LGV Lemma

Some background theory

$$\forall M_{n\times n} := (m_{i\cdot j})_{i,j=1}^{n}$$
$$\mathrm{Inv}(\sigma) := \{(i,j) | 1 \leq i < j \leq n, \sigma(i) > \sigma(j)\}$$
$$\mathrm{sgn}(\sigma) := (-1)^{|\mathrm{Inv}(\sigma)|}$$
$$\det(M) := \sum_{\sigma\in S[n]} \mathrm{sgn}(\sigma)\cdot\prod_{i=1}^{n} m_{\sigma(i)\cdot i}$$

Properties of sgn

- $\forall \sigma, \tau \ni (\forall \sigma(i) > \sigma(j), \tau(j) > \tau(i)), \mathrm{sgn}(\tau) = -\mathrm{sgn}(\sigma)$

Motivation

Suci Prastara

- Suggested by Pingala (200 BCE), is a method to compute Binomial coefficients

$$A := (a_{i\cdot j}), a_{i\cdot j} = \Big\{ 1 \text{ if } i = 0, \textstyle\sum_{k=1}^{j} a_{(i-1)\cdot k} \text{ otherwise} \quad = \binom{i+j}{i}$$

$$A := \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & ... \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & ... \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 & 36 & ... \\ 1 & 4 & 10 & 20 & 35 & 56 & 84 & 120 & ... \\ 1 & 5 & 15 & 35 & 70 & 126 & 210 & 336 & ... \\ 1 & 6 & 21 & 56 & 126 & 252 & 462 & 792 & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Every minor of this matrix is positive.

Halayudha method (aka Meru Prastara, aka Pascal's triangle)

$$B := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & ... \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & ... \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 & 0 & ... \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 & ... \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 & 0 & ... \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 & 0 & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$