Linear Model:

- OLS estimator: $\hat{\beta} = (\sum_{i=1}^{n} x_i x_i')^{-1} \sum_{i=1}^{n} x_i y_i$
- 2SLS estimator: $\hat{\beta} = [(\sum_{i=1}^{n} \mathrm{x}_i z_i')(\sum_{i=1}^{n} \mathrm{z}_i z_i')^{-1}(\sum_{i=1}^{n} \mathrm{z}_i x_i')]^{-1}(\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i x_i')(\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i z_i')^{-1}(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} \mathrm{z}_i u_i)$
- IV estimator: $\hat{\beta}[\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i x_i']^{-1}\frac{1}{n}\sum_{i=1}^{n} \mathrm{z}_i y_i$. z is the instrument which satisfies criteria uncorrelated with u and slightly correlated with x.
- Omitted Variables (IV): put omitted in error term and find instruments for any elements in the explanatory in the omitted

Nonlinear Model:

- Probit: $f(y|x_i) = [\Phi(x_i'\theta)]^y [1 - \Phi(x_i'\theta)]^{1-y}$
- Logit: $G(z) = \frac{e^z}{1+e^z}$
- Partial effect of probit: $\frac{\partial f}{\partial x}\phi$
- log-likelihood function: $l_i = y_i \log[G(x_i'\beta)] + (1 - y_i)\log[1 - G(x_i'\beta)]$
- likelihood function: $\mathcal{L} = \sum_{i=1}^{n} \mathrm{l}_i$
- multinomial logit: $P(y = j|x) = \frac{e^{x'\beta_j}}{1+\sum_{h=1}^{J} e^{x'\beta_h}}$; $P(y = j\ or\ y = h|x) = \frac{p_j(x,\beta)}{p_j(x,\beta)+p_h(x,\beta)}$

Roy:

$$y_1 = \begin{cases} 1\ if\ y_1^* > 0 \\ 0\ if\ y_1^* \leq 0 \end{cases}$$

$$y_2 = \begin{cases} y_2^*\ if\ y_1^* > 0 \\ y_3^*\ if\ y_1^* \leq 0 \end{cases}$$

$$y_1^* = x_1'\beta_1 + \epsilon_1$$

$$y_2^* = x_2'\beta_2 + \epsilon_2$$

$$y_3^* = x_3'\beta_3 + \epsilon_3$$

Heckman 2-Step:

$-\ y_{2i} = x_{2i}'\beta_2 + \sigma_{12}\lambda(x_{1i}'\hat{\beta}_1) + v_i$

$-$ v is the error, beta hat is the first step probit reg of y_1 on x_1

since $Pr[y_1^* > 0] = \lambda(x_1'\beta_1)$ and $\lambda_1(x_1'\hat{\beta}_1) = \phi(x_1'\beta_1)/\Phi(x_1'\beta_1)$ is the inverse Mills ratio

- Likelihood ratio test: $2(\mathcal{L}_{UR} - \mathcal{L}_r)$