

GATE-2020

$A \rightarrow PA$  and  $A \rightarrow XY$   
 $P.i = A.i + 2$ ,  $A.i = P.i + A.i$  and  $A.s = P.s + A.s$   
 $X.i = A.i + Y.s$  and  $Y.i = X.s + A.i$

Which of the following is True?

A) Both Rules are L-Attributed  
 B) Only R1 is L-Att.  
 C) Only R2 is L-Att.  
 D) Neither R1 nor R2 L-Attributed

3- Address Code ( $X = Y \text{ OP } Z$ )

$x = (a * b) + (c * d)$   
 $t_1 = a * b$   
 $t_2 = c * d$   
 $x = t_1 + t_2$

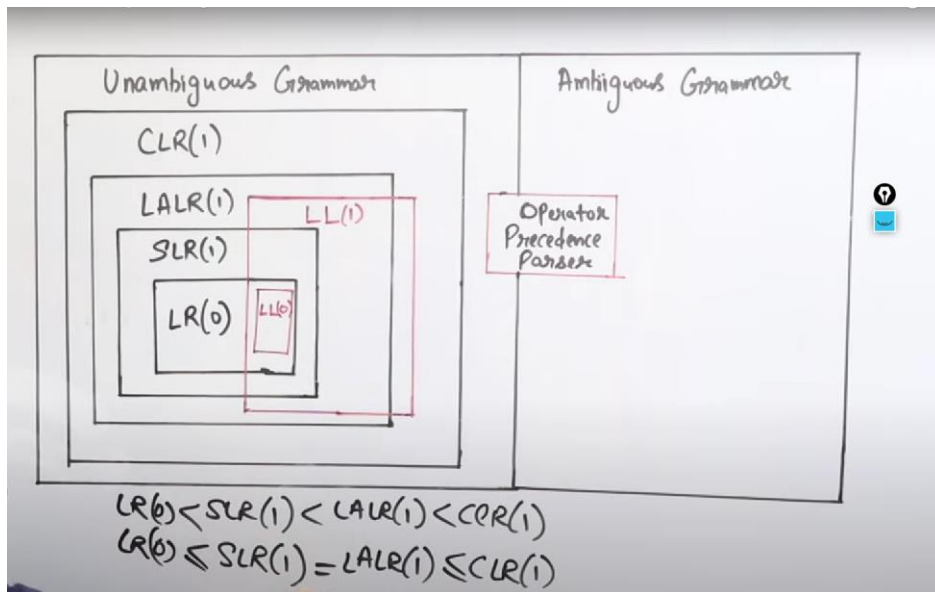
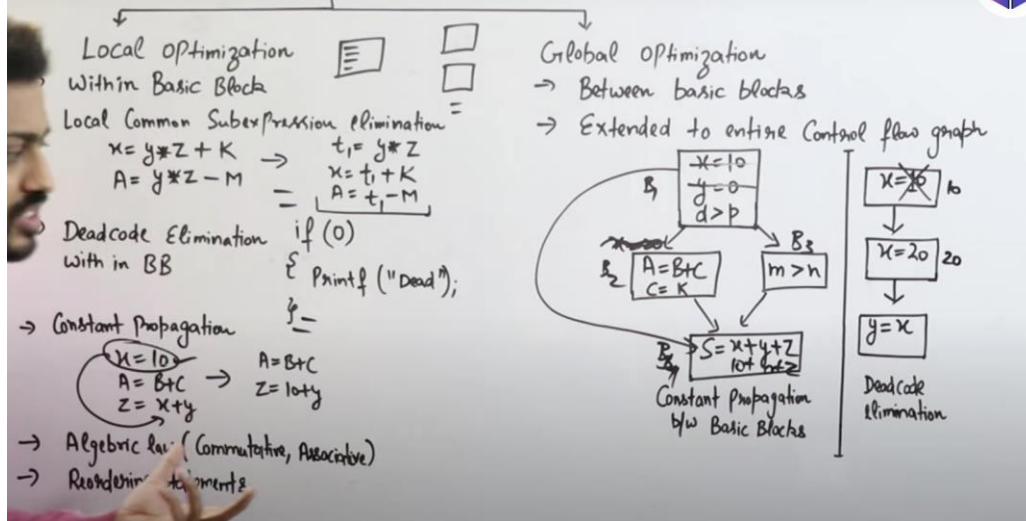
Various statements in 3-address code

1) Assignment  $\rightarrow x = y \text{ OP } z$   $x = y + z$   
 $\rightarrow x = op y$   $x = +y$   $x = x + y$   
 $\rightarrow x = y$

2) Jump  $\rightarrow$  Conditional if  $x \text{ relop } y \text{ goto } L$   $x = y$   
 $\rightarrow$  Unconditional goto  $L$  if  $x > y \text{ goto } L$

3) Array Assignment  $\rightarrow x = y[i]$   $x = y[0]$   
 $\rightarrow x[i] = y$   $x[0] = y$   
 Register, address Assign  $\rightarrow x = \&y$   
 $\rightarrow x = *y$

## Scope of Optimization



Conversion of  $RL \rightarrow RE$

Regular Expression	Regular Language
$\phi$	$\phi$
$\epsilon$	$\{\epsilon\}$
$a$	$\{a\}$
$b$	$\{b\}$
$a b$ or $a+b$	$\{a, b\}$
$ab$	$\{ab\}$
$a^*$	$\{\epsilon, a, aa, aaa, \dots\}$ Always returns even length
$(a b)^*$	$\{\epsilon, ab, abab, ababab, \dots\}$
$a^+b^+$	$\{a, aa, aaa, \dots\}$
$(a b)^+$	Any combination of $a$ and $b$

1. String Starts & Ends with different alphabet over  $\Sigma = \{a, b\}$   
 $\Rightarrow R = a(a|b)^*b + b(a|b)^*a$

2. String Starts & Ends with different alphabet over  $\Sigma = \{x, y, z\}$   
 $\Rightarrow R = (x|y)^*x|y$

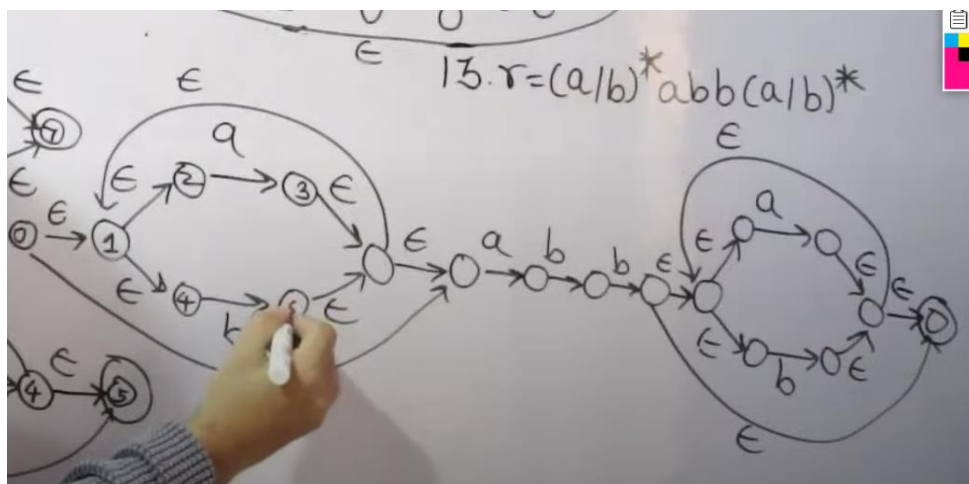
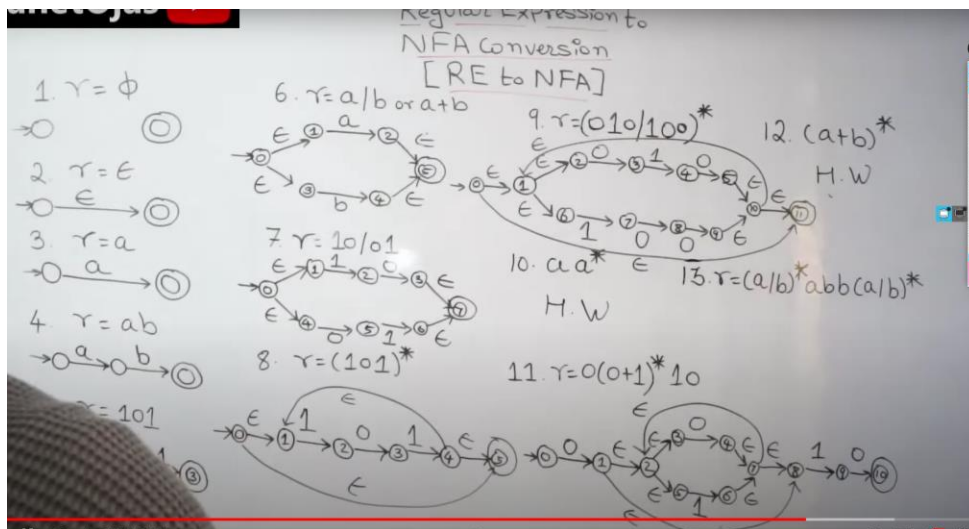
3. String Ends With 011 over  $\Sigma = \{0, 1\}$   
 $\Rightarrow R = (0|1)^*011$

4. String contains abb as a substring over  $\Sigma = \{a, b\}$   
 $\Rightarrow R = (a|b)^*abb(a|b)^*$

5. String Starts with a double occurrence of alphabet over  $\Sigma = \{0, 1\}$   
 $\Rightarrow R = (a|b)a(a|b)(a|b)$

6. Third last symbol is a over  $\Sigma = \{a, b\}$   
 $\Rightarrow R = (a|b)a(a|b)(a|b)$

7. String Starts with 1 & has an odd length or Starts with 0 & has even length  
 $\Rightarrow R = 1(00|11|01|10)^*$



NFA to DFA Conversion

RL  $\rightarrow$  RE  $\rightarrow$  NFA  $\rightarrow$  DFA  $\rightarrow$  DFAMin

$\Rightarrow$  Convert NFA to DFA:

$(\{P, Q, R, S\}, \{0, 1\}, S, P, \{S\})$

Sol:

Q \ Z	0	1
P	P, Q	P
Q	R	R
R	S	-
S	S	S

Sol:

Q \ Z	0	1
A	B	A
B	D	B
C	E	C
D	F	D
E	G	E
F	H	F
G	I	G
H	J	H
I	K	I
J	L	J
K	M	K
L	N	L
M	O	M
N	P	N
O	Q	O
P	R	P
Q	S	Q
R	T	R
S	U	S
T	V	T
U	W	U
V	X	V
W	Y	W
X	Z	X
Y	AA	Y
Z	AB	Z



RL to DFA

Design a DFA which accepts strings if it ends with abb over  $\Sigma = \{a, b\}$

Step 1: RL to RE

$$R = (a/b)^* abb$$

Step 2: RE to NFA

Step 3: NFA to DFA

State	0	1	2	3	4	5
0	0	1				
1	2	1	3			
2	1	2	3	4		
3	2	1	3	4	5	
4	1	2	3	4	5	6
5	2	1	3	4	5	6

Step 4: DFA Minimization

State	0	1	2	3	4	5	6
0	0	1					
1	2	1	3				
2	1	2	3	4			
3	2	1	3	4	5		
4	1	2	3	4	5	6	
5	2	1	3	4	5	6	
6	2	1	3	4	5	6	7

Conversion

RL  $\rightarrow$  RE  $\rightarrow$  NFA  $\rightarrow$  DFA  $\rightarrow$  DFAMin

1 Construct DFA for  $R = (11+10)^*$

Step 1: RE to NFA

Step 2: NFA to DFA

State	0	1	2	3	4	5
0	0	1				
1	2	1	3			
2	1	2	3	4		
3	2	1	3	4	5	
4	1	2	3	4	5	6
5	2	1	3	4	5	6
6	2	1	3	4	5	6

Step 3: DFAMin

State	0	1	2	3	4	5	6
0	0	1					
1	2	1	3				
2	1	2	3	4			
3	2	1	3	4	5		
4	1	2	3	4	5	6	
5	2	1	3	4	5	6	
6	2	1	3	4	5	6	7

Design minimized DFA to accept a string which ends with bcc over  $\Sigma = \{b, c\}$

Sol: RL  $\Rightarrow$  Accept string when it ends with bcc over  $\Sigma = \{b, c\}$

RE  $\Rightarrow (b/c)^* bcc$

RE to NFA with  $\epsilon$  transition

NFA with  $\epsilon$  transition to DFA

State	0	1	2	3	4	5	6	7	8	9	10
0	0	1									
1	2	1	3								
2	1	2	3	4							
3	2	1	3	4	5						
4	1	2	3	4	5	6					
5	2	1	3	4	5	6	7				
6	1	2	3	4	5	6	7	8			
7	2	1	3	4	5	6	7	8	9		
8	1	2	3	4	5	6	7	8	9	10	
9	2	1	3	4	5	6	7	8	9	10	11
10	2	1	3	4	5	6	7	8	9	10	11

DFA Minimization

State	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1										
1	2	1	3									
2	1	2	3	4								
3	2	1	3	4	5							
4	1	2	3	4	5	6						
5	2	1	3	4	5	6	7					
6	1	2	3	4	5	6	7	8				
7	2	1	3	4	5	6	7	8	9			
8	1	2	3	4	5	6	7	8	9	10		
9	2	1	3	4	5	6	7	8	9	10	11	
10	2	1	3	4	5	6	7	8	9	10	11	12
11	2	1	3	4	5	6	7	8	9	10	11	12

Direct Method

IMP State Method

RL

$\downarrow$

RE

$\downarrow$

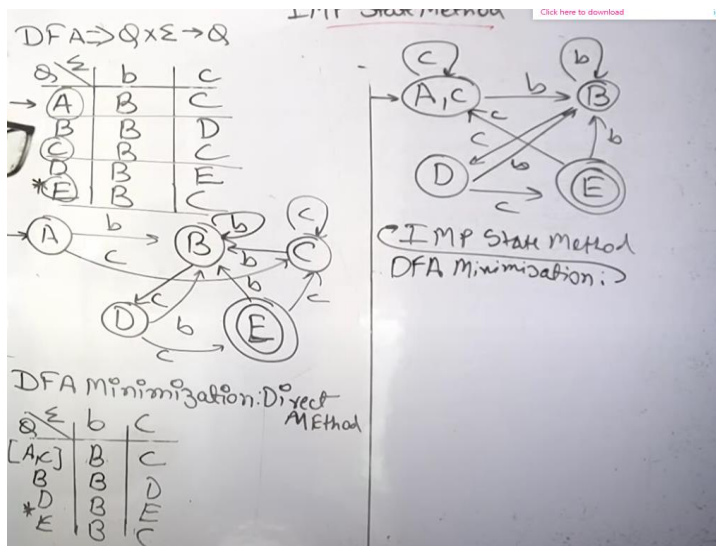
NFA

$\downarrow$

DFA

$\downarrow$

Min DFA



RE/RL to CFG

RE (r)	RL L(r)	CFG
$\emptyset$	$\emptyset$	$S \rightarrow \epsilon$
$\epsilon$	$\{\epsilon\}$	$S \rightarrow \epsilon$
$a$	$\{a\}$	$S \rightarrow a$
$y$	$\{y\}$	$S \rightarrow y$
$a.b$	$\{a.b\}$	$S \rightarrow ab$
$a/b \text{ or } (a/b)$	$\{a, b\}$	$S \rightarrow a/b$
$a^*$	$\{ \epsilon, a, aa, aaa, \dots \}$	$S \rightarrow aS/\epsilon$
$(ab)^*$	$\{ \epsilon, ab, abab, ababab, \dots \}$	$S \rightarrow abS/\epsilon$
$a^+$	$\{ a, aa, aaa, \dots \}$	$S \rightarrow aS/a$
$(ab)^+$	$\{ ab, abab, ababab, \dots \}$	$S \rightarrow abS/ab$
$(a/b)^*$	Any combination of a, b	$S \rightarrow aS/bS/\epsilon$

Convert RE/RL to CFG

- $r = a^*b^*$   
 $S \rightarrow AB$   
 $A \rightarrow aA/\epsilon$   
 $B \rightarrow bB/\epsilon$
- $r = a^+b^*$   
 $S \rightarrow AB$   
 $A \rightarrow aA/a$   
 $B \rightarrow bB/\epsilon$
- $r = a^+b^+c^+$   
 $S \rightarrow ABC$   
 $A \rightarrow aA/a$   
 $B \rightarrow bB/b$   
 $C \rightarrow cC/c$
- $r = (abb+baa)^*$   
 $S \rightarrow abbS/baaS/\epsilon$
- Starts & ends with same numbers  $\epsilon = \{0, 1\}$   
 $r = 0^*(0/1)^*0 + 1(0/1)^*1$   
 $S \rightarrow 0A0/1A1$   
 $A \rightarrow 0A/1A/\epsilon$
- Start with double occurrence  $\epsilon = \{a, b\}$   
 $S \rightarrow aSb$   
 $\rightarrow aaSbb$   
 $\rightarrow aaaabbbb$

Convert RE/RL to CFG

CFG 7. Exactly one occurrence of a  $\epsilon = \{ab\}$

$r = b^+a b^+$

$S \rightarrow A A A$   
 $A \rightarrow bA/\epsilon$

8. 3<sup>rd</sup> last symbol is a over  $\epsilon = \{a, b\}$

$\Rightarrow r = (a/b)^* a (a/b) (a/b)$

$S \rightarrow A A B B$   
 $A \rightarrow aA/bA/\epsilon$   
 $B \rightarrow a/b$

9.  $L = \{a^n b^n / n \geq 1\}$

$S \rightarrow aSb/\epsilon$

10.  $L = \{a^n b^{2n} / n \geq 1\}$

$S \rightarrow aSbb/\epsilon$

$a/b$   
 $aa/bb$   
 $aaa/bbb$   
 $a/bb$   
 $aa/bbbb$   
 $aaa/bbbbbb$

Check whether the grammar is ambiguous or not

Introduction to Theoretical Computer Science (TCS) in Hindi

Q) Consider the following grammar:  
 $E \rightarrow E + E \mid E * E \mid id$   
 Find: i) LMD ii) RMD iii) a parse tree iv) Check whether the grammar is ambiguous.

Q) Let  $G$  be the grammar. Find a Parse Tree for the string  $id + id * id$ .  
 $S \rightarrow aB \mid bA$   
 $A \rightarrow aA \mid bAA$   
 $B \rightarrow bB \mid aBB$   
 Check whether the grammar is ambiguous or not.  
 $\Rightarrow$  Let's number the strings.

1)  $E \rightarrow E + E$   
 2)  $E \rightarrow E * E$   
 3)  $E \rightarrow id$

LMD:  
 $E \rightarrow E + E (E \rightarrow E + E)$   
 $E \rightarrow id + E (E \rightarrow id)$   
 $E \rightarrow id + E * E (E \rightarrow E * E)$   
 $E \rightarrow id + id * E (E \rightarrow id)$   
 $E \rightarrow id + id * id (E \rightarrow id)$

RMD:  
 $E \rightarrow E + E (E \rightarrow E + E)$   
 $E \rightarrow E + E * E (E \rightarrow E * E)$   
 $E \rightarrow E + E * id (E \rightarrow id)$   
 $E \rightarrow E + id * id (E \rightarrow id)$   
 $E \rightarrow id + id * id (E \rightarrow id)$

Parse tree:

```

graph TD
    E1[E] --- E2[E]
    E1 --- P1[+]
    E1 --- E3[E]
    E2 --- id1[id]
    E3 --- E4[E]
    E3 --- P2[*]
    E3 --- E5[E]
    E4 --- id2[id]
    E5 --- id3[id]
  
```

Ambiguity check:  
 $E \rightarrow E * E (E \rightarrow E * E)$   
 $E \rightarrow E + E * E (E \rightarrow E + E)$   
 $E \rightarrow id + E * E (E \rightarrow id)$   
 $E \rightarrow id + id * E (E \rightarrow id)$   
 $E \rightarrow id + id * id (E \rightarrow id)$   
 The given grammar is Ambiguous.

Check whether the given grammar is ambiguous or not!

$S \rightarrow iCtS \mid iCtSeS \mid a$   
 $C \rightarrow b$  sentence "ibtibtaea"

$S \rightarrow iCtS (S \rightarrow iCtS)$   
 $\rightarrow ibtS (C \rightarrow b)$   
 $\rightarrow ibtiCtSeS (S \rightarrow iCtSeS)$   
 $\rightarrow ibtibtsS (C \rightarrow b)$   
 $\rightarrow ibtibtaes (S \rightarrow a)$   
 $\rightarrow ibtibtaea (S \rightarrow a)$

$S \rightarrow iCtSeS (S \rightarrow iCtSeS)$   
 $\rightarrow ibtSeS (C \rightarrow b)$   
 $\rightarrow ibtiCtSeS (S \rightarrow iCtS)$   
 $\rightarrow ibtibtses (C \rightarrow b)$   
 $\rightarrow ibtibtaes (S \rightarrow a)$   
 $\rightarrow ibtibtaea (S \rightarrow a)$

The given grammar is Ambiguous.