NAS UNIT - 3

Coupled Circuits

→ **Inductance** – Change in flux linking a coil due to change in current through the coil

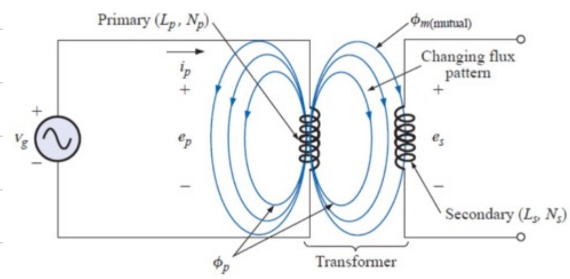
$$L_p \propto \frac{d\phi_p}{di_p} \Rightarrow \boxed{L_p = N_p \frac{d\phi_p}{di_p}}$$

→ By Faraday's Law, change in flux in one coil induces voltage across that coil

$$\boxed{e_p = N_p \frac{d\phi_p}{dt}} \Rightarrow e_p = \left(L_p \frac{di_p}{d\phi_p}\right)\frac{d\phi_p}{dt} \Rightarrow \boxed{e_p = L_p \frac{di_p}{dt}}$$

→ In Coupled circuits, we have 2 windings, in which current flows

primary	secondary
$e_p = N_p \frac{d\phi_p}{dt}$	$e_s = N_s \frac{d\phi_s}{dt}$
$e_p = L_p \frac{di_p}{dt}$	$e_s = L_s \frac{di_s}{dt}$



→ We can create a relation, $\boxed{\frac{\phi_s}{\phi_K} = K}$

K : coefficient of coupling

K = 1 for ferromagnetic cores

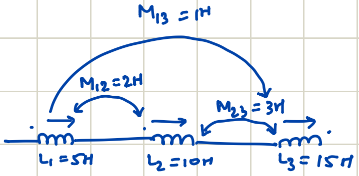
$$e_s = N_s K \frac{d\phi_p}{dt}$$

$$\boxed{M = N_s \frac{d\phi_p}{di_p}} \ (\text{or}) \ \boxed{N_p \frac{d\phi_p}{di_s}} \longrightarrow$$ Mutual Inductance

↳ Measure of change in flux linking one coil due to change in current flow in other coil

$$M = K\sqrt{L_p L_s}$$

Q.



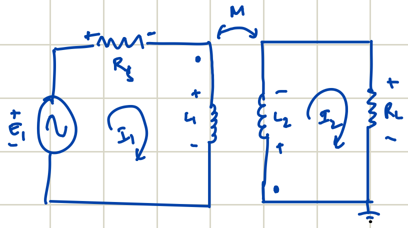
A. $L_1 + M_{12} - M_{13} = 5 + 2 - 1 = 6H$

$L_2 + M_{12} - M_{23} = 10 + 2 - 3 = 9H$

$L_3 - M_{13} - M_{23} = 15 - 1 - 3 = 11H$

Total Inductance $= 6 + 9 + 11 = 26H$

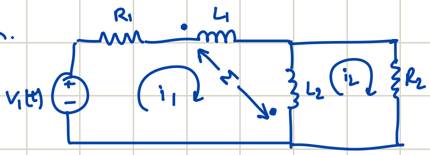
Q.



$$E_1 - L_1\frac{di_1}{dt} - M\frac{di_2}{dt} - R_s i_1 = 0$$

$$-L_2\frac{di_2}{dt} - R_L i_2 - M\frac{di_1}{dt} = 0$$

Q.



$$V_1(t) - i_1R_1 - L_1\frac{di_1}{dt} - L_2\frac{d(i_1-i_2)}{dt} + M\frac{d(i_1-i_2)}{dt} + M\frac{di_1}{dt} = 0$$

$$R_1 i_1(t) + (L_1 + L_2 - 2M)\frac{di_1}{dt} - (L_2 - M)\frac{di_2}{dt} = v_1(t)$$

$$R_1 i_1(t) + jX_{LA}\, i_1(t) - jX_{LB}\, i_2(t) = v_1(t)$$

$X_{LA} = \omega(L_1 + L_2 - 2M)$ $\quad X_{LB} = \omega(L_2 - M)$

$$L_2\frac{di_2}{dt} + i_2R_2 - L_2\frac{di_1}{dt} + M\frac{di_1}{dt}$$

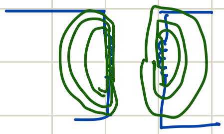
$$(-L_2 + M)\frac{di_1}{dt} + R_2 i_2 + L_2\frac{di_2}{dt}$$

$$-jX_{LC}\, i_1(t) + R_2 i_2 + jX_{LD}\, i_2(t)$$

$X_{LC} = \omega(L_2 - M)$

$X_{LD} = \omega L_2$

Q.



$L_p = 200mH$ $\quad$ $L_s = 800mH$

$N_p = 50$ $\quad$ $N_s = 100$

$K = 0.6$ $\quad$ $\frac{d\phi}{dt} = 450mWb/s$

a) $M = ?$

b) $e_p = ?$

c) $e_s = ?$

d) e_p & e_s if $\frac{di_p}{dt} = 0.2 A/ms$

A. a) $M = K\sqrt{L_pL_s} = 0.6\sqrt{200 \times 800 \times 10^{-3} \times 10^{-3}} = 0.24 H$

b) $e_p = N_p \frac{d\phi}{dt} = 50 \times 0.45 = 22.5V$

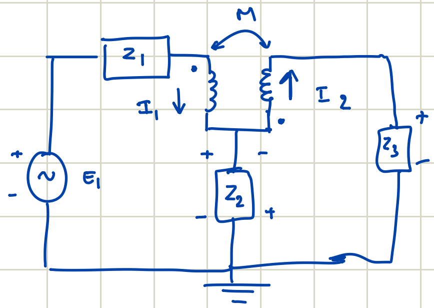
c) $e_s = N_s \frac{d\phi}{dt} = 100 \times 0.45 = 45V$

d) $\frac{di_p}{dt} = 0.2 A/ms \approx 200 A/s$

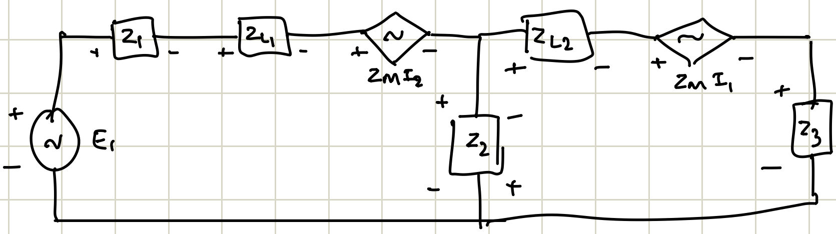
$e_p = L_p \frac{di_p}{dt} = 200 \times 10^{-3} \times 200 = 40 V$

$e_s = M \frac{di_p}{dt} = 0.24 \times 200 = 48 V$

Q.



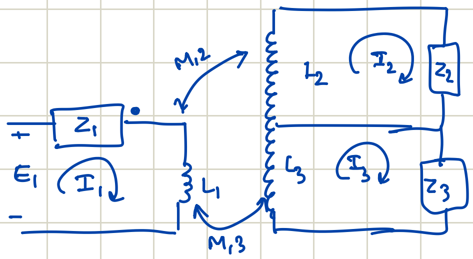
A.



$E_1 - I_1Z_1 - I_1Z_{L_1} - Z_MI_2 - Z_2I_1 + Z_2I_2 = 0 \Rightarrow (Z_1 + Z_2 + Z_{L_1})I_1 + (Z_M - Z_2)I_2 = E_1$

$Z_{L_2}I_2 + Z_2I_2 - Z_2I_1 + Z_3I_2 + Z_MI_1 = 0 \Rightarrow (Z_M - Z_2)I_1 + (Z_2 + Z_3 + Z_{L_2})I_2 = 0$

Q.



A. $$I_1 Z_1 + L_1 \frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt} - M_{13}\frac{dI_3}{dt} = E_1$$

$$L_2 \frac{dI_2}{dt} + Z_2 I_2 - M_{12}\frac{dI_1}{dt} = 0$$

$$L_3 \frac{dI_3}{dt} + Z_3 I_3 - M_{13}\frac{dI_1}{dt} = 0$$

Q. $I = 5A$ $\quad \phi_{11} = 0.2\,mWb$ $\quad \phi_{12} = 0.4\,mWb$

$N_1 = 500$ $\quad N_2 = 1500$

$L_1, L_2, M, K = ?$

A. $$\phi_1 = \phi_{11} + \phi_{12} = 0.2 + 0.4 = 0.6\ mWb$$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{500 \times 0.6}{5} \times 10^{-3} = 0.06\ H$$

$$\phi_s = K\phi_p$$

$$\phi_{12} = K\phi_1 \Rightarrow K = \frac{0.4}{0.6} = 0.67 = \frac{2}{3}$$

$$M = N_2 \frac{\phi_{12}}{I_1} = 1500 \times \frac{0.4 \times 10^{-3}}{5} = 0.12\,H$$

$$M = K\sqrt{L_1 L_2}$$

$$L_2 = \left(\frac{M}{K}\right)^2 \times \frac{1}{L_1} = \left(\frac{\overset{0.18}{\cancel{0.12}}}{\cancel{\frac{2}{3}}}\right)^2 \times \frac{1}{0.06} = 0.18 \times 3 = 0.54\,H$$

Q. $L_1 = 50\,mH$ $\quad L_2 = 200\,mH$

$K = 0.5$ $\quad N_2 = 1000$

$I_1 = 5\sin 400t$

Find $e_2 = ?$ $\quad \phi_1 = ?$

A. $$M = K\sqrt{L_1 L_2} = 0.5\sqrt{10000 \times 10^{-6}} = 0.05\ H$$

$$e_2 = M\frac{di_1}{dt} = 0.05\frac{d}{dt}(5\sin 400t)$$

$$= 0.05 \times 5\cos 400t \times 400$$

$$= 100\cos(400t)$$

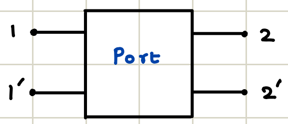
$$= 100\sin\left(400t + \frac{\pi}{2}\right)$$

$$M = N_2\frac{\phi_s}{|i_1|} = \frac{N_2 K\phi_1}{|i_1|} = \frac{0.05 \times 5}{1000 \times 0.5} = 5 \times 10^{-4}\ Wb$$

System Analysis

→ Ports in a network

→ Sometimes a circuit can't be described using branch elements, So we use port based parameters which relate I & V across different ports under special conditions applied on one or more ports



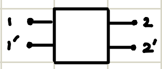
→ Port consists of terminals
& independant sources / load can be connected to a port

→ Port parameters : I & V across the ports are related using these parameters
ex: z, y, Hybrid, Transmission

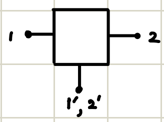
→ Configurations

→ Includes development of 2, 3, multi port models

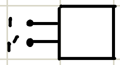
→ Some common configurations are :

1) 

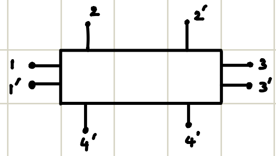
Two port network

2) 

Terminals 1' & 2' are common

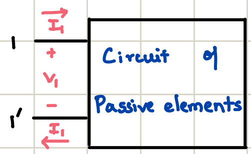
3) 

Single Port

4) 

Multiport

→ One port network



Nodal eq in terms of passive elements is given by

$$\begin{bmatrix} i_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

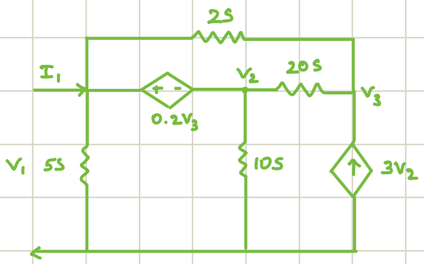
$$\Delta y = \begin{vmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{vmatrix}$$

$$\Delta_{11} = \begin{vmatrix} Y_{22} & \cdots & Y_{2N} \\ \vdots & \ddots & \vdots \\ Y_{N2} & \cdots & Y_{NN} \end{vmatrix}$$

$$V_1 = \frac{i_1 \Delta_{11}}{\Delta y}$$

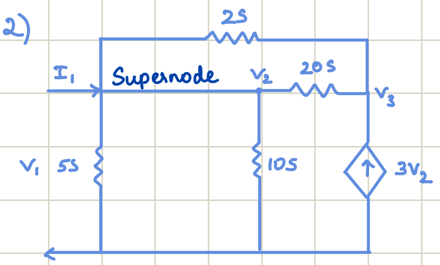
$$y_{11} = \frac{i_1}{V_1} = \frac{\Delta y}{\Delta_{11}} \Rightarrow i_1 = y_{11} V_1$$

Q. Find input impedance & input admittance



A. 1) $V_1 - V_2 = 0.2V_3 \Rightarrow 10V_1 - 10V_2 - 2V_3 = 0$

2)



3) KCL at supernode

$7V_1 + 30V_2 - 22V_3 = I_1$

4) KCL for node 3

$-2V_1 - 23V_2 + 22V_3 = 0$

5) Find Z_{11} & Y_{11} of the given one port network

$$\begin{bmatrix} 7 & 30 & -22 \\ -2 & -23 & 22 \\ 10 & -10 & -2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix}$$

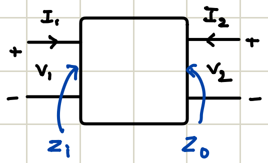
$$V_1 = \frac{I_1 \overbrace{\begin{vmatrix} -23 & 22 \\ -10 & -2 \end{vmatrix}}^{\Delta_{11}}}{\underbrace{\begin{vmatrix} 7 & 30 & -22 \\ -2 & -23 & 22 \\ 10 & -10 & -2 \end{vmatrix}}_{\Delta y}} = I_1 x$$

$$Z_{11} = \frac{V_1}{I_1} = 0.0936\ \Omega$$

$$y_{11} = \frac{1}{Z_{11}} = 10.68\ S$$

→ Two port network

→ 2 seperate ports for input & output



→ Current entering one terminal of a pair leaves the other terminal in the pair

→ Impedance parameters

Z_i : input impedance b/w Terminals 1 & 1'

Z_o : output impedance b/w Terminals 2 & 2'

$Z_i = \dfrac{E_i}{I_i}$ and $Z_o = \dfrac{E_o}{I_o}$

→ Open Circuit Analysis

If $I_2 = 0 \Rightarrow$ $Z_{11} = \left.\dfrac{E_1}{I_1}\right|_{I_2=0}$

$Z_{21} = \left.\dfrac{E_2}{I_1}\right|_{I_2=0}$

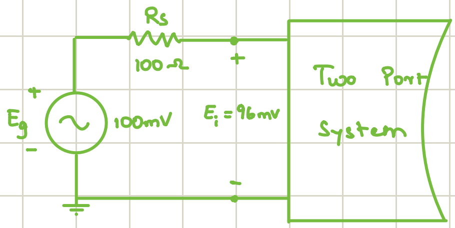
If $I_1 = 0 \Rightarrow$ $Z_{12} = \left.\dfrac{E_1}{I_2}\right|_{I_1=0}$

$Z_{22} = \left.\dfrac{E_2}{I_2}\right|_{I_2=0}$

Symmetry Condition : $Z_{11} = Z_{22}$

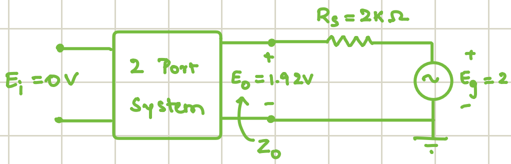
Reciprocal Condition : $Z_{12} = Z_{21}$

Q. Determine Z_i if input impedance is purely resistive



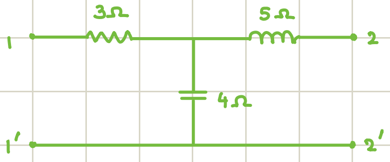
A. $Z_i = R_i = \dfrac{E_i}{I_i} = \dfrac{E_i}{\frac{V_{Rs}}{R_s}} = \dfrac{96m}{\frac{100-96m}{100}} = \dfrac{96 \times 100}{4} = 2.4\ k\Omega$

Q. Determine Z_o if output impedance is purely resistive



A. $Z_o = \dfrac{E_o}{I_o} = \dfrac{E_o}{\frac{V_{Rs}}{R_s}} = \dfrac{1.92}{\frac{2-1.92}{2000}} = 48\ k\Omega$

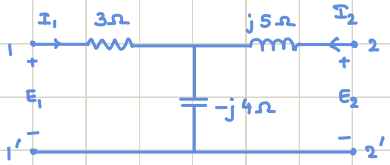
Q. Determine impedance parameters



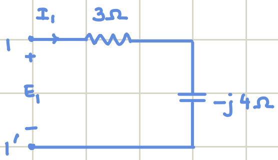
symmetry condition $z_{11} = z_{22}$

reciprocal condition $z_{12} = z_{21}$

A.



If $I_2 = 0$

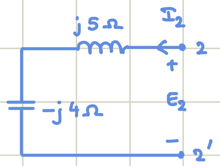


By KVL,

$$z_{11} = \left.\frac{E_1}{I_1}\right|_{I_2=0} = 3 - j4\,\Omega$$

$$z_{21} = \left.\frac{E_2}{I_1}\right|_{I_2=0} = -j4\,\Omega$$

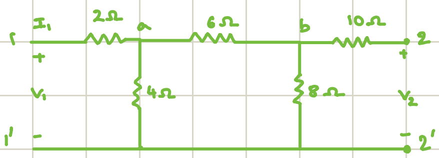
If $I_1 = 0$



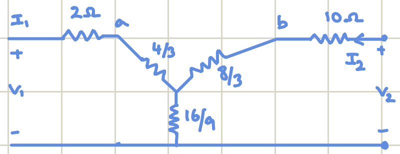
$$z_{12} = \left.\frac{E_1}{I_2}\right|_{I_1=0} = \frac{-j4 I_2}{I_2} = -j4\,\Omega$$

$$z_{22} = \left.\frac{E_2}{I_2}\right|_{I_1=0} = j1\,\Omega$$

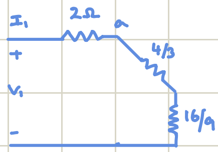
Q.



A.



If $I_2 = 0$

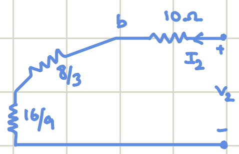


By KVL,

$$z_{11} = 2 + \frac{4}{3} + \frac{16}{9} = 5.11\,\Omega$$

$$z_{21} = \frac{16}{9} = 1.77\,\Omega$$

If $I_1 = 0$

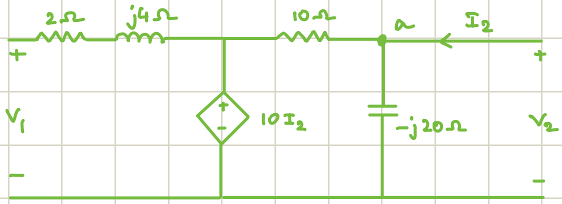


By KVL,

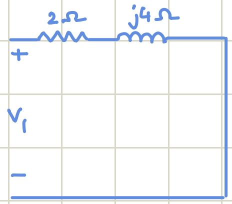
$$z_{22} = 10 + \frac{8}{3} = 12.67$$

$$z_{12} = \frac{16}{9} = 1.77\,\Omega$$

Q. Find z-parameters in the circuit



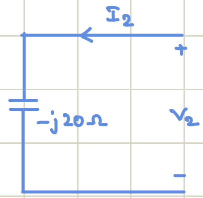
A. If $I_2 = 0$



$Z_{11} = (2 + j4)\ \Omega$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0}{I_1} = 0$

If $I_1 = 0$



$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{10I_2}{I_2} = 10\ \Omega$

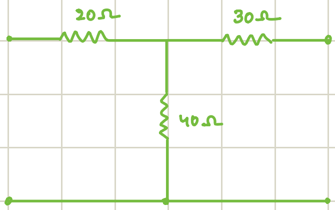
KCL at a,

$\frac{V_2 - 10I_2}{10} + \frac{V_2}{-j20} = I_2$

$V_2 = \frac{2I_2}{\left(\frac{1}{10} - \frac{1}{j20}\right)} \Rightarrow \frac{V_2}{I_2} = (16 - j8)\ \Omega$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = (16 - j8)\ \Omega$

Q.



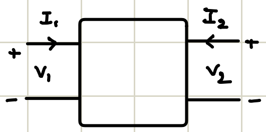
A. $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 20 + 40 = 60\ \Omega$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 40\ \Omega$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 40\ \Omega$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 30 + 40 = 70\ \Omega$

→ Admittance (Y) Parameters



Using 2 port Y parameters,

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

input admittance

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

transfer admittance (port 1 to port 2)

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

transfer admittance (port 2 to port 1)

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

output admittance

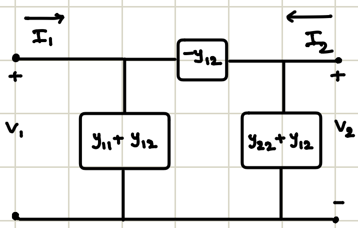
So,

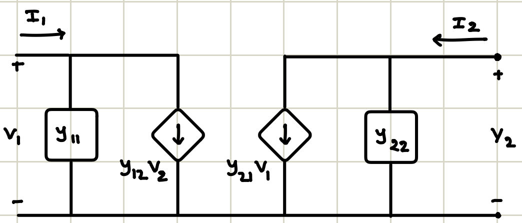
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^{-1}$$

→ For a network that is linear & no dependant sources, $y_{12} = y_{21}$

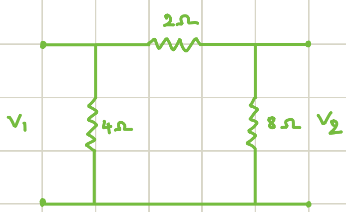
→ A reciprocal network ($y_{12} = y_{21}$) can be modelled with π-equivalent

else, a more general equivalent circuit is used





Q.



Determine y-parameters

A.

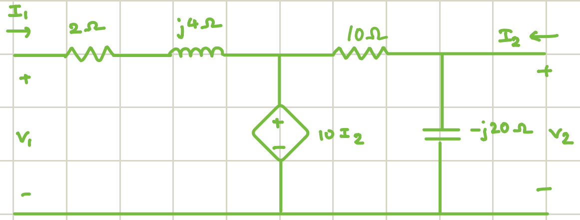
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{I_1}{(4^{-1}+2^{-1})^{-1} I_1} = \frac{1}{4}+\frac{1}{2} = 0.75\,S$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{\frac{-4I_1}{(4+2)}}{(4^{-1}+2^{-1})^{-1} I_1} = -0.5\,S$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{\frac{-8}{8+2}I_2}{(8^{-1}+2^{-1})^{-1} I_2} = -0.5\,S$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{I_2}{(8^{-1}+2^{-1})^{-1} I_2} = 0.625\,S$$

Q



A. $V_2 = 0$,

$$y_{11} = \frac{I_1}{V_1} = \frac{I_1}{I_1(2+4j)} = \frac{1}{2+4j}$$

$$\left(-V_1 + I_1(2+4j) + 10I_2 = 0 \quad V_1 = I_1(2+4j) \right)$$

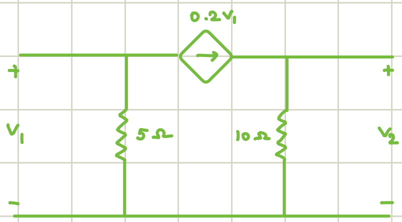
$$y_{21} = \frac{I_2}{V_1} = 0$$

$V_1 = 0$

$$y_{22} = \frac{I_2}{V_2} = \left(\frac{1}{10} - \frac{1}{j20}\right) \times \frac{1}{2} = 0.05 + j\,0.025$$

$$y_{12} = \frac{I_1}{V_2} = \frac{\frac{-10 I_2}{2+j4}}{V_2} = \frac{-10}{2+j4} \times (0.05 + j\,0.025) = -0.1 + 0.075i$$

Q.



A. $V_2 = 0$,

$$y_{11} = \frac{I_1}{V_1} = 0.4 \text{ S} \qquad \left(-I_1 + \frac{V_1}{5} + 0.2V_1 = 0 \right)$$

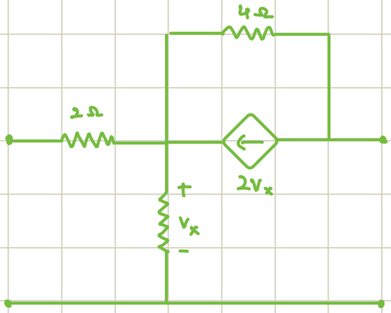
$$y_{21} = \frac{I_2}{V_1} = -0.2 \text{ S} \qquad \left(I_2 = -0.2V_1 \right)$$

$V_1 = 0$

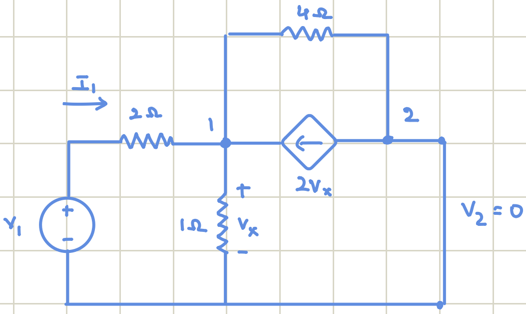
$$y_{12} = \frac{I_1}{V_2} = \frac{0.2V_1}{V_2} = \frac{0}{V_2} = 0$$

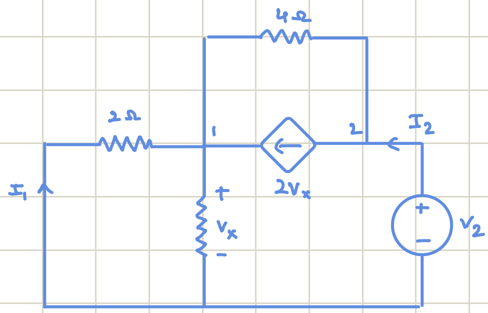
$$y_{22} = \frac{I_2}{V_2} = 0.1 \text{ S} \qquad \left(\frac{V_2}{10} = I_2 \right)$$

Q.



A.





At 1, $\frac{V_x}{1} + \frac{V_x}{4} = 2V_x + \frac{V_1 - V_x}{2} \Rightarrow 2V_1 = -V_x$

Also, $I_1 = \frac{V_1 - V_x}{2} = \frac{V_1 + 2V_1}{2} = 1.5\,V_1$

$y_{11} = 1.5\ S$

And, $I_2 + \frac{V_x}{4} = 2V_x \Rightarrow I_2 = 1.75\,V_x$

$= -3.5\,V_1$

$y_{21} = \frac{I_2}{V_1} = -3.5\ S$

At 2, $\frac{V_2 - V_x}{4} = I_2 - 2V_x$

At 1, $\frac{V_x}{2} + \frac{V_x}{1} + \frac{V_x - V_2}{4} = 2V_x$

$\hookrightarrow V_2 = -V_x$

$I_2 = -1.5\,V_2$

$y_{22} = -1.5\ S$

and, $I_1 = -\frac{V_x}{2} \Rightarrow \frac{I_1}{V_2} = 0.5S = y_{12}$

→ Hybrid Parameters

→ Mixture of I & V variables on 1 side

$E_1 = h_{11} I_1 + h_{12} E_2$

$I_2 = h_{21} I_1 + h_{22} E_2$

$$\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$$

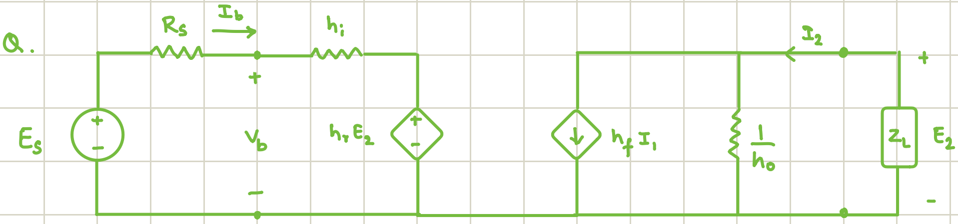
→ If $E_2 = 0$, $h_{11} = \left.\frac{E_1}{I_1}\right|_{E_2=0}$ (Short circuit input impedance)

$h_{21} = \left.\frac{I_2}{I_1}\right|_{E_2=0}$ (Short circuit current gain)

If $E_1 = 0$, $h_{12} = \left.\frac{I_1}{I_2}\right|_{E_1=0}$ (Open reverse voltage)

$h_{22} = \left.\frac{E_2}{V_2}\right|_{E_1=0}$ (Open circuit output impedance)

Q.



$h_{11} = h_i$

$h_{12} = h_f$

$h_{21} = h_r$

$h_{22} = h_o$

$E_1 = h_{11} I_1 + h_{12} E_2$

$I_2 = h_{21} I_1 + h_{22} E_2$

For the hybrid equivalent circuit, derive the following

i) Current ratio (gain) $A_i = \frac{I_2}{I_1}$

ii) Votage ratio (gain) $A_v = \frac{E_2}{E_1}$

A. i)

$$I_2 = \frac{\frac{1}{h_o} \cdot h_f I_1}{\frac{1}{h_o} + Z_L}$$

$$\frac{I_2}{I_1} = \frac{h_f}{1 + h_o Z_L} = A_i$$

ii) KVL on input side,

$$E_1 = h_i I_1 + h_f E_2$$

$$\frac{E_1 - h_f E_2}{h_i} = I_1 \longrightarrow (1)$$

$$I_2 = h_f I_1 + h_o E_2$$

$$-\frac{E_2}{Z_L} = h_f I_1 + h_o E_2 \longrightarrow (2)$$

Put (1) in (2)

$$-\frac{E_2}{Z_L} = h_f \left(\frac{E_1 - h_f E_2}{h_i} \right) + h_o E_2$$

$$-h_i E_2 = Z_L h_f E_1 - Z_L h_f E_2 + Z_L h_o h_i E_2$$

$$E_2 (Z_L h_f - Z_L h_o h_i - h_i) = Z_L h_f E_1$$

$$\frac{E_2}{E_1} = \frac{Z_L h_f}{Z_L h_f - Z_L h_o h_i - h_i} = \frac{Z_L h_f}{Z_L h_f - h_i (Z_L h_o + 1)} = A_V$$

Q. Suppose $h_i = 1 k\Omega$, $h_r = 4 \times 10^4$, $h_f = 50$, $h_o = 25 \mu S$ & $Z_L = 2 k\Omega$

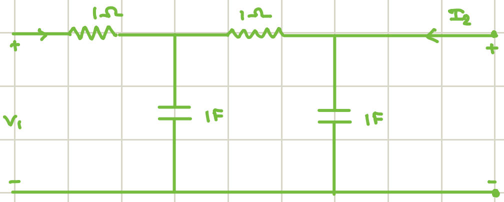
Calculare A_i & A_v

A.

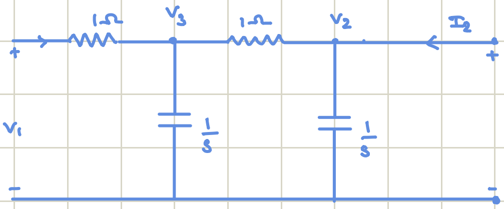
$$A_i = \frac{h_f}{1 + h_o Z_L} = \frac{50}{1 + (25 \times 10^{-6} \times 2 \times 10^3)} = 47.62$$

$$A_v = \frac{Z_L h_f}{Z_L h_f - h_i (Z_L h_o + 1)} = \frac{2000 \times 50}{(2000 \times 50) - 1000(2 \times 10^3 \times 25 \times 10^{-6} + 1)} = -99$$

Q. Find y - parameters of network system



A.



$$I_1 = \frac{V_1 - V_3}{1} \longrightarrow ①$$

at node 3, $\frac{V_3}{1/s} + \frac{V_3 - V_2}{1} = I_1 \Rightarrow (s+1)V_3 - V_2 = I_1 \longrightarrow ②$

at node 2, $\frac{V_2 - V_3}{1} + \frac{V_2}{1/s} = I_2 \Rightarrow (s+1)V_2 - V_3 = I_2 \longrightarrow ③$

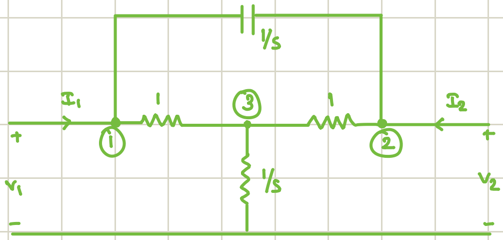
$$① = ② \Rightarrow V_1 - V_3 = (s+1)V_3 - V_2 \Rightarrow V_3 = \frac{V_1}{s+2} + \frac{V_2}{s+2}$$

$$I_1 = \frac{(s+1)(V_1+V_2)}{s+2} - V_2 = \left(\frac{s+1}{s+2}\right)V_1 - \frac{V_2}{s+2}$$

$$I_2 = (s+1)V_2 - \left(\frac{V_1+V_2}{s+2}\right) = \frac{-V_1}{s+2} + \left(\frac{s^2+3s+1}{s+2}\right)V_2$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{s+1}{s+2} & \frac{-1}{s+2} \\ \frac{-1}{s+2} & \frac{s^2+3s+1}{s+2} \end{bmatrix}$$

Q.



A. At node ①, $\dfrac{V_1 - V_3}{1} + \dfrac{V_1 - V_2}{1/s} = I_1 \Rightarrow (s+1)V_1 - sV_2 - V_3 = I_1$

At node ②, $\dfrac{V_2 - V_1}{1/s} + \dfrac{V_2 - V_3}{1} = I_2 \Rightarrow I_2 = (s+1)V_2 - sV_1 - V_3$

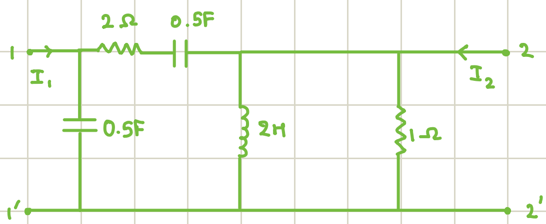
At node ③, $\dfrac{V_3 - V_1}{1} + \dfrac{V_3 - V_2}{1} + \dfrac{V_3}{1/s} = 0 \Rightarrow V_3 = \dfrac{V_1 + V_2}{s+2}$

$$I_1 = (s+1)V_1 - sV_2 - \left(\frac{V_1+V_2}{s+2}\right) = \frac{s^2+3s+1}{s+2}V_1 - \left(\frac{s^2+2s+1}{s+2}\right)V_2$$

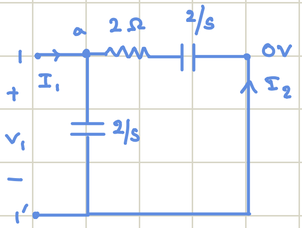
$$I_2 = (s+1)V_2 - sV_1 - \left(\frac{V_1+V_2}{s+2}\right) = -\left(\frac{s^2+2s+1}{s+2}\right)V_1 + \frac{s^2+3s+1}{s+2}V_2$$

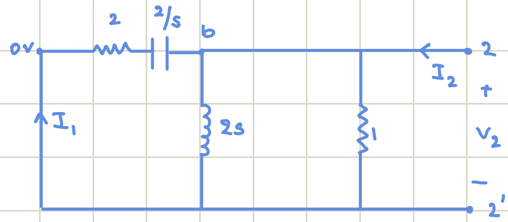
$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{s^2+3s+1}{s+2} & -\left(\dfrac{s^2+2s+1}{s+2}\right) \\ -\left(\dfrac{s^2+2s+1}{s+2}\right) & \dfrac{s^2+3s+1}{s+2} \end{bmatrix}$$

Q.



A.





For y_{11} & y_{21},

KCL at a : $\dfrac{V_1}{2/s} + \dfrac{V_1}{2+\frac{2}{s}} = I_1$

$$I_1 = V_1\left[\frac{s}{2} + \frac{s}{2s+2}\right]$$

$$= V_1\left(\frac{2s^2+4s}{4s+4}\right)$$

$$= V_1\left(\frac{s^2+2s}{2s+2}\right)$$

$$y_{11} = \frac{s^2+2s}{2s+2}$$

$$V_1 = -I_2\left(2+\frac{2}{s}\right) \Rightarrow y_{21} = \frac{-s}{2(s+1)}$$

For y_{12} & y_{22}

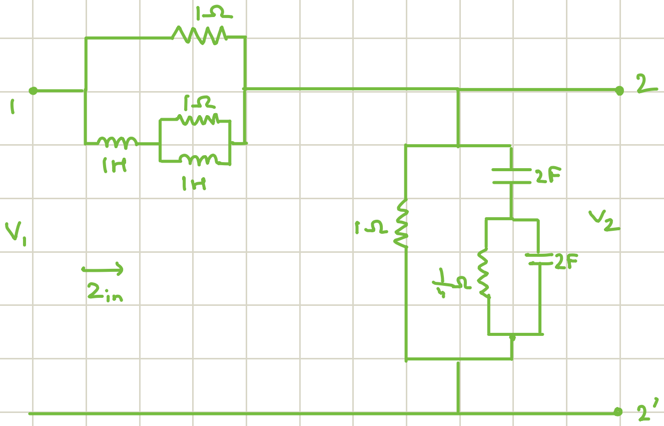
$$V_2 = -I_1\left(2+\frac{2}{s}\right) \Rightarrow y_{12} = \frac{-s}{2s+2}$$

KCL at b,

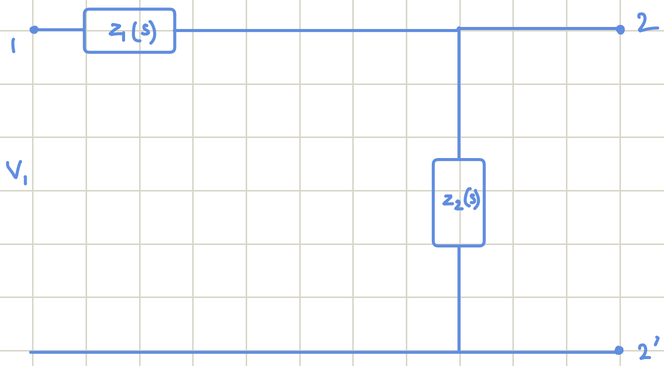
$$\frac{V_2}{2s} + \frac{V_2}{1} + \frac{V_2}{2+\frac{2}{s}} = I_2$$

$$\frac{I_2}{V_2} = \frac{1}{2s} + 1 + \frac{s}{2s+2} = \frac{4s^2+4s+2s+2+2s^2}{2s(2s+2)} = \frac{6s^2+6s+2}{4s^2+4s}$$

Q.



A.



$$z_1(s) = 1 \,||\, (s + (1 \,||\, s))$$
$$= 1 \,||\, \left(s + \frac{s}{s+1}\right)$$
$$= 1 \,||\, \left(\frac{s^2+2s}{s+1}\right) = \frac{s^2+2s}{s^2+3s+1}$$

$$z_2(s) = 1 \,||\, \left(\frac{1}{2s} + \left(\frac{1}{4} \,||\, \frac{1}{2s}\right)\right)$$
$$= 1 \,||\, \left(\frac{1}{2s} + \left(\frac{1}{4+2s}\right)\right)$$
$$= 1 \,||\, \left(\frac{4+2s+2s}{2s(4+2s)}\right)$$
$$= \frac{1 \times \frac{2+2s}{s(4+2s)}}{\frac{s(4+2s)+2+2s}{s(4+2s)}} = \frac{s+1}{s^2+3s+1}$$

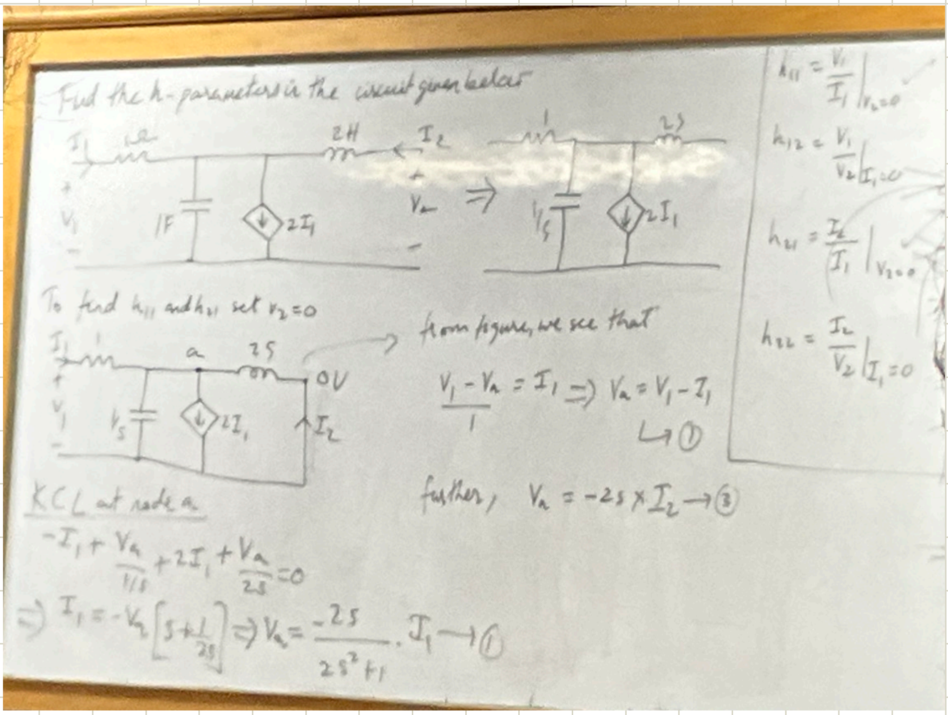
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = z_1(s) + z_2(s) = 1\Omega$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = z_2(s)$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = z_2(s)$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = z_2(s)$$

Q



→ Transmission Parameters (Relation b/w currents & voltages)

$V_1(s) = AV_2(s) - BI_2(s)$ $\quad$ $I_1(s) = CV_2(s) - DI_2(s)$ ⇒ Inductance / Capacitances

$V_1 = AV_2 - BI_2$ $\quad$ $I_1 = CV_2 - DI_2$ ⇒ Reactances

Open circuit reverse voltage gain $\quad A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$

short circuit reverse transfer circuit impedance $\quad B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$

open circuit reverse transfer admittance $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

short circuit reverse current gain $\quad D = \left.-\frac{I_1}{I_2}\right|_{V_2=0}$

In z-parameters, $V_1 = z_{11}I_1 + z_{12}I_2 \rightarrow (3)$

$V_2 = z_{21}I_1 + z_{22}I_2 \rightarrow (4)$

To derive transmission parameters in terms of z-parameters

$V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

Divide (1) by I_1 & set $I_2 = 0$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = A.\left.\frac{V_2}{I_1}\right|_{I_2=0} - B\left.\frac{I_2}{I_1}\right|_{I_2=0}$$

$$z_{11} = A.z_{21} \Rightarrow A = \frac{z_{11}}{z_{21}}$$

Divide (2) by I_1 & set $I_2 = 0$

$$\left.\frac{I_1}{I_1}\right|_{I_2=0} = C\left.\frac{V_2}{I_1}\right|_{I_2=0} - D\left.\frac{I_2}{I_1}\right|_{I_2=0}$$

$$1 = C.z_{21}$$

$$C = \frac{1}{z_{21}}$$

Divide (1) by I_2 & set $I_1 = 0$

$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = A.\left.\frac{V_2}{I_2}\right|_{I_1=0} - B\left.\frac{I_2}{I_2}\right|_{I_1=0}$$

$$z_{12} = A.z_{22} - B \Rightarrow B = \frac{z_{11}.z_{22}}{z_{21}} - z_{12} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}} \Rightarrow B = \frac{\Delta z}{z_{21}}$$

Divide (2) by I_2 & set $I_1 = 0$

$$\left.\frac{I_1}{I_2}\right|_{I_1=0} = C\left.\frac{V_2}{I_2}\right|_{I_1=0} - D\left.\frac{I_2}{I_2}\right|_{I_1=0}$$

$$0 = Cz_{22} - D \Rightarrow D = \frac{z_{22}}{z_{21}}$$

To derive transmission parameters in terms of admittance parameters

$$V_1 = AV_2 - BI_2 \longrightarrow ①$$

$$I_1 = CV_2 - DI_2 \longrightarrow ②$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Divide ① by V_1 and set $V_2 = 0$

$$\left.\frac{V_1}{V_1}\right|_{V_2=0} = A\left.\frac{V_2}{V_1}\right|_{V_2=0} - B\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$1 = -B\,y_{21} \Rightarrow B = \frac{-1}{y_{21}}$$

Divide ② by V_1 and set $V_2 = 0$

$$\left.\frac{I_1}{V_1}\right|_{V_2=0} = C\left.\frac{V_2}{V_1}\right|_{V_2=0} - D\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{11} = 0 - D\,y_{21} \Rightarrow D = -\frac{y_{11}}{y_{21}}$$

Divide ① by V_2 and set $V_1 = 0$

$$\left.\frac{V_1}{V_2}\right|_{V_1=0} = A\left.\frac{V_2}{V_2}\right|_{V_1=0} - B\left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$0 = A - B\,y_{22} \Rightarrow A = \frac{-y_{22}}{y_{21}}$$

Divide ② by V_2 and set $V_1 = 0$

$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = C\left.\frac{V_2}{V_2}\right|_{V_1=0} - D\left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y_{12} = C - D\,y_{22}$$

$$y_{12} - \frac{y_{11}y_{22}}{y_{21}} = C \Rightarrow C = \frac{\Delta y}{y_{21}}$$

To derive transimission parameters in terms of h - parameters

$V_1 = AV_2 - BI_2 \rightarrow ①$

$I_1 = CV_2 - DI_2 \rightarrow ②$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$

Divide ① by I_1 and set $V_2 = 0$

$$\left.\frac{V_1}{V_1}\right|_{V_2=0} = A\left.\frac{V_2}{V_1}\right|_{V_2=0} - 8\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$h_{11} = -B\,h_{21} \Rightarrow B = -\frac{h_{11}}{h_{21}}$$

Divide ② by I_1 and set $V_2 = 0$

$$\left.\frac{I_1}{I_1}\right|_{V_2=0} = C\left.\frac{V_2}{I_1}\right|_{V_2=0} - D\left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$1 = -D\,h_{21} \Rightarrow D = \frac{-1}{h_{21}}$$

Divide ① by V_2 and set $V_1 = 0$

$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = A\left.\frac{V_2}{V_2}\right|_{I_1=0} - B\left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$h_{12} = A - B\cdot h_{22} \Rightarrow A = \frac{-\Delta h}{h_{21}}$$

Divide ② by V_2 and set $V_1 = 0$

$$\left.\frac{I_1}{V_2}\right|_{I_1=0} = C\left.\frac{V_2}{V_2}\right|_{I_1=0} - D\left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$C = D\,h_{22} \Rightarrow C = \frac{-h_{22}}{h_{21}}$$

To derive z parameters in terms of h-parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Divide ① by I_1 and set $V_2 = 0$

$$\left.\frac{V_1}{I_1}\right|_{V_2=0} = z_{11} \left.\frac{I_1}{I_1}\right|_{V_2=0} + z_{12} \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{11} = z_{11} + z_{12} h_{21} \longrightarrow ③$$

Divide ① by V_2 and set $I_1 = 0$

$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = z_{11} \left.\frac{I_1}{V_2}\right|_{I_1=0} + z_{12} \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$h_{12} = z_{12} \cdot h_{22} \Rightarrow z_{12} = \frac{h_{12}}{h_{22}}$$

③ can be written as $z_{11} = h_{11} - z_{12} h_{21}$

$$z_{11} = h_{11} - \frac{h_{21} h_{12}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

Divide ② by V_2 and set $I_1 = 0$

$$\left.\frac{V_2}{V_2}\right|_{I_1=0} = z_{21} \left.\frac{I_1}{V_2}\right|_{I_1=0} + z_{22} \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$z_{22} = \frac{1}{h_{22}}$$

Divide ② by ① and set $V_2 = 0$

$$\left.\frac{V_2}{V_1}\right|_{V_2=0} = z_{21} \left.\frac{I_1}{I_1}\right|_{V_2=0} + z_{22} \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$z_{21} = -z_{22} h_{21} \Rightarrow z_{21} = \frac{-h_{21}}{h_{22}}$$

Condition for reciprocity : $z_{12} = z_{21}$

$$\frac{h_{12}}{h_{22}} = \frac{-h_{21}}{h_{22}} \Rightarrow h_{12} = -h_{21}$$

Condition for symmetry : $z_{11} = z_{22}$

$$\frac{\Delta h}{h_{22}} = \frac{1}{h_{22}} \Rightarrow \Delta h = 1$$

$$h_{11} h_{22} - h_{12} h_{21} = 1$$

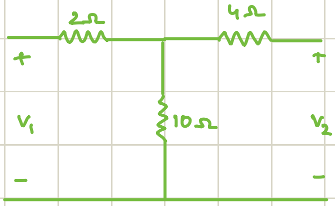
$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = z_{11} \left.\frac{I_1}{V_2}\right|_{I_1=0} + z_{12} \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$h_{12} = z_{12} \cdot h_{22} \Rightarrow z_{12} = \frac{h_{12}}{h_{22}}$$

③ can now be rewritten as

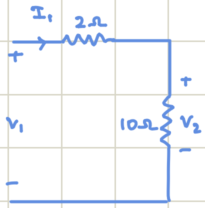
$$z_{11} = h_{11} - z_{12} h_{21}$$

Q.



Find A, B, C & D

A. Set $I_2 = 0$ to find A & C

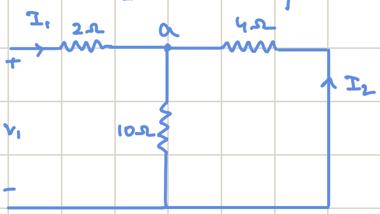


$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = \frac{12}{10}$$

$$V_2 = \frac{10V_1}{10+2} = \frac{10V_1}{12}$$

$$10 I_1 = V_2 \Rightarrow C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{1}{10}\,\Omega$$

Set $V_2 = 0$ to find B and D



$$B = \frac{-V_1}{I_2}\Big|_{V_2=0} \quad \text{and} \quad D = \frac{-I_1}{I_2}\Big|_{V_2=0}$$

$$I_1 = \frac{V_1 - V_a}{2}$$

$$I_2 = \frac{-V_a}{4}$$

By KCL at a

$$\frac{V_a - V_1}{2} + \frac{V_a}{10} + \frac{V_a}{4} = 0$$

$$\frac{V_1}{2} = V_a\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{10}\right)$$

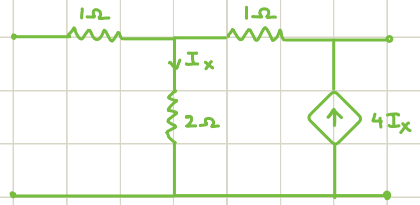
$$= V_a\left(\frac{10+5+2}{20}\right) = 17\frac{V_a}{20} \Rightarrow V_a = \frac{10V_1}{17}$$

$$I_1 = \frac{V_1}{2} - \frac{5V_1}{17} = \frac{7V_1}{34} \qquad I_2 = \frac{-10V_1}{17}$$

$$B = \frac{-V_1}{\frac{-10V_1}{17}} = 1.7 \qquad D = \frac{\frac{-7V_1}{\cancel{34}\,2}}{\frac{-10V_1}{\cancel{17}}} = \frac{7}{20}$$

Q. Determine T-parameters of the following parameters

Is the network reciprocal or symmetrical?



$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

A. Let $V_a = 2I_x$

KCL at a : $\frac{V_a - V_1}{1} + \frac{V_a}{2} + \frac{V_a - V_2}{1} = 0 \longrightarrow (1)$

KCL at b : $\frac{V_2 - V_a}{1} = 2V_a \Rightarrow V_a = \frac{V_2}{3} \longrightarrow (2)$

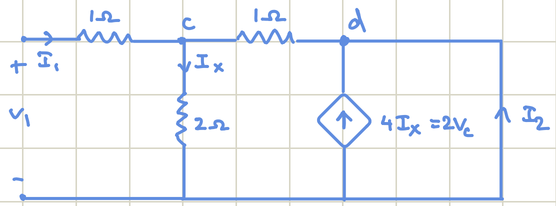
Using (1) & (2) $\Rightarrow \frac{V_2}{3} - V_1 + \frac{V_2}{6} + \frac{V_2}{3} - V_2 = 0 \Rightarrow V_1 = \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3} + -1\right] V_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0} = \frac{-1}{6}$

From figure, $\frac{V_1 - V_a}{1} = I_1 \longrightarrow (3)$

(3) can be written as $AV_2 - \frac{V_2}{3} = I_1$

$V_2\left[\frac{-1}{6} - \frac{1}{3}\right] = I_1 \Rightarrow C = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-1}{2}\,S$



KCL at c, $\frac{V_c - V_1}{1} + \frac{V_c}{2} + \frac{V_c - 0}{1} = 0$

KCL at d, $\frac{0 - V_c}{1} = 2V_c + I_2$

eliminate V_c from (3) and (4)

$V_1 = \frac{5V_c}{2} = \frac{5}{2}\left(\frac{-I_2}{3}\right)$

or $B = \left.\frac{-V_1}{I_2}\right|_{V_2=0} = \frac{5}{6}\,\Omega$

From figure, $\frac{V_1 - V_c}{1} = I_1 \longrightarrow (5)$

We can rewrite (5) as $-BI_2 - \left(-\frac{I_2}{3}\right) = I_1$

$-\frac{5}{6}I_2 + \frac{I_2}{3} = I_1 \Rightarrow \frac{I_1}{I_2} = \frac{-1}{2} \Rightarrow D = \left.\frac{-I_1}{I_2}\right|_{V_2=0} = \frac{1}{2}$