Young's Lattice

Definitions

$$\forall x \in \mathbb{P}$$

$$\Lambda_x = \left\{ (\lambda_i)_{i\in[n]} \middle| n < \infty \land \forall i, \lambda_i \in \mathbb{P} \land \forall i, \lambda_i \geq \lambda_{i+1} \land \sum_{i\leq n} \lambda_i = x \right\}$$

Ferrer's diagraph of λ

$$\forall x \in \mathbb{P}, \forall \lambda \in \Lambda_x$$

$$Y\ \lambda := \{(i,\ j)|1 \leq i \leq l, 1 \leq j \leq \lambda_i\}$$

The above set can be represented using a 2D array of dots/boxes such that each succeeding row has less (or equal) elements than the preceding row

Partial order on the union of all Λ_x

$$\mathbb{Y} = \bigcup_{x\in\mathbb{P}} \Lambda_x$$

then,

$$\forall \lambda, \mu \in \mathbb{Y}, \mu \subseteq \lambda, \iff |\mu| \leq |\lambda| \land, \forall i \in [|\mu|]\, \mu_i \leq \lambda_i$$

$$\implies \forall \lambda, \mu \in \mathbb{Y}, \mu \subseteq \lambda \iff Y\ \mu \subseteq Y\ \lambda$$

$$\implies (\mathbb{Y}, \subseteq) \longleftrightarrow (\{Y\ \rho | \rho \in \mathbb{Y}\}, \subseteq)$$

$$\forall \lambda, \mu \in \mathbb{Y}$$

$$\lambda \vee \mu = \begin{cases} () & \text{iff } \lambda = (), \mu = () \\ \mu & \text{iff } \lambda = (), \mu \neq () \\ \lambda & \text{iff } \lambda \neq (), \mu = () \\ (\lambda_1) + (\tau_{i-1} = \lambda_i)_{i\in[2,|\lambda|]} \vee \mu & \text{iff } \lambda_1 \leq \mu_1, \\ (\mu_1) + \lambda \vee (\tau_{i-1} = \mu_i)_{i\in[2,|\mu|]} & \text{iff } \lambda_1 > \mu_1. \end{cases}$$