Optimization - EE5327

Hitesh Kumar - MA17BTECH11004

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Gradient Descent Method :
Gradient descent is based on the observation that if the multi-variable function $F(\mathbf{x})$ is defined and differentiable in a neighborhood of a point $\mathbf{a}$, then $F(\mathbf{x})$ decreases fastest if one goes from $\mathbf{a}$ in the direction of the negative gradient of F at $\mathbf{a}, -\nabla F(\mathbf{a})$. It follows that, if
$\mathbf{a}_{n+1} = \mathbf{a}_n - \gamma \nabla F(\mathbf{a}_n)$
for $\gamma \in \mathbb{R}_+$ small enough, then $F(\mathbf{a_n}) \geq F(\mathbf{a_{n+1}})$
Consider the problem of finding the square root of a number c. This can be expressed as the equation

$$x^2 - c = 0$$

Problem 2.1

Sketch the function for different values of c

$$f(x) = x^3 - 3xc$$

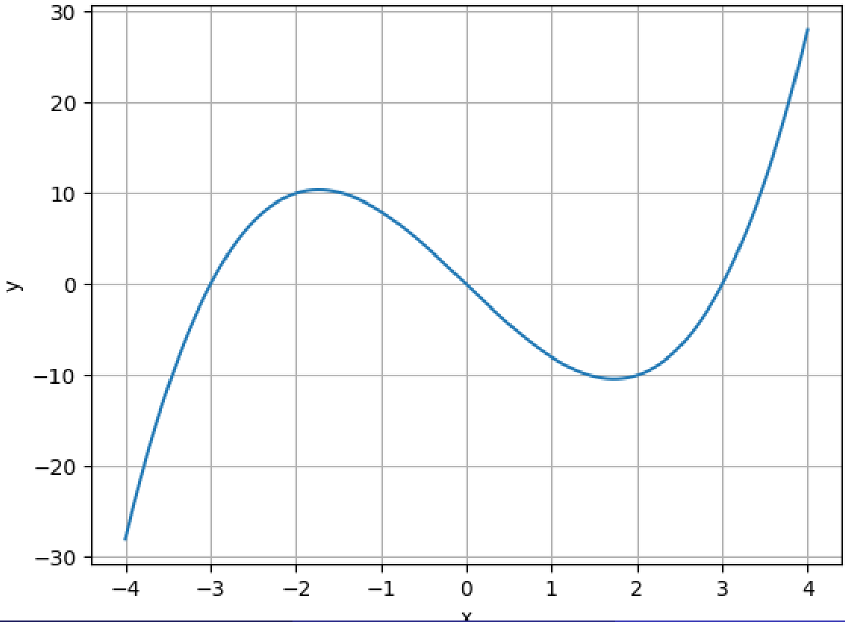
and comment upon its convexity.

Solution.

CASE 1.

$$c > 0$$

let c $=$ 3,
Clearly from the graph,
Function is convex in domain [0,∞)
And concave in domain (-∞,0]

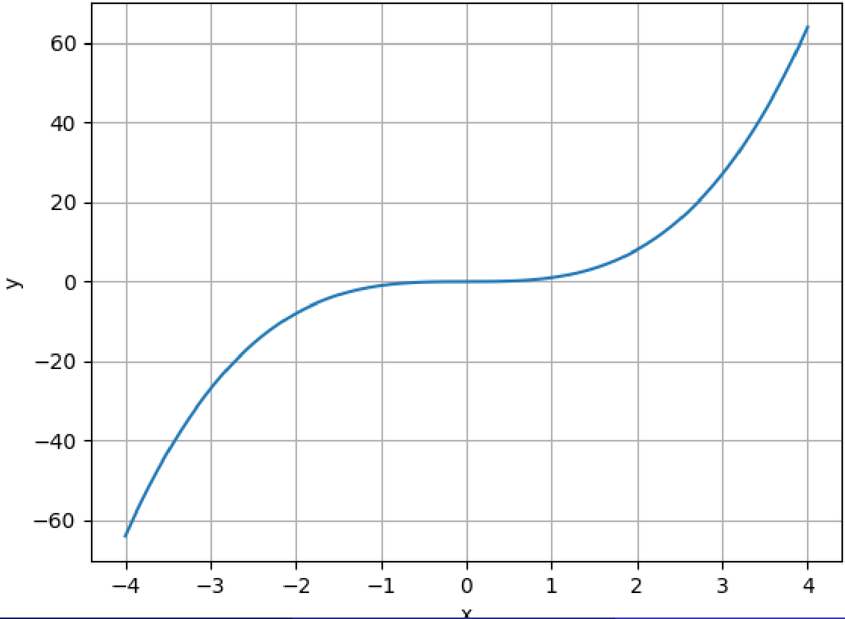

30
20
10
0
−10
−20
−30
y
−4
−3
−2
−1
0
1
2
3
4
x

Solution.

CASE 2.

$$c = 0$$

Again, from the graph,
Function is convex in domain $[0,\infty)$
And concave in domain $(-\infty,0]$

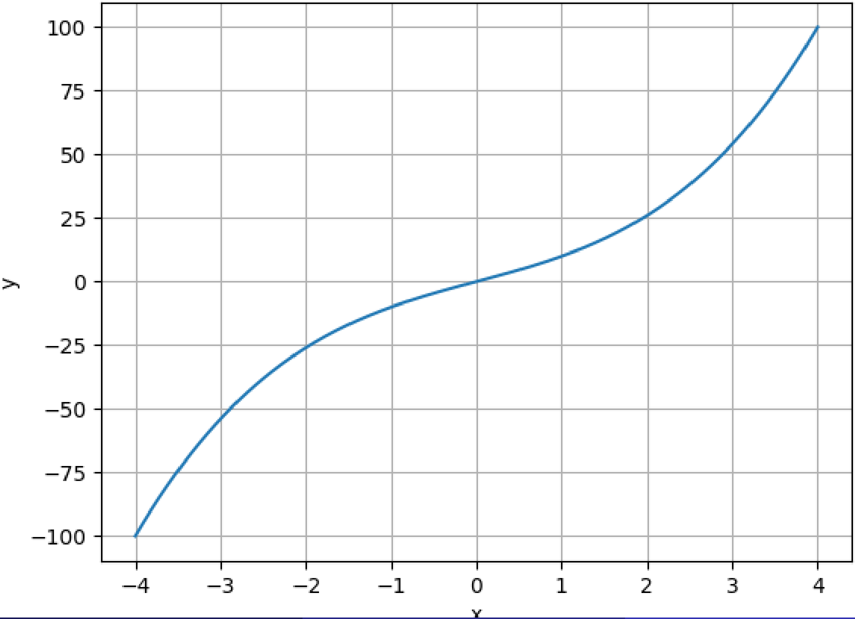

60
40
20
0
−20
−40
−60
y
−4
−3
−2
−1
0
1
2
3
4
x

Solution.

CASE 3.

$$c < 0$$

let c = - 3,
Again from the graph,
Function is convex in domain $[0,\infty)$
And concave in domain $(-\infty,0]$


100
75
50
25
0
−25
−50
−75
−100
y
−4
−3
−2
−1
0
1
2
3
4
x

Problem 2.4

Write a program to implement

$$x_{n+1} = x_n - \mu(3x_n^2 - 3c)$$

Solution.

Consider the function

$$x^3 - 3xc = 0$$

Now the minima exists if c $>$ 0 and domain is (0,∞] as function is convex in this domain.
And The minima is at square root of c. So in order to find square root of c use this function and apply gradient descent.
So, a numerical solution for

$$x^2 - c = 0$$

can be

$$x_{n+1} = x_n - \mu(3x_n^2 - 3c)$$

It will surely have a square root if c $\geq$ 0

Code Explanation.

Fix the value of c.
Set initial value x_0
Set set size multiplier $\gamma > 0$
Set some precision, difference value and Total no. of iterations.
Here difference value is abs$(x_{n+1} - x_n)$. Now initiate a while loop to form this sequence and iterate till the sequence converges i.e. either we achieve the precision or maximum no. of iterations.
So, if difference reaches below the precision value then we can say sequence converges.