

Assignment-4

UNIT-3

Ans-1. Consider a charge particle q of mass m through V .

$K = qV$, { qV is work done on charged particle by electric field }

We know that $K = E$, $K = \frac{1}{2}mv^2$

And momentum $p = mv$

$$\Rightarrow K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

$$K = qV$$

$$p = \sqrt{2mqV}$$

de broglie wavelength (λ), $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Expression for de-Broglie wavelength with a charged particle having charge & mass by potential V .

Ans-2. The group velocity is simply derivative of frequency ω w.r.t. k consⁿ $V_g = \frac{d\omega}{dk}$

$$V_g = \frac{d(2\pi\nu)}{d\left(\frac{2\pi}{\lambda}\right)} = \frac{d(\nu)}{d\left(\frac{1}{\lambda}\right)}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \frac{1}{V_g} = \frac{d(1/\lambda)}{d\nu} \quad \text{--- (1)}$$

If E and V represent that energy & p.e

$$\frac{1}{2}mv^2 = E - V$$

$$\therefore v = \left[\frac{2(E - V)}{m} \right]^{1/2}$$

Using de-broglie

$$\frac{1}{\lambda} = \frac{mv}{h} = \frac{m}{h} \left[\frac{2(E - V)}{m} \right]^{1/2}$$

$$\frac{1}{\lambda} = \frac{m}{h} = \left[\frac{2(h\nu - V)}{m} \right]^{1/2}$$

Substituting Eqⁿ (1)

$$\frac{1}{v_g} = \frac{d}{dv} \left[\frac{m}{h} \left(\frac{2(h\nu - \nu)}{m} \right)^{1/2} \right]$$

$$\frac{1}{v_g} = \frac{2m}{h} \left(\frac{2(h\nu - \nu)}{m} \right)^{-1/2} \cdot \frac{2h}{m}$$

$$\frac{1}{v_g} = \left(\frac{2(h\nu - \nu)}{m} \right)^{-1/2} = \frac{1}{v}$$

$$\boxed{\frac{1}{v_g} = \frac{1}{v}}$$

group velocity
= particle velocity

Ans-3- $P = \frac{hc}{\lambda}$ = momentum of incident photon.

$P_1 = \frac{hc}{\lambda'}$ = momentum of scattering photon.

P_2 = momentum of scattered electron at angle ϕ .

Using law of Conservation of momentum

$$P = P_1 \cos \theta + P_2 \cos \theta$$

$$P_2 \cos \theta = P - P_1 \cos \theta$$

$$P_2 \sin \theta = P_1 \sin \theta$$

Using law of conservation of momentum.

*, taking ratio

$$\frac{P_2 \sin \phi}{P_2 \cos \phi} = \frac{P_1 \sin \theta}{\frac{hf}{c} - P_1 \cos \theta}$$

$$\tan \phi = \frac{1}{\frac{hf}{P_2 c} \cot \theta} = \frac{1}{\frac{\lambda'}{\lambda \sin \theta} - \cot \theta}$$

$$\tan \phi = \frac{\lambda \sin \theta}{\lambda' - \lambda \cos \theta} = \frac{\lambda \sin \theta}{(\lambda' - \lambda \cos \theta) + \Delta \lambda}$$

$$\tan \phi = \frac{\lambda \sin \theta}{\lambda(1 - \cos \theta) + \frac{h}{mc}(1 - \cos \theta)}$$

$$\tan \phi = \frac{\sin \theta}{(1 - \cos \theta) + \frac{hf}{mc^2}(1 - \cos \theta)}$$

$$\tan \phi \left(1 + \frac{hf}{mc^2} \right) = \frac{\sin \theta}{1 - \cos \theta}$$

$$\Rightarrow 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$1 - \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}$$

$$\tan \phi \Rightarrow \frac{2 \sin \theta/2 \cos \theta/2}{2 \sin^2 \theta/2} = \cot \theta/2$$

$$\tan \theta = \frac{\cot \theta}{1 + \frac{h\nu}{mc^2}}$$

Ans-4- The photon must be near to nucleus in order to satisfy conservation of momentum as an e^- positron pair produced in free space can't satisfy conservation of both energy and momentum. Because of this when pair production occurs, atomic nucleus receives some recoil.

Ans-5- Heisenberg, Uncertainty principle:- It is also known as uncertainty principle. It states that the posⁿ & velocity of an object can't be measured exactly at same time even in theory.

Radius of nucleus of order of 10^{-5} m

8 maximum uncertainty in posⁿ of e^- i.e Δx within the nucleus will be $10^{-5}m$.

$$\text{Now } \Delta x \cdot m \Delta v \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta v = \frac{6.62 \times 10^{-34}}{4\pi \times 9.1 \times 10^{-31} \times 10^{-5}}$$

$$= \frac{6.62 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 10^{-5}}$$

$$\Rightarrow \Delta v = 5.97 \times 10^{10} m/s$$

As this level of uncertainty in velocity is impossible to achieved hence e^- can't exist in nucleus.