

TakeHomeMidterm
Econ 8 5 : Macroeconomics II
Svetlana Pashchenko

Life cycle model.

Consider the problem of a retired person who decumulates a given amount of wealth W . He solves the following problem:

$$\max_{c, k_{t-1}} \sum_{t=1}^T \beta^{t-1} u(c_t)$$

s.t.

total resources of the household:

$$res_t = k_t(1+r) + y_t - x_t$$

$$k_1 = W$$

Here k_t is savings, y_t is pension income, and x_t is medical expense shock. There is a means-tested support program that guarantees each household consumption at the level c_{min} if his resources are too low. If $res_t > c_{min}$, then $c_t = res_t - k_{t-1}$, $k_{t-1} \geq 0$. Else, $c_t = c_{min}$ and $k_{t-1} = 0$.

Solve the model using backward induction. Assume CRRA utility function with risk aversion σ : $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. Set $\beta = 0.95, r = 0.04, \sigma = 3, T = 40, c_{min} = 0.1$. For income, set $y_t = 1$ for all t . For initial wealth set $W = 10$. Assume x_t can take two values with probability 0.8 and 0.2. Download the file containing the values for x_t from the course website (xpts40.in). Discretize k using 100 gridpoints, so that $k(1) = 0$ and $k(100) = 100$. Make sure the grid is more dense around 0. When looking for optimal k_{t-1} do NOT restrict it to lie on the grid. Make sure you enforce the constraint $k_{t-1} \geq 0$. (When looking for a maximum you can use Matlab command *fminbnd*.) To find value function outside the grid of k use linear interpolation. (When doing linear interpolation command *interp* can be useful.)

1. Solve the model and plot resulting policy functions for k_{t-1} and value function for ages 10 and 30 using x_t at the 1st and 2nd grid.

Organize your graphs as follows: 2×2 matrix. Left column - savings, right column - value function. Top row - for age 10, bottom row - age 30. Each graph should have 2 lines (clearly labeled): using x_t at the 1st and 2nd grid (command subplot in Matlab can be useful).

2. Simulate $\{x_t\}$ for $t = 1 : 40$. Plot savings over the lifecycle using your policy function.

3. Increase c_{min} to 0.5 and resolve the model. Plot savings over the life cycle.

4. Go back to c_{min} equal to 0.1. Remove medical shock (set $x_t = 0$). Resolve the model and plot savings over the lifecycle.

5. Go back to the initial parametrization with medical shock and increase to 0.99. Resolve the model and plot savings over the lifecycle.

6. Combine saving profiles from questions 2-5 on the same graph and compare. Make sure to clearly label each line. Provide economic intuition.