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Social security is NOT a substitute for annuity markets ☆

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ABSTRACT

Common wisdom suggests that a fully-funded actuarially fair social security system should increase welfare when households face longevity risk and annuity markets are missing. This wisdom is based on the observation that social security pays benefits as life annuities and therefore appears to complete the market. However, we argue that common wisdom is based on a benefit-only analysis that ignores a fundamental cost—social security crowds out the bequests that households leave (and receive) in general equilibrium. We conduct a general equilibrium cost-benefit analysis of the longevity insurance role of social security, and we show that under certain conditions this decline in bequest income offsets any possible gains from access to a public annuity pool. We abstract from distortions to national income and factor prices to show that the equilibrium bequest channel is all that is needed to reach this conclusion.

1. Introduction

A normative rationale for the existence of social security is the provision of lifetime annuity insurance.[1] The private annuity market in the US is very thin, which means that few households are sharing their longevity risk in the market.[2] Given this fact, a common view is that a fully-funded actuarially fair social security system should increase welfare since social security pays benefits as life annuities that provide a higher implicit return than can be found in the market.

In this paper we claim that common wisdom about the insurance role of social security is not correct. We argue that common wisdom is based on a *benefit-only* analysis that ignores a fundamental cost—social security crowds out the bequests that households leave (and receive) in general equilibrium. We show that under certain conditions this crowding out of bequest income is in fact large enough to cancel out any welfare gains from participating in a public annuity pool with an above-market return. We abstract from distortions to national income and factor prices to show that the equilibrium bequest channel is all that is needed to reach this conclusion. This is because social security payments, unlike annuities purchased

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[1] See for example Diamond (1977, 2004) and Feldstein (2005).
[2] See Pashchenko (2013) for a survey and quantitative analysis of the leading explanations for why the annuity market is so thin.

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with one's savings on the private market, are independent of saving decisions and therefore do not affect intertemporal trade-offs (Euler equation). Thus social security only affects the lifetime income of the household. But if bequest income is carefully accounted for in general equilibrium, then any increase in lifetime income from participating in a mandatory annuity pool must be offset by a reduction in bequest income or else the aggregate resource constraint is violated.

Therefore, when we say that "social security is not a substitute for annuity markets," we mean that social security falls short of what a private (competitive) annuity market could accomplish. Unlike a private annuity market which would allow households to insure their longevity risk and reach the first-best consumption allocation, social security fails to provide any welfare gains at all even when households have no other way to insure their longevity risk. This is true even when we assume social security is actuarially fair and there are no inefficiencies in financing the system (fully funded).

We first use a stylized two-period OLG model in Section 2 to make our intuition clear. Rational households face survival uncertainty but lack access to annuity markets. They can only save in a zero-interest storage technology. The rate of population growth is also zero. Households may die with assets (accidental bequests), which are distributed anonymously to survivors in the population without regard to specific linkages between parents and children (we relax this assumption later). This assumption is motivated by a vast literature in quantitative macroeconomics that uses life-cycle OLG models (see for example Hubbard and Judd, 1987; İmrohoroğlu et al., 1995; Conesa and Krueger, 1999; Huggett and Ventura, 1999; Hong and Ríos-Rull, 2007, and Nishiyama and Smetters, 2007 among many others). To focus on the longevity insurance aspect of social security and to abstract from its effect on work incentives and retirement, we assume inelastic labor supply and exogenous retirement. In this setting we prove analytically that, as long as borrowing constraints do not bind, a fully-funded actuarially fair social security system improves welfare when bequest income is fixed but has no effect at all on consumption or welfare when bequest income is endogenous.

Next, in Section 3 we extend the two-period model to a continuous-time OLG model, but we keep the assumption that the rate of population growth equals the interest rate on savings (though they need not be zero). The continuous-time setting allows us to consider all possible equilibrium age-distributions of bequest income. We again prove analytically that, as long as borrowing constraints do not bind, social security has no effect on consumption or welfare when bequest income is endogenous.

Then in Section 4 we conduct numerical experiments to show that our result continues to hold and actually grows stronger as we expand the analysis to allow the interest rate to exceed the rate of population growth. We make a direct comparison to Hubbard and Judd (1987). By holding bequest income fixed, we replicate the results found in Hubbard and Judd (1987): a fully-funded social security system can generate large welfare gains when liquidity constraints are absent but these gains are reduced or even reversed when liquidity constraints are present. However, by allowing bequest income to adjust in general equilibrium, we confirm our analytical findings and we uncover a set of new findings not found in Hubbard and Judd (1987): now a fully-funded social security system reduces welfare for all parameterizations in which the interest rate exceeds the population growth rate and for any assumption about liquidity constraints. Reductions in equilibrium bequest income more than crowd out any gains from mandatory annuitization.

Finally, in a detailed appendix we relax the anonymous-bequest assumption and adopt an alternative that is less popular (mainly because of its analytical and computational complexity) but perhaps more realistic. We assume bequests are distributed through direct linkages between deceased parents and surviving children (as in Eckstein et al., 1985a and Abel, 1985). In this alternative case, there is heterogeneity in wealth across households due to heterogeneity in the timing of the deaths of their entire ancestry. We show that while such an assumption can lead to a welfare role for social security, the welfare gains are due to reducing the variance of the distribution of wealth. In other words, social security improves welfare because it redistributes wealth from the rich to the poor or because it provides insurance against bequest income risk. Whether the gains come through the former or the latter channel depends on the assumptions we make about the timing of information. But in either case, the gains are not coming from longevity insurance, unless we broaden longevity insurance to include insurance against which ancestral line individuals will join at birth.

1.1. Related literature

The usefulness of social security as a provider of longevity insurance has been quantitatively explored by Hubbard and Judd (1987), İmrohoroğlu et al. (1995), Conesa and Krueger (1999), Hong and Ríos-Rull (2007) among many others. These papers study the welfare implications of various social security arrangements in dynamic OLG models with multiple layers of uncertainty, market incompleteness and (possibly) multiple social insurance programs. We complement their quantitative analysis by proving analytically that under certain conditions social security, *in isolation*, fails to improve welfare when annuity markets are missing. To our knowledge, this fundamental result is new to the literature.

It has long been known (at least since Eckstein et al., 1985a and Abel, 1988) that social security does not improve welfare when households already have access to perfect annuity markets. But our contribution is to show that social security does not improve welfare even when annuity markets are totally missing. In making these statements we are, of course, focusing just on the longevity insurance role of social security while abstracting from other roles such as other forms of risk sharing, wealth redistribution, and solving the government time-inconsistency problem.

Abel (1986) and Eckstein et al. (1985b) point out that there is a welfare enhancing role for social security in the presence of adverse selection in the annuity market. Social security forces households with different mortality types to pool their survival risk and hence provides a better implicit rate of return than annuity contracts traded in the market. Our assumption that private annuity markets are completely missing biases our model in favor of social security.[3] Yet, social security fails to deliver any welfare gains due to the general equilibrium effect of bequest income.

At first glance our paper may seem like a type of Ricardian equivalence (Barro, 1974). While it is true that our paper is related to Barro's, it is certainly not an extension. For example, Barro relies on altruism and complete markets. Our paper is just the opposite: we don't need altruism or complete markets to show the irrelevance of a government program (bonds in his case and social security in ours). Indeed, conventional wisdom suggests that fully-funded social security would be welfare improving in our environment with missing annuity markets, and yet we show that social security is irrelevant.

This paper is also related to the large literature measuring the insurance value of annuitization for representative life-cycle consumers (e.g., Kotlikoff and Spivak, 1981; Mitchell et al., 1999; Brown, 2001; Davidoff et al., 2005; Lockwood, 2012). The exercise in these papers is to determine how much incremental, nonannuitized wealth would be equivalent to providing access to actuarially fair annuity markets.[4] A robust finding of this exercise is that a 65-year-old adult, with population average mortality, gains up to 30 to 50 percent of his/her retirement wealth from having access to actuarially fair insurance. A key feature of all these studies is the static comparison between full insurance and no insurance at all. A large part of this welfare gain comes from the fact that when people die their assets evaporate from the economy. On the other hand, under full insurance these assets are annuitized and hence the assets of the deceased are transferred to those who survive (the assets do not leave the economy). The conventional interpretation of this vast literature is that any public provision of annuities (e.g., social security) can have very large welfare benefits.[5] Our contribution to this literature is to point out that when the assets of the deceased are carefully accounted for these results go away. This has deep and important implications for what we think about the value of annuitization through social security.

2. A two-period model

Our goal in this section is to provide intuition for our results by using a simple two-period model. In the next section we will generalize our results to a multi-period (continuous time) setting. We use the cleanest possible environment to capture the equilibrium response of bequest income to social security. This prevents us from confounding all other roles of social security with its longevity insurance role. For simplicity we assume zero interest and no population growth, but we verify later in the paper that our results hold in a more general environment with positive interest and population growth. Finally, we do not allow the household to borrow in this simple two-period model, because borrowing would involve using social security benefits as collateral—borrowing in the first period means arriving at retirement with debts that can be repaid only with social security benefits. Therefore, in this section we will either assume the household is at an interior solution (doesn't want to borrow anyway) or is borrowing constrained, and we will be clear to signal to the reader which case we are studying.

We first demonstrate how social security can be welfare improving in a static (non-OLG) setting in which bequests are exogenous (Section 2.1). Then in Section 2.2 we demonstrate how the conclusions of the static analysis can change in an OLG environment in which bequests are endogenously determined in equilibrium.

2.1. Static model with exogenous bequests

Consider a household that lives for a maximum of two periods. We denote the probability of survival to the second period by S. Households have two sources of income in the first period. They inelastically supply one unit of labor and receive wage w. They also receive bequest income B. In the first period of life the household can hold an asset k that pays return q. We assume there is a zero-interest storage technology. Therefore, if there is a perfect annuity market the equilibrium return is $q = \frac{1}{S}$. However, if there is no annuity market the equilibrium return is $q = 1$. When annuity markets exist, the return on assets is higher since it pays only to those who are alive. On the other hand, when annuity markets do not exist, households effectively face a higher price for future consumption.

There is a fully-funded actuarially fair social security system that collects on wage income in the first period at rate τ and pays benefits b to those who are alive in the second period. A balanced budget implies that $Sb = \tau w$.

Let the household flow utility function satisfy $u' > 0$, $u'' < 0$, and the usual Inada conditions. The household problem is to choose consumption c to

$$\max u(c_1) + Su(c_2), \tag{1}$$

subject to

[3] As Hosseini (2010) shows, even in the presence of adverse selection, mandatory annuitization does come with a cost. Social security causes those with the shortest longevity to withdraw from the private annuity market, which pushes annuity prices up and reduces the welfare gains from social security. He shows that this negative welfare effect is quantitatively large.

[4] Lockwood (2012) is an exception in that he considers the comparison between no annuities and annuities that are available at actuarially unfair market rates.

[5] Once again Lockwood (2012) is an exception.

$$c_1 + k = w(1-\tau) + B, \tag{2}$$

$$c_2 = qk + b. \tag{3}$$

Note that, at an interior solution, the following Euler equation must hold

$$u'(c_1) = qSu'(c_2). \tag{4}$$

If perfect annuity markets exist, $q = \frac{1}{S}$ and the Euler equation becomes

$$u'(c_1) = u'(c_2), \tag{5}$$

and if annuity markets do not exist, then $q = 1$ and

$$u'(c_1) = Su'(c_2). \tag{6}$$

In the case when there is a perfect annuity market, household wealth is fully annuitized (no bequest income) and therefore the household lifetime budget constraint becomes

$$c_1 + Sc_2 = w(1-\tau) + Sb = w. \tag{7}$$

From the intertemporal budget (7) and Euler condition (5), the consumption allocation is

$$c_1 = c_2 = \frac{w}{1+S} \equiv c_{FB}, \tag{8}$$

where c_{FB} is the first-best allocation of the social planner's problem. From (7) and (8), it is obvious that social security does not affect consumption and welfare. This result is well known.

In the more relevant case when annuity markets are missing, we show that the effect of social security depends critically on whether bequest income is exogenous or endogenous in the model. In both cases, the Euler equation (6) holds and is not affected by social security (for interior solutions). But the lifetime budget will be different in these two cases. When bequest income is exogenous (and again focusing on an interior solution to make our point as simply as possible), the lifetime budget of the household is

$$c_1 + c_2 = w(1-\tau) + b + B = w - Sb + b + B = w + (1-S)b + B. \tag{9}$$

Taking the derivative with respect to the social security benefit,

$$\frac{\partial(c_1 + c_2)}{\partial b} = 1 - S > 0, \tag{10}$$

since we assume that bequest income B is exogenous and thus $\frac{\partial B}{\partial b} = 0$.

Note that in this static model with exogenous bequests, $(1-S)k$ units of consumption is wasted and not consumed by anyone. By taking resources from households and carrying them to the next period in the public pool of annuities, social security in effect increases household lifetime income by keeping part of the otherwise wasted assets of the deceased in the economy and paying them to survivors. Hence, social security enhances the lifetime budget and improves welfare under the assumption of exogenous bequest income.

However, in a dynamic OLG environment, bequest income is determined endogenously by those who die at the end of the first period. If a fully-funded actuarially fair social security system leads to crowding out of asset holdings, it will lead to a lower bequest level. Thus, it is possible that the negative income effect of lower bequests crowds out the positive income effect of social security. In the rest of this paper we show that under certain conditions, this crowding out is in fact large enough to cancel out any welfare gains from participating in a public annuity pool.

2.2. Dynamic model with endogenous bequests

Here we extend the static model to a dynamic OLG model. In the first period of life, the household collects wage income w and bequest income B from the deceased. Social security taxes are levied on wages at rate τ in return for social security benefits b which are paid to those who survive to the second period of life. The probability of surviving to the second period is S. Households lack access to annuities and must instead save in a zero-interest technology. They cannot hold negative assets for reasons discussed above. There is no population growth.

The household takes bequest income B and government policy as given and chooses consumption in period one c_1 and claims over consumption in period two k (and hence consumption in the second period c_2)

$$\max u(c_1) + Su(c_2), \tag{11}$$

subject to

$$c_1 + k = w(1-\tau) + B, \tag{12}$$

$$c_2 = k + b. \tag{13}$$

There is a fully-funded actuarially fair social security system. At the aggregate level, social security is self financed

$$Sb = \tau w, \tag{14}$$

and the bequests of the young must equal the assets of those who die early after just one period

$$(1-S)k = B. \tag{15}$$

A stationary equilibrium in this economy is characterized by household allocations (c_1^*, c_2^*, k^*) and aggregate bequest B^* such that given government policy and bequest B^*, allocation (c_1^*, c_2^*, k^*) solves the household problem (11)–(13) and Eqs. (14) and (15) hold.

Notice that if there is no social security program, the individual will save to prevent starvation when old. Hence, for low taxes and benefits, the individual will save in equilibrium (interior solution), but for high taxes and benefits the borrowing constraint will bind. In what follows we provide two sets of results, one for the case of an interior solution and one for the corner solution.

In our first proposition we focus on the interior solution. In this case the social security system leaves households with positive equilibrium savings. We show that a fully-funded actuarially fair social security system crowds out bequest income one for one and hence consumption and welfare in the steady state are unchanged.

Proposition 1. *For an interior solution (positive private savings), a fully-funded actuarially fair social security system does not affect consumption allocations and welfare even though private annuity markets are missing.*

Proof. If $k > 0$, the following Euler equation describes household intertemporal trade-offs

$$u'(w - \tau w + B - k) = Su'(k + b). \tag{16}$$

After imposing the social security budget constraint (14) and feasibility (15) we get the following equation

$$u'(w - Sb - Sk) = Su'(k + b). \tag{17}$$

The solution to this equation is the equilibrium allocation for a given level of social security benefit. We call this solution $k(b)$.

Consider the case when there is no social security

$$u'\big(w - Sk(0)\big) = Su'\big(k(0)\big). \tag{18}$$

We claim that $k(b) = k(0) - b$. To verify, replace this into (17)

$$u'\big(w - Sb - S\big(k(0) - b\big)\big) = Su'\big(k(0) - b + b\big), \tag{19}$$

and simplify

$$u'\big(w - Sk(0)\big) = Su'\big(k(0)\big), \tag{20}$$

which holds by definition. This implies that equilibrium consumption must be

$$c_2(b) = k(b) + b = k(0) = c_2(0), \tag{21}$$

and therefore

$$c_1(b) = c_1(0). \qquad \square \tag{22}$$

While social security is traditionally believed to improve welfare because it shares some of the properties of annuities (both make survival-contingent payments at the expense of the deceased), it cannot complete the market because payments are lump sum and do not affect intertemporal trade-offs. Instead, social security will only have a crowding out effect on saving, which in turn reduces the amount of bequest income households leave (and receive) in general equilibrium, leaving consumption allocations unchanged.

The result presented above depends on the assumption that households are not liquidity constrained. We show in our second proposition that if social security taxes and benefits are large enough that households save nothing, then social security can implement the first best (in a two-period model).

Proposition 2. *For a corner solution (zero private savings), there is a social security tax and benefit that can implement the first best in a two-period model.*

Proof. Let τ^* and b^* be such that

$$u'(w - \tau^* w) = u'(b^*), \tag{23}$$

and replacing from the social security budget

$$u'(w - \tau^* w) = u'\left(\frac{\tau^* w}{S}\right). \tag{24}$$

Note that the Inada conditions imply that there is a $0 < \tau^* < 1$ that satisfies this equation. Let $c_1 = w - \tau^* w$ and $c_2 = \frac{\tau^* w}{S}$. Note that by construction c_1 and c_2 satisfy the household budget constraint and

$$u'(c_1) = u'(c_2) > Su'(c_2), \tag{25}$$

which means under this allocation the household does not wish to save. Hence, c_1 and c_2 are the equilibrium consumption allocation and the household chooses to purchase zero claims on the second period consumption good ($k = 0$). Note that c_1 and c_2 are the first-best allocation in this economy. □

Hence, in a two-period model the only way to improve welfare is to choose taxes and benefits that are so high that they shut down all private savings. This teaches us that for social security to be a successful substitute for a missing annuity market, it may need to force at least some households to a corner for at least some part of the working period. However, in a model with many periods and realistic levels of social security taxes and benefits, we will show later that social security tends to reduce welfare with or without borrowing constraints.

Before moving on to prove our results in a more general multi-period setting, we briefly summarize what we have learned so far from the two-period model. If we account for the equilibrium determination of bequest income, then the effect of social security depends critically on whether it leaves households at an interior solution or whether it forces them to a corner:

- If social security taxes and benefits are low enough that households still want to do some private saving, then households will be indifferent between a fully-funded actuarially fair social security system and no system at all. This is because the former will ultimately lead to enough crowding out of equilibrium bequest income that all the gains from the higher implicit return from social security are canceled out. This is true even though private annuity markets are totally missing and households have no other way to insure their mortality risk.
- On the other hand, if the government is willing to consider social security taxes and benefits that are large enough that social security crowds out private saving and forces households to a corner solution, then social security can indeed provide meaningful longevity insurance and in fact it can implement the first-best consumption allocation (at least in the two-period model).

In very simple words, social security has no effect if households can undo it, and social security can increase and even implement the first best if households cannot undo it (even if they would like to).

3. A continuous-time OLG model

In this section we extend the two-period OLG model to continuous time. Age is continuous and indexed by t. Households start work at $t = 0$, retire exogenously at $t = T$, and pass away and exit the model no later than $t = \bar{T}$. At each moment an infinitely divisible cohort is born. The probability of surviving to age t, from the perspective of age 0, is $S(t)$. Thus, $S(0) = 1$ and $S(\bar{T}) = 0$. Let $N(0)$ be the fraction of newborns in the economy. Then, with population growth occurring at rate n, the share of individuals with age t is

$$N(t) = N(0)S(t)e^{-nt} \tag{26}$$

and

$$\int_0^{\bar{T}} N(t)\,dt = 1. \tag{27}$$

Note also that the fraction of individuals who die at age t is

$$M(t) = -N(t)\dot{S}(t)/S(t) = -N(0)\dot{S}(t)e^{-nt}. \tag{28}$$

As above, households lack access to private annuities. They invest in a saving technology with return r. We denote household assets by $k(t)$. Households that survive to age t receive bequest income $B(t)$. We continue to assume the flow utility function $u(c)$ satisfies $u' > 0$, $u'' < 0$ and the usual Inada conditions. Utility is discounted at rate ρ.

Rational households take bequest income $B(t)$ and government policy as given and solve the following problem

$$\max \int_0^{\bar{T}} e^{-\rho t} S(t) u\big(c(t)\big)\, dt, \tag{29}$$

subject to

$$\dot{k}(t) = rk(t) + y(t) - c(t), \tag{30}$$

$$y(t) = (1-\tau)w + B(t), \quad \text{for } t \in [0, T], \tag{31}$$

$$y(t) = b + B(t), \quad \text{for } t \in [T, \bar{T}], \tag{32}$$

$$k(0) = k(\bar{T}) = 0. \tag{33}$$

The fully-funded actuarially fair social security system is self financed

$$b \int_T^{\bar{T}} e^{-rt} S(t)\, dt = \tau w \int_0^{T} e^{-rt} S(t)\, dt, \tag{34}$$

and the total bequests that are distributed must equal the total assets of the deceased

$$\int_0^{\bar{T}} N(t) B(t)\, dt = \int_0^{\bar{T}} M(t) k(t)\, dt. \tag{35}$$

Note that we intentionally consider the entire family of $B(t)$ functions that are feasible. To see this, let $f(t)$ be the exogenously-specified age-bequest density. That is, $\int_0^t f(v)\, dv$ is the fraction of total bequest income that goes to households age t or younger. Hence,

$$\int_0^t f(v)\, dv \int_0^{\bar{T}} M(t) k(t)\, dt = \int_0^t N(v) B(v)\, dv \quad \text{for all } t \quad \Longrightarrow \quad B(t) = \frac{f(t)}{N(t)} \int_0^{\bar{T}} M(t) k(t)\, dt. \tag{36}$$

Note that we can consider any $f(t)$ density as long as it satisfies $\int_0^{\bar{T}} f(t)\, dt = 1$. For instance, if we want to assume bequests are distributed uniformly to all survivors as is often assumed for convenience in macroeconomics ($B(t) = B$ for all t), then we set

$$f(t) = S(t) e^{-nt} \Big/ \int_0^{\bar{T}} S(t) e^{-nt}\, dt \quad \Longrightarrow \quad B(t) = B = \int_0^{\bar{T}} M(t) k(t)\, dt \Big/ \int_0^{\bar{T}} N(t)\, dt, \tag{37}$$

or if we want to assume bequests are distributed to households of the same age as the deceased, then we set

$$f(t) = M(t) k(t) \Big/ \int_0^{\bar{T}} M(t) k(t)\, dt \quad \Longrightarrow \quad B(t) = M(t) k(t) / N(t), \tag{38}$$

and so on. These are just two examples of possible $f(t)$ densities. The point is that our analysis does not place any restrictions on $f(t)$.

A stationary equilibrium in this economy is household allocations $(c^*(t), k^*(t))_{t \in [0, \bar{T}]}$ and age-bequest function $(B^*(t))_{t \in [0, \bar{T}]}$ such that given government policy (τ, b) and age-bequest density $f(t)$, household allocations solve the household problem (29)–(33) and Eqs. (34) and (35) hold.

In the proposition that follows, we assume that $n = r$ and that households are not liquidity constrained. This helps us prove our point analytically. We will quantitatively explore the effect of these assumptions on our result in the next section. The following proposition establishes the same result as we proved in the two-period model: social security crowds out private saving (and hence aggregate bequests) one for one and therefore equilibrium consumption allocations are invariant to social security.

Proposition 3. *If households are not liquidity constrained and if $n = r$, then a fully-funded actuarially fair social security system does not affect equilibrium consumption allocations and welfare even though private annuity markets are missing.*

Proof. Let $\lambda = u_c(c(0))$ be the marginal utility of consumption at time zero. Intertemporal optimality implies (and note that we ignore liquidity constraints for now)

$$e^{-(\rho-r)t}S(t)u_c\big(c(t)\big) = \lambda. \tag{39}$$

Therefore

$$c(t) = u_c^{-1}\big[\lambda e^{(\rho-r)t}/S(t)\big], \tag{40}$$

where u_c^{-1} is the inverse of the marginal utility of consumption. Equilibrium consumption allocations must simultaneously satisfy the household optimization condition (40), household budget constraints (30)–(33), and the aggregate constraints (34) and (35). We begin by combining all constraints (household plus aggregate constraints) into a single equation. To do this, first integrate by parts the aggregate bequest condition (35) using the boundary conditions $k(0) = k(\bar{T}) = 0$

$$\int_0^{\bar{T}} N(t)B(t)\,dt = \int_0^{\bar{T}} N(t)\big[\dot{k}(t) - nk(t)\big]\,dt. \tag{41}$$

Insert $\dot{k}(t)$ from the household budget constraint (30) and impose $n = r$

$$\int_0^{\bar{T}} N(t)B(t)\,dt = \int_0^{\bar{T}} N(t)\big[y(t) - c(t)\big]\,dt. \tag{42}$$

Insert $y(t)$ from (31) and (32) into (42)

$$\int_0^{\bar{T}} N(t)B(t)\,dt = \int_0^{T} N(t)\big[(1-\tau)w + B(t) - c(t)\big]\,dt + \int_T^{\bar{T}} N(t)\big[b + B(t) - c(t)\big]\,dt, \tag{43}$$

and then cancel the $B(t)$ terms

$$0 = \int_0^{T} N(t)\big[(1-\tau)w - c(t)\big]\,dt + \int_T^{\bar{T}} N(t)\big[b - c(t)\big]\,dt. \tag{44}$$

Now impose the self-financing assumption of social security with the assumption $r = n$

$$0 = \int_0^{T} N(t)\big[(1-\tau)w - c(t)\big]\,dt + \int_T^{\bar{T}} N(t)\left[\tau w \frac{\int_0^T e^{-nt}S(t)\,dt}{\int_T^{\bar{T}} e^{-nt}S(t)\,dt} - c(t)\right]dt \tag{45}$$

$$= \int_0^{T} N(t)\big[(1-\tau)w - c(t)\big]\,dt + \int_0^{T} N(t)\tau w\,dt - \int_T^{\bar{T}} N(t)c(t)\,dt, \tag{46}$$

and simplify to obtain a single condition that reflects all other constraints

$$\int_0^{\bar{T}} N(t)c(t)\,dt = \int_0^{T} N(t)w\,dt. \tag{47}$$

Any equilibrium consumption allocation must simultaneously satisfy household optimization (40) and the aggregate resource constraint (47). Combine these two

$$\int_0^{\bar{T}} N(t)u_c^{-1}\big[\lambda e^{(\rho-r)t}/S(t)\big]\,dt = \int_0^{T} N(t)w\,dt. \tag{48}$$

Eq. (48) provides the equilibrium determination of λ such that household optimization and budget constraints are satisfied, social security is self financed, and aggregate bequest income equals the assets of the deceased. Because the right side of (48) is independent of τ, it must be the case that λ (and hence consumption at time zero) is also independent of τ. Therefore, equilibrium $c(t)$ for all t must be independent of τ. Consumption and welfare are invariant to social security in equilibrium, despite the fact that households face uninsurable longevity risk. □

Although the case we considered above is very special ($n = r$), it demonstrates the main mechanism at play in a neat and clean way. Any gain from the annuitization aspect of social security comes from a positive income effect (and not a better intertemporal trade-off). All of that gain is washed out by a negative income effect on equilibrium bequest income due to crowding out.[6]

We conjecture that this result continues to hold in a more general environment ($n \neq r$). Since we cannot prove this analytically we resort to numerical exercises to validate this conjecture.

4. Robustness

In the previous section we presented the main result of the paper. In order to derive clean analytical results we made two simplifying assumptions. First we assumed that households are not liquidity constrained. This assumption implies the Euler equation of the household always holds with equality. The other assumption was that the net return on capital is exactly equal to the rate of population growth. This assumption implies that whether the social security arrangement is fully funded or is pay-as-you-go, households face the same budget constraint in equilibrium.

In this section we relax both assumptions and we show that the main results still hold true, and, in a sense become stronger. In order to establish this we resort to numerical experiments in line with Hubbard and Judd (1987). We first show that when the population growth rate is less than the net return on capital, a fully-funded social security system leads to lower consumption and therefore lower welfare when bequest income is endogenous. We then go further and add liquidity constraints and verify that for an empirically relevant level of social security tax and transfer the same result holds.

We emphasize that our paper is making a theoretical point. The purpose of the numerical exercises that we present here is to show that our theoretical results are more general than is implied by the assumptions in Proposition 3. There already exists a well-developed literature that considers the welfare effect of social security in large-scale DSGE models (see our literature review above). This literature considers the usefulness of social security as a provider of longevity insurance when, in addition to survival uncertainty, households also face income uncertainty. While such models provide a broader picture of how social security affects welfare in general, we are trying to clarify the channels through which those welfare gains do and do not arise. In other words, we are making the case that the welfare gains from social security do not come from the provision of longevity insurance as is traditionally supposed, and abstracting from income risk allows us to make this point in the cleanest possible way.

Our sensitivity analysis follows Hubbard and Judd (1987) and uses the same environment and parameterization. We assume households start working at age 20, retire at age 65 and live up to a maximum age of 110. We use a CRRA utility function $u(c) = c^{1-\sigma}/(1-\sigma)$ and report results for different values of σ. We assume population growth $n = 1\%$ and construct average population survival using male and female mortality data for the cohort of 1920 (Bell and Miller, 2005, Table 7).[7] Following Hubbard and Judd (1987) the household age-earning profile is taken from Davies (1981) which is approximated by the following forth-order polynomial for $t \in [20, 65]$,[8]

$$w(t) = -36\,999.4 + 3520.22t - 101.878t^2 + 1.34816t^3 - 0.00706233t^4. \tag{49}$$

Finally, we assume an annual discount rate of $\rho = 0.015$. To isolate our findings from the factor price effects of social security we assume a small open economy with constant returns on capital, r. We report results for $r = 0$, 0.02, 0.04, and 0.06.

As we demonstrated in the previous section, the welfare effect of social security does not depend on the timing of the distribution of bequests when households are at an interior solution. But the timing does matter at corner solutions. We follow Hubbard and Judd (1987) and assume they are received when households are at age 40. We maximize the potential impact of borrowing constraints by not giving any bequest income to the very young who have the greatest need to borrow. We are therefore making borrowing constraints as tight as possible to give the greatest chance for borrowing constraints to alter consumption allocations.

We show the capital-output ratio and aggregate bequest (as a percentage of average earnings) in an economy without social security in Table 1. The top panel shows the quantities for the economy without liquidity constraints and the bottom panel shows numbers for the economy with liquidity constraints.

[6] Note that our strategy is to compare consumption allocations across different stationary equilibria, one with a social security program versus one without a program. An alternative strategy would be to identify winners and losers on the transition path when a social security program is introduced. We avoid doing that exercise because it would confound the possible welfare effects from the provision of longevity insurance with the redistributional effects that exist when a program is first introduced.

[7] Hubbard and Judd (1987) only report the source of mortality data, but they do not report which year they are using to construct their survival probabilities. For this reason our numerical results do not match theirs exactly. But they are consistent.

[8] As Hubbard and Judd (1987) claim, and we confirm, the results are not sensitive to the use of alternative age-earning profiles.

Table 1
Capital-output ratio and aggregate bequest income in steady state without social security.

	Capital-output ratio				Aggregate bequest (% of avg earnings)			
	σ				σ			
	0.9	2	4	5	0.9	2	4	5
(a) Economy without Liquidity Constraints								
$r = 0.00$	−1.13	4.78	9.12	10.35	3	10	17	19
$r = 0.02$	3.73	5.68	7.30	7.77	8	13	19	20
$r = 0.04$	7.76	6.35	6.02	6.01	19	19	20	21
$r = 0.06$	10.16	6.75	5.15	4.83	49	27	22	21
(b) Economy with Liquidity Constraints								
$r = 0.00$	3.00	6.96	10.63	11.73	5	11	18	20
$r = 0.02$	5.37	7.40	9.11	9.62	9	15	21	23
$r = 0.04$	8.29	7.74	8.23	8.50	20	22	26	28
$r = 0.06$	10.35	8.34	8.13	8.14	51	35	35	36

Table 2
Capital-output ratio, aggregate bequest income and welfare change after introduction of social security while holding bequest income fixed.

	Capital-output ratio				Aggregate bequest (% of avg earnings)				Welfare change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
(a) Economy without Liquidity Constraints												
$r = 0.00$	−14.50	−6.85	−1.41	0.10	−21	−12	−3	0	32.37	30.42	28.71	28.21
$r = 0.02$	−3.46	−0.64	1.64	2.29	−8	−2	3	5	11.57	11.04	10.54	10.39
$r = 0.04$	5.34	3.40	2.94	2.92	7	6	8	9	4.04	4.06	4.00	3.97
$r = 0.06$	9.66	5.48	3.37	2.93	40	17	12	12	1.29	1.51	1.57	1.58
(b) Economy with Liquidity Constraints												
$r = 0.00$	0.33	2.35	4.51	5.22	0	2	5	7	5.22	7.55	−0.73	−6.35
$r = 0.02$	2.88	4.73	6.50	7.06	3	6	10	11	5.28	4.04	−2.81	−5.46
$r = 0.04$	7.22	7.01	8.22	8.72	13	13	16	18	2.84	1.59	−2.44	−3.32
$r = 0.06$	10.40	9.74	10.79	11.18	44	27	27	28	1.06	0.17	−1.58	−1.76

We now consider a fully-funded actuarially fair social security system. Taxes are a constant fraction of wage income during each year of the working period and benefits are constant across the retirement period. The tax payments of those who die early are added to the benefits of survivors (this is what allows social security to pay an above-market implicit return to surviving beneficiaries). We choose social security benefits to be 60 percent of average earnings and choose social security taxes to balance the budget,

$$\tau \int_0^T e^{-rt} S(t) w(t)\, dt = b \int_T^{\bar{T}} e^{-rt} S(t)\, dt. \tag{50}$$

Note that by doing this we hold the annuity payment of social security unchanged across different parameter values, however, the tax rates that are required to balance the budget differ for different values of the interest rate, r.[9]

We first present the results with exogenous bequests in Table 2. That is, we hold the household bequest endowment fixed at the level presented in Table 1 and solve the household problem with fully-funded social security. When there is no liquidity constraint, welfare increases for all combinations of the interest rate and risk aversion parameter. However, when households are not allowed to borrow, welfare increases only for low risk aversion. The importance of liquidity constraints is in line with Hubbard and Judd (1987). Next, we show that this pattern is completely reversed when we let bequest income adjust to a new steady state.

Table 3 shows the results in the steady state with social security when aggregate bequests are determined endogenously in equilibrium. Social security causes significantly lower welfare (relative to a steady state with no social security) when the interest rate exceeds the population growth rate. This is true regardless of the risk aversion parameter and regardless of

[9] Hubbard and Judd (1987) choose a 6% social security tax rate, consistent with the tax on self-employed workers in the early 1980s. We choose a level of benefit that is more in line with today's system in the US and other OECD countries. However, the results are not qualitatively sensitive to the exact level of taxes and transfers.

Table 3
Capital-output ratio, aggregate bequest income and welfare change at the steady state with social security and endogenous bequests.

	Capital-output ratio				Aggregate bequest (% of avg earnings)				Welfare change (% of consumption)			
	σ				σ				σ			
	0.9	2	4	5	0.9	2	4	5	0.9	2	4	5
(a) Economy without Liquidity Constraints												
$r = 0.00$	−10.91	−4.43	0.33	1.68	−22	−14	−7	−4	9.67	9.67	9.67	9.67
$r = 0.02$	−1.27	0.99	2.87	3.42	−10	−5	0	2	−5.07	−5.07	−5.07	−5.07
$r = 0.04$	5.80	4.18	3.81	3.80	4	4	6	6	−8.32	−8.32	−8.32	−8.32
$r = 0.06$	9.55	5.79	4.01	4.01	35	15	10	10	−7.91	−7.91	−7.91	−7.91
(b) Economy with Liquidity Constraints												
$r = 0.00$	4.24	4.98	6.22	6.66	0	1	3	3	1.69	1.73	−4.16	−8.03
$r = 0.02$	4.30	5.23	6.10	6.39	2	4	6	7	−0.84	−3.10	−6.00	−6.82
$r = 0.04$	7.24	6.18	6.12	6.21	11	10	11	12	−4.56	−5.74	−4.81	−4.15
$r = 0.06$	9.88	7.00	6.41	6.38	40	20	18	18	−6.15	−7.23	−2.94	−2.12

whether borrowing is allowed or not.[10] Also, note that the only inefficiency in this environment is the absence of annuity insurance. The economy is dynamically efficient, and even though annuity markets are absent, a publicly provided annuity through a fully-funded social security system does not improve welfare. This system does not affect the intertemporal trade-offs of the household. It only crowds out saving more than one for one (if the interest rate is larger than the population growth rate) which in equilibrium will reduce bequests that are received by each household and therefore lowers their lifetime income. Only in the special case of $n > r$ can social security potentially improve equilibrium welfare.

To gain better intuition about the effect of social security with and without endogenous bequest income we plot the consumption allocations and asset holdings for the case $\sigma = 2$ and $r = 0.04$ in Fig. 1. The solid black lines are allocations under steady states with no social security. The green dashed line shows allocations with social security while holding bequest income fixed at its steady state level without social security. Note that consumption is higher at almost all ages. This is because the annuity through social security pays a better implicit return than non-survival-contingent assets that households hold. Also, since bequest income is fixed we don't see any crowding out effect. Social security improves welfare because it provides better longevity risk sharing.

However, once we allow for bequests to adjust to the new steady state, all the positive effect of social security is more than offset by the crowding out of bequest income. The dashed-dotted red line shows allocations in steady states with social security and endogenous bequest income. We see that consumption is lower than consumption without social security in all ages when there are no liquidity constraints and almost all ages when liquidity constraints are present. That is why we see a large decline in welfare. Social security fails to improve welfare, even though it provides actuarially fair annuities. The reason is that social security does not affect intertemporal trade-offs that households face (unless it crowds out all private savings).

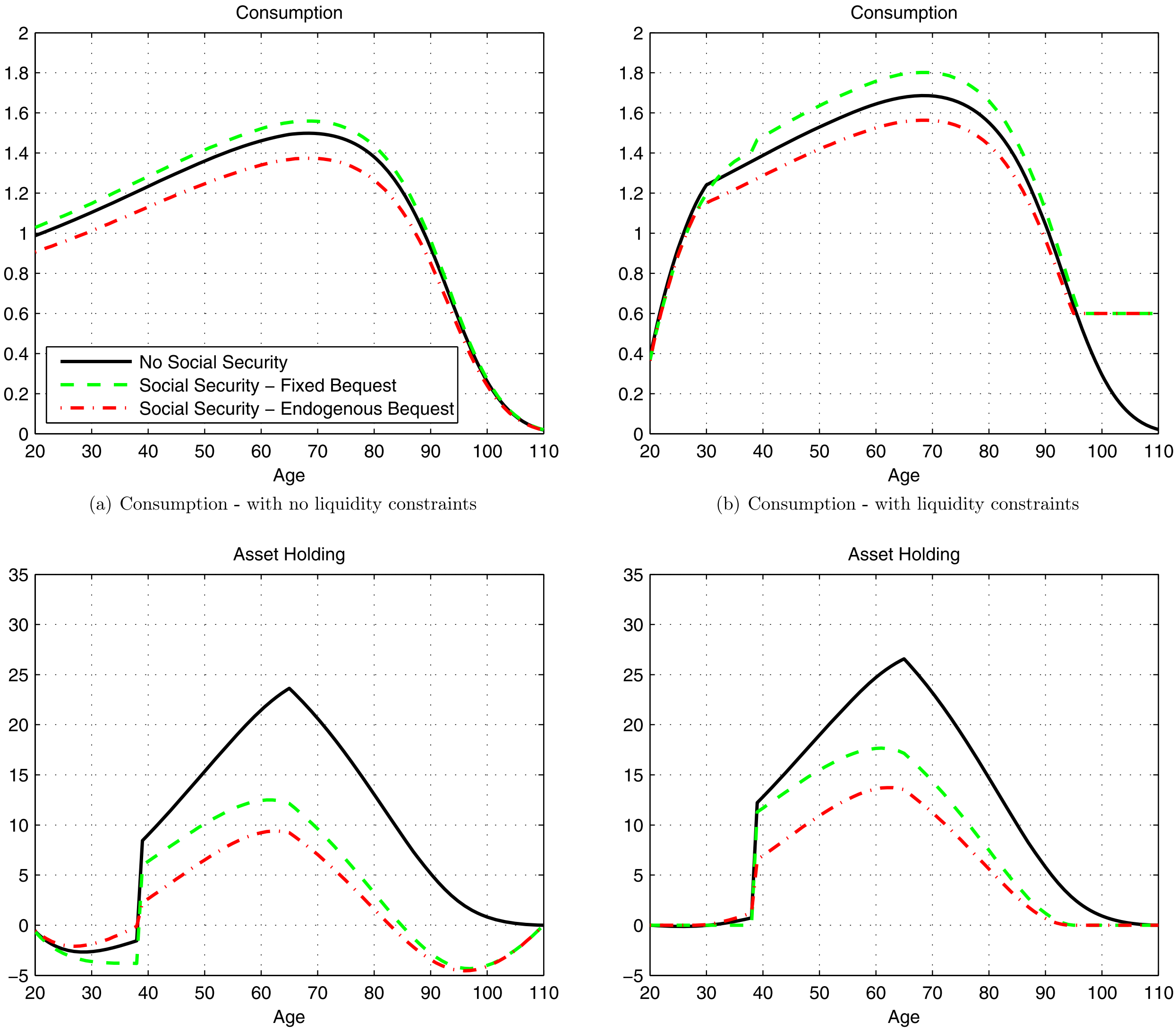
5. Conclusion

The conventional view in public economics is that social security improves welfare through the provision of longevity insurance when private annuity markets are missing. However, we show that in a general equilibrium model with closed annuity markets, social security fails to improve welfare under a wide range of conditions.

Our paper highlights the importance of including the general equilibrium effect of bequests when studying the insurance role of social security. The conventional wisdom holds only if we ignore this effect, which violates the aggregate resource constraint. Then social security is (trivially) welfare improving because of a free lunch: the welfare gains from mandatory annuitization come without the cost of lower bequest income. A proper, general equilibrium cost-benefit analysis eliminates this free lunch and eliminates the longevity insurance role for social security.

Of course, social security programs are often justified on a variety of alternative grounds. For instance, one justification says that economies can become dynamically inefficient without social security. Another says that social security provides opportunities for risk sharing across generations. Still another says that social security solves the time-inconsistency problem of the government providing a social safety net to the elderly who intentionally under save, betting that the government will take care of them even though it said it would not. In light of the findings in our paper—that the absence of private annuity markets is no longer a credible justification for a social security program once the analysis is expanded to include bequest income—the other justifications seem more plausible.

[10] For interior solutions, the welfare loss from social security is invariant to risk aversion. For the special case of $n = r$ we proved in the previous section of the paper that the equilibrium welfare effect of social security is constant (always zero) for any risk aversion. So the same result for the present cases ($n \neq r$) is not surprising because social security still does not affect the Euler equation. It is straightforward to prove this result analytically for a two-period model.



(a) Consumption - with no liquidity constraints

(b) Consumption - with liquidity constraints

(c) Asset Holding - with no liquidity constraints

(d) Asset Holding - with liquidity constraints

Fig. 1. Consumption and asset holding for parameters $\sigma = 2$ and $r = 0.04$. Figures on the left show consumption (top) and asset holding (bottom) for the economy without liquidity constraints. Figures on the right show consumption (top) and asset holding (bottom) for the economy with liquidity constraints. The black solid line is the steady state allocation without social security. The green dashed line is the allocation after the introduction of social security with bequests held fixed (at the steady state level without social security). The red dashed-dotted line is the steady state allocation with social security (and endogenous bequest income).

Finally, there is an open debate about whether households even *want* to annuitize their wealth, given their bequest motives. We bypass this debate and address a more fundamental issue. We focus on a setting in which there is no bequest motive so that households do indeed want to annuitize their wealth. And then we show that even in this setting, we still cannot use a missing annuity market as a justification for social security. This is true even though we have assumed social security is fully funded and actuarially fair.

Appendix A. Example with alternative bequest arrangement

Our result depends on the assumption that the assets of the deceased are distributed anonymously to all survivors in the economy. This assumption is motivated by common practice in the quantitative macroeconomics literature that uses life-cycle OLG models (see for example Hubbard and Judd, 1987; İmrohoroğlu et al., 1995; Conesa and Krueger, 1999; Huggett and Ventura, 1999; Hong and Ríos-Rull, 2007 and Nishiyama and Smetters, 2007 among many others). It is a convenient assumption that makes models simple and tractable. But it is not innocuous. An alternative assumption that is less popular (mainly for analytical and computational complexity) is that bequests are distributed through direct linkages between deceased parents and surviving children (as in Eckstein et al., 1985a and Abel, 1985).

In this section we show that while such an assumption can lead to a welfare role for social security, the welfare gains are due to reducing the variance of the distribution of wealth. We illustrate this point with a two-period OLG model with logarithmic utility, zero interest and no population growth.[11] We show that social security is not a Pareto move: only those whose parents live the longest (the very poorest) gain, and everyone else loses.

The household problem is

$$\max \ln(c_1) + S\ln(c_2), \tag{51}$$

subject to,

$$c_1 + k = w(1-\tau) + B, \tag{52}$$

$$c_2 = k + b. \tag{53}$$

And the first-order condition is

$$\frac{1}{w(1-\tau)+B-k} = \frac{S}{k+b}, \tag{54}$$

solving for k (and imposing $b = \frac{\tau w}{S}$)

$$(1+S)k = S(1-\tau)w + SB - b, \tag{55}$$

$$k = \frac{S}{1+S}w + \frac{S}{1+S}B - \frac{(1+S^2)\tau w}{S(1+S)}. \tag{56}$$

In this environment, although households are ex ante identical, ex post they differ by the amount of bequest income they receive. Whether a household receives a bequest or not depends on whether his/her parent has died in the first or second period.

Let ψ be the distribution of the bequest that is received by each newborn. Next we will characterize this distribution. Note that each parent survives with probability S. Therefore, independent of parents' assets, a fraction S of newborns receive zero bequest. Let $B_0 = 0$, then

$$\psi(B_0) = S. \tag{57}$$

These households have savings

$$k_0 = \frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}, \tag{58}$$

and if they die early, they leave all their assets as a bequest. So their children start the economy with

$$B_1 = \frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}, \tag{59}$$

and will have saving

$$\begin{aligned} k_1 &= \frac{S}{1+S}w + \frac{S}{1+S}\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right) - \frac{(1+S^2)\tau w}{S(1+S)} \\ &= \frac{S}{1+S}w + \left(\frac{S}{1+S}\right)^2 w - \frac{S}{1+S}\left(\frac{(1+S^2)\tau w}{S(1+S)}\right) - \frac{(1+S^2)\tau w}{S(1+S)} \\ &= \left(1+\frac{S}{1+S}\right)\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right), \end{aligned} \tag{60}$$

and if they die early their children receive

$$B_2 = \left(1+\frac{S}{1+S}\right)\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right), \tag{61}$$

and will have saving

[11] For a more general analysis we refer readers to Eckstein et al. (1985a) and Abel (1985).

$$\begin{aligned} k_2 &= \frac{S}{1+S}w + \frac{S}{1+S}B_2 - \frac{(1+S^2)\tau w}{S(1+S)} \\ &= \left(1 + \frac{S}{1+S} + \left(\frac{S}{1+S}\right)^2\right)\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right). \end{aligned} \tag{62}$$

Therefore

$$k_t = \left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right)\sum_{i=0}^{t}\left(\frac{S}{1+S}\right)^i, \tag{63}$$

and

$$B_0 = 0, \tag{64}$$

$$B_t = k_{t-1} \quad \text{for } t > 0, \tag{65}$$

also we can find the distribution ψ as

$$\psi(B_t) = S + S(1-S) + \cdots + S(1-S)^t,$$

where t denotes the number of periods since the last ancestor has survived to the second period.

We can find consumption allocations.

$$\begin{aligned} c_{1t} &= w(1-\tau) + B_t - k_t \\ &= w(1-\tau) + k_{t-1} - k_t \\ &= w(1-\tau) - \left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right)\left(\frac{S}{1+S}\right)^t \end{aligned} \tag{66}$$

and

$$c_{2t} = Sc_{1t} = Sw(1-\tau) - S\left(\frac{S}{1+S}w - \frac{(1+S^2)\tau w}{S(1+S)}\right)\left(\frac{S}{1+S}\right)^t. \tag{67}$$

It is obvious that consumption allocations are not invariant to the social security tax. And, notice that social security only increases consumption for those whose immediate ancestor survived to old age (i.e., for the poorest). We show this in the following proposition.

Proposition 4. *Social security reduces welfare for all households for whom $t > 0$.*

Proof. It is enough to show that c_{1t} decreases with τ if $t > 0$. Note that we can write c_{1t} as

$$c_{1t} = \left(1 - \left(\frac{S}{1+S}\right)^{t+1}\right)w + \left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t - 1\right)w\tau. \tag{68}$$

It is enough to show that $\frac{(1+S^2)}{S(1+S)}(\frac{S}{1+S})^t < 1$ for $t > 0$. Note that

$$\frac{\ln(\frac{(1+S^2)}{S(1+S)})}{\ln(\frac{1+S}{S})} < 1 \leqslant t, \quad t = 1, 2, \ldots \tag{69}$$

for any $S < 1$. Therefore,

$$t\ln\left(\frac{1+S}{S}\right) > \ln\left(\frac{(1+S^2)}{S(1+S)}\right) \tag{70}$$

and

$$\left(\frac{1+S}{S}\right)^t > \left(\frac{(1+S^2)}{S(1+S)}\right), \tag{71}$$

which implies

$$\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t < 1. \qquad \square \tag{72}$$

Note that for any level of social security, there is always a fraction of people who receive zero bequest. Social security does not change the bequest income for this group in the steady state. Even though there is crowding out of asset holdings and bequests in the aggregate, the bequest that these households receive does not change at all. Therefore, they benefit from the positive income effect from social security and are not harmed by the crowding out.

For those who do receive bequests in equilibrium, there will be a crowding out effect. The larger their bequest income is, the larger is the effect of crowding out on them (i.e., the effect is larger for higher t). And eventually it is so high to cancel out all the positive income effect of social security. In the case of the example presented above the crowding out effect is large enough to cancel out all the positive gains from social security for all those who receive any positive bequest in equilibrium.

Up to this point, we have focused attention strictly on uncertainty about the timing of death. However, if we back up one period before birth, then people face uncertainty not only about the timing of their own death, but they also face uncertainty about which ancestral line they will join at birth. In this case people face bequest income risk in addition to mortality risk. If we nest both layers of uncertainty in the model, then ex ante (before birth) welfare is strictly higher with social security than without. Social security raises life-cycle consumption in the lowest-income state when bequest income is zero and it reduces life-cycle consumption in all other states (of which there are infinitely many). We prove below that the gains outweigh the losses. We emphasize, however, that we can attribute the welfare gains to the provision of longevity insurance only if we take a broad view of longevity insurance, because technically it is insurance about the ancestral line that is conferring the welfare gain.

Proposition 5. *Social security strictly increases ex ante* (*behind the veil of ignorance*) *welfare when households face both mortality risk and bequest income risk.*

Proof. We will show that the consumption distribution with social security stochastically dominates (second order) the distribution without social security. We know

$$c_{1t}(\tau)=\left(1-\left(\frac{S}{1+S}\right)^{t+1}\right)w+\left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^{t}-1\right)w\tau \tag{73}$$

and

$$c_{2t}(\tau)=Sc_{1t}(\tau). \tag{74}$$

Therefore,

$$U_t(\tau)=(1+S)\ln\bigl(c_{1t}(\tau)\bigr)+S\ln(S). \tag{75}$$

Note that

$$\begin{aligned} c_{10}(\tau)&=\left(1-\left(\frac{S}{1+S}\right)\right)w+\left(\frac{(1+S^2)}{S(1+S)}-1\right)w\tau\\ &=\left(1-\left(\frac{S}{1+S}\right)\right)w+\left(\frac{1-S}{S(1+S)}\right)w\tau\\ &>\left(1-\left(\frac{S}{1+S}\right)\right)w=c_{10}(0). \end{aligned} \tag{76}$$

We have also shown in the previous proposition that for $t>0$

$$c_{1t}(\tau)<c_{1t}(0). \tag{77}$$

Let $\phi_t=S(1-S)^t$ be the density of the consumption distribution and let $C_1(\tau)$ be the average consumption of newborns

$$\begin{aligned} C_1(\tau)&=\sum_{t=0}^{\infty}S(1-S)^t\left[\left(1-\left(\frac{S}{1+S}\right)^{t+1}\right)w+\left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^{t}-1\right)w\tau\right]\\ &=w(1-\tau)-\sum_{t=0}^{\infty}S(1-S)^t\left(\frac{S}{1+S}w-\frac{(1+S^2)\tau w}{S(1+S)}\right)\left(\frac{S}{1+S}\right)^t\\ &=w(1-\tau)-S\left(\frac{S}{1+S}w-\frac{(1+S^2)\tau w}{S(1+S)}\right)\sum_{t=0}^{\infty}\left(\frac{S(1-S)}{1+S}\right)^t\\ &=w(1-\tau)-S\left(\frac{S}{1+S}w-\frac{(1+S^2)\tau w}{S(1+S)}\right)\frac{1+S}{1+S^2} \end{aligned}$$

$$= w(1-\tau) - S\left(\frac{S}{1+S^2}w - \frac{\tau w}{S}\right)$$
$$= w(1-\tau) + \tau w - \frac{S^2}{1+S^2}w$$
$$= \frac{w}{1+S^2}. \tag{78}$$

Also note that for all $\tau < \frac{S^2}{1+S^2}$

$$c_{10}(\tau) = \frac{w}{1+S} + \frac{(1-S)w}{S(1+S)}\tau$$
$$< \frac{w}{1+S} + \frac{(1-S)w}{S(1+S)}\left(\frac{S^2}{1+S^2}\right)$$
$$= \frac{w}{1+S^2} = C_1(\tau). \tag{79}$$

Recall from the previous proposition that we showed

$$\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t < 1 \quad \text{for all } t > 0. \tag{80}$$

Therefore for $t > 0$

$$c_{1t}(\tau) = \left(1 - \left(\frac{S}{1+S}\right)^{t+1}\right)w + \left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t - 1\right)w\tau$$
$$> \left(1 - \left(\frac{S}{1+S}\right)^{t+1}\right)w + \left(\frac{(1+S^2)}{S(1+S)}\left(\frac{S}{1+S}\right)^t - 1\right)w\left(\frac{S^2}{1+S^2}\right)$$
$$= \frac{w}{1+S^2} = C_1(\tau). \tag{81}$$

This implies that the vector of $c_{1t}(\tau)$ is a mean preserving reduction of spread over $c_{1t}(0)$ for all $\tau < \frac{S^2}{1+S^2}$. Therefore, it must be true that

$$\sum_{t=0}^{\infty} \phi_t\big((1+S)\ln\big(c_{1t}(\tau)\big) + S\ln(S)\big) > \sum_{t=0}^{\infty} \phi_t\big((1+S)\ln\big(c_{1t}(0)\big) + S\ln(S)\big). \quad \square \tag{82}$$

We can summarize the results in this section of the paper as follows. Social security can be welfare improving when bequest income is linked explicitly from parent to child. But these gains are either due to redistribution from the rich to the poor or they are due to insurance against bequest income risk, depending on whether welfare is evaluated ex post or ex ante. In either case, the gains are not because social security is helping to hedge the uncertainty about the timing of one's own death. Thus, social security can be said to provide longevity insurance only if we take a broad view of insurance that includes the longevity risk of one's parents and earlier ancestors.

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