The University of Georgia
TERRY COLLEGE OF BUSINESS
Department of Economics

Preliminary Exam

Macroeconomics

May 20, 2025

Instructions This exam contains total of 4 questions. Please answer all of them. You have four hours to finish the exam. Please take some time to read each question carefully before you write your answers. Show all your work, and explain all your answers. Write your answers as clearly and neatly as possible. Questions will be equally weighted. Indicate carefully if you introduce notation, or make assumptions when answering a question. Good luck!

Question 1

In each period, an individual is presented with a criminal opportunity. The value of this opportunity, or benefit, is given by the non-negative continuous random variable b with finite mean, cumulative distribution function $F(\cdot)$, and density $f(\cdot)$.

After observing the value of the current period's criminal opportunity, the individual chooses to offend or abstain. In a given period t, if he abstains, he receives flow utility $u_t = a$. If he offends and is not caught, he obtains the flow utility $u_t = a + b$. If he is caught, however, he will be incarcerated beginning immediately and perhaps for some time. Apprehension occurs with probability p. While incarcerated, he receives flow utility $a - c$, where c is a positive per-period utility cost of being incarcerated.

Once incarcerated, he is sentenced for s periods. After serving the sentence he is released and is free. Every period that he is free, he will face a random opportunity to commit crime again.

The individual knows the structure of the problem as outlined. He chooses to offend or abstain in each period he is free, seeking to maximize the expected present discounted value of utility

$$E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u_\tau \right]$$

Here, E_t is the expectation operator conditional on information available as of period t, β is the discount factor, and u_t is either $a-c$, a, or $a+b$. Let $V(b)$ be the value of an individual with the criminal opportunity b.

1. Write the expected pay off of abstaining for an individual faced with criminal opportunity b today.
2. Write the expected pay off of committing the crime for an individual faced with criminal opportunity b today.
3. Write the problem of this individual as a Bellman equation. Clearly indicate the state variable.
4. Argue that this Bellman equation has a unique solution.
5. Now assume $a = 0$ and $s = 1$. Characterize the optimal policy function. In other words, for what values of b does the individual commits the crime. How does the optimal decision depend on distribution of crime opportunity $F(\cdot)$?
6. What fraction of the free population is arrested every period?
7. In the steady state, what fraction is in jail?

Question 2

Consider an infinite horizon economy with two sectors (investment and consumption), and two types of occupations (goods producing occupations and service producing occupations).

Production of Consumption Good Consumption good is produced using capital and labor services according to the following production function

$$Y_{Ct} = K_{Ct}^{\theta} L_t^{1-\theta}$$

$$L_t = \left[\alpha \left(A_{gt} N_{gt}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(A_{st} N_{st}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, K_{Ct} is the capital used in production of consumption good. N_{gt} is labor employed in goods producing occupation. N_{st} is labor employed in service producing occupation. A_{gt} and A_{st} are labor–augmenting technological progress for goods producing and service producing occupations, respectively. L_t is labor services produced by combination of workers in various occupations.

Production of Investment Good Investment good is produced using capital

$$Y_{Xt} = A_X K_{Xt}$$

Households There is a continuum of measure one of identical households with following preferences over consumption

$$\sum_{t=0}^{\infty} \beta^t \log\left(C_t\right)$$

Households own capital and make decision about investment and consumption. Price of investment good is normalized to be 1. Households rent capital to firms and receive rental rate r_t per unit of capital rented. Capital depreciates at rate δ. Households are also endowed with 1 unit of time. They can work at either occupation and freely move between them.

1. Assume households can trade a one period asset a_t that is in zero net supply. The asset pays real interest rate i_t. Define Sequential Market Competitive Equilibrium.
2. Using equilibrium conditions find real interest rate i_t.
3. Find growth rate of total consumption expenditures in equilibrium. Under what condition on model parameters total consumption expenditure grow?
4. What is GDP in this model?

5. Show that aggregate capital, capital in each sector, total consumption expenditure, GDP, investment expenditures, and wages all grow at the same rate.

6. Assume $\sigma < 1$ (goods producing and service producing occupations are complement). Now assume A_{gt} grows faster than A_{st}. What happens to allocation of labor across occupations? Why? Give as much intuition as you can about your finding.

Question 3

Read the piece of the code below.

```
function U=ufuncW(kp, ik, iz, is, igamma)
global w r kgrid zgrid VW VR beta rho TransZ TransS tauU probR gammavec
n
if is=1
     income = rho·w·gammavec(igamma) ;
elseif is=2
     income=w·gammavec(igamma)·zgrid(iz)(1-tauU);
end
c=kgrid(ik)(1+r)-kp + income
utiltoday=ln(c)
% here the code locates kp on the grid,
% finds jlo and weight for interpolation wght
⋮
EvalW=0
for isp=1:2
for izp=1:n
     Vhat = (1-wght)·VW(jlo, izp, isp, igamma)
               +wght·VW(jlo+1, izp, isp, igamma);
     EvalW=EvalW + TransZ(iz, izp)·TransS(is, isp)·Vhat;
end
end
ValR = (1-wght)·VR(jlo, igamma)+wght·VR(jlo+1, igamma);
Eval = (1-probR)·EvalW + probR·ValR;
U=utiltoday+beta·Eval;
U=-U
```

1. Write down the dynamic programming problem that the code solves. Make sure to specify all state and control variables. Be explicit when writing expected value functions, use the information you have about the evolution of stochastic variables.

2. Suppose we now incorporate the dynamic problem from part 1 into the general equilibrium framework. The total measure of agents in the economy is one and `probR` $= 0$. Assume `gammavec` $= 1$ for all `igamma`.

   The firm has Cobb-Douglas production technology: $Y = K^{\alpha}N^{1-\alpha}$, the depreciation rate is δ. The government runs a balanced budget.

The shock z is discretized to take two values $[z_1, z_2]$. The transition matrices for s and z look as follows:

$$\texttt{TransS} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

$$\texttt{TransZ} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Denote the corresponding invariant distributions as π^s and π^z.

Suppose you want to solve for the stationary competitive equilibrium. Assume the replacement rate of the unemployment benefits ρ is exogenously fixed. As a first step of your algorithm you guessed K. Your next step should be to find w, r, and τ_u. Show how to do it in detail. Be very specific: in each case, you should provide a formula (or a set of formulas) that help you to find the variables of interest. Verbal descriptions will not be accepted.

Question 4

Consider the overlapping generations model with zero population growth rate. Consumers live for two periods. In the first period, consumers are young, and in the second period, they are old. Young consumers survive to the old age with probability θ. Every young consumer receive income y (fixed exogenously), old consumers receive no income. Consumers can save at the interest rate r and they discount the future at the rate β.

Consumers have a bequest motive, i.e., they derive utility from leaving assets after they die. The lifetime utility of consumers is a function of c^y, c^o, beq^y, and beq^o. Here, beq^y are bequests left by young consumers who didn't survive to old age, and beq^o are bequests left by old consumers.

The utility consumers derive from consumption when young is $\ln(c^y)$, from consumption when old is $\ln(c^o)$, and from leaving bequests is $\ln(beq)$. Hence the lifetime utility is:

$$V = \ln(c^y) + \beta\theta \ln(c^o) + \beta(1-\theta)\ln(beq^y) + \beta^2\theta \ln(beq^o)$$

. Consumers can save in two types of assets. They can invest in a regular bond that brings return r. Denote savings in bonds of young consumers as k^y and that of old consumers as k^o. Note that $beq^y = k^y(1+r)$ and $beq^o = k^o(1+r)$. Young consumers can also buy annuities. Denote annuities purchased by the young consumer as a. If the young consumer does not survive till old age he receives nothing, and if he does survive, he receives $a\dfrac{1+r}{\theta}$.

Assume $\beta = 1$ and $r = 0$. Also assume that bequests just disappear from the economy, i.e., living agents receive no bequests.

1. Set up and solve the consumer optimization problem for c^y, c^o, k^y, k^o, and a.

2. Consider two modifications to the above model. First, consumers no longer can invest in annuities. Second, there is now the government in this economy. The government taxes income y of young consumers at a rate τ and pays lump-sum pensions p to old consumers.

   (a) Write down the pension system balance equation (recall there is no population growth).

   (b) Find the tax rate τ^* such that each young consumer saves exactly p, i.e., $k^y = p$.

   (c) Assume the tax rate is set to τ^* you found in 2(b). Solve the consumer optimization problem for c^y, c^o, and k^o. Compare with your solution to part 2(a) (when there is no government and there is annuity market). Provide intuition.