TLWA Numericals

Hitesh Pranav 💙

UNIT 1

Q. For a Tx line, RLGC components are given by $R = 50\Omega$, $L = 1mH$, $G = 0.1$ ℧, $C = 1\mu F$ respectively. Find $Z_0, \alpha, \beta, \gamma$ at $f = 1MHz$

A. $f = 10^6$ Hz

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{5+j(2\pi\times10^6\times10^{-3})}{0.1+j(2\pi\times10^6\times10^{-6})}} = 999.87^{0.5}\angle 0.866/2 = 31.62\angle 0.433$$

$$\left(\begin{array}{l}\sqrt{r\angle\theta} = r^{0.5}\angle\theta/2 \\ (r\angle\theta)^{1/n} = r^{1/n}\angle\theta/n\end{array}\right)$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(5+j(2\pi\times10^6\times10^{-3}))(0.1+j(2\pi\times10^6\times10^{-6}))} = 39483.429^{0.5}\angle 179.04/2 = 198.7\angle 89.52 = 1.66+j198.69$$

$\gamma = \alpha + j\beta$

$\alpha = 1.66$

$\beta = 198.69$

Q. A Generator of 1V, 1KHz supplies power to a 100Km long line terminating in Z_0 and having the characteristic constants given below. Calculate characteristic impedance, propagation constant, wavelength, velocity, voltage and efficiency. $R = 10.4\ \Omega/Km$; $L = 0.00367$ H/Km ; $G = 0.8\times10^{-6}$ ℧/Km ; $C = 0.00835\times10^{-6}$ F/m

A. Given, $Z_L = Z_0$, $V_s = 1V$, $f = 1KHz = 1000$ Hz

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{482443.7481\angle -23.412} = 694.58\angle -11.706^\circ$$

$$\left(\begin{array}{l}\sqrt{r\angle\theta} = r^{0.5}\angle\theta/2 \\ (r\angle\theta)^{1/n} = r^{1/n}\angle\theta/n\end{array}\right)$$

$$Z_0 = \frac{R+j\omega L}{\gamma} \Rightarrow \gamma = \frac{10.4+j(2\pi\times f\times0.00367)}{694.58\angle -11.706} = 0.036\angle 77.42^\circ = 7.93\times10^{-3} + j0.0355 = \alpha + j\beta$$

$\Rightarrow \alpha = 0.00793$ nepers/Km

$\beta = 0.0355$ rad/km

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0355} = \frac{2\times3.14}{0.0355} = 176.90 \text{ Km}$$

$$v = \frac{\omega}{\beta} = \frac{2\pi\times1000}{0.0355} = 176901.4 \text{ Km/s}$$

$$Z_L = Z_0 \Rightarrow I_s = \frac{V_s}{Z_0} = \frac{1\angle 0}{694.58\angle -11.706} = 1.41\times10^{-3}+j2.92\times10^{-4} = 1.44\times10^{-3}\angle 11.706 \text{ A} = 1.44\angle 11.706 \text{ mA}$$

$P_s = I_s V_s = 1.44\angle 11.706$ mA $\times$ 1V $= 1.44\angle 11.706$ mW

$I_R = I_s e^{-\gamma l} = 1.44\angle 11.706 \times e^{-(0.00793+j0.0355)\times100}$ mA

$= 1.44\angle 11.706 \times e^{-0.793} \times e^{-j3.55}$ | 1 rad = 57.3°

$= 1.44 \times 0.452\ \angle 11.706 \times e^{-j(3.55\times57.3)}$

$= 1.44 \times 0.452\ \angle 11.706 \times \angle -203.415$

$= 0.65\angle -191.705$ mA

$V_R = I_R \times Z_L = I_R Z_0 = 0.65\times10^{-3}\angle -191.705 \times 694.58\angle -11.706$

$= 0.451477\angle -203.411$ V

$P_R = V_R I_R \cos\theta$

$= 0.451\angle -203.411 \times 0.65\times10^{-3}\ \angle -191.705 \times \cos(-203.411+191.705)$

$= 0.287\times10^{-3}$

$$\eta = \frac{P_R}{P_s} = \frac{0.287\times10^{-3}}{1.44\times10^{-3}} = 0.1993$$

$\eta\% = 19.93\%$

Q. A Lossless line has a characteristic impedance of $400\ \Omega$. Determine SWR if receiving impedance is $800\ \Omega$

A. $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{800-400}{800+400} = \frac{400}{1200} = \frac{1}{3}$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = \frac{4/3}{2/3} = 2$$

Q. A 50 Ω Tx line feeds 75 + j20 Ω dipole antenna. Find Γ & VSWR, R_{max} & R_{min}

A. $Z_L = 75 + j20\ \Omega$

$Z_0 = 50\ \Omega$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + j20 - 50}{75 + j20 + 50} = \frac{25 + j20}{125 + j20} = 0.2529 \angle 29.56$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.2529}{1 - 0.2529} = 1.677$$

$$R_{max} = VSWR \cdot Z_0 = 1.677 \times 50 = 83.85$$

$$R_{min} = \frac{Z_0}{VSWR} = \frac{50}{1.677} = 29.815$$

Q. Determine the input impedance & SWR for a 1.25λ long transmission line at the sending end with $Z_0 = 50\ \Omega$ & the load impedance $Z_L = 30 + j40\ \Omega$

A. $Z_0 = 50\ \Omega$

$Z_L = 30 + j40\ \Omega$

$L = 1.25\lambda$

i) Find normalized load impedance

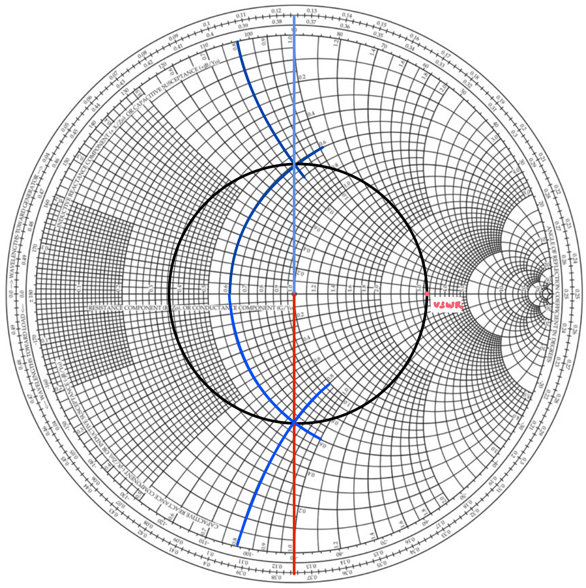
$$z_L = \frac{Z_L}{Z_0} = \frac{30 + j40}{50} = 0.6 + j0.8$$

ii) Plot normalized z_L on smith chart & draw impedance circle

$$\frac{z_i}{z_0} = 0.6 - j0.8$$

$$Z_i = 30 - j40\ \Omega$$

$SWR = 3.1$



Q. A 50 Ω Lossless Tx Line is terminated in a load impedance of $Z_L = 25 + j50\ \Omega$. Find VSWR, Γ, Z_{in} given line is 3.3λ long, Y_{in}

A. $Z_L = 25 + j50\ \Omega$ & $Z_0 = 50\ \Omega$

$$z_L = \frac{25 + j50}{50} = 0.5 + j2.06$$

$z_i = 0.26 - j0$

$Z_i = 50(0.4 - j0.8) = 20 - j40\ \Omega$

$VSWR = 4.6$

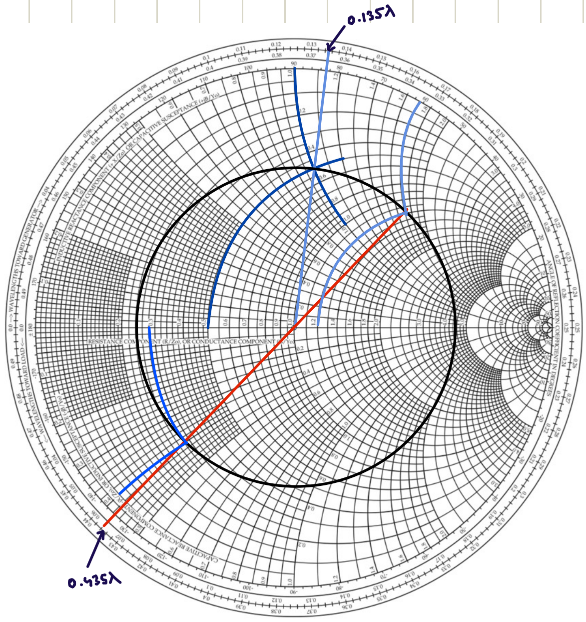
$$VSWR = \frac{1 + \Gamma}{1 - \Gamma} \Rightarrow 4.6 - 4.6\Gamma = 1 + \Gamma \Rightarrow \Gamma = \frac{3.6}{5.6} = 0.643$$

length $= 0.135\lambda + 3.3\lambda = 3.435\lambda$

$\Rightarrow 3.435\lambda - (6 \times 0.5\lambda) = 0.435\lambda \Rightarrow$ Draw Line from origin to 0.435

$z_{in} = 0.28 - j0.4$

$Z_{in} = z_{in} \cdot Z_0 = 14 - j20\ \Omega$

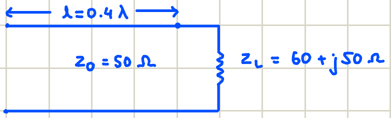


To find admittance, stretch the z_{in} line other side & find intersection

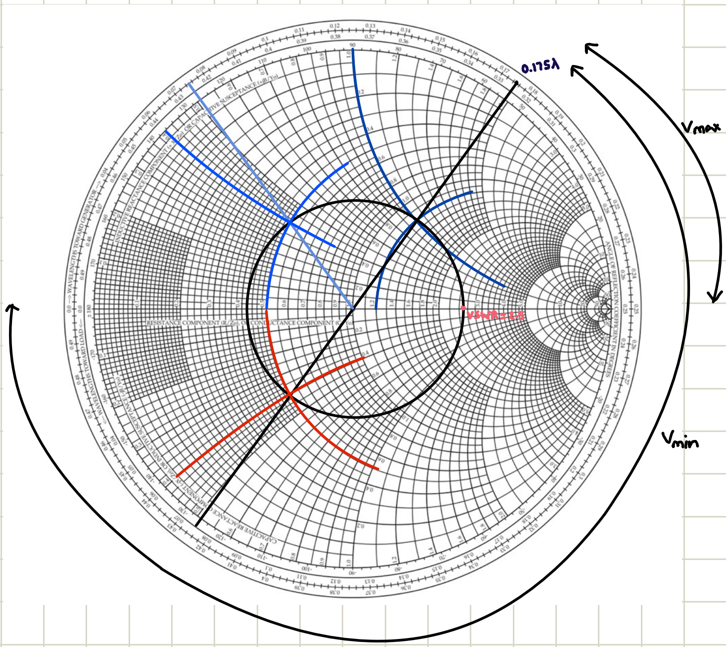
$y_{in} = 1.2 + j1.7$

$$Y_{in} = y_{in} \cdot G_0 = y_{in} \cdot \frac{1}{R_0} = (1.2 + j1.7) \times \frac{1}{50} = 0.024 + j0.034\ \mho$$

Q. Use the smith chart to find the following quantities for the Tx line ckt shown in the figure. Find VSWR, Γ, Y_L, Z_{in},



A. $z_L = \frac{Z_L}{Z_0} = \frac{60+j50}{50} = 1.2 + j1$



$0.175\lambda + 0.4\lambda = 0.575\lambda$

$0.575\lambda - (1 \times 0.5\lambda) = 0.075\lambda$

VSWR = 2.5

$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow 2.5 - 2.5\Gamma = 1 + \Gamma \Rightarrow \Gamma = \frac{1.5}{3.5} = 0.43$

$y_L = 0.5 - j0.4$ (Stretch opposite to z_L, not z_{in})

$Y_L = y_L \times \frac{1}{R_0} = \frac{0.5 - j0.4}{50} = 0.01 - j0.008 \; ℧$

$z_{in} = 0.5 + j0.4$

$Z_{in} = z_{in} \cdot z_0 = (0.5 + j0.4) \times 50 = 25 + j20 \,\Omega$

$dV_{min} = |0.5 - 0.175|\lambda = 0.325\lambda \longrightarrow$ 1st minima

$(0.325 + 0.5)\lambda = 0.825 \longrightarrow$ 2nd minima

$dV_{max} = |0.25 - 0.175|\lambda = 0.075\lambda$

Q. Let incident voltage $= V_0^+$ & Reflected voltage $= V_0^-$, current $= i_0^+$ in a Tx line. Normalize this by characteristic impedance z_0

A. $V = V_0^+ + V_0^-$

$I = \frac{1}{z_0}\left[V_0^+ - V_0^-\right]$

$V_0^+ = \frac{V + Iz_0}{2}$

$V_0^- = \frac{V - Iz_0}{2}$

The normalized incident vector $a = \frac{V_0^+}{\sqrt{z_0}} = \frac{V + Iz_0}{2\sqrt{z_0}} = I_0^+\sqrt{z_0}$

The normalized reflected vector $b = \frac{V_0^-}{\sqrt{z_0}} = \frac{V - Iz_0}{2\sqrt{z_0}} = I_0^-\sqrt{z_0}$

Now in terms of a & b, the incident power $P^+ = |a|^2$

the reflected power $P^- = |b|^2$

the reflected coefficient $\Gamma = \frac{b}{a}$

The actual power delivered to the load $P = |a|^2 - |b|^2$

VSWR of the termination $\rho = \frac{|a| + |b|}{|a| - |b|}$

For n-port network, $a_i = \frac{1}{2}\left(\frac{V_i + z_0 I_i}{\sqrt{z_0}}\right)$

$b_j = \frac{1}{2}\left(\frac{V_j - z_0 I_j}{\sqrt{z_0}}\right)$

Q. Consider 75 Ω Tx line that is terminated with a $35 + j120\ \Omega$ load. Find the reflection coefficient at load & VSWR using wave vectors.

A. Normalizing the voltage source to 1, $V_s = 1$

then $V_0^+ = \frac{V_s}{2} = \frac{1}{2}$

$$a = \frac{V_0^+}{\sqrt{Z_0}} = \frac{1}{2\sqrt{75}} = 0.0577$$

$$\frac{V_0^-}{V_0^+} = \Gamma_L = \left(\frac{Z_0 - Z_L}{Z_0 + Z_L}\right) = \frac{40 - j120}{40 + j120} = -0.377 - j0.679$$

$$V_0^- = V_0^+ (\Gamma_L) = \frac{-0.377 - j0.679}{2} = -0.188 - j0.34$$

$$b = \frac{V_0^-}{\sqrt{Z_0}} = \frac{-0.188 - j0.34}{\sqrt{75}} = -0.0217 - j0.039$$

$$\text{VSWR},\ \rho = \frac{|a| + |b|}{|a| - |b|} = 7.97$$

Q. Given, $Z_0 = 50\ \Omega$, $Z_L = 25 + j25\ \Omega$, $f = 1\text{ GHz}$. Calculate normalized load impedance, admittance, admittance at a distance d stub susceptance and stub length for a simple short circuited shunt stub.

A. $Z_0 = 50\ \Omega$

$Z_L = 25 + j25\ \Omega$

$f = 1\text{ GHz}$

$$z_l = \frac{Z_L}{Z_0} = \frac{25 + j25}{50} = 0.5 + j0.5$$

$$y_l = \frac{1}{z_l} = 1 - j1$$

$$Z_{in} = \frac{Z_0(Z_L + jZ_0 \tan \beta l)}{(Z_0 + jZ_L \tan \beta l)} = 1$$

So,

$$\beta l = 45° = \pi/4$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi \times 1\times10^9}{3\times10^8} = \frac{20\pi}{3} \Rightarrow l = \frac{\pi/4}{\beta} = \frac{\pi/4}{20\pi/3} = \frac{3}{80} = 0.0375\text{ m}$$

Q. Find voltage & current at $z = 0.25\lambda$, when the characteristic impedance is 50 Ω & incident voltage is 10 V with reflection coefficient of -0.5

A. $V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$

$$\beta z = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$V = 10\left(e^{-j\pi/2} + \Gamma e^{j\pi/2}\right) = 10\left(-j - 0.5(j)\right) = -j15\text{ V} \Rightarrow |V| = 15\text{ V}$$

$$I = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

$$= \frac{10}{50}\left(e^{-j\pi/2} - (-0.5)e^{j\pi/2}\right) = 0.2(-j + 0.5j) = -0.2 \times -0.5j = -0.1j \Rightarrow |I| = 0.1\text{ A}$$

Q. 5 Q. A 50 Ω Tx line is connected to a load impedance of $60 + j80\ \Omega$. The stub is connected shunt with the load. Find the position & length of the short ckt stub required to match the line using Smith chart.

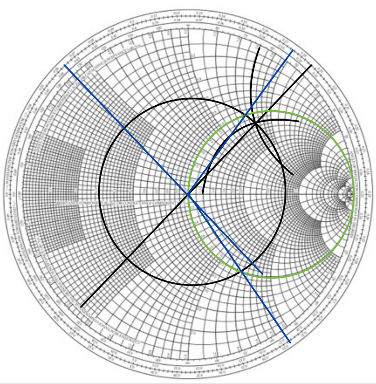
A.

* $z_L = \frac{60 + j80}{50} = 1.2 + j1.6$
* $y_L = 0.3 - j0.4\ \Omega$
* $+j1.5$ at 0.175λ

  $-j1.5$ at 0.325λ
* l_1 :- $|0.175\lambda - 0.435\lambda| = 0.26\lambda$

  (or)

  $0.175\lambda + 0.065\lambda = 0.24\lambda$
* l_s :- $0.32 - 0.25 = 0.07\lambda$



Q. A Tx. Line has $\Gamma = 0.6$. Find SWR

A. $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.6}{1-0.6} = 4$

Q. Given, $L = 2.54\ \mu H/m$ & $C = 100\ pF/m$. For lossless line, find z_0 & v_p

A. $z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.5\times10^{-6}}{100\times10^{-12}}} = 158\ \Omega$

$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5\times10^{-6}\times100\times10^{-12}}} = 2\times10^{8}\ m/s$

Q. A lossless line of length $\lambda/4$ is SC. Find z_{in}

A. $z_{in} = j z_0 \tan\beta l$

$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$z_{in} = j z_0 \tan\frac{\pi}{2} = \infty \Rightarrow$ Open circuit $(\tan 90 = \infty)$

Q. A lossless Tx line of characteristic impedance $z_0 = 50\ \Omega$ & length $\frac{\lambda}{4}$ is terminated with $z_L = 200\ \Omega$. Find z_{in}

A. $z_0 = 50\ \Omega$, $z_L = 200\ \Omega$

$z_{in} = \frac{z_0^2}{z_L} = \frac{50\times50}{200} = 12.5\ \Omega$

Q. A lossless $\frac{\lambda}{4}$ line has $z_0 = 100\ \Omega$. The load is SC. Find z_{in}

A. SC $\Rightarrow z_L = 0$

$z_{in} = \frac{z_0^2}{z_L} = \frac{100^2}{0} = \infty \Rightarrow$ Open Circuit

Q. A $300\ \Omega$ Tx line is connected to load impedance of $450 - j600\ \Omega$ at 10 MHz. Find the position & length of SC stub to match the line using smith chart

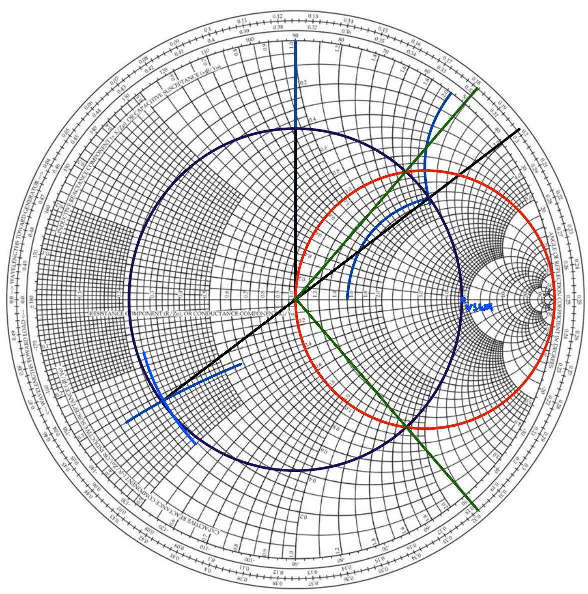
A. $Z_L = 450 - j600$

$z_0 = 300$

$z_l = 1.5 - j2$

$y_l = 0.24 - j0.32$

$Y_L = 0.24 - j0.32 \times \frac{1}{300}$

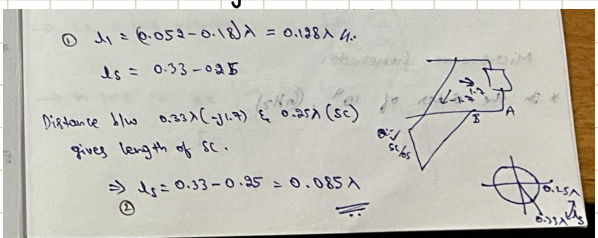


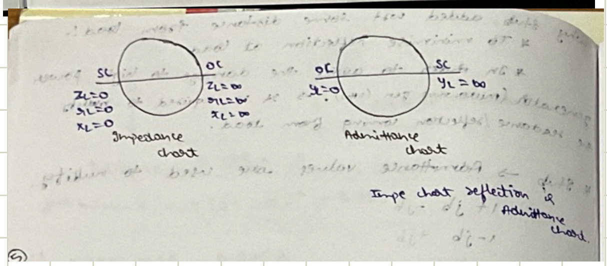
VSWR = 4.6

Now draw $1+j0$ circle (In Red Colour)

Intersection point of SWR circle & $1+j$ circle at $\pm j1.7$

Stub location $= +j1.7$





UNIT 2

Q. An air filled rectangular waveguide of inside dimension 7 x 3.5 cm operates in a dominant mode. Find f_c, determine phase velocity of the wave in a waveguide operates at a frequency 3.5 GHz. Determine guide wavelength at same frequency

A. $a = 7\text{ cm},\ b = 3.5\text{ cm}$

$$TE_{01} \Rightarrow f_c = \frac{c}{2a} = \frac{3\times 10^8}{2\times 7\times 10^{-2}} = 0.214\times 10^{10} = 2.14\text{ GHz}$$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times 10^8}{\sqrt{1-\left(\frac{2.14\times 10^9}{3.5\times 10^9}\right)^2}} = 3.79\times 10^8\text{ m/s} =$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c/f}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{v_p}{f} = \frac{3.79\times 10^8}{3.5\times 10^9} = 0.108\text{ m} = 10.8\text{cm}$$

Q. When a dominant node is propagated in an airfilled rectangular waveguide, the guide wavelength for a freq of 9000 MHz is 4cm. Calculate breadth of the wave.

A. $TE_{10} \Rightarrow m=1,\ n=0$

$$\lambda_g = 4\text{cm} = 0.04\text{ m}$$

$$\lambda_0 = \frac{c}{f} = \frac{3\times 10^8}{9\times 10^9} = 0.033\text{ m}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{0.033}{\sqrt{1-\left(\frac{0.033}{\lambda_c}\right)^2}} = 0.04 \Rightarrow \lambda_c = 0.0584\text{ m}$$

$$\lambda_c = 2a \Rightarrow a = \frac{\lambda_c}{2} = 0.0292\text{ m}$$

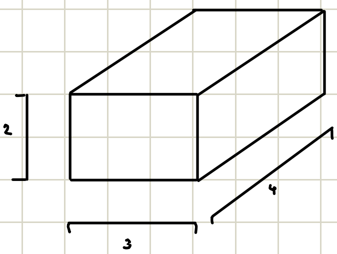
$$a = 2b \Rightarrow b = \frac{a}{2} = 0.0146\text{ m} = 1.46\text{ cm}$$

Q. Calculate resonant frequency of rectangular cavity resonator of dimensions 3 x 2 x 4 cm when filled with air & mode of operation is TE_{101}

A. $d = 4\text{cm}$

$a = 3\text{cm}$

$b = 2\text{cm}$



& since TE_{101}, $m=1$ $n=0$ $p=1$

$$f_r = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{d}\right)^2}$$

$$= \frac{3\times 10^8\,\pi}{2\pi}\sqrt{\left(\frac{1}{3\times 10^{-2}}\right)^2+\left(\frac{1}{4\times 10^{-2}}\right)^2}$$

$$= 6.25\text{ GHz}$$

Q. A rectangular metal waveguide filled with dielectric material of relative permittivity $\varepsilon_r = 4$ has inside dimensions 3 x 1.2 cm. Calculate cutoff freq for dominant mode

A. Dominant mode $\Rightarrow TE_{10}$ (2D cuz rectangular metal waveguide)

$a = 3\text{ cm}$ $b = 1.2\text{ cm}$

$$f_c = \frac{c}{\sqrt{\varepsilon_r}\cdot 2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$= \frac{3\times 10^8}{\sqrt{4}\times 2}\sqrt{\left(\frac{1}{3\times 10^{-2}}\right)^{\cancel{2}}} = 2.5\times 10^9\text{ Hz}$$

$$v = \frac{c}{\sqrt{\varepsilon_r}} = 1.5\times 10^8\text{ ms}^{-1}$$

Q. For the dominant mode propagation along +ve z direction in air filled rectangular waveguide, the longitudinal magnetic field $H_z = \cos(100x)\, e^{-j\beta z}$. Determine dimensions of waveguide if the ratio a:b = 2:1

A. $H_z = H_{oz} \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$

$= \cos(100x)\, e^{-j\beta z}$

So, $H_{oz} = 1$, $\frac{m\pi}{a} = 100$

$TE_{10} \Rightarrow m=1,\ n=0$

So, $a = \frac{\pi}{100}$ Then $\frac{a}{b} = \frac{2}{1} \Rightarrow b = \frac{a}{2} = \frac{\pi}{200}$

Q. The rectangular waveguide with dimensions a = 1.07 cm b = 0.43 cm filled with certain dielectric exhibits $f_c = 31.03$ GHz for TM_{21} mode. Find the relative dielectric constant.

A. $f_c = \frac{c}{\sqrt{\epsilon_r} \cdot 2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$31.03 \times 10^9 = \frac{3\times10^8}{\sqrt{\epsilon_r} \times 2} \sqrt{\left(\frac{2}{1.07\times10^{-2}}\right)^2 + \left(\frac{1}{0.43\times10^{-2}}\right)^2}$

$\epsilon_r = \left(\frac{3\times10^8}{31.03\times10^9\times2}\right)^2 \times \left(\left(\frac{2}{1.07\times10^{-2}}\right)^2 + \left(\frac{1}{0.43\times10^{-2}}\right)^2\right)$

$= 2.08$

Q. The dimensions of waveguide are 2.5 × 1 cm & frequency is 8.6 GHz, Find the following
i) Possible modes of operation ii) Cutoff freq. iii) Wavelength

A. i) Condition for propagation $\Rightarrow \lambda_c > \lambda_0$ (or) $f_c < f_0$

Here $\lambda_0 = \frac{c}{f_0} = \frac{3\times10^8}{8.6\times10^9} = 0.034$ m

$TE_{01} \Rightarrow m=0,\ n=1$ $\lambda_c = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}} = \frac{5\times10^{-4}}{2.5\times10^{-2}} = 2\times10^{-2} < \lambda_0$ ✗

$TE_{10} \Rightarrow m=1,\ n=0$ $\lambda_c = \frac{5\times10^{-4}}{1\times10^{-2}} = 5\times10^{-2}$ cm $> \lambda_0$ ✓

$TE_{11} \Rightarrow m=1,\ n=1$ $\lambda_c = \frac{5\times10^{-2}}{\sqrt{6.25+1}} = 1.85\times10^{-2}$ cm $< \lambda_0$ ✗

ii) f_c (10 Mode) $= \frac{c}{\lambda_c} = \frac{3\times10^8}{5\times10^{-2}} = 6\times10^9$ Hz $= 6$ GHz

iii) $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{0.034}{\sqrt{1-\left(\frac{0.034}{0.05}\right)^2}} = 4.637$ cm

Q. An air filled rectangular cavity resonator is operating in dominant mode. Determine resonant freq. of resonator if the field inside the resonator is $H_z = \cos(20\pi x) \cdot \sin(5\pi z)$. Now if the resonator is filled with dielectric $\epsilon_r = 4$, determine new resonant freq.

A. $H_z = H_{oz} \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$

$= \cos(20\pi x) \cdot \sin(5\pi z)$

$m=1,\ n=0,\ p=1$

$\frac{m\pi}{a} = 20\pi \Rightarrow a = \frac{1}{20} = 0.05\text{m} = 5\text{cm}$

$\frac{p\pi}{d} = 5\pi \Rightarrow d = \frac{1}{5}$

$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.05}\right)^2 + \left(\frac{1}{1/5}\right)^2} = 3.1$ GHz

At ϵ_r, $f_r = \frac{f_c}{\sqrt{\epsilon_r}} = \frac{3.1}{\sqrt{4}} = 1.55$ GHz

Q. For a dominant mode propagation, in an air filled rectangular waveguide, the longitudinal magnetic wavefield is given by $H_z = \cos\left(\frac{100\pi}{3}x\right) \cdot e^{-j\beta z}$. If operating freq. of wave is $f = 8.333$ GHz, determine phase constant β

A. $H_z = H_{0z} \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \cdot e^{-j\beta z}$

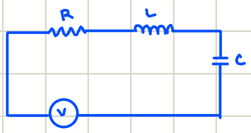
TE_{10} $\Rightarrow m=1,\ n=0$

$\frac{m\pi}{a} = \frac{100\pi}{3} \Rightarrow a = \frac{3}{100}$

$f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times\frac{3}{100}} = 5\times10^9 = 5$GHz

$\beta = \frac{2\pi}{\lambda} = \sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi f}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 139.54$ rad/m

Q. The series RLC circuit has $R = 100\Omega$, $L = 0.5H$, $C = 0.4\mu F$. Find resonant freq, half power freq & bandwidth



A. $f_r = \frac{1}{2\pi\sqrt{LC}} = 355.88$ Hz

$f_1 = f_r - \frac{R}{4\pi L} = 339.96$ Hz

$f_2 = f_r + \frac{R}{4\pi L} = 371.79$ Hz

$BW = f_2 - f_1 = 31.83$ Hz