Midterm

Econ 8050: Macroeconomics II

Svetlana Pashchenko

Problem

Consider the retirement stage of Modigliani-Brumberg life-cycle model. The consumer enters the model at age $t = 1$ with initial assets a_1 and he never receives any income. The consumer lives for T periods. He can save/borrow at the interest rate r. Denote assets the consumer holds at time t as a_t. The consumer discounts the future at the rate β. The consumer derives utility not only from consumption but also from asset holding (he has the so-called taste for wealth). The flow utility of the consumer of age t can be represented as follows:

$$u(c_t, a_t) = ln(c_t) + \eta\, ln(a_t),$$

where η indicates the strength of the taste for wealth.

Assume that $\beta = 1$ and $r = 0$.

1. Solve for optimal consumption c_t and optimal savings $a_{t\;1}$. (Hint: you should start with age $t = T \quad 1$ and move backwards. After iterating for several periods you should be able to see the pattern and derive a general expression).

2. Find the optimal consumption growth $\frac{c_{t\;1}}{c_t}$. How does it compare with the optimal consumption growth when there is no taste for wealth? Provide intuition.

Problem 2

Consider a life-cycle model where agents live from age $t = 1$ to T. Agents can save at the interest rate r and discount the future at the rate β. Denote asset holdings of an agent at age t as a_t.

Every period, agents are endowed with one unit of time that they allocate between leisure l_t, work n_t, exercise time e_t, and time to take medication (described later).

An agent derives utility from leisure and consumption of food. Food consumption has two categories: carbohydrates, c_t^C (sugar, bread, etc.), and non-carbohydrates, c_t^{NC} (everything else). The price of c_t^C is normalized to one, and the price of c_t^{NC} is denoted as p. Agents also care about their body mass index (BMI), b_t. Denote utility from BMI as $\nu(b_t)$.

An agent who exercises, experiences disutility that depends on health status h_t (described later), and is denoted $\phi(e_t, h_t)$. The function $\phi(\cdot)$ strictly increases in its first argument.

The utility function looks as follows:

$$u(c_t^C, c_t^{NC}, l_t, e_t, b_t) = \underbrace{\frac{\left.(c_t^C \quad g_t)^{\xi}(c_t^{NC})^{1 \quad \xi}\right]^{1 \quad \sigma}}{1 \quad \sigma}}_{\text{utility from food consumption}} + \frac{l_t^{1}}{1+\gamma} + \nu(b_t) \qquad \phi(e_t, h_t)$$

Here, ξ determines the relative weight of carbohydrates in total utility from food, σ is risk aversion, and γ determines labor supply elasticity. The utility from consuming carbohydrates depends on habits, where the habit stock g_t evolves as follows:

$$g_{t \quad 1} = (1 \qquad) \, g_t + \quad c_t^C,$$

where g_1 is the same for all agents. Thus, the level of habit is the weighted average of past carbohydrates consumption.

Every period, agents can develop diabetes. The probability that an age-t agent will have diabetes at $t+1$ is $PD(t, g_t, b_t, \mu)$. The probability depends on age t, habitual carbohydrate consumption g_t, BMI b_t, and genetic predisposition to diabetes μ. Genetic predisposition is different across agents. Diabetes is irreversible: an agent who develops it never recovers.

An agent with diabetes decides every period whether to take medications or not. Taking medications involves fixed time costs mt and monetary costs mp. Denote this decision as iM_t: $iM_t = 1$ if an agent takes medication this period, and $iM_t = 0$ otherwise.

Agents with diabetes do not have any symptoms or disutility. However, diabetes matters for health evolution. Health can be either good ($h_t = 1$) or bad ($h_t = 0$). *gents without diabetes always have good health.* For agents with diabetes, health evolves stochastically. Agents who newly developed diabetes draw h_t from the invariant distribution $\quad(h_t)$. After that, health evolves based on the transition matrix $TH(h_{t \quad 1}|h_t, iM_t)$. Agents who take medications ($iM_t = 1$) have better health transitions.

Health affects agents' disutility from exercising with $\phi(e_t, 1) < \phi(e_t, 0)$. Health also affects labor productivity, which is a deterministic function of age and health, $z(t, h_t)$. Hence, labor income is equal to $z(t, h_t)n_t$.

BMI of an agent evolves as follows:

$$b_{t \quad 1} = b_t + f(TC_t) \qquad \psi(e_t)$$

Here $TC_t = c_t^C + c_t^{NC}$ is the total caloric intake, and $f(\cdot)$ and $\psi(\cdot)$ are strictly increasing functions.

Questions

1. Write down the budget constraint and the time constraint of an agent. (Hint: make use of an indicator function.)

2. Write down the problem of an agent in a recursive form. Make sure to specify all state and control variables, constraints, and the law of motion for non-stochastic variables. Use information you have about the evolution of the stochastic variables (do not just use expectation operator E). Denote as $V^{ND}(\cdot)$ the value function of an agent who does not have diabetes, and as $V^{D}(\cdot)$ the value function of an agent with diabetes.

3. Consider an age-t *diabetic* agent. Using the first-order conditions, compare the optimal choice of carbohydrate versus non-carbohydrate consumption. Discuss the intuition.

(To simplify notations, denote the derivative of the utility function $u(c_t^C, c_t^{NC}, l_t, e_t)$ with respect to the first argument at time t as u_{1t}, and the derivative with respect to second argument at time t as u_{2t}.)