Mathematics for Electronics Engineers

Unit - 1

Data – Individual facts, statistics or items of information, often numeric, collected through observation

- Raw facts, unorganized
- Structured Data, Semi-Structured Data, Unstructured Data

Structured Data: Data whose elements are addressable for effective analysis

Semi-Structured Data: Data that doesn't reside in relational database but has some organizational properties

Unstructured Data: Data which isn't organized in predefined data model, so not good fit for mainstraim relational database

Information – Collection of facts that are organized to have additional value beyond the data they hold

Statistics – Science of data. Involves collecting, classifying, organizing, summarizing, analyzing & interpreting numerical information.

S. No	Descriptive Statistics	Inferential Statistics
1	Concerned with the describing the target population	Make inferences from the sample and generalize them to the population.
2	Organize, analyze and present the data in a meaningful manner	Compares, test and predicts future outcomes.
3	Final results are shown in form of charts, tables and Graphs	Final result is the probability scores.
4	Describes the data which is already known	Tries to make conclusions about the population that is beyond the data available.
5	Tools- Measures of central tendency (mean/median/ mode), Spread of data (range,	Tools- hypothesis tests, Analysis of variance

Measures of Central Tendency

→ Mean = $\frac{\text{Sum of all values}}{\text{total no. of values}}$

→ Median = Middle value in an arranged data set

→ Mode = Most repeated value

Mean

→ Weighted Mean = average where certain values of data set contribute more to the mean value

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

→ Trimmed Mean = The mean that is computed by arranging sample values in order & trimming an equal number of them from each end and computing the mean of the remaining

Median

→ In a data set, the middle value is the median

→ Steps to Calculate Median:

i) Arrange values of data set in ascending order

ii) Find middle position

iii) If n is total no. of values,

When n = odd, median = $\left(\frac{n+1}{2}\right)^{th}$ element

When n = even, median = $\dfrac{\frac{n}{2}^{th} \text{ element} + \left(\frac{n}{2}+1\right)^{th} \text{ element}}{2}$

Mode

→ Value that appears the most often in a data set

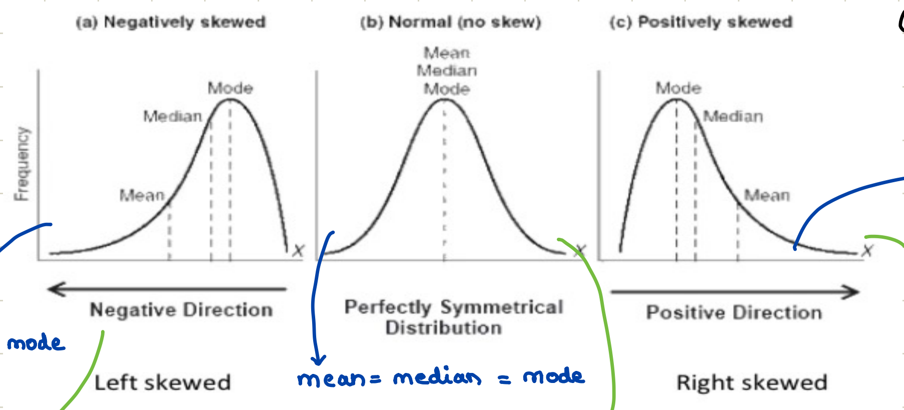
→ A set is bimodal if it has 2 or more modes

→ Emperical Formula ⇒ mode = 3 median − 2 mean

Skewness

→ Measure of the asymmetry of the distribution of about its mean

→ Can be



(d) undefined

mean > median > mode

mean < median < mode

mean = median = mode

Skewed Distribution

Symmetric Distribution

Skewed Distribution

Measure of Spread / Dispersion

→ Helps to interpret variability of data to know whether data is homogeneous or heterogeneous

→ 2 Main types of dispersion:

i) Absolute Measure of Dispersion

→ Contains same unit as original data set.

ex: Range, Standard Deviation, Quartile Deviation etc.,

ii) Relative Measure of Dispersion

→ Used to compare distribution of 2 or more data sets

→ Measure compares values w/o units

ex: Coefficient of Range, Coefficient of Variation, Coefficient of Standard Deviation etc.,

Range

→ Difference b/w 2 extreme observations of data set

Range = $x_{max} - x_{min}$

Quartiles → Splits data into 4 equal groups
Quintiles → Splits data into 5 equal groups
Deciles → Splits data into 10 equal groups
Percentiles → Splits data into 100 equal groups

→ Percentile Rank, $R = \left(\frac{P}{100}\right) * (n+1)$

If R is not a whole number (lies b/w 2 whole numbers), Find the average of the 2 that it lies between

Variance

→ Squared deviation from the mean

→ Population Variance ⇒ $\sigma^2 = \frac{\Sigma (x-\mu)^2}{N}$

Sample Variance ⇒ $s^2 = \frac{\Sigma (x-\bar{x})^2}{n-1}$

Standard Deviation

→ Signifies the deviation of elements of data set from mean value of distribution

→ Square root of Variance

SD for Discrete Data ⇒ $\sigma = \sqrt{\frac{\Sigma f_i (x_i-\bar{x})^2}{N}}$

Population vs Sample

Population	Sample
The population is a **complete set**.	The sample is a **subset** of the population
Population is **hard to define and observe** in real life.	A sample is much **easier to contact and observe**.
It is **time consuming and costly** to study a population	It is relatively **less time consuming and low cost** to study a sample.
Population **contains all members of a specified group**.	Sample is a **subset that represents the entire population**.
Reports on a **population are a true representation of opinion**.	Reports on a sample are **have a margin of error**.

Sampling

→ Refers to the study of how we select members from the population to be included in the study

→ **Probability Sampling** – Every unit in population has a probability of being selected

→ **Non-probability Sampling** – Every unit in population doesn't have a probability of being selected

→ Probability $= \frac{n}{N} \times 100$

Probability Sampling

i) **Simple Random Sampling**

→ Every individual or item in population has equal chance of being selected

ii) **Systematic Sampling**

→ You select every K^{th} individual or item from population after starting from a random point

iii) **Stratified Sampling**

→ Population divided into $\overbrace{\text{groups}}^{\text{strata}}$ based on characteristics & then sampled

iv) **Cluster Sampling**

→ Population divided into $\overbrace{\text{groups}}^{\text{clusters}}$, & random selection of clusters is made, & then survey everyone

Non-Probability Sampling

i) Convenience

ii) Judgement

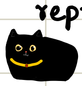
iii) Quota

iv) Snowball

Error in Sampling

i) **Sampling / Random Error**

→ Occurs when sample isn't representative of the population

(2024 AP Elections Exit Poll )

ii) **Non-Sampling / Systematic Error**

→ Occurs during data collection, causing data to differ from true values

→ 2 Major Sources : Sampling bias , Non-response bias

Sampling bias: It is a bias in which a sample is collected in such a way that some members of intended population have lower or higher sampling probability than others

Non-response bias: Type of sampling bias that occurs because of the absence of certain objects or subjects from a sample

Random Variable

→ It is a function (mapping)

$X : \Omega \longrightarrow R$ such that it follows some condition

CDF - Cumulative Distribution Function

→ $F_X(x) \triangleq P(X \leq x)$

$$= \sum_{i=1}^{N} P\{X = x_i\}\, u(x - x_i)$$

→ Properties :

i) CDF is monotonically non-decreasing function

$$F_X(x_2) \geq F_X(x_1) \quad , \text{ if } x_2 > x_1 \quad \forall x_1, x_2 \in R$$

ii) $\lim_{x \to -\infty} F_X(x) = 0 \quad \& \quad \lim_{x \to \infty} F_X(x) = 1$

iii) $P\{x_1 < X \leq x_2\} = F_X(x_2) - F_X(x_1)$

PMF - Probability Mass Function

→ $p_X(x) \triangleq P(X = x) \quad , \quad \forall x \in Z$

$$= \sum_{i=1}^{N} P\{X = x_i\}\, \delta(x - x_i)$$

→ Properties :

i) $p_X(x) \geq 0 \, , \quad \forall x \in Z$

ii) $\sum_{x=-\infty}^{\infty} p_X(x) = 1$

iii) $\sum_{a=-\infty}^{x} p_X(a) = F_X(x)$

→ $p_X(x) = \dfrac{dF_X(x)}{dx}$

Well Known Discrete RV

i) Bernoulli R.V

ii) Binomial R.V

iii) Poisson R.V

Bernoulli R V

→ 2 Outcomes : Success or Failure

→ $P(X = 1) = 1 - P(X = 0) = P$

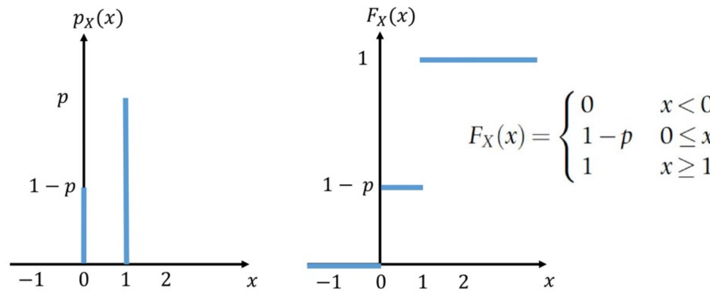
$P_X(x) = p^x (1-p)^{1-x}$, $x = 0, 1$

$X \sim Ber(p)$

→ For n Bernoulli trials

$P(A) = p$

$P(\bar{A}) = 1 - p$



$P(A \text{ occuring } k \text{ times}) = \dfrac{n!}{(n-k)!\,k!} p^k (1-p)^{n-k}$, $\mu = p$, $\sigma^2 = p(1-p)$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

Binomial R.V

→ Obtained when bernoulli R.V is repeated n times

→ $P(X = x) = \binom{n}{k} p^x (1-p)^{n-x}$, $\mu = np$, $\sigma^2 = npq$

Poisson R.V

→ $P(X = x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$, $\mu = \lambda$, $\sigma^2 = \lambda$

Limiting Case of Binomial Variable

→ Let $X \sim Bin(n, p)$

Then, PMF of X approaches a Poisson PMF with parameter λ , as $n \to \infty$ & $p \to 0$

Such that $\lambda = np$ is constant

$$\lim_{\substack{n \to \infty \\ p \to 0 \\ \lambda = np}} P_X(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Probability Density Function

→ $f_X(x) = \lim_{\varepsilon \to 0} \dfrac{P(x \le X < x + \varepsilon)}{\varepsilon}$

→ Properties:

i) $f_X(x) \geq 0$, $\forall x \in R$

ii) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

iii) $\int_{-\infty}^{x} f_X(x)\,dx = F_X(x)$, $\forall x \in R$

iv) $P\{x_1 < X \le x_2\} = \int_{x_1}^{x_2} f_X(x)\,dx$

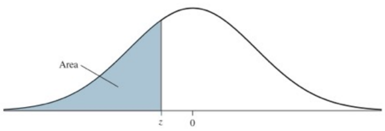
Gaussian / Normal RV

$\rightarrow \quad X \sim N(\mu, \sigma^2)$

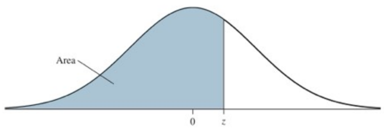
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

$$F_X(x) = F\left(\frac{x-\mu}{\sigma}\right)$$



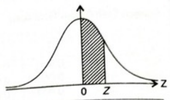
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.7 or less	.0001									
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7 or more	.9999									

A.12 — STATISTICAL TABLES

3. Areas under the Standard Normal Curve from 0 to z
(Normal Tables)



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1256	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1916	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2649
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4654	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4979	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4098	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Exponential R.V

$\rightarrow \quad f_X(x) = \lambda e^{-\lambda x}$

$$F_X(x) = 1 - e^{-\lambda x}$$

$$\mu = \frac{1}{\lambda}, \quad \sigma^2 = \frac{1}{\lambda^2}$$

Expectation of RV

→ Process of averaging a R.V

→ For Continuous RV

$$E(X) \triangleq \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

For Discrete RV

$$E(X) \triangleq \sum_{-\infty}^{\infty} x\, p_X(x)$$

Non Central / Raw Moments

→ $m_n = E[x^n] = \sum_i x_i^n\, p_X(x_i)$ (D.V)

$= \int x_i^n\, f_X(x_i)\, dx$ (C.V)

→ 0^{th} non-central moment $= 1$ (always)

$m_0 = E[x^0] = \sum_i p_X(x_i) = 1$

$= \int f_X(x_i)\, dx = 1$

→ 1st raw moment gives the average value of the R.V

Central Moments / Moments about Origin

→ $\mu_n = E[(x-\bar{x})^n] = \sum (x_i - \bar{x})\, p_X(x_i)$

$= \int (x-\bar{x})\, f_X(x)\, dx$

→ $E[x-\bar{x}] = E[x] - \bar{x}$

$= \bar{x} - \bar{x} = 0$

→ For $n = 3$,

$$E[(x-\bar{x})^3] = E[x^3 - 3x^2\bar{x} + 3x\bar{x}^2 - \bar{x}^3]$$

$$= E[x^3] - 3\bar{x}\,E[x^2] + 3\bar{x}^2\,E[x] - \bar{x}^3$$

$$= m_3 - 3m_1 m_2 + 3m_1^2(m_1) - m_1^3$$

$$= m_3 - 3m_1 m_2 + 2m_1^3$$

→ Properties :

i) Variance is non-negative

ii) Variance of a constant $= 0$

iii) Variance is not a linear operator

iv) $Var(\alpha X + \beta) = \alpha^2\, Var(X)$

v) $Var(X+Y) = Var(X) + Var(Y)$

vi) $Var(X) = E[X^2] - E^2[X] = m_2 - m_1^2$

$Var(X) = E[(X-\bar{X})^2]$

$Var(\alpha X + \beta) = E[\{(\alpha X+\beta) - (\alpha\bar{X}+\beta)\}^2]$

$= E[(\alpha X + \beta - \alpha\bar{X} - \beta)^2]$

$= \alpha^2\, E[(X-\bar{X})^2]$

Bernoulli R.V

→ $X \sim Ber(p)$

$p\{X=0\} = 1-p$

$p\{X=1\} = p$

→ $E[x] = \sum x_i P_x(x_i)$

$= 0(1-p) + 1(p) = p$

$E[x^2] = \sum x_i^2 P_x(x_i)$

$= 0^2(1-p) + 1^2(p) = p$

$Var(x) = p - p^2 = p(1-p)$

Binomial R.V

→ $X \sim Bin(n, p)$

$X = \sum_{k=1}^{n} x_k$

$E[x] = E\left[\sum_{n=1}^{k} x_k\right] = \sum_{k=1}^{n} E[x_k] = \sum_{k=1}^{n} p = np$

$Var[x] = Var\left[\sum_{n=1}^{k} x_k\right] = \sum_{k=1}^{n} Var[x_k] = npq$

Poisson RV

→ $X \sim Poi(\lambda)$

$P_x(x) = \sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!}$

$E[x] = \sum_k \frac{k\lambda^k e^{-\lambda}}{k!} = e^{-\lambda}\lambda \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!} = e^{-\lambda}\lambda . \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} = e^{-\lambda}.\lambda.e^{\lambda} = \lambda$

$E[x^2] = \sum_k \frac{k^2\lambda^k e^{-\lambda}}{k!} = e^{-\lambda}\sum_k \frac{k(k-1)\lambda^k}{k!} + e^{-\lambda}\sum \frac{k\lambda^k}{k!} = e^{-\lambda}\lambda^2 \sum_{j=0}^{\infty}\frac{\lambda^j}{j} + \lambda = \lambda^2 + \lambda$

$var[x] = E[x^2] - E^2[x] = \lambda^2 + \lambda - \lambda^2 = \lambda$

Normal RV

→ $f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$E[x] = \int_{-\infty}^{\infty} x \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

$z = x - \mu \Rightarrow dz = dx$

$E[x] = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty}(z+\mu)\, e^{\frac{-z^2}{2\sigma^2}} dz = \cancel{\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} z e^{\frac{-z^2}{2\sigma^2}} dz} \xrightarrow{} 0 + \frac{\mu}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} e^{\frac{-z^2}{2\sigma^2}} dz = \mu(1) = \mu$

odd func^n

$Var(x) = E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

Unit - 2

→ Chebyshev's Inequality

→ The probability that a RV differs from its mean by K standard deviations or more is never greater than $\frac{1}{K^2}$

$$P(|X-\mu| \geq K\sigma) \leq \frac{1}{K^2} \qquad (K>1)$$

→ Chebyshev Bound is generally much larger than actual probability. Only should be used when distribution of RV is unknown.

Moment Generating Function (MGF)

→ All the moments in 1 shot

$M_x(t) = E(e^{tx})$, for $t \in R$

→ D.R.V ⇒ $M_x(t) = E(e^{tx}) = \sum_{x=-\infty}^{\infty} e^{tx} p_x(x)$

C.R.V ⇒ $M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f_x(x)\,dx$

$$e^{tx} = 1 + tx + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \dots$$

$$M_x(t) = E[e^{tx}] = E\left[1 + tx + \frac{t^2x^2}{2!} + \dots\right]$$

$$= 1 + tE[x] + \frac{t^2}{2!}E[x^2] + \dots = \sum_{K=0}^{\infty} \frac{t^K E(x^K)}{K!}$$

$$\left.\frac{dE[e^{tx}]}{dt}\right|_{t=0} = E[x]$$

$$\left.\frac{d^2E[e^{tx}]}{dt^2}\right|_{t=0} = E[x^2]$$

$$\Rightarrow \left.\frac{d^nE[e^{tx}]}{dt^n}\right|_{t=0} = E[x^n]$$

MGF for Poisson

$$M_x(t) = E[e^{tx}] = \sum_{K=0}^{\infty} e^{tK} \frac{e^{-\lambda}\lambda^K}{K!} = e^{-\lambda} \sum \frac{(e^t\lambda)^K}{K!} = e^{-\lambda} e^{e^t\lambda}$$

$$= e^{(e^t-1)\lambda}$$

$$\left.\frac{dE[e^{tx}]}{dt}\right|_{t=0} = \left(e^{(e^t-1)\lambda} \times (e^t)\lambda\right)_{t=0} = \lambda$$

$$\frac{d^2E[e^{tx}]}{dt} = \left((e^t)\lambda e^{(e^t-1)\lambda}\cdot(e^t)\lambda + e^{(e^t-1)\lambda}\cdot(e^t)\lambda\right)_{t=0} = \lambda^2+\lambda$$

$$Var(x) = E[x^2] - E^2[x] = \lambda^2+\lambda-\lambda^2 = \lambda$$

MGF of Binomial R.V

$$P_X(X=K) = \binom{n}{K} p^K (1-p)^{n-K}$$

$$M_X(t) = E[e^{tX}] = \sum_{K=0}^{\infty} e^{tK} \binom{N}{K} p^K (1-p)^{N-K}$$

$$= \sum_{K=0}^{\infty} \binom{N}{K} (e^t p)^K (1-p)^{N-K}$$

$$= (e^t p + 1 - p)^N$$

$$\frac{d\,E[e^{tX}]}{dt}\Big|_{t=0} = \left(N(e^t p + 1 - p)^{N-1}(e^t p)\right)_{t=0} = N(p+1-p)(p) = Np$$

$$\frac{d^2 E[e^{tX}]}{dt^2} = \left(N(N-1)(e^t p + 1 - p)^{N-2}(e^t p)^2 + N(e^t p + 1 - p)^{N-1}(e^t p)\right)_{t=0}$$

$$= N(N-1)(1)(p)^2 + N(p+1-p)(p)$$

$$= N^2 p^2 - Np^2 + Np$$

$$Var(X) = E[X^2] - E^2[X]$$

$$= \cancel{N^2 p^2} - Np^2 + Np - \cancel{N^2 p^2}$$

$$= N(p - p^2)$$

$$= N(p)(1-p)$$

MGF of Normal R.V

$$\rightarrow \quad f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f_X(x)\,dx = E[e^{tX}]$$

$$= \int_{-\infty}^{\infty} \frac{e^{tx - x^2/2}}{\sqrt{2\pi}}\,dx = \frac{1}{\sqrt{2\pi}} \int e^{\frac{-(x^2 - 2tx)}{2}}\,dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{-(x^2 - 2tx + t^2 - t^2)}{2}}\,dx = \frac{e^{+t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x - t)^2}{2}}\,dx = e^{t^2/2}(1)$$

→ Area Under Gausian Curve = 1

→ Consider a R.V $U \sim N(0,1)$

Consider a transformation $X = \mu + \sigma U$

$$M_X(t) = E[e^{tX}]$$

$$= E[e^{t(\mu + \sigma U)}] = E[e^{t\mu + t\sigma u}] = e^{t\mu} E[e^{t\sigma u}] = e^{t\mu} M_U[t\sigma]$$

$$= e^{t\mu} e^{(t\sigma)^2/2} = e^{\left(\frac{(t\sigma)^2}{2} + \mu t\right)}$$

$$M_X'(t)\Big|_{t=0} = (\mu + t\sigma^2) e^{\left(\frac{(t\sigma)^2}{2} + \mu t\right)} = \mu$$

$$M_X''(t)\Big|_{t=0} = \sigma^2 e^{\left(\frac{(t\sigma)^2}{2} + \mu t\right)} + (\mu + t\sigma^2)^2 e^{\left(\frac{(t\sigma)^2}{2} + \mu t\right)} = \sigma^2 + \mu^2$$

$$Var(X) = M_X''(t) - M_X'^2(t) = \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$

Characteristics Function

→ $\phi_x(\omega) = E(e^{j\omega x})$ for $\omega \in R$, $j = \sqrt{-1}$

→ For D.R.V, $\phi_x(\omega) = E(e^{j\omega x}) = \sum_{x=-\infty}^{\infty} e^{j\omega x} p_x(x)$

$\phi_x(0) = 1$ ⇒ Test whether given func^n of ω is true Charac funct^n of a R.V

→ For C.R.V, $\phi_x(\omega) = E(e^{j\omega x}) = \int_{-\infty}^{\infty} e^{j\omega x} p_x(x)$

→ $\phi_x(\omega) = E(e^{j\omega x}) = 1 + j\omega E(X) - \frac{\omega^2 E(x^2)}{2!} + \dots$

$\left.\frac{d\phi_x(\omega)}{d\omega}\right|_{\omega=0} = jE(X)$; $\left.\frac{d^2\phi_x(\omega)}{d\omega^2}\right|_{\omega=0} = j^2 E(x^2)$; $\frac{d^n\phi_x(\omega)}{d\omega^n} = j^n E(x^n)$

Transformation of Single R.V

→ $f_Y(y) = \dfrac{f_X(x)}{\left|\frac{dy}{dx}\right|}$

Conditional Distribution Function

→ Similar to $P(A \mid B) = \dfrac{P\{A \cap B\}}{P[B]}$

We have $F_X(x \mid B) = \dfrac{P\{(X \leq x) \cap B\}}{P[B]}$

$= \dfrac{F_X(x) \cap F_X(b)}{F_X(b)}$

$= \begin{cases} \dfrac{F_X(x)}{F_X(b)}, & x < b \\ 1, & x \geq b \end{cases}$

$$f_X(x \mid (X \leq b)) = \begin{cases} \dfrac{f_X(x)}{\int_{-\infty}^{b} f_X(x)\,dx}, & x < b \\ 0, & x \geq b \end{cases}$$

Joint CDF

→ $F_{XY}(x,y) = P(X \leq x, Y \leq y)$

$$\lim_{y\to\infty} F_{XY}(x,y) = \lim_{y\to\infty} P(X \leq x, Y \leq y) = P(X \leq x) = F_X(x)$$

$$\lim_{x\to\infty} F_{XY}(x,y) = \lim_{x\to\infty} P(X \leq x, Y \leq y) = P(Y \leq y) = F_Y(y)$$

Marginal CDFs

→ Properties :

1) Monotonically non-decreasing function
2) $0 \leq F_{XY}(x,y) \leq 1$ for $-\infty < x, y < \infty$
3) $x_1 \leq x_2$ & $y_1 \leq y_2$

   $F_{XY}(x_1, y_1) \leq F_{XY}(x_1, y_2)$ (or) $F_{XY}(x_2, y_1) \leq F_{XY}(x_2, y_2)$
4) $F_{XY}(-\infty, \infty) = 0$

   $F_{XY}(-\infty, y) = 0$

   $F_{XY}(x, -\infty) = 0$

   $F_{XY}(\infty, \infty) = 1$
5) $P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1) \geq 0$

Joint PMF

→ $P_{X_1,\dots,X_N}(x_1,\dots x_N) = P(X_1 = x_1, \dots, X_N = x_N)$

→ $$\sum_{y=-\infty}^{\infty} P_{XY}(x,y) = \sum_{y=-\infty}^{\infty} P(X=x, Y=y) = P(X=x) = P_X(x)$$

$$\sum_{x=-\infty}^{\infty} P_{XY}(x,y) = \sum_{x=-\infty}^{\infty} P(X=x, Y=y) = P(Y=y) = P_Y(y)$$

Marginal PMFs

→ $\sum_x \sum_y P_{XY}(x,y) = 1$

→ $\sum_{x\leq a} \sum_{y \leq b} P_{XY}(x,y) = F_{XY}(a,b)$

→ $P_{XY}(x,y) = \dfrac{\partial^2}{\partial x \partial y} F_{XY}(x,y)$

Joint PDF

→ $P(a \leq X \leq b, c \leq Y \leq d) = \int_{x=a}^{b} \int_{y=c}^{d} f_{XY}(x,y)\, dx\, dy$

→ $\int_{y=-\infty}^{\infty} f_{XY}(x,y)\, dy = f_X(x)$

$\int_{x=-\infty}^{\infty} f_{XY}(x,y)\, dx = f_Y(y)$

Marginal PDFs

→ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx\, dy = 1$ → $f_{XY}(x,y) = \dfrac{\partial^2}{\partial x \partial y} F_{XY}(x,y)$

Correlation (R_{XY})

→ $R_{XY} = m_{11} = E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{XY}(x,y)\,dx\,dy$

If $R_{XY} = 0 \Rightarrow$ orthogonal

$$E[g(X,Y)] = E[(X+Y)^2] = E[X^2+Y^2+2XY] = E[X^2] + E[Y^2] + E[2XY]$$

→ Statistical independance

$P(A \cap B) = 0$ / $P(A) \cdot P(B)$

2 RV are SI if Joint density func^n = product of individual density func^n

Statistical Impedance

→ $P(A \cap B) = P(A)\,P(B)$ ⇒ if 2 events are independant of each other

→ $P(X \leq x, Y \leq y) = P(X \leq x) \cdot P(Y \leq y)$

→ $F_{X_1,\dots X_N}(x_1,\dots,x_n) = F_{X_1}(x_1) \dots F_{X_N}(x_N) = \prod_{n=1}^{N} F_{X_n}(x_n)$

$P_{X_1,\dots X_N}(x_1,\dots,x_n) = P_{X_1}(x_1) \dots P_{X_N}(x_N) = \prod_{n=1}^{N} P_{X_n}(x_n)$

→ $f_{XY}(x,y) = \dfrac{\partial^2 F_{XY}(x,y)}{\partial x\, \partial y} = \dfrac{\partial^2 F_X(x)\, F_Y(y)}{\partial x\, \partial y} = \dfrac{\partial F_X(x)}{\partial x} \cdot \dfrac{\partial F_Y(y)}{\partial y} = f_X(x) \cdot f_Y(y)$

Conditional Distribution Function

→ $F_{X|B}(x) = P\{x \leq X | B\}$

$= \dfrac{P\{X \leq x \cap B\}}{P(B)}$

$F_{X|(a<X\leq b)}(x) = \dfrac{P\{X \leq x \cap (a < X \leq b)\}}{P\{a < X \leq b\}} = \dfrac{F_X(x) - F_X(a)}{F_X(b) - F_X(a)}$

Conditional Density function

→ $f_{X|(a<X\leq b)}(x) = \dfrac{f_X(x)}{F_X(b) - F_X(a)}$

Vector R.V

→ For linear transformation of vector R.V of the form $Y = AX + b$

X & Y are related by $\mu_Y = A\mu_X + b$

→ Correlation matrices ⇒ $R_{YY} = AR_{XX}A^T + A\mu_X b^T + b\mu_X^T A^T + bb^T$

→ Covariance matrices ⇒ $C_{YY} = AC_{XX}A^T$

Central Limit Theorem

→ Let $x_1, x_2, \dots$ be a sequence of independant & identically distribution R.V, each having finite μ & σ^2

Also let $S_n = \dfrac{x_1 + x_2 + \dots + x_n}{\sqrt{n\sigma^2}}$

Then $\lim_{n\to\infty} S_n \sim N(\mu, \sigma^2)$

$$\lim_{n\to\infty} P(S_n \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

Random Processes

→ It is a collection of time functions or signals corresponding to various outcomes of experiment. For each outcome s, a time function $x(t,s)$ is assigned. The family of all such functions denoted $X(t,s)$ is called random process