TakeHomeMidterm
Econ 8050: Macroeconomics II
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Life-cycle model.

Consider the problem of a retired person who decumulates a given amount of wealth W. He solves the following problem:

$$\sum_{t=1}^{T} \beta^t u(c_t) \longrightarrow \max_{c_t, k_{t+1}}$$

s.t.

total resources of the household:

$$res_t = k_t(1+r) + y_t - x_t$$

$$k_1 = W$$

Here k_t is savings, y_t is pension income, and x_t is medical expense shock. There is a means-tested support program that guarantees each household consumption at the level c_{min} if his resources are too low. If $res_t > c_{\min}$, then $c_t = res_t - k_{t+1},\ k_{t+1} \geq 0$. Else, $c_t = c_{\min}$ and $k_{t+1} = 0$.

Solve the model using backward induction. Assume CRRA utility function with risk aversion σ: $u(c_t) = \dfrac{c_t^{1-\sigma}}{1-\sigma}$. Set $\beta = 0.95, r = 0.04, \sigma = 3, T = 40, c_{\min} = 0.1$. For income, set $y_t = 1$ for all t. For initial wealth set $W = 10$. Assume x_t can take two values with probability 0.8 and 0.2. Download the file containing the values for x_t from the course website (xpts40.in). Discretize k using 100 gridpoints, so that $k(1) = 0$ and $k(100) = 100$. Make sure the grid is more dense around 0. When looking for optimal k_{t+1} do NOT restrict it to lie on the grid. Make sure you enforce the constraint $k_{t+1} \geq 0$. (When looking for a maximum you can use Matlab command *fminbnd*.) To find value function outside the grid of k use linear interpolation. (When doing linear interpolation command *find* can be useful.)

1. Solve the model and plot resulting policy functions for k_{t+1} and value function for ages 10 and 30 fixing x_t at the 1st and 2nd grid.

Organize your graphs as follows: 2×2 matrix. Left column - savings, right column - value function. Top row - for age 10, bottom row - age 30. Each graph should have 2 lines (clearly labeled): fixing x_t at the 1st and 2nd grid (command subplot in Matlab can be useful).

2. Simulate $\{x_t\}$ for $t = 1 : 40$. Plot savings over the lifecycle using your policy function.

3. Increase c_{min} to 0.5 and resolve the model. Plot savings over the life cycle.

4. Go back to c_{min} equal to 0.1. Remove medical shok (set $x_t = 0$). Resolve the model and plot savings over the lifecycle.

5. Go back to the initial parametrization with medical shok and increase β to 0.99. Resolve the model and plot savings over the lifecycle.

6. Combine saving profiles from questions 2-5 on the same graph and compare. Make sure to clearly label each line. Provide economic intuition.