ASSIGNMENT UNIT:1

Q1 (i) $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix}$

$R_2 \to R_1 + R_2$
$R_3 \to R_1 - R_3$
$R_4 \to R_1 - R_4$
$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & -4 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 2 & -4 & -2 \end{bmatrix}$

$R_4 \to R_2 - R_4$
$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & -4 & -2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2 \to R_2 - R_3$
$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_2 - R_3$
$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore$ rank$=2$

2) $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

$R_2 \longrightarrow 4R_1 - R_2$
$R_3 \longrightarrow 3R_1 - R_3$
$R_4 \longrightarrow R_1 - R_4$
$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 7 & -6 & 11 \\ 0 & 7 & -4 & 7 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

$R_2 \longrightarrow R_2/7$ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -6/7 & 11/7 \\ 0 & 7 & -4 & 7 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

$R_3 \longrightarrow R_3 - 7R_2$ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -6/7 & 11/7 \\ 0 & 0 & 2 & -4 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

$R_2 \longrightarrow R_2 - 6/7\, R_4$ $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & -1/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

$R_4 \to R_3 + R_4$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & -1/7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + 2C_3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & -1/7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to \frac{1}{7} C_2 + C_4$

$$\begin{bmatrix} 1 & 2 & 1 & 9/7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to \frac{9}{7} C_1 - C_4$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \longrightarrow R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore \text{rank}(A) = 3$

Q2 (i) $x_1 = [3, 1, -4]$ $x_2 = [2, 2, -3]$

So,

$\lambda x_1 + \lambda x_2 = 0$

$\lambda(3, 1, -4) + \lambda_2(2, 2, -3) = 0$

Now,

$3\lambda_1 + 2\lambda_2 = 0$

$\lambda_1 + 2\lambda_2 = 0$

$-4\lambda_1 - 3\lambda_2 = 0$

$$\begin{bmatrix} 3 & 2 \\ 1 & 2 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2 \quad \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -4 & -3 \end{bmatrix}$

$R_2 \longrightarrow R_2 - 3R_1$
$R_3 \longrightarrow R_3 + 4R_1 \quad \begin{bmatrix} 1 & 2 \\ 0 & -4 \\ 0 & 5 \end{bmatrix}$

$R_3 \longrightarrow 4R_3 + 5R_2 \quad \begin{bmatrix} 1 & 2 \\ 0 & -4 \\ 0 & 0 \end{bmatrix}$

So,

$$-4\lambda_2 = 0 \Rightarrow \lambda_2 = 0$$

and

$$\lambda_1 + 2\lambda_2 = 0 \Longrightarrow \lambda_1 = 0$$

As,

$$\lambda_2 = \lambda_1 = 0$$

$\therefore$ vectors x_1 and x_2 are linearly independent

2 $x_1 = [3, 1, 4]$ $x_2 = [2, 2; 3]$

$x_3 = [0, -4, 1]$

So, $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$\lambda_1 (3, 1, -4) + \lambda_2 (2, 2, 3) + \lambda_3 (0, -4, 1) = 0$

now, $3\lambda_1 + 2\lambda_2 = 0$

$\lambda_1 + 2\lambda_2 - 4\lambda_3 = 0$

$-4\lambda - 3\lambda_2 + \lambda_3 = 0$

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$ $\begin{bmatrix} 1 & 2 & -4 \\ 3 & 2 & 0 \\ -4 & -3 & 1 \end{bmatrix}$

$R_2 \longrightarrow R_2 - 3R_1$

$R_3 \longrightarrow R_3 + 4R_1$ $\begin{bmatrix} 1 & 2 & -4 \\ 0 & -4 & 12 \\ 0 & 5 & -15 \end{bmatrix}$

$R_3 \rightarrow 4R_3 + 5R_2$ $\begin{bmatrix} 1 & 2 & -4 \\ 0 & -4 & 12 \\ 0 & 0 & 0 \end{bmatrix}$

$R_2 \rightarrow R_2/(-4)$ $\begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$

$\lambda_1 + 2\lambda_2 - 4\lambda_3 = 0$

$\lambda_2 - 3\lambda_3 = 0$

Put $\lambda_3 = t$

$\lambda_2 = 3t$

and, $\lambda_1 + 6t - 4t = 0$

$\rightarrow \lambda = -2t$

As, λ_1 or λ_2 or $\lambda_3 \neq 0$, $\therefore$ vectors x_1, x_2, x_3 are linearly dependent

As, $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 = 0$

$\rightarrow -2tx_1 + 3tx_2 + tx_3 = 0$

$2\lambda_1 - 3\lambda_2 - \lambda_3 = 0$

$\rightarrow \lambda_1 = \dfrac{3\lambda_2 + \lambda_3}{2}$

Q3 As, $A_{4\times3} = I_4 A I_3$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 \to R_1$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 4 & 4 \\ 0 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$, $R_4 \to R_4 - 3R_2$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ -1 & -2 & 1 & 0 \\ 1/2 & -3/2 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \longleftrightarrow R_4$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1/2 & 0 & 0 \\ 1/2 & -3/2 & 0 & 0 \\ -1 & -2 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & 0 & 0 \\ -1 & -2 & 1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + C_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & 0 & 0 \\ -1 & -2 & 1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{2} & 0 & 0 \\ -1 & -2 & 1 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ \frac{1}{2} & -\frac{3}{4} & 0 & 0 \\ -1 & -2 & 1 & 0 \end{bmatrix} \qquad \therefore Q = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q4 $\begin{bmatrix} (3K-8) & 3 & 3 \\ 3 & (3K-8) & 3 \\ 3 & 3 & (3K-8) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$Ax = 0$

For ∞ many solution $|A| = 0$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$(3K-2) \begin{vmatrix} 1 & 1 & 1 \\ 3 & (3K-8) & 3 \\ 3 & 3 & (3K-8) \end{vmatrix} = 0$$

$C_2 \rightarrow C_2 - C_3$

$$(3K-2) \begin{vmatrix} 1 & 0 & 1 \\ 3 & (3K-11) & 3 \\ 3 & (11-3K) & (3K-8) \end{vmatrix} = 0$$

$$(3K-2) \begin{vmatrix} 1 & 0 & 1 \\ 3 & (3K-11) & 3 \\ 3 & (11-3K) & (3K-8) \end{vmatrix} = 0$$

$(3K-2)(3K-11)^2 = 0$ [expanding from R_1]

$3K - 2 = 0$ or $3K - 11 = 0$

$K = \frac{2}{3}$ $K = \frac{11}{3}$

Q5 $$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ b \end{bmatrix}$$

$$\therefore [A:B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & a & b \end{array}\right]$$

$R_1 - R_1/2$ $$\left[\begin{array}{ccc|c} 1 & 3/2 & 5/2 & 9/2 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & a & b \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$ $$\left[\begin{array}{ccc|c} 1 & 3/2 & 5/2 & 9/2 \\ 0 & -15/2 & -39/2 & -7/2 \\ 0 & 0 & a-5 & b-9 \end{array}\right]$$

Now,

① For no solution:

$R(A) \neq R([A:B])$

$\therefore a - 5 = 0$

$a = 5$

and

$\therefore b - 9 \neq 0$

$b \neq 9$

② for unique solution:

$R(A) = R[(A:B)] =$ no of unknowns

$\therefore a - 5 \neq 0$

$a \neq 5$

$\therefore b$ can take any value

③ for ∞ solution:

$R(A) = R[(A:B)] <$ no of unknows

$\therefore a - 5 = 0$

$a = 5$

$\therefore b - 9 = 0$

$b = 9$

06 $$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

So, $\because [A:B] = \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right]$

$R_1 \leftrightarrow R_2$ $\left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ -2 & 1 & 1 & a \\ 1 & 1 & -2 & c \end{array}\right]$

$R_2 \rightarrow 3R_1 + R_2$
$R_3 \rightarrow R_3 - R_1$ $\left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ 0 & -3 & 3 & 2b+a \\ 0 & 3 & -3 & c-b \end{array}\right]$

$R_3 \rightarrow R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & -2 & 1 & b \\ 0 & -3 & 3 & 2b+a \\ 0 & 3 & -3 & a+b+c \end{array}\right]$

now (i) for ∞ solution

$R(A) = R[(A:B)] <$ no of unknowns

$\therefore a+b+c=0$

(ii) for no solution

$R(A) \neq R[(A:B)]$

$\therefore a+b+c \neq 0$

Q7 $C = (8, 2, 1)$

$T = (10, 3, 2)$

$S = (16, 5, 3)$

$V = (240, 69, 41)$

So,

$x_1 C + x_2 T + x_3 S = 0$

$$x_1 \begin{bmatrix} 8 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 10 \\ 3 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 16 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 240 \\ 69 \\ 41 \end{bmatrix}$$

$8x_1 + 10x_2 + 16x_3 = 240$

$2x_1 + 3x_2 + 5x_3 = 69$

$x_1 + 2x_2 + 3x_3 = 41$

$$\begin{bmatrix} 8 & 10 & 16 \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 240 \\ 69 \\ 41 \end{bmatrix}$$

$$R_2 \longleftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 41 \\ 2 & 3 & 5 & 69 \\ 8 & 10 & 16 & 240 \end{array}\right]$$

$$\begin{array}{l} R_2 \longrightarrow R_2 - 2R_1 \\ R_3 \longrightarrow R_3 - 8R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 41 \\ 0 & -1 & -1 & -13 \\ 0 & -6 & -8 & -88 \end{array}\right]$$

$$R_3 \rightarrow R_3 - 6R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 41 \\ 0 & -1 & -1 & -13 \\ 0 & 0 & -2 & -10 \end{array}\right]$$

So, $x_1 + 2x_2 + 3x_2 = 41$

$-x_2 - x_3 = -13$

$-2x_3 = -10 \longrightarrow x_3 = 5$

and, $\therefore -x_2 - x_3 = -13 \longrightarrow -x_2 - 5 = -13$

$x_2 = 8$

and, $\therefore x_1 + 16 + 15 = 41$

$x_1 = 41 - 31$

$= 10$

$\therefore$ the company will produce 10 centuion, 8 tribune and 5 senator

Q8 the characterstic eqn:

$$[A^2 - \lambda I] = 0$$

$$\begin{bmatrix} 56-\lambda & -40 \\ 20 & -4-\lambda \end{bmatrix} = 0$$

$= (56-\lambda)(-4-\lambda) + 800 = 0$

$-224 - 56\lambda + 4\lambda + \lambda^2 + 800 = 0$

$\lambda^2 - 52\lambda + 576 = 0$

$\lambda(\lambda - 36) - 16(\lambda - 36) = 0$

$(\lambda - 36)(\lambda - 16) = 0$

Now, $\lambda - 36 = 0$ or $\lambda - 16 = 0$

$\lambda = 36$ or $\lambda = 16$

By the relation

$$A^n = \lambda^n$$

$$A^2 = \lambda^2 \longrightarrow A = \lambda$$

$\therefore$ Eigen value of A are 6, 4

Now, let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$A^2 = A.A$$

$$\begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix} = \begin{bmatrix} p^2+q^2 & pq+qs \\ pr+sr & rq+s^2 \end{bmatrix}$$

On comparing we get

$$p^2 + qr = 56 \text{ —— } 1$$

$$pq + qs = -40 \text{ —— } 2$$

$$pr + sr = 20 \text{ —— } 3$$

$$s^2 + qr = -4 \text{ —— } 4$$

$$iv - i \longrightarrow s^2 - p^2 = -60 \text{ —— } 5$$

Also, Sum of principle diagnol = Sum of eigen values

$p+s=6+4$

$p+s=10$

$p=10-s$ —— (6)

Put (6) in (5)

$(10-s)^2-s^2=60$

$100+s^2-20s-s^2=60$

$-20s=-40$

$s=2$

$\therefore p=10-2=8$

from eq (2)

$8q+2q=-40$

$10q=-40$

$q=-4$

from eq (3)

$8r+2r=20$

$10r=20$

$r=20/10=2$

$\therefore A = \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

So, for eigen vector, we have characterstic value $(A - \lambda I)X = 0$

① for $\lambda = 6$

$[A - 6I]\lambda = 0$

$$\begin{bmatrix} 8-6 & -4 \\ 2 & 2-6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2x - 4y = 0$

and, $2x - 4y = 0$

So, $2x = 4y$

$x = 2y$

$y = x/2$

let $x = t$

$y = \frac{t}{2}$ $\longrightarrow$ Eigen vector, $X_1 = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$

② for $\lambda = 4$;

$[A - 4\lambda]X = 0$

$$\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x - 4y = 0 \longrightarrow x - y = 0$

$2x - 2y = 0 \longrightarrow x - y = 0$

So, $x = y$

let, $x = t$

$y = t$

$\therefore$ Eigen vector, $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

09 $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$\therefore$ characterstic eqn

$[A - \lambda I] = 0$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 5 \\ 3 & 5 & 6-\lambda \end{bmatrix} = 0$$

We have,

$\lambda^3 - (\text{sum of diagnol})\lambda^2 + (\text{sum of co-factors of diagnol})x - |A| = 0$

$\rightarrow \lambda^3 - 11\lambda^2 + 4\lambda + 1 = 0$

To verify cayley-hamilton theorm

$$A^3 - 11A^2 - 4A + 1 = 0$$

$$\because A^2 = \begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix}$$

$$\because A^3 = A^2 - A$$

$$A^3 = \begin{bmatrix} 157 & 283 & 353 \\ 283 & 570 & 636 \\ 353 & 636 & 793 \end{bmatrix}$$

Now,

$$A^3 - 11A^2 - 4A + I$$

$$\begin{bmatrix} 157 & 283 & 353 \\ 283 & 570 & 636 \\ 353 & 636 & 793 \end{bmatrix} + \begin{bmatrix} -157 & -275 & -341 \\ -275 & -495 & -616 \\ -341 & -616 & -770 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -12 \\ -8 & -16 & -20 \\ -12 & 20 & -24 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence cayley hamilton theorm is verified

$A^3 - 11A^2 - 4A + I = 0$

Now, multiply both sides by A^{-1}

$$A^2 - 11A - 4I + A^{-1} = A^{-1}(0)$$

$$A^{-1} = A^2 - 11A - 4I$$

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Now,

$$A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$$

$$\rightarrow A^5[A^3 - 11A^2 - 4A + I] + A[A^3 - 11A^2 - 4A + I]$$

$$A^2 + A + I \qquad + A^2 + A + I$$

$$\begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 27 & 34 \\ 27 & 50 & 61 \\ 34 & 61 & 77 \end{bmatrix}$$

Q10 ① $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$[A - \lambda I]\, \lambda = 0$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} = 0$$

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$(\lambda-2)(\lambda^2 - 10\lambda + 16) = 0$

$\rightarrow (\lambda-2)(\lambda(\lambda-8) - 2(\lambda-8)] = 0$

$(\lambda-2)(\lambda-8)(\lambda-2) = 0$

$\rightarrow \lambda = 2, 8, 2$

for $\lambda = 2$

$[A - 2I]\, \lambda = 0$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$4x - 2y + 2z = 0$

$-2x + y - z = 0$

$2x - y + z = 0$

taking, $2x - y + z = 0$

let, $z = 0$, $y = t$

$2x - t = 0$

$x = t/2$

$\therefore \lambda_1 = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}$

let $t = 1$, $y = -1$

$2x - 1 + 1 = 0$

$\longrightarrow x = 0$

$\therefore \lambda_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

let $\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x - 2y + 2z = 0$ ------ 1

$-2x - 5y - z = 0$ ------- 2

$2x - y - 5z = 0$ ------ 3

Solving 1 and 2

$$\frac{x}{\begin{bmatrix} -2 & 2 \\ -5 & -1 \end{bmatrix}} = \frac{-y}{\begin{bmatrix} -2 & 2 \\ -2 & 1 \end{bmatrix}} = \frac{z}{\begin{bmatrix} -2 & -2 \\ -2 & -5 \end{bmatrix}} = t$$

$$\frac{x}{12} = \frac{-y}{6} = \frac{z}{6} = t$$

$x = 12t$, $y = -6t$, $z = 6t$

$$\therefore x_3 = \begin{bmatrix} 12 \\ -6 \\ 6 \end{bmatrix}$$

(2) $A = \begin{bmatrix} 2 & 0 & 2 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

Characterstic eqn : $[A - \lambda I]x = 0$

$$\begin{bmatrix} 2-\lambda & 0 & 2 \\ -1 & 3-\lambda & 1 \\ 1 & -1 & 3-\lambda \end{bmatrix} = 0$$

$\lambda^3 - 8\lambda^2 + 20\lambda - 16 = 0$

$(\lambda - 4)(\lambda^2 - 4\lambda + 4) = 0$

$\therefore \lambda = 2, 2, 4$

Now, for $\lambda = 2$

$[A - 2I]\lambda = 0$

$$\begin{bmatrix} 0 & 0 & 2 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $2z = 0 \longrightarrow z = 0$ —— 1

and, $-x + y + z = 0$ —— 2

and, $x - y + z = 0$ —— 3

from 1 and 2

$= x + y = 0$

$x = y$

let $x = t \longrightarrow y = t$

$[A - 4I]x = 0$

$$\begin{bmatrix} -2 & 0 & 2 \\ -1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x + 2z = 0$ —— 1

$-x + y + z = 0$ —— 2

$x - y - z = 0$ —— 3

from 1 $-2x + 2z = 0$

$x = z$

from 2 $-x - y + z = 0$

$y = 0$

$$\therefore x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$