Homework 5

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An ECON - 8070 Homework Assignment

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Question 15.3

Problem

Let $\mathbf{z}_i$ be a vector of variables. Let z_2 be a continuous variable, and let d_1 be a dummy variable.

(a)

In the model

$$\mathbb{P}(y = 1|\mathbf{z}_1, z_2) = \Phi(\mathbf{z}_1\boldsymbol{\delta}_1 + \gamma_1 z_2 + \gamma_2 z_2^2),$$

find the partial effects of z_2 on the response probability. How would you estimate this partial effect?

(b)

In the model

$$\mathbb{P}(y = 1|\mathbf{z}_1, z_2, d_1) = \Phi(\mathbf{z}_1\boldsymbol{\delta}_1 + \gamma_1 z_2 + \gamma_2 d_1 + \gamma_3 z_2 d_1),$$

find the partial effects of z_2. How would you estimate the effect of d_1 on the response probability? How would you estimate these effects?

(c)

Describe how you would obtain the standard errors of the estimated partial effects from parts a and b.

Solution

(a)

To get the partial effect of z_2 on the response probability, we will take the partial derivative with respect to z_2

$$G(z) = \mathbb{P}(y = 1|\mathbf{z}_1, z_2)$$
$$G(z) = \Phi\mathbf{z}_1\boldsymbol{\delta}_1 + \Phi\gamma_1 z_2 + \Phi\gamma_2 z_2^2$$
$$\boxed{\frac{\partial G(z)}{\partial z_2} = \Phi(\gamma_1 + 2\gamma_2 z_2)}$$

This partial effect can be estimated by Probit regression.

(b)

To get the partial effect of z_2, we follow the same protocol as in (a)

$$\frac{\partial G(z)}{\partial z_2} = \Phi(\gamma_1 + \gamma_3 d_1)$$

To estimate the effects of d_1, we would then evaluate the partial effects at $d_1 = 1, 0$.

$$d_1 = 1 \rightarrow \Phi(\gamma_1 + \gamma_3)$$
$$d_1 = 0 \rightarrow \Phi\gamma_1$$

This would be estimated, again, by a Probit model, being sure to include the interaction term $z_2 \times d_1$

Question 15.5

Problem

Consider the probit model

$$\mathbb{P}(y = 1|\mathbf{z}, q) = \Phi(\mathbf{z}_1\boldsymbol{\delta}_1 + \gamma z_2 q),$$

where q is independent of $\mathbf{z}$ and distributed as $N(0, 1)$; the vector $\mathbf{z}$ is observed but the scalar $\mathbf{q}$ is not.

(a)

Find the partial effect of z_2 on the response probability, namely,

$$\frac{\partial \mathbb{P}(y = 1|\mathbf{z}, q)}{\partial z_2}$$

(b)

Show that $\mathbb{P}(y = 1|\mathbf{z}) = \Phi[\mathbf{z}_1\boldsymbol{\delta}_1/(1 + \gamma_1\gamma_2)^{\frac{1}{2}}]$

(c)

Define $\rho_1 \equiv \gamma_1^2$. How would you test $\mathrm{H}_0 : \rho_1 = 0$?

(d)

If you have a reason to believe $\rho_1 > 0$, how would you estimate $\boldsymbol{\delta}_1$ along with ρ_1?

Question 19.7

Problem

Suppose in section 19.6.1, we replace assumption 19.1d with

$$\mathbb{E}(u_1 \mid v_2) = \gamma_1 v_2 + \gamma_2(v_2^2 - 1).$$

(We subtract unity from v_2^2 to ensure that the second term has zero expectation.)

(a)

Using the fact that $\text{Var}(v_2 \mid v_2 > -a) = 1 - \lambda(a)[\lambda(a) + a]$, show that

$$\text{E}(y_1 \mid \mathbf{x}, y_2 = 1) = \mathbf{x}_1\boldsymbol{\beta}_1 + \gamma_1\lambda(\mathbf{x}\boldsymbol{\delta}_2) - \gamma_2\lambda(\mathbf{x}\boldsymbol{\delta}_2)\mathbf{x}\boldsymbol{\delta}_2.$$

[Hint: Take $a = \mathbf{x}\boldsymbol{\delta}_2$ and use the fact that $\text{E}(v_2^2 \mid v_2 > -a) = \text{Var}(v_2 \mid v_2 > -a) + [\text{E}(v_2 \mid v_2 > -a)]^2$.]

(b)

Explain how to correct for sample selection in this case.

(c)

How would you test for the presence of sample selection bias?