The fundamental theorem of finite distributive lattices (Birkhoff)

$$\forall(\Lambda, \leq, \vee, \wedge) \ni |\Lambda| < \infty \wedge \forall a, b, c \in \Lambda$$

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$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

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$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

Theorem: -

$$P \subseteq \{x \in \Lambda | \forall y \in \Lambda, x \vee y = x\}$$

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$$\Lambda \cong J(P)$$

Proof:
Let $\phi_P : \Lambda \longrightarrow J(P)$

$$\phi_P = \lambda x. \{y \in P | y \leq_\Lambda x\}$$

Here, ϕ_P is an order preserving injection.

Lemma:

$$\forall x \in \Lambda \exists Y \subset \{z \in \Lambda | \forall y \in \Lambda, z \leq y\}, \bigvee_{y \in Y} y$$