TakeHomeMidterm

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Life cycle model

Consider the problem of a retired person who decumulates a given amount of wealth W. He solves the following problem:

$$\sum_{t=1}^{T} \beta^t u(c_t) \to \max_{c,k}$$

s.t.
total resources of the household:

$$res_t = k_t(1+r) + y_t - x_t$$
$$k_1 = W$$

Here k_t is savings, y_t is pension income, and x_t is medical expense shock. There is a means-tested support program that guarantees each household consumption at the level c_{min} if his resources are too low. If $res_t > c_{min}$, then $c_t = res_t - k_{t+1}$, $k_{t+1} \geq 0$. Else, $c_t = c_{min}$ and $k_{t+1} = 0$.

Solve the model using backward induction. Assume CRRA utility function with risk aversion σ: $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. Set $\beta = 0.95$, $r = 0.04$, $\sigma = 3$, $T = 40$, $c_{min} = 0.1$. For income, set $y_t = 1$ for all t. For initial wealth set $W = 10$. Assume x_t can take two values with probability 0.8 and 0.2. Download the file containing the values for x_t from the course website (xpts40.in). Discretize k using 100 gridpoints, so that $k(1) = 0$ and $k(100) = 100$. Make sure the grid is more dense around 0. When looking for optimal k_{t+1} do NOT restrict it to lie on the grid. Make sure you enforce the constraint $k_{t+1} \geq 0$. (When looking for a maximum you can use Matlab command fminbnd.) To find value function outside the grid of k use linear interpolation. (When doing linear interpolation command findnearest can be useful.)

1. Solve the model and plot resulting policy functions for k_{t+1} and value function for ages 10 and 30 fixing x_t at the 1st and 2nd grid. Organize your graphs as follows: 2×2 matrix. Left column - savings, right column - value function. Top row - for age 10, bottom row - age 30. Each graph should have 2 lines (clearly labeled): fixing x_t at the 1st and 2nd grid (command subplot in Matlab can be useful).

2. Simulate $\{x_t\}$ for $t = 1:40$. Plot savings over the lifecycle using your policy function.

3. Increase c_{min} to 0.5 and resolve the model. Plot savings over the life cycle.

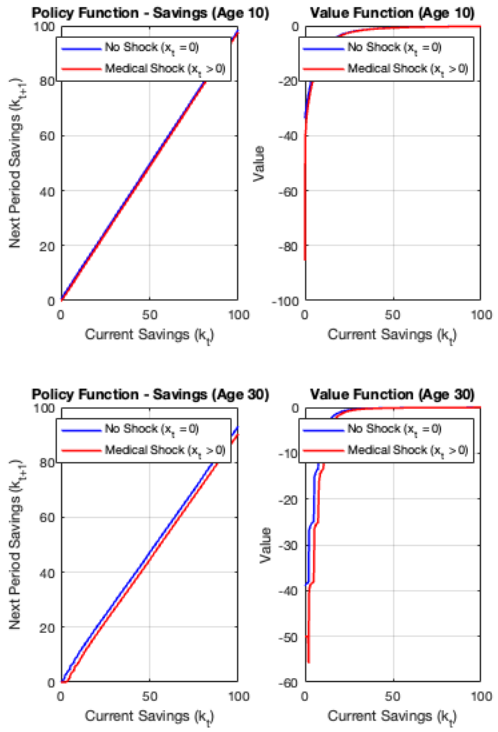
4. Go back to c_{min} equal to 0.1. Remove medical shock (set $x_t = 0$). Resolve the model and plot savings over the lifecycle.

5. Go back to the initial parametrization with medical shock and increase β to 0.99. Resolve the model and plot savings over the lifecycle.

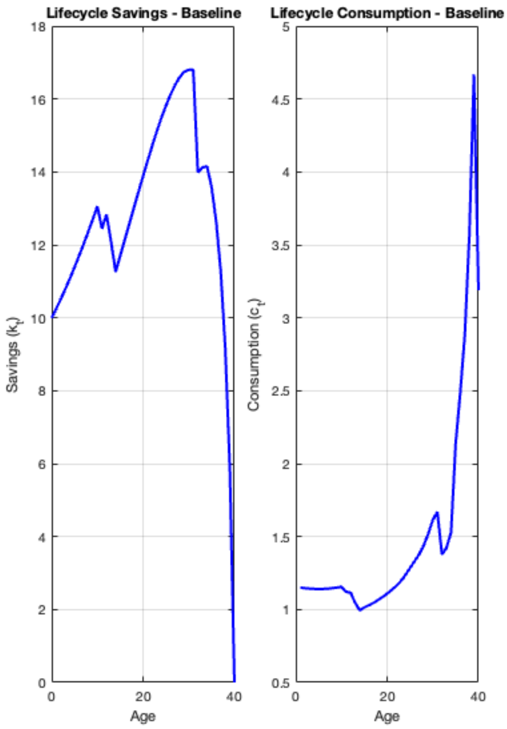
6. Combine saving profiles from questions 2-5 on the same graph and compare. Make sure to clearly label each line. Provide economic intuition.

Solutions

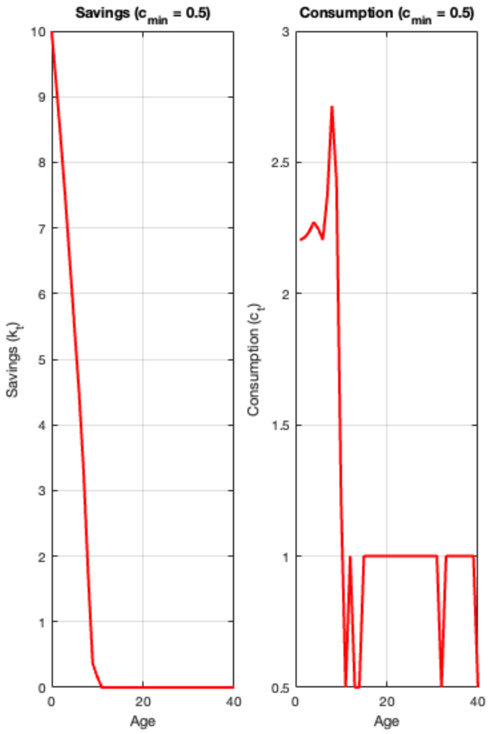
Part 1



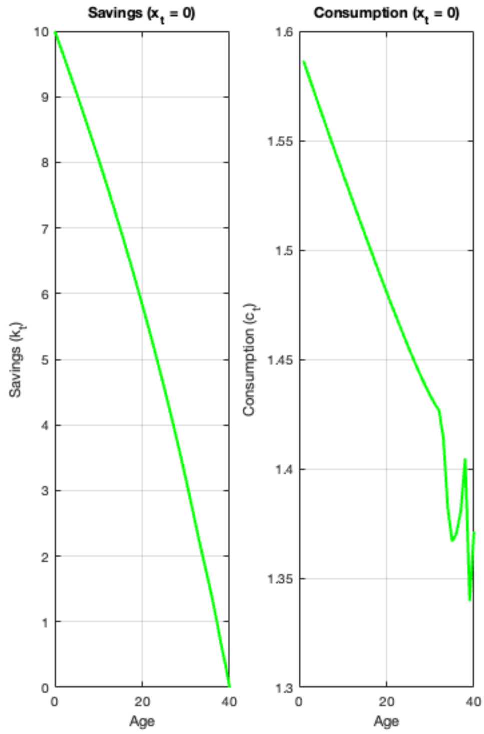
Part 2


Lifecycle Savings - Baseline
Savings (k_t)
Age
Lifecycle Consumption - Baseline
Consumption (c_t)
Age

Part 3

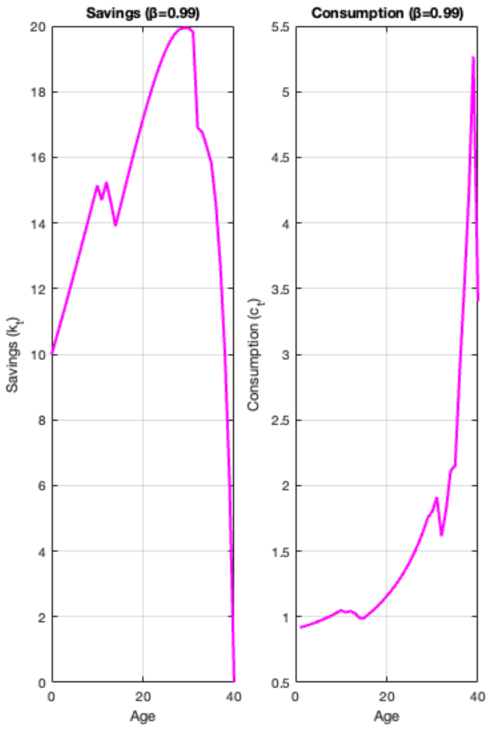

Savings (c_{min} = 0.5)
Savings (k_t)
Age
Consumption (c_{min} = 0.5)
Consumption (c_t)
Age

Part 4



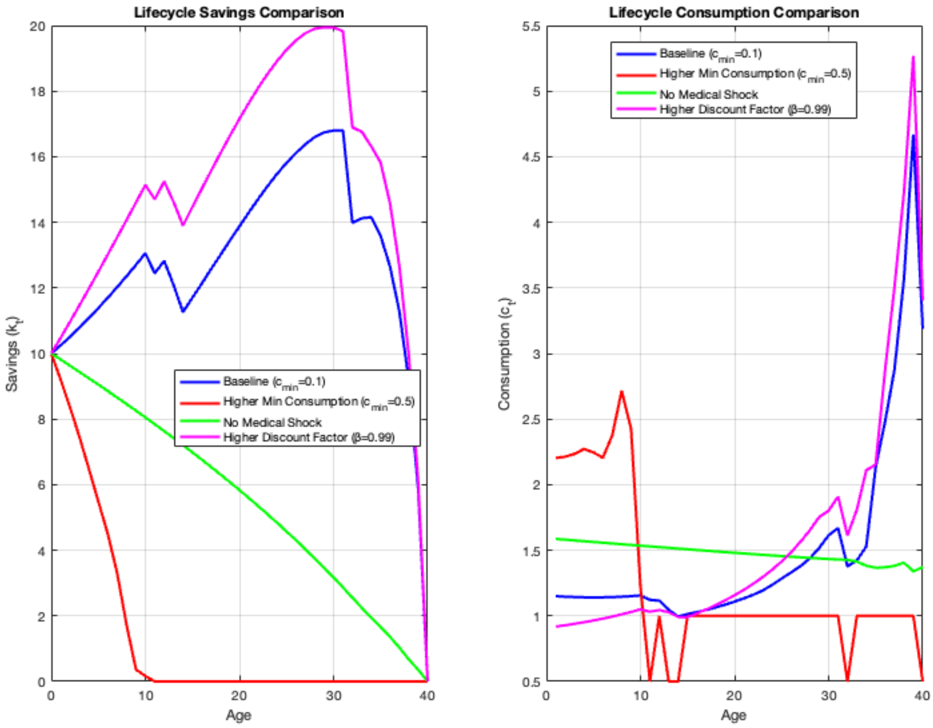
Savings (x_t = 0)
Savings (k_t)
Age
Consumption (x_t = 0)
Consumption (c_t)
Age

Part 5



Part 6

As can be seen in the graph above, raising minimum consumption leads to less precautionary saving over the lifetime. As the floor is higher, there is less incentive to hold wealth, instead, agents will consume to meet their needs. In the case of getting rid of the medical shock, there is now incentive to save earlier due to the lack of uncertainty, and a much more linear curve for both savings and consumption. Now, when we raise the discounting rate, we can see an emphasized curve of the one from part 2. This makes sense as, with a higher level of patience (for lack of a better term), they will save more throughout, leading to a steeper decline as period $t = T$ approaches.


Lifecycle Savings Comparison
Savings (k_t)
Age
Baseline (c_{min}=0.1)
Higher Min Consumption (c_{min}=0.5)
No Medical Shock
Higher Discount Factor (β=0.99)
Lifecycle Consumption Comparison
Consumption (c_t)