Numerical Problems
on
Quantum Mechanics

1. Find maximum wavelength that can liberate electron from potassium. Work function of potassium is 2.24 eV. **Ans: 5524Å**

Given

Solution

Work function = $\emptyset = \frac{hc}{\lambda} \rightarrow$ find λ

2. The stopping Potential for a metal is 4.6 V when light of frequency 2×10^{13} Hz is incident on it. When light of frequency 2×10^{15} is used, the stopping potential is 12.9 V. Calculate the Planck's constant. (Given the charge of electron $e = 1.6 \times 10^{-19}$ C). **Ans:6.64×10^{-34} Js**

Solution

3. A metallic surface, when illuminated with light of wavelength λ_1, emits electrons with energies upto a maximum value E_1, and when illuminated with light of wavelength λ_2, where $\lambda_2 < \lambda_1$, it emits electrons with energies upto a maximum value E_2. Prove that plank's constant h and the work function φ of the metal are given by

$$h = \frac{(E_2 - E_1)\lambda_1\lambda_2}{C(\lambda_1 - \lambda_2)} \quad \text{and} \quad \varphi = \frac{E_2\lambda_2 - E_1\lambda_1}{(\lambda_1 - \lambda_2)}$$

Solution

4. Light of wavelength 800 nm is shone on a metal surface connected to a battery. The work function of the metal is 1.25 eV. Find the extinction voltage or retarding voltage at which the photoelectron current stops. Find the highest speed of the emitted photoelectrons at this incident frequency.

Ans: $V_r = 0.3$ volt, 3.2×10^5m/s

Solution

5. Calculate the maximum percentage change in wavelength due to Compton scattering for incident photons of wavelength 1Å and 10Å. What information do you draw from the result?

Ans: 4.8%, 0.48%, the change is difficult to detect in sufficient larger wavelengths.

Solution

6. X-ray of photon of wavelength 0.3 Å is scattered through an angle of 60° by a free electron. Find the wavelength of scattered photon and the recoil energy of the electron.

Ans: 3.01212 Å, 2.57×10^{-16} J

Solution

7. A photon of energy E is scattered by an electron initially at rest (rest mass energy, E_0) (Compton scattering problem). Show that the maximum kinetic energy (KE_{max}) of the recoil electron can be calculated as $KE_{max} = \frac{2E^2/E_0}{\left(1+ 2E/E_0\right)}$

Solution

8. Show that the minimum energy of incident radiation should be ~ 256 KeV in order to transfer half of its energy to recoiled electron.

Solution

9. Calculate the de-Broglie wavelength of α-particle accelerated through a potential difference of 2000 volt.

Ans: 2.3×10^{-3} Å

Solution

10. What will be the percentage error if an electron moves with a speed of equivalent KE of 100KeV if someone does not treat the electron relativistically? **Ans: $\lambda_R < \lambda_{non\text{-}R}$ by 8%**

Solution

11. Show that deBroglie wavelength associated with an electron accelerated from rest with a voltage V is approximately $\frac{12.27}{\sqrt{V}}$.

Solution

12. Calculate the ratio between the wavelength of an electron λ_e and a proton λ_p if the proton is moving with half the velocity of the electron. **Ans: 981**

Solution

13. Compute the ratio of Compton wavelength to de Broglie wavelength in the form of

$$\frac{\lambda_c}{\lambda_d} = \frac{\varphi_0}{(1-\varphi_0)^{1/2}} \text{ where } \varphi_0 = \frac{v}{c}$$

Solution

14. If an excited atom has maximum uncertainty of 10 ns in its life time, what would be the inherent fractional line broadening? Assume emitted wavelength = l (in meter unit).

Ans: λ/12π

Solution

15. Establish the relation $v_g \times v_p = c^2$ for a relativistically moving particle and its associated waves (consider particle velocity = group velocity). Find the phase and group velocity of the associated de Broglie wave of a moving electron (speed 0.9c).

Ans: (1st case) $v_p = w/k = E/p = mc^2/mv = c^2/v = c^2/v_g$ and simplify. (2nd case), $v_p = c/0.9$, and $v_g = 0.9c$

Solution

16. An electron has a speed of 1.05×10^{4} m/sec within the accuracy of 0.01%. Calculate the uncertainty in the position of the electron. **Ans: 5.5×10^{-5} m**

Solution

17. Which of the following wave functions cannot be solutions of Schrodinger's equation for any values of x? Why not? (a) $Y = A \sec x$, (b) $Y = A \tan x$, (c) $Y = A \exp(x^2)$, (d) $Y = \exp(-x^2)$?

Ans: (a-c) not solutions because Y is undefined for x= π /2, π /2, and ∞, respectively (d) is a solution as Y is finite in the range x = 0 to ∞

Solution

18. An electron is trapped in an infinitely deep well of width L. If the electron is in its ground state, what fraction of the time does it spend in central one-third of the well.

Ans: 0.61 or 61%

Solution

19. The formula $y = A\cos(wt - kx)$, where $k = \frac{w}{v}$, describes a wave that moves in the $+x$ direction along a stretched string. Show that this formula represents a solution of the wave equation

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2}$$

Solution

20. Find the value of the normalization constant N for the wave function

(a) $Y = Nx \exp(-x^2/2)$, (b) $Y = N \exp(-x^2/2a^2) \exp(-ikx)$.

Ans: N = [2/√π] $^{0.5}$, N = [1/a√ π] $^{0.5}$

Solution

21. According to the corresponding principle, quantum theory should give the same result as classical physics in the limits of large quantum numbers. Show that as $n \to \infty$, the probability of finding the trapped particle between x and $x + \Delta x$ is $\Delta x / L$. Also show that it is independent of x, which is the classical expectation.

Solution

22. Find the total number of maxima and minima (in probability) for a particle in a box having boundary between 0 and L for $n = 2$ state.

Ans: between 0 and L there are 2 maxima and 1 minimum (excluding boundary points).

Solution

23. (a) Calculate the lowest energy of an electron confined in a cubical box of each side 1Å.
(b) Find the temperature at which the average energy of the molecules of a perfect gas would be equal to the lowest energy of the electron, $k_B = 1.38\times10^{-23}$ J/K. **Ans: 6.02 $\times10^{-18}$ J, 2.9$\times10^{5}$ K**

Solution

24. An electron is confined in a 1D infinite potential box of boundary between 0 and 2 nm. If the particle has 6 nodes find the particle energy in eV. **Ans: 4.57 eV**

Solution

25. Calculate the work function, stopping potential, and maximum velocity of photoelectrons for a light of ***wavelength 4350Å*** when it incidents on sodium surface. Consider the ***threshold wavelength*** of photoelectrons to be ***5420Å***.

Given	Solution

$$\Phi = \frac{12375}{\lambda_0} = \frac{12375}{5420} = 2.28eV$$

$$K.E._{max} = h\nu - \Phi = hc/\lambda - \Phi$$