Unit - 2

Equations Reducible to Exact Form

1) $IF = e^{\int f(x)\,dx}$ (or) $IF = e^{-\int f(x)\,dx}$

↳ Integrating factors

Then,

i) If ($M\,dx + N\,dy = 0$) is not exact, find $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$

ii) Difference should be close to M (or) N

iii) If difference close to M,

Find $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = f(y)$ then $IF = e^{-\int f(y)\,dy}$

iv) if difference close to N,

Find $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ then $IF = e^{\int f(x)\,dx}$

v) Multiply IF in the eq. $M\,dx + N\,dy = 0$ which will give the exact eq & proceed to find the solⁿ.

Q. $(5x^3 + 12x^2 + 6y^2)\,dx + (6xy)\,dy = 0$

$$\frac{\partial M}{\partial y} = 12y \qquad \frac{\partial N}{\partial x} = 6y$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 6y \longrightarrow \text{Close to } N$$

$$\Rightarrow f(x) = \frac{6y}{6xy} = \frac{1}{x}$$

$$IF = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$\Rightarrow$ Multiplying with IF

$$\Rightarrow M = 5x^4 + 12x^3 + 6xy^2$$

$$N = 6x^2y$$

$$\frac{\partial M}{\partial y} = 12xy \qquad \frac{\partial N}{\partial x} = 12xy \Longrightarrow \text{It is exact}$$

$$\int M(x)\,dx + \int N\,dy = c$$

$$\int (5x^4 + 12x^3)\,dx + \int 6x^2y\,dy = c$$

$$\Rightarrow \left[\frac{5x^5}{5} + \frac{12x^4}{4}\right] + \frac{6x^2y^2}{2} = c$$

$$\boxed{\Rightarrow x^5 + 3x^4 + 3x^2y^2 = c}$$

Q. $(xy + y^2)\,dx + (x+2y-1)\,dy = 0$

$M = xy + y^2$ | $N = x + 2y - 1$

$\frac{\partial M}{\partial y} = x + 2y$ | $\frac{\partial N}{\partial x} = 1$

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = x + 2y - 1 \Rightarrow$ Close to N

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = 1 = f(x)$$

$$IF = e^{\int f(x)\,dx} = e^{x}$$

Now, $M.IF = (xy + y^2)\,e^{x} = xy.e^{x} + y^2.e^{x}$

$N.IF = (x+2y-1)\,e^{x} = x.e^{x} + 2y.e^{x} - e^{x}$

$\int M(x)\,dx + \int N\,dy = c$

$\int 0\,dx + \int (xe^{x} + 2y.e^{x} - e^{x})\,dy$

$= e^{x}\int (x + 2y - 1)\,dy$

$= e^{x}\,[xy + y^2 - y]$

$= e^{x}\,[xy + y^2 - y] \Rightarrow \boxed{e^{x}y\,[x + y - 1] = c}$

Type 2 : In given DE $Mdx + Ndy = 0$

if $M(x, y)$ & $N(x, y)$ are homogeneous of same degree

$$IF = \frac{1}{Mx + Ny} \text{ . Provided } Mx + Ny \neq 0$$

Note : if $Mx + Ny = 0$, $IF = \frac{1}{x^2}$ or $\frac{1}{y^2}$ or $\frac{1}{xy}$

Type 3 : In DE of the form,

$y f(xy) dx + x g(xy) dy = 0$, then $IF = \frac{1}{Mx - Ny}$

where $M = y f(xy)$ & $N = x g(xy)$

provided $Mx - Ny \neq 0$

If $Mx - Ny = 0$, $Mx = Ny$ (or) $\frac{M}{N} = \frac{y}{x}$

Q. $(y + x) dx + (y - x) dy = 0$

$$IF = \frac{1}{yx + x^2 + y^2 - xy} = \frac{1}{x^2 + y^2}$$

$M.IF = \frac{x+y}{x^2+y^2}$

$N.IF = \frac{y-x}{x^2+y^2}$

$\int M dx + \int N(y) dy$

$t = x^2 + y^2$

$\frac{dt}{dx} = 2x$

$$= \int \frac{y}{x^2+y^2} dx + \int \frac{x}{x^2+y^2} dx + 0$$

$$= y \int \frac{1}{x^2+y^2} dx + \frac{1}{2} \int \frac{2x}{x^2+y^2} dx = \tan^{-1}\frac{x}{y} + \frac{1}{2}\int \frac{dt}{t} = \tan^{-1}\frac{x}{y} + \frac{1}{2}\log t$$

$$\Rightarrow \tan^{-1}\frac{x}{y} + \frac{1}{2}\log(x^2+y^2) = c$$

Q. $(2xy + x^2)\frac{dy}{dx} = (3y^2 + 2xy)$

$-(2xy + x^2)dy + (3y^2 + 2xy)dx = 0$

$M = +3y^2 + 2xy$ | $N = -2xy - x^2$

$\frac{\partial M}{\partial y} = 6y + 2x$ | $\frac{\partial N}{\partial y} = -2x$

→ Homogeneous

$IF = \frac{1}{Mx + Ny} = \frac{1}{3xy^2 + 2x^2y - 2xy^2 - x^2y}$

$= \frac{1}{x^2y + xy^2} = \frac{1}{xy(x+y)}$

$M.IF = \frac{3y^2 + 2xy}{xy(x+y)} = \frac{y(3y + 2x)}{xy(x+y)} = \frac{3y + 2x}{x(x+y)}$

$N \cdot IF = \frac{-2xy - x^2}{xy(x+y)} = \frac{-x(2y + x)}{xy(y+x)} = \frac{-(2y + x)}{y(y+x)}$

$\Rightarrow M \cdot IF = \cancel{\frac{3y}{x(x+y)}}\ \frac{3y}{x(x+y)} + \frac{2}{x+y}$

$N.IF = \frac{-2}{y+x} - \frac{x}{y(y+x)} \rightarrow \frac{A}{y} + \frac{B}{x+y}$

$Ax + Ay + By$

$A = 1 \ ; A + B = -1$

$\cancel{A = 0 ; A + B = 3}$

$\Rightarrow \int M dx + \int N(y) dy = c$

$\Rightarrow \int \frac{3y}{x(x+y)} dx + \int \frac{2\,dx}{x+y} + 0 \int \frac{1}{y} dy + 0 \rightarrow \log y$

$= \frac{A}{x} + \frac{B}{x+y} = \frac{Ax + Ay + Bx}{x(x+y)}$

$A = 3 \quad B = -3$

$\Rightarrow \int \frac{3dx}{x} - \cancel{\frac{3}{x+y}dx} + \int \frac{2dx}{x+y} - \log y = c \Rightarrow \boxed{\log x^3 - \log(x+y) - \log y}$

Ques based on Type - 3

1. $(xy \sin xy + \cos xy)\, y\,dx + (xy \sin xy - \cos xy)\, x\,dy = 0$

$$IF = \frac{1}{x^2y^2 \cancel{\sin xy} + xy\cos xy - x^2y^2\cancel{\sin xy} + xy\cos xy}$$

$$IF = \frac{1}{2xy\cos xy}$$

$$M \cdot IF = \frac{xy^2 \sin xy + y\cos xy}{2xy\cos xy}$$

$$= \frac{y}{2}\tan xy + \frac{1}{2x}$$

$$N \cdot IF = \frac{x^2y \sin xy - x\cos xy}{2xy\cos xy}$$

$$= \frac{x}{2}\tan xy - \frac{1}{2y}$$

$$\frac{\partial M}{\partial y} = \frac{\tan xy}{2} + \frac{\sec^2 xy \cdot xy}{2}$$

$$\frac{\partial N}{\partial x} = \frac{\tan xy}{2} + \frac{\sec^2 xy \cdot xy}{2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M(x)\,dx + \int N\,dy = C$$

$$\int \frac{1}{2x}\,dx + \int \frac{x\tan xy}{2}\,dy - \int \frac{1}{2y}\,dy = C$$

$xy = t$

$x = \frac{dt}{dy}$

$$\frac{1}{2}\log x + \frac{1}{2}\int \tan t\,dt - \frac{1}{2}\log y = C$$

$$\frac{1}{2}\left(\log x + \log(\sec t) - \log y\right) = C$$

$$\frac{1}{2}\left(\log x + \log(\sec xy) - \log y\right) = C$$

$$= \frac{1}{2}\log\left(\frac{x \cdot \sec xy}{y}\right) = C$$

$$\frac{x \cdot \sec xy}{y} = C'$$

Q. $y(1+xy)\,dx + (1-xy)\,x\,dy = 0$

$M = y + xy^2 \qquad N = x - x^2y$

$$IF = \frac{1}{Mx - Ny} = \frac{1}{\cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2} = \frac{1}{2x^2y^2}$$

$$M.IF = \frac{1}{2x^2y} + \frac{1}{2x} = \frac{1}{2x}\left(\frac{1}{xy} + 1\right)$$

$$N.IF = \frac{1}{2xy^2} - \frac{1}{2y} = \frac{1}{2y}\left(\frac{1}{xy} - 1\right)$$

$$\int M(x)\,dx + \int N\,dy = c$$

$$\int \frac{1}{2x}\,dx + \int \frac{dy}{2xy^2} - \int \frac{dy}{2y} = c$$

$$\frac{1}{2}\left(\log x - \log y - \frac{1}{xy}\right) = c$$

$$\log\frac{x}{y} - \frac{1}{xy} = c$$

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Orthogonal Trajectories

Definition 1

→ 2 families of curves are said to be orthogonal trajectories of each other if every members of one family intersects every member of of the other family orthogonally

ex: $x^2+y^2=a^2$ & $y=mx$ are orthogonal trajectories

Procedure :

Step 1: Differentiate $f(x,y,c)=0$ wrt x & eliminate c between y and y'

Step 2: Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ to obtain DE of required orthogonal family of curves

Step 3: Solving the DE, orthogonal family of curves can be obtained.

Definition 2

→ A given family of curves is said to be self Orthogonal if its family of Orthogonal trajectories are same as given family of curves

ex: $x^2=4c(y+c)$

↳ if DE remains same when $\frac{dy}{dx}$ is replaced by $-\frac{dx}{dy}$, it is self orthogonal

Q. $x^2+y^2=c$

$$2x + 2y\cdot\frac{dy}{dx} = 0$$

$$x + y\cdot\frac{dy}{dx} = 0$$

$$x - y\cdot\frac{dx}{dy} = 0$$

$$x = \frac{y\cdot dx}{dy} \Rightarrow x\,dy - y\,dx = 0$$

$$M = -y \qquad N = x$$

$$\frac{\partial M}{\partial y} = -1 \qquad \frac{\partial N}{\partial x} = 1$$

$\hookrightarrow$ Homogeneous

$$IF = \frac{1}{Mx+Ny} = \frac{1}{-xy+xy} = \frac{1}{0}.$$

$$IF = \cancel{\frac{1}{y^2}} \text{ (or) } \cancel{\frac{1}{x^2}} \text{ (or) } \cancel{\frac{1}{xy}}$$

$$M.IF = \frac{-1}{x} \qquad N.IF = \frac{1}{y}$$

$$\int M(x)\,dx + \int N\,dy = c$$

$$\int \frac{-1}{x}\,dx + \int \frac{1}{y}\,dy = c$$

$$-\log x + \log y = c$$

$$\frac{y}{x} = \log c$$

$$\boxed{y = c'x}$$

Q. $x^2 - y^2 = c$

$$2x - 2y \cdot \frac{dy}{dx} = 0$$

$$x = y \cdot \frac{dy}{dx}$$

$$x = -y \cdot \frac{dx}{dy}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$+\log y = -\log x + \log c \Rightarrow \log\left(\frac{c}{x}\right)$$

$$\log y = \log \frac{c}{x}$$

$$\boxed{y = \frac{c}{x}}$$

Q. $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1 \Rightarrow \frac{y^2}{b^2+\lambda} = 1 - \frac{x^2}{a^2} \Rightarrow \frac{1}{b^2+\lambda} = \frac{1}{y^2} - \frac{x^2}{a^2y^2}$

$$= \frac{a^2 - x^2}{a^2y^2}$$

$$\frac{2x}{a^2} + \frac{2y}{b^2+\lambda} \cdot \frac{dy}{dx} = 0$$

$$\frac{x}{a^2} - \frac{y}{b^2+\lambda} \cdot \frac{dx}{dy} = 0$$

$$\frac{x}{a^2} = \frac{y}{b^2+\lambda} \cdot \frac{dx}{dy} \Rightarrow \cancel{\int \frac{dy}{y} \cdot \frac{(b^2+\lambda)}{a^2} = \int \frac{dx}{x}}$$

$$\cancel{\log y \times \frac{b^2+\lambda}{a^2} = \log x}$$

$$\cancel{\log y \times}$$

$$\frac{x}{a^2} = \frac{y}{\frac{a^2-x^2}{a^2y^2}} \cdot \frac{dx}{dy} \Rightarrow \frac{a^2-x^2}{a^2y^2} \times \frac{x}{a^2} = y\frac{dx}{dy}$$

$$\frac{dx}{dy} = \frac{xy}{a^2-x^2} \Rightarrow y\,dy = \frac{a^2-x^2}{x}\,dx$$

$$y^2 \quad a^2 \log x - x^2 + c$$

$y^2 = 4xc + 4c^2$

$2y \cdot \frac{dy}{dx} = 4c$

$y \cdot \left(\frac{dy}{dx}\right) = 2c$

$\frac{y}{2}\left(\frac{dy}{dx}\right) = c \Rightarrow y^2 = \frac{4xy}{2}\left(\frac{dy}{dx}\right) + \frac{y^2}{4}\left(\frac{dy}{dx}\right)^2$

$y = 2xy' + \frac{y}{} \cdot (y')^2$

$y = 2x\left(\frac{-1}{y'}\right) + y \cdot \left(\frac{-1}{y'}\right)^2$

$y = \frac{-2x}{y'} + \frac{y}{(y')^2} = \frac{-2xy' + y}{(y')^2}$

$yy'^2 = -2xy' + y \Rightarrow y = yy'^2 + 2xy'$

$\cancel{y = y + 2xy'}$

5. $\frac{x^2}{a} + \frac{y^2}{a-b} = 1$

$\frac{2x}{a} + \frac{2y}{a-b} = 0$

$\frac{x}{a} + \frac{yy'}{a-b} = 0$

$\frac{yy'}{a-b} = 0 - \frac{x}{a}$

$ayy' = -ax + bx$

$a(x+yy') = bx$

$a = \frac{bx}{x+yy'}$

$\Rightarrow \frac{x^2(x+yy')}{bx} + \frac{y^2}{\frac{bx}{x+yy'} - b} = 1$

$\frac{x^2(x+yy')}{bx} + \frac{y^2x + y^3y'}{\cancel{bx} - \cancel{bx} - byy'} = 1$

$\frac{x^2 + xyy'}{b} + \frac{y^2x + y^3y'}{-byy'} = 1$

$x^2y' - xy^2y' + y^2x + y^3y' + byy' = 0$

$\Rightarrow xy + y^2y' + by' - x^2y' - xy^2y' = 0$

$\Rightarrow xy - \frac{y^2}{y'} - \frac{b}{y'} + \frac{x^2}{y'} + \frac{xy^2}{y'} = 0$

$\Rightarrow \frac{xyy' - y^2 - b + x^2 + xy^2}{y'} = 0$

$(x+yy')(xy'-y) = by'$

$\Rightarrow \left(x + \frac{y}{y'}\right)\left(\frac{-x}{y'} - y\right) = \frac{-b}{y'} \Rightarrow \left(\frac{xy'-y}{y'}\right)\left(+\left(\frac{x+yy'}{\cancel{y'}}\right)\right) = \frac{+b}{\cancel{y'}}$

$\Rightarrow (xy'-y)(x+yy') = by'$

→ For finding Orthogonal Trajectory of Polar family of curv

Step 1 : Take log on both sides on eq $f(r, \theta, c) = 0$

Step 2 : Differentiate wrt θ & eliminate c

Step 3 : Replace $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ to obtain DE of required orthogonal family of curves

Step 4 : Solve DE

Note : If DE remains same after replacing $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ then it is self orthogon

ex: $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$

Q. i) $r^2 = c\sin 2\theta$

$\log r^2 = \log(c\sin 2\theta)$

$2\log r = \log c + \log\sin 2\theta$

$\frac{2}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\sin 2\theta}\times 2\cos 2\theta$

$\Rightarrow \frac{dr}{d\theta}\times\frac{1}{r} = \cot 2\theta$

$\Rightarrow -r^2\cdot\frac{d\theta}{dr}\times\frac{1}{r} = \cot 2\theta$

$\int\frac{dr}{r} = -\int d\theta\cdot\tan 2\theta$

$\log r = -\frac{\log(\sec 2\theta)}{2} + \log c$

$\log r = \log\left((\sec 2\theta)^{-1/2}\times c\right)$

$r^2\sec 2\theta = c^2$

$\boxed{r^2 = k\cos 2\theta}$

ii) $r = c(\sec\theta + \tan\theta)$

$\log r = \log c + \log(\sec\theta + \tan\theta)$

$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{\sec\theta+\tan\theta}\times((\sec\theta\cdot\tan\theta) + \sec^2\theta)$

$\frac{1}{r}\cdot\frac{dr}{d\theta} = \sec\theta$

$-\frac{1}{r}\times r^2\frac{d\theta}{dr} = \sec\theta$

$\int\frac{dr}{r} = -\int\cos\theta\, d\theta \Rightarrow \log r = -\sin\theta + c$

$\log r + \sin\theta = c$

$r = e^{-\sin\theta}\cdot e^c$

$\boxed{r = ke^{-\sin\theta}}$

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Q. $r^n \cos n\theta = a^n$

$\log(r^n \cos n\theta) = \log a^n$

$n \log r + \log \cos n\theta = \log a^n$

$\frac{n}{r}\frac{dr}{d\theta} - \frac{n \sin n\theta}{\cos n\theta} = 0$

$-\frac{1}{r} \times r^2 \frac{d\theta}{dr} = \tan(n\theta)$

$\int \frac{dr}{r} = -\int \cot n\theta$

$\log r = -\frac{\log(\sin n\theta)}{n} + \log K$

$r^n = \frac{K^n}{\sin n\theta}$

$$\boxed{r^n \sin n\theta = C}$$

Q. $r = 4a \sec\theta \tan\theta$

$\log r = \log 4a + \log \sec\theta + \log \tan\theta$

$\frac{1}{r} = \frac{1}{\sec\theta} \cdot \sec\theta \tan\theta + \frac{1}{\tan\theta} \times \sec^2\theta$

$= \frac{\tan^2\theta + \sec^2\theta}{\tan\theta}$

$= \frac{\tan^2\theta + 1 + \tan^2\theta}{\tan\theta}$

$\frac{1}{r}\frac{dr}{d\theta} = \cot\theta + 2\tan\theta$

$-\frac{1}{r} r^2 \cdot \frac{d\theta}{dr} = \cot\theta + 2\tan\theta \Rightarrow -\frac{r}{dr} = \left(\frac{1}{\tan\theta} + 2\tan\theta\right)$

$\cancel{\frac{dr}{r} = -d\theta\left(\frac{1}{\tan\theta} + 2\tan\theta\right)}$

$$-\frac{dr}{r} = \frac{\tan\theta}{1+2\tan^2\theta} \cdot d\theta$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1+\frac{2\sin^2\theta}{\cos^2\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+2\sin^2\theta}{\cos^2\theta}}$$

$$\int \frac{dr}{r} = -\frac{2\sin\theta\cos\theta}{2(1+\sin^2\theta)} \cdot d\theta$$

$$\log r = -\frac{1}{2}\log(1+\sin^2\theta) + \log k$$

$$\log r + \frac{1}{2}\log(1+\sin^2\theta) = \log k$$

$$\boxed{r^2(1+\sin^2\theta) = C}$$

- D.E
  - linear: $O = 1$, $D = 1$
    - Variable seperable
    - Homo
    - Linear DE
    - Exact DE
  - Non-linear: $O = 1$, $D > 1$
    - Solvable for p
    - Solvable for y
    - Solvable for x

Non-linear DE

- First order non-linear DE of nth degree of form

$$p^n + a_1 p^{n-1} + a_2 p^{n-2} + \dots + a_{n-1} p' + a_n = 0$$

$a_1, a_2, \dots a_{n-1}, a_n \rightarrow$ co-eff

$x, y \Rightarrow$ functions

$p = \frac{dy}{dx}$ $\quad y = f(x)$

$\hookrightarrow$ Such non-linear eq can be reduced to diff. eq of $O = 1$ & $D = 1$

i) solving for p

ii) solving for y

iii) solving for x

$\textcircled{f}$ Eq. solvable for p

1. $p^2 + 2xp - 3x^2 = 0$

$$\left(\frac{dy}{dx}\right)^2 + 2x \cdot \frac{dy}{dx} - 3x^2 = 0$$

$$\frac{-2 \pm \sqrt{4+12}}{2}$$

$$= \frac{-2 \pm 4}{2} = -3 \text{ (or) } 1$$

$$(p+3x)(p-x) = 0$$

$$\frac{dy}{dx} = -3x \quad \text{(or)} \quad \frac{dy}{dx} = x$$

$$\int dy = -\int 3x \cdot dx \quad \text{(or)} \quad \int dy = \int x\, dx$$

$$y = -\frac{3x^2}{2} + c \qquad y = \frac{x^2}{2} + c$$

$$y + \frac{3x^2}{2} - c = 0 \qquad \left(y - \frac{x^2}{2} - c\right) = 0$$

→ General Sol^n is

$$\left(y + \frac{3x^2}{2} - c\right)\left(y - \frac{x^2}{2} - c\right) = 0$$

2. $p(p+y) = x(x+y)$

$$\left(\frac{dy}{dx}\right)^2 + y \cdot \frac{dy}{dx} - x^2 - xy = 0$$

$$p^2 + py - x^2 - xy$$

$$(p+x)(p-x) + (p-x)y$$

$$(p+x+y)(p-x) = 0$$

~~$\frac{d}{}$~~

$$\frac{dy}{dx} + x + y = 0$$

$$\frac{dy}{dx} + y = -x$$

$$P = e^{\int 1\, dx} = e^x$$

$$Q = -x$$

$$y \cdot IF = \int Q \cdot IF\, dx + c$$

$$ye^x = \int -x \cdot e^x\, dx + c$$

$$= -\left[xe^{+x} - e^x\right] + c$$

$$\frac{dy}{dx} = x$$

$$y = \frac{x^2}{2} + c$$

Q. $p^3 + 2xp^2 - p^2y^2 - 2xy^2p = 0$

$\Rightarrow$ ~~$p(p^2 + 2xp - py^2 - 2xy^2) = 0$~~

$p^2(p+2x) - py^2(p+2x) = 0$

$p(p-y^2)(p+2x)$

$p = 0$; $p = y^2$; $p + 2x = 0$

$\int \frac{dy}{dx} = 0$	$\frac{dy}{dx} = y^2$	$\frac{dy}{dx} = -2x$
$y = c$	$\int \frac{dy}{y^2} = \int dx$	$dy = -2x\,dx$
	$-\frac{1}{y} = x + c$	$y = -x^2 + c$

$(y-c)\left(x+\frac{1}{y}+c\right)(y+x^2-c) = 0$

Q. $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$

~~$xy(p^2-6) + p(3x^2 - 2y^2) = 0$~~

$xyp^2 + 3x^2p - 2y^2p - 6xy = 0$

$xp(yp + 3x) - 2y(yp + 3x) = 0$

$(xp - 2y)(yp + 3x) = 0$

$xp = 2y$	$yp = -3x$	$(y - x^2c)(y^2 + 3x^2 - 2$
$x.\frac{dy}{dx} = 2y$	$y.\frac{dy}{dx} = -3x$	
$\int \frac{dy}{y} = \int 2.\frac{dx}{x}$	$y.dy = -3x\,dx$	
$\log y = \log x^2 + \log c$	$\frac{y^2}{2} = -\frac{3x^2}{2} + c$	
$(y = x^2c)$		

Method 2 Solvable for y

Procedure :

1) It is solvable for y, if its possible to express y in terms of x & p explicity

$y = F(x, p) \longrightarrow (1)$

2) Differentiate (1) wrt x to obtain the eq of form $p = \phi\left(x, p, \frac{dp}{dx}\right) \longrightarrow (2)$

which is 1st order 1st degree D.E in variable p

3) Solve (2)

solution of the form $G(x, p, c) = 0$

4) Eliminate p from (1) & (3),

required solution of DE (1) is obtained

Note : Whenever it is possible to eliminate p from ① & ③ solution of DE ① is given by parametric equations

$x = x(p, c)$ & $y = y(p, c)$

Q. $x^2 \left(\frac{dy}{dx}\right)^4 + 2x\left(\frac{dy}{dx}\right) - y = 0$

$x^2 p^4 + 2xp - y = 0 \longrightarrow$ ①

$y = x^2 p^4 + 2xp$

$\cancel{2xp(xp^3+2)}$

→ diff wrt x

$\frac{dy}{dx} = 2xp^4 + 4x^2p^3 \cdot \frac{dp}{dx} + 2p + 2x\frac{dp}{dx}$

$p = 2x\frac{dp}{dx}(2xp^3 + 1) + 2p + 2xp^4$

$0 = 2x\frac{dp}{dx}(2xp^3 + 1) + p + 2xp^4$

$2x\frac{dp}{dx}(2xp^3 + 1) = -p - 2xp^4$

$= -p\,\cancel{(1 + 2xp^3)}$

$2x \cdot \frac{dp}{dx}\,\cancel{(2xp^3 + 1)}$

$2x \cdot \frac{dp}{dx} = -p$

$\int \frac{dp}{p} = \int \frac{dx}{2x} \Rightarrow \log p = -\log x^{1/2}$

$p = cx^{-1/2} = c$

$$\cancel{x^2} \times \left(\frac{c}{\cancel{\sqrt{x}}}\right)^4 + \frac{2x \cdot c}{\sqrt{x}} = y$$

$$c^4 + 2c\sqrt{x} = y$$

$$\boxed{y = 2c\sqrt{x} + c^4}$$

Q. $xp^2 + ax = 2yp$

$$\frac{xp^2 + ax}{2p} = y$$

$$y = \frac{xp^2}{2p} + \frac{ax}{2p}$$

$$= \frac{xp}{2} + \frac{ax}{2p}$$

$$y = f(x, p)$$

$$\frac{dy}{dx} = \frac{p}{2} + \frac{x}{2}\frac{dp}{dx} + \frac{a}{2p} + \frac{ax}{2}\cdot\left(\frac{-1}{p^2}\right)\cdot\frac{dp}{dx}$$

$$p = \frac{x}{2}\frac{dp}{dx}\left(1 - \frac{a}{p^2}\right) + \frac{1}{2}\left(p + \frac{a}{p}\right)$$

$$\frac{p}{2} - \frac{a}{2p} = \frac{x}{2}\cdot\frac{dp}{dx}\left(1 - \frac{a}{p^2}\right)$$

$$\frac{\cancel{p^2 - a}}{\cancel{2}p} = \frac{x}{\cancel{2}}\cdot\frac{dp}{dx}\left(\frac{\cancel{p^2 - a}}{p^{\cancel{2}}}\right)$$

$$\int\frac{dp}{p} = \int\frac{dx}{x}$$

$$\log p = \log x + \log c$$

$$\boxed{p = cx} \Rightarrow$$

in (1),

$$x(cx)^2 + ax = 2y(cx)$$

$$\frac{c^2x^3 + ax}{2cx} = y$$

$$\frac{c^2x^2 + a}{2c} = y$$

Q. $y = 2px + p^n \longrightarrow (1)$

$$\cancel{\frac{dy}{dx} = 2p + n.p^{n-1}\frac{dp}{dx}}$$

$$\cancel{0 = p + n\,p^{n-1}.\frac{dp}{dx}}$$

$$\cancel{p\left(1 + n.p^{n-1}\frac{dp}{dx}\right)}$$

$$\frac{dy}{dx} = 2p + 2x.\frac{dp}{dx} + n.p^{n-1}.\frac{dp}{dx}$$

$$p = (2x + n.p^{n-1})\frac{dp}{dx} + 2p$$

$$-p = (2x + n.p^{n-1})\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{-p}{2x + n.p^{n-1}}$$

$$\frac{dx}{dp} = \frac{-2x - n.p^{n-1}}{+p}$$

$$\frac{dx}{dp} = \frac{-2x}{p} - n.p^{n-2}$$

$$\frac{dx}{dp} + \frac{2x}{p} = -n.p^{n-2}$$

In the form $\frac{dx}{dp} + P'x = Q'$

$$P' = \frac{2}{p} \Rightarrow IF = e^{\int P'dp} = e^{\int \frac{2}{p}dp} = p^2$$

$$x.IF = \int Q'.IF.dp + c$$

$$xp^2 = \int -n.p^{n-2} \times p^2 .dp + c$$

$$= -\int n.p^n\,dp + c$$

$$xp^2 = \frac{-np^{n+1}}{n+1} + c \Rightarrow \boxed{x = \frac{-np^{n-1}}{n+1} + \frac{c}{p^2}} \rightarrow (2)$$

Substitute in (1)

$$y = 2p\left(\frac{-np^{n-1}}{n+1} + \frac{c}{p^2}\right) + p^n$$

$$\boxed{y = \left(\frac{-2np^n}{n+1} + \frac{2c}{p}\right) + p^n} \rightarrow (3)$$

(2) & (3) are the parametric solutions.

Newton's Law of Cooling

$\rightarrow \frac{dT}{dt} = -K(T - t_2)$

$\Rightarrow \int \frac{dT}{T - t_2} = -\int K\, dt$

$\ln(T - t_2) = -Kt + \alpha$

$T - t_2 = e^{-Kt} \cdot e^{\alpha}$

$T = t_2 + Ce^{-Kt}$

at $t = 0$, $T = t_1$

$t_1 = t_2 + Ce^{-K \times 0}$

$\boxed{C = t_1 - t_2}$ → initial temp

$\boxed{T = t_2 + (t_1 - t_2)e^{-Kt}}$

T → Temp of body at time t

t_2 → temp of surr.

Q. Water at $10°C$ takes 5 min to warm up to $20°C$ in room at temp $40°C$.

a) Find temp after 20 min & 30 min

b) When will be temp at $25°C$

A.

~~$40 = 20 + (10)e^{-K \times 5}$~~

~~$\frac{20}{10} = e^{-5K}$~~

~~$\log 2 = -5K \Rightarrow \boxed{-0.138 = K}$~~

~~$40 = t_2 + t_1$~~

~~a) $40 = t_2 + (10 - t_2)e^{-1.6218}$~~

~~$40 - t_2 = (10 - t_2)e^{-1.6218}$~~

~~$40 - t_2 = (10 - t_2)(0.1975)$~~

$20 = 40 + (-30)e^{-5K}$

$\frac{+2}{3} = e^{-5K}$

$\log 2 - \log 3 = -5K$

$\frac{\log 3 - \log 2}{5} = K$

$K = 0.0811$

a) i) $T = 40 - 30e^{-1.6218}$

$= 34.0737°C$

ii) $T = 40 - 30e^{-2.433}$

$= 37.3668°C$

b) $T = 25° \Rightarrow 25 = 40 - 30e^{-(0.0811)t}$

$\frac{}{} e^{-0.0811t} \Rightarrow \log 1 - \log 2 = -0.0811t \Rightarrow t = 8.546$

Q. A thermometer is removed from room where the temp is 70°F to outside, where temp is 10°F. After $\frac{1}{2}$ min, thermometer reads 50°F. What is reading at t = 1 min. How long to reach 15°F?

A. $t_1 = 70°F$; $t_2 = 10°F$; $T = 50°F$

$t = 0.5$

$T = t_2 + (t_1 - t_2)e^{-kt}$

$50 = 10 + 60\, e^{-k/2}$

$\frac{2}{3} = e^{-k/2}$

$-0.4054 = \frac{-k}{2}$

$k = 0.8108$

linear

- **Higher Order DE of order n :**

$\rightarrow a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = X$

where $a_0, a_1, a_2 \dots a_{n-1}, a_n$ are functions of x

$\rightarrow D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}, \quad D^3 = \frac{d^3}{dx^3} \dots \quad D^n = \frac{d^n}{dx^n}$

$D^n y + a_1 D^{n-1} y + \dots + a_{n-1} Dy + a_n y = X$

So, $f(D) y = X$ where $f(D) = a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n$

- **2 forms of Higher order linear DE of order n :**

1) Linear DE with constant coeff :

ex : $6 \frac{d^3 y}{dx^3} + 3 \frac{d^2 y}{dx^2} + 9 \frac{dy}{dx} + y = \sin x$

$(6D^3 + 3D^2 + 9D + 1) y = \sin x$

2) Linear DE with variable coeff :

ex : $(2x-5)^2 \frac{d^2 y}{dx^2} - (2x-5) \frac{dy}{dx} - 12y = 6x^2$

• Higher Order linear DE of order n :

$\to a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = X$

where $a_0, a_1, a_2 \dots a_{n-1}, a_n$ are functions of x

$\to D = \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2}$, $D^3 = \frac{d^3}{dx^3}$ ... $D^n = \frac{d^n}{dx^n}$

$D^n y + a_1 D^{n-1} y + \dots + a_{n-1} Dy + a_n y = X$

So, $f(D) y = X$ where $f(D) = a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D^{n-1} + a_n$

• 2 forms of Higher order linear DE of order n :

1) linear DE with constant coeff :

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2) linear DE with variable coeff :

ex : $(2x-5)^2 \frac{d^2 y}{dx^2} - (2x-5) \frac{dy}{dx} - 12y = 6x^2$

LDE

Homogeneous LDE

RHS $X = 0$

ex: $2D^2y - 3Dy + 9y = 0$

Non - Homogeneous LDE

RHS X is non-zero func^n of x

ex: $2D^2y - 3Dy + 9y = x \cdot \sinh x$

Note: $\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Solution of homogeneous LDE w constant coeff

ex: Consider $D^2y + a_1 Dy + a_2 y = 0 \longrightarrow (1)$

Then y_1 & y_2 are linearly independant solutions of 2nd order homogeneous LDE, then

$\underbrace{c_1y_1 + c_2y_2}$ is also sol^n of the DE

where c_1 & c_2 are arbitrary constant

$\Rightarrow y = c_1y_1 + c_2y_2$ is complete solution of DE

Working procedure:

1) Write given eq in symbolic form $\Rightarrow (D^2 + a_1 D + a_2)y = 0$

2) Write the auxillary eq by replacing D by m

$m^2 + a_1 m + a_2 = 0$

3) Find roots, say m_1 & m_2

Case 1 : roots of m_1 & m_2 are real & distinct, $\Rightarrow y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

ex: $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

$D^2y + 5Dy + 6y = 0$

$m^2 + 5m + 6 = 0$

$\frac{-5 \pm \sqrt{25-24}}{2} = -3, -2$

$m_1 = -3$

$m_2 = -2$

$y = c_1 e^{-3x} + c_2 e^{-2x}$ $\Rightarrow$ general solution

Case 2 : If $m_1 = m_2 = m$, then $y = (c_1 + c_2 x) e^{mx}$

ex: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

$m^2 + 4m + 4 = 0$

$m = -2$

$y = (c_1 + c_2 x) e^{-2x}$

Note : suppose m_1, m_2, m_3 are real & equal & remaining $(n-3)$ are real & distinct, then,

$y = \left((c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{mx} + c_5 e^{m_5 x} + c_6 e^{m_6 x} + \dots + c_n e^{m_n x} \right)$

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Case 3 : If roots m_1 & m_2 are complex

say $m = \alpha \pm i\beta$

Let roots be $m_1 = \alpha + i\beta$ & $m_2 = \alpha - i\beta$

Then, general soln,

$$y = e^{\alpha x}(c_1 \cos\beta . x + c_2 \sin\beta x)$$

ex: Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

$$m^2 - 4m + 13 = 0$$

$\boxed{\begin{matrix} 2+3i \\ 2-3i \end{matrix}}$ $\alpha = 2$, $\beta = 3$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$$

Note 1: If there are 2 pairs of imaginary roots $\alpha \pm \beta i$, $\gamma \pm i\delta$, then

$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x) + e^{\gamma x}(c_3 \cos\delta x + c_4 \sin\delta x) + c_5 e^{m_5 x} + \dots + c_n e^{m_n x}$$

Note 2: If a pair of complex roots say $\alpha \pm i\beta$ occuring twice, then GS is

$$y = e^{\alpha x}\left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right] + c_5 e^{m_5 x} + \dots + c_n e^{m_n x}$$

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Q. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$

$m^2 - 4m + 5 = 0$

$m = 2 \pm i$

$y = e^{2x}(c_1 \cos x + c_2 \sin x) \Rightarrow y' = e^{2x} \times 2(c_1 \cos x + c_2 \sin x) + e^{2x}(-c_1 \sin x + c_2 \cos x)$

$y(0) = 2$

$y'(0) = -1$

$2 = e^0 (c_1)$

$\boxed{c_1 = 2}$

$-1 = 2(c_1) + (c_2)$

$-1 = 2c_1 + c_2$

$-1 = 4 + c_2$

$\boxed{c_2 = -5}$

$$\boxed{y = e^{2x}(2\cos x - 5\sin x)}$$

Q. $\frac{d^4x}{dt^4} + 4x = 0$

$m^4 + 4 = 0 \Rightarrow m_1 = 1 + i; \quad m_2 = 1 - i$

$m_3 = -1 + i; \quad m_4 = -1 - i$

$x = e^{t}(c_1 \cos x + c_2 \sin x) + e^{-t}(c_3 \cos x + c_4 \sin x)$

Q. $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$

$m_1 = 2$

$m_2 = 3/2$

$m_3 = -1/2$

$m_4 = -1$

$Dy = X$

$y = c_1 e^{2x} + c_2 e^{3/2 x} + c_3 e^{-x/2} + c_4 e^{-x}$

Q. $y'''' + 50y'' + 625y = 0$

$m = 5i, -5i, 5i, -5i$

$y = e^0 [(c_1 + c_2 x)\cos 5x + (c_3 + c_4 x)\sin 5x]$

$= [(c_1 + c_2 x)\cos 5x + (c_3 + c_4 x)\sin 5x]$

Q. $y''' - 6y'' + 11y' - 6y$; $y(0) = 0$; $y'(0) = -4$

$y''(0) = -18$

$m = 1, 2, 3$

$\Rightarrow y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$

$0 = c_1 + c_2 + c_3 \longrightarrow (1)$

$\Rightarrow y' = c_1 e^x + 2c_2 e^{2x} + 3c_3 e^{3x}$

$-4 = c_1 + 2c_2 + 3c_3 \longrightarrow (2)$

$\Rightarrow y'' = c_1 e^x + 4c_2 e^{2x} + 9c_3 e^{3x}$

$-18 = c_1 + 4c_2 + 9c_3 \longrightarrow (3)$

From (1), (2),

$c_2 + 2c_3 = -4$

$2c_2 + 6c_3 = -14$

$2c_3 = -6$

$c_3 = -3$

$c_2 = 2$

$c_1 = 1$

$y = e^x + 2e^{2x} - 3e^{3x}$

Non-homo linear DE of n^{th} order w const. coeff

$\Rightarrow (a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n) y = X \longrightarrow (1)$

$\Rightarrow (a_0 D^n + a_1 D^{n-1} + \dots + a_{n-1} D + a_n) y = 0 \longrightarrow (2)$

y = complementary func^n + Particular integral

$y = y_c + y_p$

Particular Integral

- Imp. Results :

1) $\frac{1}{D}(x) = \int X \, dx$

2) $\frac{1}{D^2}(X) = \int\left(\int X \, dx\right) dx$

3) $\frac{1}{D-a}(X) = e^{ax} \int e^{-ax} X \, dx$

4) if $f(D) = ((D-a)(D-b))$

then $\frac{1}{f(D)}(X) = \frac{1}{(D-a)(D-b)}(X) = \frac{1}{D-a}\left(\frac{1}{D-b}(X)\right)$

Rules to find P. I

Type 1: when $X = e^{ax}$

$$y_p = \frac{1}{f(D)} e^{ax} \quad (D \to a)$$

$$= \frac{1}{f(a)} e^{ax} \quad \text{provided } f(a) \neq 0$$

Case of failure:

i) when $f(a) = 0$,

then $y_p = x \cdot \frac{1}{f'(D)} e^{ax} = x \cdot \frac{1}{f'(a)} e^{ax}$

provided $f'(a) \neq 0$

ii) when $f'(a) = 0$,

then $y_p = x^2 \cdot \frac{1}{f''(D)} e^{ax} = x^2 \cdot \frac{1}{f''(a)} e^{ax}$

provided $f''(a) \neq 0$

Note: also entended for $X = e^{ax+b}$, ke^{ax}, K, a^x,

$\sinh x$, $\cosh x$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = e^x + 4e^{4x}$$

$$(D^2y - 3Dy - 4y) = e^x + 4e^{4x}$$

$\Rightarrow m^2 - 3m - 4$

$m = 4, -1$

$c_1 e^{4x} + c_2 e^{-x}$

Now, $PI = \frac{1}{f(D)} \cdot X$

$$= \frac{1}{D^2 - 3D - 4} \cdot (e^x + 4e^{4x})$$

$PI = PI_1 + PI_2$

$$= \frac{e^x}{D^2 - 3D - 4} + \frac{4e^{4x}}{D^2 - 3D - 4}$$

$$\frac{e^x}{1 - 3 - 4} + \frac{4e^{4x}}{16 - 12 - 4}$$

diff

$$\Rightarrow \frac{x \cdot 4e^{4x}}{\cancel{2D - 3}\ 5}$$

$$\Rightarrow \frac{4e^{4x}x}{5}$$

$$\frac{e^x}{-6} + \frac{4e^{4x}x}{5}$$

General solution, $y = CF + PI$

$$= (c_1 e^{4x} + c_2 e^{-x}) + \left(\frac{e^x}{-6} + \frac{4e^{4x}x}{5}\right)$$

2. $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 2\sinh x$

CF $\Rightarrow$ $D^3y - 3Dy + 2y$

$m = -2, 1, 1$

$CF = c_1 e^{-2x} + (c_2 + c_3 x)e^{x}$

PI $\Rightarrow$ $PI = \frac{1}{f(D)} \cdot X$

$= \frac{1}{m^3 - 3m + 2} \times 2\sinh x$

$= \frac{1}{m^3 - 3m + 2} \times \cancel{2} \times \left(\frac{e^x - e^{-x}}{\cancel{2}}\right)$

$= \frac{e^x - e^{-x}}{m^3 - 3m + 2} = \frac{e^x}{m^3 - 3m + 2} - \frac{e^{-x}}{m^3 - 3m + 2}$

$= \frac{e^x}{\cancel{1 - 3 + 2}} - \frac{e^{-x}}{\cancel{-1 + 3} + 2}$

$0 \qquad 4$

$\frac{e^x \cdot x}{\cancel{3m^2 - 3}}$

0

$\frac{e^x x^2}{6m} - \frac{e^{-x}}{4}$

$PI = \frac{e^x x^2}{6} - \frac{e^{-x}}{4}$

$y = c_1 e^{-2x} + (c_2 + c_3 x)e^x + \frac{e^x x^2}{6} - \frac{e^{-x}}{4}$

3. $(D^4+D^3-3D^2-5D-2)y = (e^{-x}+2)^2 + e^{-x}\cosh x$

$CF \Rightarrow m^4+m^3-3m^2-5m-2$

$m = 2, -1, -1, -1$

$C.F = c_1 e^{2x} + (c_2 + c_3 x + c_4 x^2) e^{-x}$

$$PI_1 = \cancel{\frac{2e^{-2x} + 5 + 4e^{-4x} + \dots}{D^4+D^3-3D^2-5D-2}}$$

$$= \cancel{\frac{2e^{-2x}}{16-8-12+10-2} + \frac{5}{-2} + \frac{4e^{-4x}}{256-64-48+20-2}}$$

$$= \cancel{\frac{2e^{-2x}}{4} - \frac{5}{2} + 4e^{-4x}}$$

$$= \frac{e^{-2x} + 4 + 4e^{-x} + e^{-x}\left(\frac{e^{x}+e^{-x}}{2}\right)}{D^4+D^3-3D^2-5D-2}$$

$$= \frac{2e^{-2x} + 8 + 8e^{-x} + 1 + e^{-2x}}{2D^4+2D^3-6D^2-10D-4}$$

$$= \frac{3e^{-2x}}{2(16)+2(-8)-24+20-4} + \frac{-2}{8}e^{-x}x^3 + \frac{9}{-4}$$

$$= \frac{3e^{-2x}}{32-16-24+20-4} - \frac{2x^3}{9}e^{-x} - \frac{9}{4}$$

$e^{2x} + (c_2 + c_3 x + c_4 x^2)e^{-x}$

Q. $(D^2-4)y = \cosh(2x-1) + 3^x$

C.F $= m^2 - 4 = 0 \Rightarrow CF = c_1 e^{2x} + c_2 e^{-2x}$

$m = \pm 2$

$$PI = \frac{e^{2x-1} + e^{1-2x}}{2(D^2-4)} + \frac{e^{\log 3 . x}}{(D^2-4)}$$

$$= \frac{1}{2}\left[\frac{e^{2x} . e^{-1} + e . e^{-2x}}{D^2 - 4}\right] + \frac{e^{\log 3 . x}}{(\log 3)^2 - 4}$$

$$= \frac{1}{2}\left[\frac{x . e^{2x-1}}{4} + \frac{x e^{1-2x}}{4}\right] + \frac{e^{\log 3 . x}}{(\log 3)^2 - 4}$$

Type (2) : When $X = \sin(ax+b)$ (or) $\cos(ax+b)$

$$y_p = \frac{1}{f(D)} \sin(ax+b) \qquad D^2 \rightarrow -a^2$$

$$= \frac{1}{f(-a^2)} \sin(ax+b) \quad \text{provided } f(-a^2) \neq 0$$

Same rule holds when $X = \cos(ax+b)$

Note : i) $X = \sin ax . \cos bx$ (or) $\cos ax . \cos bx$

(or) $\sin ax . \sin bx$

ii) $X = \sin^2 ax$ (or) $\cos^2 ax$

(or) $\sin^3 ax$ (or) $\cos^3 ax$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$\cos^2\frac{x}{2} = \frac{1+\cos x}{2}$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$\cos^3 x = \frac{3\cos x + \cos 3x}{4}$$

Q. $y''' - y'' + 4y' - 4y = \sin 3x$

$(D^3 - D^2 + 4D - 4)y = \sin 3x$

$\Rightarrow m^3 - m^2 + 4m - 4 = 0$

$m = 1, 2i, -2i$

$CF = c_1 e^x + e^{0x}(c_2 \cos 2x + c_3 \sin 2x)$

$CF = c_1 e^x + (c_2 \cos 2x + c_3 \sin 2x)$

$$PI = \frac{1}{D^3 - D^2 + 4D - 4} \cdot \sin 3x$$

$$= \frac{1}{-9D - (-9) + 4D - 4} \times \sin 3x$$

$$= \frac{\sin 3x}{-9D + 9 - 4 + 4D} = \frac{\sin 3x}{5 - 5D}$$

$$= \frac{\sin 3x}{5} \times \left[\frac{1}{1-D} \times \frac{1+D}{1+D}\right]$$

$$= \frac{1+D}{1-D^2} \times \frac{\sin 3x}{5} = \frac{(1+D)\sin 3x}{10 \times 5} = \frac{1+D}{50} \times \sin 3x$$

$$= \frac{\sin 3x + D\sin 3x}{50} = \boxed{\frac{\sin 3x + 3\cos 3x}{50}}$$

$y = CF + PI$

$$= c_1 e^x + (c_2 \cos 2x + c_3 \sin 2x) + \frac{\sin 3x + 3\cos 3x}{50}$$

~~$(D^2+3y)y$~~

Q. $(D^2+3)y = \cos\sqrt{3}x$

$m^2+3=0$

$m = \sqrt{3}i, -\sqrt{3}i$

$CF = e^{0x}(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)$

$= (c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)$

$PI = \frac{1}{D^2+3} \cdot \cos\sqrt{3}x$ $\qquad D^2 = (+\sqrt{3})^2 = -3$

$= \frac{1}{-3+3} \cdot \cos\sqrt{3}x$

Differentiate denominator wrt D

$\Rightarrow \frac{x}{2D} \cdot \cos\sqrt{3}x$

$\Rightarrow \frac{x}{2}\int \cos\sqrt{3}x\,dx$

$PI = \frac{x}{2}\frac{\sin\sqrt{3}x}{\sqrt{3}}$

$y = CF + PI = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x + \frac{x\sin\sqrt{3}x}{2\sqrt{3}}$

Q. $(D^3+1)y = \cos^2\left(\frac{x}{2}\right) + e^{-x}$

$CF = D^3+1$

$= m^3+1$

$m = -1, \ \frac{1 \pm \sqrt{3}i}{2}$

$CF = c_1 e^{-x} + e^{\frac{x}{2}}\left(c_2 \cos\frac{\sqrt{3}x}{2} + c_3 \sin\frac{\sqrt{3}x}{2}\right)$

$PI = \underbrace{\frac{1}{D^3+1} \cdot \cos^2\frac{x}{2}}_{PI_1} + \underbrace{\frac{1}{D^3+1} \cdot e^{-x}}_{PI_2}$

$PI_2 = \frac{1}{D^3+1} \cdot e^{-x}$

$= \frac{1}{-1+1} e^{-x} = \frac{1}{0} \times$

$\Rightarrow \frac{x \cdot e^{-x}}{3D^2} = \frac{xe^{-x}}{3}$

$PI_1 = \frac{1}{D^2 \cdot D+1} \cdot \cos^2\frac{x}{2}$

$= \frac{1}{D^2 \cdot D+1} \cdot \frac{1+\cos x}{2}$

$= \frac{1}{2}\left[\frac{1}{D^3+1} \cdot e^0 + \frac{1}{D^2 \cdot D+1} \cos x\right]$

$= \frac{1}{2}\left[\frac{1}{1} \cdot e^0 + \frac{1}{-D+1} \cos x\right]$

$= \frac{1}{2}\left[1 + \cos x\left(\frac{1}{1-D} \times \frac{1+D}{1+D}\right)\right]$

$= \frac{1}{2}\left[1 + \cos x\left(\frac{1+D}{1-D^2}\right)\right]$

$= \frac{1}{2}\left[1 + \frac{\cos x}{2} - \frac{\sin x}{2}\right] = \frac{1}{2} + \frac{\cos x}{4} - \frac{\sin x}{4}$

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$PI = PI_1 + PI_2$

$= \frac{1}{2} + \frac{\cos x}{4} - \frac{\sin x}{4} + \frac{xe^{-x}}{3}$

$y = CF + PI$

$= c_1 e^{-x} + e^{\frac{x}{2}} \left(c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}}{2} \right) + \frac{1}{2} + \frac{\cos x}{4}$

$- \frac{\sin x}{4} + \frac{xe^{-x}}{3}$

Q. $y'' + 5y' - 6y = \sin 4x . \sin x$

$(D^2 + 5D - 6) y = \sin 4x . \sin x$

$m^2 + 5m - 6 = 0$

$m = 1, -6$

$CF = c_1 e^{x} + c_2 e^{-6x}$

$PI = \frac{1}{D^2 + 5D - 6} . \sin 4x . \sin x$

$= \frac{1}{D^2 + 5D - 6} . \frac{1}{2} [\cos (3)x - \cos 5x]$

$= \frac{1}{2} . \frac{1}{D^2 + 5D - 6} . \cos 3x - \frac{1}{2} . \frac{1}{D^2 + 5D - 6} . \cos 5x$

$PI_1 = \frac{1}{2} . \frac{1}{-9 + 5D - 6} . \cos 3x$

$= \frac{1}{2} . \frac{1}{5D - 15} . \cos 3x$

$= \frac{1}{10} . \frac{1}{D - 3} \cos 3x$

$= \frac{1}{10} \times \frac{D + 3}{D^2 - 9} \cos 3x = \frac{1}{\cancel{180}\ 60} \times (\cancel{3}\cos 3x \overset{+}{-} \cancel{3} \sin 3x)$

$$PI_2 = -\frac{1}{2} \cdot \frac{1}{D^2+5D-6} \cdot \cos 5x$$

$$= -\frac{1}{2} \cdot \frac{1}{-25+5D-6} \cdot \cos 5x$$

$$= \frac{-1}{2} \cdot \frac{1}{5D-31} \cdot \cos 5x$$

$$= \frac{-1}{2} \cdot \frac{1}{25D^2-961} \times 5D+31 \times \cos 5x$$

$$= \frac{-1}{2} \cdot \frac{1}{-1586} \times (5D\cos 5x + 31\cos 5x)$$

$$= \frac{1}{3172} \times -25\sin 5x + 31\cos 5x$$

$$PI_2 = \frac{-25\sin 5x + 31\cos 5x}{3172}$$

$$PI_1 + PI_2 = \frac{\sin 3x - \cos 3x}{60} + \frac{31\cos 5x - 25\sin 5x}{3172}$$

$$y = CF + PI = c_1 e^{x} + c_2 e^{-6x} + \frac{\sin 3x - \cos 3x}{60} + \frac{31\cos 5x - 25\sin 5x}{3172}$$

Type 3 :

when $X = \phi(x)$ is a polynomial in x,

Then $PI = \frac{1}{f(D)} \cdot \phi(x)$

Method 1 :

- Divide $\phi(x)$ by $f(D)$ by writing $\phi(x)$ in decreasing powers of x and $f(D)$ in increasing powers of D

Quotient = PI and remainder will be zero.

Method 2 :

- Take lowest degree term common from $f(D)$ to get an expression of the form $(1 \pm \phi(D))$ in the denominator.
Take this factor to the numerator to get $(1 \pm \phi(D))^{-1}$

- Expand $(1 \pm \phi(D))^{-1}$ using BT upto nth power as $(n+1)$th derivative of x^n is zero

- Operate on numerator term by term by taking $D = \frac{d}{dx}$.

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

Q. $(D^2 - 2D - 3)y = 2x^2 + 6x$

Method 1

$m^2 - 2m - 3 = 0$

$m = 3, -1$

$CF = c_1 e^{3x} + c_2 e^{-x}$

$PI = \frac{1}{D^2 - 2D - 3} \cdot (2x^2 + 6x)$

$$\begin{array}{r|l}
 & -\frac{2x^2}{3} - \frac{10x}{9} + \frac{8}{27} \\
\hline
-3 - 2D + D^2 & 2x^2 + 6x \\
 & \underline{2x^2 + \frac{8x}{3} - \frac{4}{3}} \\
 & \quad - \quad - \quad + \\
 & \frac{10x}{3} + \frac{4}{3} \\
 & \underline{\frac{10x}{3} + \frac{20}{9} + 0} \\
 & \quad - \quad - \\
 & -\frac{8}{9} \\
 & \underline{-\frac{8}{9}} \\
 & \quad + \\
 & 0
\end{array}$$

$\Rightarrow PI = -\frac{2x^2}{3} - \frac{10x}{9} + \frac{8}{27}$

$= \frac{-18x^2 - 30x + 8}{27}$

$y = CF + PI$

Method 2

$$PI = \frac{1}{D^2 - 2D - 3} \times 3x^2 + 6x$$

$$= \frac{1}{-3\left(1 + \frac{2D}{3} - \frac{D^2}{3}\right)} \times (3x^2 + 6x)$$

$$= -\frac{1}{3}\left[1 + \left(\frac{2D}{3} - \frac{D^2}{3}\right)\right]^{-1} \times (3x^2 + 6x)$$

$$= -\frac{1}{3}\left[1 - \left(\frac{2D}{3} - \frac{D^2}{3}\right) + \left(\frac{2D}{3} - \frac{D^2}{3}\right)^2\right] \times (3x^2 + 6x)$$

$$= -\frac{1}{3}\left[1 - \left(\frac{2D}{3} - \frac{D^2}{3}\right) + \left(\frac{4D^2}{9} + \cancel{\frac{D^4}{9}} - \cancel{\frac{4D^3}{9}}\right)\right](3x^2 + 6x)$$

$$= -\frac{1}{3}\left[3x^2 + 6x - \frac{2}{3}(6x + 6) + \frac{1}{3}(6) + \frac{4}{9}(6)\right]$$

$$= -\frac{1}{3}\left[3x^2 + 6x - \frac{8x}{3} - 4 + \frac{6}{3} + \frac{24}{9}\right]$$

$$= -\frac{1}{3}\left[3x^2 + \frac{10x}{3} + \left(\frac{-36 + 18 + 24}{9}\right)\right]$$

$$= -\frac{1}{3}\left(3x^2 + \frac{10x}{3} + \frac{-6}{9}\right)$$

$$= \frac{-18x^2 - 30x - 6}{27}$$

Q. $D^3y - y = x^5 + 3x^4 - 2x^3$

$(D^3 - 1)\,y = x^5 + 3x^4 - 2x^3$

$m^3 - 1$

$m = 1, \ \frac{-1 \pm \sqrt{3}i}{2}$

$CF = c_1 e^{x} + e^{-x/2}\left(c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2}\right)$

$PI = \frac{1}{D^3 - 1} \times (x^5 + 3x^4 - 2x^3)$

$D_1 x^5 = D_2 5x^4 = D_3 20x^3 = 60x^2$

$D_1 3x^4 = D_2 12x^3 = D_3 36x^2 = 72x$

$D_1 2x^3 = D_2 6x^2 = D_3 12x = 12$

$-x^5 - 3x^4 + 2x^3 - 60x^2 - 72x + 12$

$-1 + D^3 \,\big)\, x^5 + 3x^4 - 2x^3$

$\cancel{x^5} \qquad 60x^2$

$3x^4 - 2x^3 + 60x^2$

$\cancel{3x^4} \qquad -72x$

$-2x^3 + 60x^2 + 72x$

$-\cancel{2x^3} \qquad \pm 12$

$60x^2 + 72x - 12$

$-\cancel{60x^2} \qquad +0$

$+72x - 12$

$-\cancel{72x}$

-12

-12

0

$PI = -x^5 - 3x^4 + 2x^3 - 60x^2 - 72x + 12$

$y = CF + PI$

$= c_1 e^{x} + e^{-x/2}\left(c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2}\right)$

$- x^5 - 3x^4 + 2x^3 - 60x^2 - 72x + 12$ //

Type 4 : Shifting

When $X = e^{ax} V(x) = \sin bx$ (or) $\cos bx$ (or) polynomial in x

Then $y_p = \frac{1}{f(D)} e^{ax} V(x)$ $\quad D \to D+a$

$= e^{ax} \frac{1}{f(D+a)} V(x)$

& shift e^{ax} to front

Replace D with $D+a$ & Proceed with type 2 or 3

Q. Solve $(D^2+4D+3)y = e^{2x}\cos x$

$a = 2 \quad b = 1$

CF $\Rightarrow D^2+4D+3$

$\Rightarrow -3, -1$

$CF = c_1 e^{-3x} + c_2 e^{-x}$

PI $\Rightarrow \frac{1}{D^2+4D+3} \cdot e^{2x}\cos x$

$\Rightarrow$ replace D with $D+2$ & shift e^{2x} to front

$e^{2x} \cdot \frac{1}{(D+2)^2+4(D+2)+3} \cos x$

$= e^{2x} \cdot \frac{1}{D^2+4+4D+4D+8+3} \cos x$

$= e^{2x} \cdot \frac{1}{D^2+8D+15} \cos x$

$(D^2 \to -1)$

$= e^{2x} \frac{1}{-1+8D+15} \cos x$

$= e^{2x} \frac{1}{8D+14} \times \frac{14-8D}{14-8D} \cos x$

$= e^{2x} \frac{7-4D}{\ldots} \cos x = \frac{e^{2x}(7-4D)}{130} \cos x$

$\frac{e^{2x}}{65}(7\cos x + 4\sin x)$

$y = \frac{e^{2x}}{65}(7\cos x + 4\sin x) + c_1 e^{-3x} + c_2 e^{-x}$

Q. $(D^3-2D^2+D)y = x^2e^{-x} + \sin^2 x$

m^3-2m^2+m

$m = 1, 1, 0$

$CF = c_1e^x + c_2xe^x + c_3$

$= (c_1+c_2x)e^x + c_3$

$$PI = \frac{1}{D^3-2D^2+D}\,x^2e^{-x} + \frac{1}{D^3-2D^2+D}\left(\frac{1-\cos 2x}{2}\right)$$

$$= e^{-x}\cdot\frac{1}{(D-1)^3-2(D-1)^2+(D-1)}\cdot x^2 + \frac{1}{D^3-2D^2+D}\left(\frac{1}{2}-\frac{\cos 2x}{2}\right)$$

$$= e^{-x}\frac{1}{D^3-1-3D^2+3D-2D^2-2+4D+D-1} + \frac{1}{2}\cdot\frac{1}{D^3-2D^2+D}e^0 - \frac{1}{2}\frac{1}{D^3-2D^2+D}\cos$$

$$= e^{-x}\cdot\frac{1}{D^3-5D^2+8D-4}x^2 + \frac{1}{2}\cdot\frac{x}{\cancel{3D^2-4D+1}}\,1\,e^0 - \frac{1}{2}\frac{1}{-4D+8+D}\cos$$

$= e^{-x}$

$$\begin{array}{r|l} & -\frac{x^2}{4} - x - \frac{11}{8} \\ \hline -4+8D-5D^2+D^3 & x^2 \\ & x^2 - 4x + \frac{5}{2} \\ \hline & 0 + 4x - \frac{5}{2} \\ & +4x - 8 \\ \hline & \frac{11}{2} \\ & \frac{11}{2} \\ \hline & \boxed{0} \end{array}$$

$$= e^{-x}\cdot\left(-\frac{x^2}{4}-x-\frac{11}{8}\right) + \frac{x}{2} - \frac{\cos 2x}{2}\cdot\frac{1}{8-3D}\times\frac{8+3D}{8+3D}$$

$$= e^{-x}\left(-\frac{x^2}{4}-x-\frac{11}{8}\right) + \frac{x}{2} - \frac{\cos 2x}{2}\cdot\frac{8+3D}{64-9D^2}$$

$$= e^{-x}\left(-\frac{x^2}{4}-x-\frac{11}{8}\right) + \frac{x}{2} \cancel{\frac{\cos 2x}{40}}_{10} + \cancel{\frac{x\sin 2x}{40}}_{20}$$

PI $$= e^{-x}\left(-\frac{x^2}{4}-x-\frac{11}{8}\right) + \frac{x}{2} - \cancel{\frac{\cos 2x}{10}} + \cancel{\frac{\sin 2x}{20}}$$

$$\boxed{y = (c_1+c_2x)e^x + c_3 + e^{-x}\left(-\frac{x^2}{4}-x-\frac{11}{8}\right) + \frac{x}{2} - \frac{\cos 2x}{25} + \frac{3\sin 2x}{50}}$$

Type 5 : if $X = x\phi(x)$ where $\phi(x) = \sin ax$ (or) $\cos ax$

then, $PI = \left(x - \frac{f'(D)}{f(D)}\right) \frac{\phi(x)}{f(D)}$

1. $(D^2 - 1)y = x \sin 3x$

$m = \pm 1$

$CF = c_1 e^{x} + c_2 e^{-x}$

$$PI = \left(x - \frac{2D}{D^2 - 1}\right) \cdot \frac{\sin 3x}{D^2 - 1} \qquad (D^2 = -a)$$

$$= \left(x - \frac{2D}{-10}\right) \cdot \frac{\sin 3x}{-10}$$

$$= \left(x + \frac{2D}{10}\right) \cdot \frac{\sin 3x}{-10}$$

$$= \frac{x \sin 3x}{-10} + \frac{6 \cos 3x}{-100}$$

$$= -\left(\frac{x \sin 3x}{10} + \frac{\cancel{6}^{3} \cos 3x}{\cancel{100}\, 50}\right) = -\left(\frac{x \sin 3x}{10} + \frac{3\cos 3x}{50}\right)$$

$=$

$$y = c_1 e^{x} + c_2 e^{-x} - \left(\frac{5x \sin 3x + 3\cos 3x}{50}\right)$$

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Q. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x\cos 2x$

$(D^2 - 5D + 6)y = x\cos 2x$

$(m^2 - 5m + 6)$

$m = 2, 3$

$\Rightarrow CF = c_1 e^{2x} + c_2 e^{3x}$

$$PI = \frac{x\cos 2x}{D^2 - 5D + 6}$$

$$= \left(x - \left(\frac{2D - 5}{D^2 - 5D + 6}\right)\right) \cdot \frac{\cos 2x}{D^2 - 5D + 6}$$

$$= \left(x - \left(\frac{2D - 5}{-4 - 5D + 6}\right)\right) \frac{\cos 2x}{-4 - 5D + 6}$$

$$= \left(x - \left(\frac{2D - 5}{2 - 5D}\right)\right) \cdot \frac{\cos 2x}{2 - 5D}$$

$$\cancel{= \left(x - \left(\frac{2D - 5}{2 - 5D}\right)\right) \cdot \frac{\cos 2x}{2 - 5D} \times \frac{2 + 5D}{2 + 5D}}$$

$$\cancel{= \left(x - \left(\frac{2D - 5}{2 - 5D}\right)\right) \cdot \frac{\cos 2x \, (2 + 5D)}{4 - 25D^2}} \quad 104$$

$$\cancel{= \left(x - \left(\frac{2D - 5}{2 - 5D}\right)\right) \cdot \left(\frac{2\cos 2x - 10\sin 2x}{104}\right)}$$

$$\cancel{= x - \frac{2D - 5}{4 - 25D} \times \frac{(2 + 5D)}{104}}$$

$$\cancel{= \frac{x\cos 2x}{2 - 5D} - \frac{(2D - 5)\cos 2x}{}}$$

Unit 1	Unit 2
$8 \times 1 = 8$	$8 \times 1 = 8$
$2 \times 2 = 4$	$2 \times 2 = 4$
$2 \times 4 = 8$	$2 \times 4 = 8$
20	20

40

$= x - \left(\frac{2D-5}{2-5D}\right) \cdot \left(\frac{2\cos 2x - 10\cos 2x}{104}\right)$

$= x - \frac{(2D-5)(2+5D)}{104}\left(\frac{2\cos 2x - 10\sin 2x}{104}\right)$

$= x - \frac{(4D-10-25D+10D^2)}{104}\left(\frac{2\cos 2x - 10\sin 2x}{4}\right)$

$= \frac{x\cdot\cos 2x}{2-5D} - \frac{(2D-5)}{(2-5D)^2}\cdot\cos 2x$

$= x\frac{(2+5D)\cdot\cos 2x}{104} - \frac{2D-5}{\cancel{4+25D^2}-20D}\cos 2x$

-96

$= x\cdot\frac{2\cos 2x - 10\sin 2x}{104} + \left(\frac{-4\sin 2x - 5\cos 2x}{+96+20D}\times\frac{96-20D}{96-20D}\right)$

$= x\left(\frac{2\cos 2x - 10\sin 2x}{104}\right) + \left(\frac{-384\sin 2x - 480\cos 2x + 160\cos 2x - 200\sin 2x}{\cancel{9216} - 400D^2}\right)$

10816

$PI = x\left(\frac{2\cos 2x - 10\sin 2x}{104}\right) + \left(\frac{-320\cos 2x - 584\sin 2x}{10816}\right)$

$y = CF + PI$

$= c_1 e^{2x} + c_2 e^{3x} + x\left(\frac{2\cos 2x - 10\sin 2x}{104}\right) - \left(\frac{320\cos 2x + 584\sin 2x}{10816}\right)$

Q. $(D-2)^2 y = 8\left(e^{2x} + \sin 2x + x^2\right)$

$D^2 + 4 - 40$

$m = 2$

$CF = (c_1 + c_2 x)e^{2x}$

$PI = \frac{1}{(D-2)^2} \cdot 8\left(e^{2x} + \sin 2x + x^2\right)$

$= \frac{8 \cdot x^2 e^{2x}}{2} + \frac{8 \sin 2x}{D^2 + 4 - 40} + \frac{x^2}{(D-2)^2}$

$= 4x^2 e^{x} + 8 \frac{1}{-4 + 4 - 40} \sin 2x + \frac{x^2}{D^2 + 4 - 40}$

$= 4x^2 e^{x} + \frac{8}{40} \cdot \frac{8\cos 2x}{(\sin 2x)} + (4 - 4D + D^2)^{-1} x^2$

$PI = 4x^2 e^{x} + \cos 2x + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}$

$\frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}$

x^2

$x^2 - 2x + \frac{1}{2}$

$2x - \frac{1}{2}$

$2x - 2$

$3/2$

$y = CF + PI$

$= (c_1 + c_2 x)e^{2x} + 4x^2 e^{x} + \cos 2x + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}$

Q. $(D-1)^2(D+1)^2 = \sin^2\frac{x}{2} + 2^{-x} + x$

$(D^2-1)^2 y = \sin^2\frac{x}{2} + 2^{-x} + x$

$2^{-x} = e^{-x(\log 2)}$

$D^4 + 1 - 2D^2$

$m^4 - 2m^4 + 1 = 0$

$m = 1, -1, 1, -1$

$CF = e^x(c_1 + c_2 x) + e^{-x}(c_3 + c_4 x)$

$$PI = \frac{1}{D^4 - 2D^2 + 1} \times \left(\sin^2\frac{x}{2} + 2^{-x} + x\right)$$

$$= \frac{1}{D^4 - 2D^2 + 1} \left(\frac{1-\cos x}{2} + e^{-x(\log 2)} + x\right)$$

$$= \frac{1}{D^4 - 2D^2 + 1} \left(\frac{e^0}{2} - \frac{\cos x}{2} + e^{-x\log 2} + x\right)$$

$$= \frac{1}{2} - \frac{\cos x}{2}\left(\frac{1}{\cancel{+1+2+1}}_{4}\right) + \frac{e^{-x\log 2}}{(-\log 2)^4 - 2(-\log 2)^2 + 1} + \frac{x}{D^4 - 2D^2 + 1}$$

$$PI = \frac{1}{2} - \frac{\cos x}{8} + \frac{e^{\log 2^{-x}}}{(\log 2)^4 - 2(\log 2)^2 + 1} + x$$

$1 - 2D^2 + D^4 \overline{)\, x \,}$ (x
$\underline{x + 0}$
0

$$y = e^x(c_1 + c_2 x) + e^{-x}(c_3 + c_4 x) + \frac{1}{2} - \frac{\cos x}{8} + \frac{e^{\log 2^{-x}}}{(\log 2)^4 - 2(\log 2)^2 + 1}$$

Q. $(D^2+2)y = (x^2+1)e^{3x} + e^x\cos 2x$

$m^2+2=0$

$m = \pm\sqrt{2}\,i$

$CF = e^0(c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x)$

$PI = \dfrac{(x^2+1)e^{3x} + e^x\cos 2x}{D^2+2}$

$= \dfrac{x^2e^{3x}}{D^2+2} + \dfrac{e^{3x}}{D^2+2} + \dfrac{e^x\cos 2x}{D^2+2}$

$= \dfrac{x^2e^{3x}}{(D+3)^2+2} + \dfrac{e^{3x}}{9+2} + \dfrac{e^x\cos 2x}{(D+1)^2+2}$

$= \dfrac{x^2e^{3x}}{D^2+6D+11} + \dfrac{e^{3x}}{11} + \dfrac{e^x\cos 2x}{D^2+2D+3}$

$= e^{3x}\dfrac{x^2}{D^2+6D+11} + \dfrac{e^{3x}}{11} + e^x\dfrac{\cos 2x}{\cancel{-4+3+2D}\,(2D-1)}$

$$11+6D+D^2 \,\big)\, x^2 \,\big(\, \frac{x^2}{11} - \frac{12x}{121} + \frac{50}{1331}$$

$x^2 + \frac{12x}{11} + \frac{2}{11}$

$-\frac{12x}{11} - \frac{2}{11}$

$-\frac{12x}{11} - \frac{72}{121}$

$\frac{50}{121}$

$\frac{50}{121}$

0

$= e^{3x}\left(\dfrac{x^2}{11} - \dfrac{12x}{121} + \dfrac{50}{1331}\right) + \dfrac{e^{3x}}{11} + e^x\cos 2x \dfrac{2D\ldots}{\ldots}$

$= e^{3x}\left(\dfrac{x^2}{11} - \dfrac{12x}{121} + \dfrac{171}{1331}\right) + \dfrac{e^x(4\sin 2x)}{17}$

$= e^{3x}\left(\dfrac{x^2}{11} - \dfrac{12x}{121} + \dfrac{171}{1331}\right) + \dfrac{e^x}{17}(4\sin 2x \ldots$

$y = (c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x) + e^{3x}\left(\dfrac{x^2}{11} - \dfrac{12x}{121} + \dfrac{171}{1331}\right) + \dfrac{e^x}{17}(4\sin 2x \ldots$

1. $8\frac{d^3y}{dx^3} - 12\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - y = 0$

$(8D^3 - 12D^2 + 6D - 1)y = 0$

$m = \frac{1}{2}$

$$\boxed{y = e^{\frac{x}{2}}(c_1 + c_2x + c_3x^2)}$$

2. $(x^2 - 3xy + 2y^2)\,dx + (3x^2 - 2xy)\,dy = 0$

$M = x^2 - 3xy + 2y^2$ | $N = 3x^2 - 2xy$

$\frac{\partial M}{\partial y} = -3x + 4y$ | $\frac{\partial N}{\partial x} = 6x - 2y$

Not exact

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -9x + 6y$ ⇒ Close to N

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-9x + 6y}{3x^2 - 2xy} = \frac{-3\cancel{(3x - 2y)}}{x\cancel{(3x - 2y)}} = \frac{-3}{x}$$

$$IF = e^{\int -\frac{3}{x}dx} = e^{-3\log x} = x^{-3} \,//$$

3Q. $xy\left(\frac{dy}{dx}\right)^2 - (3x^2 - 2y^2)\frac{dy}{dx} - 6xy = 0$

$xyp^2 - (3x^2 - 2y^2)p - 6xy = 0$

$xyp^2 - 3x^2p + 2y^2p - 6xy = 0$

$py(xp + 2y) - 3x(xp + 2y) = 0$

$py = 3x$

$\int dy\,y = \int 3x\,dx$

$\frac{y^2}{2} = \frac{3x^2}{2} + c$

$\boxed{(y - cx^2)(y^2 - 3x^2 - 2) = 0}$

$\cancel{\frac{dy}{dx} - \frac{3x^2}{2} - dc = 0}$

$\boxed{y^2 - 3x^2 - 2C = 0}$

$xp + 2y = 0$

$x\frac{dy}{dx} = +2y$

$\int \frac{dy}{y} = +2\int \frac{dx}{x}$

$\log y = +2\log x + \log c$

$y = cx^{+2}$

$\cancel{y - cx^2 = 0}$

$y - cx^2 =$

4Q. $16x^2 + 2p^2y - p^3x = 0$

$\frac{16x^2 - p^3x}{-2p^2} = y \Rightarrow y = \frac{-16x^2 + p^3x}{2p^2}$

$\frac{dy}{dx} = \frac{(-32x + 3p^2.p'x + p^3)2p^2 - 4pp'(-16x^2 + p^3x)}{4p^4}$

$= \frac{-64xp^2 + \cancel{6p^4p'x} + 2p^5 + 64x^2pp' - \cancel{4p^4p'x}}{4p^4}$ ($2p^4p'x$)

$\frac{4p^5}{4p^4} = -\frac{64xp^2}{4p^4} + \frac{2p^4p'x}{4p^4} + \frac{64x^2pp'}{4p^4} + \frac{2p^5}{4p^4}$

$p =$ $\cancel{\frac{16x}{p^2}}$ $\quad 0 = -\frac{16x}{p^2} + \frac{p'x}{2} + \frac{16x^2p'}{p^3}$

$\frac{p}{2} + \frac{16x}{p^2} = \frac{p'x}{2} + \frac{16x^2p'}{p^3}$