Homework 3

Tate Mason

An ECON - 8010 Homework Assignment

September 29, 2024

1 Question 3.I.5

1.1 Problem

Show that if $u(x)$ is quasilinear with respect to the first good (p_1 fixed at 1), then $CV(p^0, p^1, w) = EV(p^0, p^1, w)$ for any (p^0, p^1, w).

1.2 Solution

2 Question 5.C.9

2.1 Problem

Derive the profit function $\pi(p)$ and supply function $y(p)$ for the single output technologies whose production functions $f(z)$ are given by:
(b) $f(z) = \sqrt{\min\{z_1, z_2\}}$
(c) $f(z) = (z_1^\rho z_2^\rho)^{\frac{1}{\rho}}$ for $\rho \leq 1$

2.2 Solution

3 Question 5.C.10

3.1 Problem

Derive the cost function $c(w, q)$ and conditional function demand functions (or correspondences) $z(w, q)$ for each of the following single-output constant return technologies with production functions:
(b) $f(z) = \min\{z_1, z_2\}$ (Leontief technology)
(c) $f(z) = (z_1^\rho z_2^\rho)^{\frac{1}{\rho}}$ for $\rho \leq 1$ (CES technology)

3.2 Solution

(c) Derive cost function and conditional demand functions:

$$\max_{\vec{z} \geq 0} - \vec{w} \cdot \vec{z}$$

s.t.

$$-(z_1^\rho z_2^\rho)^{\frac{1}{\rho}} \geq q$$
$$-\vec{z} \leq 0$$

$$\vec{w} = \lambda \begin{bmatrix} p(z_1^\rho z_2^\rho)^{\frac{1}{\rho}-1} \cdot (z_1^{\rho-1} z_2^\rho) \\ p(z_1^\rho z_2^\rho)^{\frac{1}{\rho}-1} \cdot (z_1^\rho z_2^{\rho-1}) \end{bmatrix} + \vec{\mu}$$

s.t.

$$\vec{\mu} \cdot \vec{z} = 0$$
$$\lambda(q - f(\vec{z})) = 0$$

From FOC's

$$\frac{w_1}{w_2} = \frac{z_2}{z_1} \Rightarrow z_2 = z_1(\frac{w_1}{w_2})$$

Now, plug into $f(\vec{z})$

$$z_1^\rho(z_1(\frac{w_1}{w_2}))^\rho = q$$
$$z_1^2 = q^{\frac{1}{\rho}}(\frac{w_2}{w_1})$$
$$z_1(w,q) = q^{\frac{2}{\rho}}(\frac{w_2}{w_1})^{\frac{1}{2}}$$
$$z_2(w,q) = q^{\frac{2}{\rho}}$$

4 Question 5.C.11

4.1 Problem

Show that $\frac{\partial z_l(w,q)}{\partial q} > 0$ if and only if marginal cost at q is increasing in w_l.

4.2 Solution

Proof. This can be proven using Shephard's Lemma such that $z_l(w,q) = \frac{\partial C(w,q)}{\partial w_l}$. Marginal cost, then, is given as $\frac{\partial c(w,q)}{\partial q}$. So, we should show that $\frac{\partial z_l(w,q)}{\partial q} \Leftrightarrow \frac{\partial^2 c(w,q)}{\partial q \partial w_l}$. Using Shephard's Lemma and symmetry of second derivatives, it can be shown that $\frac{\partial z_l(w,q)}{\partial q} = \frac{\partial^2 c(w,q)}{\partial w_l \partial q} = \frac{\partial^2 c(w,q)}{q \partial w_l}$. Thus, $\frac{\partial z_l(w,q)}{\partial q} > 0 \Leftrightarrow \frac{\partial^2 c(w,q)}{\partial q \partial w_l} > 0$, showing that conditional factor demand for input l increases with output if and only if the marginal cost is increasing in the price of input l. $\square$

5 Question 5

5.1 Problem

A firm uses 2 inputs, z_1 and z_2, which it purchases at prices w_1 and w_2 to produce a single output. The firm's technology is described by production function f which is strictly increasing and obeys the Inada conditions $\lim_{z_1 0} \frac{\partial f(z_1,z_2)}{\partial z_1} = \lim_{z_1 \to 0} \frac{\partial f(z_1,z_2)}{\partial z_2} = \infty$ for each x. (Hence, the firm will always choose to use a strictly positive quantity of each input.)

(a) Set up firm's cost minimization problem, write down its Lagrangian, find firm's first order conditions for cost minimization.

(b) Use the envelope theorem to find an expression (possibly involving a Lagrange multiplier) for the firm's marginal cost $\frac{\partial c(w,q)}{\partial q}$.

(c) An economist wishes to measure the firm's markup-ratio of price of output, p, to its marginal cost $\frac{\partial c(w,q)}{\partial q}$. However, she does not know what kind of competition the firm faces inthe production market. In fact, the only data she has are:

- the marginal product of input 1 at the input fix selected by the firm:

$$\frac{\partial f(z(w,q))}{\partial z_1}$$

- the price of input 1, w_1
- the price of firm's output p.

How can she use these data to recover the firm's markup?

5.2 Solution

6 Question 6.B.2

6.1 Problem

Show that if the preference relation $\succeq$ on $\mathcal{L}$ is represented by a utility function $U(\cdot)$ that has the expected utility form, then $\succeq$ satisfies the independence axiom.