Homework 4

ECON 8050: Macroeconomics II
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The process for $y = log(income)$ is:

$$y_{t+1} = \mu + \rho y_t + \sigma \varepsilon_{t+1}$$

where $\varepsilon \sim N(0, 1)$

(1) Set $\mu = 0$, $\rho = 0.9$ and $\sigma = 0.0242$. Discretize the process for y with 9 points. Download the Matlab code ghquad.m to compute Gauss-Hermit grids and weights. Use $10,000$ as maxit input. As an output, print out the vector of discretized y and the transition matrix.

(2) Simulate the Markov chain and compute the implied autocorrelation coefficient ($\hat{\rho}$). Note: use 1 million observations to simulate a persistent AR process. Disregard first 1000 observations. Report both $\hat{\rho}$ and $\hat{\sigma}$ computed from the simulated data.

Solution 1

y - vector:

$$\begin{pmatrix} -0.1092 & -0.776 & -0.503 & -0.0248 & 0.0000 & 0.0248 & 0.0503 & 0.0776 & 0.1092 \end{pmatrix}$$

Transition Matrix:

$$\begin{bmatrix}
0.5755 & 0.3551 & 0.0649 & 0.0044 & 0.0001 & 0.0000 & 0.000 & 0.0000 & 0.0000 \\
0.1572 & 0.4517 & 0.3116 & 0.0729 & 0.0063 & 0.0002 & 0.0000 & 0.0000 & 0.0000 \\
0.0179 & 0.1945 & 0.4222 & 0.2881 & 0.0708 & 0.0063 & 0.0002 & 0.0000 & 0.0000 \\
0.0009 & 0.0349 & 0.2212 & 0.4099 & 0.2669 & 0.0623 & 0.0048 & 0.0001 & 0.0000 \\
0.0000 & 0.0028 & 0.0499 & 0.2441 & 0.4063 & 0.2441 & 0.0499 & 0.0028 & 0.0000 \\
0.0000 & 0.0001 & 0.0048 & 0.0623 & 0.2659 & 0.4099 & 0.2212 & 0.0349 & 0.0009 \\
0.0000 & 0.0000 & 0.0002 & 0.0063 & 0.0708 & 0.2881 & 0.4222 & 0.1945 & 0.0179 \\
0.0000 & 0.0000 & 0.0000 & 0.0002 & 0.0063 & 0.0729 & 0.3116 & 0.4517 & 0.1572 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0044 & 0.0649 & 0.3551 & 0.5755
\end{bmatrix}$$

Solution 2

$$\hat{\rho} = 0.8857$$
$$\hat{\sigma} = 0.0239$$

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%% PS4 - Tauchen-Hussey
clear;
clc;

%% Setting parameters
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```
mu = 0;
rho = 0.9;
sigma = 0.0242;
n = 9;

maxit = 10000;

%% Calling GHQuad
[x,w] = ghquad(n, maxit);

%% Problem 1

%Calculating Unconditional Distribution
uncon_mean = mu / (1-rho);
uncon_sd = sigma / sqrt(1-rho^2);

%Calculate Steps and Discretize Space
m = 3;

y_state = zeros(n,1);
for i = 1:n
  y_state(i) = uncon_mean + sqrt(2)*sigma*x(i);
end

% Transition Probability
P = zeros(n, n);

for i = 1:n
  mean_next = mu + rho*y_state(i);
  for j = 1:n
    P(i,j) = w(j)/sqrt(pi)*exp(-(0.5)*((y_state(j) - rho*y_state(i))/sigma)^2)/ex
  end
  dens = sum(P(i,:));
  P(i,:) = P(i,:)/dens;
end

%Print results
disp('Discretized State Space for y: ');
disp(y_state);

disp('Transition Matrix: ');
disp(P);

%% Problem 2

n = 1000000;
dis = 1000;

y_sim = zeros(n, 1);

init = randi(length(y_state));
y_sim(1) = y_state(init);
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current_state = init;

for t = 2:n
  cum_prob = cumsum(P(current_state, :));
  r = rand();
  next_state = find(r <= cum_prob, 1, 'first');
  y_sim(t) = y_state(next_state);
  current_state = next_state;
end

y_sim_final = y_sim(dis+1:end);

y_t = y_sim_final(1:end-1);
y_tp1 = y_sim_final(2:end);

y_mean = mean(y_sim_final);
y_demean = y_t - y_mean;
y_tp1_demean = y_tp1 - y_mean;

rho_hat = (y_demean' * y_tp1_demean)/(y_demean'*y_demean);

e_hat = y_tp1 - (mu + rho_hat*y_t);
sigma_hat = sqrt(mean(e_hat.^2));

fprintf('Estimated autocorrelation coefficient (rho_hat): %.6f\n', rho_hat);
fprintf('Estimated variance (sigma_hat): %.6f\n', sigma_hat);
```