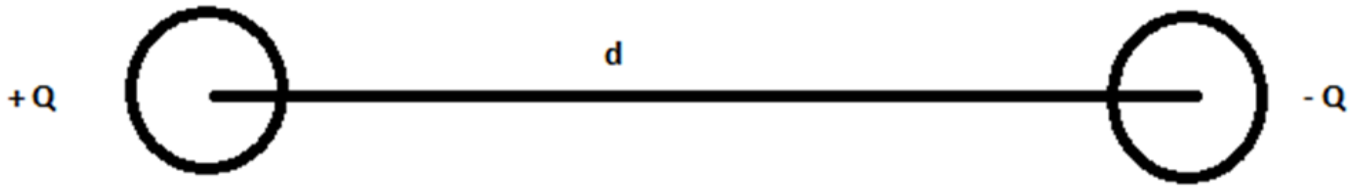
Polarization in Dielectric

Important points

Electric dipole



$$\vec{p} = Q\,\vec{d}$$

Materials

Metals

Insulators

While classifying solids on the basis of their band structure, we refer to the group of *solids as insulators which have energy gap of 3 e.v. or more*. The large magnitude of energy gap in an ideal insulator excludes the possibility of electrons being excited from valence band to conduction band by **thermal means** much less so by the application of an external electric field. It is due to this reason that insulators are poor conductors of electricity and are also known as **dielectrics.** Dielectric materials find extensive use in electric industry and as capacitors.

POLAR AND NON-POLAR MOLECULES

In the case of polar molecules the *centre of gravity of the positive charges (i.e., protons) and negative charges (i.e., electrons) do not coincide.* Such molecules are called **permanent electric dipoles** as these have permanent dipole moments. Some examples of polar molecules are N_2O, H_2O and HC1. In the absence of an electric field, the *electric dipole moments* of these polar molecules are in random directions and cancel each other.

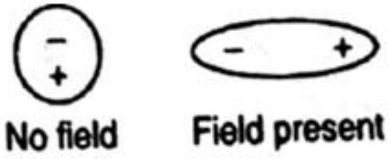


Fig. 5.2

So even though each of molecule has a dipole moment, the average moment per unit volume is zero: on the application of an electric field, the dipole moments of these molecules align themselves parallel to the direction of electric field · However, this alignment is not complete because of thermal vibrations of the molecules.

- In case of ***non polar molecules***, the ***centre of gravity of positive and negative charges coincide***, so these molecules do not have any permanent dipole moment. Some common examples of non-polar molecules are ***H_2, N_2, O_2***.

- When a ***non polar molecule*** is placed in an electric field, the ***centers of positive and negative charges get displaced*** and the molecules are said to have been ***polarized***.

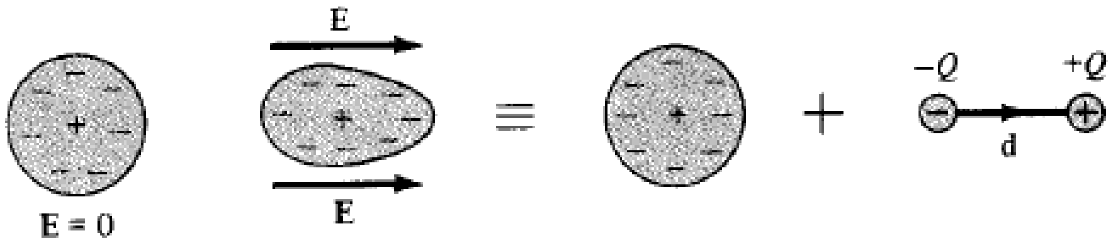
- Such a molecule is called ***induced electric dipole*** and its electric dipole moment is called ***induced electric dipole moment***.

- So the ***polarization*** is a phenomenon in which an alignment of ***positive and negative charges*** takes place with in the dielectric resulting ***no net increase*** in the charge of the dielectric.

Polarization in Dielectrics

- Consider an atom of the dielectric consisting of an electron cloud (-Q) and a positive nucleus ***(+Q)***.

- When an electric field $\vec{E}$ is applied, the positive charge is displaced from its equilibrium position in the direction of $\vec{E}$ by $\vec{F_+} = Q\vec{E}$ while the negative charge is displaced by $\vec{F_-} = Q\vec{E}$ in the opposite direction.



- A dipole results from the ***displacement of charges*** and the dielectric is polarized. In polarized the electron cloud is distorted by the applied electric field.

- This distorted charge distribution is equivalent to the original distribution plus the dipole whose moment is

$$\vec{p} = Q\vec{d}$$

where $\vec{d}$ is the distance vector between -Q to +Q.

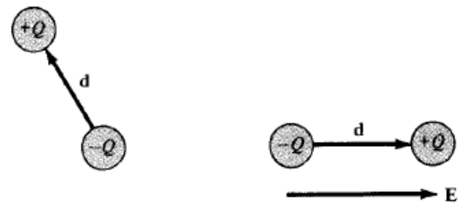
- If there are N dipoles in a volume Δv of the dielectric, the total dipole moment due to the electric field

$$Q_1\mathbf{d}_1 + Q_2\mathbf{d}_2 + \cdots + Q_N\mathbf{d}_N = \sum_{k=1}^{N} Q_k\mathbf{d}_k$$

- For the measurement of intensity of polarization, we define polarization $\vec{P}$(coulomb per square meter) as dipole moment per unit volume

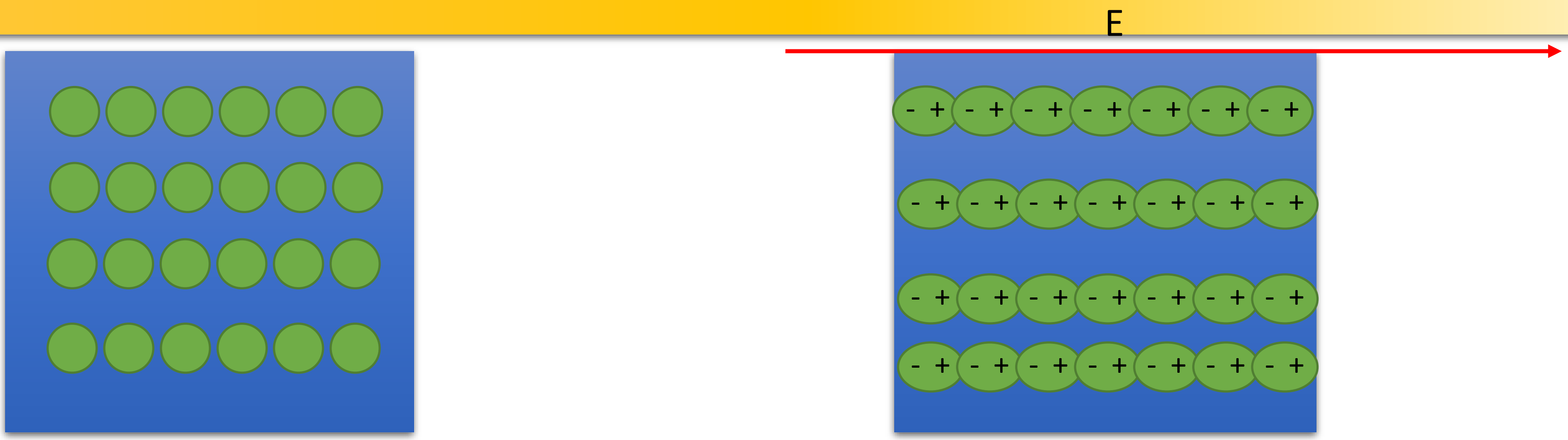
$$\mathbf{P} = \frac{\lim_{\Delta v \to 0} \sum_{k=1}^{N} Q_k\mathbf{d}_k}{\Delta v}$$

- The major effect of the electric field on the dielectric is the creation of dipole moments that align themselves in the direction of electric field.
- This type of dielectrics are said to be non-polar. eg: H_2, N_2, O_2
- Other types of molecules that have in-built permanent dipole moments are called polar. eg: H_2O, HCl
- When electric field is applied to a polar material then its permanent dipole experiences a torque that tends to align its dipole moment in the direction of the electric field.

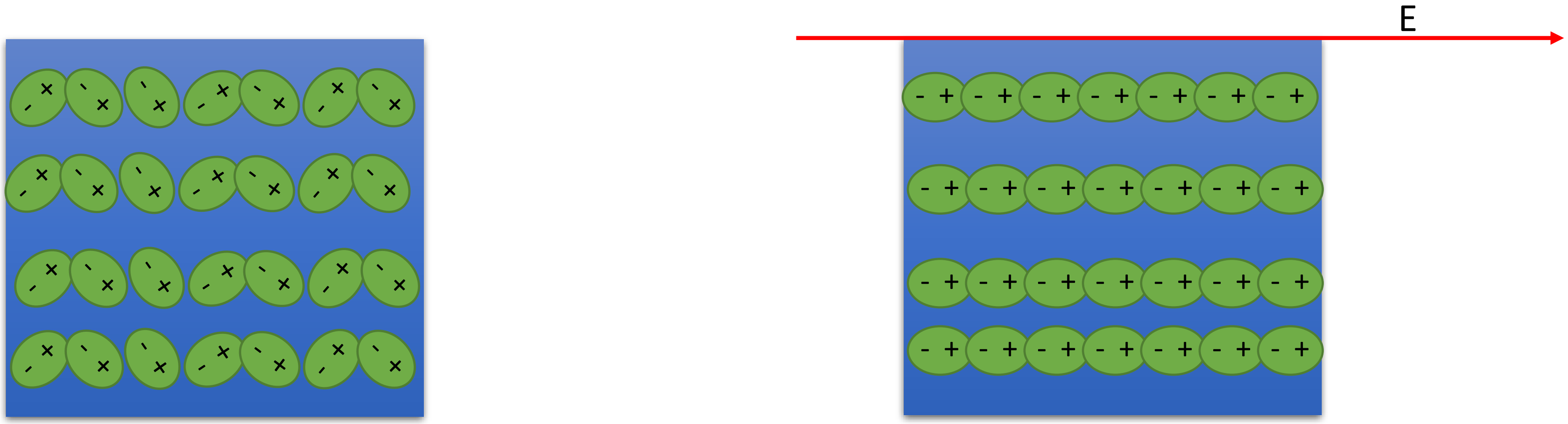


Dielectric in Electric Field

Non-polar



Polar



Problem

Let $\mathbf{r} = (x, y, z)$ so that $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$. Show that $\nabla(r^n) = nr^{n-2}\mathbf{r}$,

for any integer n and deduce the values of $\text{grad}(r)$, $\text{grad}(r^2)$ and $\text{grad}(1/r)$.

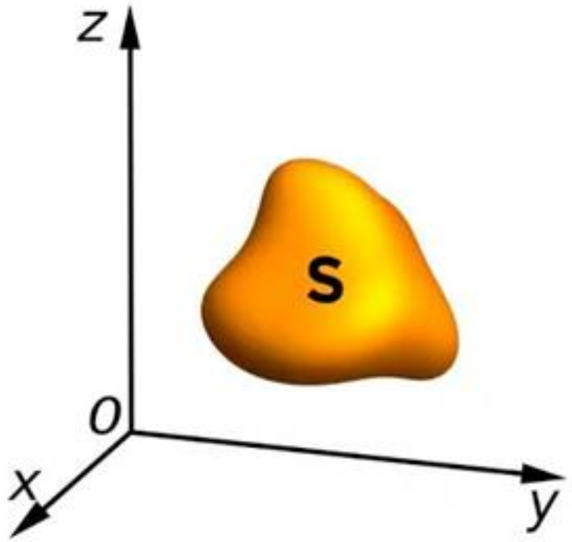
Your task

if $r^2 = (x-x')^2+(y-y')^2+(z-z')^2$ $\quad \nabla'(r^n) = n r^{n-2}\mathbf{r},$

for any integer n and deduce the values of grad(r), grad(r^2) and grad$(1/r)$.

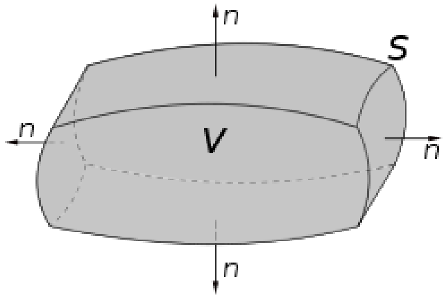
The Divergence Theorem

The divergence theorem states that the surface integral of the normal component of a vector point function **"F"** over a closed surface **"S"** is equal to the volume integral of the divergence of **"F"** taken over the volume "V" enclosed by the surface S.



$$\iint_S \vec{F}.\hat{n}\, dS = \iiint_V \nabla.\vec{F}\, dV$$

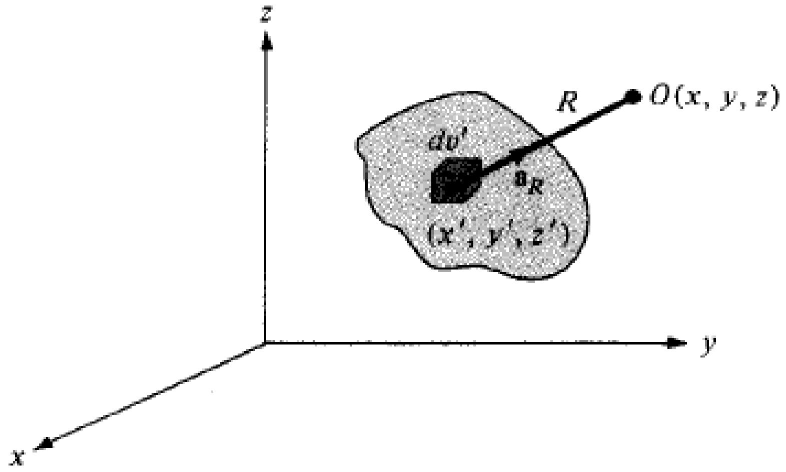
$\hat{n}$ is the outward pointing unit normal at each point on the boundary dV.



Field due to a Polarized Dielectric

Consider a dielectric material consisting of dipoles with Dipole moment $\vec{P}$ per unit volume.

The potential dV at an external point O due to $\vec{P}dv'$



$$dV = \frac{\mathbf{P} \cdot \mathbf{a}_R \, dv'}{4\pi\varepsilon_o R^2} \qquad \text{(i)}$$

where $R^2 = (x-x')^2+(y-y')^2+(z-z')^2$ and R is the distance between volume element dv' and the point O.

But $$\nabla' \cdot \frac{1}{R} = \frac{\mathbf{a}_R}{R^2}$$

So $$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \mathbf{P} \cdot \nabla'\left(\frac{1}{R}\right)$$

Applying the vector identity

$$\nabla' \cdot f\mathbf{A} = f\nabla' \cdot \mathbf{A} + \mathbf{A} \cdot \nabla' f$$

$$\mathbf{A} \cdot \nabla' f = \nabla' \cdot f\mathbf{A} - f\nabla' \cdot \mathbf{A}$$

$$\frac{\mathbf{P} \cdot \mathbf{a}_R}{R^2} = \nabla' \cdot \frac{\mathbf{P}}{R} - \frac{\nabla' \cdot \mathbf{P}}{R}$$

Put this in (i) and integrate over the entire volume v' of the dielectric

$$V = \int_{v'} \frac{1}{4\pi\varepsilon_o}\left[\nabla' \cdot \frac{\mathbf{P}}{R} - \frac{1}{R}\nabla' \cdot \mathbf{P}\right]dv'$$

Applying Divergence Theorem to the first term

$$V = \int_{S'} \frac{\mathbf{P} \cdot \mathbf{a}'_n}{4\pi\varepsilon_o R}\, dS' + \int_{v'} \frac{-\nabla' \cdot \mathbf{P}}{4\pi\varepsilon_o R}\, dv' \qquad \text{(ii)}$$

where $\boldsymbol{a}_n$' is the outward unit normal to the surface dS' of the dielectric

The two terms in (ii) denote the potential due to ***surface and volume charge*** distributions with densities;

$$\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n$$
$$\rho_{pv} = -\nabla \cdot \mathbf{P}$$

where ρ_{ps} and ρ_{pv} are the ***bound surface and volume charge densities***.

Bound charges are those which are not free to move in the dielectric material.

Equation (ii) says that where polarization occurs, an equivalent volume charge density, ρ_{pv} is formed throughout the dielectric while an equivalent surface charge density, ρ_{ps} is formed over the surface of dielectric.

The total positive bound charge on surface S bounding the dielectric is

$$Q_b = \oint \mathbf{P} \cdot d\mathbf{S} = \int \rho_{ps}\, dS$$

while the charge that remains inside S is

$$-Q_b = \int_v \rho_{pv}\, dv = -\int_v \nabla \cdot \mathbf{P}\, dv$$

Total charge on dielectric remains zero (as was prior to the application of an electric field).

Total charge = $$\oint_S \rho_{ps}\, dS + \int_v \rho_{pv}\, dv = Q_b - Q_b = 0$$

We now consider the case in which dielectric ***contains free charge***. If ρ_v is the free volume charge density then the total volume charge density ρ_t is given by

Hence
$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_o \mathbf{E}$$

$$\rho_v = \nabla \cdot \varepsilon_o \mathbf{E} - \rho_{pv}$$
$$= \nabla \cdot (\varepsilon_o \mathbf{E} + \mathbf{P})$$
$$= \nabla \cdot \mathbf{D}$$

Where
$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

So, we may say that, the net effect of the dielectric on the electric field $\vec{E}$ is to increase $\vec{D}$ inside it by an amount $\vec{P}$.

The polarization would vary directly as the applied electric field.

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

Where χ_e is known as the electric susceptibility of the material

It is a measure of how susceptible a given dielectric is to electric fields.

We know that $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ and $\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$

Thus
$$\mathbf{D} = \varepsilon_0 (1 + \chi_e)\,\mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

or
$$\mathbf{D} = \varepsilon \mathbf{E}$$

where
$$\varepsilon = \varepsilon_o \varepsilon_r$$

and
$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$$

- where ε ***is the permittivity*** of the dielectric, ε_o ***is the permittivity of the free space*** and ε_r ***is the dielectric constant or relative permittivity***.
- So, dielectric constant or ***relative permittivity*** ε_r is the ratio of permittivity of the dielectric (ε_o) to that of free space.
- ***No dielectric is ideal.*** When the electric field in a dielectric is sufficiently high then it begins to pull electrons completely out of the molecules, and ***the dielectric becomes conducting***.
- When a dielectric becomes conducting then it is called ***dielectric breakdown***. It depends on the type of material, humidity, temperature and the amount of time for which the field is applied.
- The minimum value of the electric field at which the dielectric breakdown occurs is called the ***dielectric strength of the dielectric material***. Or
- The ***dielectric strength*** is the maximum value of the electric field that a dielectric can tolerate or withstand without breakdown.