NAS UNIT - 1

→ Circuit

→ Interconnection of active & passive elements

→ Circuit Analysis

→ Process of finding voltages & currents across the circuit using linear algebra or Laplace Transform

→ Active elements :

Diodes, Transistors, Operational Amplifiers, Batteries, Thermal Generators, BJTs, MOSFETs, Voltage & Current regulators, Photodetectors, LEDs, LASERs, Piezoelectric Devices, Thermocouples

→ Passive elements :

Resistors, Inductors, Capacitors, Light Bulb, Heating elements

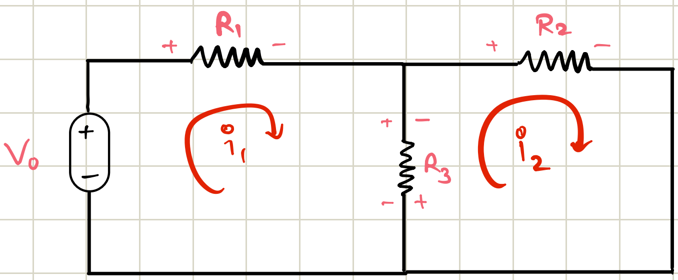
→ Reference Current Polarity

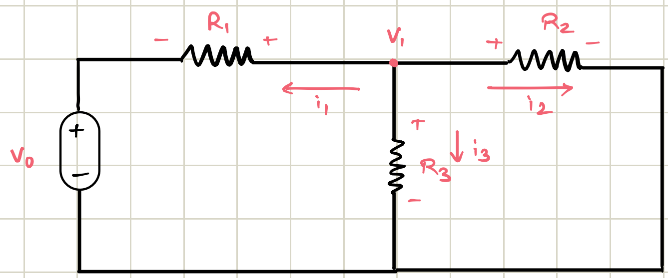
→ They are chosen for purpose of analysis

→ Polarity of passive elements in marked wrt assumed current direction but actual current may or may not be same as assumed current direction

→ Types of reference current directions

Current Type	Direction	Analysis
Mesh Current	Clockwise	Mesh Analysis
Branch Current	Leaving Node	Nodal Analysis





By mesh analysis,

$$V_0 - i_1R_1 - (i_1 - i_2)R_3 = 0$$

By nodal analysis

$$i_1 + i_2 + i_3 = 0$$

$$\frac{(V_1 - V_0)}{R_1} + \frac{V_1}{R_2} + \frac{V_1}{R_3} = 0$$

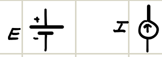
→ Independant Source

- → They produce voltage / current at specified value
- → Independant sources can be AC / DC

→ Voltage Sources

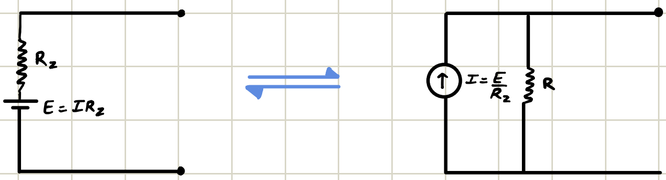
- → Produces fixed voltage at terminals but current can vary with time
- → Usually accompanied with series impedance

ex: DC Voltage Sources – Batteries, DC Generators, Power Suppliers
AC Voltage Sources – AC Generator

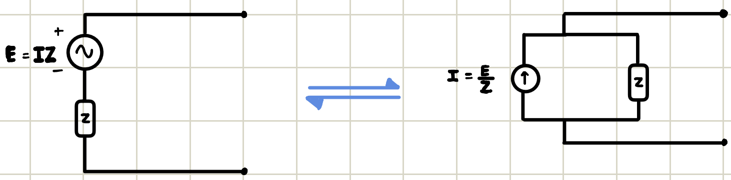
- → Interest in current sources is due to semiconductor devices such as transistors
- → Ideal Voltage sources have 0 internal impedance
- → Ideal Current sources have 0 internal admittance
- → DC Current / Voltage expressed in *italics* 

  AC Current / Voltage expressed in **bold** 

→ DC Source Conversion

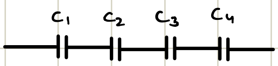


→ AC Source Conversion

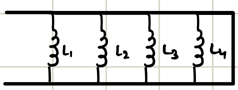


Note :

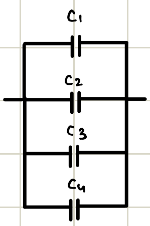
$L_T = L_1 + L_2 + L_3$



$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$

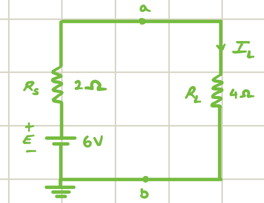


$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}$



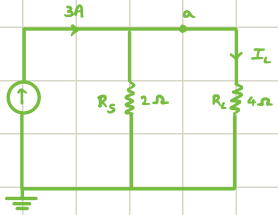
$C_T = C_1 + C_2 + C_3 + C_4$

Q.



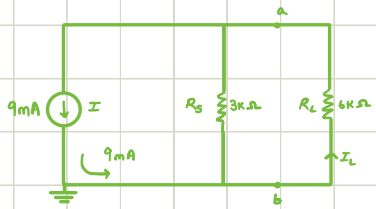
A. $I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4} = 1A$

Q.



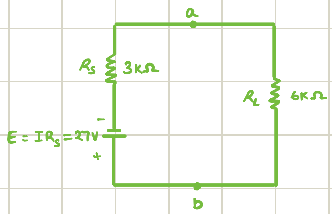
A. $I_L = \frac{R_s I}{R_s + R_L} = \frac{2 \times 3}{2+4} = 1A$

Q.



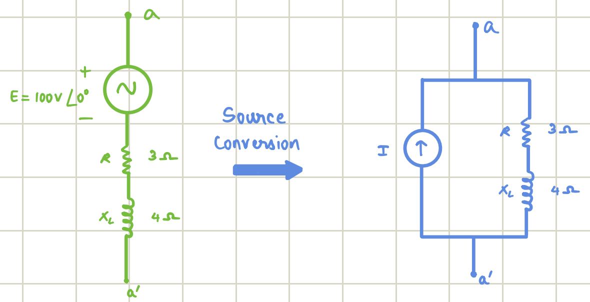
A. $I_L = \frac{R_s I}{R_s + R_L} = \frac{3000 \times 9 \times 10^{-3}}{3000 + 6000} = 3mA$

Q.



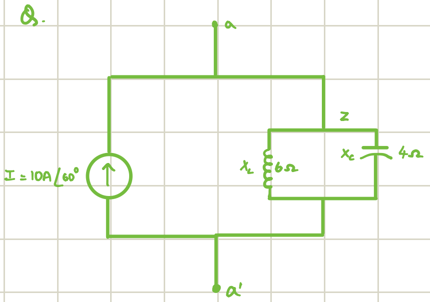
A. $I_L = \frac{E}{R_s + R_L} = \frac{27}{3000 + 6000} = 3mA$

Q.



$I = \frac{E}{Z} = \frac{100V\angle 0^\circ}{3+4i} = 20A\angle -53.13^\circ$

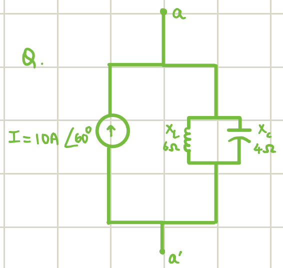
Q.



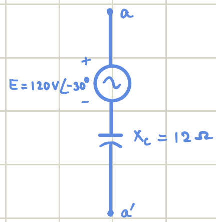
$Z = \frac{Z_c Z_L}{Z_c + Z_L} = \frac{(X_c\angle -90^\circ)(X_L\angle 90^\circ)}{-jX_c + jX_L} = \frac{(4\angle -90^\circ)(6\angle 90^\circ)}{-4j + 6j} = \frac{24\angle 0}{2\angle 90} = 12\,\Omega\angle -90$

$E = IZ = (10A\angle 60^\circ)(12\,\Omega\angle -90^\circ) = 120\,V\angle -30^\circ$

Q.



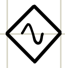
$\frac{1}{\frac{1}{6i} - \frac{1}{4i}} = -12i$

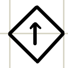


→ Dependant Sources

→ Magnitude of dependant source is controlled by current or voltage present elsewhere in the circuit

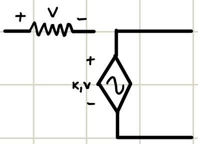
→ Circuit symbol is given by diamond shape with sine for AC or w/o sine for DC



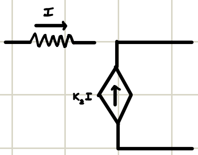


→ Dependant sources are classified as

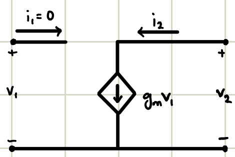
- Voltage Controlled Voltage Source — **VCVS**
- Current Controlled Current Source — **CCCS**
- Voltage Controlled Current Source — **VCCS**
- Current Controlled Voltage Source — **CCVS**



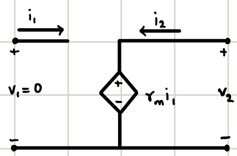
K_1 : Voltage gain



K_2 : Current gain



g_m : conductance



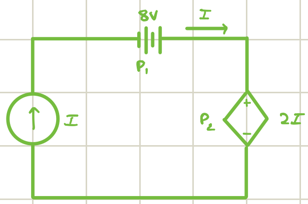
r_m : resistance

ex: Differential Amplifier

ex: Common Base Configuration

ex: MOSFET drain characteristics

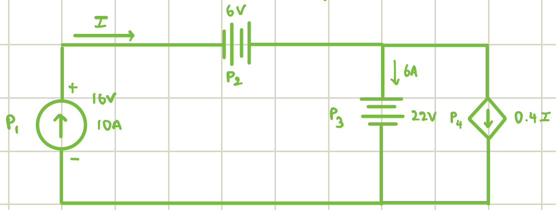
Q. Calculate the powers P_1 and P_2 for given cases $I = 4A$ and $I = -3A$



A. i) $I = 4A$, $P_1 = -8 \times I = -32W$, $P_2 = 2I \times I = 32W$

ii) $I = -3A$, $P_1 = -8 \times I = 24W$, $P_2 = 2I \times I = 18W$

Q. Calculate the powers P_1 and P_2 , P_3 and P_4 given in the circuit



A. $P_1 = -16 \times 10 = -160W$

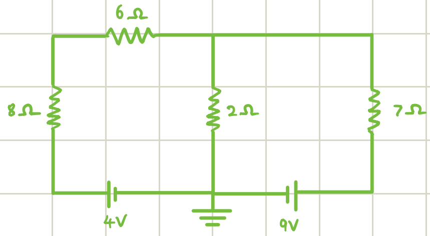
$P_2 = -6 \times 10 = -60W$

$P_3 = 22 \times 6 = 132W$

$P_4 = 22 \times 0.4I = 88W$

→ Mesh Analysis : DC Circuits

Q. Write mesh equations and find current flowing in the $7\,\Omega$ resistor



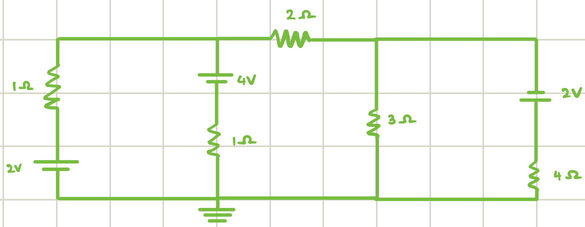
A. $(8+6+2)\,i_1 - 2i_2 = 4$

$-2i_2 + (2+7)\,i_2 = -9$

$\Big\} \rightarrow$ $i_1 = 0.128\,A$

$i_2 = -0.971\,A = i_{7\Omega}$

Q.



A. $(1+1)i_1 - (1)i_2 + 0i_3 = 2-4$

$-(1)i_1 + (1+2+3)i_2 - 3i_3 = 4$

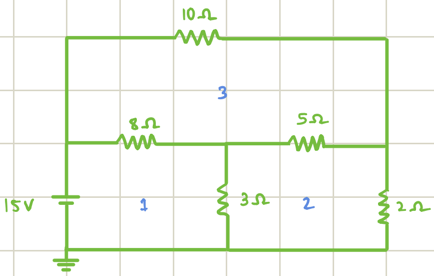
$0i_1 - 3i_2 + (3+4)i_3 = 2$

$\Big\} \rightarrow$ $i_1 = -0.542\,A$

$i_2 = 0.915\,A$

$i_3 = 0.678\,A$

Q. Determine the current through $10\,\Omega$ resistor



A. $(8+3)\,i_1 - (3)i_2 - 8i_3 = 15$

$-3i_1 + 10i_2 - 5i_3 = 0$

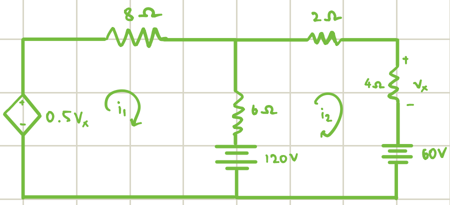
$-8i_1 - 5i_2 + (8+5+10)i_3 = 0$

$\Big\} \rightarrow$ $i_1 = 2.632\,A$

$i_2 = 1.4\,A$

$i_3 = 1.22\,A = i_{10\Omega}$

Q.



Determine the mesh currents I_1 and I_2

A. $14i_1 - 6i_2 = -120 + 0.5V_x \Rightarrow 14i_1 - 8i_2 = -120$

$-6i_1 + 12i_2 = 120 - 60$

also, $V_x = 4i_2$

$\Big\} \rightarrow$ $i_1 = -8\,A$

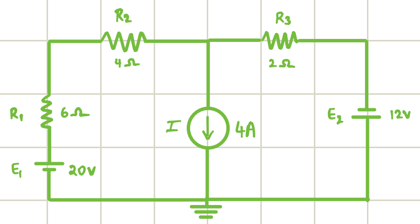
$i_2 = 1\,A$

→ Supermesh Analysis

→ Steps :

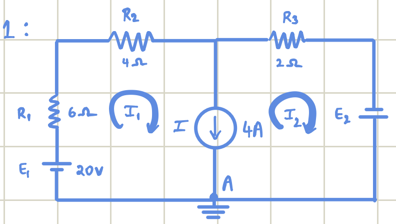
1) Clockwise mesh currents are drawn as usual
2) Current source is removed from the circuit
3) This results in a supermesh
4) Write KVL for supermesh & represent the current source in terms of corresponding mesh currents

Q.



Determine the currents in the network using mesh analysis

A. Step 1 :



Step 2 : KCL at A : $I + I_2 = I_1$

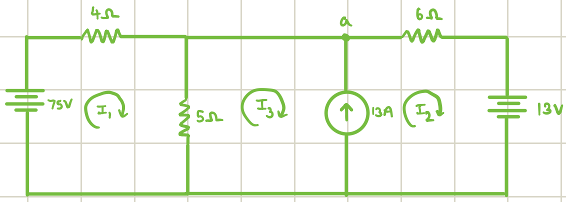
$4 = I_1 - I_2$

Step 3 : $20 - 6I_1 - 4I_1 - 2I_2 + 12 = 0$

$10I_1 + 2I_2 = 32$

$\Rightarrow I_1 = 3.33A \,,\; I_2 = -0.67A$

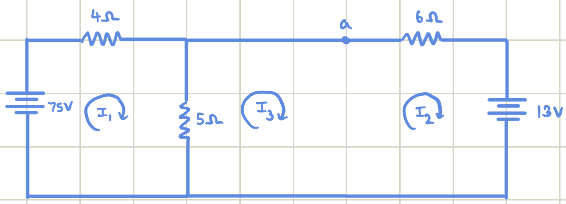
Q.



Find the mesh currents in the circuit given.

A. Step 1: KCL at a : $13 + I_3 = I_2$

Step 2:

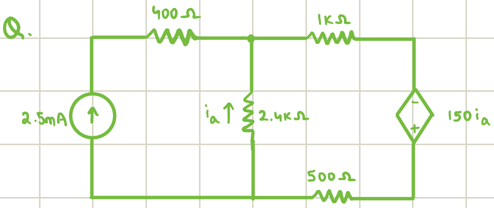


Step 3: $9I_1 - 5I_3 = 75$

$-5I_1 + 6I_2 + 5I_3 = -13$

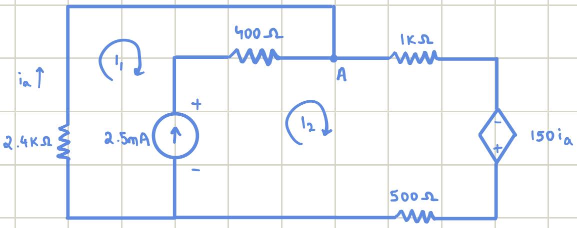
$\Rightarrow I_1 = 5A \,,\; I_2 = 7A \,,\; I_3 = -6A$

Q.



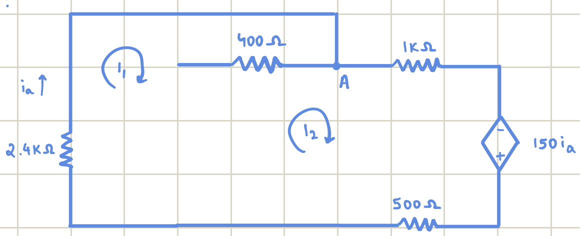
Find the current i_a in the circuit given below

A. Step 1 : Find dependant source & represent in terms of mesh current



Step 2 : KCL at A : $2.5mA + I_1 = I_2$

Step 3 :



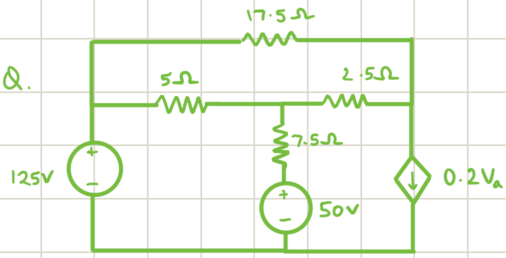
Step 4 : $2.4K\, I_1 + (1K + 0.5K)\, I_2 = 150\, i_a$

$I_1 = i_a$

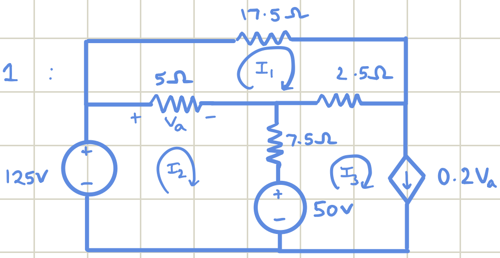
So, $2.25K\, I_1 + 1.5K\, I_2 = 0$

$\Rightarrow$ $I_1 = -1mA$ $\quad I_2 = 1.5\, mA$

Q.



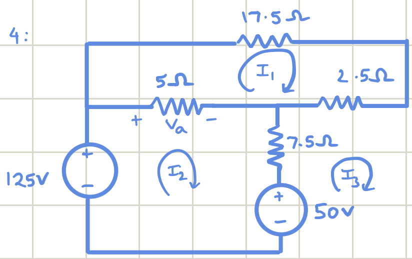
Step 1 :



Step 2 : $V_a = 5(I_2 - I_1)$

Step 3 : $I_3 = 0.2 V_a = 0.2 \times 5 (I_2 - I_1)$ $\Rightarrow$ $I_3 = I_2 - I_1$

Step 4 :



$25\, I_1 - 5 I_2 - 2.5\, I_3 = 0$

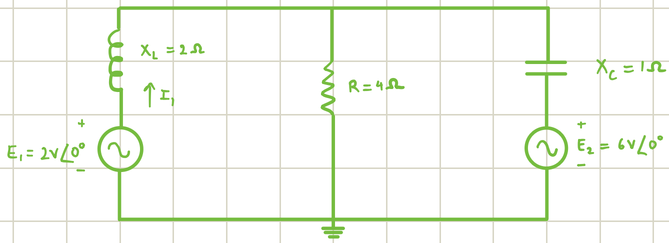
$-5\, I_1 + 12.5 I_2 - 7.5\, I_3 = -50 + 125$

$\Downarrow$

$I_1 = 3.6A$, $I_2 = 13.2A$, $I_3 = 9.6A$

→ Mesh Analysis : AC Circuits

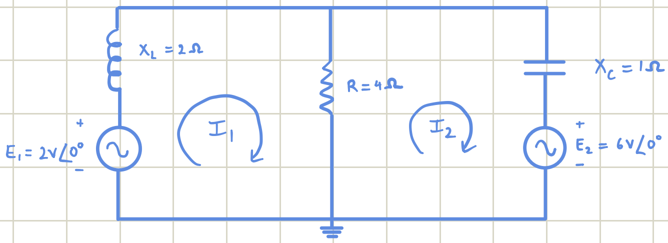
Q. Determine the mesh current I_1 in the circuit given below



A. $Z_1 = +jX_L = +j2\Omega = 2\angle 90°$

$Z_2 = R = 4\Omega$

$Z_3 = -jX_C = -j1\Omega = 1\angle -90°$

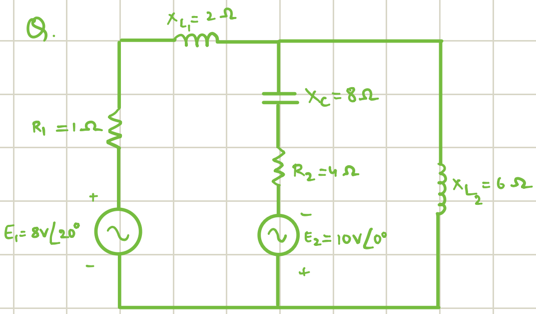


Mesh 1: $(Z_1+Z_2)I_1 - Z_2 I_2 = E_1$

Mesh 2: $-Z_2 I_1 + (Z_2+Z_3) I_2 = -E_2$

$$I_1 = \frac{\begin{vmatrix} E_1 & -Z_2 \\ -E_2 & Z_2+Z_3 \end{vmatrix}}{\begin{vmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & Z_2+Z_3 \end{vmatrix}} = \frac{E_1(Z_2+Z_3) - E_2 Z_2}{Z_1Z_2 + Z_1Z_3 + \cancel{Z_2^2} + Z_2Z_3 - \cancel{Z_2^2}} = \frac{2(4-i) - 6(4)}{8i + 2 - 4i} = \frac{-16-2i}{2+4i} = 3.605\angle 123.69°$$

Q.



Determine I_2

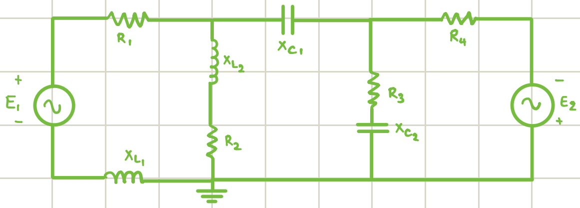
A. $Z_1 = R_1 + jX_{L_1} = 1 + j2\ \Omega$; $Z_2 = R_2 - jX_C = 4 - j8\ \Omega$; $Z_3 = jX_{L_2} = j6\ \Omega$

Mesh 1: $(Z_1+Z_2)I_1 - Z_2 I_2 = E_1 + E_2$

Mesh 2: $-Z_2 I_1 + (Z_2+Z_3) I_2 = -E_2$

$$I_2 = \frac{\begin{vmatrix} Z_1+Z_2 & E_1+E_2 \\ -Z_2 & -E_2 \end{vmatrix}}{\begin{vmatrix} Z_1+Z_2 & -Z_2 \\ -Z_2 & Z_2+Z_3 \end{vmatrix}} = \frac{-(Z_1+Z_2)E_2 + Z_2(E_1+E_2)}{(Z_1+Z_2)(Z_2+Z_3) - Z_2^2} = \frac{E_1 Z_2 - E_2 Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} = 1.27\angle -86.95°$$

Q. Write the KVL equations for the network given below



A. $Z_1 = R_1 + jX_{L_1}$; $Z_2 = R_2 + jX_{L_2}$; $Z_3 = -jX_{C_1}$; $Z_4 = R_3 - jX_{C_2}$; $Z_5 = R_4$

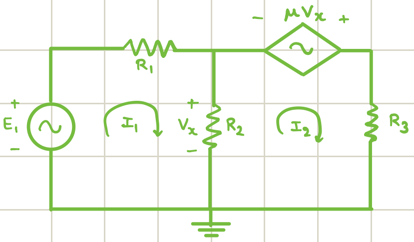
Mesh 1: $(Z_1+Z_2)I_1 - Z_2 I_2 = E_1$

Mesh 2: $-Z_2 I_1 + (Z_2+Z_3+Z_4)I_2 - Z_4 I_3 = 0$

Mesh 3: $-Z_4 I_2 + (Z_4+Z_5)I_3 = E_2$

→ Mesh Analysis : Dependant sources

Q. Write the mesh currents for the circuit below



A. 1) The dependant source expressed in terms of branch currents as

$V_x = R_2(I_1 - I_2) \Rightarrow \mu V_x = \mu R_2(I_1 - I_2)$

2) Write KVL for each mesh

Mesh 1: $(R_1+R_2)I_1 - R_2 I_2 = E_1$

Mesh 2: $-R_2 I_1 + (R_2+R_3)I_2 = \mu V_x = \mu R_2(I_1 - I_2)$

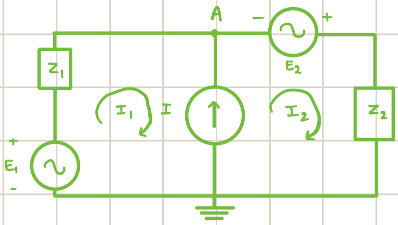
$-R_2 I_1 + R_2 I_2 + R_3 I_2 = \mu R_2 I_1 - \mu R_2 I_2$

$-(1+\mu)R_2 I_1 + (1+\mu)R_2 I_2 + R_3 I_2 = 0$

$\Rightarrow -(1+\mu)R_2 I_1 + ((1+\mu)R_2 + R_3)I_2 = 0$

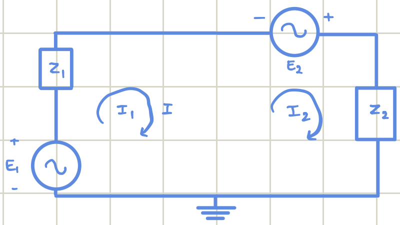
→ Mesh Analysis : AC Circuits

Q. Write the mesh currents for the circuit given below



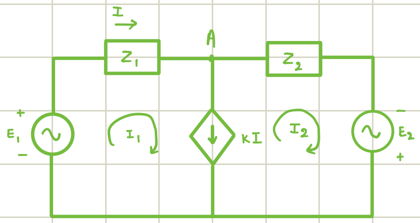
A. 1) KCL at A : $I_1 + I = I_2$

2) Remove current source & write KVL for supermesh



$E_1 - I_1 Z_1 + E_2 - I_2 Z_2 = 0 \Rightarrow E_1 + E_2 = I_1 Z_1 + I_2 Z_2$

Q.



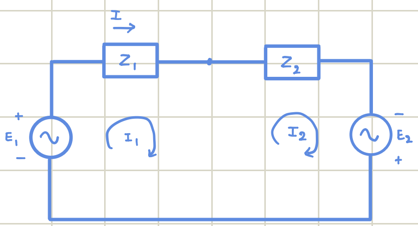
Write the mesh currents for the circuit given

A. 1) KCL at A : $I_1 = I_2 + K I$

also $I = I_1$,

So, $(K-1) I_1 + I_2 = 0$

2)

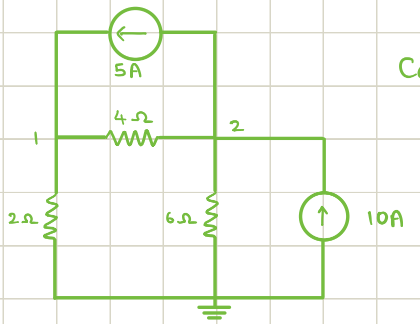


$E_1 - I_1 Z_1 - I_2 Z_2 + E_2 = 0$

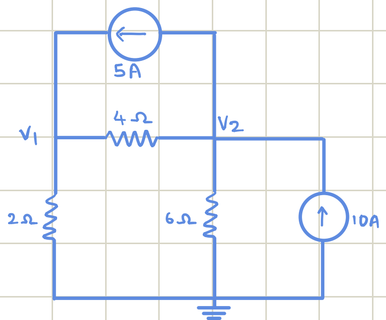
$Z_1 I_1 + Z_2 I_2 = E_1 + E_2$

→ Nodal Analysis : DC Circuits

Q. Calculate the node voltages for the circuit shown



A.



1) Introduce nodal voltages V_1 and V_2

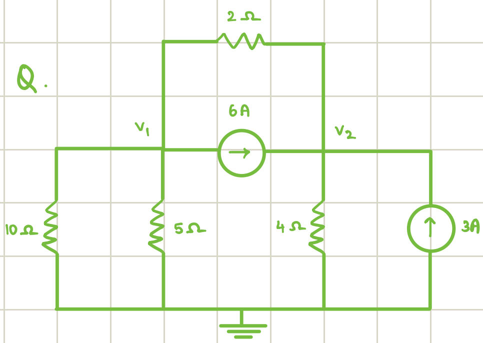
2) KCL at 1: $\frac{V_1}{2} + \frac{V_1 - V_2}{4} = 5 \Rightarrow V_1\left(\frac{1}{2}+\frac{1}{4}\right) + V_2\left(\frac{-1}{4}\right) = 5$

KCL at 2: $\frac{V_2 - V_1}{4} + \frac{V_2}{6} = 10 - 5 \Rightarrow V_2\left(\frac{1}{4}+\frac{1}{6}\right) + V_1\left(\frac{-1}{4}\right) = 5$

$V_1 = 13.33\text{ V}$

$V_2 = 20\text{ V}$

Q. Find the nodal voltages equations for the circuit shown

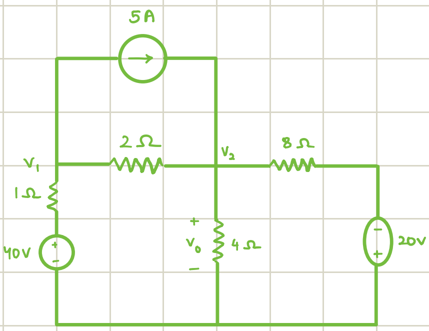


A. Node 1: $\frac{V_1}{10} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = -6 \Rightarrow V_1\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{2}\right) + V_2\left(-\frac{1}{2}\right) = -6$

Node 2: $\frac{V_2}{4} + \frac{V_2 - V_1}{2} = 6 + 3 \Rightarrow V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2}+\frac{1}{4}\right) = 9$

$V_1 = 0\text{ V}, \quad V_2 = 12\text{ V}$

Q. For the circuit shown, determine the node voltage V_o



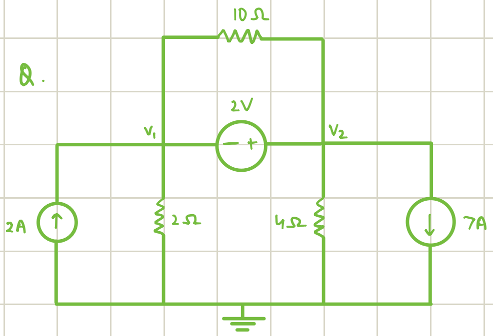
A. Node 1, $\frac{V_1 - 40}{1} + \frac{V_1 - V_2}{2} = -5 \Rightarrow V_1\left(1+\frac{1}{2}\right) + V_2\left(\frac{-1}{2}\right) = 35$

Node 2, $\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 + 20}{8} = 5 \Rightarrow V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right) = 5 - \frac{20}{8}$

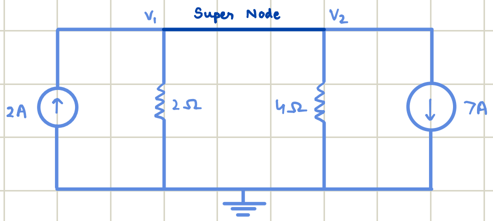
$V_1 = 30\text{ V}$

$V_2 = 20\text{ V} = V_o$

Q. For the circuit shown find the node voltage



A. $V_2 - V_1 = 2\,V$



→ We eliminate floating voltage source from the circuit

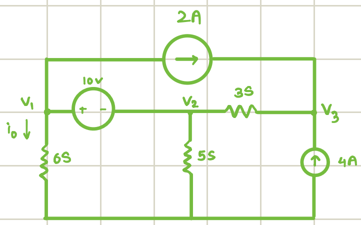
→ Redraw circuit by replacing the floating voltage circuit with short circuit

KCL for super node : $\frac{V_1}{2} + \frac{V_2}{4} = 2 - 7$

$V_1 = -7.33V$

$V_2 = -5.33V$

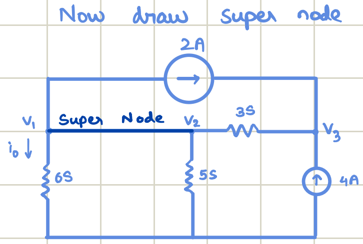
Q.



Apply nodal analysis to find the current i_o

A. $V_1 - V_2 = 10$

Now draw super node



KCL for super node : $6V_1 + 5V_2 + 3(V_2 - V_3) = -2$

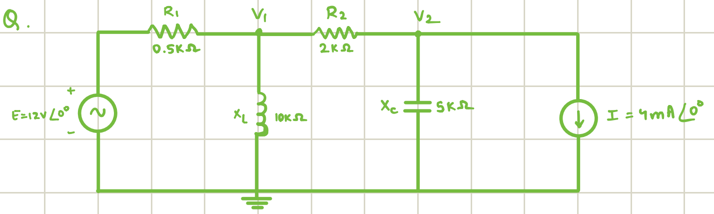
KCL at node 3 : $3(V_3 - V_2) = 4 + 2$

$V_1 = 4.91V$; $V_2 = -5.09\,V$; $V_3 = -3.09\,V$

$i_o = 6V_1 = 29.46\,A$

→ Nodal Analysis : AC Circuits

Q.



Determine Nodal Voltages

A.

1) $Z_1 = R_1$; $Z_2 = jX_L$; $Z_3 = R_2$; $Z_4 = -jX_C$

2) Node 1: $\frac{V_1 - E}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0 \Rightarrow V_1\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right) + V_2\left(\frac{-1}{Z_3}\right) = \frac{E}{Z_1}$

$$\left(2.5\times10^{-3} - 10^{-4} i\right) V_1 \quad -5\times10^{-4} V_2 = 0.024$$

$$\underset{A}{2.502\times10^{-3}\angle -2.29}\, V_1 - \underset{B}{5\times10^{-4}}\, V_2 = \underset{E}{0.024}$$

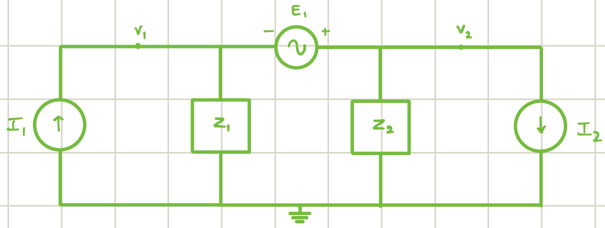
Node 2: $\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} = -I \Rightarrow V_1\left(\frac{-1}{Z_3}\right) + V_2\left(\frac{1}{Z_3} + \frac{1}{Z_4}\right) = -I$

$$-5\times10^{-4} V_1 + \left(5\times10^{-4} + 2\times10^{-4} i\right) V_2 = -0.004$$

$$\underset{C}{5\times10^{-4}}\, V_1 - \underset{D}{5.39\times10^{-4}\angle 21.8} = \underset{F}{0.004}$$

$$V_1 = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{ED - BF}{AD - BC} = 9.95\angle 1.83^\circ$$

Q.

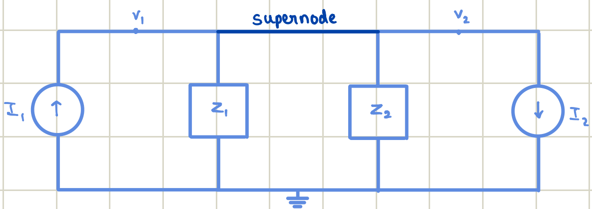


Find nodal voltages V_1 & V_2

A

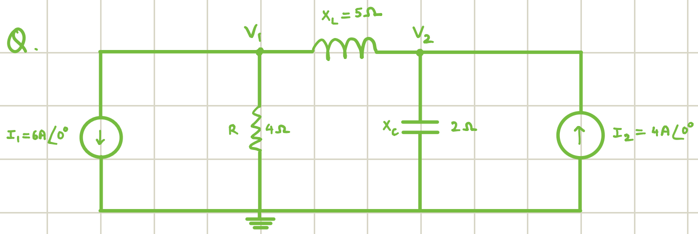
1) $V_2 - V_1 = E_1$

2)



KCL for supernode: $\frac{V_1}{Z_1} + \frac{V_2}{Z_2} = I_1 - I_2$

Q.



Determine the current through 4Ω resistor using nodal analysis

A. $Z_1 = R = 4\Omega$; $Z_2 = jX_L = j5\Omega$; $Z_3 = -jX_c = -j2\Omega$

Node 1: $\frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} = -I_1 \Rightarrow V_1\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) + V_2\left(\frac{-1}{Z_2}\right) = -I_1$

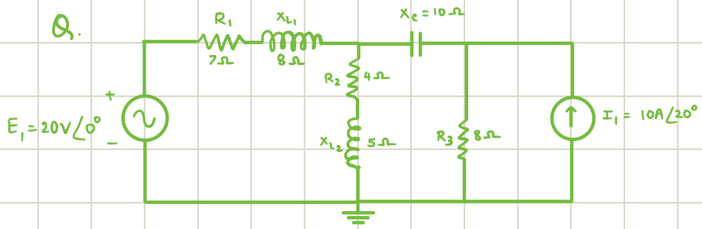
$$\underset{A}{0.32\angle -38.66}\, V_1 + \underset{B}{0.2\angle 90}\, V_2 = \underset{E}{-6}$$

Node 2: $\frac{(V_2 - V_1)}{Z_2} + \frac{V_2}{Z_3} = I_2 \Rightarrow V_1\left(\frac{-1}{Z_2}\right) + V_2\left(\frac{1}{Z_2} + \frac{1}{Z_3}\right) = I_2$

$$\underset{C}{0.2\angle 90}\, V_1 + \underset{D}{0.3\angle 90}\, V_2 = \underset{F}{4}$$

$$V_1 = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{ED - BF}{AD - BC} = 20.8\angle -126.87^\circ$$

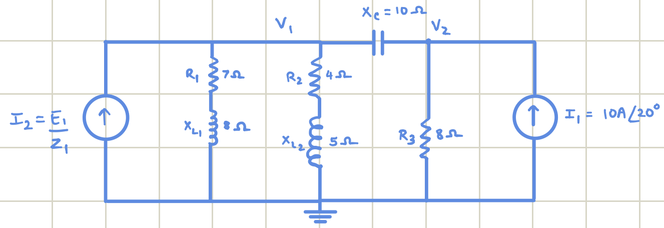
Q.



Find the nodal voltages

A. 1) $Z_1 = 7 + j8\ \Omega$; $Z_2 = 4 + j5\ \Omega$; $Z_3 = -j10\ \Omega$; $Z_4 = 8\Omega$

2) Apply source transformation to get Z_1 in parallel



$I_2 = \frac{20}{7 + j8} = 1.88\angle -48.81^\circ A$

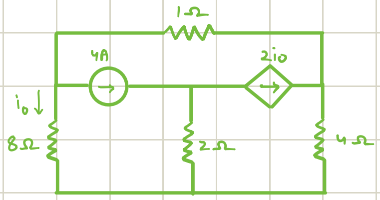
node 1: $\frac{V_1}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = I_2 \Rightarrow V_1\underset{A}{\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)} + V_2\underset{B}{\left(\frac{-1}{Z_3}\right)} = \underset{E}{I_2}$

node 2: $\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} = I_1 \Rightarrow V_1\underset{C}{\left(\frac{-1}{Z_3}\right)} + V_2\underset{D}{\left(\frac{1}{Z_3} + \frac{1}{Z_4}\right)} = \underset{F}{I_1}$

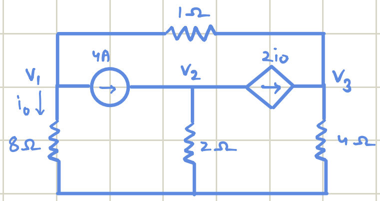
$$V_1 = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = 22.5\angle 86.6^\circ \qquad V_2 = \frac{\begin{vmatrix} A & E \\ C & F \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = 49.88\angle -12.23^\circ$$

→ Nodal Analysis – Dependant Sources

Q.



A.



$V_1 = 8i_0 \Rightarrow i_0 = \frac{V_1}{8}$

Output of dependant current source $2i_0 = \frac{V_1}{4}$

node 1 : $\frac{V_1}{8} + \frac{V_1 - V_3}{1} = -4A \Rightarrow V_1\left(1+\frac{1}{8}\right) + V_3(-1) = -4$

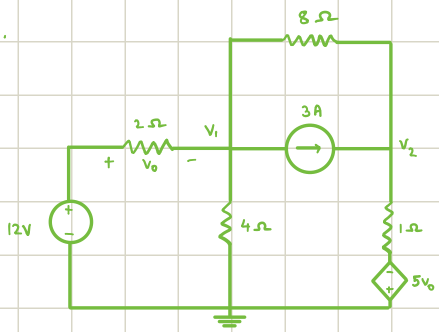
node 2 : $\frac{V_2}{2} = 4 - 2i_0 \Rightarrow \frac{V_1}{4} + \frac{V_2}{2} = 4$

node 3 : $\frac{V_3 - V_1}{1} - 2i_0 + \frac{V_3}{4} = 0 \Rightarrow V_1\left(-1-\frac{1}{4}\right) + V_3\left(1+\frac{1}{4}\right) = 0$

$V_1 = -32V$, $V_2 = 24V$, $V_3 = -32V$

$i_0 = \frac{V_1}{8} = -4V$

Q. Determine the nodal voltages for the circuit shown



A $V_0 = 12 - V_1$

and $5V_0 = 60 - 5V_1$

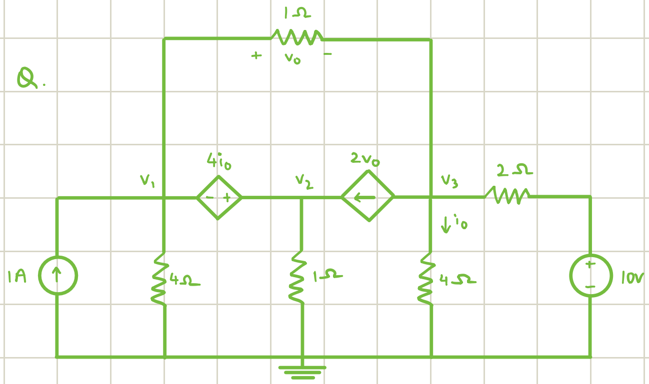
node 1: $\frac{V_1 - 12}{2} + \frac{V_1}{4} + \frac{V_1 - V_2}{8} = -3 \Rightarrow V_1\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right) + V_2\left(\frac{-1}{8}\right) = 3$

node 2: $\frac{V_2 - V_1}{8} + \frac{V_2 + 60 - 5V_1}{1} = 3 \Rightarrow V_1\left(-5-\frac{1}{8}\right) + V_2\left(\frac{1}{8}+1\right) = -57$

$V_1 = -10.91\,V$

$V_2 = -100.36\,V$

Q.

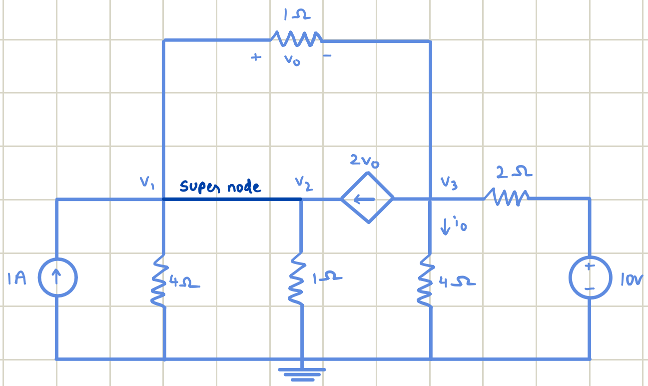


A. $i_o = \frac{V_3}{4}$

$V_o = V_1 - V_3 \Rightarrow 2v_o = 2(V_1 - V_3)$

$4i_o = V_3$

$V_2 - V_1 = 4i_o = V_3 \Rightarrow -V_1 + V_2 - V_3 = 0$



KCL for supernode:

$\frac{V_1}{4} + \frac{V_1 - V_3}{1} + \frac{V_2}{1} = 1 + 2V_o \Rightarrow \frac{V_1}{4} + V_1 - V_3 + V_2 - 2V_1 + 2V_3 = 1$

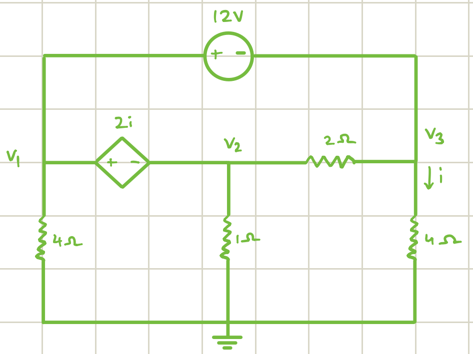
$-\frac{3}{4}V_1 + V_2 + V_3 = 1$

KCL for node 3:

$\frac{V_3}{4} + \frac{V_3 - 10}{2} + \frac{V_3 - V_1}{1} = -2V_o \Rightarrow V_1 - \frac{V_3}{4} = 5$

$V_1 = 4.969V \qquad V_2 = 4.848V \qquad V_3 = -0.121V$

Q.

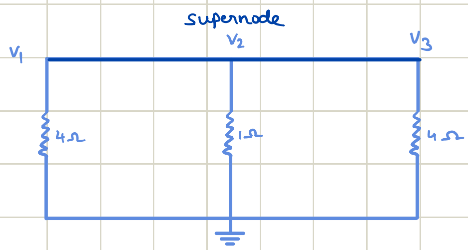


A. $i = \frac{V_3}{4}$

$2i = \frac{2V_3}{4} = \frac{V_3}{2}$

$V_1 - V_2 = 2i \Rightarrow V_1 - V_2 - \frac{V_3}{2} = 0$

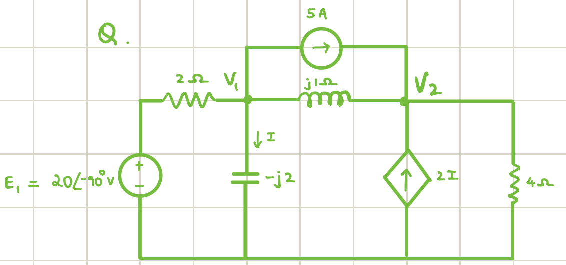
$V_1 - V_3 = 12$



$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0$

$V_1 = -3V,\ V_2 = 4.5V,\ V_3 = -15V$

Q.



Solve for I

A. $Z_1 = 2\,\Omega \qquad Z_2 = -j2\,\Omega \qquad Z_3 = j1\,\Omega \qquad Z_4 = 4\,\Omega$

$I = \frac{V_1}{Z_2} \Rightarrow 2I = \frac{2V_1}{Z_2}$

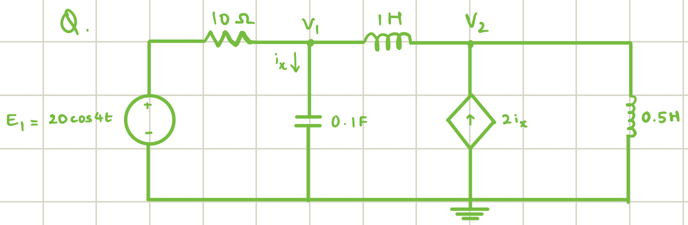
node 1: $\frac{(V_1 - E_1)}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = -5 \Rightarrow V_1\underbrace{\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)}_{A} + V_2\underbrace{\left(\frac{-1}{Z_3}\right)}_{B} = \underbrace{\frac{E_1}{Z_1} - 5}_{E}$

node 2: $\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} - 2I = 5 \Rightarrow V_1\underbrace{\left(\frac{2}{Z_2} - \frac{1}{Z_3}\right)}_{C} + V_2\underbrace{\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right)}_{D} = \underbrace{5}_{F}$

$$V_1 = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{ED - BF}{AD - BC} = 6.62\angle{-146.43}$$

$$I = \frac{V_1}{Z_2} = 3.31\angle{-56.43}$$

Q.



A. $20\cos 4t \Rightarrow \omega = 4$ rad/s

$Z_1 = 10\,\Omega$

$Z_2 = \frac{1}{j\omega C} = -j2.5\,\Omega$

$Z_3 = j\omega L = j4$

$Z_4 = j\omega L = j2$

$i_x = \frac{V_1}{Z_2} \Rightarrow 2i_x = \frac{2V_1}{Z_2}$

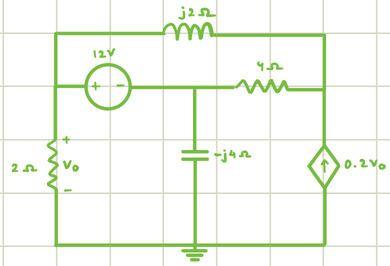
node 1: $\frac{V_1 - E_1}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0 \Rightarrow V_1\underbrace{\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)}_{A} + V_2\underbrace{\left(-\frac{1}{Z_3}\right)}_{B} = \underbrace{\frac{E_1}{Z_1}}_{E}$

node 2: $\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} = 2i_x \Rightarrow V_1\underbrace{\left(\frac{-1}{Z_3} - \frac{2}{Z_2}\right)}_{C} + V_2\underbrace{\left(\frac{1}{Z_3}+\frac{1}{Z_4}\right)}_{D} = \underbrace{0}_{F}$

$$V_1 = \frac{\begin{vmatrix} E & B \\ F & D \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = \frac{ED - BF}{AD - BC} = 18.97\angle{18.43} \qquad V_2 = \frac{\begin{vmatrix} A & E \\ C & F \end{vmatrix}}{\begin{vmatrix} A & B \\ C & D \end{vmatrix}} = 13.91\angle{-161.56}$$

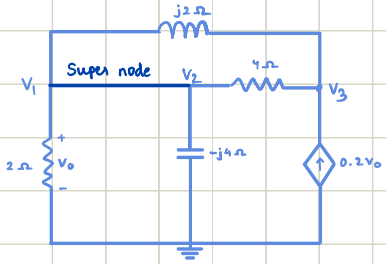
$$I_x = \frac{V_1}{Z_1} = 1.89\angle{18.43} \Rightarrow 1.89\cos(4t + 18.43)$$

Q.



Use nodal analysis to find V_o

A. $Z_1 = j2$ $\quad Z_2 = 4\Omega$ $\quad Z_3 = 2\Omega$ $\quad Z_4 = -j4\,\Omega$ $\quad E_1 = 12V$



$V_0 = V_1$

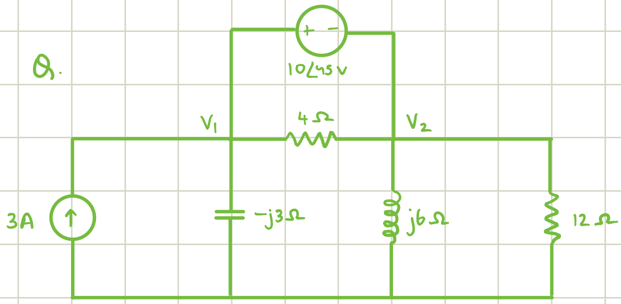
$V_1 - V_2 = E_1$

KCL at supernode: $\dfrac{V_1 - V_3}{Z_1} + \dfrac{V_1}{Z_4} + \dfrac{V_2}{Z_3} + \dfrac{V_2 - V_3}{Z_2} = 0 \Rightarrow V_1\underbrace{\left(\frac{1}{Z_1}+\frac{1}{Z_4}\right)}_{A} + V_2\underbrace{\left(\frac{1}{Z_2}+\frac{1}{Z_3}\right)}_{B} + V_3\underbrace{\left(\frac{-1}{Z_1}-\frac{1}{Z_2}\right)}_{C} = 0$

node 3: $\dfrac{V_3 - V_2}{Z_2} + \dfrac{V_3 - V_1}{Z_1} = 0.2V_0 \Rightarrow V_1\underbrace{\left(\frac{1}{Z_1}-0.2\right)}_{D} + V_2\underbrace{\left(\frac{-1}{Z_2}\right)}_{E} + V_3\underbrace{\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)}_{F} = 0$

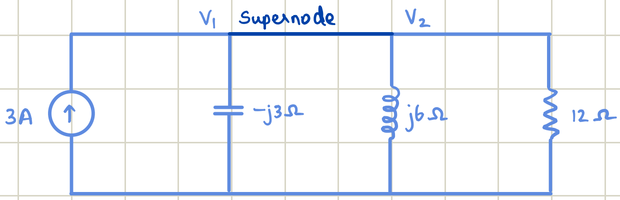
$$V_0 = V_1 = \frac{\begin{vmatrix} 12 & -1 & 0 \\ 0 & B & C \\ 0 & E & F \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 0 \\ A & B & C \\ D & E & F \end{vmatrix}} = \frac{12(BF-CE)}{(BF-CE)+(AF-CD)} = 7.42\angle 68.2$$

Q.



A. $Z_1 = -j3\Omega$ $\quad Z_2 = j6\Omega$ $\quad Z_3 = 12\Omega$

$V_1 - V_2 = 10\angle{-45}$



Supernode: $\dfrac{V_1}{Z_1} + \dfrac{V_2}{Z_2} + \dfrac{V_2}{Z_3} = 3$

$$V_1 = \frac{\begin{vmatrix} 10\angle{-45} & -1 \\ 3 & \frac{1}{Z_2}+\frac{1}{Z_3} \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ \frac{1}{Z_1} & \frac{1}{Z_2}+\frac{1}{Z_3} \end{vmatrix}} = 25.78\angle{-70.48^0} \checkmark$$

$$V_2 = \frac{\begin{vmatrix} 1 & 10\angle{-45} \\ \frac{1}{Z_1} & 3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ \frac{1}{Z_1} & \frac{1}{Z_2}+\frac{1}{Z_3} \end{vmatrix}} = 31.41\angle{-87.18^0} \checkmark$$

Superposition Theorem

→ The current through, or voltage across, an element in a linear bilateral network is equal to algeabric sum of the currents or voltages produced independantly by each independant source

→ Applicable for bilateral linear AC networks as well

Advantages –

- Simplifies calculation
- Complex computation like calculation of determinants can be avoided

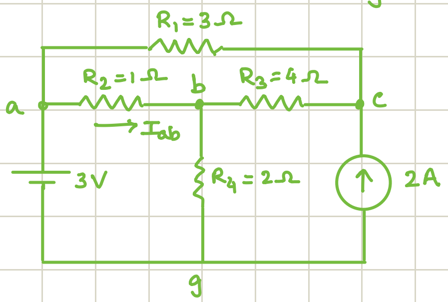
Disadvantages –

- Can't be applied to circuits with passive elements with non-linear IV characteristics
- Superposition of branch powers w different independant sources isn't allowed

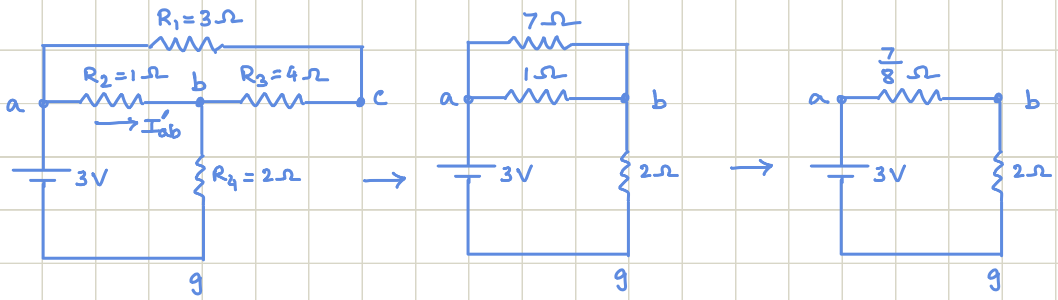
→ Procedure

1) Replace all but one of the sources by their internal resistances
2) Determine current/voltage of various branches using basic laws
3) Repeat it for all sources one by one
4) Add all the currents/voltages for each branch due to different independant sources resulting the actual current/voltage for that branch when all sources acting on the circuit simultaneously

Q. Calculate I_{ab} & V_{cg} using superposition theorem



A. 1)



$$I = \frac{3}{2.875} = 1.043\,A$$

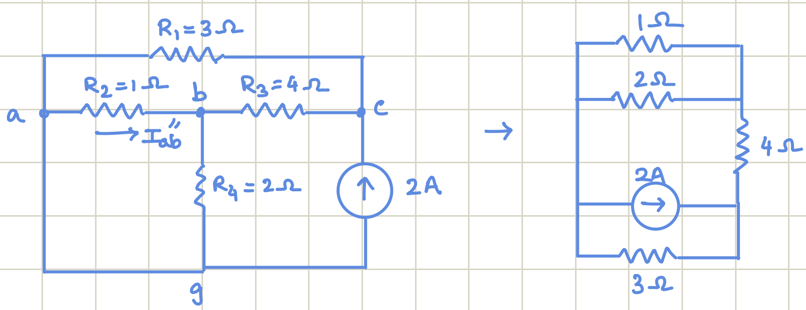
$$V'_{bg} = 2I = 2 \times 1.043 = 2.086V$$

$$I'_{ab} = \frac{3 - 2.086}{1} = 0.914\,A$$

$$I_{bc} = I'_{ab} - I = -0.129A$$

$$V'_{cg} = V'_{bg} + V'_{cb} = 2.086 - (4 \times -0.129) = 2.606\,V$$

2)



$$I = \frac{2 \times 3}{3 + \left(4 + \left(\frac{1}{1} + \frac{1}{2}\right)^{-1}\right)} = 0.783A$$

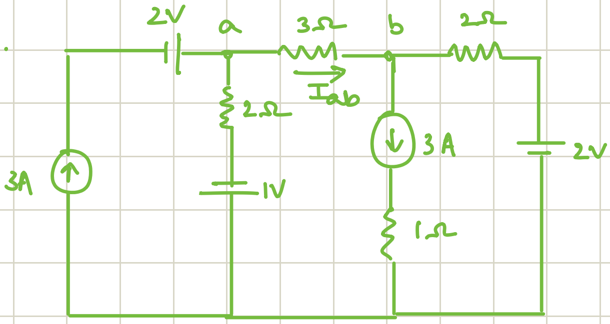
$$I''_{ab} = -\frac{I \times 2}{1+2} = -0.522\,A$$

$$V''_{cg} = 3 \times (2 - I) = 3.652\,V$$

$$I_{ab} = I'_{ab} + I''_{ab} = 0.913 - 0.522 = 0.391\,A$$

$$V_{cg} = V'_{cg} + V''_{cg} = 2.606 + 3.651 = 6.257V$$

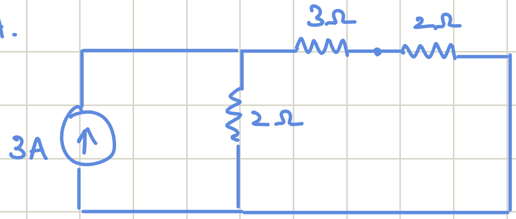
Q.



Find I_{ab}

A.

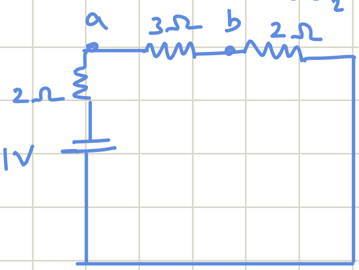
1)



$$I_{ab_1} = \frac{3 \times 2}{7} = 0.857\,A$$

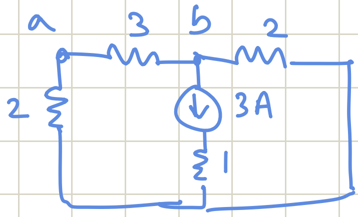
2) 2V $\Rightarrow$ $I_{ab_2} = 0\,A$

3)



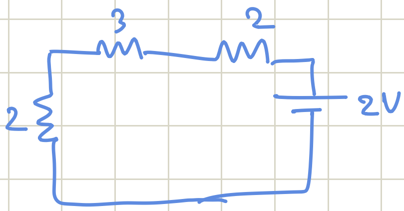
$$I_{ab_3} = -\frac{1}{7} = -0.142\,A$$

4)



$$I_{ab_4} = \frac{3 \times 2}{5+2} = 0.857\,A$$

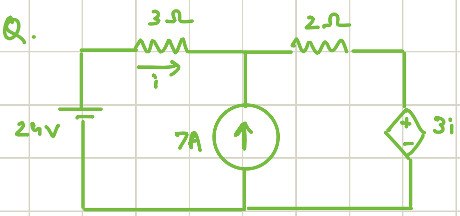
5)



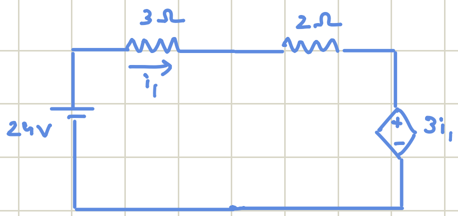
$$I_{ab_5} = -\frac{2}{7} = -0.285\,A$$

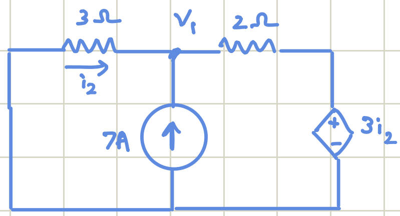
$$I_{ab} = I_{ab_1} + I_{ab_2} + I_{ab_3} + I_{ab_4} + I_{ab_5} = 1.285\,A$$

Q.



A.





$$24 - 3i_1 - 2i_1 - 3i_1 = 0$$

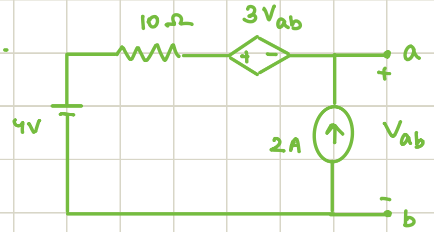
$$i_1 = 3A$$

$$i_2 = -\frac{V_1}{3}$$

$$\frac{V_1}{3} + \frac{V_1 - 3\left(\frac{-V_1}{3}\right)}{2} = 7 \Rightarrow V_1 = \frac{21}{4} \longrightarrow i_2 = \frac{-7}{4}\,A$$

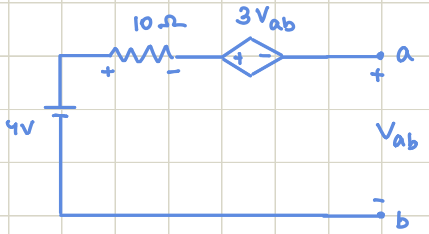
Then, $i_1 + i_2 = 3 - \frac{7}{4} = \frac{5}{4}\,A$

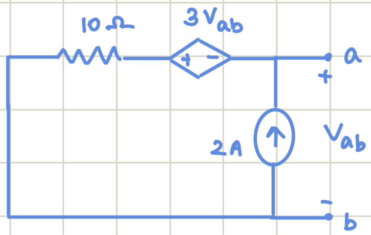
Q.



Find V_{AB}

A.





$$V_{ab_1} + 3V_{ab_1} + 10 i_1 - 4 = 0$$

$$4V_{ab_1} + 10(0) = 4$$

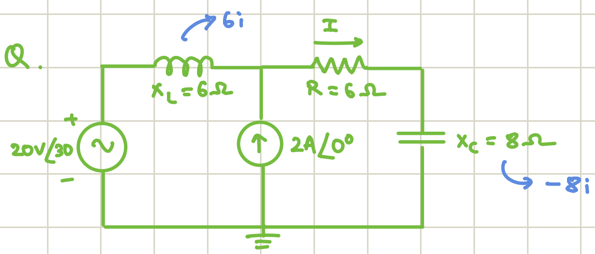
$$V_{ab_1} = 1\,V$$

$$\frac{V_{ab_2} - (-3V_{ab_2})}{10} = 2$$

$$V_{ab_2} = 5\,V$$

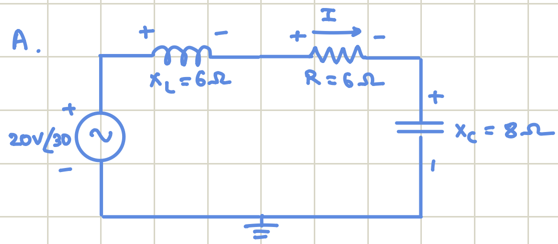
$$V_{ab} = V_{ab_1} + V_{ab_2} = 6V$$

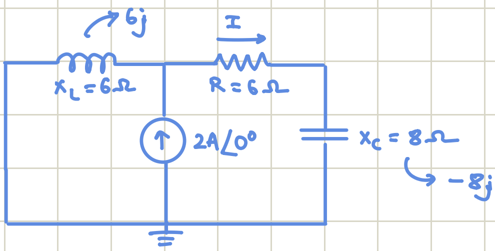
Q.



Find I

A.



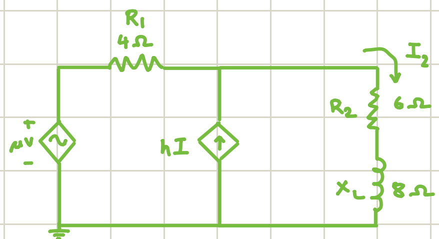


$$20V\angle 30 = (6-2j)\, i_1 \Rightarrow i_1 = 3.16\angle 48.43\ A$$

$$i_2 = \frac{2 \times 6j}{6-2j} = 1.89\angle 108.43$$

$$i = i_1 + i_2 = 4.42\angle 70.22\ A$$

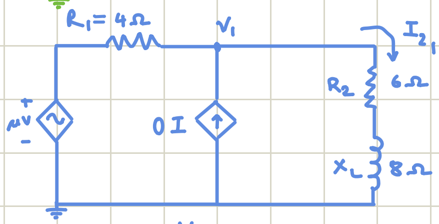
Q.



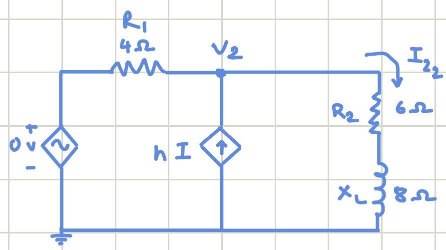
Find I_2

μ & h are constants

A.



$$I_{2_1} = \frac{V_1}{6+8j} = \frac{\mu V}{10+8j}$$



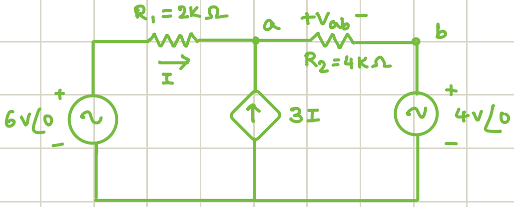
$$\frac{V_1 - \mu V}{4} + \frac{V_1}{6+8j} = 0$$

$$V_1 = \frac{\mu V}{4}\left(\frac{1}{\frac{1}{4} + \frac{1}{6+8j}}\right) = \mu V\,\frac{(6+8j)}{10+8j}$$

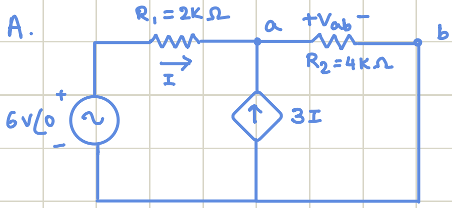
$$I_{2_2} = \frac{hI \times 4}{10+8j}$$

$$I_2 = I_{2_1} + I_{2_2} = \frac{\mu V + 4hI}{10+8j}$$

Q. Determine V_{ab}



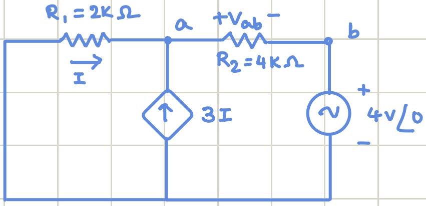
A.



at a, $\frac{V_a'-6}{2000}+\frac{V_a'}{4000}=3I \Rightarrow \frac{3V_a'-12}{4000}=3I \Rightarrow I=\frac{V_a'-4}{4000}$

also, $I=\frac{6-V_a'}{2000}=\frac{V_a'-4}{4000} \Rightarrow 12-2V_a'=V_a'-4 \Rightarrow V_a'=\frac{16}{3}$ V

$V_{ab}'=V_a'=\frac{16}{3}$ V



at a, $\frac{V_a''}{2000}+\frac{V_a''-4}{4000}=3I \Rightarrow I=\frac{3V_a''-4}{12000}$

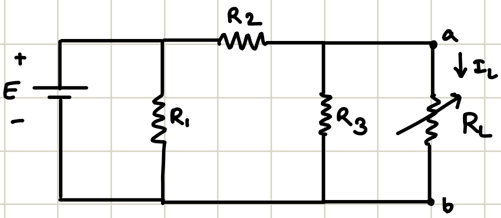
also, $I=\frac{-V_a''}{2000}=\frac{3V_a''-4}{12000} \Rightarrow V_a''=\frac{4}{9}$ V

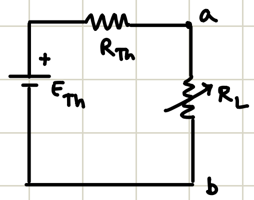
$V_{ab}''=V_a''-4=-\frac{32}{9}$ V

$\Rightarrow V_{ab}=V_{ab}'+V_{ab}''=\frac{16}{9}$ V

Thevenin's Theorem

→ Same current passes through the terminals after replacing the original circuit across the terminals with the thevenin's equivalent circuit

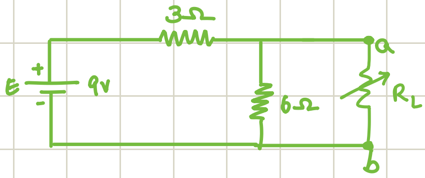




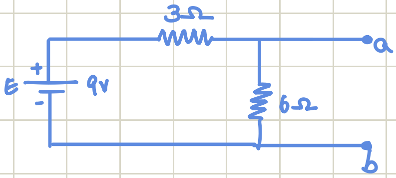
→ Procedure

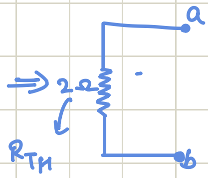
1) Remove that portion of the network across which Thevenin's equivalent circuit is to be found
2) Mark the terminals of the remaining 2-terminal network
3) Calculate R_{Th} by first setting all sources to zero & then finding resultant resistance b/w 2 marked terminals
4) Voltage sources are replaced by short circuits & current sources by open circuits
5) Calculate E_{Th} by first returning all sources to original position & finding open-circuit voltage b/w marked terminals
6) Draw Thevenin equivalent circuit

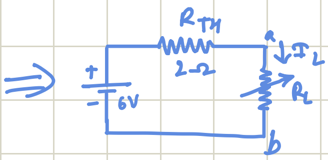
Q)



Find R_L & I_L for $R_L = 100\,\Omega$





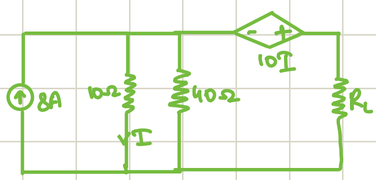


$$I_L = \frac{E_{TH}}{R_{TH}+R_L} = \frac{6}{2+100}$$

$$= 0.058A$$

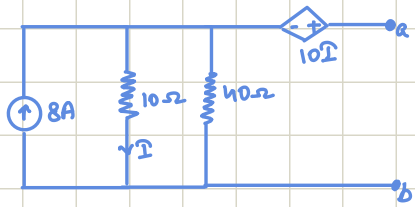
$$E_{TH} = E_{R_2} = \frac{9\times 6}{3+6} = 6V$$

Q)



Find V_{TH} accross R_L

A.



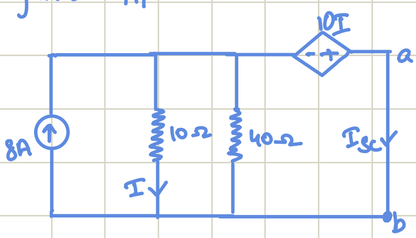
$$I = \frac{8\times 4\cancel{0}}{5\cancel{0}} = 6.4A$$

$$V_{ab} - 10I - 10I = 0$$

$$\Rightarrow V_{ab} = 20\times 6.4$$

$$= 128V$$

To find R_{TH}:



WKT, $R_{TH} = \frac{E_{Th}}{I_{sc}}$ $\quad E_{Th} = 128V$

All branches are parallel

$V_{10\Omega} = 10\times I \longrightarrow ①$

But drop voltage source is parallel to 10Ω. $\therefore V_{10\Omega} = -10I$ —

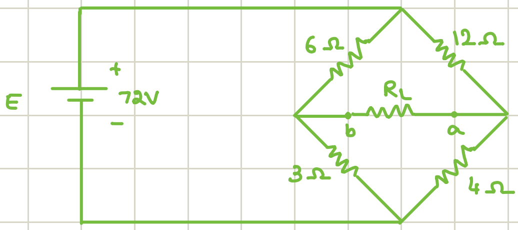
So from ① & ②,

$10\times I = -10I$ or $I=0$

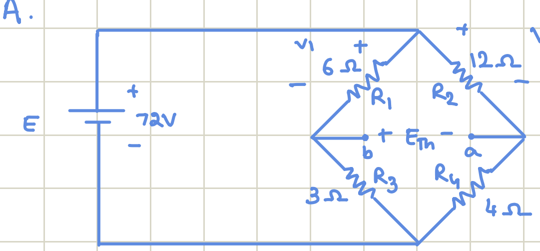
Therefore the nodal voltage $V_c = 0V$

By KCL, $I_{sc} = 8A \Rightarrow R_{TH} = \frac{128}{8} = 16\Omega$

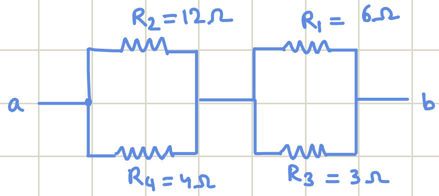
Q. Find Thevenin's equivalent across a-b



A.



⇒

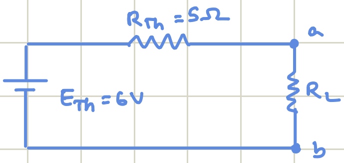


⇒ $R_{Th} = 3 + 2 = 5\Omega$

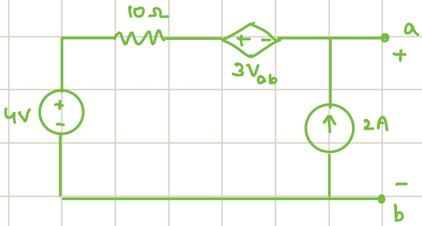
$$V_1 = \frac{E \times R_1}{R_1 + R_3} = \frac{72 \times 6}{6 + 3} = 48V$$

$$V_2 = \frac{E \times R_2}{R_2 + R_4} = \frac{72 \times 12}{12 + 4} = 54V$$

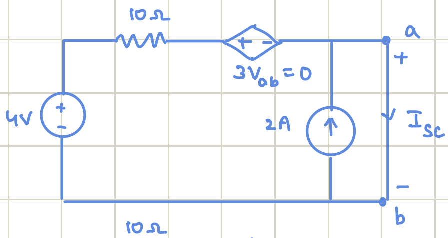
KVL for mesh 1 ⇒ $E_{Th} + V_1 - V_2 = 0$ ⇒ $E_{Th} = 54 - 48 = 6V$

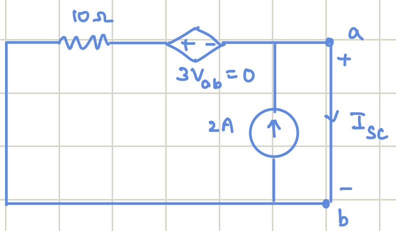


Q.

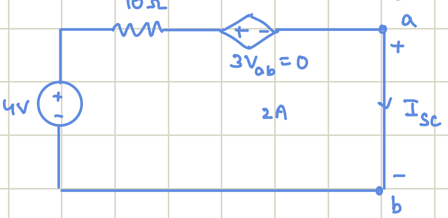


A. $V_{ab} = 6V$





$I_{sc_2} = 2A$

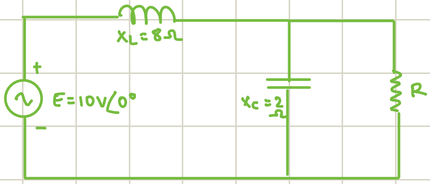


$-4 + 10 I_{sc_1} + 0 = 0$ ⇒ $I_{sc_1} = 0.4A$

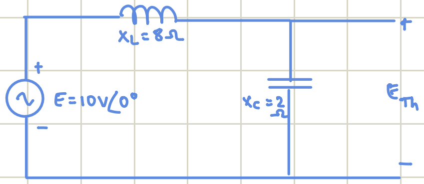
$I_{sc} = I_{sc_1} + I_{sc_2} = 2.4A$

$$R_{Th} = \frac{E_{Th}}{I_{sc}} = \frac{6}{2.4} = 2.5\Omega$$

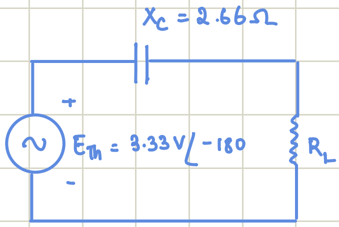
Q.



A.



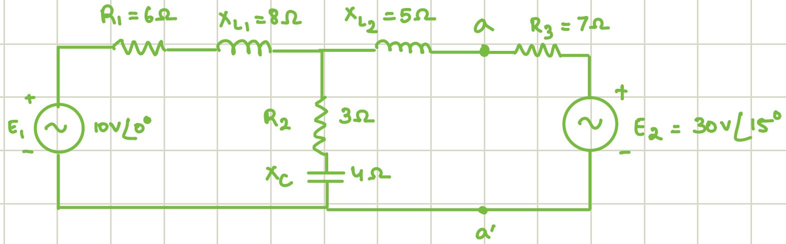
$\Rightarrow$



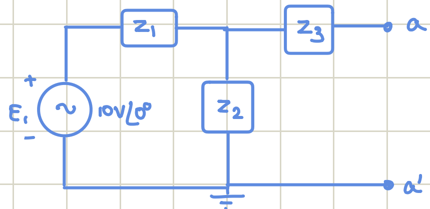
$$E_{Th} = \frac{E \times (-2j)}{8j - 2j} = 3.33V \angle -180$$

$$Z_{Th} = \left(\frac{1}{8j} - \frac{1}{2j}\right)^{-1} = 2.60\ \Omega \angle -90$$

Q.



A.



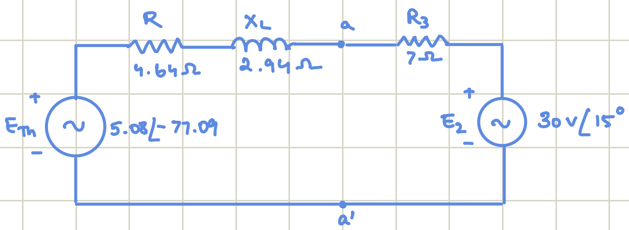
$Z_1 = 6 + j8\ \Omega$

$Z_2 = 3 - j4\ \Omega$

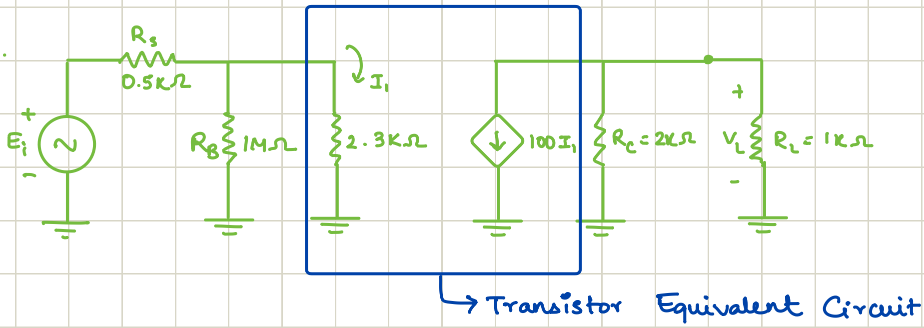
$Z_3 = j5\ \Omega$

$$Z_{Th} = Z_3 + (Z_1 \| Z_2) = j5 + \frac{(6+j8)(3-j4)}{9+j4} = 5.49 \angle 32.36°\ \Omega$$

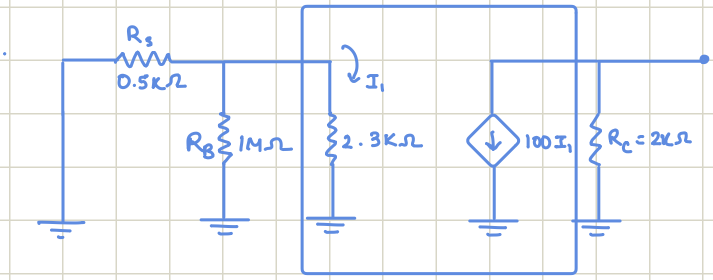
$$E_{Th} = \frac{E_1 Z_2}{Z_1 + Z_2} = \frac{10 \times (3-j4)}{9+j4} = 5.08 \angle -77.09\ V$$



Q.



A.

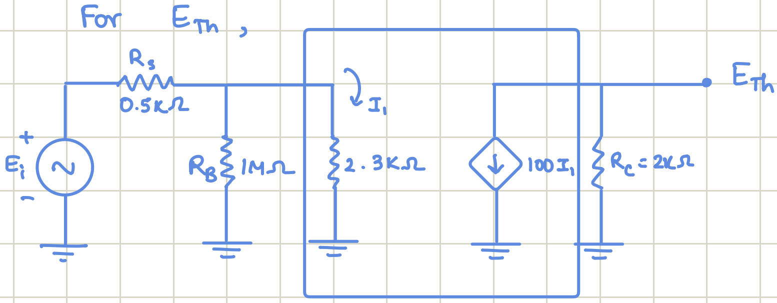


$I_1 = 0$ cuz No voltage Source

Dependant Source also is 0

$Z_{Th} = 2K\Omega$

For E_{Th},



By KCL $\Rightarrow$ $100 I_1 + \frac{E_{Th}}{2000} = 0$

$$I_1 = \frac{E_i}{R_s + 2300} = 0.357 \times 10^{-3} E_i$$

$$E_{Th} = -2000 \times 100 I_1 = -71.4 E_i$$

Norton's Theorem

→ Any 2-terminal linear bilateral DC network can be replaced by an equivalent circuit consisting of a current source (I_N) & a parallel resistor

(Norton's & Thevenin's are the dual of each other)

→ Procedure

→ Remove portion across which norton equivalent circuit is found

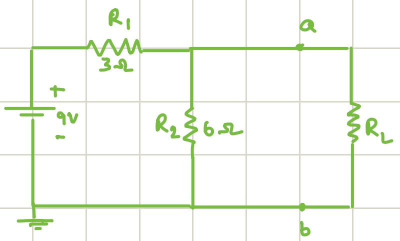
→ Mark the terminals of remaining '2-terminal network'

→ Calculate R_N by setting all sources to zero & finding resultant resistance b/w marked 2 terminals

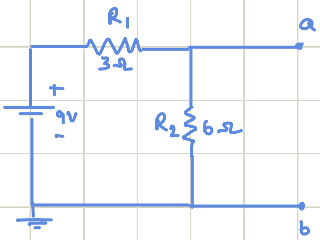
→ Calculate I_N by setting all sources back to original & finding short circuit b/w marked terminals

→ Draw Norton equivalent with portion of circuit initially removed replaced b/w terminals of equivalent circuit

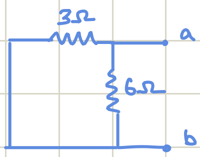
Q.



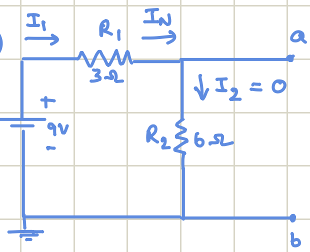
A. 1) Remove R_L b/w a & b



2) $R_N \Rightarrow 3 \| 6 = 2\Omega$

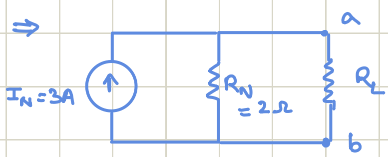


3)

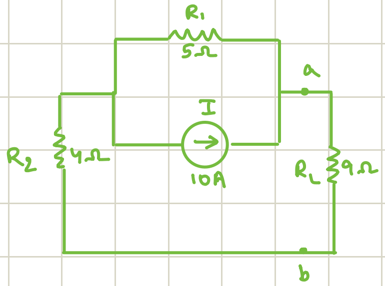


$I_N = \frac{9}{3} = 3A$

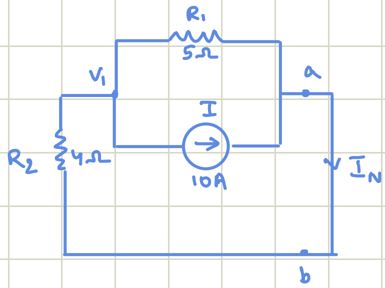
Norton's equivalent ⇒



Q.



A. To find Norton's current



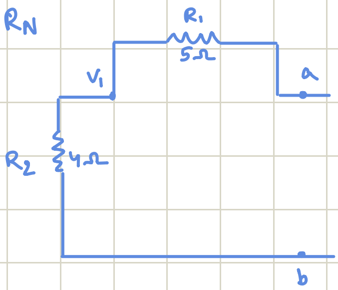
at V_1,

$\Rightarrow \frac{V_1}{4} + \frac{V_1}{5} = -10 \Rightarrow V_1 = \frac{-200}{9}$

$\Rightarrow$ at a,

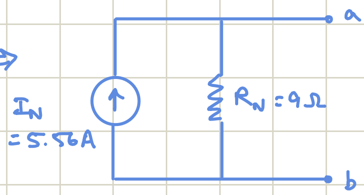
$-\frac{V_1}{5} - 10 + I_N = 0 \Rightarrow I_N = 5.56A$

To find R_N

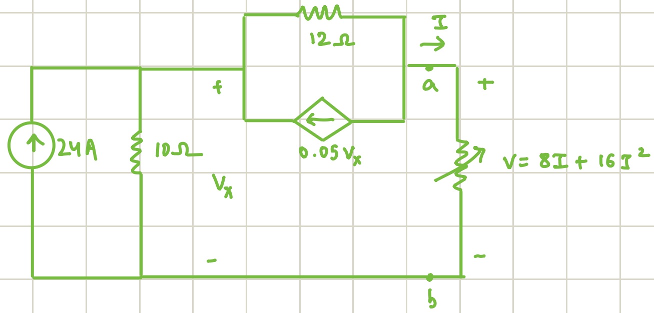


$R_N = 9\Omega$

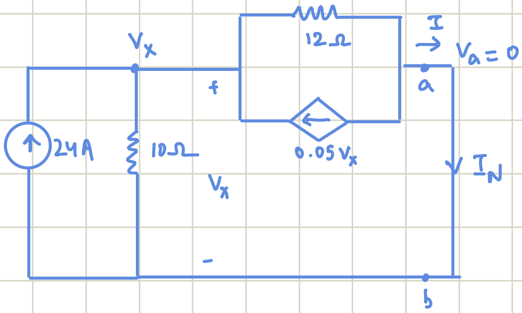
$\Rightarrow$



Q.



A.



at node x,

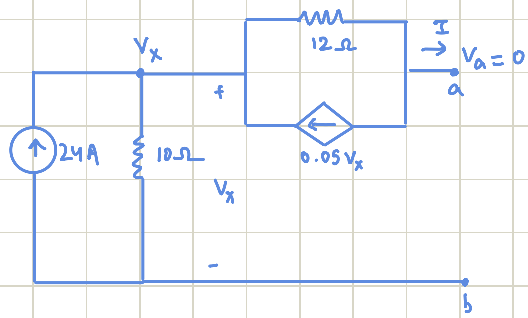
$\frac{V_x}{10} + \frac{V_x}{12} = 24 + 0.05V_x$

$V_x = 180V$

at node a,

$\frac{0 - V_x}{12} = -I_N - 0.05V_x \Rightarrow I_N = \frac{V_x}{30} = 6A$

Find V_{oc} b/w a-b



$V_{oc} + V_{12} - V_x = 0$

To find V_{oc},

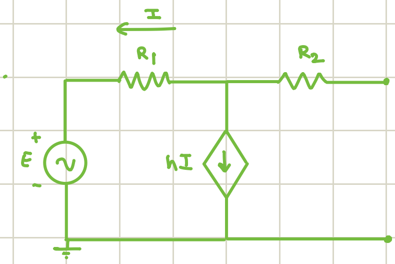
KCL at x: $-24 + \frac{V_x}{10} = 0 \Rightarrow V_x = 240V$

KCL at a: $\frac{V_{oc} - V_x}{12} = -0.05V_x$

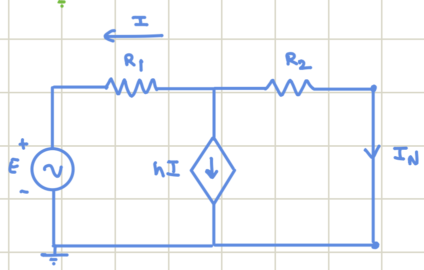
$\Rightarrow V_{oc} = 12\left(\frac{1}{12} - 0.05\right)V_x$

$= 96V$

Q.



A.



To find I_N,

$$I = \frac{V_1 - E}{R_1}$$

$$\Rightarrow hI = \frac{h(V_1 - E)}{R_1}$$

KCL at node 1,

$$\frac{V_1 - E}{R_1} + \frac{h(V_1 - E)}{R_1} + \frac{V_1}{R_2} = 0$$

$$V_1\left[\frac{1}{R_1} + \frac{h}{R_1} + \frac{1}{R_2}\right] = \frac{(h+1)E}{R_1}$$

$$V_1\left[\frac{R_2 + hR_2 + R_1}{R_1R_2}\right] = (1+h)\frac{E}{R_1}$$

$$V_1 = \frac{(1+h)ER_2}{R_2 + hR_2 + R_1}$$

As $I_N = \frac{V_1}{R_2} = \frac{(1+h)E}{R_1 + R_2(1+h)}$

$$\frac{V_{oc} - E}{R_1} + \frac{h(V_{oc} - E)}{R_1} = 0$$

$$V_{oc}\left[\frac{1}{R_1} + \frac{h}{R_1}\right] = \frac{1+h}{R_1} \cdot E$$

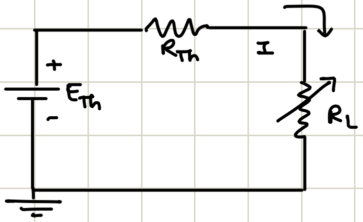
$$V_{oc} = E$$

$$Z_N = \frac{V_{oc}}{I_N} = \frac{E}{\frac{(1+h)E}{R_1 + R_2(1+h)}}$$

$$Z_N = \frac{R_1 + R_2(1+h)}{1+h}$$

Maximum Power Transfer Theorem

→ A load will receive max power from a linear bilateral DC/AC network when its total resistive value is exactly equal to Thevenin's Resistance/Impedance of the DC/AC network as 'seen' by the load



$R_L = R_{Th} = R_N$

→ $P_L = I^2 R_L = \left(\frac{E_{Th}}{R_{Th}+R_L}\right)^2 R_L$

$P_{max} = I^2 R_{L(opt)}$

→ P_L is concave function of R_L

$$\frac{dP_L}{dR_L} = E_{Th}^2 \left(\frac{(R_L+R_{Th})^2 - 2R_L(R_{Th}+R_L)}{(R_L+R_{Th})^4} \right) = 0$$

$$\boxed{\underbrace{R_L = R_{Th}}_{\text{Condition}} \Rightarrow P_{L(max)} = \underbrace{\frac{E_{Th}^2}{4R_{Th}} = \frac{I_N^2 R_{Th}}{4}}_{\text{Max Power transferred}}}$$

Q. $E_{Th} = 60V$, $R_{Th} = 9\Omega$

Analyze I_L , V_L and P_L

A. $I_L = \frac{E_{Th}}{R_{Th}+R_L} = \frac{60}{9+R_L}$

$V_L = \frac{R_L E_{Th}}{R_{Th}+R_L} = \frac{R_L(60)}{9+R_L}$

$P_L = V_L I_L = \frac{3600 R_L}{(9+R_L)^2}$

DC Operating Efficiency (η)

→ Ratio of power delivered to the load to power supplied by source

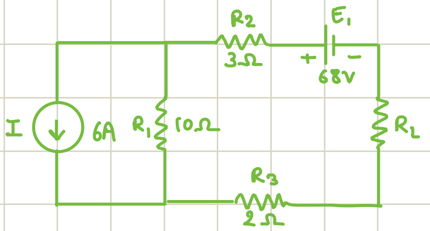
$\eta\% = \frac{P_L}{P_s} \times 100\% = \frac{I_L^2 R_L}{E_{Th} I_L} \times 100\% = \frac{R_L}{R_{Th}+R_L} \times 100\%$

→ Under max power transfer condition $(R_L = R_{Th})$, only 50% efficiency achieved by DC Circuits

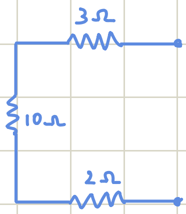
→ R_L for given efficiency & Thevenin's circuit is given by

$R_L = \frac{\eta R_{Th}}{1-\eta}$

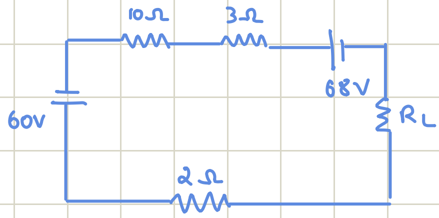
Q. For the below circuit, Find R_L that results to maximum power transfer & P_{Lmax}?



A. $R_{Th} \Rightarrow$



$R_{Th} = 15\Omega = R_L$



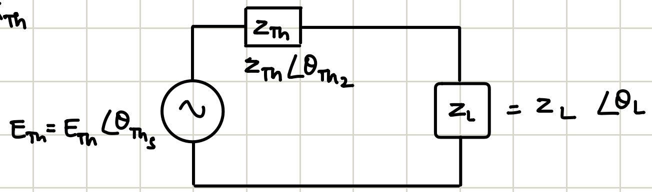
$E_{Th} = -128V$

$$P_{Lmax} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128)^2}{4 \times 15} = 273.067\ W$$

Maximum Power Transfer Theorem (AC)

→ A load will receive Max power transfer when variable impendance is equal to conjugate of Thevenin's resistance

$$Z_L = Z_{Th}^*$$
$$Z_L = R_L \pm jX_L = R_{Th} \mp jX_{Th}$$
$$Z_L = Z_L \angle\theta_L = Z_{Th} \angle -\theta_{Th}$$



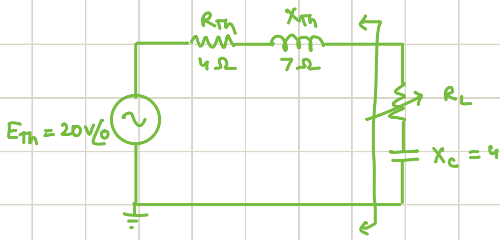
→ $P_{Lmax} = \frac{E_{Th}^2}{4R_{Th}}$

→ Suppose only load resistance (R_L) is variable & load reactance (X_L) is fixed then the condition for maximum power is given by

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} \pm X_L)^2} \Rightarrow P_{Lmax} = \frac{E_{Th}^2}{4R_{avg}} \quad \text{where} \quad R_{avg} = \frac{R_L + R_{Th}}{2}$$

Q. Determine optimal R_L for Max P.T when load reactance is fixed to 4Ω. What is P_{Lmax}?

Suppose load reactance can also be varied. What is the optimal load impedance for maximum power transfer? P_{Lmax}?



A. $R_L = \sqrt{R_{Th}^2 + (X_{Th} \pm X_L)^2}$

$= \sqrt{4^2 + (7-4)^2} = 5\Omega$

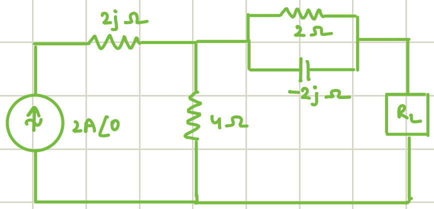
$R_{avg} = \frac{R_{Th} + R_L}{2} = 4.5\Omega$

$P_{Lmax} = \frac{E_{Th}^2}{4R_{avg}} = \frac{400}{4 \times 4.5} \sim 22W$

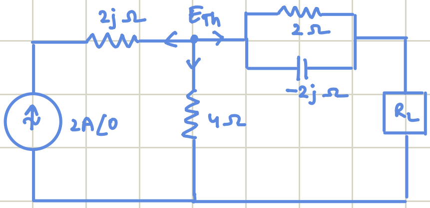
$Z_L = Z_{Th}^* = (4 - 7j)\Omega$

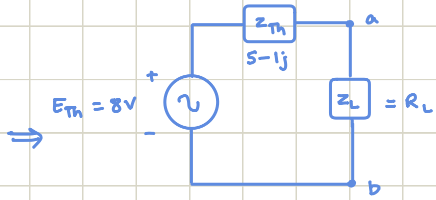
$P_{Lmax} = \frac{E_{Th}^2}{4R_{Th}} = 25W$

Q. Find Optimal R_L.



A. $R_{Th} = 4 + \left(\frac{1}{2} - \frac{1}{2i}\right)^{-1} = (5 - 1j)\ \Omega$





$\frac{E_{Th}}{4} = 2 \Rightarrow E_{Th} = 8V$

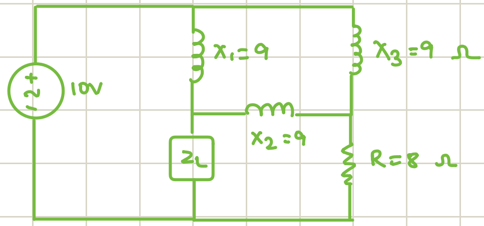
$X_L = 0$; $R_L = \sqrt{R_{Th}^2 + (X_L \pm X_{Th})^2}$

$= \sqrt{5^2 + (0-1)^2} = 5.1\ \Omega$

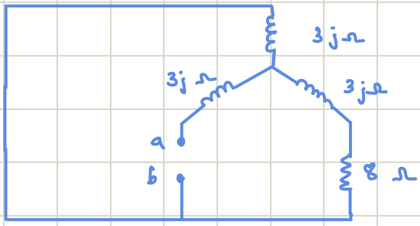
$R_{avg} = \frac{R_{Th} + R_L}{2} = \frac{5 + 5.1}{2} = 5.05\ \Omega$

$P_{L\,max} = \frac{|E_{Th}|^2}{4R_{avg}} = 0.627W$

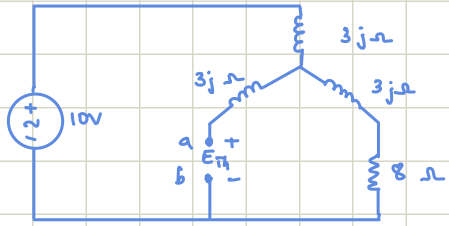
Q.



A.



$R_{Th} = 3j + \left(\frac{1}{3j} + (8 + 3j)^{-1}\right)^{-1} = \frac{18}{25} + \frac{273}{50}j = 0.72 + j\,5.46\ \Omega$



$E_{Th} = 10\angle 0 \times \frac{(8+3j)}{8+3j+3j} = 8.54\ \angle -16.31°$

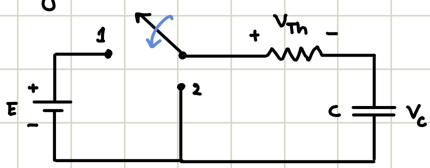
$P_{RL} = \frac{|E_{Th}|^2}{4R_{Th}} = 25.34\ W$

NAS UNIT - 2

Transients : RC Charging & discharging phase

→ Charging Phase

→ Voltage across the capacitor doesn't change instantaneously



i_c = current through the capacitor

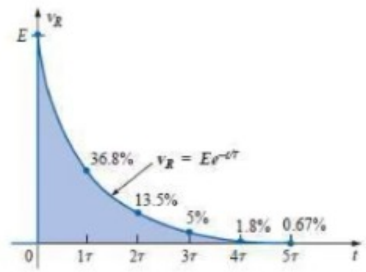
a) $t = 0^-$ ⇒ time before switching ⇒ $i_c(0^-) = 0A$
$V_c(0^-) = 0V$

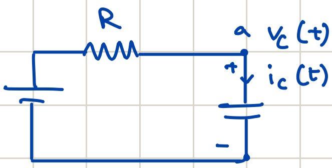
b) $t = 0$ ⇒ time of switching ⇒ $i_c(0) = E/R$
$V_c(0) = V_c(0^-)$

c) $t \geq 0$ ⇒ $V_c(t)$, $i_c(t) = C\dfrac{dV_c(t)}{dt}$

At $t = 0$: C acts as short circuit
$t = \infty$: C acts as open circuit

At $t = \infty$, $V_c(\infty) = E$
$i_c(\infty) = 0A$





→ KCL at a,

$$i_c(t) + \frac{V_c(t) - E}{R} = 0$$

$$C\frac{dV_c(t)}{dt} = \frac{E - V_c(t)}{R}$$

$$\int_0^{V_c} \frac{dV_c(t)}{E - V_c(t)} = \int_0^{t} \frac{dt}{RC}$$

$$\Big[-\ln(E - V_c)\Big]_0^{V_c} = t/RC$$

$$-\ln(E - V_c) + \ln E = t/RC$$

$$\ln\left(\frac{E}{E - V_c}\right) = t/RC$$

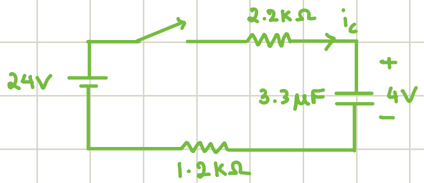
$$\frac{E}{E - V_c} = e^{t/RC}$$

$$\frac{E - V_c}{E} = e^{-t/RC}$$

$$1 - \frac{V_c}{E} = e^{-t/RC} \Rightarrow \boxed{V_c(t) = E\left[1 - e^{-t/RC}\right]} \Rightarrow \boxed{i_c(t) = \frac{E}{R}\left[1 - e^{-t/RC}\right]}$$

$$\boxed{V_c(t) = V_f + (V_i - V_f)\,e^{-t/RC}} \Longrightarrow \boxed{t = \tau \ln\left(\frac{V_i - V_f}{V_c(t) - V_f}\right)}$$

Q. Find $V_c(t)$ & $i_c(t)$. Find t at which $V = 21V$



A. $\tau = RC = (1.2 + 2.2) \times 10^{3} \times 3.3 \times 10^{-6} = 11.22 \times 10^{-3}$ s

$i_c(0) = \dfrac{24 - 4}{(2.2 + 1.2) \times 10^{3}} = 5.88$ mA , $V_c(0) = 4V$

(After a long period of time when switch is on, capacitor behaves like open ckt)

$V_c(\infty) = 24V$, $i_c(\infty) = 0A$

$$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty))e^{-t/\tau}$$
$$= 24 + (4 - 24)e^{-t/11.22\times10^{-3}}$$
$$= 24 - 20e^{\frac{-t}{11.22m}} \; V$$

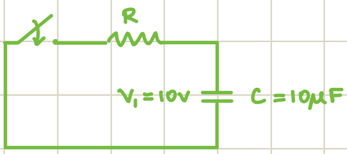
$$i_c(t) = \frac{20e^{-t/11.22m}}{3.4 \times 10^{-3}} = 5.88e^{\frac{-t}{11.22m}} \; mA$$

If $V = 21V$, $21 = 24 - 20e^{-t/11.22m}$

$$\ln\left(\frac{20}{3}\right) = \frac{t}{11.22 \times 10^{-3}}$$

$$\Rightarrow t = 0.021 \text{ s}$$

Q. Capacitor was charged $V_1 = 10V$. What should be value of R such that capacitor is 90% discharged at 5ms



A. $\tau = RC = 10^{-5}R$

$V_c(t) = 1V$ @ 5ms

$V_c(0) = V_c(0^-) = 10V$

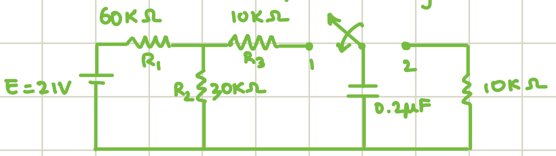
$V_c(\infty) = 0V$

$$\tau = \frac{t}{\ln\left(\frac{V_c(0) - V_c(\infty)}{V_c(t) - V_c(\infty)}\right)} = \frac{5 \times 10^{-3}}{\ln\left(\frac{10 - 0}{1 - 0}\right)} = 2.17 \times 10^{-3}$$

$$R = \frac{\tau}{C} = \frac{2.17 \times 10^{-3}}{10^{-5}} = 217\Omega$$

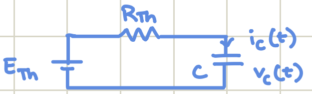
Q. a) Find mathematical expression for the transient behaviour of voltage V_c & i_c by closing switch pos 1. $V_c(0) = 0V$

b) Find mathematical expression for the transient behaviour of voltage V_c & i_c by closing switch pos 2 at $t = 9ms$



A. a) $E_{Th} = \dfrac{E\,R_2}{R_1+R_2} = \dfrac{21\times30}{60+30} = 7V$

$R_{Th} = R_3 + \left(R_1^{-1}+R_2^{-1}\right)^{-1} = 10 + \left(60^{-1}+30^{-1}\right)^{-1} = 30\,K\Omega$



Using thevenin's eq circuit,

$v_c(0^-) = 0V$

$V_c(0) = V_c(0^-) = 0V$

$i_c(0) = \dfrac{E_{Th}}{R_{Th}} = 0.23\,mA$

$V_c(\infty) = 7V \qquad i_c(\infty) = 0A$

$\tau = RC = 30\times10^{3}\times0.2\times10^{-6} = 6ms$

$V_c(t) = V_c(\infty) + \left(V_c(0) - V_c(\infty)\right)e^{-t/\tau}$

$= 7 + (0-7)e^{-t/6m} = 7 - 7e^{-t/6m}$

$i_c(t) = i_c(\infty) + \left(i_c(0) - i_c(\infty)\right)e^{-t/\tau}$

$= 0 + (0.23m - 0)e^{-t/6m}$

$= 0.23mA.\ e^{-t/6m}$

b) at $t = 9ms$

$V_i = 7 - 7e^{-1.5}$

$= 5.44\,V$

$V_f = 0V$

$\tau_2 = R_4\times C = 10\times10^{3}\times0.2\times10^{-6} = 2ms$

$V_{c_2}(t) = V_f + (V_i - V_f)e^{-t/\tau_2}$

$= -5.44\,e^{-t/2m}\ V$

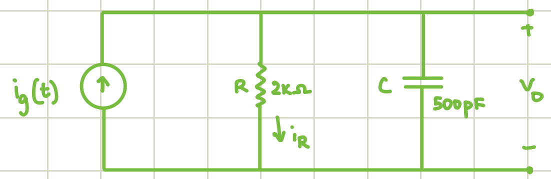
$i_i = \dfrac{-V_i}{R_4} = \dfrac{-5.44}{10^4} = -0.544\,mA$

$i_f = 0A$

$i_{c_2}(t) = i_f + (i_i - i_f)e^{-t/\tau}$

$= -0.544\,mA\ .\ e^{-t/2m}$

Q. After being at zero for a long time, value of $i_g(t)$ changes to 15mA at $t=0$ (and remains the same till $t=\infty$). Find $V_o(t)$ for $t\geq0$. Find i_R expression.



A. $\tau = R_{Th}C = 2\times10^{3}\times500\times10^{-12} = 10^{-6}\,s$

$V_o(0) = 0V$, $V_o(\infty) = 2000\times15\times10^{-3} = 30V$

$V_o(t) = V_o(\infty) + \left(V_o(0) - V_o(\infty)\right)e^{-t/\tau}$

$= 30 + (0-30)e^{-t/10^{-6}}$

$= 30\left(1 - e^{-t/10^{-6}}\right)\ V$

$i_c(0) = 15mA$; $i_R(0) = 0A$

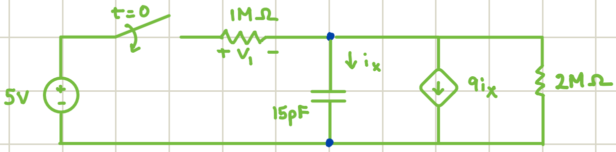
$i_c + i_R = i_g = 15mA$

$i_c(\infty) = 0A \Rightarrow i_R(\infty) = 15A$

$i_R(t) = i_R(\infty) + \left(i_R(0) - i_R(\infty)\right)e^{-t/\tau} = 15\left(1 - e^{-t/10^{-6}}\right)A$

Q. $V_1(t)$ for $t \geq 0$

(To find R_{Th}, find I_{sc} & E_{Th} across the blue terminals)



A. i) $V_1(0^-) = 0V$, $V_c(0^-) = 0V$

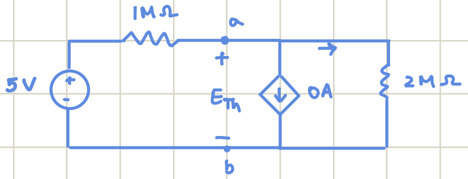
At $t=0$, $V_c(0) = V_c(0^-) = 0V$, capacitor voltage doesn't change instantaneously

By KVL, $V_1(0) + V_c(0) = 5V \Rightarrow V_1(0) + 0 = 5V \Rightarrow V_1(0) = 5V$

At $t=\infty$, $V_c(\infty) = 5 \times \frac{2}{1+2} = 3.33V$

has fully charged By KVL : $V_1(\infty) = 5 \times \frac{1}{1+2} = 1.67V$ & acts as open circuit

ii) For $t \geq 0$

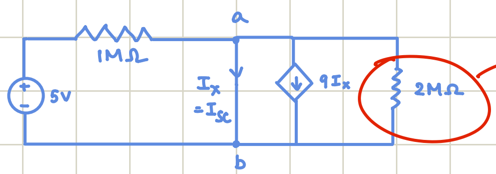


⟶ Thevenin's

KCL at a, $\frac{E_{Th} - 5}{1M} + 0 + \frac{E_{Th}}{2M} = 0$

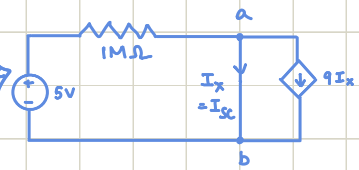
$2E_{Th} - 10 + E_{Th} = 0 \Rightarrow E_{Th} = 3.33V$

To find I_{sc} :



Eliminated cuz in $\|^{el}$ with short circuit

$\Rightarrow$



KCL at node a : $\frac{0-5}{1M} = -I_x - 9I_x \Rightarrow I_x = 0.5\,\mu A$

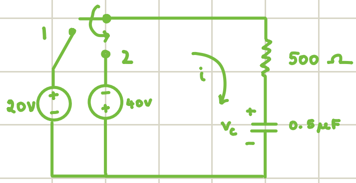
$R_{Th} = \frac{E_{Th}}{I_{sc}} = \frac{3.33}{0.5 \times 10^{-6}} = 6.66\,M\Omega$

iii) $\tau = R_{Th}C = 6.66M \times 15pF = 99.9\,\mu s \approx 100\,\mu s$

iv) $V_1(t) = V_1(\infty) + (V_1(0) - V_1(\infty))\, e^{-t/\tau}$

$= 1.67 + 3.33\, e^{-t/100\times10^{-6}}\ V$

Q. The switch in the circuit is closed on position 1 at $t=0$ & then moved to 2 after one time constant, at $t = \tau = 250\,ms$. Obtain the current for $t > 0$



A.

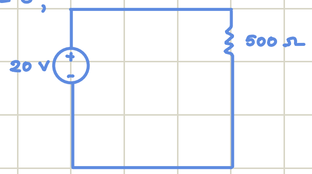
$$i(t) = \begin{cases} 0.04\, e^{-t/250\mu}\ A, & t \leq \tau = 250\,\mu s \\ -0.105\, e^{-t/250\mu}\ A, & t \geq \tau \end{cases}$$

i) for $t \geq 0$ & switch at position 1

$\tau = RC = 500 \times 0.5 \times 10^{-6} = 250\,\mu s$

$i(0^-) = 0A$, $i(0) = \dfrac{20}{500} = 0.04\ A$

at $t = 0$,



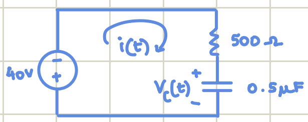
at $t = \infty$, $i(\infty) = 0A$

$$i(t) = i(\infty) + (i(0) - i(\infty))e^{-t/\tau}$$
$$= 0.04\, e^{-t/250\times10^{-6}}\ A$$
$$i(\tau^-) = 0.04\, e^{-1} = 0.014\ A$$

$V_c(0^-) = 0V$, $V_c(0) = V_c(0^-) = 0\,V$

$V_c(\infty) = 20V \Rightarrow V_c(t) = 20\left(1 - e^{-t/\tau}\right)$

at $t = \tau$



Voltage across capacitor does not change instantaneously upon switching but the current does

By KVL at $t = \tau$,

$$40 + 500\, i(\tau) + V_c(\tau) = 0$$

$$i(\tau) = \frac{-12.64 - 40}{500} = -0.105\ A$$

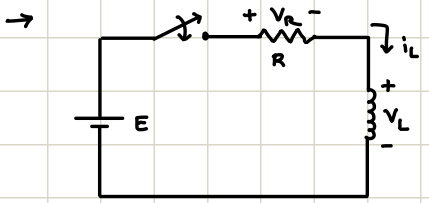
$V_c(\tau) = 20(1 - e^{-1})$

$= 12.64\ V$

$i(\infty) = 0A$ because capacitor acts as open circuit

RL Transients – Storage Phase

→



By KVL, $E - Ri_L(t) - V_L(t) = 0$

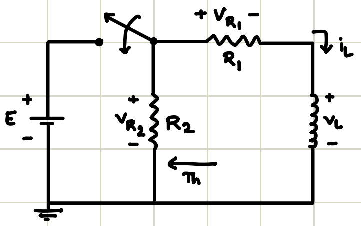
$$E - Ri_L(t) + L\frac{di_L(t)}{dt} = 0$$

$$i_L R + L\frac{di_L}{dt} = E \Rightarrow \frac{di_L}{i_L - \frac{E}{R}} = \frac{-R}{L}dt \Rightarrow \int_0^{i_L(t)} \frac{di_L}{i_L - \frac{E}{R}} = \frac{-R}{L}\int_0^t dt$$

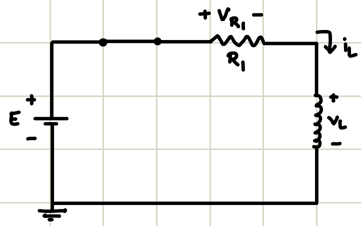
$$\ln\left(i_L - \frac{E}{R}\right)_0^{i_L} = \frac{-Rt}{L} \Rightarrow \ln\left(\frac{i_L(t) - E/R}{-E/R}\right) = -Rt/L \Rightarrow \ln\left(\frac{-i_L(t)}{E/R} + 1\right) = -Rt/L$$

$$\frac{i_L(t)}{E/R} = 1 - e^{-Rt/L} \Rightarrow \boxed{i_L(t) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)} \quad \& \quad \boxed{V_L(t) = L\frac{di_L(t)}{dt} = E\left(1 - e^{-\frac{Rt}{L}}\right)}$$

RL Transients – Decay Phase



⇒



$V_L(0^-)$; $i_L(0^-) = E/R_1$

$$I_L = i_L(0) = i_L(0^-) = E/R_1$$

$$V_i = V_L(0) = -\left(\frac{R_1 + R_2}{R_1}\right)E$$

$$V_L(0) + i_L(0)(R_1 + R_2) = 0$$

$$V_L(0) = -(R_1 + R_2)\frac{E}{R_1}$$

$$V_L(0) + i_L(0)(R_1 + R_2) = 0$$

$$L\frac{di_L}{dt} = -i_L(0)(R_1 + R_2)$$

$$\int_{E/R_1}^{i_L(t)} \frac{di_L}{i_L} = -\int_0^t \frac{R_1 + R_2}{L}dt \Rightarrow \ln(i_L)_{E/R_1}^{i_L(t)} = -\frac{(R_1 + R_2)}{L}(dt)_0^t$$

$$\ln\left(\frac{i_L(t)}{E/R_1}\right) = -\left(\frac{R_1 + R_2}{L}\right) \times t$$

$$\boxed{i_L(t) = \frac{E}{R_1}e^{-\left(\frac{R_1 + R_2}{L}\right)t}}$$

$$\Rightarrow V_L(t) = L\frac{di_L(t)}{dt} = -L \times \frac{E}{R_1} \times \left(\frac{R_1 + R_2}{L}\right)e^{-\left(\frac{R_1 + R_2}{L}\right)t} \Rightarrow \boxed{V_L(t) = -\left(1 + \frac{R_2}{R_1}\right)Ee^{-t/\tau}}$$

Decay Phase

$\rightarrow \quad i_L(t) = \frac{E}{R_1} e^{-t/\tau_d}$

$\rightarrow \quad v_L(t) = -\left(1 + \frac{R_2}{R_1}\right) E e^{-t/\tau_d}$

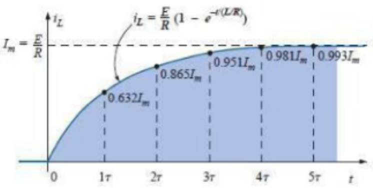
$\left(\tau_d = \frac{L}{R_1 + R_2}\right)$

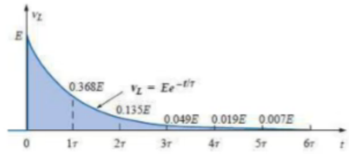
Storage Phase

$\rightarrow \quad i_L(t) = \frac{E}{R_1}\left(1 - e^{-t/\tau_s}\right)$

$\rightarrow \quad v_L(t) = E\left(e^{-t/\tau_s}\right)$

$\left(\tau_s = \frac{L}{R_1}\right)$





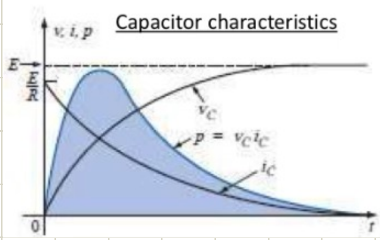
$$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right) e^{-t/\tau}$$

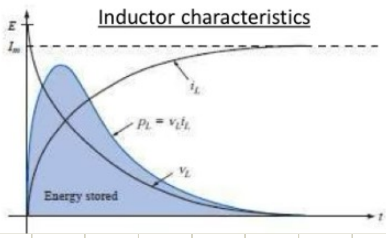
$$v_L(t) = v_L(\infty) + \left(v_L(0) - v_L(\infty)\right) e^{-t/\tau}$$

Energy

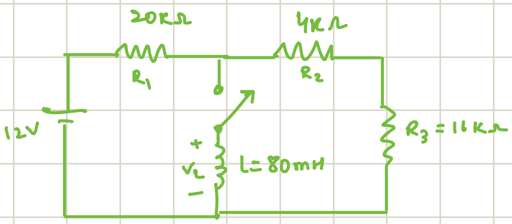
$\rightarrow \quad W_C = \frac{1}{2} C V^2 = \frac{1}{2} C \left(\frac{Q}{C}\right)^2 = \frac{Q^2}{2C}$

$\rightarrow \quad W_L = \frac{L I_m^2}{2}$





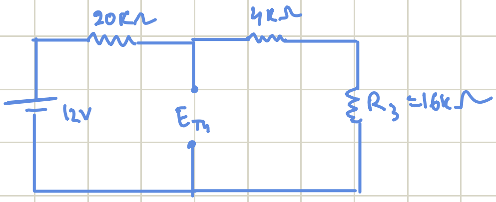
Q.



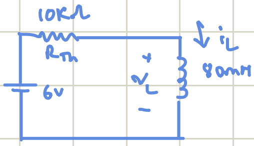
A.

$R_{Th} = R_1 \parallel (R_2 + R_3) = 10K\Omega$

$E_{Th} = \frac{E \times (R_2 + R_3)}{R_1 + R_2 + R_3} = 6V$



$\Rightarrow$

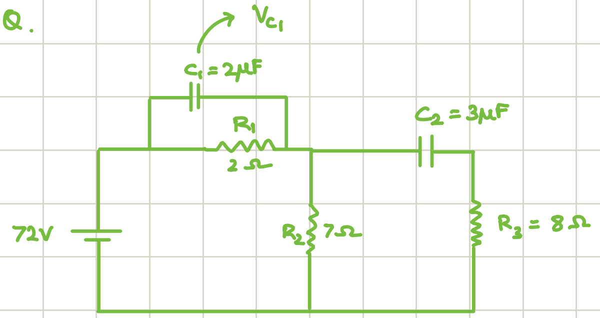


$I_i = i_L(0) = i_L(0^-) = 0$

$I_f = i_L(\infty) = E/R_{Th} = 0.6mA$

$V_i = v_L(0) = 6V$

Q.



A.

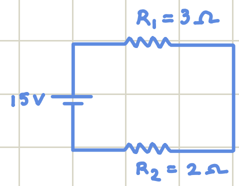
$$V_{C_1}(\infty) = V_{R_1} = \frac{E \times R_1}{R_1+R_2} = \frac{72\times 2}{2+7} = 16\text{ V}$$

$$V_{C_2}(\infty) = V_{R_2} = \frac{E \times R_2}{R_1+R_2} = \frac{72\times 7}{2+7} = 56\text{ V}$$

$$E = \frac{1}{2}C_1 V_1^2(\infty) + \frac{1}{2}C_2 V_{C_2}^2(\infty)$$

$$= \left(\frac{1}{2}\times 2\times 10^{-6}\times 16^2\right) + \left(\frac{1}{2}\times 3\times 10^{-6}\times 56^2\right)$$

$$= 4960 \times 10^{-6}\text{ J} = 4.96\text{ mJ}$$

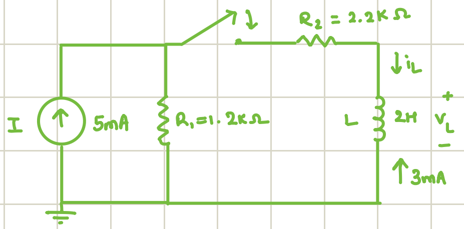


inductor acts as short circuit at $t = \infty$ in storage phase

$$i_L(\infty) = \frac{E}{R_1+R_2} = \frac{15}{2+3} = 3\text{ A}$$

$$E = \frac{1}{2}L i_L^2(\infty) = \frac{1}{2}\times 6\times 10^{-3}\times 3^2 = 27\text{ mJ}$$

Q



A.

$$I_L = i_L(0^-) = -3\text{mA}$$

$$I_f = i_L(\infty) = \frac{6}{R_1+R_2} = 1.76\text{mA}$$

$$V_{R_1}(0) + V_{R_2}(0) + v_L(0) = 0$$

$$-1.2K\, I_{R_1} + 2.2K\, I_L(0) + v_L(0) = 0$$

$$I_{R_1}(0) - I + I_L(0) = 0$$

$$I_{R_1}(0) - 5m - 3m = 0$$

$$I_{R_1}(0) = 8\text{mA}$$

$$-1.2K(8m) + 2.2K(-3m) + v_L(0) = 0 \Rightarrow V_L = 16.2\text{V}$$

$$v_L(\infty) = 0\text{V}$$

$$\tau = \frac{L}{R_1+R_2} = \frac{2}{3400} = 588\,\mu s$$

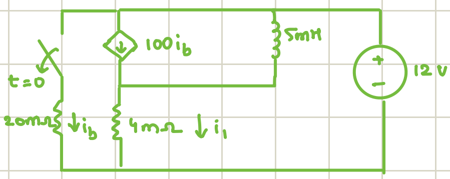
$$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)e^{-t/\tau}$$

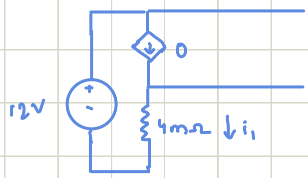
$$= 1.76 + 4.76\, e^{-\frac{t}{588\mu s}}\text{ mA}$$

$$v_L(t) = v_L(\infty) + \left(v_L(0) - v_L(\infty)\right)e^{-t/\tau}$$

$$= 16.2\, e^{-t/588\mu s}\text{ V}$$

Q.





A.

$$-i_1(0^-) + i_L(0^-) + 100\, i_b(0^-)$$

$$i_L(0^-) = i_1(0^-) - 100\, i_b(0^-)$$

$$= \frac{12}{4m} - 100\left(\frac{12}{20m}\right)$$

$$= -57\text{kA}$$

$$i_L(0) = i_L(0^-) = -57\text{kA}$$

$$i_1(\infty) = \frac{12}{4m} = 3\text{kA}$$

$$E_{Th} = 12\text{V}, \quad I_{sc} = \frac{12}{4m}$$

$$R_{Th} = \frac{12}{\frac{12}{4m}} = 4m\Omega$$

$$\tau = \frac{L}{R_{Th}} = \frac{5n}{4m} = 1.25\,\mu s$$

$$i_1(t) = i_1(\infty) + \left(i_1(0) - i_1(\infty)\right)e^{-t/\tau}$$

$$= 3 - 60\, e^{-t/1.25\mu s}\text{ kA}$$

Laplace Transform

→ $L[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$

$$f(t) = L^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)\, e^{st}\, ds \qquad \text{where } s = \sigma + j\omega$$

$$i(t) \xrightarrow{\text{Laplace}} I(s)$$

→ Laplace transform converts differential equation from time domain into algebraic equations in complex frequency domain

→ $f(t)$ to have laplace transform must obey

$$\int_{0^-}^{\infty} |f(t)|\, e^{-st}\, dt \leq \infty$$

→

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{-at}	$\frac{1}{s+a}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

→ $L\left[\sum_i f_i(t)\right] = \sum_i L[f_i(t)]$ (Linearity)

→ If $L[f(t)] = F(s)$, $L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-)$ (Real Differentiation)

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) - \dots - f^{n-1}(0^-)$$

→ If $L[f(t) = F(s)$, $L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$ (Real integration)

$L[t^n f(t)] = (-1)^n \frac{dF(s)}{ds}$ (Differentiation by s)

$F(s-a) = L[e^{at} f(t)]$ (Complex Translation)

$L[f(t-a)\, u(t-a)] = e^{-as} F(s)$ (Shifting Theorem)

Partial Fraction

→ **Real roots**

$$F(s) = \frac{N(s)}{(s-s_0)(s-s_1)(s-s_2)}$$

where s_0, s_1, s_2 are distinct & real and degree of $N(s) < 3$

Then, $F(s) = \frac{K_0}{s-s_0} + \frac{K_1}{s-s_1} + \frac{K_2}{s-s_2}$

→ To solve,

$$K_i = (s-s_i) \cdot F(s)\Big|_{s=s_i}$$

→ **Complex Roots**

$$F(s) = \frac{N(s)}{D_1(s)\,(s+\sigma+j\omega)(s+\sigma-j\omega)}$$

$$F(s) = \frac{N_1(s)}{D_1(s)} + \frac{A_1}{s+\sigma+j\omega} + \frac{A_2}{s+\sigma-j\omega}$$

$$A_1 = (s+\sigma+j\omega)\,F(s)\Big|_{s=\sigma-j\omega}$$

$$= \alpha + j\beta \quad (\text{or}) \quad Ke^{j\phi}$$

$$K = |A_1| = \sqrt{\alpha^2+\beta^2} \quad \& \quad \phi = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

$$f_1(t) = L^{-1}\left(\frac{Ke^{j\phi}}{s+\sigma+j\omega} + \frac{Ke^{-j\phi}}{s+\sigma-j\omega}\right)$$

$$= K\left(e^{j\phi}e^{-(\sigma+j\omega)t} + e^{-j\phi}e^{-(\sigma-j\omega)t}\right)$$

$$= Ke^{-\sigma t}\left(e^{-j(\omega t-\phi)} + e^{j(\omega t-\phi)}\right)$$

$$= 2Ke^{-\sigma t}\cos(\omega t-\phi)$$

Q. $F(s) = \frac{s+3}{(s+5)(s^2+4s+5)}$

A. $F(s) = \frac{A_0}{s+5} + \frac{A_1}{s+2+j} + \frac{A_2}{s+2-j}$

$$A_0 = F(s)\,(s+5)\Big|_{s=-5} = \frac{s+3}{s^2+4s+5} = \frac{-2}{25-20+5} = \frac{-1}{5}$$

$$A_1 = F(s)\,(s+2+j)\Big|_{s=-2-j} = \frac{s+3}{(s+5)(s+2-j)} = \frac{1-j}{(3-j)(-2j)} = 0.1+0.2j$$

$$A_2 = A_1^* = 0.1-0.2j$$

$$F(s) = \frac{-1}{5(s+5)} + \frac{0.1+0.2j}{s+2+j} + \frac{0.1-0.2j}{s+2-j}$$

Inverse Laplace of $F(s) \Rightarrow L^{-1}[F(s)] = \frac{-1}{5}e^{-5t} + 2Ke^{-\sigma t}\cos(\omega t-\phi)$

$K = \sqrt{0.2^2+0.1^2} = 0.223$ $\qquad \phi = \tan^{-1}\left(\frac{0.2}{0.1}\right) = 63.43^\circ$

$$f(t) = \frac{-1}{5}e^{-5t} + 0.446\,e^{-2t}\cos(t-63.43)$$

Q. $\dfrac{s+3}{(s+5)(s^2+4s+5)}$

A. $\dfrac{A}{s+5} + \dfrac{Bs+C}{s^2+4s+5} = \dfrac{A(s^2+4s+5) + (Bs+C)(s+5)}{(s+5)(s^2+4s+5)}$

$$\left.\begin{aligned} A+B &= 0 \\ 4A+5B+C &= 1 \\ 5A+5C &= 3 \end{aligned}\right\} \rightarrow \begin{aligned} A &= -0.2 \\ B &= 0.2 \\ C &= 0.8 \end{aligned}$$

$\Rightarrow \dfrac{-0.2}{s+5} + \dfrac{0.2s+0.8}{(s+2)^2+1^2}$

$L^{-1}[F(s)] = -0.2e^{-5t} + L^{-1}\left[\dfrac{0.2(s+2)}{(s+2)^2+1^2} + \dfrac{0.4}{(s+2)^2+1^2}\right]$

$= -0.2e^{-5t} + 0.2e^{-2t}.\cos t + 0.4e^{-2t}\sin t$

$= -0.2e^{-5t} + 0.2e^{-2t}(\cos t + 2\sin t)$

Q. $Y(s) = \dfrac{3s^2-28}{(s-4)(s^2+4)}$

A. $Y(s) = \dfrac{A}{s-4} + \dfrac{Bs+C}{s^2+4} = \dfrac{A(s^2+4) + (Bs+C)(s-4)}{(s-4)(s^2+4)}$

$$\left.\begin{aligned} A+B &= 3 \\ -4B+C &= 0 \\ 4A-4C &= -28 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 1 \\ B &= 2 \\ C &= 8 \end{aligned}$$

$Y(s) = \dfrac{1}{s-4} + \dfrac{2s+8}{s^2+4}$

$L^{-1}[Y(s)] = e^{4t} + 2\cos 2t + 4\sin 2t$

$= e^{4t} + 2(\cos 2t + 2\sin 2t)$

(OR)

$Y(s) = \dfrac{A_0}{s-4} + \dfrac{A_1}{s+2j} + \dfrac{A_1^*}{s-2j}$

$A_0 = Y(s).(s-4)\big|_{s=4} = \dfrac{3s^2-28}{s^2+4} = \dfrac{20}{20} = 1$

$A_1 = Y(s)(s+2j)\big|_{s=-2j} = \dfrac{3s^2-28}{(s-4)(s-2j)} = \dfrac{-12-28}{(-2j-4)(-4j)} = 1+2j$

$A_2 = 1-2j$

$K = \sqrt{1^2+2^2} = \sqrt{5} = 2.236$ $\qquad \omega = 2 \quad , \sigma = 0$

$\phi = \tan^{-1}\left(\frac{1}{2}\right) = 63.43$

$f(t) = e^{4t} + 2K\cos(2t-\phi)$

$= e^{4t} + 4.472\cos(2t-63.43)$

Q. $F(s) = \dfrac{16}{(s-2)(s^2-7s+12)} + \dfrac{6s-38}{s^2-7s+12}$

A. $F(s) = \dfrac{16+(6s-38)(s-2)}{(s-2)(s^2-7s+12)} = \dfrac{16+6s^2-12s-38s+76}{(s-2)(s-3)(s-4)} = \dfrac{6s^2-50s+92}{(s-2)(s-3)(s-4)}$

$F(s) = \dfrac{A}{s-2} + \dfrac{B}{s-3} + \dfrac{C}{s-4}$

$= \dfrac{A(s-3)(s-4) + B(s-2)(s-4) + C(s-2)(s-3)}{(s-2)(s-3)(s-4)}$

$= \dfrac{A(s^2-7s+12) + B(s^2-6s+8) + C(s^2-5s+6)}{(s-2)(s-3)(s-4)}$

$$\left.\begin{aligned} s^2 &\Rightarrow A + B + C = 6 \\ s &\Rightarrow -7A - 6B - 5C = -50 \\ 1 &\Rightarrow 12A + 8B + 6C = 92 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 8 \\ B &= 4 \\ C &= -6 \end{aligned}$$

$F(s) = \dfrac{8}{s-2} + \dfrac{4}{s-3} - \dfrac{6}{s-4}$

$L^{-1}[F(s)] = f(t) = 8e^{2t} + 4e^{3t} - 6e^{4t}$

Multiple roots

$\rightarrow$ $$F(s) = \frac{N(s)}{(s-s_0)^n D_1(s)}$$

$$= \underbrace{\frac{K_0}{(s-s_0)^n} + \frac{K_1}{(s-s_0)^{n-1}} + \dots + \frac{K_{n-1}}{s-s_0}}_{F_1(s)} + \frac{N_1(s)}{D_1(s)}$$

(Deals only with real roots in syllabus)

$$K_0 = (s-s_0)^n \cdot F(s) \Big|_{s=s_0} \Rightarrow \text{This leads to } \frac{0}{0}$$

So,

$$F_1(s) = (s-s_0)^n F(s)$$

$$= K_0 + K_1(s-s_0) + \dots + K_{n-1}(s-s_0)^{n-1} + \frac{N_1(s)(s-s_0)^n}{D_1(s)}$$

$$\frac{dF_1(s)}{ds} = K_1 + 2K_2(s-s_0) + \dots + K_{n-1}(n-1)(s-s_0)^{n-2}$$

$$(s-s_0)^1 K_1 = \frac{dF_1(s)}{ds}\Big|_{s=s_0} , \quad K_2(s-s_0)^2 = \frac{1}{2}\frac{d^2F_2(s)}{ds^2}\Big|_{s=s_0}$$

$$(s-s_0)^j K_j = \frac{1}{j!} \cdot \frac{d_j F(s)}{ds^j}\Big|_{s=s_0} \qquad j = 0,1,2\dots n-1$$

Q. $$\frac{s-2}{s(s+1)^3}$$

A. $$\frac{A}{s} + \frac{K_0}{(s+1)^3} + \frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)}$$

$$F_1 = \frac{K_0}{(s+1)^3} + \frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)}$$

$$A = sF(s)\Big|_{s=0} = \frac{-2}{1} = -2$$

$$K_0 = (s-1)^3 F(s)\Big|_{s=-1} = \frac{s-2}{s} = 3$$

$$K_1 = \frac{1}{1!}\frac{d}{ds}\left(F_1(s)\right)\Big|_{s=-1} = \frac{2}{s^2}\Big|_{s=-1} = 2$$

$$K_2 = \frac{1}{2!}\frac{d^2}{ds^2}\left(F_1(s)\right)\Big|_{s=-1} = \frac{-2}{2s^3}\Big|_{s=-1} = 1$$

$$F(s) = \frac{-2}{s} + \frac{3}{(s+1)^3} + \frac{2}{(s+1)^2} + \frac{2}{s+1}$$

$$f(t) = -2u(t) + (3t^2 + 2t + 2)e^{-t}u(t)$$

Q. $F(s) = \frac{2s^2}{(s-6)^3}$. $f(t) = ?$

A. $F(s) = \frac{K_1}{(s-6)^3} + \frac{K_2}{(s-6)^2} + \frac{K_3}{(s-6)}$

$(s-6)^3 F(s) = K_1 + K_2(s-6) + K_3(s-6)^2$

$2s^2 \Big|_{s=6} = K_1 = 72$

$\frac{d((s-6)^3 F(s))}{ds}\Big|_{s=6} = K_2 = 4s = 24$

$\frac{d^2}{ds^2}((s-6)^3 F(s))\Big|_{s=6} = K_3 = 4$

$F(s) = \frac{72}{(s-6)^3} + \frac{24}{(s-6)^2} + \frac{4}{s-6}$

$= (72t^2 + 24t + 4)\, e^{6t}\, u(t)$

Initial & Final Value Theorems

→ Initial Value Theorem

→ Relates initial value of $f(t)$ at $t = 0^+$ to limiting value of $sF(s)$ as s approaches ∞

$$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$$

($f(t)$ must be continuous (or) step discontinuity at $t=0$)

→ If $f(t)$ is continuous at $t=0$

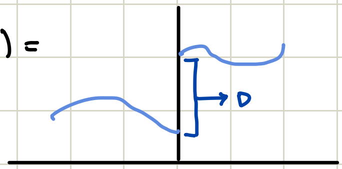
$$f(0^-) = f(0^+)$$

We know, $$L[f'(t)] = \int_{0^-}^{\infty} f'(t)\, e^{st}\, dt = sF(s) - f(0^-)$$

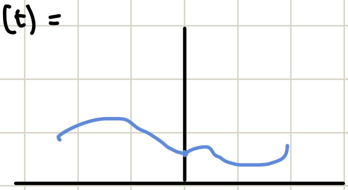
$$\lim_{s\to\infty} L[f'(t)] = \lim_{s\to\infty} sF(s) - f(0^-) = 0 \Rightarrow \boxed{\lim_{s\to\infty} sF(s) = f(0^-) = f(0^+)}$$

→ If $f(t)$ is discontinuous at $t=0$

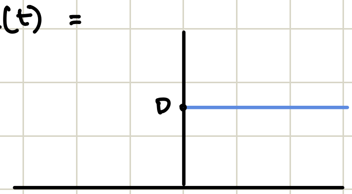
Let $f(t) =$



Then $f_1(t) =$



and $D\,u(t) =$



$$f(t) = f_1(t) + D\,u(t)$$

where $D = f(0^+) - f(0^-)$

Then, $f'(t) = f_1'(t) + D\,\delta(t)$

Since $f_1(t)$ is continuous at $t=0$

$$\lim_{s\to\infty} sF_1(s) = f_1(0^-) = f(0^-)$$

Laplace on B.S

$$L[f'(t)] = L[f_1'(t)] + L[D\,\delta(t)]$$

$$sF(s) - f(\cancel{0^-}) = sF_1(s) - f_1(\cancel{0^-}) + D$$

$$sF(s) = sF_1(s) + D$$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \cancel{s}F_1(s) + f(0^+) - \cancel{f}(0^-)$$

$$\boxed{\lim_{s\to\infty} sF(s) = f(0^+)}$$

→ Final Value Theorem

→ Theorem states that

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

(Poles of denominator of $F(s)$ must not be in right half of complex frequency plane)

→ $$\int_{0^-}^{\infty} f'(t) e^{-st} dt = sF(s) - f(0^-)$$

Taking limit as $s \to 0$

$$\int_{0^-}^{\infty} f'(t) dt = \lim_{s \to 0} sF(s) - f(0^-)$$

$$f(\infty) - f(0^-) = \lim_{s \to 0} sF(s) - f(0^-)$$

$$\boxed{f(\infty) = \lim_{s \to \infty} sF(s)}$$

Q. Find initial value of $f(t)$ if $F(s) = \dfrac{2(s+1)}{s^2+2s+5}$

A. $f(0^+) = \lim_{s \to \infty} sF(s)$

$$= \lim_{s \to \infty} \frac{2s(s+1)}{s^2+2s+5} = \lim_{s \to \infty} \frac{\cancel{s^2}}{\cancel{s^2}} \left(\frac{2\left(1+\frac{1}{s}\right)}{1+\frac{2}{s}+\frac{5}{s^2}} \right) = \frac{2(1+0)}{1+0+0} = \underline{\underline{2}}$$

Verification,

$$f(t) = L^{-1}\{F(s)\} = L^{-1}\left\{ \frac{2(s+1)}{s^2+2s+5} \right\} = L^{-1}\left\{ \frac{2(s+1)}{(s+1)^2+2^2} \right\}$$

$$= 2e^{-t}\cos t$$

$$f(0^+) = 2$$

Q. Find final value of $f(t)$ if $F(s) = \frac{5s+3}{s(s+1)}$

A. $f(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{5s+3}{s+1} = 3$

Verification, $f(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\frac{5s+3}{s(s+1)}\right\} = L^{-1}\left\{\frac{A}{s} + \frac{B}{s+1}\right\} = L^{-1}\left\{\frac{3}{s} + \frac{2}{s+1}\right\}$

$f(t) = 3u(t) + 2e^{-t} \Rightarrow f(\infty) = 3$

→ Failures of initial / final value theorem

→ Initial value theorem fails when function $f(t)$ is discontinuous

$f(t) = \delta(t) + 3e^{-t}$ then $f(0^+) = 3$

$F(s) = 1 + \frac{3}{s+1}$ then $\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} s + \frac{3s}{s+1} = \lim_{s \to \infty} s + \frac{3}{1+\frac{1}{s}} = \infty$

→ Final value theorem fails when roots of denominator (poles) are on right half of s-plane

$f(t) = 2e^{t}$ then $f(\infty) = \infty$

$F(s) = \frac{2}{s-1}$ then $\lim_{s \to 0} sF(s) = \frac{2s}{s-1} = 0$

The Transformed Circuit

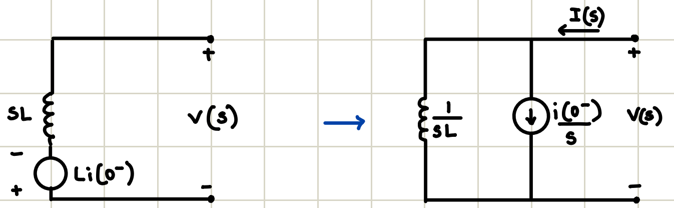
→ The V-I relationships of network elements in time domain maybe represented in complex frequency domain

$$v(t) \longrightarrow L[v(t)] = V(s)$$
$$i(t) \longrightarrow L[i(t)] = I(s)$$

So, $v(t) = R\,i(t) \longrightarrow V(s) = R\,I(s)$

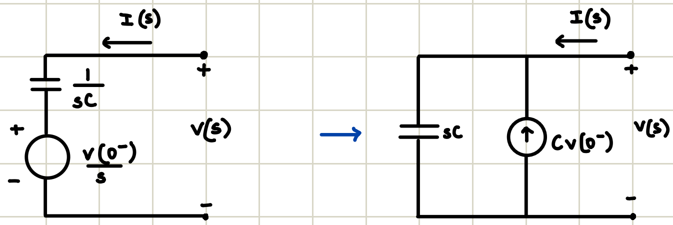
$v(t) = L\dfrac{di(t)}{dt} \longrightarrow V(s) = sL\,I(s) - L\,i(0^-)$

$i(t) = \dfrac{1}{L}\displaystyle\int_{0^-}^{t} v(\tau)\,d\tau + i(0^-) \longrightarrow I(s) = \dfrac{1}{sL}V(s) + \dfrac{i(0^-)}{s}$

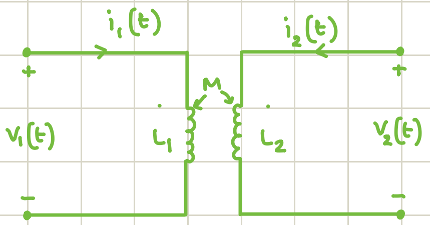


$v(t) = \dfrac{1}{C}\displaystyle\int_{0^-}^{t} i(\tau)\,d\tau + v(0^-) \longrightarrow V(s) = \dfrac{I(s)}{sC} + \dfrac{v(0^-)}{s}$

$i(t) = C\dfrac{dV}{dt} \longrightarrow I(s) = sC\,V(s) - C\,v(0^-)$



Q. Consider the following transformer & transform it

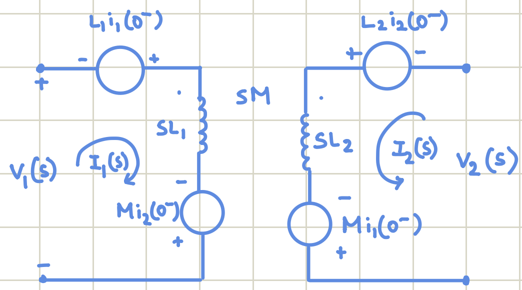


A. $v_1(t) = L_1\dfrac{di_1}{dt} + M\dfrac{di_2}{dt}$ & $v_2(t) = L_2\dfrac{di_2}{dt} + M\dfrac{di_1}{dt}$

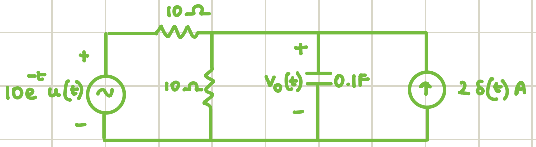
Transformed equations ⇒ $V_1(s) = sL_1I_1(s) - L_1i_1(0^-) + sMI_2(s) - Mi_2(0^-)$

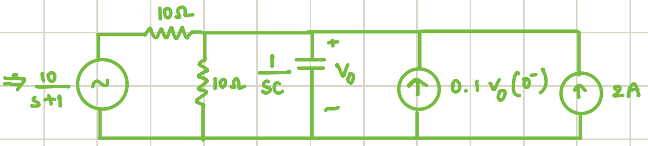
$V_2(s) = sL_2I_2(s) - L_2i_2(0^-) + sMI_1(s) - Mi_1(0^-)$

Transformed circuit ⇒



Q. Find $v_o(t)$ given $v_o(0^-) = 5V$





A. Applying KCL,

$$\frac{v_o(t)}{10} + \frac{v_o(t) - 10e^{-t}u(t)}{10} + 0.1\frac{d\,v_o(t)}{dt} = 2\delta(t)$$

Transforming,
$$\frac{V_o(s)}{10} + \frac{V_o(s)}{10} - \frac{1}{s+1} + 0.1\left(sV_o(s) - v(0^-)\right) = 2$$

$$V_o(s)\left(\frac{1}{10} + \frac{1}{10} + \frac{s}{10}\right) = 2 + \frac{1}{s+1} + 0.5$$

$$V_o(s)\left(\frac{s+2}{10}\right) = \frac{2.5(s+1)+1}{s+1}$$

$$V_o(s) = \frac{25(s+1)+10}{(s+1)(s+2)} = \frac{25s+35}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

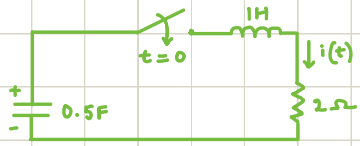
$$As + 2A + Bs + B = 25s + 35$$

$$\left.\begin{aligned} A+B &= 25 \\ 2A+B &= 35 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 10 \\ B &= 15 \end{aligned}$$

$$V_o(s) = \frac{10}{s+1} + \frac{15}{s+2}$$

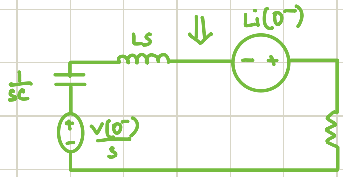
$$v_o(t) = 10e^{-t} + 15e^{-2t}$$

Q. Find $i(t)$ for $t \geq 0$. Assume initial capacitor voltage of 1V



A. By KVL,

$$-v_c(0^-) + L\frac{di(t)}{dt} + Ri(t) = 0 \Rightarrow -1 + \frac{di(t)}{dt} + 2i(t) = 0$$



Applying transform,

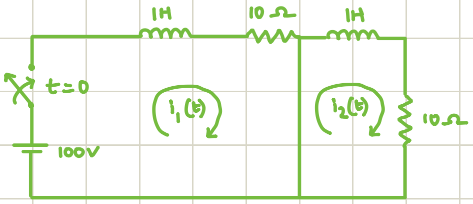
$$sI(s) - \underbrace{i(0^-)}_{0} + 2I(s) = \frac{1}{s}$$

$$sI(s) + 2I(s) = 1/s \Rightarrow I(s) = \frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$As + 2A + Bs = 1 \Rightarrow A = 1/2 \quad B = -1/2$$

$$I(s) = \frac{1}{2s} - \frac{1}{2(s+2)}$$

$$i(t) = 0.5u(t) - 0.5e^{-2t}$$

Q. 

Find the mesh currents in the circuit. The switch is open for a long time before closing at $t=0$

A. $i_1(0^-) = i_2(0^-) = 0A$

$1\dfrac{di_1(t)}{dt} + 10i_1(t) = 100$ and $\dfrac{di_2(t)}{dt} + 10i_2(t) = 0$

Applying Transform,

$sI_1(s) - i_1(0^-) + 10\,I(s) = \dfrac{100}{s}$

$I_1(s) = \dfrac{100}{s(s+10)} = \dfrac{10}{s} - \dfrac{10}{s+10}$

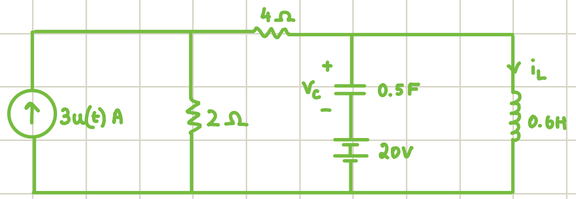
$i_1(t) = 10u(t) - 10e^{-10t}$

$sI_2(s) + i_2(0^-) + 10I_2(s) = 0$

$I_2(s) = 0$

$i_2(t) = 0$

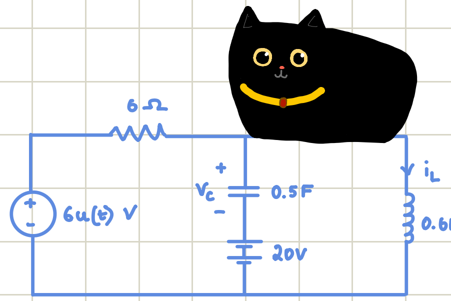
Q. Find $v_c(t)$ for $t \geq 0$. The current source is activated at $t=0$



A. $V_c(0^-) = 20V$ & $i_L(0^-) = 0A$

At $t=0$, we can apply Thevenin's Theorem

And by source transformation



By KCL,

$$\frac{V_1(t) - 6u(t)}{6} + 0.5\frac{dV_c(t)}{dt} + \frac{1}{0.6}\int_{-\infty}^{t} v_1(\tau)\,d\tau = 0$$

$$\frac{V_1(t)}{6} - u(t) + 0.5\frac{d(v_1(t) - 20)}{dt} + \frac{5}{3}\int_{-\infty}^{t} v_1(\tau)\,d\tau = 0$$

$$\frac{V_1(t)}{6} - u(t) + \frac{1}{2}\frac{dV_1(t)}{dt} - 0 + \frac{5}{3}\int_{-\infty}^{t} v_1(\tau)\,d\tau = 0$$

Applying Transform, $\dfrac{V_1(s)}{6} - \dfrac{1}{s} + \dfrac{1}{2}(sV_1(s) - v_1(0^-)) + \dfrac{5}{3}\dfrac{V_1(s)}{s} + i_L(0^-) = 0$

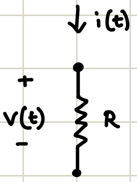
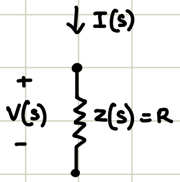
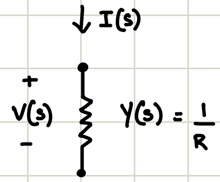
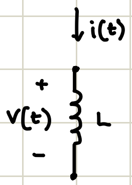
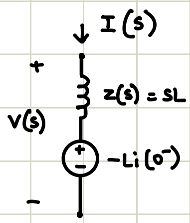
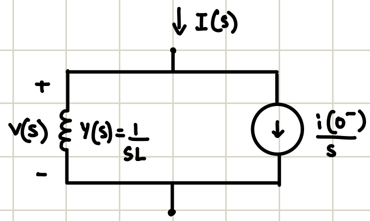
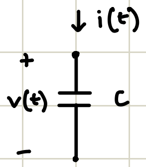
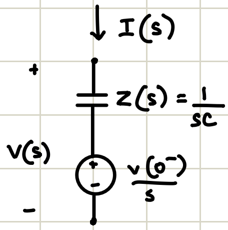
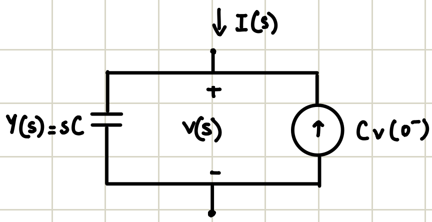
$$V_1(s)\left(\frac{1}{6} - \frac{s}{2} + \frac{5}{3s}\right) = \frac{1}{s} \Rightarrow V_1(s) = \frac{2}{s^2 + \frac{s}{3} + \frac{10}{3}}$$

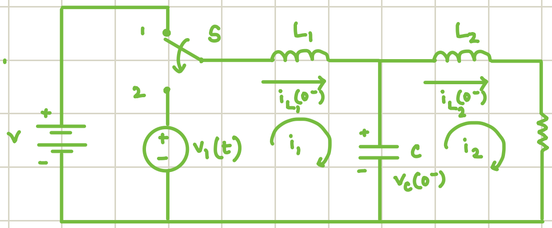
$$V_1(s) = \frac{2}{\left(s+\frac{1}{6}\right)^2 + \left(\frac{\sqrt{119}}{6}\right)^2} = \frac{2}{(s+0.167)^2 + (1.82)^2}$$

$$\mathcal{L}^{-1}[V_1(s)] = v_1(t) = 2e^{-0.167t}\sin(1.82t)$$

$$V_c(t) = V_1(t) - 20u(t) = 2e^{-0.167t}\sin(1.82t) - 20u(t)$$

Table for Constructing Transformed Circuits

Resistor  $v(t) = Ri(t)$	 $V(s) = RI(s)$	 $I(s) = \frac{V(s)}{R}$
Inductor  $v(t) = L\frac{di(t)}{dt}$	 $V(s) = sLI(s) - Li(0^-)$	 $I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$
Capacitor  $i(t) = C\frac{dv}{dt}$	 $V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}$	 $I(s) = sCV(s) - Cv(0^-)$

Q. 

Assuming switch is thrown from pos^n 1 to 2 at $t = 0$, write mesh equations

A. mesh 1,

$$v_1(t) - L_1\frac{di_1(t)}{dt} - \frac{1}{C}\int_{-\infty}^{t}(i_1(\tau) - i_2(\tau))d\tau = 0$$

$$V_1(s) - L_1(sI_1(s) - i_1(0^-)) - \frac{1}{C}\left(\frac{I_1(s)}{s} - \frac{I_2(s)}{s}\right) + \frac{V_c(0^-)}{s} = 0$$

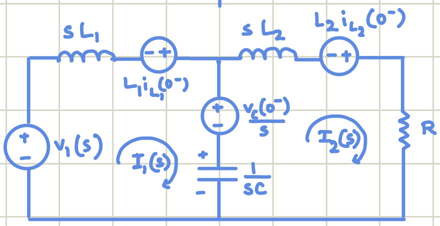
$$\left(sL_1 + \frac{1}{Cs}\right)I_1(s) - \frac{I_2(s)}{sC} = V_1(s) + L_1 i_1(0^-) - \frac{V_c(0^-)}{s}$$

mesh 2,

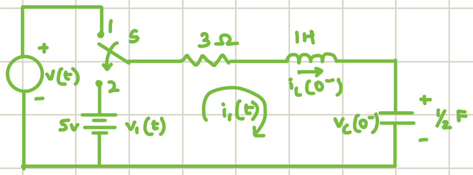
$$L_2\frac{di_2(t)}{dt} + Ri_2(t) + \frac{1}{C}\int_{-\infty}^{t}(i_2(\tau) - i_1(\tau))d\tau$$

$$L_2(sI_2(s) - i_2(0^-)) + RI_2(s) + \frac{1}{C}\left(\frac{I_2}{s} - \frac{I_1}{s}\right) - \frac{V_c(0^-)}{s} = 0$$

$$-\frac{I_1(s)}{sC} + \left(sL_2 + \frac{1}{Cs} + R\right)I_2(s) = L_2 i_2(0^-) + \frac{V_c(0^-)}{s}$$



Q. Assuming switch is thrown from posⁿ 1 to 2 at $t=0$, write $i(t)$ for $t \geq 0$.
Here, $i_L(0^-) = 2A$ & $v_c(0^-) = 2V$



A.

KVL, $5 - 3i_1(t) - 1\dfrac{di_1(t)}{dt} - \dfrac{1}{1/2}\displaystyle\int_{-\infty}^{t} i_1(t)dt = 0$

Transform, $\dfrac{5}{s} - 3I_1(s) - sI_1(s) + i_L(0^-) - 2\dfrac{I_1(s)}{s} - \dfrac{v_c(0^-)}{s} = 0$

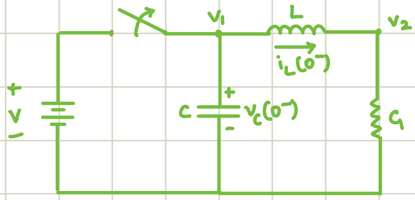
$$I_1(s)\left(s+3+\frac{2}{s}\right) = \frac{5}{s} - \frac{2}{s} + 2 = \frac{3}{s} + 2$$

$$I_1(s) = \frac{3+2s}{s\left(s+3+\frac{2}{s}\right)} = \frac{3+2s}{s^2+3s+2}$$

Q. Assume switch opened at $t=0$, write nodal voltages for $t \geq 0$

$L = \frac{1}{2}$ H $\qquad C = 1$ F

$G = 1$ mho $\qquad V = 1$V



A. Node 1, $C\dfrac{dv_1(t)}{dt} + \dfrac{1}{L}\displaystyle\int_{-\infty}^{t}(v_1(t) - v_2(t))\,dt = 0$ $\qquad i_L(0^-) = 1A$

$v_c(0^-) = 1V$

Laplace, $C\left(sv_1(s) - v_c(0^-)\right) + \dfrac{1}{L}\left(\dfrac{v_1(s)}{s} - \dfrac{v_2(s)}{s}\right) + \dfrac{i(0^-)}{s} = 0$

$$v_1(s)\left(\frac{2}{s}+s\right) - \frac{2v_2(s)}{s} = 1 - \frac{1}{s} \Rightarrow (2+s^2)\,v_1(s) - 2v_2(s) = s-1$$

Node 2, $\dfrac{1}{L}\displaystyle\int_{-\infty}^{t}(v_2(t) - v_1(t))\,dt + Gv_2(s) = 0$

Laplace, $2\left(\dfrac{v_2(s)}{s} - \dfrac{v_1(s)}{s}\right) - \dfrac{i_L(0^-)}{s} + v_2(s) = 0$

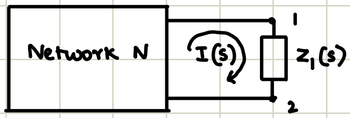
$$v_2(s)\left(\frac{2}{s}+1\right) - \frac{2v_1(s)}{s} = \frac{1}{s} \Rightarrow -2v_1(s) + (2+s)v_2(s) = 1$$

By Cramer's Rule, $v_1(s) = \dfrac{\begin{vmatrix} s-1 & -2 \\ 1 & s+2 \end{vmatrix}}{\begin{vmatrix} s^2+2 & -2 \\ -2 & s+2 \end{vmatrix}} = \dfrac{s+1}{(s+1)^2+1}$ $\qquad v_2(s) = \dfrac{\begin{vmatrix} s^2+2 & s-1 \\ -2 & 1 \end{vmatrix}}{\begin{vmatrix} s^2+2 & -2 \\ -2 & s+2 \end{vmatrix}} = \dfrac{(s+1)+1}{(s+1)^2+1}$

$$v_1(t) = e^{-t}\cos t$$
$$v_2(t) = e^{-t}(\cos t + \sin t)$$

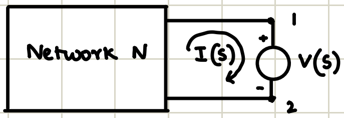
Thevenin's Theorem

→ From the standpoint of determining the current $I(s)$ through an element of impedance $Z_1(s)$, The rest of the network N can be replaced by an equivalent impedance $Z_e(s)$ in series with an equivalent voltage source $V_e(s)$



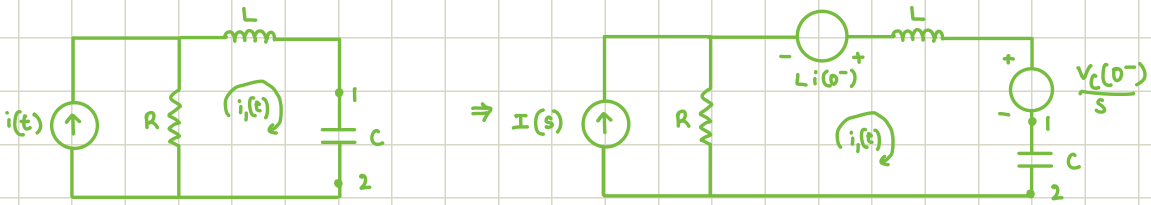
(elements in Z_1 must not be magnetically coupled to any element N)

By compensation theorem, replace $Z_1(s)$ by Voltage Source

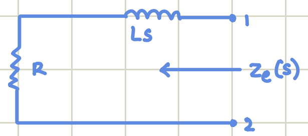


and by superposition principle, $I(s) = I_1(s) + I_2(s)$

Q. Find current through capacitor using Thevenin's theorem

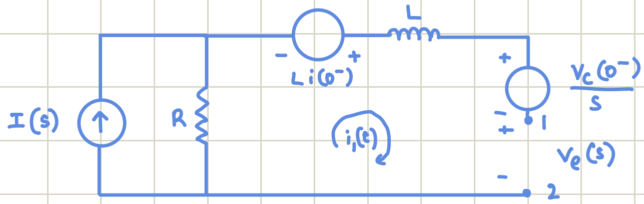


A. Removing Capacitor, find $Z_e(s)$



$$Z_e(s) = R + sL$$

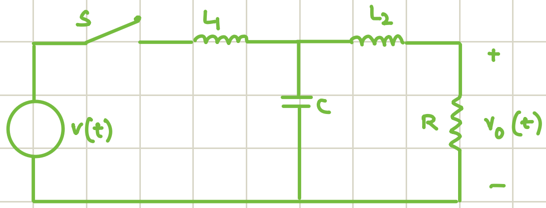
Find Thevenin's voltage across Terminals 1 & 2



$$V_e(s) + \frac{V_c(0^-)}{s} - RI(s) - Li(0^-) = 0$$

$$I_1(s) = \frac{V_e(s)}{Z_e(s) + Z(s)} = \frac{RI(s) + Li(0^-) - \frac{V_c(0^-)}{s}}{R + sL + \frac{1}{sC}}$$

Q. Find $V_o(t)$ by Thevenin's Theorem. Assume all initial values to be zero at by the time switch closes at $t = 0$



A. Thevenin's Impedance $\quad Z_e(s) = L_2 s + \left(L_1 s \parallel \frac{1}{sC}\right)$

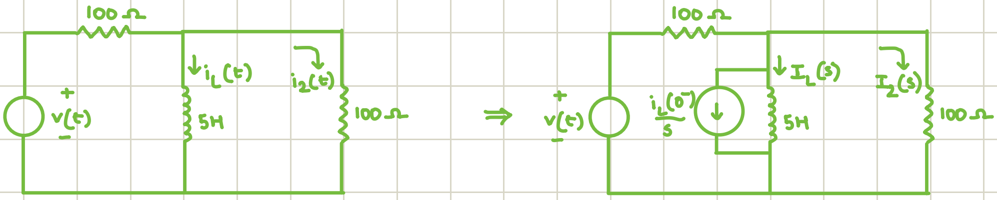
Thevenin's Voltage $\quad V_e(s) = \frac{V(s)\left(\frac{1}{sC}\right)}{L_2 s + \frac{1}{sC}}$

By voltage division, $\quad V_o(s) = \frac{V_e(s)\, R}{R + Z_e(s)} = \frac{R\, V(s)\left(\frac{1}{sC}\right)}{\left(L_2 s + \frac{1}{sC}\right)\left(R + L_2 s + \frac{L_1 s \times \frac{1}{sC}}{L_2 s + \frac{1}{sC}}\right)}$

$$= \frac{R\, V(s) \times \frac{1}{sC}}{\left((R + L_2 s)\left(L_2 s + \frac{1}{sC}\right)\right) + \frac{L_1}{C}}$$

$$v_o(t) = L^{-1}\{V_o(s)\}$$

Q. Find $i_2(t)$ given $v(t) = 2u(t)$ & $i_L(0^-) = 2A$



A. Thevenin's impedance, $Z_e(s) = \dfrac{100(5s)}{5s+100}$

Thevenin's voltage, $V_e'(s) = V(s) \times \dfrac{5s}{5s+100} = \dfrac{10}{5s+100}$

$$V_e''(s) = -\frac{i_L(0^-)}{s} \times \frac{5s(100)}{5s+100} = -\frac{1000}{5s+100}$$

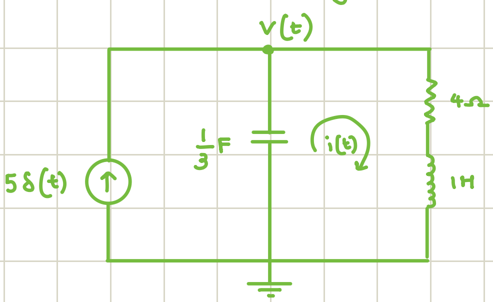
$$V_e(s) = V_e'(s) + V_e''(s) = \frac{-990}{5s+100}$$

$$I_2(s) = \frac{V_e(s)}{100+Z_e(s)} = \frac{-990}{(5s+100)\left(100+\frac{500s}{5s+100}\right)} = \frac{-990}{500s+10000+500s} = \frac{-99}{100s+1000}$$

$$= \frac{-0.99}{s+10}$$

$$i_2(t) = L^{-1}[I_2(s)] = L^{-1}\left[\frac{-0.99}{s+10}\right] = -0.99\,e^{-10t}$$

Q. Find $i(t)$ using Thevenin's theorem assuming initial values as zeroes



A. Thevenin's Impedance $Z_e(s) = 4 \,||\, \dfrac{3}{s} = \dfrac{4 \times 3}{s\left(4+\frac{3}{s}\right)} = \dfrac{12}{4s+3}$ Ω

Thevenin's Voltage $V_e(s) = \dfrac{5}{s} \times \dfrac{3/s}{4+\frac{3}{s}} = \dfrac{15}{4s^2+3s}$

Voltage across capacitor

$$I(s) = \frac{V_e(s)}{Z_e(s)+s} = \frac{\frac{15}{(4s+3)s}}{\frac{12}{4s+3}+s} = \frac{15}{\frac{(12+4s^2+3s)\,s\,(4s+3)}{4s+3}} = \frac{15}{s(4s^2+3s+12)} = \frac{15/4}{s\left(s^2+\frac{3s}{4}+3\right)}$$

Solve it yourself!

$$i(t) = 1.25u(t) - e^{-0.375t}(1.25\cos 1.69t + 0.277\sin 1.69t)$$