

Algebra of Matrices

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① Practice Examples :-

(a) Test for consistency and solve.

(ii)  $2x - 3y + 7z = 5$ ,

$3x + y - 3z = 13$ ,

$2x + 19y - 47z = 32$ .

Here,  $A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}$ ,  $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$

Solu:-  $\bar{T}$  can be written in form of  $AX = B$ .  $\left. \begin{array}{l} \text{if } \text{rank } AX = B \\ \therefore \text{non-homogeneous} \end{array} \right\}$ 

$\therefore [A:B] = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 19 & -47 & : & 32 \end{bmatrix}$

$\approx \begin{bmatrix} 3 & 1 & -3 & 13 \\ 2 & -3 & 7 & 5 \\ 2 & 19 & -47 & 32 \end{bmatrix}$

$\begin{bmatrix} 1 & \frac{1}{3} & -1 & 13 \\ 2 & -3 & 7 & 5 \\ 2 & 19 & -47 & 32 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{bmatrix} 1 & \frac{1}{3} & -1 & 13 \\ 0 & -22 & 54 & -27 \\ 2 & 19 & -47 & 32 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \begin{bmatrix} 1 & \frac{1}{3} & -1 & 13 \\ 0 & -22 & 54 & -27 \\ 0 & 55 & -44 & -7 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$\begin{bmatrix} 1 & \frac{1}{3} & -1 & 13 \\ 0 & -22 & 54 & -27 \\ -55 & 0 & 11 & -708 \end{bmatrix} \xrightarrow[R_3 \rightarrow R_3 - 5R_1]{CR_3} \begin{bmatrix} 1 & \frac{1}{3} & -1 & 13 \\ 0 & -22 & 54 & -27 \\ 0 & 0 & 11 & -708 \end{bmatrix}$

$\therefore [A:B] = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 19 & -47 & : & 32 \end{bmatrix} \xrightarrow[R_1 \rightarrow \frac{1}{2}R_1]{R_2 \rightarrow R_2 - R_1} \begin{bmatrix} 1 & -\frac{3}{2} & \frac{7}{2} & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{bmatrix} \xrightarrow[R_2 \rightarrow R_2 - 3R_1]{R_3 \rightarrow R_3 - 4R_1} \begin{bmatrix} 1 & -\frac{3}{2} & \frac{7}{2} & 5 \\ 0 & \frac{11}{2} & -\frac{27}{2} & -2 \\ 0 & 0 & 3 & 3 \end{bmatrix} \xrightarrow[R_3 \rightarrow R_3 - 2R_1]{R_3 \rightarrow R_3 - 2R_2} \begin{bmatrix} 1 & -\frac{3}{2} & \frac{7}{2} & 5 \\ 0 & \frac{11}{2} & -\frac{27}{2} & -2 \\ 0 & 0 & -54 & 22 \end{bmatrix}$

Here,  $P(A:B) = 3$ ,  $P(A) = 2$   $\neq$  no. of unknown.  
 $P(A) \neq P(A:B)$ 

∴ The given system is inconsistent (No Solution)

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$$(ii) \quad 2x - y + 3z = 8$$

$$-x + 2y + z = 4$$

$$3x + y - 4z = 0$$

Solve:- Given eq. can be written in form of.  $AX=B$ .

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

Augmented matrix.

$$[A:B] = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{2}R_1} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 4 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 4 \\ 0 & \frac{3}{2} & \frac{1}{2} & 8 \\ 3 & 1 & -4 & 0 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - 3R_1} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 4 \\ 0 & \frac{3}{2} & \frac{1}{2} & 8 \\ 0 & \frac{5}{2} & -\frac{1}{2} & -12 \end{bmatrix}$$

Here, Rank,  $P(A:B) = P(A) = 3$

$\therefore$  System is consistent. ( $\because$  Unique soln)

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} & 4 \\ 0 & \frac{3}{2} & \frac{1}{2} & 8 \\ 0 & 0 & -4 & -\frac{76}{3} \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - \frac{5}{3}R_2}$$

Eqs are:-  $\therefore -\frac{4}{3}z = -\frac{76}{3} \Rightarrow z = 19$

$$\frac{3}{2}y + \frac{1}{2}z = 8 \Rightarrow \frac{3}{2}y + \frac{19}{2} = 8 \Rightarrow \frac{3}{2}y = -\frac{3}{2} \Rightarrow y = -1$$

$$x - \frac{1}{2}y + \frac{3}{2}z = 4 \Rightarrow x + \frac{1}{2} + \frac{57}{2} = 4 \Rightarrow 2x + 58 = 8 \Rightarrow 2x = -50 \Rightarrow x = -25$$

$$(iii) \quad 4x - y = 12$$

$$-x + 5y - 2z = 0$$

$$-2x + 4z = -8$$

Solve:- Given eq. can be in form of  $AX=B$ ,  $\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 8 \end{bmatrix}$

$$\therefore [A:B] = \begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ 2 & 0 & 4 & 8 \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{4}R_1} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ -1 & 5 & -2 & 0 \\ 2 & 0 & 4 & 8 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 6 & -2 & 3 \\ 2 & 0 & 4 & 8 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 6 & -2 & 3 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

Here,  $P(A|B) = P(A) = 3 = n$ .

$\therefore$  Consistent & Unique

$$\begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 6 & -2 & 3 \\ 0 & 0 & 4 & 2 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - \frac{2}{3}R_2} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 6 & -2 & 3 \\ 0 & 0 & 0 & \frac{10}{3} \end{bmatrix}$$

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$$\text{Now, } \frac{80}{19}z = \frac{32}{19} \Rightarrow z = \frac{32}{80} \xrightarrow{\cancel{8}} \frac{2}{5} \Rightarrow z = \frac{2}{5}$$

$$\frac{19}{4}y - 2z = 3 \Rightarrow \frac{19}{4}y - \frac{4}{5} = 3 \Rightarrow \frac{19}{4}y = \frac{19}{5} \Rightarrow y = \frac{4}{5}$$

$$x + \frac{1}{4}y = 3 \Rightarrow x + \frac{1}{5} = 3 \Rightarrow x = 3 - \frac{1}{5} \Rightarrow x = \frac{14}{5}$$

(b) For what values of  $\lambda$  in  $M$  the given sys of eqn

$$x+y+z=6 \quad \text{has (i) no solution.}$$

$$x+2y-3z=10 \quad \text{(ii) a unique soln.}$$

$$x+2y+\lambda z=M \quad \text{(iii) infinite no. of soln.}$$

Soln:- Given eq in form of  $AX=B$ ,  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ M \end{bmatrix}$

$$\therefore [A:B] = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & -3 & 10 \\ 1 & 2 & \lambda & M \end{array} \right] \xrightarrow[R_3 \rightarrow R_3 - R_2]{\cancel{R_1 \rightarrow R_1}} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & -3 & 10 \\ 0 & 0 & (\lambda+3)(M-10) & 0 \end{array} \right] \xrightarrow[R_2 \rightarrow R_2 - R_1]{\downarrow} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & (\lambda+3)(M-10) & 0 \end{array} \right]$$

Here,  $P(A:B) = P(A) = 3 = \text{no. of unknown.}$

$\therefore$  Consistent & Unique soln.

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & (\lambda+3)(M-10) & 0 \end{array} \right]$$

Echlon form

(i) No Soln., if  $\lambda=3, M \neq 10$ ; rank  $P(A)=2, P(A:B)=3$ .  
 $P(A) \neq P(A:B)$ .

$\therefore$  inconsistent and no soln.

(ii) Unique Soln; if  $\lambda \neq 3$ , then  $P(A)=P(A:B)=n$ .

$\therefore$  unique soln.

(iii) Infinite Soln; if  $\lambda=3, M=10$ , then,  $P(A)=P(A:B)=2 \neq n$ .

$\therefore$  infinite soln.

(C) Find for what value of  $\lambda$  the given eq.

$$x+y+z=1$$

have a sol<sup>n</sup> and solve them completely in each case

$$x+2y+4z=\lambda$$

$$x+4y+10z=\lambda^2$$

Solu:- Given eq. in form of  $AX=B$ ,  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$

$$\begin{bmatrix} A:B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \xrightarrow[R_2 \rightarrow R_2 - R_1]{R_3 \rightarrow R_3 - R_1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 3 & 9 & (\lambda^2-1) \end{bmatrix} \xrightarrow[R_3 \rightarrow R_3 - 3R_2]{R_3 \rightarrow R_3 - 3R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 0 & 0 & 0 & (\lambda^2-3\lambda+2) \end{bmatrix}$$

$$\text{Here, } P(A:B) = 3, P(A) = 2$$

$P(A:B) \neq P(A) \therefore \text{Inconsistent}$

In order that given eq. has sol<sup>n</sup>, it should be consistent

$$P(A:B) \text{ should } = P(A)$$

Rank-A = 2, Since rank of  $[A:B]$  should also be 2, it is necessary that

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda + \lambda + 2 = 0 \Rightarrow \lambda(\lambda-2) - 1(\lambda-2) = 0 \Rightarrow (\lambda-1)(\lambda-2) = 0$$

$$\Rightarrow \boxed{\lambda = 1, 2}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Case 1 :- For  $\lambda = 1$ ,  $x+y+z = 1$ .

$$y+3z = 0$$

No. of unknown (3)  $>$  Rank of A(2)

$\therefore$  Equations have infinite solution and we have to assign 3-2 = 1 param  
 say k.

$$\text{Let } z=k, \therefore y+3k=0 \Rightarrow \boxed{y=-3k}$$

$$\& x+y+z=1 \Rightarrow x-3k+k=1 \Rightarrow \boxed{x=2k+1}$$

Case 2

$$\text{For } \lambda=2 \quad \therefore \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore x+y+z=1.$$

$$y+3z=1.$$

Here, also we have to assign  $3-2=1$  para. say  $1k$ .

Let  $z=k$ .

$$\therefore y=1-3k$$

$$\therefore x+y+z=1.$$

$$\Rightarrow x+1-3k+k=1 \Rightarrow x=2k$$

- (d) Find the soln. of system of eqn  $x+3y-2z=0$ ,  $2x-y+4z=0$ ,  $x-11y+14z=0$ .

Soln:- Here, given eq. are.

$$x+3y-2z=0 \quad \text{it can be expressed in } AX=B$$

$$2x-y+4z=0 \quad \cdot \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x-11y+14z=0$$

$$\text{Now, } [A \mid B] = \left[ \begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1}} \left[ \begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 1 & -11 & 14 & 0 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_3 - R_1}} \left[ \begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -8 & 12 & 0 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_3 - \frac{8}{7}R_2}} \left[ \begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & -\frac{40}{7} & 0 \end{array} \right]$$

$$P(A) = 3 = \text{no. of unknowns}$$

$\therefore$  Consistent and has Unique Soln. Ans.

- (e) Find for what values of  $\lambda$  the given eqs  $3x+y-\lambda z=0$ ,

$4x-2y-3z=0$ ,  $2\lambda x+4y+\lambda z=0$  may possess non-trivial solution. and solve them completely in each case.

Soln:- Given eqs can be in form of  $AX = B$

$$3x - y - \lambda z = 0$$

$$4x - 2y - 3z = 0$$

$$2\lambda x + 4y + z = 0$$

$$\begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here,  $AX=0$  ( $\because$  System is consistent (Unique / Infinite)).

$$\text{Now, } A = \begin{bmatrix} 3 & -1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & 1 \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{3}R_1} \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{\lambda}{3} \\ 4 & -2 & -3 \\ 2\lambda & 4 & 1 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 4R_1} \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{\lambda}{3} \\ 0 & -10 & -3-2\lambda \\ 2\lambda & 4 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 - 2\lambda R_1} \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{\lambda}{3} \\ 0 & -10 & -3-2\lambda \\ 0 & 14 & 5\lambda \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 + 10R_3} \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{\lambda}{3} \\ 0 & 1 & -3-2\lambda \\ 0 & 14 & 5\lambda \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 - 14R_2} \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{\lambda}{3} \\ 0 & 1 & -3-2\lambda \\ 0 & 0 & 11\lambda+21 \end{bmatrix}$$

Here,  $P(A) = \text{no. of unknown} = 3$ ,

$\therefore$  Consistent & unique.

For system to possess non-trivial soln. (infinite soln).

$P(A) < \text{No. of unknown}$

$$\therefore \frac{11\lambda+21}{11} = 0 \Rightarrow 11\lambda = 21 \Rightarrow \boxed{\lambda = \frac{21}{11}}$$

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# Linear Algebra

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## Assignment - 2

Problems:-

Are the following sets of vectors • linearly independent or dependent?

1)  $[1 \ 0 \ 0], [1 \ 1 \ 0], [1 \ 1 \ 1]$

Solve:- Let  $v_1 = (1, 0, 0)$ ,  $v_2 = (1, 1, 0)$ ,  $v_3 = (1, 1, 1)$

Now, Matrix (A) =  $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \text{Det}(A) = 1(1-0) - 0 + 0 = 1 \neq 0$

$\therefore$  It is linearly independent.

Note:- Different methods to find LI & LD :-

① Definition check:

• if the only sol<sup>n</sup> is  $c_1v_1 + c_2v_2 + c_3v_3 + \dots + c_nv_n = 0$  is  
 $c_1 = c_2 = \dots = c_n = 0$ , then LI.

② Matrix Determinant method: [vectors as columns]

If  $\text{det}(A) \neq 0$ , then LI.

③ Row Reduction (Gaussian Elimination)

$A : B \left[ \begin{array}{cc|c} a & b & 0 \\ c & d & 0 \end{array} \right] \quad r(A) =$

④ Rank Nullity Theorem

If the rank of the matrix formed by the vectors equal no. of vectors, they are linearly independent.

Eg:-  $v_1 = [1, 2]$ ,  $v_2 = [3, 4]$ , Form matrix & calc. its rank

$$\boxed{P(A) + N = D}$$

$\downarrow \quad \downarrow \quad \downarrow$   
Rank Nullity Dimension/  
no. of col.

$$A = \left[ \begin{array}{cc} 1 & 3 \\ 2 & 4 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \left[ \begin{array}{cc} 1 & 3 \\ 0 & -2 \end{array} \right]$$

$$P(A) = 2.$$

$$\text{Dimension (no. of col)} = 2$$

$$\therefore N = D - P(A)$$

$$= 2 - 2$$

$$= \boxed{0}$$

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$$2. \begin{bmatrix} 7 & -3 & 11 & -6 \end{bmatrix}, \begin{bmatrix} -56 & 24 & -88 & 48 \end{bmatrix}.$$

Soln:- Let  $a \begin{bmatrix} 7 & -3 & 11 & -6 \end{bmatrix} + b \begin{bmatrix} -56 & 24 & -88 & 48 \end{bmatrix} = 0$ .

$$\therefore 7a - 56b = 0 \Rightarrow a = 8b$$

$$-3a + 24b = 0$$

$$11a - 88b = 0 \rightarrow \text{Put } a = 8b \Rightarrow 11(8b) - 88b = 0 \Rightarrow 0 = 0.$$

$$-6a + 48b = 0 \quad (\text{non-trivial soln})$$

Let  $b = k$ , then  $a = 8k$   $\therefore \text{Eigen Space} = K \begin{bmatrix} 8 \\ 1 \end{bmatrix}$

Also, the second vector is 8 times the first vector.

$\therefore$  It is Linearly Dependent.

$$3. \begin{bmatrix} -1 & 5 & 0 \end{bmatrix}, \begin{bmatrix} 16 & 8 & -3 \end{bmatrix}, \begin{bmatrix} -64 & 56 & 9 \end{bmatrix}$$

Soln:- Now,  $A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$

$$\det(A) = -1(72 + 168) - 16(45 - 0) + (-64)(-15 - 0)$$

$$= -96 - 720 + 960$$

$$= \boxed{144} \neq 0 \quad \therefore \text{It is linearly Independent}$$

$$4. \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

Soln:- Let  ~~$(A:B)$~~   $A:B = \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + R_2} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right]$

$$\rightarrow A:B = \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 + R_1} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{array} \right]$$

$$\text{Here, } P(A) = 3 = P(A:B)$$

$\therefore$  It is Linearly Independent. [also vectors are not scalar multiple of each other]

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$$5) [2 - 4], [1 9], [3 5]$$

Solve - Let  $a[2 - 4] + b[1, 9] + c[3 5] = 0$

$$\begin{aligned} \therefore 2a + b + 3c &= 0 \quad \text{--- (i)} \\ -4a + 9b + 5c &= 0 \quad \text{--- (ii)} \end{aligned}$$

$$\text{Put } b \text{ in (i)}, \quad -4a + 9(-3c - 2a) + 5c = 0$$

$$\Rightarrow -4a - 27c - 18a + 5c = 0$$

$$\Rightarrow -22a - 22c = 0$$

$$\Rightarrow \boxed{a = -c}$$

$$\text{So, from (i), } b = -3a - 2a = -5a$$

if  $a = k$ , then

Now, here we have.  $x = \begin{bmatrix} k \\ -5k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$  is a non-trivial soln

Hence, it's a linearly dependent (set of vectors).

$$6) [3 - 2 0 4], [5 0 0 1], [-6 1 0 1], [2 0 0 3]$$

Solve - Here,  $c_1v_1 + c_2v_2 + c_3v_3 = 0$

For scalars  $c_1, c_2, c_3$  where  $v_1, v_2, v_3$  are given vectors.

$$\therefore c_1 \begin{bmatrix} 3 \\ -2 \\ -6 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_4 \begin{bmatrix} 4 \\ 1 \\ 1 \\ 3 \end{bmatrix} = 0$$

$$\therefore 3c_1 - 2c_2 + 4c_4 = 0 \quad \text{--- (i)}$$

$$5c_2 + c_4 = 0 \Rightarrow \boxed{c_4 = -5c_2}$$

$$-6c_1 + c_2 + c_4 = 0 \quad \text{--- (ii)}$$

$$2c_1 + 3c_4 = 0 \quad \text{--- (iii)}$$

$$\text{Now, Put } c_4 \text{ in (i), } 3c_1 - 2(-5c_2) + 4c_4 = 0 \Rightarrow 3c_1 + 10c_2 = 0 \Rightarrow \boxed{c_1 = 0}$$

$$\text{Now, from (ii) \& (iii), } 2(3c_1 - 2c_2 + 4c_4) = 0$$

$$-6c_1 + c_2 + c_4 = 0$$

$$-c_2 + 5c_4 = 0 \Rightarrow \boxed{c_2 = 5c_4}$$

$$\therefore c_4 = 0 \Rightarrow \boxed{c_2 = 0}$$

Since,  $c_1 = c_2 = c_3 = c_4 = 0$  Teacher's Signature.....  
 $\therefore$  Linearly Independent

7)  $[3 \ 4 \ 7], [2 \ 0 \ 3], [8 \ 2 \ 3], [5 \ 5 \ 6]$

Soln -

$$\text{let } A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$$

Now, we perform elementary row operations, to convert into row echelon form.

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_3} \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 3 & 3 & 1 & 1 \end{bmatrix} \xrightarrow{\substack{R_3 \rightarrow R_3 - R_1 \\ R_2 \rightarrow R_2 - R_1}} \begin{bmatrix} 3 & 2 & 8 & 5 \\ 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 8 & 26 & 5 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 8 & 26 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 8R_1} \begin{bmatrix} 1 & -2 & -6 & 0 \\ 3 & 2 & 8 & 5 \\ 0 & 1 & -7 & -4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2}$$

$$\downarrow R_3 \rightarrow R_3 - 8R_2$$

$$\begin{bmatrix} 1 & -2 & -6 & 0 \\ 0 & 1 & -7 & -4 \\ 0 & 0 & 82 & 37 \end{bmatrix}$$

Here By Rank Nullity Theorem,

$$r(A) + N = D$$

$$\Rightarrow 3 + N = 4 \Rightarrow N = 1$$

Here, no. of unknown = 4 = n

But  $r(A) = 3 \neq n$

$\therefore$  It is infinite sol. and linearly Independent.

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$$8) \begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 2 & 7 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 12 & 3 & 0 & -19 & 8 & -11 \end{bmatrix}$$

Solve: Let  $c_1 \begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 & -1 & 2 & 7 & 0 & 5 \end{bmatrix} + c_3 \begin{bmatrix} 12 & 3 & 0 & -19 & 8 & -11 \end{bmatrix} = 0$

$$\textcircled{1} \quad \because 6c_1 + 12c_3 = 0 \Rightarrow c_1 = -2c_3$$

$$\textcircled{2} \quad -c_2 + 3c_3 = 0 \Rightarrow c_2 = 3c_3$$

$$\textcircled{3} \quad 3c_1 + 2c_2 = 0 \Rightarrow c_2 = -\frac{3}{2}c_1$$

$$\textcircled{4} \quad c_1 + 7c_2 - 19c_3 = 0 \Rightarrow c_1 - \frac{21}{2}c_1 - 19c_3 = 0 \Rightarrow -\frac{19}{2}c_1 - 19c_3 = 0$$

$$\Rightarrow -\frac{19}{2}(-2c_3) - 19c_3 = 0 \quad [\because c_1 = -2c_3]$$

$$\Rightarrow \frac{38}{2}c_3 - 19c_3 = 0 \Rightarrow 38c_3 - 38c_3 = 0 \Rightarrow [0 = 0]$$

$$\textcircled{5} \quad 4c_1 + 8c_3 = 0 \Rightarrow 4(-2c_3) + 8c_3 = 0 \Rightarrow -8c_3 + 8c_3 = 0 \quad [0 = 0]$$

$$\textcircled{6} \quad 2c_1 + 5c_2 - 11c_3 = 0$$

$$\Rightarrow 2(-2c_3) + 5(3c_3) - 11c_3 = 0$$

$$\Rightarrow -4c_3 + 15c_3 - 11c_3 = 0$$

$$\Rightarrow -15c_3 + 15c_3 = 0$$

$$\Rightarrow [0 = 0]$$

Let  $c_3 = k$ , then,

$$c_1 = -2k, \quad c_2 = 3k$$

$$\therefore X = \begin{bmatrix} -2k \\ 3k \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \text{ this is infinite soln.}$$

$\therefore$  It is linearly Dependent

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# Linear Algebra

Hiteendra Deoveryoung Date \_\_\_\_\_

## Assignment - 4

Q. 1. Find the rank of the matrix A by reducing in Row reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Solve:- Performing row operations :

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow{\substack{R_4 \rightarrow R_4 - 2R_3 \\ R_2 \rightarrow R_2 - 2R_1}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \xrightarrow{\substack{R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 + R_3}} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$$\xleftarrow{R_3 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_4 \rightarrow R_4 - R_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xleftarrow{R_4 \rightarrow R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, no. of non-zero rows = 3

$$\therefore P(A) = 3 \quad \text{Ans}$$

Q. 2. Let W be the vector space of all symmetric  $2 \times 2$  matrices and let  $T: W \rightarrow P_2$  be the linear transformation defined by  $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)x + (b-c)x^2 + (c-a)x^3$ . Find the rank and nullity of T.

Solve:- According to Question,

every matrix and vector space W is of the form  $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

and,

the linear transformation,  $T: W \rightarrow P_2$  defined by :

$$T \left( \begin{bmatrix} a & b \\ b & c \end{bmatrix} \right) = (a-b)x + (b-c)x^2 + (c-a)x^3$$

Here, we have three coefficients corresponding to 1, x and  $x^2$ . ( $R^3$ )

$$\therefore \text{Dimension} = 2+1 = 3$$

Now, The standard basis for W is :

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \quad (\because W \text{ is } 2 \times 2 \text{ matrix space, which is symmetric})$$

Also if  $T$  is defined as  $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a-b & c \\ b-c & d-a \end{bmatrix}$

So, we can represent  $T$  in terms of the standard basis elements:

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Now, these column vectors form the matrix  $A$ :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Now, we have to convert this into the row-echelon form.

Applying row elementary operation.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \rightarrow R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Here,  $\boxed{P(A) = 3}$ , linearly independent.

$$\text{Now, } P(A) + N = D \Rightarrow 3 + N = 3 \Rightarrow N = 3 - 3 \Rightarrow \boxed{N=0}$$

Hence, Rank,  $P(A) = 3$

Nullity ( $N$ ) = 0

Q. 3) Let  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ . Find the eigenvalues and eigenvectors of  $A^{-1}$  and  $A+4I$ .

Solve) To get the eigenvalues we need to solve characteristic equation.

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0 \Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0 \Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3 = 0$$

$$\Rightarrow \lambda(\lambda-3) - 1(\lambda-3) = 0 \quad \text{for } A^{-1} = \frac{1}{\lambda}$$

$$\Rightarrow \boxed{\lambda = 1, 3} \text{ Ans.} \rightarrow \text{Eigen values.} \quad \therefore A^{-1} = 1, \frac{1}{3}$$

Now, we find eigen space corresponding to  $\lambda=1$ , &  $\lambda=3$

For  $\lambda=1$ ,  $[A - \lambda I]x = [0] \Rightarrow [A - I]x = [0]$

$$\Rightarrow \left( \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\Rightarrow x - y = 0 \Rightarrow \boxed{x=y}$$

So, if  $x=k$ , then  $\boxed{x=y=k}$

$$\therefore \text{Eigen Space} = \begin{bmatrix} k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{Ker } A^{-1}$$

For  $\lambda=3$ ,  $[A - 3I]x = [0]$

$$\Rightarrow \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x - y = 0 \Rightarrow \boxed{-x=y}$$

if  $x=k$ , then,  $\boxed{x=y=k}$

Eigen values for  $A+4I$ .

$\Rightarrow \lambda_1 + 4, \lambda_2 + 4$

$\Rightarrow 1+4, 3+4$

$\Rightarrow \boxed{5, 7}$

$$\therefore \text{Eigen Space} = \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(3) Solution—Continue — We know if  $\lambda$  is eigen value of matrix A, then  $\frac{1}{\lambda}$  ( $\lambda \neq 0$ ) is eigen values of  $A^{-1}$

$\therefore$  Eigen values of  $A^{-1} = 1, \frac{1}{3}$ , [for 5 we have already found.]

eigen vector

For  $\lambda = \frac{1}{3}$ ,  $\left[ \tilde{A}^{-1} - \frac{1}{3} I \right] [x] = [0]$



Here,

$$|A| = \lambda_1 \lambda_2 = (2)(3) = [3 \neq 0] \therefore A^{-1} \text{ exist}$$

and  $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj. } A) = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$$

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③

Solution:- Continue:- Now For eigen values of  $A^T = (1, \frac{1}{3})$ .

and  $A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

• Find Eigen vectors.—

① For  $\lambda = 1$ ,  $[A^T - I][x] = [0]$

$$\Rightarrow \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} [x] = [0]$$

$$\Rightarrow \frac{2}{3}x + \frac{1}{3}y = 0 \Rightarrow x = -\frac{1}{2}y \times \frac{8}{2} \Rightarrow \boxed{x = -\frac{1}{2}y}$$

$$\& \frac{1}{3}x + \frac{2}{3}y = 0 \Rightarrow x = -\frac{2}{3}y \cancel{\times \frac{3}{3}} \Rightarrow \boxed{x = -2y}$$

This is not possible

Q. 4) Solve by Gauss-Siedel Method (Take three iterations)

$$3x + 0.1y - 0.2z = 7.85$$

$$0.1x + 7y - 0.3z = -19.3$$

$$0.3x - 0.2y + 10z = 71.4$$

$$\begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

with initial values  $x(0) = 0, y(0) = 0, z(0) = 0$ .

Soln:- Now, Using Gauss Siedel Method, initial  $\boxed{k=0}$  ( $\because k = \text{iteration}$ )

$$x^{(k+1)} = \frac{1}{a_{11}} (b_1 - a_{12}y^{(k)} - a_{13}z^{(k)})$$

$$y^{(k+1)} = \frac{1}{a_{22}} (b_2 - a_{21}x^{(k+1)} - a_{23}z^{(k)})$$

$$z^{(k+1)} = \frac{1}{a_{33}} (b_3 - a_{31}x^{(k+1)} - a_{32}y^{(k+1)})$$

Iteration 1<sup>st</sup> :-  $x^{(1)} = \frac{1}{3} (7.85 + 0.1 \cdot 0 + 0.2 \cdot 0) = \frac{7.85}{3} = \boxed{2.6167}$

$$y^{(1)} = \frac{1}{7} (-19.3 - 0.1 \cdot 2.6167 + 0.3 \cdot 0) = \frac{1}{7} (-19.3 + 0.785) = \frac{-18.515}{7} = \boxed{-2.645}$$

$$z^{(1)} = \frac{1}{10} (-71.4 - 0.1 \cdot 2.6167 + 0.3 \cdot -2.645) = \frac{1}{10} (-71.4 - 0.26167 + 0.79349) = \frac{-71.85833}{10} = \boxed{-7.185833}$$

$$= \frac{+198.1505}{30} = \boxed{6.605}$$

Iteration 2<sup>nd</sup> :-  $x^{(2)} = \frac{1}{3} (-7.85 + 0.1 \cdot 6.605 + 0.2 \cdot -7.185833) = \frac{1}{3} (-7.85 + 0.1 \cdot 56.115 + 0.2 \cdot -198.1505) = \frac{-286.101}{30} = \boxed{1.7908}$

$$y^{(2)} = \frac{1}{7} (-19.3 - 0.1 \cdot 1.7908 + 0.3 \cdot 6.605) = \boxed{-3.3377}$$

$$z^{(2)} = \frac{1}{10} (71.4 - 0.3 \cdot 1.7908 + 0.2 \cdot -3.3377) = \boxed{7.2991}$$

$$\text{Iteration } 3^{\text{rd}} : - \quad x^{(3)} = \frac{1}{3} (7.85 + 0.1(-3.3377) + 0.2(7.2991)) = [1.6792]$$

$$y^{(3)} = \frac{1}{7} (-19.3 - 0.1(1.6792) + 0.3(7.2991)) = [-3.2744]$$

$$z^{(3)} = \frac{1}{10} (71.4 - 0.3(1.6792) + 0.2(-3.2744)) = [7.3651]$$

(Q.5) Define consistent & inconsistent system of eqn. Hence solve the following system of eqn. if consistent  $x+3y+2z=0$ ,  $2x-y+3z=0$ ,  $3x-5y+4z=0$ ,  $x+17y+4z=0$

Sol:- Here, in the form of  $AX = B$ ,

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(Homogeneous)

Since,  $B=0$   $\therefore$  Our system of eqn is consistent

Now, let's convert this into row-echelon form.

$$\left[ \begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array} \right] \xrightarrow{R_4 \rightarrow R_4 - R_1} \left[ \begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & 1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 + R_1} \left[ \begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & 1 & 0 \\ 0 & -14 & -1 & 0 \\ 0 & 14 & 2 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left[ \begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Here,  $(\rho(A) = 2) < (n=3)$

$\therefore$  This system of eqn has infinite soln

Now, we have,  $1x + 3y + 2z = 0 \quad \text{--- (1)}$

$$-7y - z = 0 \Rightarrow z = -7y$$

$$\text{From (1), } 1x + 3y + 14y = 0$$

$$x + 17y = 0$$

$$x = -17y$$

$$\text{if } y = k, \quad x = -17k, \quad z = -7k \quad \Rightarrow \quad x = \begin{bmatrix} -17k \\ k \\ 7k \end{bmatrix} = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$$

Q.6. Determine whether the Function  $T: P_2 \rightarrow P_2$  is linear transformation or not. Where  $T(ax+bx+cx^2) = (a+1)x + (b+1)x + (c+1)x^2$

Soln:- To check if  $T$  is a linear Transformation, we need to verify two properties

(1) Additivity:  $T(u+v) = T(u) + T(v)$  ( $u, v \in \text{Polynomial}$ )

(2) Homogeneity:  $T(cu) = cT(u)$  (For any polynomial  $u$  and scalar  $c$ )

Let's Check. (can take any constant to add.  
not necessary "1")

(1) Additivity:  $T((a_1+b_1)x + (c_1)x^2) + T(a_2+b_2)x + (c_2)x^2)$   
 $= T((a_1+a_2) + (b_1+b_2)x + (c_1+c_2)x^2)$   
 $= (a_1+a_2+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2 = \text{LHS}$

now,  $T(a_1+b_1)x + b_2x^2) + T(a_2+b_2)x + c_2x^2)$   
 $= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$   
 $= (a_1+a_2+2) + (b_1+b_2+2)x + (c_1+c_2+2)x^2 = \text{RHS}$ .

$\therefore \text{LHS} \neq \text{RHS}$ ,  $T$  is not additive.

(2) Homogeneity:  $T(cu) = T(c(a+bx+cx^2)) = (ca+1) + (cb+1) + (cc+1)x^2 = \text{LHS}$ .  
 $\& \text{RHS} = cT(u) = cT(a+bx+cx^2) = c((a+1) + (b+1)x + (c+1)x^2)$   
 $= (ca+c) + (cb+c)x + (cc+c)x^2$

$\therefore \text{LHS} \neq \text{RHS}$ ,  $T$  is not homogeneous.

Therefore,  $T$  is not a linear Transformation. D.N.

Q.7 Determine whether the set  $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$  is a basis of  $V_3(\mathbb{R})$ . If  $S$  is not a basis determine the dimension and the basis of the subspace spanned by  $S$ .

Solution: To determine if the set  $S$  is a basis for  $V_3(\mathbb{R})$ , we need to check 2 things:

1. Linear Independence: Confirm that none of the vectors in  $S$  can be written as a linear combination of the others.

2. Spanning: Verify that the set  $S$  spans  $V_3(\mathbb{R})$ , meaning that every vector in  $V_3(\mathbb{R})$  can be expressed as linear combination of vectors from  $S$ .

Now, Form a matrix with vectors of  $S$  as its columns.

& now reduce to check for linear independence:

$$\left[ \begin{array}{ccc} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_3 - 3R_1 \\ R_2 \rightarrow R_2 - 2R_1}} \left[ \begin{array}{ccc} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - \frac{9}{5}R_2} \left[ \begin{array}{ccc} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{array} \right]$$

$$\text{Now, } \rho(A) = 2 < (n=3)$$

so, it is infinite, linearly dependent, Hence don't span  $V_3(\mathbb{R})$ .  
and not basis

Next let's Determine dimension & basis of subspace spanned by  $S$ .

Dimension  $\rightarrow$  no. of linearly independent vectors in  $S$ .

From above, we see that the first and second row are L.I but row 3 not.

$\therefore$  Dimension = 2, and basis for the ~~subspace~~ spanned by  $S$  is given by L.I. vectors. =  $\{(1, 2, 3), (3, 1, 0)\}$ .  $\approx$  Ans

Q.8. Using Jacobi's method (perform 3 iterations), solve.

$$\begin{aligned} 3x + 6y + 2z &= 23 \\ -4x + y - z &= -15 \\ x - 3y + 7z &= 16 \end{aligned} \Rightarrow \left[ \begin{array}{ccc} 3 & 6 & 2 \\ -4 & 1 & -1 \\ 1 & -3 & 7 \end{array} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ -15 \\ 16 \end{bmatrix}$$

with initial values  $x_0 = 1, y_0 = 1, z_0 = 1$ .

# Hitenra Dewangan.

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Solve:- Initialize  $x^{(0)} = 1, y^{(0)} = 1, z^{(0)} = 1$ , ,  $\begin{cases} x^k = \frac{1}{a_{11}}(b_1 + a_{12}y^{(k-1)} + a_{13}z^{(k-1)}) \\ y^k = \frac{1}{a_{22}}(b_2 + a_{21}x^{(k-1)} + a_{23}z^{(k-1)}) \\ similarly z. \end{cases}$

1st Iteration:-  $x^{(1)} = \frac{1}{3}(23 + 6 \cdot 1 - 2 \cdot 1) = \boxed{9}$

$$y^{(1)} = \frac{1}{7}(-15 + 1 + 1) = \boxed{-10}$$

$$z^{(1)} = \frac{1}{7}(16 + 1 - 3 + 1) = \boxed{2}$$

2nd Iteration:-  $x^{(2)} = \frac{1}{3}(23 + 6(-10) - 2 \cdot 2) = \boxed{-36}$

$$y^{(2)} = \frac{1}{7}(-15 + 4(9) + 2) = \boxed{26}$$

$$z^{(2)} = \frac{1}{7}(16 - (-36) + 3(-10)) = \boxed{8}$$

3rd Iteration:-  $x^{(3)} = \frac{1}{3}(23 + 6(26) - 2(8)) = \boxed{47}$

$$y^{(3)} = \frac{1}{7}(-15 + 4(-36) + 8) = \boxed{-163}$$

$$z^{(3)} = \frac{1}{7}(16 - 47 + 3(26)) = \boxed{11}$$

After 3 iterations,  $x=47, y=-163, z=11$

Q. 19 Explain one application of matrix operation in image processing with example.

Solve:- Application:- Image Transformation through affine transformation.

Affine transformation involve translation, scaling, rotation, scaling and shearing of images, and these operations can be efficiently represented by using matrices.

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Example Scaling Operation :-

Let's take a 2D image represented as  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

To scale the image by a factor of  $(s)$  in the x-direction and  $(t)$  in the y-direction, the transformation matrix  $T$  is :

$$T = \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}$$

The resulting transformed image  $I'$  is obtained by multiplying the original image matrix  $I$  with the transformation matrix  $T$  :

$$I' = T \cdot I$$

This multiplication efficiently applies scaling to each pixel in the image.

For exm we have image represented by.  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

we want to scale it by a factor of 2 in x-direction and 3 in y-direction the transformation would be :

$$T = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

The resulting transformed image  $I'$  would be :

$$I' = \begin{bmatrix} 2 & 4 \\ 6 & 12 \end{bmatrix}$$

This demonstrates how matrix operations facilitate efficient and systematic manipulation of images in various ways within the field of ~~process~~ image processing.

Q. 10 Give a brief description of Linear Transformation For computer Vision for rotating 2D image.

Solve- Linear transformation plays a crucial role in computer vision, particularly in the context of rotating 2D images. A linear transformation can be presented by a matrix that operates on the coordinates of each pixel in the image, producing a transformed image. For rotation specifically, the rotation matrix is employed.

Rotation matrix for 2D images —

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

If  $I$  = original image matrix, after multiplying  $I$  with  $R$ , we obtain the rotated image  $I'$ , reflecting the rotation.

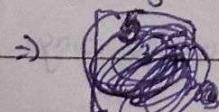
This linear transformation is fundamental in computer vision application allowing for efficient manipulation and analysis of images, including rotation to correct orientation, or align objects in the image.

(3)

Solution — Continue — We know if  $\lambda$  is eigen value of matrix  $A$ , then  $\frac{1}{\lambda} (\lambda \neq 0)$  is eigen value of  $A^{-1}$

$\therefore$  Eigen values of  $A^{-1} = \frac{1}{1}, \frac{1}{3}$  , [ ~~for  $\lambda = 1$  we have already found~~ ]

For  $\lambda = \frac{1}{3}$ ,  $\left[ \bar{A}^{-1} - \frac{1}{3} I \right] [x] = [0]$



Here,

$$|A| = \lambda_1 \lambda_2 = (1)(3) = [3 \neq 0] \therefore A^{-1} \text{ exist}$$

and  $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore \bar{A}^{-1} = \frac{1}{|A|} (\text{adj. } A) = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$$

Assignment - 3

Exercise: Find the Eigen Values and Eigen Vectors of following matrices:

1.  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Solve:- ① Find Eigen Values ( $\lambda$ ) by solving characteristic eqn.

$$\det(A - \lambda I) = 0 \Rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$$

$$\therefore -2-\lambda(-\lambda+\lambda^2-12) - 2(-2\lambda-6) + (-3)(-4+1-\lambda) = 0$$

$$\Rightarrow +2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 4\lambda + \cancel{12} + \cancel{9} + (+3\lambda) = 0$$

$$\Rightarrow (+2\lambda + 12\lambda + 4\lambda + 3\lambda) + (2\lambda^2 + \lambda^2) + (24 + \cancel{9} + \cancel{12}) - \lambda^3 = 0$$

$$\Rightarrow 21\lambda + (\cancel{\lambda^2}) + 45 - \lambda^3 = 0$$

$$\Rightarrow -\lambda^3 + (\cancel{\lambda^2}) + 21\lambda + 45 = 0$$

$$\Rightarrow -\lambda^3 - \lambda^2 + 21\lambda = -45$$

$$\Rightarrow -\lambda(\lambda^2 + \lambda + 21) = -45$$

$$\Rightarrow -\lambda(\lambda^2 + 1 - 21) = -45 \Rightarrow \boxed{\lambda = 45}$$

17x5  
517x5  
✓

By Quadratic eqn,  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore \frac{-1 \pm \sqrt{1 + 84}}{2} = \frac{-1 \pm \sqrt{85}}{2} = \frac{-1 \pm 9}{2},$$

$$\therefore \boxed{\lambda = 4, -5, 45}$$

Now, Find Eigen Vectors :-

② For  $\lambda = 4$ ,  $[A - 4I][x] = [0]$

$$\Rightarrow \begin{bmatrix} -2-4 & 2 & -3 \\ 2 & 1-4 & -6 \\ -1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6 & 2 & -3 \\ 2 & -3 & -6 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore -6x + 2y - 3z = 0 \quad \text{--- (i)}$$

$$2x - 3y - 6z = 0 \quad \text{--- (ii)}$$

$$-x - 2y + 4z = 0 \quad \text{--- (iii)}$$

$$\left[ \begin{array}{ccc|c} 8 & 2 & 3 & - \\ 2 & 1 & 8 & 0 \\ 0 & 5 & 1 & 0 \end{array} \right]$$

$$\text{From (i) & (ii), } 2x - 3y - 6z = 0$$

$$2x(-x - 2y + 4z) = 0$$

$$-7y + 2z = 0 \Rightarrow 7y = 2z \Rightarrow y = \frac{2z}{7}$$

$$\text{Put in eq (iii), } -x - \frac{4z}{7} + 4z = 0$$

$$\Rightarrow -7x - 4z + 28z = 0$$

$$\Rightarrow -7x + 24z = 0$$

$$\Rightarrow 7x = 24z$$

$$\Rightarrow x = \frac{24z}{7}$$

$$\text{Let } z = k, \text{ then } x = \frac{24k}{7}, y = \frac{2k}{7}$$

$$\text{Figure space} = \begin{bmatrix} \frac{24}{7}k \\ \frac{2}{7}k \\ k \end{bmatrix} = k \begin{bmatrix} \frac{24}{7} \\ \frac{2}{7} \\ 1 \end{bmatrix}$$

• For  $\lambda = -5$ ,  $[A + 5\lambda][x] = [0]$

$$\Rightarrow \begin{bmatrix} -2+5 & 2 & -3 \\ 2 & 1+5 & -6 \\ -1 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & -3 \\ 2 & 6 & -6 \\ -1 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore 3x + 2y - 3z = 0 \quad \text{--- (i)}$$

$$2x + 6y - 6z = 0 \quad \text{--- (ii)}$$

$$-x - 2y + 5z = 0 \quad \text{--- (iii)}$$

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From ⑪ & ⑫,  $2x + 6y - 6z = 0$

$$2x(-x - 2y + 5z = 0)$$

$$2y + 4z = 0 \Rightarrow [y = -2z]$$

Put in eq ⑬,

$$-(x - 2(-2z) + 5z) = 0 \Rightarrow -x + 4z + 5z = 0$$

$$\Rightarrow -x + 9z = 0 \Rightarrow [x = 9z]$$

Let  $z = k$ , then,  $x = 9k$ ,  $y = -2k$ .

$$\therefore \text{Eigen Space} = \begin{bmatrix} 9k \\ -2k \\ k \end{bmatrix} = k \begin{bmatrix} 9 \\ -2 \\ 1 \end{bmatrix} = A_n$$

For  $\lambda = 4s$ ,  $[A - 4sI][x] = [0]$

$$\begin{bmatrix} -2-4s & 2 & -3 \\ 2 & 1-4s & -6 \\ -1 & -2 & -4s \end{bmatrix} [x] = [0]$$

$$\Rightarrow \begin{bmatrix} -4s & 2 & -3 \\ 2 & -4s & -6 \\ -1 & -2 & -4s \end{bmatrix} [x] = [0]$$

$$\therefore \Rightarrow -4s x + 2y + 3z = 0 \quad \text{--- ①}$$

$$2x - 4sy - 6z = 0 \quad \text{--- ②}$$

$$-x - 2y - 4sz = 0 \quad \text{--- ③}$$

From ① & ②,  $2x - 14sy - 6z = 0$

$$-2x - 4y - 4sz = 0$$

$$-14sy = sz \Rightarrow [y = \frac{s}{148}z]$$

Put in eq ③

$$-x - 2\left(\frac{s}{148}z\right) - 4sz = 0$$

$$\Rightarrow -x - \frac{102z}{148} - 4sz = 0 \Rightarrow -x - \frac{s}{74}z - \frac{3380z}{74} = 0$$

$$\Rightarrow [x = -\frac{3381z}{74}]$$

Let  $z = k \Rightarrow x = -\frac{3381}{74}k$ ,  $y = \frac{s}{148}k$ ,

$$\text{Eigen space} = k \begin{bmatrix} -\frac{3381}{74} \\ \frac{s}{148} \\ k \end{bmatrix}$$

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Eigen vector :-

$$\begin{bmatrix} \frac{24}{7} & 9 & -\frac{3381}{74} \\ \frac{2}{7} & -2 & \frac{51}{148} \\ 1 & 1 & 1 \end{bmatrix}$$

Ans

### Assignment - 3

2.

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Solve:-  $\det(A - \lambda I) = 0 \rightarrow \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$

$$-2(\lambda-1) + (1-\lambda)((4-\lambda)(1-\lambda)) = 0$$

~~$$(1-\lambda)(6+\lambda^2-5\lambda) = 0$$~~

$$(\lambda-1)(\lambda-3)(\lambda-2) = 0$$

$$\boxed{\lambda_1 = 1}, \boxed{\lambda_2 = 3}, \boxed{\lambda_3 = 2}$$

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For  $\lambda=1$ ,  $[A-\lambda I][x] = [0]$ .

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1. \quad 3x_1 + x_3 = 0 \Rightarrow x_1 = -3x_3 \Rightarrow \boxed{x_1 = 0}$$

$$-2x_3 = 0 \Rightarrow \boxed{x_3 = 0}$$

Let  $k \in \mathbb{R}$ ,  $x_2 = \underline{\underline{k}}$

$$x = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{Ans.}$$

For  $\lambda=3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad n(A)=1$$

$$x_1 + x_3 = 0$$

$$-2x_2 + 2x_3 = 0$$

$$\Rightarrow \text{let } x_3 = k$$

$$\therefore x_1 = -k$$

$$x_2 = k.$$

$$\therefore x = k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \text{Ans.}$$

For  $\lambda=2$ ,

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + x_3 = 0$$

$$-x_2 + x_3 = 0$$

$$\text{let } x_3 = k \Rightarrow \boxed{x_2 = k} \quad \& \quad \boxed{x_1 = -\frac{k}{2}}$$

$$x = k \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix} \quad \text{Ans.}$$

3.  $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$  It's a lower triangular matrix.  
So, Diagonal Elements will be the Eigen values.

Hence,  $\lambda = 5, 0, 3$

For  $\lambda = 5$ ,  $\begin{bmatrix} 0 & 5 & 0 \\ 0 & -5 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\therefore -x_1 - 2x_3 = 0$$

$$\text{&} -5x_2 + 3x_3 = 0$$

$$\Rightarrow \det, x_3 = k$$

$$x_1 = -2k$$

$$x_2 = \frac{3}{5}k$$

$$\therefore x = k \begin{bmatrix} -2 \\ \frac{3}{5} \\ 1 \end{bmatrix}$$

For  $\lambda = 0$ ,

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 = 0 \Rightarrow x_1 = 0$$

$$-x_1 + 3x_3 = 0 \Rightarrow 3x_3 = 0 \Rightarrow x_3 = 0$$

$$\text{let } x_2 = k$$

$$\therefore x = \begin{bmatrix} 0 \\ k \\ 0 \end{bmatrix} \Rightarrow x = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

For  $\lambda = 3$ ,

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 = 0 \Rightarrow x_1 = 0$$

$$-3x_2 = 0 \Rightarrow x_2 = 0$$

$$\Rightarrow \det x_3 = k,$$

$$x_1 = 0 \text{ & } x_2 = 0$$

$$\therefore x = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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4.  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}$  Eigen values  $\Rightarrow \lambda_1 = 0, \lambda_2 = 3, \lambda_3 = -2$ .

Solu:- For  $\lambda=0$

$$\begin{bmatrix} 0 & 0 & -2 \\ 0 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_3 = 0$$

$$3x_2 + 4x_3 = 0$$

$$\Rightarrow \text{let } x_1 = k, \therefore x_2 = 0, x_3 = 0$$

$$\therefore x = k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\approx$  Ans.

For  $\lambda=3$ ,

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3x_1 = 0$$

$$4x_3 = 0$$

$$-5x_3 = 0$$

$$\text{let } x_2 = k \rightarrow \therefore x = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\approx$  Ans.

For  $\lambda=-2$ ,

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 = 0$$

$$5x_2 + 4x_3 = 0$$

$$\Rightarrow \text{let } x_3 = k$$

$$x_1 = 0 \quad \& \quad x_2 = -\frac{4}{5}k$$

$$\Rightarrow x = k \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$$

$\approx$  Ans.

$$5. \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

Solver - Because there are 3 identical rows in this matrix.  
 So, 2 eigen values must be 0.

~~0 X 0~~

Completed

~~0 X 0~~

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x = 0$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$