Assignment - 3

EC-201-3B (Aug-Dec-2024)

Q① Consider a sinusoidal signal with binary random phase given as $X(t) = \cos(t+\phi)$ where ϕ is a random variable determined by

$$\phi = \begin{cases} 0 & ; P\{\phi=0\} = 1/2 \\ \pi & ; P\{\phi=\pi\} = 1/2 \end{cases}$$

Determine the following :

(a) Is $X(t)$ discrete or continuous ?

(b) Is $X(t)$ SSS or WSS ?

(c) Is it white process ?

(d) Is it ergodic process ?

Q② Consider a random process $X(t)$ defined by $X(t) = \sin(2\pi f t)$ in which the frequency 'f' is a random variable uniformly distributed over the interval $(0, W)$. Show that $X(t)$ is non-stationary.

Q③ Describe the following :

(a) SSS process (b) WSS process

(c) Independent process (d) Ergodic process

(e) White noise process (f) Probability mass function

(g) Gaussian process

Q④ Consider a sinusoidal signal with binary random phase $X(t) = \cos(t + \Theta)$ where Θ is the random variable determined by

$$\Theta = \begin{cases} 0 & ; \; P\{\Theta = 0\} = 1/3 \\ \pi/2 & ; \; P\{\Theta = \pi/2\} = 1/3 \\ \pi & ; \; P\{\Theta = \pi\} = 1/3 \end{cases}$$

Determine the following :

(a) Is $X(t)$ a SSS or WSS process ?

(b) Is $X(t)$ a white process ?

(c) Is $X(t)$ an ergodic process ?

Q⑤ Let C be the covariance matrix of three random variables X_1, X_2, X_3 and a linear transformation $Y = AX$ is made where

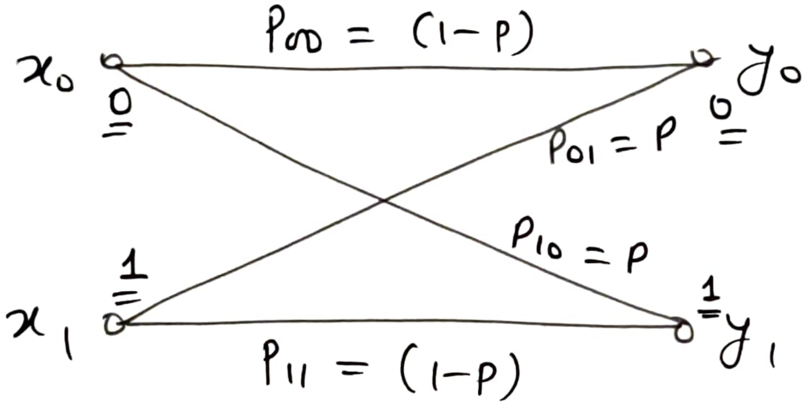
$$C = \begin{bmatrix} C_{11} & 0 & C_{13} \\ 0 & C_{22} & 0 \\ C_{31} & 0 & C_{33} \end{bmatrix} \quad \& \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Determine the covariance matrix of Y.

Q⑥ Discuss the difference between discrete random variables, continuous random variables and mixed random variables with suitable applications.

Q7 Consider the binary communication channel shown in figure. Let (X, Y) be a bivariate random variable, where X is the input to the channel and Y is the output to the channel.

Let $P\{X=0\} = 0.5$, $P\{Y=1 \mid X=0\} = 0.2$, and $P\{Y=0 \mid X=1\} = 0.2$.



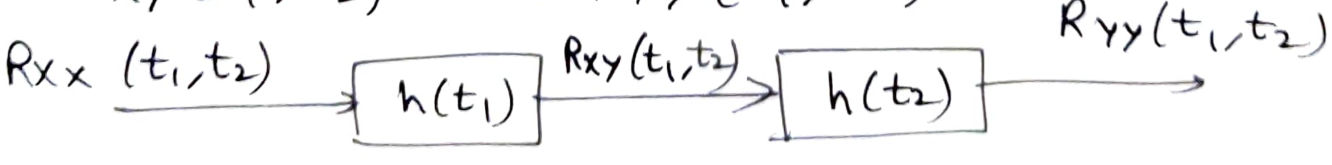
Determine the following:

(a) Find the joint probability mass function of (X, Y).

(b) Find the marginal probability mass function of X and Y.

(c) Are X and Y independent?

Q8 For the linear system shown below, find out the expression of autocorrelations $R_{XY}(t_1, t_2)$ and $R_{YY}(t_1, t_2)$.



Q 9 Consider the following Rayleigh distribution function

$$f_X(x) = \frac{x^{n-1}}{2^{(n-2)/2}\, \sigma^n\, \Gamma\left(\frac{n}{2}\right)} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

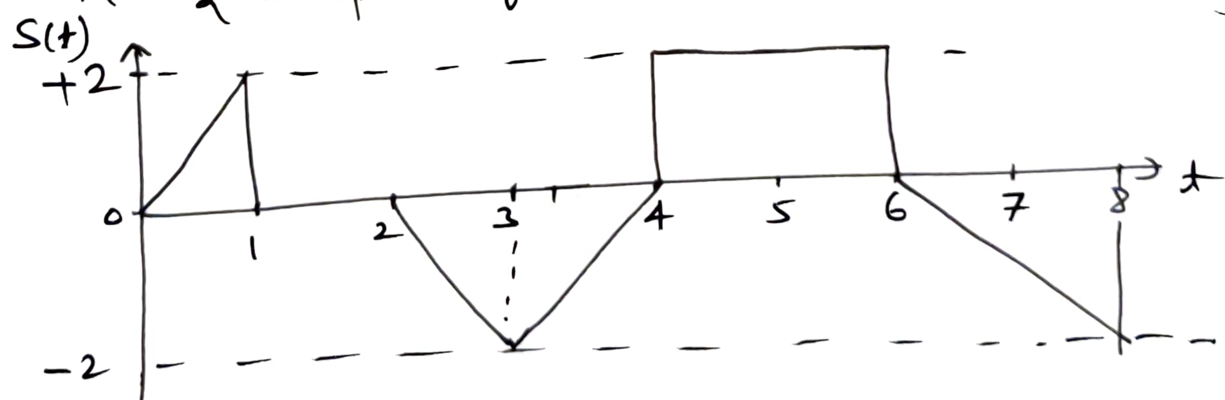
Consider variance to be 4 and $n = 2$.

(a) Derive the CDF for this pdf, $f_X(x)$.

(b) Compute the probability $P\{X \le 10\}$.

Q 10 An aperiodic signal of finite duration 8 seconds is shown below. This is the output of the signal generator. This output is sampled arbitrarily at a time instant $t = c$. Hence the random variable (rv) is defined as

$X = \{\text{sample of } s(t) \text{ at time instant } t = c\}$



Determine the following:

(a) PDF (b) CDF (c) $P\{-2 < X < 2\}$

& $P(-0.5 < X < -1.5\}$

Q (11) A random process $X(t)$ has the following member functions :

$x_1(t) = -2\cos(t)$, $x_2(t) = -2\sin(t)$,

$x_3(t) = 2[\cos(t) + \sin(t)]$,

$x_4(t) = [\cos(t) - \sin(t)]$

$x_5(t) = [\sin(t) - \cos(t)]$

Each member function occur with equal probability. Determine the following :

(a) Mean function, $\mu_x(t)$.

(b) $R_{xx}(t_1, t_2)$

(c) Is the process WSS ? Is it a SSS ?

Q (12) Let $X(n)$ be a WSS, discrete random process with autocorrelation function $R_{xx}(n)$, and let c be a constant.

(a) Determine the autocorrelation function for the discrete random process

$$Y(n) = X(n) + c$$

(b) Are $X(n)$ and $Y(n)$ independent ? Uncorrelated ? orthogonal ?

Q 13 A random process is defined by $X(t) = \exp(-At)\, u(t)$ where A is a random variable with pdf, $f_A(a)$.

(a) find the pdf of $X(t)$ in terms of $f_A(a)$.

(b) If A is an exponential random variable with $f_A(a) = e^{-a} u(a)$, find $\mu_X(t)$ and $R_{XX}(t_1, t_2)$. Is the $X(t)$ a WSS process ?

Q 14 Let $X(t)$ be a WSS Gaussian random process and form a new process according to $Y(t) = X(t) \cos(\omega t + \theta)$ where ω is a constant and θ is a random variable uniformly distributed over $[0, 2\pi)$ and independent of $X(t)$.

(a) Is $Y(t)$ WSS process ?

(b) Is $Y(t)$ a Gaussian random process ?

Q 15 Find the autocorrelation function corresponding to the power spectrum

$$S_{XX}(\omega) = \frac{8}{(9+\omega^2)^2}$$

Q 16 A random process has the power-density spectrum

$$S_{XX}(\omega) = \frac{6\omega^2}{(1+\omega^4)}$$

Find the average power in the process.

Q 17 Assume a random process has a power spectrum

$$S_{XX}(\omega) = \begin{cases} 4 - (\omega^2/9) & ; |\omega| \le 6 \\ 0 & ; \text{elsewhere} \end{cases}$$

find (a) the average power

(b) the rms bandwidth

(c) the autocorrelation function of the process.

Q 18 If $X(t)$ is a stationary process, find the power spectrum of

$$Y(t) = A_0 + B_0 X(t)$$

in terms of the power spectrum of $X(t)$ if A_0 and B_0 are real constants.

Q (19) Let A_0 and B_0 be random variables. The random process $X(t)$ is defined as

$$X(t) = A_0 \cos(\omega_0 t) + B_0 \sin(\omega_0 t)$$

where ω_0 is a real constant.

(a) Show that if A_0 and B_0 are uncorrelated with zero means and equal variances, then $X(t)$ is wide-sense stationary.

(b) Find the autocorrelation function of $X(t)$.

(c) find the power density spectrum.

Q (20) Two jointly stationary random process $X(t)$ and $Y(t)$ are defined as:

$$X(t) = 2\cos(5t + \phi)$$

$$Y(t) = 10\sin(5t + \phi)$$

where ϕ is a random variable that is uniformly distributed between 0 and 2π. Determine the cross correlation functions, $R_{XY}(\tau)$ and $R_{YX}(\tau)$.