularly sensitive to the interest rate and the productivity growth rate of the economy.

The first row of Table 9 summarizes the main findings of the benchmark economy. In the remaining rows we report the outcome of several alternative parameter

TABLE 9
SENSITIVITY ANALYSIS

β	γ	δ	α	σ	$corr$	τ_k	r^*	K/Y	$TrWe/K$	HH	HL	LH	LL
0.988	0.0165	0.044	0.31	2	0.4	0.4	0.045	2.5	1.79	80	100	0	44
0.981	0.0165	0.044	0.31	2	0.4	0.3	0.053	2.46	2.47	30	100	0	10
0.988	0.0165	0.044	0.31	2	0.7	0.4	0.0465	2.47	1.84	44	100	0	60
0.988	0.0165	0.044	0.31	2	0.2	0.4	0.046	2.48	1.79	80	100	0	10
0.999	0.0165	0.06	0.36	2	0.4	0.4	0.035	3.0	1.15	100	100	0	100
1.008	0.021	0.06	0.36	2	0.4	0.4	0.035	3.0	0.95	100	100	100	100
1.003	0.0165	0.044	0.31	3	0.4	0.4	0.046	2.48	1.79	80	100	0	5
0.965	0.0165	0.044	0.31	0.5	0.4	0.4	0.046	2.47	1.78	80	100	0	100

NOTE: r^* is the aftertax interest rate $r(1 - \tau_k)$.

configurations. For all the cases, the first seven columns of the table display the parameter choices whereas the rest of the columns report the equilibrium quantities that are determined by the model. The last four columns report (100 times) the optimal social security replacement rates for the four types of households.

The second row examines the impact of a lower capital income tax rate on optimal replacement rates. Notice that we change β slightly in order to keep the capital output ratio close to 2.5, but obtain a higher after tax return on capital as a consequence of a lower tax rate. Using this parameterization, we conduct the experiment that was carried out in Table 2, where we gradually change the social security replacement rate. Note that the transfer wealth to capital stock ratio is higher than that in the benchmark economy because the aftertax interest rate is relatively higher. In general, households in this environment prefer a lower replacement rate than those in the benchmark calibration because the higher aftertax interest rate lowers the attractiveness of social security. However, the newborn HL type still prefers the maximum replacement rate.

In the third and the fourth rows, we examine the sensitivity of our results to the correlation coefficient of abilities between the father and the children. In the benchmark computations, we used an estimate of 0.4 based on the findings of Zimmerman (1992) and Solon (1992). Mulligan (1997), on the other hand, estimates the correlation between the income of the parent and the son as 0.7. His income definition includes the income from capital, which may not be the appropriate measure for us to use. Nevertheless, we examine the sensitivity of our results to this parameter by choosing Mulligan’s estimate of 0.7 as well as a lower estimate of 0.2.²⁹ Results in Table 9 indicate qualitatively similar findings for these cases.

In the fifth and sixth rows we change the parameters β , γ , and δ in order to generate economies with different ratios of transfer wealth to capital stock. The comparison of these two rows with the benchmark case is intended to shed some light on the sensitivity of our results to the size of the transfer wealth in these economies. Both of these experiments generate higher optimal social security replacement rates compared to the benchmark calibration, even though they imply

²⁹ Notice that in these experiments we are changing the measures of types HH, HL, LH, LL, while holding the measure of newborns constant at $\mu(H) = 0.28$ and $\mu(L) = 0.72$.

arguably more realistic amounts of transfer wealth. In these experiments, consumption profiles are steeper and children receive fewer transfers, which have negative effects on the ratio of transfer wealth to capital stock. In the case in which the productivity growth rate is set to 2.1%, we observe that even the LH type of households prefers to be born into an economy with social security. The higher growth rate used in this calibration, and the resulting lower return to capital, make social security a better saving device by increasing the return of social security and decreasing the return to capital.³⁰

In the last two rows we examine the sensitivity of our results to the elasticity of intertemporal substitution. In both cases we find that most of the households continue to prefer economies with positive social security replacement rates at the steady state.

Overall, our model with two-sided altruism generates positive optimal replacement rates for most households when steady-state comparisons are carried out. This result is robust to the changes in the calibration examined above.

5. CONCLUSIONS

In this article, we examine the welfare effects of social security in an environment with two-sided altruism in which agents face differential lifespan uncertainty. The general equilibrium model we construct is populated with overlapping generations of finite but random-lived individuals facing borrowing constraints and individual ability shocks. The realization of the generationally persistent ability shock determines the age–efficiency profile and the type-dependent vector of conditional survival probabilities of a newborn individual. Contrary to previous research, our results indicate that steady-state welfare increases with social security for *most* household types, although by very different amounts. In this environment with bequests, the crowding out of capital due to a U.S.-type social security system is 6%, which is significantly smaller than that obtained in pure life-cycle models. When individuals are altruistic, the impact of social security on the saving rate is negligible since the decrease in saving motivated by life-cycle reasons is compensated by an increase in saving motivated by intergenerational transfers.

Our results indicate that households differ in terms of the value they assign to the insurance role of social security due to differential age–earnings profiles and mortality risk. There are differences in the effective rate of return on social security contributions for different households. We show that the return to social security increases with the age and decreases with the ability of the taxpayer. So the household type where the children pay the highest taxes and the father has the shortest lifetime (LH) is one that likes social security the least. Households that like social security in this framework are those for which family insurance plays an important role in undoing the borrowing constraints and government policy.

³⁰ In order to understand this result we can again examine the rate of return to social security for a given life expectancy. For example, conditional on the father having survived to age 14, the return of social security for the LH household is 4.44%, which is much higher than the return to capital (3.5%) in this case. In this economy, 40% of households have a father who reaches model age 14.

These are mainly households where the father is alive and transfers income to his children. Our results also indicate that, in addition to altruism, incorporating mortality risk and ability shocks is very important in order to obtain differences in induced preferences for social security. Abstracting from either one of these features or simply examining the average utility in an economy with heterogenous agents may give the false impression that the majority of the population prefers to be born into economies without social security.

APPENDIX

A.1. Utility Function. The expected utility of a dynasty as the household begins with a father of ability z_0 and his children of ability z_1 is

$$\sum_{j=0}^{\infty} E_{z_{j+1}/z_j} \left[\prod_{i=1}^T \psi_i(z_j) \right] \hat{\beta}^T m \sum_{t=1}^T \hat{\beta}^{t-1} \times \left\{ \left[\prod_{i=1}^t \psi_{i+T}(z_j) \right] \frac{c_f(t, j)^{1-\sigma}}{1-\sigma} + m \left[\prod_{i=1}^t \psi_i(z_{j+1}) \right] \frac{c_s(t, j+1)^{1-\sigma}}{1-\sigma} \right\}$$

where $\hat{\beta} \equiv \beta(1 + \gamma)^{1-\sigma}$, j indicates the father's generation, and $j + 1$ indicates the children's generation in the dynasty where $j = \{0, 1, 2, \dots\}$. $c_f(t, j)$ is the consumption of the father, and $c_s(t, j + 1)$ is the consumption of the son in an age- t household. The expected utility of a new dynasty as the household begins with m age-1 individuals of generation 0 and with ability z_0 is

$$\sum_{t=1}^T \hat{\beta}^{t-1} \left\{ \left[\prod_{i=1}^t \psi_i(z_0) \right] m \frac{c_s(t, 0)^{1-\sigma}}{1-\sigma} \right\} + \sum_{j=0}^{\infty} E_{z_{j+1}/z_j} \left[\prod_{i=1}^T \psi_i(z_j) \right] \hat{\beta}^T m \sum_{t=1}^T \hat{\beta}^{t-1} \times \left\{ \left[\prod_{i=1}^t \psi_{i+T}(z_j) \right] \frac{c_f(t, j)^{1-\sigma}}{1-\sigma} + m \left[\prod_{i=1}^t \psi_i(z_{j+1}) \right] \frac{c_s(t, j+1)^{1-\sigma}}{1-\sigma} \right\}$$

A.2. Computation of Life-Cycle Wealth and Transfer Wealth.

A.2.1. First stage: Computing inter vivos transfers and bequests. The assumptions on preferences imply that the father and the son have the same objective function. For this reason, we have solved a joint maximization problem in which the father and the children pool their resources and choose the optimal paths of consumption and asset holdings. We can use the optimal paths for consumption and assets, and individuals' budget constraints to construct the path of inter vivos transfers and individuals' assets. In order to do this, we have to assume a distribution of the household's initial assets across father and son. We assume that a newborn individual owns zero wealth and, thus, the initial assets of a household are owned by the father.

Let us denote the assets of a father by $a_f(j)$ where j is the age of the household, and let $a_s(j)$ be the assets of a son. The assets of the father and the son are

determined by the budget constraints

$$\begin{aligned}(1 + \gamma)a_f(j) &= (1 + r)a_f(j - 1) + e_f(j) - m[nt(j)] - c_j \\ (1 + \gamma)a_s(j) &= (1 + r)a_s(j - 1) + e_s(j) + nt(j) - c_j \\ m[a_s(j)] + a_f(j) &= a(j) \quad \text{for all } j, \text{ and } a_s(0) = 0\end{aligned}$$

where c_j is the optimal consumption, $e_i(j)$, $i \in \{s, f\}$, denotes aftertax earnings plus government transfers, $nt(j)$ denotes net intervivos transfer from the father to each of his m children. A negative net intervivos transfer is a gift from the son to the father.

Net transfers are computed as follows. When an individual's consumption is larger than his aftertax income plus lump-sum transfers, we assume that he has received a transfer from the other member of the household and we set his assets for next period at zero. For example, if

$$(1 + r)a_s(j - 1) + e_s(j) - c_j < 0$$

then the son needs a transfer in order to finance his consumption. Therefore we assume that

$$nt(j) = c_j - (1 + r)a_s(j - 1) - e_s(j)$$

Since we also assume that

$$a_s(j) = 0$$

all assets of the household in the next period belong to the father ($a_f(j) = a(j)$). If neither the father nor the son needs a transfer to finance their consumption, we set $nt(j) = 0$ and use the budget constraints of the father and the son to distribute the household assets across its members.

We define planned bequest as the next period assets of a father when he is $2T$ periods old ($a_f(T + 1) = \text{bequest}$).

A.2.2. Second stage: Computing life-cycle wealth and transfer wealth. We follow Kotlikoff and Summers (1981) to measure life-cycle wealth and transfer wealth. For simplicity, consider the case of certain lifetimes. The budget constraint of an age-1 individual is

$$(1 + \gamma)a_s(1) = e_s(1) - c_1 + nt(1)$$

This shows that an individual's assets can be divided into a life-cycle component (government's lump-sum transfer plus aftertax earnings minus consumption) and a transfer component (net intervivos transfers). If we substitute recursively the budget constraint into itself, we obtain that the assets of a son of age j are

$$(1 + \gamma)a_s(j) = \sum_{i=1}^j \left[\frac{1+r}{1+\gamma} \right]^{j-i} (e_s(i) - c_i) + \sum_{i=1}^j \left[\frac{1+r}{1+\gamma} \right]^{j-i} nt(i)$$

In this way we divide individuals' assets into a life-cycle component (first term) and a transfer component (second term). In order to compute aggregate life-cycle wealth and aggregate transfer wealth we sum over all individuals in the economy.

We use the same methodology to compute life-cycle and transfer wealth in the case of uncertain lifetimes. The only difference with the previous case is that there are accidental bequests when a member of the household dies early and there is an additional lump-sum transfer from the government that redistributes the asset holdings and capital income of individuals that die without descendents, ξ_1 . We assume that accidental bequests are transfer wealth, whereas the lump-sum transfer from the government, ξ_1 , is life-cycle wealth as we assumed for ξ_2 .

A.3. Computation of the Rate of Return of Social Security. We compute the return of social security for a newborn household, that is, a household composed by a father of age $T + 1$ (8) and m children of age 1 (type 3 household). Since the father and the children face uncertain lifetimes, the rate of return of social security depends on the length of their lifetime. The longer the father lives, the higher is the present value of the pension benefits received by the household. The shorter the children live, the lower is the present value of the taxes paid by the household.

The return of social security is defined as the rate of return that equates the present value of tax payments to the present value of pension benefits (at the initial period of the household). The present value of the taxes paid by the children is given by

$$\sum_{t=1}^{\min\{d_s, T\}} \frac{\tau \omega (1 + \gamma)^{t-1} m \varepsilon_t(z')}{(1 + r_{ss})^{t-1}}$$

where d_s denotes the age of death of the children, ω is the wage per effective labor at the initial period of the household, and r_{ss} is the rate of return of social security. The sum of tax payments has at most T terms because we only count taxes paid by the current household. The present value of the taxes paid by the father, assuming that he survives to the retirement age, is given by

$$\sum_{t=T+1}^{R-1} \frac{\tau \omega (1 + \gamma)^{t-(T+1)} \varepsilon_t(z)}{(1 + r_{ss})^{t-(T+1)}}$$

The present value of the benefits received by the household is given by

$$\sum_{t=R}^{d_f} \frac{(1 + \gamma)^{R-(T+1)} B_R(z)}{(1 + r_{ss})^{t-(T+1)}}$$

where d_f is the age of death of the father. Thus, the rate of return of social security is defined as the value of r_{ss} for which the following equation holds:

$$\begin{aligned} & \sum_{t=1}^{\min\{d_s, T\}} \frac{\tau \omega (1 + \gamma)^{t-1} m \varepsilon_t(z')}{(1 + r_{ss})^{t-1}} + \sum_{t=T+1}^{R-1} \frac{\tau \omega (1 + \gamma)^{t-(T+1)} \varepsilon_t(z)}{(1 + r_{ss})^{t-(T+1)}} \\ &= \sum_{t=R}^{d_f} \frac{(1 + \gamma)^{R-(T+1)} B_R(z)}{(1 + r_{ss})^{t-(T+1)}} \end{aligned}$$

In Table 3 we assume $d_s > T$ and $d_f \geq R$. Notice that these mortality histories are the most likely ones.

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