Overview of Asymptotic Notations Examples

This video demonstrates applying Big O, Omega, and Theta notations to specific functions through bounding techniques.

Key principle: Compare **$f(n) \leq c \cdot g(n)$** for **Big O** (upper bound), **$f(n) \geq c \cdot g(n)$** for **Omega** (lower bound), and both for **Theta** (tight bound), for some **$c > 0$** and **$n \geq n_0$**.

Example 1: f(n) = 2n^2 + 3n + 4

Upper Bound (Big O)

1. **Bound above**: $$2n^2 + 3n + 4 \leq 2n^2 + 3n^2 + 4n^2 = 9n^2$$ for **$n \geq 1$**.
2. Thus, **$f(n) = O(n^2)$** (with **$c=9$**, **$g(n)=n^2$**).

Lower Bound (Omega)

1. **Bound below**: $$2n^2 + 3n + 4 \geq 1 \cdot n^2$$ (dominant term).
2. Thus, **$f(n) = \Omega(n^2)$** (with **$c=1$**, **$g(n)=n^2$**).

Tight Bound (Theta)

- **$$n^2 \leq f(n) \leq 9n^2$$**, so **$f(n) = \Theta(n^2)$**.

Example 2: f(n) = n^2 \log n + n

All Three Notations

1. **Upper bound**: $$n^2 \log n + n \leq 10 n^2 \log n$$ (for large n, constant absorbs lower term).
2. **Lower bound**: $$n^2 \log n + n \geq 1 \cdot n^2 \log n$$.
3. **Sandwiched**: **$$n^2 \log n \leq f(n) \leq 10 n^2 \log n$$**, so **$f(n) = O(n^2 \log n) = \Omega(n^2 \log n) = \Theta(n^2 \log n)$**.

Position in hierarchy: **$n^2 < n^2 \log n < n^3$**.

Example 3: f(n) = n!(n factorial = n \times (n-1) \times \cdots \times 1)

Bounding Technique

- Rewrite: **1 × 2 ×… × n**.
- **Lower bound**: $$n! \geq 1$$ (trivially, or all 1s).
- **Upper bound**: Replace terms with n: $$n! \leq n \times n \times \cdots \times n = n^n$$ (n terms).

Results

- **f(n) = O(n^n)** (upper bound).
- **f(n) = Ω(1)** (lower bound).
- **No Theta**: Bounds don't match; **n!** grows between **1** and **$$n^n$$**, no tight class.

Growth behavior: Starts near lower functions for small n, approaches **$$n^n$$** for large n.

Example 4: f(n) = \log(n!)

Bounding Technique

- **$$ \log(n!) = \log(1 \times 2 \times \cdots \times n) $$**.
- **Lower bound**: **≥ 1** (or log of all 1s ≈ 0, but use 1).
- **Upper bound**: **≤ log($$n \times n \times \cdots \times n$$) = log($$n^n$$) = n log n**.

Results

- **f(n) = O(n log n)**.
- **f(n) = Ω(1)**.
- **No Theta**: Similar to n!, no tight bound.

Key Guidelines for Choosing Notations 💡

Prefer Theta When Possible

- **Theta** gives **exact growth rate** (tight bound on both sides).
- Example: For **n^2**, use **$\Theta(n^2)$**, not looser **$O(n^3)$** or **$\Omega(n)$**.

When to Use Big O / Omega

- Use when **no tight bound** exists (e.g., **n!**, **log n!**).
- **Big O**: Loose upper bound OK, but prefer tight (e.g., not **O(n^{100})** for **n^2**).
- **Omega**: Loose lower bound OK, but prefer tight.