# Overview of Classes of Functions Comparison

**The video compares common classes of functions by their growth rates (asymptotic behavior) for large values of n, arranged in increasing order: constant < log n < n < n log n < n² < n^k (any k) < 2^n.**

This order shows how functions grow relative to each other as **n** increases significantly (typically n → ∞).

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## Step 1: The Ordered Hierarchy

Functions are listed from **slowest-growing** to **fastest-growing**:

1. **Constant** (e.g., 1) - Always fixed.

2. **Log n** (e.g., $\log_2 n$) - Very slow growth.

3. **Linear (n)** - Steady linear increase.

4. **n log n** - Linear times log (faster than linear).

5. **Polynomial (n², n³,..., n^k for any fixed k)** - Quadratic, cubic, etc.

6. **Exponential (2^n)** - Extremely rapid growth.

**Key inequality**: For large n, each grows slower than the next (e.g., $n^{100} < 2^n$ eventually).

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## Step 2: Proving with Numerical Examples

**Demonstration uses specific n values to show when faster functions overtake slower ones.**

### Table of Values (Selected Examples from Video)

| n | $\log_2 n$ | n | $n^2$ | $2^n$ |
|---|---|---|---|---|
| **1** | 0 | 1 | 1 | 2 |
| **2** | 1 | 2 | 4 | 4 |
| **4** | 2 | 4 | 16 | 16 |

| n | $\log_2 n$ | n | $n^2$ | 2^n |
|---|---|---|---|---|
| **8** | 3 | 8 | 64 | 256 |

**Observations**:

- Up to n=4, some values equal (e.g., $n^2$ = 2^n = 16).
- At n=8, **exponential surges ahead** (256 >> 64).
- For n=9: $\log_2 9 \approx 3.17$, $n^2 = 81$, 2^9=512 - gap widens.

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## Step 3: Growth Rate Insights

**Slower functions may lead initially but get dominated for large n.**

- **Log n**: Slowest after constant.
- **Polynomials (n^k)**: Even high powers like **n^100** eventually lose to **2^n**.
- **Exponential**: "Sudden change" and "very faster growth."

**For small n (1-10)**: Order may fluctuate (less > greater sometimes).

**For large n**: Order holds strictly.
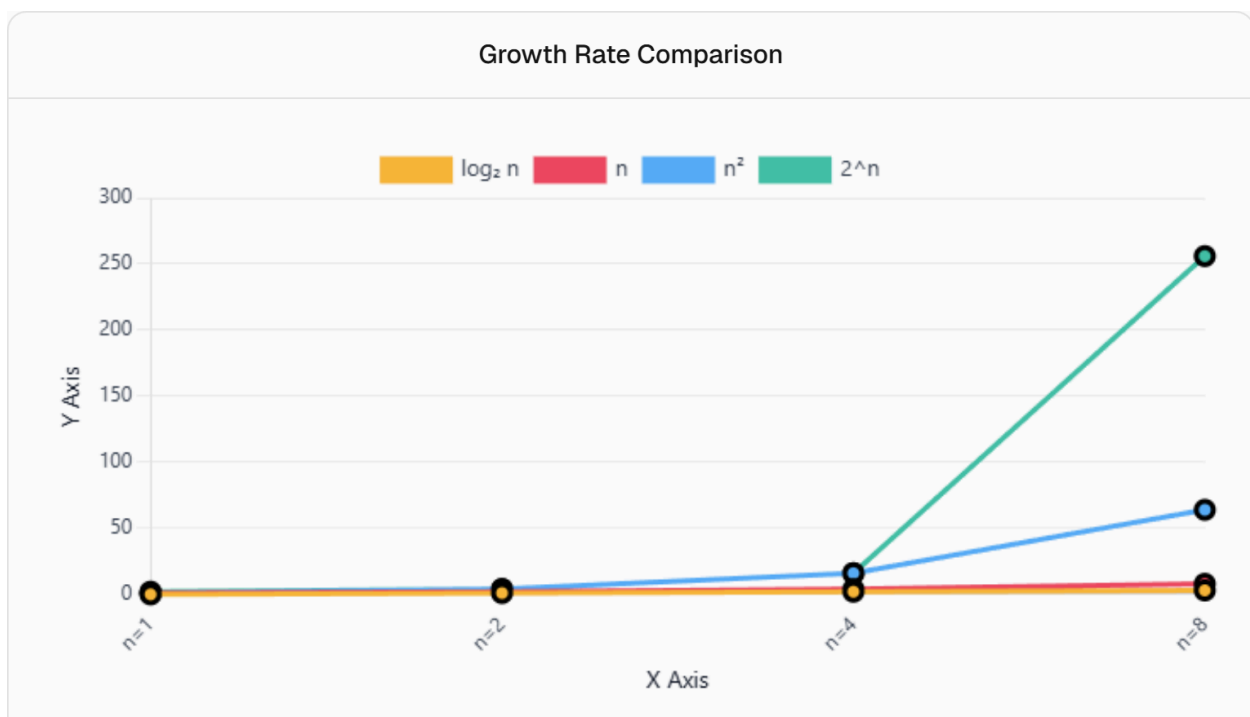


Growth Rate Comparison

**Chart shows**: Linear/log stay low; n² grows steadily; 2^n explodes at n=8.

## Step 4: Graphical Intuition

**Imagine plotting on a graph (n on x-axis, f(n) on y-axis):**

- **Log n**: Shallow curve.

- **n**: Straight line (45°).

- **n²**: Steeper parabola.

- **2^n**: Hockey stick - flat then vertical rise.

**Initially**: Higher powers may be below exponentials.

**Eventually**: Exponential overtakes everything.