UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	B.Tech (All SoCSBranches)	Semester	I
Course	Engineering Mathematics	Course Code	MATH1036

1. (i) Reduce the matrix $A=\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ to row echelon form and find its rank.

(ii) Find the rank of the matrix of the following matrix by reducing it to normal form.

$$A=\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

2. If $X_1 = [3, 1, -4]$, $X_2 = [2, 2, -3]$ and $X_3 = [0, -4, 1]$, then show that

(i) The vectors X_1 and X_2 are linearly independent.

(ii) The vectors X_1, X_2 and X_3 are linearly dependent. Write one vector in terms of the others.

3. Find P and Q such that the normal form of the matrix

$$A=\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$ is PAQ.

4. Find the values of k for which the system of equations

$(3k-8)\,x+3y+3z=0,\ 3x+(3k-8)\,y+3z=0, 3x+3y+(3k-8)\,z=0$

has a non-trivial solution.

5. Investigate for what values of a and b so that the system of equations

$2x+3y+5z=9, 7x+3y-2z=8,\ 2x+3y+az=b$ has

No solution (ii) Unique solution (iii) Infinite solutions.

6. Find the conditions on a, b, c for which the given system

$-2x+y+z=a$

$x-2y+z=b$

$x+y-2z=c$

has (i) solution (ii) no solution.

where a, b, c are constants.

7. The manufacturing of an automobile requires painting, drying and polishing. The Rome Motor Company produces three types of cars: the Centurion, the Tribune, and the Senator. Each Centurion requires 8 hours for painting, 2 hours for drying, and 1 hour for polishing. A Tribune needs 10 hours for painting, 3 hours of drying and 2 hours for polishing. It takes 16 hours of painting, 5 hours of drying and 3 hours of polishing to prepare a Senator. If the company uses 240 hours for painting, 69 hours for drying, and 41 hours for polishing in a given month, how many of each type of car are produced?

8. If $A^2 = \begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix}$ then find the eigen value and eigen vectors of A.

9. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Hence compute A^{-1} and find $A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$

10. Find the Eigen values and corresponding Eigen vectors of the following matrices:

(i) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 0 & 2 \\ -1 & 3 & 1 \\ 1 & -1 & 3 \end{bmatrix}$