Derivatives of a species

Definition

Let F be a species of structures, then we define it's derivative $F' = \frac{d}{dx}F := F[U^+] \ni U^+ = U \cup \{*_U\}$

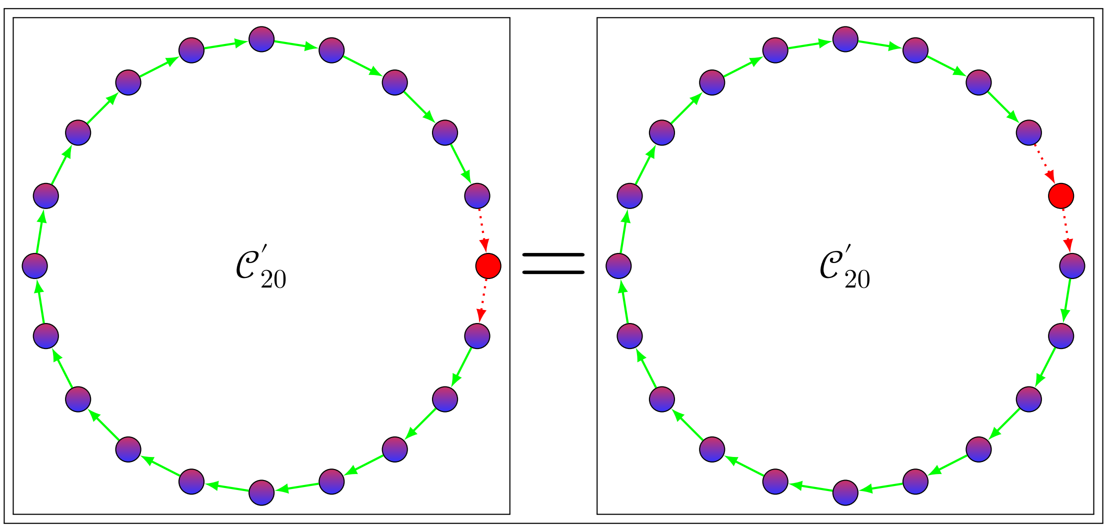
$$\forall, \sigma : U \longrightarrow V, \sigma^+ : U^+ \longrightarrow V^+ := \begin{cases} \sigma(u) & \text{iff } u \in U \\ *_v & \text{otherwise.} \end{cases}$$

Example 1

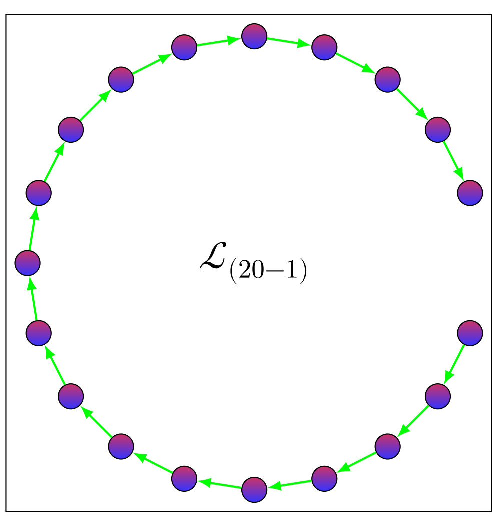
$$\mathcal{C}' \cong \mathcal{L}$$

Proof 1

Let $\mathcal{C}_n$ be species of cycles with n nodes,
then, for $\mathcal{C}'_n$ we just "point out" a node from the $\mathcal{C}_n$
$\because$ it doesn't really matter which node you "point out" from the cycle $(\because, [n] = \langle n, 1, ..., n-1 \rangle)$
We consider the resulting set as just a singleton set consisting of only the Cycle with n nodes,
Which as you can see visually in the following figure is isomorphic to a Linear Ordering with $n-1$ nodes.



$$\cong$$



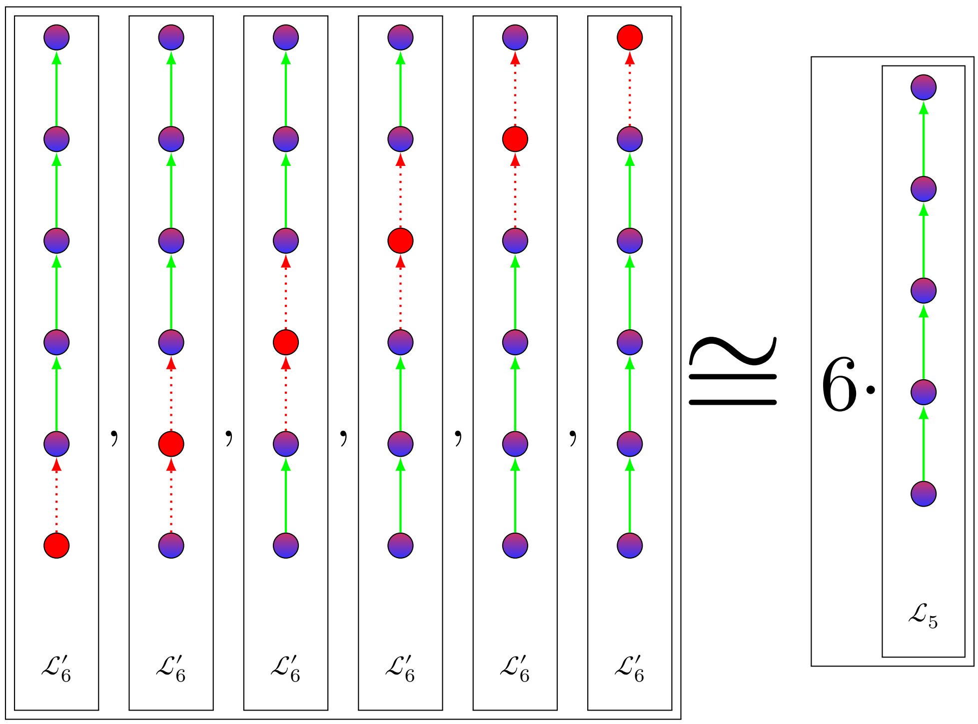
Example 2

$$(X^n)' \cong n \cdot X^{n-1}$$

Proof 2

$$X^n \cong \mathcal{L}_n$$

This assertion is trivial because X is the species of singletons, and if you create an ordered tuple of n singletons, that is the same as having a linear order with n nodes. $\implies \mathcal{L}'_n \cong n \cdot \mathcal{L}_{n-1}$ Following is the visual proof of this.



Note that in a Linear order, the selected node **does** change the original order, because unlike in case of cycles, rotations of a Linear order are not the same. Hence we end with a set of n distinct structures in the set. HOWEVER all of these structures are isomorphic to $\mathcal{L}_{n-1}$ so, after performing the transformation, we're left with n structures of type $\mathcal{L}_{n-1}$ which are equal.