Homework 7

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Collaboration to varying degrees with Timothy Duhon, Josephine Hughes, Abdul Khan, Kasra Lak, Rachel Lobo, Mingzhou Wang, Wenyi Wang

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An ECON - 8040 Homework Assignment

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Question 1

Problem

Consider the following infinite horizon production economy with a household sector and a business sector:

Business Sector: Firms in the economy produce a composite good that can be used for either consumption or investment purposes according to the following technology:

$$Y_t = K_{Mt}^{\alpha} N_{Mt}^{1-\alpha}$$

where K_{Mt} is the amount of capital rented by the firm at date t and N_{Mt} is the amount of labor hired by the firm at date t.

Household Sector: There is a continuum of measure 1 of infinitely lived households.

Preferences: Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

where the variable c_t is an aggregator of the good produced by the business sector and a good produced by the household. More specifically:

$$c_t = [\mu c_{Mt}^{\rho} + (1-\mu) c_{Ht}^{\rho}]^{\frac{1}{\rho}}$$

where c_{Mt} is the good produced in the business sector and c_{Ht} is the good produced at home.

Home Production: Each household has access to the same technology to produce the home good. The use of this technology by a particular household requires that household's own capital and labor. This technology is:

$$c_{Ht} = k_{Ht}^{\alpha} n_{Ht}^{1-\alpha}$$

Endowments: Each household is endowed with one unit of time. Additionally, each household is endowed with k_{M0} units of capital it can rent out to firms in the economy and k_{H0} units of capital that it can use to produce the home good. The two capital stocks depreciate at their respective rates δ_K and δ_H. Capital is sector specific so home capital cannot be used in the business sector and vice versa.

Questions

1. Write down the Social Planner's problem.
2. Write down the Social Planner's problem in recursive form (Bellman equation) – what are the state variables?
3. Write down FOCs and envelope conditions for this Bellman equation.
4. Write down equations that characterize the steady state.

Solutions

(a)

$$\max_{c_t, n_t, N_t, Y_t, k_{Ht}, K_{Mt}} \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

$$s.t.$$

$$c_t = [\mu c_{Mt}^{\rho} + (1-\mu)c_{Ht}^{\rho}]^{\frac{1}{\rho}}$$

$$c_{Ht} = k_{Ht}^{\alpha} n_{Ht}^{1-\alpha}$$

$$Y_t = K_{Mt}^{\alpha} N_{Mt}^{1-\alpha}$$

$$Y_t = c_{Mt} + x_{Mt} + x_{Ht}$$

$$K_{Mt+1} = (1-\delta_M)K_{Mt} + x_{Mt}$$

$$k_{Ht+1} = (1-\delta_H)k_{Ht} + xHt$$

$$n_{Ht} + N_{Mt} = 1$$

$$k_{H0}, K_{M0} \ given$$

(b)

$$v(k_{Ht}, K_{Mt}) = \max_{N_t, k_{Ht+1}, K_{Mt+1}} \log(Z^{\frac{1}{\rho}}) + \beta v(k_{Ht+1}, K_{Mt+1})$$

$$s.t.$$

$$Z \equiv \mu c_M^{\rho} + (1-\mu)[k_{Ht}^{\alpha}(1-N_{Mt})^{1-\alpha}]^{\rho}$$

$$c_t = Z^{\frac{1}{\rho}}$$

$$c_{Ht} = k_{Ht}^{\alpha}(1-N_{Mt})^{1-\alpha} c_{Mt} + k_{Ht+1} + K_{Mt+1} = K_{MT}^{\alpha} N_{MT}^{1-\alpha} + (1-\delta_H)k_{Ht} + (1-\delta_M)K_{Mt}$$

$$K_{Mt+1} = (1-\delta_M)K_{Mt} + x_{Mt}$$

$$k_{Ht+1} = (1-\delta_H)k_{Ht} + xHt$$

In this case, the state variables are K_{Mt} and k_{Ht} as we have some sort of control over labor, consumption, and investment.