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B. Tech (AI)
7AI-1

CSE 471

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Time Series Analysis
ASSIGNMENT

Ans 1) a) $C = \frac{n - f + 2P}{s} + 1$

$n = 28$
$f = 3$
$P = 0$
$s = 2$

$$C = \frac{28 - 3 + 0}{2} + 1$$

$$= \frac{25}{2} + 1$$

$$= \frac{27}{2} = 13.5$$

Since, the output size must be a whole number,
$\therefore$ the size of the Convoluted matrix is 13×13

b) The vanishing gradient problem is a challenge that arises during the training of deep neural networks, particularly in Recurrent neural networks (RNNs). It occurs when the gradients used to update the model's weights during the backpropagation process become very small, effectively causing the network's weights to stop updating. This problem hinders the ability of deep networks to learn and can lead to poor performance

The vanishing gradient problem is closely related to the structure and architecture of the neural network, and in particular, how information is propagated through the network during backpropagation

Vanishing gradient related to RNNs as follows-

1) Sequential nature of RNNs
RNNs are designed for sequential data, effectively capturing dependencies between time steps in tasks like time series prediction and language processing.

2) Backpropagation through Time (BPTT)
RNN training uses backpropagation through time to adapt weights for sequential data, over time to enable learning.

3) Repetition of Weight Matrices
RNNs reuse weight matrices at each time step to facilitate the flow of information between steps, a fundamental feature that also contributes to the vanishing gradient problem.

4) Vanishing Gradient Problem
It arises when gradients become tiny during backpropagation in RNN, especially when weights are shared across time steps, hindering learning.

5) Effect on Learning
This issue severely limits RNNs in learning long-range dependencies and extracting context

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over extended sequences, which is vital for tasks like language understanding and time series analysis.

6) Solutions
To address the vanishing gradient problem in RNNs, advanced technologies like LSTM & GRU have been developed, improving gradient flow and enhancing the ability to learn from sequential data.

Ans 2) AR(1) Model,

$$K_{t+1} = \rho_0 + \rho_1 K_t + \varepsilon_{t+1}$$

where,

$$E(K_{t+1}) = 2$$
$$Var(K_{t+1}) = 1/3$$
$$\rho_0 = 1$$
$$\varepsilon_{t+1} \sim NIID(0, \sigma_\varepsilon^2)$$

Normally, Independently & Identically Distributed

$$E(K_{t+1}) = \rho_0 + \rho_1 E(K_t) + E(\varepsilon_{t+1})$$

Since $\rho_0 = 1$
and, $E(\varepsilon_{t+1}) = 0$ [$\because$ it follows a normal distribution with a mean of 0]

Now,

$$2 = 1 + \rho_1 E(K_t) + 0$$

$E(K_{t+1}) = E(K_t)$ [$\because$ it is IID and thus, AR(1) being stable & stationary]

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$$2 = 1 + \rho_1 (2)$$
$$\rho_1 = \frac{1}{2}$$
$$\boxed{\rho_1 = 0.5}$$

H.P.

Now,

$$Var(K_{t+1}) = Var(\rho_0 + \rho_1 K_t + \varepsilon_{t+1})$$

$Var(\varepsilon_{t+1}) = \sigma_\varepsilon^2$ (Given)

and K_t & ε_{t+1} are indipendent

So,

$$Var(K_{t+1}) = Var(\rho_0) + Var(\rho_1 K_t) + Var(\varepsilon_{t+1})$$

Since, ρ_0 is constant,

$$Var(\rho_0) = 0$$

and $Var(K_t) = Var(K_{t+1}) = 1/3$

Now, eq^n becomes,

$$1/3 = 0 + \rho_1^2 \, Var(K_t) + \sigma_\varepsilon^2$$
$$1/3 = \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{3}\right) + \sigma_\varepsilon^2$$

$$\sigma_\varepsilon^2 = 1/3 - \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$$
$$\sigma_\varepsilon^2 = \frac{4-1}{12}$$
$$\sigma_\varepsilon^2 = \frac{3}{12}$$
$$\boxed{\sigma_\varepsilon^2 = 1/4}$$

H.P.

Ans 3) AR(1) Model

$y_t = 0.2 + 0.4\, y_{t-1} + \mu_t$, with disturbance having zero mean & unit variance

$E[\mu_t] = 0$ [Given]

& $\sigma_E^2 = 1$

a) Unconditional Mean of y

$y_t = y$ (steady state mean) (In long run)

Taking expected value of AR(1) model,

$$E[y_t] = E[0.2 + 0.4\, y_{t-1} + \mu_t]$$

$\because$ μ_t has 0 mean & independent of y_t

$E[\mu_t] = 0$

So,

$$E[y_t] = 0.2 + 0.4\, E[y_{t-1}]$$

Since, we are in steady state (unconditional)

So, $E[y_t] = E[y_{t-1}] = y$

Now, eq^n becomes,

$$E[y] = 0.2 + 0.4\, E[y]$$

$$(1 - 0.4)\, E[y] = 0.2$$

$$E[y] = \frac{0.2}{0.6}$$

$$\boxed{E[y] = 1/3}$$

$\therefore$ Unconditional mean of y is $1/3$

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(b) Unconditional Variance of AR(1)

$$Var[y] = \frac{\sigma_{\mu_t}^2}{(1-\phi^2)}$$

where,

$\sigma_{\mu_t}^2 \Rightarrow Var(\mu_t) = 1$ (given)

$\phi \Rightarrow$ coefficient of lagged value $= 0.4$ (given)

Now,

$$Var[y] = \frac{1}{[1-(0.4)^2]}$$

$$Var[y] = \frac{1}{(1-0.16)}$$

$$Var[y] = \frac{1}{0.84}$$

$$\boxed{Var[y] = 1.1905}$$

$\therefore$ Unconditional variance of AR(1) process is 1.1905 (approx.)

Ans 4) ID3 Algorithm,

$P(\text{Species} = \text{Human}) = \frac{5}{10} = \frac{1}{2}$

$P(\text{Species} = \text{Alien}) = \frac{5}{10} = \frac{1}{2}$

$S = [5_H, 5_A]$

$$\text{Entropy}(S) = -P(A)\log_2(P(A)) - P(H)\log_2(P(H))$$

$$= -0.5 \times \log_2(0.5) - 0.5 \log_2(0.5)$$

$$= -2 \times 0.5 \times (-1)$$

$$E(S) = 1$$

Split 1

For Blue

$S_{Blue} \leftarrow [4+, 6-]$

$E(S_{Blue}) = -\frac{4}{10} \log_2 \frac{4}{10} -$

$S_{Blue=Y} \leftarrow [1H, 3A]$

$$E(S_{Blue=Y}) = -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{4}\log_2\left(\frac{3}{4}\right)$$
$$= -0.25 \times \log_2(0.25) - 0.75 \times \log_2(0.75)$$
$$= -0.25(-2) - 0.75(-0.415)$$
$$= 0.5 + 0.31125$$
$$= 0.8112$$

$S_{Blue=N} \leftarrow [4H, 2A]$

$$E(S_{Blue=N}) = -\frac{4}{6}\log_2\left(\frac{4}{6}\right) - \frac{2}{6}\log_2\left(\frac{2}{6}\right)$$
$$= 0.9182$$

$$IG(S, Blue) = E(S) - \text{Weighted Entropy}$$
$$= 1 - \frac{4}{10} \times E(Blue = Y) - \frac{6}{10} \times E(Blue = N)$$
$$= 1 - \frac{4}{10} \times 0.8112 - \frac{6}{10} \times 0.9182$$

$$= 0.1246$$

For Legs

$S_{Legs=3} \leftarrow [0H, 3A]$

$$E(S_{Legs=3}) = -\frac{0}{3}\log_2\left(\frac{0}{3}\right) - \frac{3}{3}\log_2\left(\frac{3}{3}\right)$$
$$= 0$$

$S_{Legs=2} \leftarrow [5H, 2A]$

$$E(S_{Legs=2}) = -\frac{5}{7}\log_2\left(\frac{5}{7}\right) - \frac{2}{7}\log_2\left(\frac{2}{7}\right)$$
$$= 0.8631$$

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$IG(S, Legs) = 1 - \frac{3}{10} \times 0 - \frac{7}{10} \times 0.8631$
$= 0.39583$

For Height

$S_{Ht=T} \leftarrow [3H, 3A]$

$E(S_{Ht=T}) = -\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \frac{3}{6}\log_2\left(\frac{3}{6}\right)$
$= 1$

$S_{Ht=S} \leftarrow [2H, 2A]$

$E(S_{Ht=S}) = -\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)$
$= 1$

$IG(S, Height) = 1 - \frac{6}{10} \times 1 - \frac{4}{10} \times 1$
$= 0$

For Smelly

$S_{Yes} \leftarrow [2H, 2A]$

$E(S_{Smelly=Y}) = -\frac{2}{4}\log_2\left(\frac{2}{4}\right) - \frac{2}{4}\log_2\left(\frac{2}{4}\right)$
$= 1$

$S_{Smelly=N} \leftarrow [3H, 3A]$

$E(S_{Smelly=N}) = -\frac{3}{6}\log_2\left(\frac{3}{6}\right) - \frac{3}{6}\log_2\left(\frac{3}{6}\right)$
$= 1$

$IG(S, Smelly) = 1 - \frac{4}{10} \times 1 - \frac{6}{10} \times 1$
$= 0$

From the gains,

$IG(S, Blue) = 0.1246$
$IG(S, Legs) = 0.39583$
$IG(S, Height) = 0$
$IG(S, Smelly) = 0$

So, the Root node is Legs

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Split 2

When root node 'Legs' = 2

$E(S_{Legs=2}) = 0.8631$

For Blue

$S_{Blue=Y} \leftarrow [1H, 1A]$

$E(S_{Blue=Y}) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right)$

$= 1$

$S_{Blue=N} \leftarrow [4H, 1A]$

$E(S_{Blue=N}) = -\frac{4}{5}\log_2\left(\frac{4}{5}\right) - \frac{1}{5}\log_2\left(\frac{1}{5}\right)$

$= 0.7219$

$IG(S_{Legs=2}, Blue) = E(S_{Legs=2}) - \frac{2}{7} \times E(S_{Blue=Y})$

$- \frac{5}{7} \times E(S_{Blue=N})$

$= 0.8631 - \frac{2}{7} \times 1 - \frac{5}{7} \times 0.7219$

$= 0.0617$

For Height

$S_{Ht=T} \leftarrow [3H, 1A]$

$E(S_{Ht=T}) = -\frac{3}{4}\log_2\frac{3}{4} - \frac{1}{4}\log_2\left(\frac{1}{4}\right)$

$= 0.8112$

$S_{Ht=S} \leftarrow [2H, 1A]$

$E(S_{Ht=S}) = -\frac{2}{3}\log_2\left(\frac{2}{3}\right) - \frac{1}{3}\log_2\left(\frac{1}{3}\right)$

$= 0.9182$

$IG(S_{Legs=2}, Height) = 0.8631 - \frac{4}{7} \times 0.8112$

$- \frac{3}{7} \times 0.9182$

$= 0.006042$

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For Smelly

$S_{Smelly=Y} \leftarrow [2H, 1A]$

$E(S_{Smelly=Y}) = -\frac{2}{3}\log_2(\frac{2}{3}) - \frac{1}{3}\log_2(\frac{1}{3})$

$= 0.9182$

$S_{Smelly=N} \leftarrow [3H, 1A]$

$E(S_{Smelly=N}) = -\frac{3}{4}\log_2(\frac{3}{4}) - \frac{1}{4}\log_2(\frac{1}{4})$

$= 0.8112$

$IG(S_{Legs=2}, Smelly) = 0.8631 - \frac{3}{7} \times 0.9182 - \frac{4}{7} \times 0.8112$

$= 0.006042$

From the gains,

$IG(S_{Legs=2}, Blue) = 0.0617$

$IG(S_{Legs=2}, Height) = 0.00604$

$IG(S_{Legs=2}, Smelly) = 0.00604$

the next decision node for $S_{Legs=2}$ part is Blue

When root node 'Legs' is 3,

Species is Alien

No further splitting for this part

Split 3

When decision node 'Blue' is Yes

$E(S_{Legs=2, B=Y}) = 1$

For Height

$S_{Ht=T} \leftarrow [0H, 1A]$

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$E(S_{Ht=T}) = 0$

$S_{Ht=S} \leftarrow [1_H, 0_A]$

$E(S_{Ht=S}) = 0$

$IG(S_{Legs=2, B=Y}, \text{Height}) = 1 - \frac{1}{2} \times 0 - \frac{1}{2} \times 0$
$= 1$

For Smelly

$S_{Smelly=Y} \leftarrow [0_H, 0_A]$

$E(S_{Smelly=Y}) = 0$

$S_{Smelly=N} \leftarrow [1_H, 1_A]$

$E(S_{Smelly=N}) = 1$

$IG(S_{Legs=2, B=Y}, \text{Smelly}) = 1 - 0 \times 0 - \frac{2}{2} \times 1$
$= 0$

From gains,

$IG(\text{Height}) = 1$

$IG(\text{Smelly}) = 0$

we choose next decision node as height

where $S_{Legs=2, B=Y}$

When decision node 'Blue' is No

$E(S_{Legs=2, B=N}) = 0.7219$

For Height

$S_{Ht=T} \leftarrow [3H, 0A]$

$E(S_{Ht=T}) = -\frac{3}{3} \log_2(1) - 0 \log_2(0)$
$= 0$

$S_{Ht=S} \leftarrow [1H, 1A]$

$E(S_{Ht=S}) = 1$

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$IG(\text{Height}) = 0.7219 - \frac{3}{5} \times 0 - \frac{2}{5} \times 1$
$= 0.3219$

For Smelly

$S_{Smelly=Y} \leftarrow [2H, 1A]$

$E(S_{Smelly=Y}) = -\frac{2}{3}\log_2(\frac{2}{3}) - \frac{1}{3}\log_2(\frac{1}{3})$
$= 0.9182$

$S_{Smelly=N} \leftarrow [2H, 0A]$

$E(S_{Smelly=N}) = -\frac{2}{2}\log_2(1) - 0$
$= 0$

$IG(\text{Smelly}) = 0.7219 - \frac{3}{5} \times 0.9182 - \frac{2}{5} \times 0$
$= 0.1709$

From Gains,

$IG(\text{Height}) = 0.3219$

$IG(\text{Smelly}) = 0.1709$

So, we choose 'Height' as the next decision node for when $S_{Legs=2,\ Blue=N}$

Split 4

When decision node 'height' = T with $S_{Legs=2,\ B=N}$

then Species = Human
No further splitting

But when decision node 'height' = S

$E(S_{Legs=2,\ B=N,\ ht=S}) = 1$

For Smelly

$S_{Smelly=Y} \leftarrow [0H, 1A]$

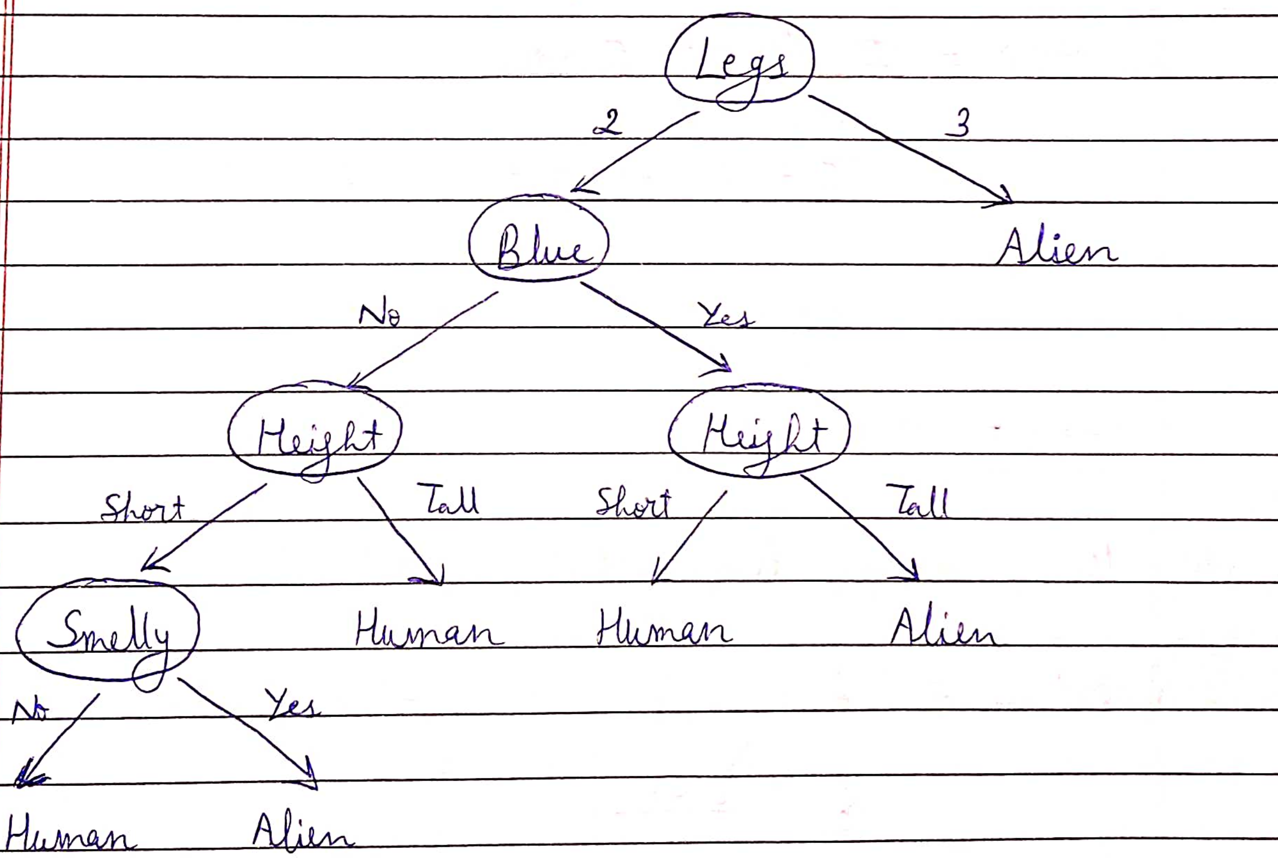
$E(S_{Smelly=Y}) = 0$

$S_{Smelly=N} \leftarrow [1H, 0A]$

$E(S_{Smelly=N}) = 0$

$$IG = 1 - \tfrac{1}{2} \times 0 - \tfrac{1}{2} \times 0$$
$$= 1$$

From the above splits, we can construct the decision tree as follows–



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Ans 5)

a)

Period	Actual Demand (D_i)	Forecast Demand (F_i)	Absolute Error $E_i = \lvert D_i - F_i \rvert$
April	225	240	15
May	220	200	20
June	285	270	15
July	290	300	10
August	250	230	20
September	276	250	26

a) Mean Absolute Deviation (MAD)

$$MAD = \frac{\Sigma \text{ Absolute Errors}}{\text{No. of Periods}}$$

$$= \frac{15 + 20 + 15 + 10 + 20 + 26}{6}$$

$$= \frac{106}{6}$$

$$\boxed{MAD = 17.667}$$

b) Mean Absolute Percent Error (MAPE)

$$MAPE = \frac{\Sigma \text{ Absolute Percent Error}}{\text{No. of Periods}} \times 100$$

$$= \frac{\sum_{i=1}^{n} \left| \frac{D_i - F_i}{D_i} \right|}{\text{No. of Periods} = (n)} \times 100$$

$$= 100\left(\frac{15/225 + 20/220 + 15/285 + 10/290 + 20/250 + 26/276}{6} \right)$$

$$= 100\left(\frac{0.067 + 0.09 + 0.052 + 0.034 + 0.08 + 0.094}{6} \right)$$

$$= 0.0695 \times 100$$

$$\boxed{MAPE = 6.95\%}$$

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c) Mean Squared Error (MSE)

$$MSE = \frac{\Sigma\,(\text{Absolute Error})^2}{\text{No. of Periods}}$$

$$= \frac{\sum_{i=1}^{n=6} (D_i - F_i)^2}{n}$$

$$= \frac{15^2 + 20^2 + 15^2 + 10^2 + 20^2 + 26^2}{6}$$

$$= \frac{2026}{6}$$

$$\boxed{MSE = 337.67}$$

Ans 6)

a) $P(q_2 = \text{Happy} \mid q_1 = \text{Happy}) = 0.8$

b) $P(O_2 = \text{frown}) = P(O_2 = \text{frown} \mid q_2 = \text{Happy}) + P(O_2 = \text{frown} \mid q_2 = \text{Angry})$

$= P(\text{Happy} \mid \text{Happy}) \times P(\text{frown} \mid \text{Happy}) + P(\text{Angry} \mid \text{Happy}) \times P(\text{frown} \mid \text{Angry})$

$= (0.8 \times 0.1) + (0.2 \times 0.5)$

$= 0.18$

c) $P(q_2 = \text{Happy} \mid O_2 = \text{frown}) = \dfrac{P(O_2 = f \mid q_2 = H) \times P(q_2 = H)}{P(O_2 = f)}$

$$= \frac{P(H \mid H) \times P(F \mid H)}{0.18}$$

$$= \frac{0.8 \times 0.1}{0.18}$$

$$= 0.4444$$

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d) $P(f, f, f, f, f, H, A, A, A, A)$

$\Rightarrow P(f|H) \times P(f|A) \times P(f|A) \times P(f|A) \times P(f|A)$
$\times P(H) \times P(A|H) \times P(A|H) \times P(A|H) \times P(A|H)$

$\Rightarrow 0.1 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.7 \times 0.2 \times 0.2$
$\times 0.2 \times 0.2$

$\Rightarrow 0.000007$