Physics Notes (U1-4) Complete

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Unit - 1

- Gauss Law $\Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

- Gauss law for Magnetism $\Rightarrow \nabla \cdot \vec{B} = 0$

- Faraday's law $\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

- Ampere-Maxwell Law $\Rightarrow \nabla \times \vec{B} = \mu_0 \hat{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

- Wave equation for Electric Field

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(\frac{-\partial \vec{B}}{\partial t}\right)$$

$$\nabla (\cancel{\nabla \cdot \vec{E}})^{0} - \nabla^2 \vec{E} = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$$

$$-\nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

Wave Equation for

$$\nabla \times \vec{B} = \mu_0 \hat{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{B}) = \nabla \times \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \nabla \vec{B} - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$0 - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \vec{E}}{\partial t}\right)$$

$$\boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

- Average Energy (Poynting Vector)

- Energy in inductor,

$$E_m = \frac{LI^2}{2} = \frac{1}{2}\left(\frac{N^2 \mu_0 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{B^2 A l}{2 \varepsilon_0}$$

per unit volume,

$$\boxed{E_M = \frac{B^2}{2\mu_0}}$$

- Energy in Capacitor,

$$E_c = \frac{CV^2}{2} = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\left(E^2 d^2\right) = \frac{\varepsilon_0 A d E^2}{2}$$

per unit volume

$$\boxed{E_c = \frac{\varepsilon_0 E^2}{2}}$$

$$E_T = E_c + E_m = \frac{B^2}{2\mu_0} + \frac{\varepsilon_0 E^2}{2} = \left(\frac{E}{c}\right)^2 \times \frac{1}{2\mu_0} + \frac{\varepsilon_0 E^2}{2}$$

$$\boxed{E_T = \varepsilon_0 E^2}$$

$$\text{flux} = \frac{E_T \times cAt}{At}$$

$$= E_T c$$

$$= E^2 \varepsilon_0 c$$

$$= E B c \varepsilon_0 c$$

$$= E \varepsilon_0 B c^2$$

$$\boxed{\vec{S} = c^2 \varepsilon_0 \vec{E} \times \vec{B}}$$

$$\left(c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

The electric field associated with an EM radiation (light) is given by,

$E(x, t) = 10^3 \cos(\omega t - \pi x 3x10^6 z)$

Evaluate

1. Speed of the Electric vector
2. Wavelength
3. Frequency
4. Period of the wave
5. Magnetic field associated with the wave
6. Direction of propagation of the magnetic transverse wave
7. Amplitude of the electric field vector
8. Amplitude and direction of the transverse magnetic wave

$$E(x,t) = E_m \cos(\omega t - kx)$$

A. 1. $v = 3 \times 10^8 \, ms^{-1}$

2. $\lambda = \frac{2\pi}{K} = \frac{2\pi}{3\pi \times 10^6} = \frac{2}{3} \times 10^{-6} \, m$

3. $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{\frac{2}{3} \times 10^{-6}} = 4.5 \times 10^{14} \, Hz$

4. $T = \frac{1}{\nu} = 2.2 \times 10^{-15} \, s$

5. $B = \frac{E_0}{c} = \frac{10^3}{3 \times 10^8} = 0.33 \times 10^{-5}$

6. $E_0 = 10^3$

7. $B_0 = 3.33 \times 10^{-6}$ in y direction

- **Classical EM Wave Theory**

Failed to explain :

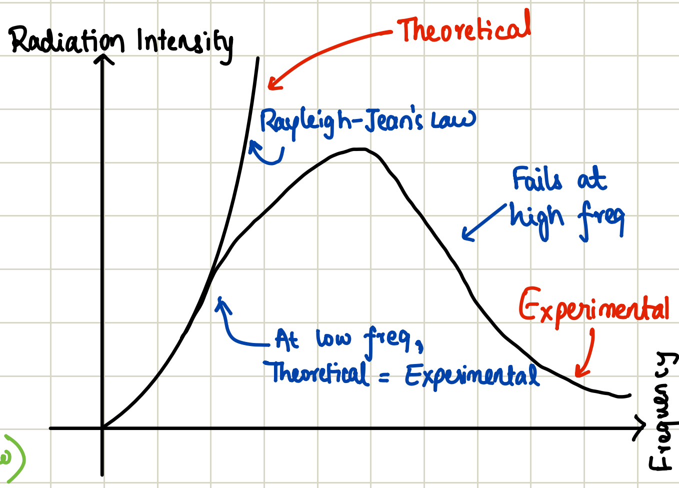
i) Photo-electric effect

ii) Atomic Spectra

iii) Blackbody Radiation Spectrum

iv) Compton Scattering

- **Rayleigh - Jean's Law**



dN between ν & $\nu + \delta\nu$

$$dN = \frac{8\pi\nu^3}{c^3}$$

dN per unit volume

$$dN = \frac{8\pi\nu^2 d\nu}{c^3}$$

Avg. energy of oscillator (Maxwell Boltzmann Law)

$$\langle E \rangle = K_B T$$

Energy density

$$\rho(\nu) d\nu = \langle E \rangle dN = \frac{8\pi\nu^2 d\nu}{c^3} \times K_B T$$

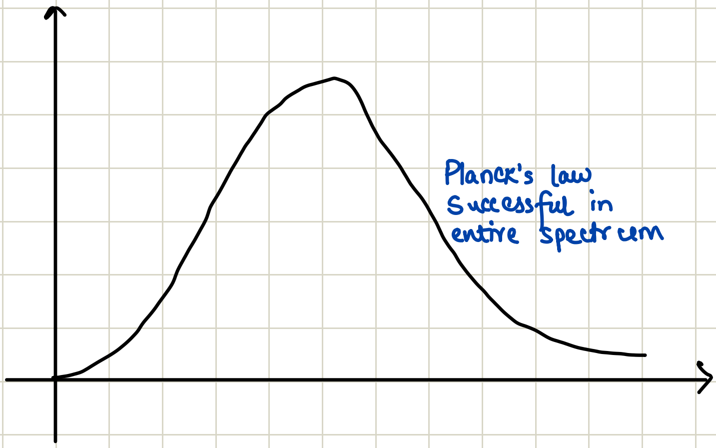
- Max Planck's Radiation Law

↳ Energy of oscillators are multiples of frequencies times a constant ($E = nh\nu$)

Avg Energy of oscillators

$$\langle E \rangle = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}$$

$$\langle E \rangle dN = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{KT}} - 1}$$



The frequency of harmonic oscillator at 50C is 6.2x10^12 per sec. Estimate the average energy of the oscillator as per Planck's idea of cavity oscillator, also compare the same with classical average energy and average energy by R-J law.

A. $T = 50^\circ C = 323\,K$

$\nu = 6.2 \times 10^{12}\ Hz$

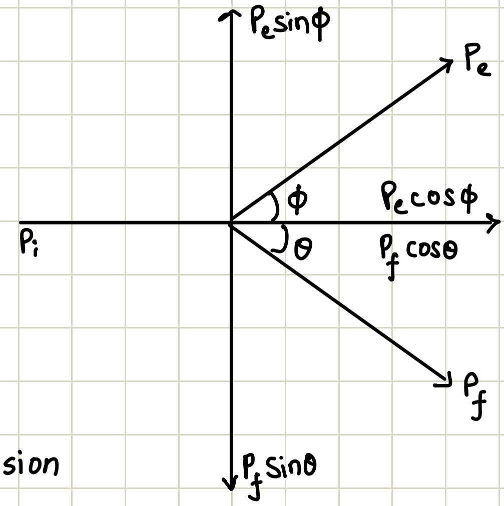
According to Planck, $E = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1} = 2.716 \times 10^{-21}\,J = 0.017\,eV$

According to R-J, $E = KT = 4.46 \times 10^{-21}\,J = 0.028\,eV$

- Compton Shift

Rest mass $\Rightarrow E = m_0 c^2$

Total energy of particle $\Rightarrow E_T = \sqrt{P_e^2 c^2 + m_0^2 c^4}$



Momentum conservation along incident direction

$$P_i + 0 = P_f \cos\theta + P_e \cos\phi$$

Momentum conservation along perpendicular direction

$$0 = P_e \sin\phi - P_f \sin\theta$$

Conservation of momentum before & after collision

$$P_e^2 = P_i^2 + P_f^2 - 2P_i P_f \cos\theta \longrightarrow (1)$$

Conservation of energy before & after collision

$$P_i c + m_0 c^2 = P_f c + \sqrt{P_e^2 c^2 + m_0^2 c^4}$$

$$P_e^2 = P_i^2 + P_f^2 - 2P_i P_f + 2m_0 c (P_i - P_f) \longrightarrow (2)$$

From (1) & (2),

$$-2P_i P_f + 2m_0 c (P_i - P_f) = -2P_i P_f \cos\theta$$

With $P_i = h/\lambda_i$ & $P_f = h/\lambda_f$

$$m_0 c h \left(\frac{\lambda_f - \lambda_i}{\lambda_f \lambda_i} \right) = \frac{h^2}{\lambda_f \lambda_f} (1 - \cos\theta) \Rightarrow \boxed{\lambda_f - \lambda_i = \frac{h}{m_0 c} (1 - \cos\theta)}$$

X-rays of wavelength 0.112 nm is scattered from a carbon target. Calculate the wavelength of X-rays scattered at an angle 90o with respect to the original direction. What is the energy lost by the X-ray photons? What is the energy gained by the electrons? If the incident x-ray retraces back what will be the shift?

A. $\lambda_i = 0.112 \times 10^{-9}$

$\theta = 90^\circ$

$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$

$\lambda_f = 1.144$ Å

Energy lost $= \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$

$= hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$

$= 3.72 \times 10^{-17}$ J

$= 232.23$ eV

If it retraces back, shift $= 180^\circ$

- De-broglie hypothesis

→ Moving matter should also exhibit wave characteristics

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}} = \frac{h}{mv}}$$

→ By Davisson & Germer's experiment, electron waves satisfied bragg's law ($\lambda = 2d\sin\theta$)

Find the de Broglie wavelength of electrons moving with a speed of 10^7 m/s

A. $\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{m_e \times 10^7} = 7.27 \times 10^{-11}$ m

$= 0.072$ nm

An alpha particle is accelerated through a potential difference of 1 kV. Find its de Broglie wavelength.

A. $\left.\begin{array}{l} V = 1\text{ kV} \\ m_\alpha = 4m_p \\ q = 2e \end{array}\right\} \rightarrow \lambda = \frac{h}{\sqrt{2m_\alpha qV}} = \frac{h}{\sqrt{16 m_\alpha eV}}$

$\lambda = 3.2 \times 10^{-13}$ m

Compare the momenta and energy of an electron and photon whose de Broglie wavelength is 650nm

A. electron & photon $\Rightarrow \lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = 1.02 \times 10^{-27}$ kgms^{-1}

$e^- \Rightarrow p^2/2m_e = 5.7 \times 10^{-25}$ kgm^{2}s^{-2}

photon ⇒ has no rest mass, So, $E = \frac{hc}{\lambda_p} = 3.056 \times 10^{-19}$ kgm^{2}s^{-2}

Calculate the de Broglie wavelength of electrons and protons if their kinetic energies are
i) 1% and ii) 5% of their rest mass energies.

A. rest mass energy of e^- $= m_e c^2 = 8.2 \times 10^{-14}$ J
rest mass energy of p^+ $= m_p c^2 = 1.5 \times 10^{-10}$ J

i) $KE_e = 0.01 \times 8.2 \times 10^{-14} = 8.2 \times 10^{-16}$ J

$$\lambda_e = \frac{h}{\sqrt{2m_e KE_e}} = 1.71 \times 10^{-11} \text{ m}$$

$KE_p = 0.01 \times 1.5 \times 10^{-10} = 1.5 \times 10^{-12}$

$$\lambda_p = \frac{h}{\sqrt{2m_p KE_p}} = 9.35 \times 10^{-15} \text{ m}$$

ii) $KE_e = 0.05 \times 8.2 \times 10^{-14} = 4.1 \times 10^{-15}$

$$\lambda_e = \frac{h}{\sqrt{2m_e KE}} = 7.66 \times 10^{-12} \text{ m}$$

$KE_p = 0.05 \times 1.5 \times 10^{-10} = 7.5 \times 10^{-12}$

$$\lambda_p = \frac{h}{\sqrt{2m_p KE_p}} = 4.18 \times 10^{-15} \text{ m}$$

An electron and a photon have a wavelength of 2A. Calculate their momenta and total energies.

A.

$$P_e = \frac{h}{\lambda} = 3.313 \times 10^{-24} \text{ Kgms}^{-1} = P_p$$

$$TE_e = KE + PE$$
$$= m_e c^2 + \frac{p^2}{2m} = 8.2 \times 10^{-14} \text{ J}$$

$$TE_p = \frac{hc}{\lambda} = 9.945 \times 10^{-16} \text{ J}$$ (Cuz no rest mass for photon)

What is the wavelength of an hydrogen atom moving with a mean velocity corresponding to the average kinetic energy of hydrogen atoms under thermal equilibrium at 293K?

A.

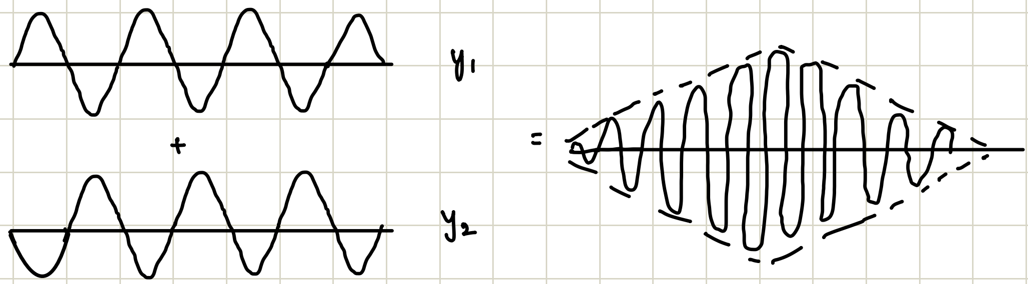
$$\lambda = \frac{h}{\sqrt{2m\,KE}} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{\cancel{2}m \times \frac{3K_BT}{\cancel{2}}}} = \frac{h}{\sqrt{3mK_BT}}$$

$m = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{\sqrt{3 \times 1.67 \times 10^{-27} \times K_B \times 293}} = 1.47 \times 10^{-10} \text{ m}$$

- ## Matter Waves representation

They are represented using matter waves with defined wavelength & amplitude maximum (unlike sin & cos)



Wave packet , $y = y_1 + y_2$

$$y_1 = A\sin(\omega t + Kx)$$
$$y_2 = A\sin\left((\omega+\Delta\omega)t + (K+\Delta K)x\right)$$

$$y = y_1 + y_2 = 2\cos\left(\frac{\omega t + Kx - (\omega+\Delta\omega)t - (K+\Delta K)x}{2}\right)\sin\left(\frac{\omega t + Kx + (\omega+\Delta\omega)t + (K+\Delta K)x}{2}\right)$$

$$= 2\cos\left(\frac{-(\Delta\omega t + \Delta K x)}{2}\right)\sin\left(\frac{(2\omega+\Delta\omega)t + (2K+\Delta K)x}{2}\right)$$

$$\boxed{y \simeq 2\cos\left(\frac{\Delta\omega t + \Delta K x}{2}\right)\sin(\omega t + Kx)}$$

- ## Phase Velocity & Group Velocity

↳ velocity of a representative point on the packet

velocity of common velocity of superposed wave group

$$V_p = \frac{\omega}{K} \qquad V_g = \frac{d\omega}{dK}$$

$$V_g = \frac{d}{dK}(V_p \cdot K) = K\cdot\frac{dV_p}{dK} + V_p\cdot\frac{dK}{dK}$$

$$= K\frac{dV_p}{dK} + V_p = K\cdot\frac{dV_p}{d\lambda}\cdot\frac{d\lambda}{dK} + V_p$$

$$= \frac{2\pi}{\lambda}\cdot\left(\frac{dV_p}{d\lambda}\right)\cdot\left(\frac{-2\pi}{K^2}\right) + V_p \qquad \left(K = \frac{2\pi}{\lambda}\right)$$

$$V_g = \frac{2\pi}{\lambda}\left(\frac{dV_p}{d\lambda}\right)\left(\frac{-2\pi}{\frac{(2\pi)^2}{\lambda^2}}\right) + V_p = V_p - \lambda\frac{dV_p}{d\lambda}$$

$$\boxed{V_g = V_k - \lambda\frac{dV_p}{d\lambda}}$$

V_g depends on V_p & phase vel. change with wavelength

Case(i) : $V_p = V_g$

Only in non dispersive medium when waves independant of λ

Case (ii) $V_g = V_p/2$ $(V_g < V_p)$

Then, $\frac{dV_p}{V_p} = \frac{d\lambda}{2\lambda}$

$\ln V_p \propto \ln\sqrt{\lambda} \Rightarrow V_p \propto \sqrt{\lambda}$

Case (iii) $V_g = 2V_p$ $(V_g > V_p)$

Then, $\frac{dV_p}{V_p} = \frac{-d\lambda}{\lambda}$

$\ln V_p \propto \ln\left(\frac{1}{\lambda}\right) \Rightarrow V_p \propto \frac{1}{\lambda}$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar} \Rightarrow \boxed{p = \hbar K}$$

$$\omega = 2\pi\nu = \frac{2\pi E}{h} = \frac{E}{\hbar} \Rightarrow \boxed{E = \hbar\omega}$$

$$V_g = \frac{d\omega}{dK} = \frac{d(E/\hbar)}{d(p/\hbar)} = \frac{dE}{dp}$$

also, $E = KE + PE$

$$E = \frac{p^2}{2m} + V(x)$$

$$\frac{dE}{dp} = \frac{p}{m} + 0 = \frac{mv}{m} = v$$

$$V_g = \frac{dE}{dp} = v$$

A wave packet is represented as, $y = 10\, sin(30t - 40x)\,.\, cos(0.3t - 0.5\,x)$
Find the phase and group velocities

A. $V_g = \frac{\omega}{K} = \frac{30}{40} = 0.75\ ms^{-1}$

$V_p = \frac{d\omega}{dK} = \frac{0.3}{0.5} = 0.6\ ms^{-1}$

- Heisenberg's Analysis & Uncertainity Principle

Wave packets describe matter waves
So, they have inherent components of uncertainities

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$x \propto \frac{1}{h}$$

$$\Delta K \propto \frac{1}{\Delta x}$$

$\Rightarrow$ Product of deviations $\boxed{\Delta x . \Delta K \geq \frac{1}{2}}$

$$\Delta x . \Delta\left(\frac{2\pi}{\lambda}\right) \geq \frac{1}{2}$$

$$\Delta x . \Delta\left(\frac{2\pi}{h}\right) . p \geq \frac{1}{2}$$

$$\Delta x . \Delta p \geq \frac{h}{4\pi} \text{ or } \frac{\hbar}{2}$$ (Position - Momentum Uncertainity)

$$\Delta E . \Delta t \geq \frac{h}{4\pi}$$ (Energy - Time Uncertainity)

$$\Delta \theta . \Delta L \geq \frac{h}{4\pi}$$ (Uncertainity for Circular motion)

- ## Applications of Uncertainity Principle

1) Non-existence of e^- inside nuclei

→ e^- don't exist in nucleus but, during β decay, e^- are emitted from nucleus with energies of order of 8 MeV

→ Nuclear diameter $\simeq 10^{-14}$ m $= \Delta x$

Using H.U.P,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{4\pi \times 10^{-14}} \simeq 5.28 \times 10^{-21} \text{ kgms}^{-1}$$

→ So, $E = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{(5.28 \times 10^{-21})^2}{2m_e} = 96 \text{ MeV}$

→ This means, energy of e^- should be quite high to be integral member of nuclei

→ But this is far from reality, because energy of e^- is very less compared to above estimate

2) Gamma Ray Microscope

→ to observe an e^-, wavelength should be comparable to size of e^-

So, γ- rays wavelength $= 10^{-12}$ m

$\Delta x = \frac{\lambda}{\sin\theta}$ (Resolution of microscope)

High energy γ-rays impart momentum to e^-

→ $P_x = \pm \frac{h}{\lambda} \sin\theta$

$\Delta P_x = \frac{2h\sin\theta}{\lambda}$

$\Delta x \cdot \Delta p_x = \frac{\lambda}{\sin\theta} \times \frac{2h\sin\theta}{\lambda} = 2h > \frac{h}{4\pi}$

Hence, simultaneous determination of position & momentum leads to uncertainity

- Other forms of Uncertainity principle

1) $\Delta x . \Delta \lambda \geq \left| \frac{\lambda^2}{4\pi} \right|$ Position - Wavelength Uncertainity

2) $\Delta x . \Delta v \geq \frac{h}{4\pi m}$ Position - Velocity Uncertainity

3) $\Delta x . \Delta K \geq \frac{1}{2}$ Position - Propagation vector Uncertainity

4) $\Delta x_{min} \Delta P_{max} \geq \frac{h}{4\pi}$ Min & Max Uncertainity

5) $\Delta v = v \times accuracy$ Accuracy & Uncertainity

6) $\Delta t . \Delta \lambda \geq \left| \frac{\lambda^2}{4\pi c} \right|$ Wavelength - time Uncertainity

7) $\Delta E . \Delta v \geq \frac{1}{4\pi}$ Energy - time Uncertainity

The speed of an electron is measured to be 1 km/s with an accuracy of 0.005%. Estimate the uncertainty in the position of the particle.

A. $v = 1 \, Km/s = 1000 \, ms^{-1}$

$\Delta v = 1000 \times \frac{0.005}{100} = 0.05 \, ms^{-1}$

$\Delta x \geq \frac{h}{4\pi \Delta v \, m_e} = 1.157 \times 10^{-3} \, m$

The spectral line of Hg green is 546.1 nm has a width of 10-5 nm. Evaluate the minimum time spent by the electrons in the upper state before de excitation to the lower state.

A. $\Delta t . \Delta \lambda \geq \left| \frac{\lambda^2}{4\pi c} \right|$

$\Delta t \geq \left| \frac{\lambda^2}{4\pi c} \right| \times \frac{1}{\Delta \lambda} = \frac{(546.1 \times 10^{-9})^2}{4\pi c} \times \frac{1}{10^{-5} \times 10^{-9}} = 7.91 \times 10^{-9} \, s$

The uncertainty in the location of a particle is equal to it's de Broglie wavelength. Show that the corresponding uncertainty in its velocity is approx one tenth of it's velocity.

A. $\Delta x = \lambda = h/p$

$p = mv$

$\Delta p = m \Delta v$

$\Delta x . \Delta p \geq \frac{h}{4\pi} \Rightarrow \frac{h}{p} . \Delta p \geq \frac{h}{4\pi} \Rightarrow \frac{\cancel{m} \Delta v}{\cancel{m} v} \geq \frac{1}{4\pi} \Rightarrow \Delta v \geq \frac{v}{4\pi}$

$\simeq \Delta v \geq v/10$

A proton is confined to a box of length 2 nm. What is the minimum uncertainty in its velocity?

A. $\Delta x . \Delta p \geq h/4\pi$

$\Delta x . m \Delta v \geq h/4\pi \Rightarrow \Delta v \geq \frac{h}{4\pi m_p . \times 2 \times 10^{-9}} \Rightarrow \Delta v \geq 15.76 \, ms^{-1}$

- **Wave function**

→ The representation of matter waves of moving bodies
$\psi(x, y, z, t)$

→ It can be +ve / -ve / complex

→ Square of wave function = Probability Density

$$\underbrace{\psi^*}_{\text{Complex Conjugate}} \overbrace{\psi}^{\text{Wave function}} = \underbrace{|\psi|^2}_{\text{Probability Density}}$$

- **Characteristics of Acceptable Wave Function**

1) F C S ⇒ Finite, Continuous, Single Valued
2) d F C S ⇒ Derivatives : Finite, Continuous, Single Valued
3) Normalizable ⇒ $\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$

$$\psi = A e^{\frac{i}{\hbar}(px - Et)}$$

ψ_1 ⇒ wave func^n of photons from slit 1
ψ_2 ⇒ wave func^n of photons from slit 2

$I_1 = |\psi_1|^2$; $I_2 = |\psi_2|^2$

$\psi_3 = \psi_1 + \psi_2$

$I_3 = |\psi_3|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_1 \psi_2^*$

$|\psi_1 + \psi_2|^2 = (\psi_1^* + \psi_2^*)(\psi_1 + \psi_2)$

- **Expectation Values of Observable**

operator $\hat{G}$ of observable g

$$\langle g \rangle = \frac{\int \psi^* \hat{G} \psi \, dx}{\int \psi^* \psi \, dx}$$

In 3D,

$$\langle g \rangle = \frac{\int \psi^* \hat{G} \psi \, dV}{\int \psi^* \psi \, dV}$$

- **Eigen Value equation**

$$\hat{G} \psi = G \psi$$

A value upon operation gives a value similar to initial eigen value

(like $\frac{d^2}{dx^2}(e^{4x}) = 16 \underbrace{e^{4x}}_{\text{similar}}$)

- Momentum Operator $(\hat{P}) = -i\hbar \frac{\partial}{\partial x}$

- Kinetic Energy Operator $(\widehat{KE}) = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}$

- Total Energy Operator / Hamiltonian Operator $(\hat{E})$ / $(\hat{H})$ $= i\hbar \frac{\partial}{\partial t}$

- Position Operator $(\hat{x}) = x$

- Potential Operator $(\hat{V})$ ⇒ Difference b/w TE & KE

- ## Schrodinger's Time Dependant Equation

General form of wave eqn $\Rightarrow$ $\psi(x,t) = Ae^{\frac{i}{\hbar}(px-Et)}$

Total Energy $\Rightarrow$ $T = K + P$

$E\,\psi(x,t) = KE\,\psi(x,t) + V\psi(x,t)$

In terms of Operators $\Rightarrow$ $\hat{E}\,\psi(x,t) = \widehat{KE}\,\psi(x,t) + \hat{V}\,\psi(x,t)$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\boxed{\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + i\hbar\frac{\partial\psi}{\partial t} - V\psi = 0}$$

- ## Schrodinger's Time Independant Wave Equation

→ For steady state system,

$$\psi(x,t) = Ae^{\frac{i}{\hbar}(px-Et)}$$

$$\psi(x,t) = Ae^{\frac{ipx}{\hbar}} \cdot Ae^{\frac{-iEt}{\hbar}}$$

$$\psi(x,t) = \psi(x) \cdot \psi(t)$$

Now Substitute in Schrodinger's Time independant eqn

$$\frac{\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2}\big(\psi(x)\cdot\psi(t)\big) + i\hbar\frac{\partial}{\partial t}\big(\psi(x)\cdot\psi(t)\big) - V\,\psi(x)\cdot\psi(t) = 0$$

$$\left(\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + i\hbar\frac{\partial\psi(x)}{\partial t} - V\psi(x)\right)\psi(t) = 0$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + E\psi(x) - V\psi(x) = 0$$

(or)

$$\psi(t) = 0$$

$$\boxed{\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0}$$

Unit – 2

- **Free Particle**

→ Particle of mass m & energy E moves free when no force acts on it

$$F = 0 \Rightarrow -\frac{dV}{dx} = 0 \Rightarrow V = \text{constant (or) } 0$$

→ General Schrodinger's eq^n

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi(x) = 0$$

If $V = 0$,

$$\frac{d^2\psi(x)}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \quad \left(K^2 = \frac{2mE}{\hbar^2}\right)$$

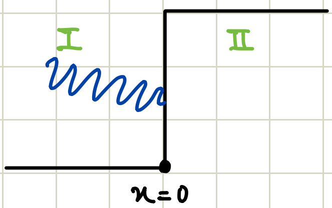
General Sol^n for this type of eq^n is $\psi = \underbrace{Ae^{iKx}}_{\text{moves in +ve x-direction}} + \underbrace{Be^{-iKx}}_{\text{moves in -ve x-direction}}$

- **Step Potential**

→ particle of mass m & energy E moving from R I to R II

R I → $x < 0$ & $V = 0$

R II → $x > 0$ & $V = V_0$



So problem can split into 2 cases, R I & R II

Region I ($x < 0$, $V = 0$)

$$\frac{d^2\psi_1(x)}{dx^2} + \frac{2m}{\hbar^2}E\psi(x) = 0 \Rightarrow \frac{d^2\psi_1(x)}{dx^2} + K^2E\psi(x) = 0$$

$$\boxed{\psi_1 = \underbrace{Ae^{iKx}}_{\text{incident wave}} + \underbrace{Be^{-iKx}}_{\text{reflected wave}}}$$

Region II ($x > 0$, $V \neq 0$)

→ This has 2 cases again V_0)

Case I : $E > V_0$

$$\frac{d^2\psi_2(x)}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_2(x) = 0$$

$$\frac{d^2\psi_2(x)}{dx^2} + K_2^2\psi_2(x) = 0 \quad \left(K = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)$$

$$\psi_2(x) = \underbrace{Ce^{iK_2x}}_{\text{Transmitted wave}} + \underbrace{De^{-iK_2x}}_{\text{Neglected}} \Rightarrow \boxed{\psi_2(x) = Ce^{iK_2x}}$$

Applying Boundary conditions, $(\phi_1(0) = \phi_2(0))$

$(\phi_1'(0) = \phi_2'(0))$

$$\boxed{B = \frac{A(K_1-K_2)}{K_1+K_2}} \quad \boxed{C = \frac{2K_1A}{K_1+K_2}}$$

Case II : $E < V_0$ (Quantum Tunneling)

$$\frac{\partial^2\psi_2(x)}{\partial x^2} - \frac{2m}{\hbar^2}(V_0-E)\psi_2(x) = 0$$

$$\frac{\partial^2\psi_2(x)}{\partial x^2} - \beta^2\psi_2(x) = 0 \quad \left(\beta = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}\right)$$

$$\psi_2(x) = \underbrace{Ce^{-\beta x}}_{\text{Transmitted wave}} + \underbrace{De^{\beta x}}_{\text{Neglected}} \Rightarrow \psi_2(x) = Ce^{-\beta x} = Ce^{-1}$$

Then,

$$\Delta x = \frac{1}{\beta} = \frac{\hbar}{\sqrt{2m(V_0-E)}} \Rightarrow \Delta E = \frac{p^2}{2m} = \frac{2m(V_0-E)}{2m} = V_0 - E$$

$$R = \frac{\text{Prob. flux of Reflected Wave}}{\text{Prob. flux of Incident Wave}}$$

$$= \frac{|B|^2 v_1}{|A|^2 v_1} = \frac{|B|^2}{|A|^2} = \left|\frac{K_1 - K_2}{K_1 + K_2}\right|^2$$

$$T = \frac{\text{Prob. flux of Transmitted Wave}}{\text{Prob. flux of Incident Wave}}$$

$$= \frac{|C|^2 v_2}{|A|^2 v_1} = \frac{|C|^2}{|A|^2} \cdot \frac{\frac{\hbar K_2}{m}}{\frac{\hbar K_1}{m}} = \left|\frac{2K_1}{K_1+K_2}\right|^2 \times \frac{K_2}{K_1} = \frac{4K_1K_2}{|K_1+K_2|^2}$$

$$R + T = 1$$

$$R = \frac{|B|^2}{|A|^2}\frac{v_1}{v_1} = \left|\frac{K_1-K_2}{K_1+K_2}\right|^2 = \left|\frac{K_1 - i\beta}{K_1 + i\beta}\right|^2$$

$$\frac{B^*B}{A^*A} = \frac{K_1+i\beta}{K_1-i\beta} \times \frac{K_1-i\beta}{K_1+i\beta} = 1$$

$$T = 0$$

$$R + T = 1$$

10^4 electrons incident on a potential step of height 8 eV.
If the energy of each electron is 10 eV, estimate the reflection coefficient and probability of reflection

A. $$R = \frac{|B|^2 v_1}{|A|^2 v_2} = \left|\frac{K_1 - K_2}{K_1+K_2}\right|^2 = \left(\frac{\sqrt{\frac{2mE}{\hbar^2}} - \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}} + \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}\right)^2 = \frac{\sqrt{\cancel{\frac{2m}{\hbar^2}}}}{\sqrt{\cancel{\frac{2m}{\hbar^2}}}}\left(\frac{\sqrt{E} - \sqrt{E-V_0}}{\sqrt{E}+\sqrt{E-V_0}}\right)$$

$$R = \left(\frac{\sqrt{10} - \sqrt{10-8}}{\sqrt{10} + \sqrt{10-8}}\right)^2 = 0.146 \Rightarrow 14.6\%$$

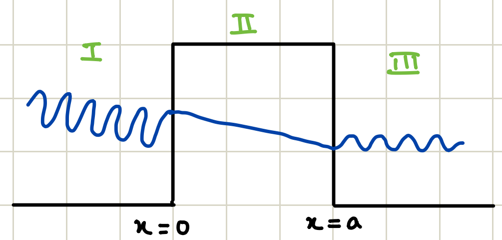
No. of e^- that are probably to reflect $= 10^4 \times 0.146 = 1460\ e^-$

A beam of electron of energy 1.5 eV approaches a potential step of height 4 eV. To
what depth it can penetrate into the classically forbidden region?

A. $$\Delta x = \frac{1}{\beta} = \frac{\hbar}{\sqrt{2m(V_0 - E)}} = \frac{\hbar}{\sqrt{2m(4-1.5)\times 1.6\times 10^{-19}}} = 1.23\times 10^{-10}\ m$$

- Potential Barrier

1



$$\psi_1 = Ae^{iK_1x} + Be^{-iK_1x} \qquad \psi_3 = De^{iK_1x}$$

$$\psi_2 = Ce^{-\beta x}$$

at $x = 0$,

$$\psi_1 = \psi_2$$

$$Ae^{iK_1x} + Be^{-iK_1x} = Ce^{-\beta x}$$

$$A + B = C$$

At $x = a$

$$\psi_2 = \psi_3$$

$$Ce^{-\beta a} = De^{iK_1 a}$$

$$C = De^{a(iK_1+\beta)} = De^{ai(K_1-K_2)} \qquad (\beta = -iK_2)$$

2

$$T = \frac{|D|^2}{|A|^2} = \left[1 + \frac{\sinh^2(\beta a)}{4\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)}\right]^{-1}$$

$$= 16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)e^{-2\beta L} \simeq e^{-2\beta L}$$

where $\beta = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

A conduction electron in a copper wire moving 240eV impinges on a potential of 50eV barrier. Find Transmission and Reflection Coefficient

A. $K_1 = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{2m_e \times 240 \times 1.6 \times 10^{-19}}{\hbar^2}} = 7.936 \times 10^{10}$

$K_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} = \sqrt{\frac{2m_e \times 190 \times 1.6 \times 10^{-19}}{\hbar^2}} = 7.06 \times 10^{10}$

$R = \left|\frac{K_1 - K_2}{K_1 + K_2}\right|^2 = \left|\frac{\sqrt{240} - \sqrt{190}}{\sqrt{240} + \sqrt{190}}\right|^2 = 3.4 \times 10^{-3}$

$T = \frac{4K_1K_2}{(K_1+K_2)^2} = \frac{4 \times 7.936 \times 7.06 \times 10^{20}}{((7.936 + 7.06) \times 10^{10})^2} = 0.996$

A beam of identical electrons is incident in a barrier 6eV high and 2nm wide
Find energy of electrons if 1% of electron are to get through barrier

A. $T = e^{-2\beta L}$

$T = 0.01$

$0.01 = e^{-2\beta \times 2 \times 10^{-9}}$

$\ln(0.01) = -2\beta \times 2 \times 10^{-9}$

$\beta = 1.15 \times 10^{9}$

$\beta = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

$E = V_0 - \frac{(\beta\hbar)^2}{2m}$

$= 6 \times 1.6 \times 10^{-19} - \frac{(1.15 \times 10^{9} \times \hbar)^2}{2m_e}$

$= 9.5 \times 10^{-19}$

$= 5.95$ eV

Electrons of some energy E are incident on barrier potential of height 6eV and L=0.5nm.
The transmitted electrons are found to have de-broglie wavelength = 3nm.
Find energy of the incident electrons, Hence, find T.

A. $V_0 = 6eV \quad l = 0.5 \times 10^{-9} m$

$\lambda = 3 \times 10^{-9} m$

$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{\lambda^2 \times 2m_e} = 2.67 \times 10^{-20} J$

$V = 6 \times 1.6 \times 10^{-19} = 9.6 \times 10^{-19} J$

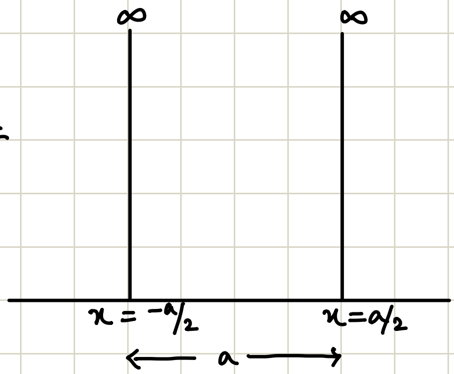
$\beta = \frac{\sqrt{2m_e(V_0 - E)}}{\hbar} = 1.236 \times 10^{10}$

$T = e^{-2\beta L} = 4.286 \times 10^{-6}$

$L = 1 - T = 0.99999$

- Applications of Barrier Tunnelling
  1) Radioactive Alpha Decay
  2) Radioactive Nuclear Decay
  3) Josephson Junction

Particles in 1D Infinite Potential Well



→ Particle has 0 Probability outside $x = \frac{-a}{2}$ & $x = \frac{a}{2}$

$V = 0 \quad ; \quad \frac{-a}{2} < x < \frac{a}{2}$

$V = \infty \quad ; \quad x = \frac{-a}{2}$ & $x = \frac{a}{2}$

→ $$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\,\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}E\,\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \qquad \left(K^2 = \frac{2mE}{\hbar^2}\right)$$

$$\psi(x) = A\sin Kx + B\cos Kx$$

→ Applying boundary conditions,

⇒ Boundary I, $x = -a/2$

$$\psi\left(\frac{-a}{2}\right) = -A\sin\left(\frac{Ka}{2}\right) + B\cos\left(\frac{Ka}{2}\right) = 0 \quad \rightarrow ①$$

⇒ Boundary II, $x = a/2$

$$\psi\left(\frac{a}{2}\right) = A\sin\left(\frac{Ka}{2}\right) + B\cos\left(\frac{Ka}{2}\right) = 0 \quad \rightarrow ②$$

① + ② ⇒ $2B\cos\left(\frac{Ka}{2}\right) = 0$

$A = 0$

$\frac{Ka}{2} = (2n+1)\frac{\pi}{2}$

$K = \frac{(2n+1)\pi}{a}$

① − ② ⇒ $2A\sin\left(\frac{Ka}{2}\right) = 0$

$B = 0$

$\frac{Ka}{2} = n\pi$

$K = \frac{2n\pi}{a}$

$n = 0, 1, 2, 3, \ldots$

→ $$K^2 = \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$\Rightarrow \frac{8mE}{h^2} = \frac{n^2}{a^2} \Rightarrow \boxed{E = \frac{n^2h^2}{8ma^2}}$$

$n = 1, 2, 3 \ldots$

$n \neq 0$ (atom is dead at $n = 0$)

→ $\boxed{E_{n-1} = n^2E_0}$ $\quad n = 2, 3, 4 \ldots$

→ Eigen energies

→ $$\psi(x) = \begin{cases} B\cos\left(\frac{n\pi x}{a}\right), & n = 1, 3, 5, 7\ldots \\ A\sin\left(\frac{n\pi x}{a}\right), & n = 2, 4, 6, 8\ldots \end{cases}$$

$$= \begin{cases} \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi x}{a}\right), & n = 1, 3, 5, 7\ldots \\ \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right), & n = 2, 4, 6, 8\ldots \end{cases}$$

Applying normalisation, $\left(\int_{-\infty}^{\infty}\varphi^*\varphi\, dV = 1\right)$

$$\int_{-a/2}^{a/2} A^2\sin^2\left(\frac{n\pi x}{a}\right)dx = 1$$

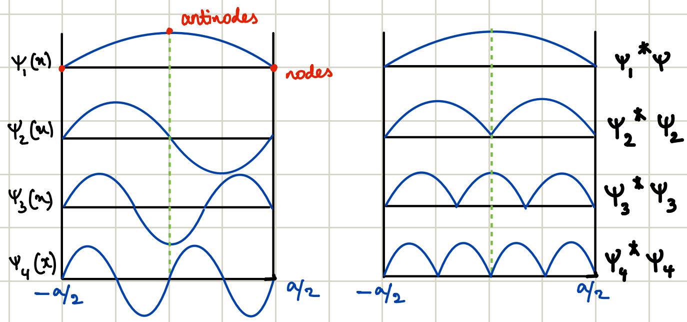
$$\frac{A^2a}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{a}}$$

Similarly, $B = \sqrt{\frac{2}{a}}$

- Eigen Functions & Probabilities for 1 D Infinite Potential Wall

$$E = \frac{n^2h^2}{8ma^2}$$

$E_1 = \frac{h^2}{8ma^2} = E_0$	$\psi_1(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi x}{a}\right)$	$\lvert\psi_1(x)\rvert^2 = \frac{2}{a}\cos^2\left(\frac{\pi x}{a}\right)$
$E_2 = \frac{4h^2}{8ma^2} = 4E_0$	$\psi_2(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right)$	$\lvert\psi_2(x)\rvert^2 = \frac{2}{a}\sin^2\left(\frac{2\pi x}{a}\right)$
$E_3 = \frac{9h^2}{8ma^2} = 9E_0$	$\psi_3(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{3\pi x}{a}\right)$	$\lvert\psi_3(x)\rvert^2 = \frac{2}{a}\cos^2\left(\frac{3\pi x}{a}\right)$
$E_4 = \frac{16h^2}{8ma^2} = 16E_0$	$\psi_4(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{4\pi x}{a}\right)$	$\lvert\psi_4(x)\rvert^2 = \frac{2}{a}\sin^2\left(\frac{4\pi x}{a}\right)$



- Particles in 2D Potential Wall

x-direction $\Rightarrow \frac{d^2\psi}{dx^2} + k_x^2\psi = 0$

y-direction $\Rightarrow \frac{d^2\psi}{dy^2} + k_y^2\psi = 0$

$$\psi_{n_x n_y} = \psi_{n_x} \times \psi_{n_y}$$

$$\psi_{11} = \frac{2}{a}\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{a}\right)$$

$$\psi_{12} = \frac{2}{a}\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)$$

$$\psi_{21} = \frac{2}{a}\sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{\pi y}{a}\right)$$

$$\psi_{22} = \frac{2}{a}\sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{2\pi y}{a}\right)$$

- Energy Eigen values

$$E_{n_x n_y} = \frac{h^2}{8ma^2}\left(n_x^2 + n_y^2\right)$$

$$E_{11} = \frac{2h^2}{8ma^2} = 2E_0$$

Non-degenerate

$$E_{21} = E_{12} = \frac{5h^2}{8ma^2} = 5E_0$$

Degenerate

$$E_{22} = \frac{8h^2}{8ma^2} = 8E_0$$

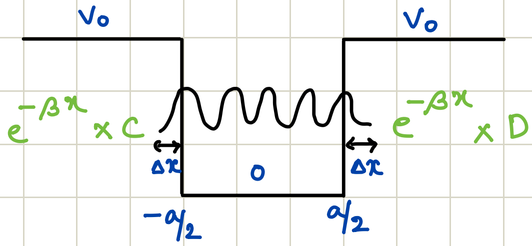
Non-degenerate

- Particles in 3D Potential Well

$$\psi_{n_x n_y n_z} = \psi_{n_x} \times \psi_{n_y} \times \psi_{n_z}$$

$$E_n = \frac{h^2}{8ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

- Particle in 1D Finite Potential Well



$$E_{finite} = \frac{n^2h^2}{8m(a+2\Delta x)^2} \longrightarrow (E_{finite} < E_\infty)$$

$$\left(\Delta x = \frac{\hbar}{\sqrt{2m(V_0-E)}}\right)$$

$$\beta = K\tan\left(\frac{Ka}{2}\right)$$

$$= -K\cot\left(\frac{Ka}{2}\right)$$

Evaluate the probability of locating a particle trapped in an infinite potential well between the limits L/4 and 3L/4 assuming that the particle is in the 3 rd excited state and the well is defined between 0 to L.

A. $\text{Probability} = \int \psi^* \psi \, dx$

$$= \int_{L/4}^{3L/4} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{4\pi x}{L}\right)\right)^2 dx = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2 \frac{4\pi x}{L} \, dx$$

$$= \frac{\cancel{2}}{L}\int_{L/4}^{3L/4} \frac{1-\cos\frac{8\pi x}{L}}{\cancel{2}} \, dx = \frac{1}{L}\left(x - \cancelto{0}{\frac{\sin\frac{8\pi x}{L}}{\frac{8\pi}{L}}}\right)_{L/4}^{3L/4} = \frac{3L-L}{4L} = \frac{1}{2}$$

1000 protons are incident on a potential barrier of width 1 pm and height 20 eV. If the energy of each proton is 10 eV, how many protons are likely to be detected on the other side of the barrier.

A. $T = e^{-2\beta L}$

$L = 10^{-12} m$

$V_0 = 20 \, eV$

$$\beta = \frac{\sqrt{2m(V_0-E)}}{\hbar} = \frac{\sqrt{2m \times (20-10) \times e}}{\hbar} = 6.942 \times 10^{11} \, m^{-1}$$

$$T = e^{-2\times 6.942 \times 10^{11} \times 10^{-12}} = 0.25$$

$T \Rightarrow 25\%$ $\longrightarrow$ No of protons tunnelling through $= 1000 \times 0.25$
$= 250$

A current beam 10 pico amperes (of identical electrons) is incident on a barrier 5.0 eV high and 1 nm wide. Find the transmitted current strength if the energy of the electrons is 4.9ev.

A. $L = 10^{-9}\ m$

$V_0 = 5eV$

$E = 4.9eV$

$\beta = \frac{\sqrt{2m(5-4.9)\times e}}{\hbar} = 1.62\times10^{+9}\ m^{-1}$

$T = e^{-2\beta L} = e^{-3.24} = 0.0392 = 3.92\%$

Transmitted Current $= 10\times10^{-12}\times0.0392 = 0.392\ pA$

Show that the probability of finding the particle in an infinite potential well of width L in the n th state in an interval L/n (between 0 to L/n) is 1/n.

A. Probability $= \int \psi^*\psi\, dx = \int_0^{L/n} \left(\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right)^2 dx = \frac{2}{L}\int_0^{L/n} \sin^2\left(\frac{n\pi x}{L}\right) dx$

$= \frac{2}{L}\int_0^{L/n} \left(\frac{1-\cos\frac{2n\pi x}{L}}{2}\right) dx$

$= \frac{1}{L}\left(x - \cancel{\frac{\sin\frac{2n\pi x}{L}}{\frac{2n\pi}{L}}}^{\,0}\right)_0^{L/n} = \frac{1}{L}\left(\frac{L}{n} - 0\right) = \frac{1}{n}$

What is the minimum energy of an electron trapped in a one dimensional region of the size of an atomic nucleus?

A. $E = \frac{n^2h^2}{8m_ea^2}$ Size of atomic nucleus $= 10^{-14} m = a$

$= \frac{1^2 \times h^2}{8m_e \times 10^{-28}} = 6.024 \times 10^{-10}\ J = 3.76 \times 10^9\ eV$

Evaluate the lowest excited state energy for an electron in 1D box of 0.1 nm width

A. $E = \frac{n^2h^2}{8m_ea^2}$

$n = 2 \qquad a = 0.1\times10^{-9}$

$E = 2.41\times10^{-17}\ J = 150.41\ eV$

An electron moving in a one dimensional potential of width 8 Ao and depth 12 eV. Find the number of bound states present.

A. $a = 8\times10^{-10}\ m$

$E = 12eV$

$12\times1.6\times10^{-19} = \frac{n^2h^2}{8m_e\times64\times10^{-20}} \Rightarrow n = 4.519$ ✗ Round off to nearest whole number

$n = 4$ ✓

Find out what would be minimum energy of the electron confined to an infinite potential well of width 0.52A. Also if height of potential is reduced to 150eV, what would be the change in energy

A. For ∞ Potential well,

$$E_\infty = \frac{n^2h^2}{8m_e a^2} = \frac{1 \times h^2}{8m_e \times (0.52 \times 10^{-10})^2} = 2.23 \times 10^{-17} J = 139 eV$$

$$E_{Finite} = \frac{n^2h^2}{8m_e(a+2\Delta x)^2}$$

$$\Delta x = \frac{\hbar}{\sqrt{2m(V_0-E)}} = \frac{\hbar}{\sqrt{2m(150-139)\times 1.6\times 10^{-19}}} = 0.58 \text{ Å}$$

$$E_{finite} = \frac{h^2}{8m_e(0.52+1.17)^2 \times (10^{-10})^2} = 3.565 \times 10^{-18} = 22.25 eV$$

Difference = 116.75 eV

- **Density of States**

$$n(E)\,dE = g(E)\,dE \times f(E)$$

$$n = \frac{\text{no. of free } e^- \text{ per atom} \times \text{Avagadro No} \times \text{Density}}{\text{Atomic Wt.}} = \frac{\pi}{3}\left(\frac{8mE_f}{h^2}\right)^{3/2}$$

$$= 7.082 \times 10^{53} E_f^{3/2}$$

$$f(E) = \frac{1}{e^{\frac{E-E_f}{KT}}+1} \longrightarrow 3 \text{ Cases}$$

i) $T = 0K$, $E > E_f$

$$f(E) = \frac{1}{e^{\frac{+ve}{0}}+1} = 0$$

ii) $T = 0K$, $E < E_f$

$$f(E) = \frac{1}{e^{\frac{-ve}{0}}+1} = 1$$

iii) $T > 0K$, $E = E_f = \frac{1}{e^0+1} = \frac{1}{2}$

- **Fermi level** – Highest occupied energy level at 0K Temperature
- **Fermi energy** – Energy corresponding to fermilevel

$$E_f = \left(\frac{3}{\pi}\right)^{2/3} \cdot \left(\frac{h^2}{8m}\right) n^{2/3}$$

Also, $n_{eff} = n \times \frac{KT}{E_F}$

- **Fermi velocity** – Velocity of e^- occupying fermilevel

$$V_F = \sqrt{\frac{2E_F}{m}}$$

- **Fermi Temperature** – Temperature at which Thermal energy of e^- = Fermi energy

$$T_F = \frac{E_F}{K_B}$$

- ## Density of States Derivation

→ $\frac{\hbar^2}{2m} \cdot \frac{d^2\psi(x)}{dx^2} + E\psi(x) = 0$ $\qquad E_{nx} = \frac{h^2 n_x^2}{8ma^2}$

$\frac{\hbar^2}{2m} \cdot \frac{d^2\psi(y)}{dy^2} + E\psi(y) = 0$ $\qquad E_{ny} = \frac{h^2 n_y^2}{8ma^2}$

$\frac{\hbar^2}{2m} \cdot \frac{d^2\psi(z)}{dz^2} + E(z) = 0$ $\qquad E_{nz} = \frac{h^2 n_z^2}{8ma^2}$

→ $E_n = \frac{h^2}{8ma^2}(n_x^2 + n_y^2 + n_z^2) = \frac{h^2 n^2}{8ma^2}$

→ Volume of shell of thickness dn $= \frac{\pi n^2 dn}{2}$

→ Max no. of e^- occupying these states $= 2 \times \frac{1}{2}\pi n^2 dn = \pi n^2 dn \longrightarrow$ (1)

Now, $E = n^2 E_0 \Rightarrow n = \sqrt{\frac{E}{E_0}} \longrightarrow$ (2)

and, $\frac{dE}{E_0} = 2n\,dn \longrightarrow$ (3)

(2) & (3) in (1)

$\Rightarrow \pi n \times n\,dn = \pi\sqrt{\frac{E}{E_0}} \times \frac{dE}{2E_0} = \frac{\pi\sqrt{E}\,dE}{2E_0^{3/2}}$

$E_0 = \frac{E}{n^2} = \frac{h^2}{8ma^2}$

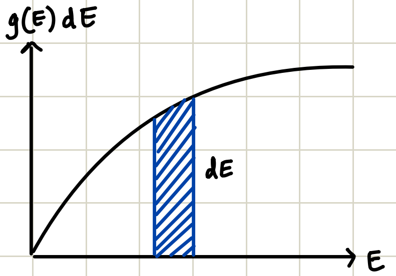
So, max no. of e^- occupying these states $= \frac{\pi}{2} \times \frac{\sqrt{E}\,dE}{\left(\frac{h^2}{8ma^2}\right)^{3/2}}$

$\Rightarrow$ No. of energy states b/w E & dE per unit volume,

$= \frac{\frac{\pi}{2} \times \left(\frac{8m}{h^2}\right)^{3/2} a^3 \sqrt{E}\,dE}{a^3} = \frac{\pi}{2} \times \left(\frac{8m}{h^2}\right)^{3/2} \sqrt{E}\,dE$ = Density of States

$\Rightarrow$ No. of e^- in solid $= \frac{\pi}{2} \times \left(\frac{8m}{h^2}\right)^{3/2} \sqrt{E}\,dE \times \frac{1}{e^{\frac{E-E_F}{KT}} + 1}$

$g(E)\,dE$ vs E graph



$n = \int_{n_{min}}^{n_{max}} n(E)\,dE \Rightarrow n = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} \int_0^{E_f} \sqrt{E}\,dE$

$= \frac{\pi}{3}\left(\frac{8mE_f}{h^2}\right)^{3/2}$

(no. density of e^-)

$n = \frac{\pi}{3}\left(\frac{8mE_f}{h^2}\right)^{3/2} = 7.0822 \times 10^{55} E_f^{3/2}$

$E_f = \left(\frac{n}{7.0822 \times 10^{55}}\right)^{2/3}$

If Ef of Copper is 7eV, Find VF and TF

A. $T_F = \frac{7eV}{K_B} = 81231.62\,K$

$V_F = \sqrt{\frac{2 \times 7eV}{m_e}} = 1.57 \times 10^6\, ms^{-1}$

But, $E_F \propto N$

$K_B T \propto n_{eff}$

So, $\frac{n_{eff}}{N} \propto \frac{K_B T}{E_F}$ (at 300K Temp)

$n_{eff} \propto \frac{K_B T N}{E_F}$

$n_{eff} \propto \frac{1.38 \times 10^{-23} \times 300 \times N}{7 \times 1.6 \times 10^{-19}}$

$n_{eff} \propto 3.69 \times 10^{-3}\, N$

- Average Energy

$$\langle E \rangle = \frac{\int_0^{E_f} g(E) \times E \times F_d \times dE}{\int_0^{E_f} g(E) \times F_d \times dE}$$

$$= \frac{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} \int_0^{E_f} \sqrt{E}\, dE \times E}{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} \int_0^{E_f} \sqrt{E}\, dE}$$

$$\langle E \rangle = \frac{3E_F}{5}$$

$$\boxed{\langle E \rangle = 0.6\, E_F}$$

- Quantum Harmonic Oscillator

→ Mechanical oscillator oscillates b/w PE & KE

$F = ma = m\frac{d^2x}{dt^2} = -Kx$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{Kx}{m} = 0 \Rightarrow \frac{d^2x}{dt^2} + w^2 x = 0 \quad \left(w = \sqrt{\frac{K}{m}}\right)$

$x(t) = A\sin wt + B\cos wt$

→ PE of system

$V(x) = \frac{Kx^2}{2} = \frac{mw^2x^2}{2}$

$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$\frac{d^2\psi(x)}{dx^2} + \frac{2\mu}{\hbar^2}\left(E - \frac{\mu w^2 x^2}{2}\right)\psi(x) = 0 \Rightarrow$ Hermite Equation

→ Eigen function of Harmonic Oscillator

$\psi_n(x) = N H_n(x)\, e^{-x^2/2}$

where $N = \frac{1}{\sqrt{2^n n!}\,\pi^{1/4}}$

N → Normalisation constant

H_n → Hermite Polynomial

Not important, but,

$H_0(x) = 1$

$H_1(x) = 2x$

$H_2(x) = -2(1 - x^2)$

→ $\frac{2E_n}{\hbar w} = 2n + 1 \Rightarrow E_n = \left(n + \frac{1}{2}\right) w\hbar$

A mono valent metal has 5 x 10^28 valence electrons per m^3 . Estimate the number of electron energy states per unit volume in the metal between 2eV and 2.005eV from the concept of the density of states

A. $$g(E)dE = \frac{\pi}{2}\left(\frac{8m_e}{h^2}\right)^{3/2}\sqrt{E}\,dE$$

$$= \frac{\pi}{2}\left(\frac{8m_e}{h^2}\right)^{3/2}\sqrt{2\times1.6\times10^{-19}} \times 0.05\times1.6\times10^{-19}$$

$$= 4.81 \times 10^{25}\ m^{-3}$$

Find the temperatures at which the occupancy of an energy state 0.3 eV above the Fermi level has an occupancy probability of 0.05

A. $$F_d = \frac{1}{e^{\frac{E-E_F}{K_BT}}+1}$$

$$\Rightarrow e^{\frac{E-E_f}{K_BT}} = \frac{1}{F_d}-1$$

$$\frac{E-E_f}{K_BT} = \ln\left(\frac{1}{F_d}-1\right)$$

$$T = \frac{E-E_f}{K_B\times\ln\left(\frac{1}{F_d}-1\right)} = \frac{0.3\times1.6\times10^{-19}}{K_B\times\ln\left(\frac{1}{0.05}-1\right)} = 1182.35\ K$$

Estimate the energy for which the probability of occupation at 300K is 0.1 for copper with Fermi energy of 7.0eV. Comment on the probability of this level to be 0.5.

A. $$\frac{E-E_f}{K_BT} = \ln\left(\frac{1}{F_d}-1\right) \Rightarrow E = E_f + K_BT\ln\left(\frac{1}{F_d}-1\right)$$

$$E = 7\times1.6\times10^{-19} + K_B\times300\ \ln\left(\frac{1}{0.1}-1\right) = 1.133\times10^{-18}\ J = 7.076\ eV$$

Determine the free electron concentration, the Fermi velocity for electrons in a metal with Fermi energy of 5.10 eV.

A. $$n = \frac{\pi}{3}\left(\frac{8m}{h^2}\right)^{3/2} E_f^{3/2}$$

$$= \frac{\pi}{3}\left(\frac{8m_e}{h^2}\right)^{3/2}\cdot\left(5.1\times1.6\times10^{-19}\right)^{3/2}$$

$$= 5.3\times10^{28}\ m^{-3}$$

$$v_f = \sqrt{\frac{2E_f}{m}} = \sqrt{\frac{2\times5.1\times1.6\times10^{-19}}{m_e}} = 1.34\times10^{6}\ ms^{-1}$$

Determine the Fermi energy and Fermi temperature for copper with 8.5 x 10^28 free electrons per unit volume

A. $n = 8.5 \times 10^{28}\ m^{-3}$

$$n = \frac{\pi}{3}\left(\frac{8m_e E_f}{h^2}\right)^{3/2}$$

$$E_f = \left(\frac{3n}{\pi}\right)^{2/3} \times \frac{h^2}{8m_e} = 1.13 \times 10^{-18}\ J = 7.05\ eV$$

$$E_f = K_B T_f \Rightarrow T_f = 81804.4\ K$$

Find the electron density in a metal having Fermi energy of 5.5 eV.

A. $E_f = 5.5\ eV$

$$n = \frac{\pi}{3}\left(\frac{8m_e}{h^2}\right)^{3/2} \times E_f^{3/2}$$

$$= \frac{\pi}{3}\left(\frac{8m_e}{h^2}\right)^{3/2} \times \left(5.5 \times 1.6 \times 10^{-19}\right)^{3/2}$$

$$= 5.857 \times 10^{28}\ m^{-3}$$

The Fermi energy of silver is 5.5 eV. What is the mean energy of a free electron in sliver at 0K? Also, at what temperature a classical free particle will have this kinetic energy?

A. $E_f = 5.5\ eV$

$$\text{Avg. Energy} = \frac{3E_f}{5} = 3.3\ eV$$

$$\text{Also, } \langle E \rangle = \frac{3K_B T}{2}$$

$$\text{So, } T = \frac{\langle E \rangle \times 2}{3K_B} = 25529.94\ K$$

The Fermi temperature of two metals A and B are in the ratio 1.103. If the electron concentration of metal A is 5.86x10^28 m^-3 , find the Fermi velocity of electrons in metal B

A. $\frac{T_{FA}}{T_{FB}} = 1.103$

$n_A = 5.86 \times 10^{28}\ m^{-3}$

$\Rightarrow T_F \propto E_F$

$$\text{So, } \frac{E_{FA}}{E_{FB}} = 1.103$$

$$n = \frac{\pi}{3}\left(\frac{8m_e}{h^2} \times E_F\right)^{3/2} \Rightarrow E_{F_A} = \left(\frac{3n}{\pi}\right)^{2/3} \times \frac{h^2}{8m_e} = 8.814 \times 10^{-19}\ J$$

$$E_{F_B} = \frac{E_{FA}}{1.103} = 7.991 \times 10^{-19}\ J$$

$$V_{F_B} = \sqrt{\frac{2E_{FB}}{m_e}}$$

$$= 1.324 \times 10^{6}\ ms^{-1}$$

- Hydrogen atom Problem

→ Hydrogen is simplest atom with 1 proton in nucleus & 1 electron orbiting

→ The schrödinger wave eq for Hydrogen is complex

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2\mu}{\hbar^2}(E-V)\,\psi(x,y,z) = 0$$

because $V = \frac{-e^2}{4\pi\varepsilon_0 r}$ which is direct function of r

→ We use spherical coordinates,

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

Then wave eqⁿ becomes,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2\mu}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right)\psi = 0$$

Thus solution will be,

$$\psi(r,\theta,\phi) = R(r)\,.\,\Theta(\theta).\,\Phi(\phi)$$

Solving them seperately, we get 3 quantum no.s

1) n, Principal Quantum No.

$n = 1, 2, 3$

2) l, Orbital Quantum No

$l = 0, 1, 2, 3 \ldots n-1$

3) m_l, Magnetic

$m_l = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm l$

→ Eigen energy values

$$E_n = \frac{-R_H}{n^2} = -\left[\frac{\mu}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right].\frac{1}{n^2}$$

where $n = 1, 2, 3, \ldots$

$$\Delta E = E_{n_2} - E_{n_1}$$

$$= -\left(\frac{\mu}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Unit - 3

- **Classical Free Electron Theory** (Drude & Lorentz Theory)

→ Assumptions made :

Valence e^-
- Don't repel each other
- Treated like Conduction e^-
- Treated like gas molecules
- +ve ion cores influence is neglected
- Follow Maxwell - Boltzmann Statistics

- **Thermal Velocity** → The avg speed at which particles move due to thermal energy in a material

→ $\frac{1}{2}mv_{Th}^2 = \frac{3}{2}K_BT$

$$V_{Th} = \sqrt{\frac{3K_BT}{m}}$$

→ This random motion has no contribution to net drift of e^- & hence no current flows

- **Drift Velocity** → In presence of Electric field, free e^- shows net drift across metal in opposite direction to Electric field. This velocity is drift velocity

$$\boxed{V_d = \frac{e\tau E}{m} = \frac{eE}{Km}}$$

$\left(\tau = \frac{1}{K}\right)$ K : Coefficient of Scattering loss
τ : Relaxation time

- **Relaxation Time** → The time b/w successive collissions

$$\boxed{\tau = \frac{\lambda}{V_{Th}}}$$

→ The time for drift velocity to fall $\frac{1}{e}$ times its steady value in presence of electric field.

- **Mobility** → Drift velocity per unit electric field

$$\boxed{\mu = \frac{V_d}{E} = \frac{e\tau}{m} = \frac{\sigma}{ne}}$$

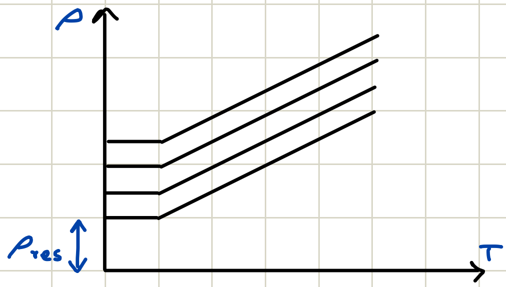
- **Electrical Conductivity**

$$I = neAV_d \Rightarrow J = \frac{I}{A} = neV_d$$

$$\sigma = \frac{J}{E} = \frac{neV_d}{E} = \frac{ne.eE\tau}{mE} = \boxed{\frac{ne^2\tau}{m} = \sigma} \quad \& \quad \boxed{\rho = \frac{m}{ne^2\tau}} \qquad \rho = \frac{1}{\sigma}$$

Conductivity
Resistivity

- **Matthiessen's Rule**



$\rightarrow \rho = \rho_{res} + \rho_{sc}$ $\left(\rho = \frac{m}{ne^2\tau}\right)$

$$\frac{m}{ne^2\tau} = \frac{m}{ne^2\tau_{res}} + \frac{m}{ne^2\tau_{sc}} \Rightarrow \frac{1}{\tau} = \frac{1}{\tau_{res}} + \frac{1}{\tau_{sc}}$$

- **Failure of CFET**

1) Temp. dependance of resistivity of metals

Theoretically, $\rho \propto \sqrt{T}$
Experimentally, $\rho \propto T$

2) Specific heat dependance of e^- in metals

Theoretically, $C_{el} = \frac{dU}{dt} = \frac{3}{2} K_B N_{avg} = \frac{3R}{2}$
Experimentally, Only 1% of this value & Temperature dependant

3) Dependance of e^- conc. on conductivity

Theoretically, $\sigma \propto n$
Experimentally, no linear dependance

Calculate the free electron concentration, mobility and drift velocity of electrons in an Al wire of diameter 0.5mm, length 5m, resistance of 60 milliohms that carries a current of 15A . Al has 3 free electrons At wt of Al=26.98 and density 2.7 x 10^3 kg m^-3.

A. $n = \dfrac{\text{no. of free } e^- \times N_A \times 10^3 \times \text{density}}{\text{Atomic Wt}} = 1.808 \times 10^{29}\ m^{-3}$

$R = \dfrac{\rho l}{A} \Rightarrow \rho = \dfrac{RA}{l} = \dfrac{60\times10^{-3}}{5} \times \dfrac{\pi}{4} \times (0.5\times10^{-3})^2 = 2.35\times10^{-9}\ \Omega m$

$\mu = \dfrac{1}{\rho n e} = \dfrac{1}{2.35\times10^{-9} \times 1.808\times10^{29} \times e} = 0.014\ m^2 V^{-1} s^{-1}$

$V_d = \dfrac{I}{neA} = \dfrac{15}{1.808\times10^{29} \times e \times \frac{\pi}{4}(0.5\times10^{-3})^2} = 2.637 \times 10^{-3}\ ms^{-1}$

Silver has a density of 10.5 x 10^3 Kgm^-3 and atomic weight of 107.9. If conductivity of silver at 27° C is 6.8 x 10^7 (ohm-m)^-1, find the mean free path of electrons as per the classical free electron theory.

A. $n = \dfrac{\text{no. of free } e^- \times N_A \times 10^3 \times \text{density}}{\text{Atomic. Wt}}$

$= \dfrac{1 \times N_A \times 10^3 \times 10.5 \times 10^3}{107.9} = 5.86 \times 10^{28}\ m^{-3}$

$V_{Th} = \sqrt{\dfrac{3K_BT}{m}} = \sqrt{\dfrac{3\ K_B \times 300}{m_e}} = 1.168 \times 10^5\ ms^{-1}$

$\sigma = \dfrac{ne^2\tau}{m_e} \Rightarrow \tau = \dfrac{\sigma m_e}{ne^2} = \dfrac{6.8\times10^7 \times m_e}{5.86\times10^{28} \times e^2} = 4.118 \times 10^{-14}\ s$

$\lambda = \tau \times V_{Th} = 4.118\times10^{-14} \times 1.168 \times 10^5 = 4.81 \times 10^{-9}\ m$

A certain conductor has a free electron concentration of 5.9 x 10^28 m^-3. What current density in the conductor will correspond to a drift velocity of 1/1.6 mm s^-1? Calculate the mobility of charge carriers given conductivity as 6.22x10^7 (Ωm)-1.

A. $n = 5.9 \times 10^{28}\ m^{-3}$

$V_d = \dfrac{1}{1.6}\ mm s^{-1} = \dfrac{10^{-3}}{1.6}\ ms^{-1}$

$\sigma = 6.22 \times 10^7 (\Omega m)^{-1}$

$J = \dfrac{I}{A} = neV_d = 5.9\times10^{28} \times e \times \dfrac{10^{-3}}{1.6} = 5.9\times10^6\ A/m^2$

$\mu = \dfrac{\sigma}{ne} = \dfrac{6.22\times10^7}{5.9\times10^{28} \times e} = 6.58 \times 10^{-3}\ m^2 V^{-1} s^{-1}$

- Quantum Free Electron Theory (Sommerfeld)
  → Assumptions made :
  - Valence e^- fill discrete energy states & only those close to Fermi level conduct electricity
  - Occupation probability is given by Fermi-dirac distribution
  - Free e^- move through a grid of +ve ions bumping into ions & e^- causing electrical resistance
  - electron - electron & electron - ion interactions are negligible

- Merits of QFET
  1) Conductivity variance with electron concentration

$$\sigma_{QFET} = \left(\frac{n_{eff} e^2}{m}\right) \frac{\lambda}{v_F} \Rightarrow \sigma \propto n$$

  2) Temperature dependance on resistivity

$$\rho \propto T$$

  3) Relation b/w K & σ

$$\frac{K}{\sigma} = LT$$

$$L = 2.26 \times 10^{-8} \ W \Omega K^{-2}$$

K: Thermal Conductivity
σ: Electrical Conductivity
L: Lorenz Number Constant

  4) Heat capacity due to e^-

$$L_{CFET} = \frac{K}{\sigma T} = \frac{3}{2} \cdot \left(\frac{K_B}{e}\right)^2 = 1.11 \times 10^{-8} \ W \Omega K^{-2}$$

$$L_{QFET} = \frac{K}{\sigma T} = \frac{\pi^2 K_B^2}{3 e^2} = 2.44 \times 10^{-8} \ W \Omega K^{-2}$$

- Drawbacks of QFET

  1) Can't differentiate b/w metals, semiconductors & insulators

  2) Can't explain +ve Hall co-efficient (existance of poles)

Calculate the ratio of the thermal conductivity of a metal to the electrical conductivity of the metal at 500 K.

A. $$\frac{K}{\sigma T} = \frac{\pi^2 K_B^2}{3e^2}$$

$$\frac{K}{\sigma} = \frac{\pi^2 K_B^2 \times 500}{3e^2} = 1.22 \times 10^{-5}$$

Discuss the molar specific heat of free electrons as per quantum free electron theory. Estimate the molar specific heat of free electron gas at 500 K as per the quantum free electron theory if the Fermi energy of the metal is 3.75 eV.

A. Correct evaluation of electronic specific heat – Considering contribution from valence e^- close to fermi level

Heat absorption happens due to this small fraction of e^-

$$U = \frac{3}{2} \cdot \frac{N_A}{E_F} K_B^2 T^2$$

$$C_{el} = \frac{dU}{dT} = \frac{3 N_A K_B^2 T}{E_F} = \frac{3 N_A K_B^2 \times 500}{3.75 \times e} = 0.2866 \quad J K^{-1} mol^{-1}$$

- Wiedmann – Franz Law (Relation b/w Thermal & Electrical conductivity)

$\rightarrow$ $$K = \frac{1}{3} \frac{C_V}{V} \times v L$$

$$= \frac{1}{3} \times \frac{1}{V} \times \frac{\pi^2}{2} N_{eff} \frac{K_B^2 T}{E_F} v_F \cdot v_F \tau$$

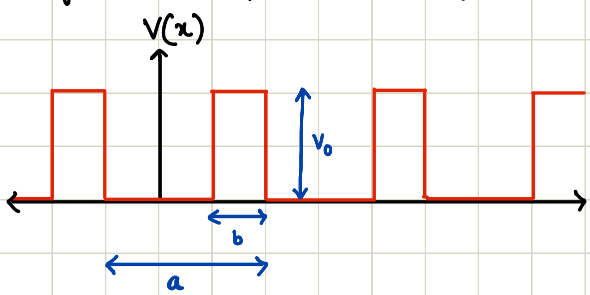
$$= \frac{\pi^2 n_{eff} K_B^2 T \; v_F^2 \tau}{6 E_F}$$

$$= \frac{\pi^2 n_{eff} K_B^2 T \tau}{3m}$$

$$= \frac{\pi^2 K_B^2 T}{3e^2} \cdot \sigma \qquad \left(\sigma = \frac{n_{eff} e^2 \tau}{m} \right)$$

- Bloch's Theorem

$\rightarrow$ The atomic structure in solids is regular, so, e^- waves are also regular & predictable, resulting in accurate analysis of wave function



$$\psi(x) = e^{ikx}$$

$$\psi(x+a) = e^{ik(x+a)} = e^{ikx} \cdot e^{ika}$$

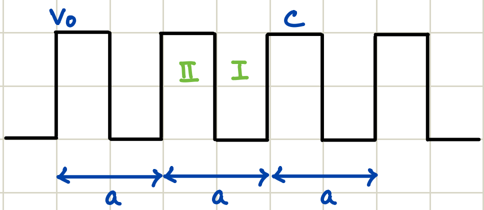
$$\psi(x) = U_K(x) \cdot e^{ikx}$$

$$= U_K(x+a) \cdot e^{ikx}$$

Hence, $\boxed{U_K(x) = U_K(x+a)}$

- Kronig - Penny Model

→ Approximates periodic potential as long chain of coupled finite rectangular wells



→ $\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}E\psi_1 = 0$

$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}(V_0-E)\psi_2 = 0 \xrightarrow{\alpha^2 = \frac{2m}{\hbar^2}(V_0-E)} \frac{d^2\psi_2}{dx^2} - \alpha^2\psi_2 = 0$

→ Using Bloch's Theorem & Boundary Condition, Obtained solution is a transcendantal equation giving variation of E with K & has discontinuities (forbidden gap)

→ In allowed region, e^- move with $E = \frac{\hbar^2 K^2}{2m_e}$

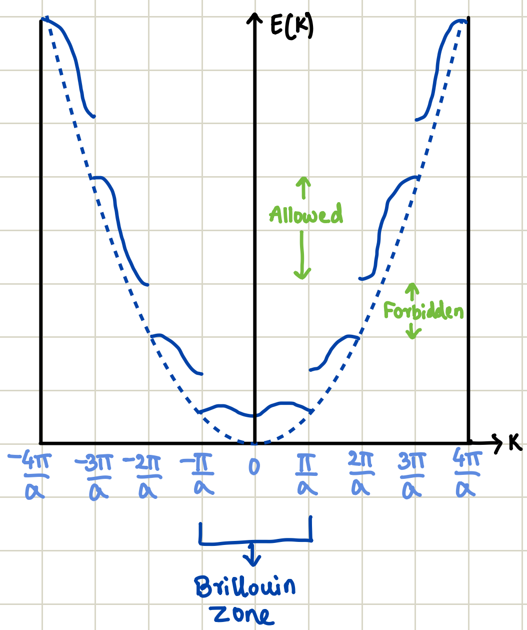
→ Condition under which they would be periodic

$$\frac{m}{\alpha\hbar^2} V_0 b \sin\alpha a + \cos\alpha a = \cos Ka$$

$$P \sin\alpha a + \cos\alpha a = \cos Ka \qquad \left(P = \frac{maV_0 b}{\hbar^2}\right)$$

Longer P, steeper the curve

→ E - K diagram shows discontinuities at boundary zones $\left(K = \pm\frac{n\pi}{a}\right)$



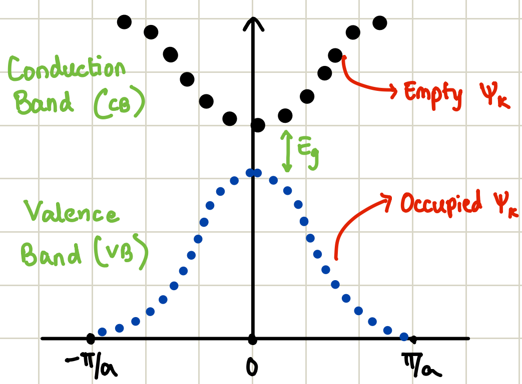
- KP Model Classification

→ Conductors - Have partially filled conduction gap & no band gap

→ Semi-conductors - Completely filled valence band & completely empty conduction band

Energy gap 3 - 5 eV

→ Insulators - Energy gap > 5eV



- # Effective Mass

→ The apparent mass an e^- gains / loses due to motion along band structure

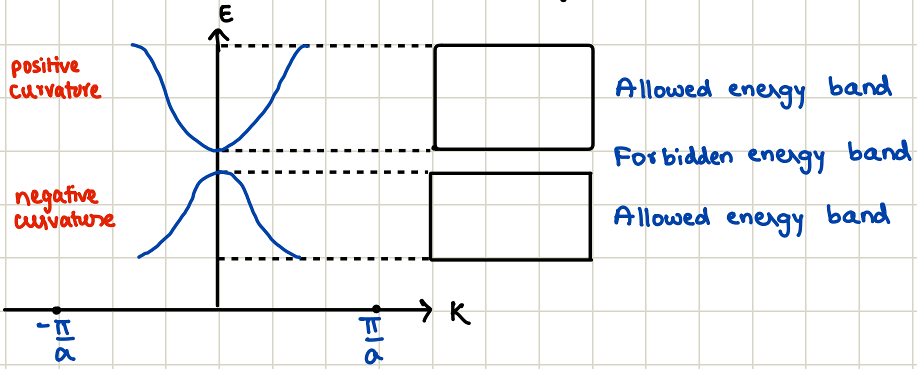
→ It is a way to describe how an e^- behaves in the crystal

$$E = \frac{\hbar^2 K^2}{2m}$$

$$V_g = \left(\frac{dE}{dK}\right) = \frac{\hbar^2 K}{m}$$

$$a = \frac{d^2E}{dK^2} = \frac{\hbar^2}{m^*}$$

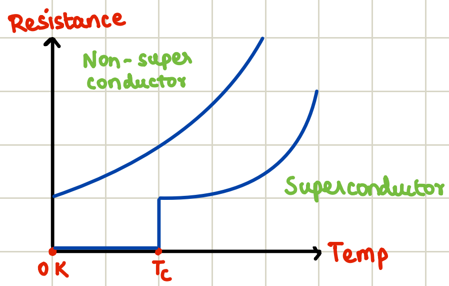
$$\boxed{m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dK^2}\right)}}$$



⇒ m^* isn't constant & depends on non-linearity of E

- # Superconductivity

→ Property of certain metals, alloys & ceramics where electrical resistance drops to $0\,\Omega$ when temperature is reduced below a critical value



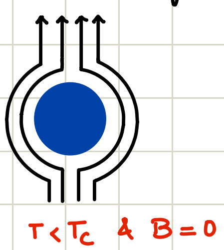
applications :

→ MAGLEV vehicles

→ MRI scan

- # Meissner Effect

→ The ability of superconductors expel magnetic fields during transition to superconducting state (like a perfect diamagnetic material) is Meissner effect

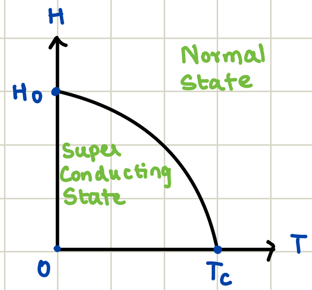


- # Critical Field

→ The magnetic field at which material loses its superconducting state

→ It is temperature dependant

$$H_c = H_0\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$



Classification of Superconductors

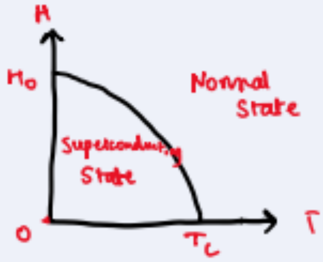
1) Soft Superconductors

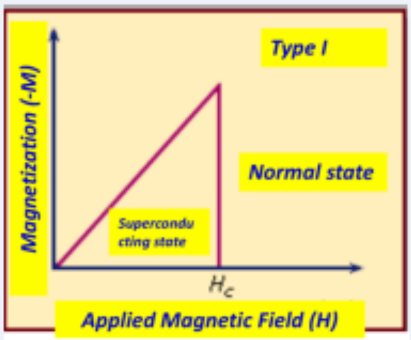
→ Exhibit complete Meissner effect

→ In presence of external magnetic field $H < H_c$, material in superconducting state

→ As soon as H exceeds H_c, material becomes normal (Have low values of H_c)

ex: Al, Pb, In etc.,





Superconductors are dogmatic!
↳ Neha Shah

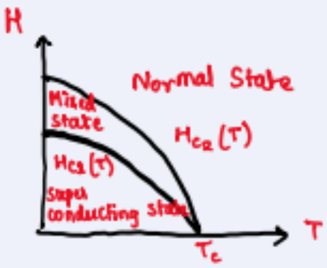
Superconductivity or Nothing

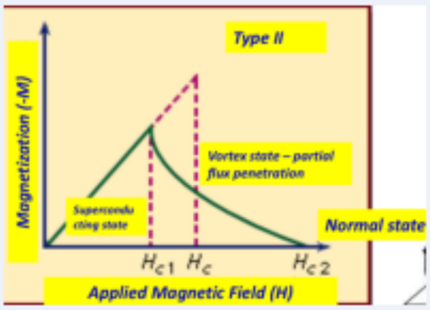
2) Hard Semiconductors

→ Characterized by 2 Critical magnetic fields

→ Below H_{c1}, Semiconductor shows complete Meissner effect
Above H_{c2}, It goes to normal state

→ ex: Transition metals, Metal alloys containing Niobium, Si, Vanadium





Superconductors are pragmatic

adjust & compromise

→ Vortex state: Partial flux penetration through filaments

- BCS Theory
  (Bardeen, Cooper, Schrieffer)

→ In Superconductors, If 2 e^- have opposite spin & opposite momenta, they overcome repulsion & form a pair called Cooper pair

→ These pairs move together through material w/o getting scattered

→ But while moving, it slightly distorts atomic lattice & creates small +ve charge region attracting another e^- to form cooper pair!

→ An energy gap exists in semiconductors to prevent scattering & loss of energy in e^- maintaing superconductivity

→ Because the cooper pairs move in collective coordinated way, it smoothly allows semiconductor to carry current, resistance-free.

Superconducting tin has a critical temperature of 3.7 K with critical field of 0.0306 T. Find the critical field at 2 K.

A. $T_c = 3.7K$; $H_0 = 0.0306\,T$; $T = 2K$

$$H_c = H_0\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

$$H_c = 0.0306\left(1 - \left(\frac{2}{3.7}\right)^2\right)$$

$$H_c = 0.0216\,T$$

Calculate the critical field for a Superconducting wire at 4.2 K. Critical temperature for lead is 7.18 K and Hc(0) is 6.5 x 104 T.

$H_0 = 6.5 \times 10^4\,T$; $T_c = 7.18\,K$; $T = 4.2K$

$$H_c = H_0\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

$$= 6.5\times10^4\left(1 - \left(\frac{4.2}{7.18}\right)^2\right)$$

$$= 4.276 \times 10^4\,T$$

For Nb, the critical fields are 1x10^5 A/m at 8K and 2x10^5 A/m at 0K. Calculate Transition Temperature. Also if we want to keep the Nb wire superconductivity at an applied magnetic field of 1.5x10^5 A/m, What precaution must be taken?

A. $H_c = H_0\left(1 - \left(\frac{T}{T_c}\right)^2\right) \Rightarrow T_c = \dfrac{T}{\sqrt{1 - \frac{H_c}{H_0}}} = \dfrac{8}{\sqrt{1 - \frac{10^5}{2\times10^5}}} = 11.31\,K$

$$T = T_c\sqrt{1 - \frac{H_c}{H_0}} = 11.31\sqrt{1 - \frac{1.5\times10^5}{2\times10^5}} = 5.657\,K$$

- ## Magnetics

→ $\boxed{M = \frac{\mu}{V}}$ → Magnetic Dipole Moment → Volume

Intensity of Magnetization (Am^{-1})

→ $\boxed{M = \chi H}$ → Magnetic Field strength (Am^{-1})

Magnetic susceptibility

→ $\boxed{H = \frac{nI}{l}}$

→ $\boxed{B = \mu_0 H}$ $\qquad (\mu_0 = 4\pi \times 10^{-7}\ H/m)$

Flux Density $(wb\ m^{-2})$

→ $\boxed{B = \mu_0 (1 + \chi_m) H}$ ⇒ When material of susceptibility χ_m introduced in solenoid

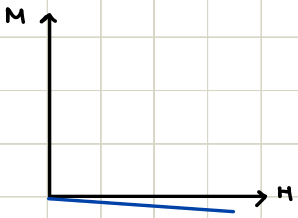
$\boxed{B = \mu_0 (M + H)}$

→ $\boxed{\mu_r = 1 + \chi_m}$

Relative Permeability

- ## Classification of materials by susceptibility & permeability

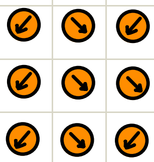
1) Diamagnetic



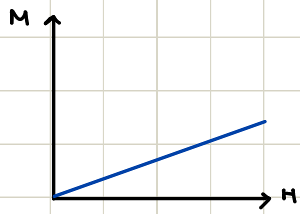
$\chi_m = -ve$

$(-10^{-3}$ to $-10^{-6})$

$\mu_r < 1$



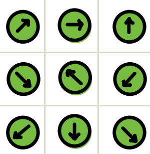
2) Paramagnetic



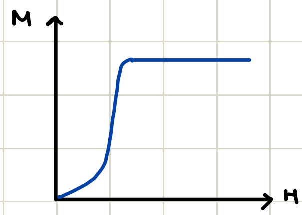
$\chi_m = +ve$

$(10^{-4}$ to $10^{-5})$

$\mu_r > 1$



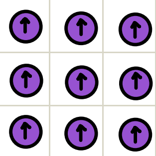
3) Ferromagnetic



$\chi_m = +ve$

$(10^{3}$ to $10^{6})$

$\mu_r >> 1$



A magnetic material has a magnetization of 2300 Am-1 and produces a flux density of 0.00314 Wb m-2. Calculate the magnetizing force and the relative permeability of the material.

A. $M = 2300\ Am^{-1}$ $\qquad H = ?$ $\qquad \mu_r = ?$

$B = 0.00314\ Wbm^{-2}$

$B = \mu_0 (M + H)$

$H = \frac{B}{\mu_0} - M = \frac{0.00314}{4\pi \times 10^{-7}} - 2300 = 198.73\ Am^{-1}$

$\mu_r = \chi + 1 = \frac{M}{H} + 1 = 12.57$

- Magnetic Moment

$\rightarrow$ $I = \frac{-e}{T} = \frac{-e\omega}{2\pi} = \frac{-ev}{2\pi r}$

$$\mu = IA$$
$$= \frac{-ev}{2\pi r} \times \pi r^2$$
$$= -\frac{evr}{2}$$
$$= -\frac{evr}{2} \times \frac{m_e}{m_e}$$
$$= \frac{-eL}{2m_e} \qquad (mvr = L)$$

$$\boxed{\frac{\mu_{orb}}{L} = \frac{e}{2m_e}} \longrightarrow \text{Gyromagnetic Ratio} = 8.794 \times 10^{10}\ s^{-1}\ T^{-1}$$

$\rightarrow$ $J = S + L$

Total Magnetic Moment — Spin — Orbital

$$\mu_e = \mu_s + \mu_L$$
$$= \sqrt{s(s+1)}\,\frac{e\hbar}{2m_e} + \sqrt{l(l+1)}\,\frac{e\hbar}{2m_e}$$

$$\boxed{\mu_e = g\,\frac{e\hbar}{2m_e}}$$

$g \rightarrow$ Lande's g-factor

$$= 1 + \frac{j(j+1) + s(s-1) - l(l+1)}{2j(j+1)}$$
$$= 1 + 1 = 2$$

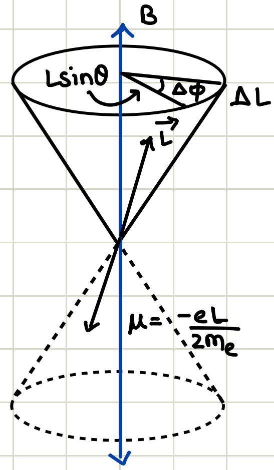
$\rightarrow$ For smallest non-zero value of μ_s due to e^- ($s = j$ & $l = 0$) is $\mu_s = \frac{e\hbar}{m} = 2 \times \mu_{orb}$

- Larmor Precession

→ The torque produces a change in angular momentum ΔL & magnetic moment around applied field called Larmor Precession

→ $\tau = \frac{\Delta L}{\Delta t}$

& $\tau = \mu \times B$
$= \mu B \sin\theta$
$= -\nu L B \sin\theta$ $\quad (\mu = -\nu L)$
$= -\frac{eB}{2m} \times L\sin\theta$ $\quad \left(\nu = \frac{e}{2m}\right)$
$= \omega_L \cdot L\sin\theta$



- Magnetic Moment due to Precessing Charge

→ $\mu_e = IA = \frac{e\omega r^2}{2}$

→ $\mu_{ind} = \frac{e\omega_L r^2}{2}$

Larmor freq $\omega_L = \frac{eB}{2m_e}$

Then, $\mu_{ind} = -\frac{e^2 B r^2}{4m_e}$ (-ve cuz of Lenz's law)

→ Total induced Magnetization,

$$M = N_a Z \mu_{ind} = -\frac{N_a Z B e^2 r^2}{4m_e} = -\frac{N_a Z \mu_0 H e^2 r^2}{4m_e}$$

$$\chi = -\frac{N_a Z \mu_0 e^2 r^2}{4m_e} \quad \left(\chi = \frac{M}{H}\right)$$

A magnetic field of 1T is applied to an electron undergoing orbital motion. Calculate the precessional frequency

A. $\omega_L = \frac{eB}{2m_e} = \frac{e \times 1}{2\,m_e} = 8.794 \times 10^{10}$ rad/s

Estimate the magnetic field required to produce a precessional frequency 67.5 MHz in proton

A. $\omega_L = 67.5 \times 10^6$ Hz

$$B = \omega_L \times \frac{2m_p}{e} = \frac{67.5 \times 10^6 \times 2 \times m_p}{e} = 1.409\text{ T}$$

Classification of Magnetic Materials

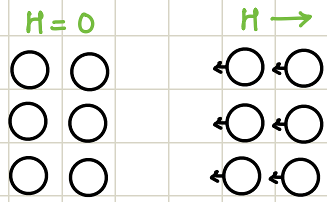
→ Diamagnetic

→ Materials whose magnetic moments of orbiting e^- adds upto 0 in absence of external magnetic field & allign in opposite direction in presence of it.

→ $\chi_{dia} = -ve$

$\mu_r < 1$ $\quad (\mu_r = 1 + \chi_m)$

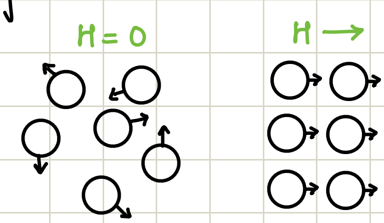
$$\chi_{dia} \propto \frac{N_a Z e^2 \langle r^2 \rangle \mu_0}{m}$$



→ ex: H_2O, NaCl, Plastics etc.,

→ Paramagnetic

→ Materials where atoms with unpaired e^- are weakly attracted by magnetic fields & randomly oriented magnetic dipoles in absence of magnetic field



→ $\underbrace{K_B T}_{\text{Thermal energy}} > \underbrace{\mu B}_{\text{Magnetic energy}}$

→ $\chi_{para} > 0$ (Depends on Temp)

ex: Al, O, Na

→ Ferromagnetic

→ Shows spontaneous magnetization & easily magnetized

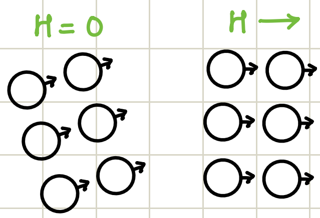
→ $10^3 < \chi_{Ferro} < 10^6$

→ High ordering of unpaired e^- spins – magnetic domains

→ Show sharp hysteresis characteristics

→ Phase transition from ferro to para occurs above curie Temp.

ex: Fe, Co, Ni



• Spin Ordered Magnetic Materials

→ Ferromagnetic

→ Shows spontaneous magnetization & easily magnetized

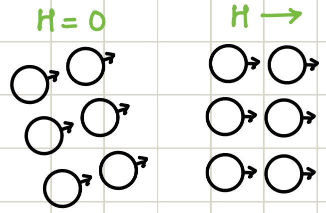
→ $10^3 < \chi_{Ferro} < 10^6$

→ High ordering of unpaired e^- spins – magnetic domains

→ Show sharp hysteresis characteristics

→ Phase transition from ferro to para occurs above curie Temp

ex : Fe , Co, Ni



→ Anti- Ferromagnetic

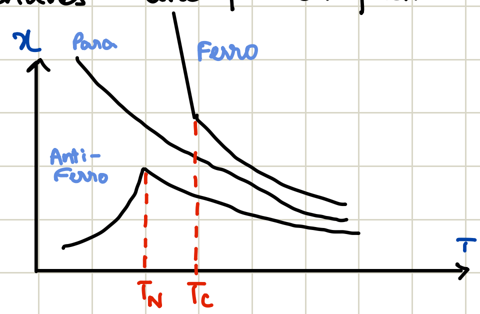
→ e^- spins exhibit degree of ordering within domain

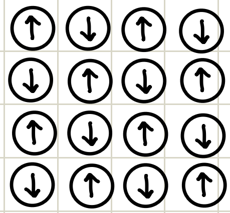
→ Adjacent domains are alligned opposite to each other (equal & anti-parallel)

→ Net magnetization = 0 below certain temp called Neel Temperature

→ Above Neel temperature, it behaves like paramagnetic

ex: MnO, NiO, CoO etc.,





→ Ferrimagnetic

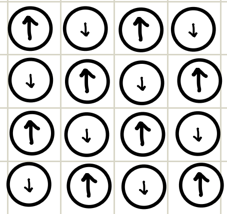
→ Material with e^- spin are ordered but anti-parallel & unequal

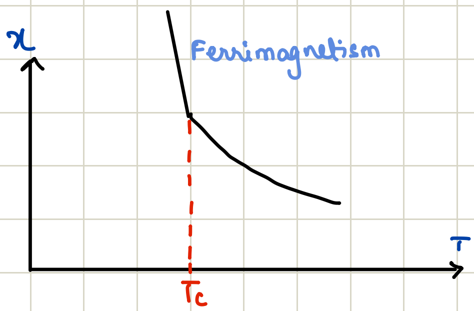
→ Don't show net zero magnetization

→ Contains cations with 2 different magnetic moments

→ Exhibits spontaneous magnetization

ex: $NiFe_2O_4$, $CoFe_3O_4$, $BaFe_{12}O_{19}$ etc.,

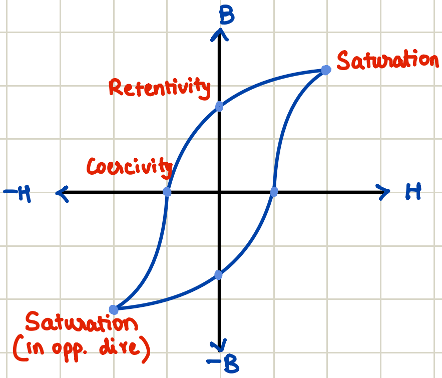




- Hysteresis
  - → Magnetization in presence of external magnetic field forms hysteresis loop
  - → When all spins are alligned, magnetization shows saturation value

  - → Retentivity – Net magnetization even when external magnetic field is removed
  - → Coercivity – Remnant magnetization can be removed if coercive field applied in reverse

  - → Memory Effect – Prev. experience of external fields & can be used as memory materials
  ex : Fe, Co, Ni



Classification of ferromagnetic materials (based on nature of Hysteresis)

Hard Ferromagnetic Material
- → Difficult to demagnetise
- → High retentivity & coercivity
- → Low permeability
- → Large hysteresis curve (more losses)
- → Hard, difficult to shape

Applications : perm. magnets, Memory devices

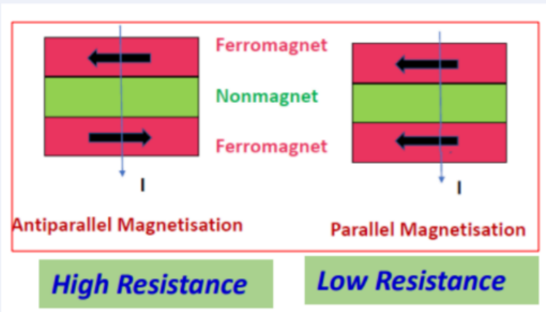
Soft Ferromagnetic
- → Easy to magnetise & demagnetise
- → low coercivity
- → Large saturation magnetisation
- → Narrow hysteresis curve (less losses)

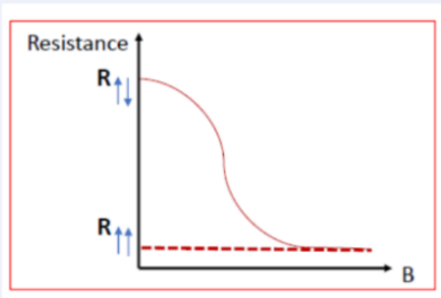
Applications : Transformers, Motors or Inductors

Giant Magneto Resistance effect – GMR
- → Large change in electrical resistance with applied magnetic field
- → Observed in layered magnetic materials

ex : Magnetic metallic multilayers : Fe/Cr , Co/Cu

- → Spin scattering of e^- in non-magnetic layer is less when magnetization is parallel
- → Resistance increases when magnetization is anti-parallel
- → Resistance to current flow depends on direction of magnetization of the 2 layers
- → Most scattering occurs at interfaces of ferromagnetic & conduction layers





$$GMR = \frac{R_{\uparrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\uparrow}}$$

→ Principal Sources for magnetic moment of a free atom

i) Electron's spin

ii) Electron's Orbital Angular momentum about the nucleus

iii) Change in orbital moment induced by an applied magnetic field

→ According to quantum theory, magnetic moments are quantized & can't have arbitrary values.

→ $\mu_j = j g_e \mu_B$

$\mu_z = \underbrace{m_j}_{\text{total angular momentum quantum no.}} \underbrace{g_e}_{\text{Lande g-factor}} \mu_B \longrightarrow$ Bohr Magneton $= 9.28 \times 10^{-24}$

→ $E_j = \mu_j B$

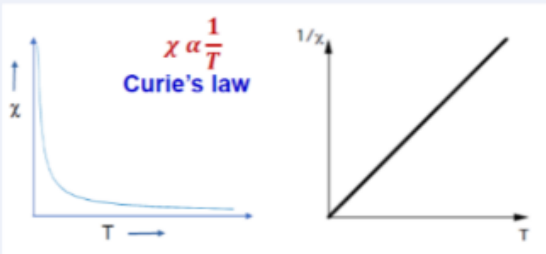
$= g_e \mu_B m_j \mu_0 H$

→ $$M = N\left[\frac{\sum_{m_j=-j}^{j} m_j g \mu_B e^{\frac{m_j g \mu_B \mu_0 H}{KT}}}{\sum_{m_j=-j}^{j} e^{\frac{m_j g \mu_B \mu_0 H}{KT}}}\right]$$

Case 1 : $\underbrace{m_j g \mu_B \mu_0 H}_{\text{Magnetic Interaction Energy}} << \underbrace{KT}_{\text{Thermal Energy}}$

$$M \propto \frac{N g \mu_B \mu_0 H}{KT}$$

$$\chi = \frac{M}{H} \propto \frac{N g \mu_B \mu_0 H}{KT} \propto \frac{C}{T}$$



Case 2: $m_j g \mu_B \mu_0 H >> KT$

$M = N g \mu_B J\, B_j(a)$, where $a = e^{\frac{g \mu_B \mu_0 H}{KT}}$

$$B_j(a) = \left[\frac{2j+1}{2j}\coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j}\coth\left(\frac{a}{2j}\right)\right]$$

→ Brillouin Function

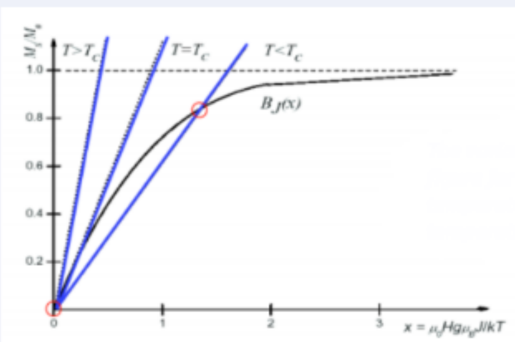
Thus when, $\frac{g \mu_B \mu_0 H}{KT} >> 1$ (saturation magnetization)

$B_j(a) \simeq 1$

and, $M = N g \mu_B J \cdot B_j(a) = N g \mu_B J = M_s$

$$B_j(a) = \left[\frac{2j+1}{2j}\coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j}\coth\left(\frac{a}{2j}\right)\right] = \coth(a) - \frac{1}{a} = L \text{ (Langevin Function)}$$

(For large no. of allowed orientation of magnetic dipole, in limit of $j = \infty$, Brillouin function reduces to Langevin's function L(a)]



Weiss Molecular Field

→ Variation of $\frac{1}{\chi}$ vs T is expected to be a straight line through origin while some others show +ve intercept T_c on temp. axis

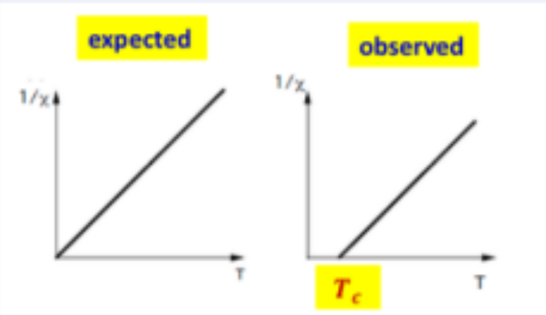
→ This is due to molecular field from neighbouring dipoles (as λM)

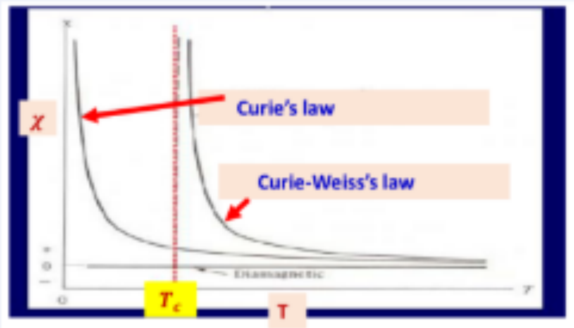
$$\chi = \frac{M}{H(\text{Total})} = \frac{M}{H+\lambda M} = \frac{C}{T} \Rightarrow M(T-\lambda C) = CH \quad \text{(Curie's law)}$$

$$\chi = \frac{C}{T-T_c} \Rightarrow \chi = \frac{M}{H} = \frac{C}{T-\lambda C} \quad \text{(Curie - Weiss law)}$$

$$\lambda C = T_c$$

→ The field represents the interaction of dipoles which leads to spontaneous magnetization





Unit - 4

- **LASER**

Light Amplification by Stimulated Emission of Radiation

$$\rho(\nu) = \frac{8\pi h \nu^3}{c^3} \times \frac{1}{e^{h\nu/KT} - 1}$$

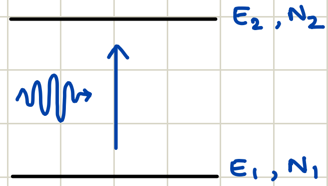
- **3 Quantum Transitions**

i) Induced Absorption

→ Radiation is incident on ground state atom E_1, absorbs photon of energy $h\nu$ & raised to excited state of energy E_2

$E_1 + h\nu \longrightarrow E_2$

→ Rate of Induced Absorption = $B_{12} * N_1 * \rho(\nu)$

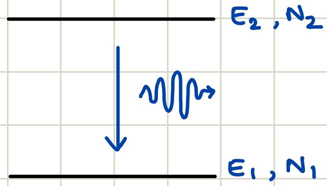


2) Spontaneous Absorption

→ Excited e^- emits photons while falling to ground level w/o external aid

$E_2 \longrightarrow E_1 + h\nu$

→ Rate of Spontaneous Emission = $A_{21} * N_2$

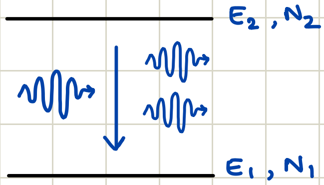


3) Stimulated Absorption

→ Excited e^- emits photons while falling to ground level with external aid

$E_2 + h\nu \longrightarrow E_1 + 2h\nu$

→ Rate of Stimulated emission = $B_{21} * N_2 * \rho(\nu)$



- **Einstein's Equation**

Rate of Induced Absorption = Rate of Spontaneous Emission + Rate of Stimulated emission

$$B_{12} N_1 \rho(\nu) = A_{21} N_2 + B_{21} N_2 \rho(\nu)$$

$$\rho(\nu)(B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$\rho(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \qquad \left(\frac{N_2}{N_1} = e^{\frac{\Delta E}{KT}} \right)$$

$$\rho(\nu) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} e^{\frac{\Delta E}{KT}} - 1} = \frac{\frac{8\pi h \nu^3}{c^3}}{e^{\frac{\Delta E}{KT}} - 1} \Rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \quad \& \quad \frac{B_{12}}{B_{21}} = 1$$

→ Generally, $e^{-\frac{\Delta E}{KT}} < 1 \Rightarrow N_2 < N_1$

and, $R_{sp.em} > R_{st.em}$

→ $R_{sp.em} = A_{21} * N_2$
$= A_{21} * N_2(0) . e^{-A_{21}t}$
$\Rightarrow A_{21} = \frac{1}{\tau}$

(τ: Avg life time of $\bar{e}$ in upper energy state for spontaneous emission)

An emission system has two levels which gives rise to an emission wavelength of 546.1 nm. If the population of the lower state is 4x1022 at 600 K, estimate the population of the higher energy state.

A.

$$\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{KT}} \Rightarrow N_2 = N_1 e^{\frac{-h\nu}{KT}} = N_1 e^{\frac{-hc}{\lambda KT}}$$

$$N_2 = 4 \times 10^{22} \times e^{\frac{-hc}{546.1 \times 10^{-9} \times K \times 600}}$$
$$= 3403$$

The ratio of population between the high energy states to the lower energy state is 5x 10-19 at 400K. Find the emission wavelength between two states and the ratio A/B.

A.

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{KT}}$$

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda KT}}$$

$$5 \times 10^{-19} = e^{\frac{-hc}{\lambda K \times 400}}$$

$$\ln(5 \times 10^{-19}) = \frac{-hc}{\lambda K \times 400} \Rightarrow \lambda = \frac{-hc}{\ln(5 \times 10^{-19}) \times K \times 400} = 853.5 \text{ nm}$$

$$\frac{A}{B} = \frac{8\pi h}{\lambda^3} = 2.67 \times 10^{-14}$$

The ratio of population of the upper excited state to the lower energy state in a system at 300K is found to be 1.2 x 10-19. Find the wavelength of the radiation emitted and the energy density of radiation.

A. $\frac{N_2}{N_1} = 1.2 \times 10^{-19}$

$$\frac{N_2}{N_1} = e^{\frac{-hc}{\lambda KT}} \Rightarrow \lambda = \frac{-hc}{\ln(1.2 \times 10^{-19}) \times K \times 300} = 1.1 \times 10^{-6} \text{ m}$$

$$\rho(\nu) = \frac{8\pi h}{\lambda^3} . \frac{1}{e^{\frac{hc}{\lambda KT}} - 1} = 1.453 \times 10^{-33} \text{ Js/m}^3$$

An hypothetical atom has energy levels uniformly separated by 1.2 eV. Find the ratio of the no of atoms in the 7th excited state to that in the 5th excited state.

A.

$$\frac{N_1}{N_2} = e^{\frac{\Delta E}{KT}} = e^{\frac{1.2 \times e}{K \times 300}} = 1.442 \times 10^{20}$$

$$\frac{N_7}{N_5} = e^{\frac{-\Delta E}{KT}} = e^{\frac{-2 \times 1.2 \times e}{K \times 300}} = 5.22 \times 10^{-41}$$

Calculate the population of the excited state at a temperature of 350 K, if the ground state has 1028 atoms and the transition corresponds to 700 nm radiation.

A. $T = 350K$

$\lambda = 700 \times 10^{-9} m$

$N_1 = 10^{28}$

$$\frac{N_1}{N_2} = e^{\frac{+hc}{\lambda KT}} \Rightarrow N_2 = N_1 \times e^{\frac{-hc}{\lambda KT}} = 10^{28} \times e^{\frac{-hc}{700 \times 10^{-9} \times K \times 350}} = 313.85 \simeq 314$$

The wavelength of emission is 600 Ao and the life time is 10-6 s. Determine the coefficient for stimulated emission.

A. $\lambda = 600 \times 10^{-10} m$

$\tau = 10^{-6} s$

$A = \frac{1}{\tau} = 10^{6}$

$$\frac{A}{B} = \frac{8\pi h}{\lambda^3} \Rightarrow B = \frac{A\lambda^3}{8\pi h} = 1.29 \times 10^{16}$$

- Condition for LASER action

→ It must allow stimulated emission to dominate ($N_2 > N_1$)

↳ Population Inversion

- Population Inversion

→ For stimulated emission to dominate, $N_2 > N_1$

→ It can be achieved in matter containing meta-stable states

→ If population inversion is established, the system isn't in equilibrium

- Requirements of a LASER System

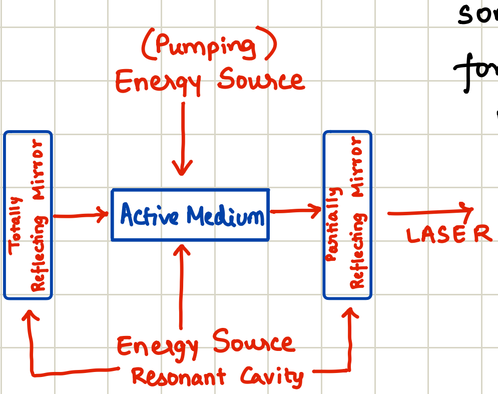
→ Active Medium - The medium in which population inversion is achieved

→ Energy Pump - For pumping action to ensure $N_2 > N_1$

→ Resonating Cavity - 2 ||el mirrors placed so light bounces back & forth through active medium multiple times

- It increases no. of photons & amplifies light while some light escapes through partially reflective mirror forming coherent laser beam

$L = \frac{n\lambda}{2}$ (L : length of cavity)



- **2 Level System**

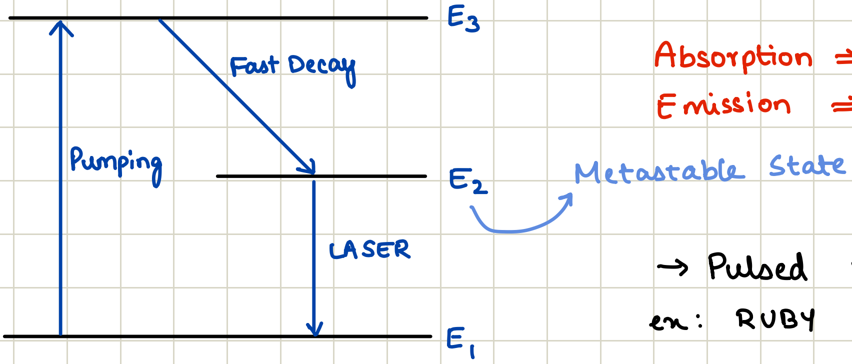
→ For population inversion $N_2 > N_1$

but $\frac{N_2}{N_1} = e^{\frac{-h\nu}{KT}} < 1 \Rightarrow N_2 < N_1$

So population Inversion doesn't occur

- **3 Level System**

→ Intermediate level b/w ground & upper excited state can result in decoupling emission & absorption processes.



Absorption ⇒ b/w E_1 & E_3

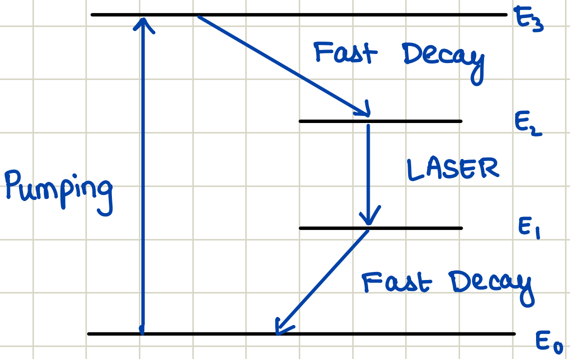
Emission ⇒ b/w E_2 & E_1

→ Pulsed Laser

ex: RUBY Laser

- **4 Level System**

→ Can effectively decouple absorption levels & emission levels



Absorption ⇒ E_0 & E_3

Emission ⇒ E_2 & E_1

→ Continuous laser

ex: He-Ne Laser

A laser emission from a certain laser has an output power of 10 milli watts. If the wavelength of the emission is 632.8nm, find the rate of emission of the stimulated photons.

A. $P = \frac{E}{t} = \frac{nh\nu}{t} = \frac{nhc}{\lambda t}$ (n : rate of stimulated emission) ($t = 1s$)

$n = \frac{P\lambda}{hc} = \frac{10 \times 10^{-3} \times 632.8 \times 10^{-9}}{hc} = 3.18 \times 10^{16}\ s^{-1}$

A pulsed laser has a power of 1mW and lasts for 10 ns. If the no. of photons emitted per second is 3.491 x 107, calculate the wavelength of the photons.

A. $P = 1mW \quad t = 10 \times 10^{-9}\ s$

$n = 3.491 \times 10^{7}\ s^{-1}$

$\lambda = ?$

$P = \frac{E}{t} = \frac{nh\nu}{t} = \frac{nhc}{\lambda t}$

$\lambda = \frac{nhc}{Pt} = \frac{3.491 \times 10^{7} \times hc}{10^{-3} \times 10 \times 10^{-9}} = 693.46\ nm$

If R1 is the rate of stimulated emission and R2 is the rate of spontaneous emission between two energy levels, show that λ= hc / [kT ln{(R2/R1)+1}].

A. $\frac{R_2}{R_1} = e^{\frac{h\nu}{KT}} - 1$

$\frac{R_2}{R_1} + 1 = e^{h\nu/KT}$

$\ln\left(1 + \frac{R_2}{R_1}\right) = \frac{h\nu}{KT} = \frac{hc}{\lambda KT}$

$\lambda = \frac{hc}{KT} \times \frac{1}{\ln\left(1 + \frac{R_2}{R_1}\right)}$

If B10 = 2.7 x 1019m3/W-s3 for a particular atom, find the life time of the 1 to 0 transition at 550nm

A. $B_{10} = B_{01}$

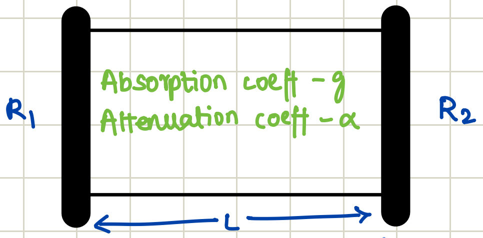
$\frac{A}{B} = \frac{8\pi h}{\lambda^3}$

$A = \frac{8\pi h \times 2.7 \times 10^{19}}{(550 \times 10^{-9})^3} = 2.7 \times 10^{6}$

$\tau = \frac{1}{A} = 390\ ns$

- Losses in Cavity
  - → Scattering
  - → Absorption in beam path
  - → Diffraction loses
  - → Mirror losses

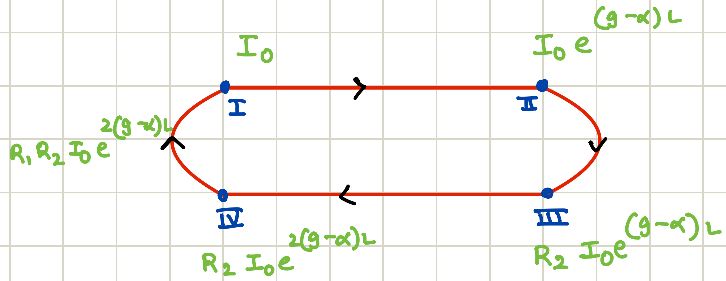
- Round Trip Gain & Loss



gain , $I = I_0 e^{gx}$

loss , $I = I_0 e^{-\alpha x}$

result , $I = I_0 e^{(g-\alpha)x}$



→ Amplification factor ⇒ $\dfrac{I_{Final}}{I_0} = \dfrac{I_0 R_1 R_2 e^{2(g-\alpha)L}}{I_0}$

→ Threshold condition ⇒ $R_1 R_2 e^{2(g-\alpha)L} = 1$

$$\Rightarrow e^{2(g-\alpha)L} = \frac{1}{R_1 R_2}$$

$$\Rightarrow 2(g-\alpha)L = \ln\left(\frac{1}{R_1 R_2}\right)$$

$$\Rightarrow g - \alpha = \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$\Rightarrow g = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

Calculate the threshold gain factors of a laser which has a loss factor of 0.5 cm-1if the configuration of the system is as follows
(a) A 50 cm tube with one mirror 99% reflecting and the output coupler 90% reflecting
(b) A 20 cm tube with one mirror 99% reflecting and the output coupler 97% reflecting
Comment on the results obtained.

A. $\alpha = 0.5\ cm^{-1}$

i) $L = 50cm$

$R_1 = 0.99$

$R_2 = 0.9$

$$g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$= 0.5 + \frac{1}{100}\ln\left(\frac{1}{0.99 \times 0.9}\right)$$

$$= 0.501\ cm^{-1}$$

ii) $L = 20cm$

$R_1 = 0.99$

$R_2 = 0.97$

$$g_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

$$= 0.5 + \frac{1}{40}\ln\left(\frac{1}{0.99 \times 0.97}\right)$$

$$= 0.501\ cm^{-1}$$

- Properties of LASERs

1) Monochromatic

→ Light from laser has a single wavelength, so it has 1 spectral colour

→ $\Delta E = \Delta h\nu = h\Delta\left(\frac{c}{\lambda}\right) = hc \cdot \left|\frac{\Delta\lambda}{\lambda^2}\right|$

2) Coherence

→ Describes relationship with other lasers in terms of phase

→ Temporal Coherence

→ Describes the well-defined consistent phase correlation when lasers are sampled at different times

→ Coherence Time, $\tau_c = \frac{1}{\underbrace{\Delta\nu}_{\text{Spread in frequency}}}$ (Time duration upto which temporal coherence can be maintained)

→ Coherence length, $l_c = \tau_c \cdot c$ (Distance upto which beam exhibits temporal coherence)

→ Spatial Coherence

→ Describes the consistency of phase of laser across different points in space

→ Coherence width, $l_w = \frac{\lambda}{\theta}$

3) Divergence

→ Very low divergence, of the order of milliradians

$$\phi_d = \frac{d_2 - d_1}{z_2 - z_1}$$

→ $\theta = \frac{\lambda_0}{\pi w_0}$

4) Intensity

→ High Intensity laser because of monochromaticity, coherence & low divergence

Calculate the coherence length of a laser beam for which the bandwidth Δv = 3000 Hz

A. $\tau_c = \frac{1}{\Delta v} = \frac{1}{3000}$

$l_c = c \times \tau_c = \frac{3 \times 10^8}{3000} = 10^5 \text{ m} = 100 \text{ Km}$

The lifetime of transitions in a Na atoms emitting wavelength of 589.6nm is estimated to be 16.4ns. Calculate the Einstein's coefficients A and B. Calculate spectral broadening and the coherence length of radiations.

A. $\lambda = 589.6 \times 10^{-9} \text{ m}$ $\quad \tau = 16.4 \times 10^{-9} \text{ s}$

$A = \frac{1}{\tau} = \frac{10^9}{16.4} = 6.1 \times 10^7 \text{ m}^3 \omega^{-1} s^{-3}$

$\frac{A}{B} = \frac{8\pi h}{\lambda^3} \Rightarrow B = \frac{A\lambda^3}{8\pi h}$

$B = \frac{6.1 \times 10^7 \times (589.6 \times 10^{-9})^3}{8\pi h} = 7.5 \times 10^{20} \text{ m}^3 \omega^{-1} s^{-3}$

$\Delta\lambda = \frac{\lambda^2}{4\pi c \tau} = \frac{(589.6 \times 10^{-9})^2}{4\pi c \times 16.4 \times 10^{-9}} = 5.6 \times 10^{-15} \text{ m}$

$\Delta\nu = \frac{c \cdot \Delta\lambda}{\lambda^2} = \frac{c \times 5.6 \times 10^{-15}}{(589.6 \times 10^{-9})^2} = 4.8 \times 10^6 \text{ s}^{-1}$

$\tau_c = \frac{1}{\Delta\nu} = 2.06 \times 10^{-7} \text{ s}$

$l_c = c \cdot \tau_c = 61.8 \text{ m}$

For an ordinary source, the coherence time τc = 10–10 second. Obtain the degree of non-monochromaticity for wavelength λ0 = 5400 Å.

A. $\lambda_0 = 5400 \times 10^{-10} \text{ m}$

$\tau = 10^{-10} \text{ s}$

$\tau_c = \frac{1}{\Delta\nu} \Rightarrow \Delta\nu = 10^{10} \text{ Hz}$

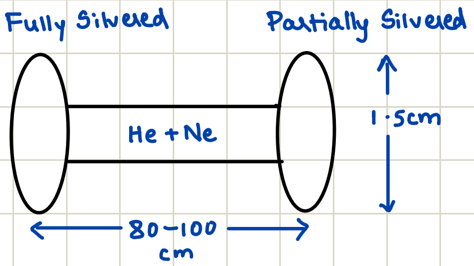
$\Delta\nu = \frac{c\,\Delta\lambda}{\lambda^2} \Rightarrow \Delta\lambda = \frac{\Delta\nu\,\lambda^2}{c} = \frac{10^{10} \times (5400 \times 10^{-10})^2}{c} = 9.7 \times 10^{-12} \text{ m}$

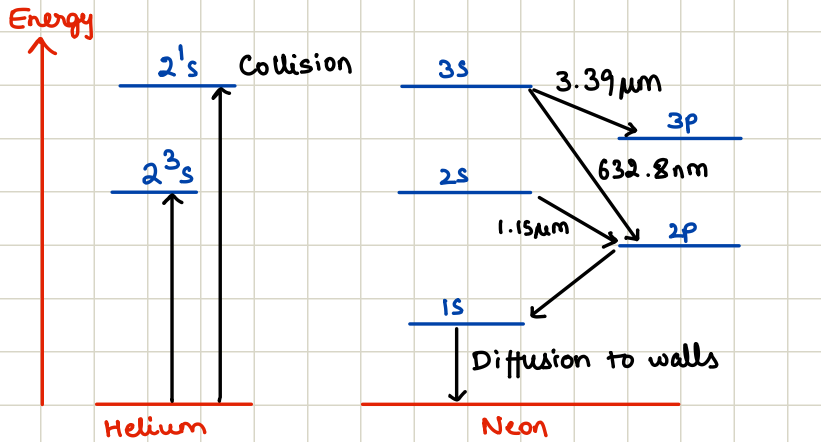
- He – Ne Laser

→ Active Medium – He–Ne gas mixture in 10 : 1 ratio

→ Energy Pump – Electrical Discharge of high voltage DC / RF Source

→ Resonating Cavity – Optical Resonator made of Ge, Zn Selenide to avoid IR absorption





→ 4 level pump

→ Continuous lasing

→ Low efficiency

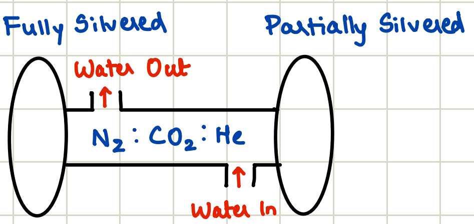
→ low power output

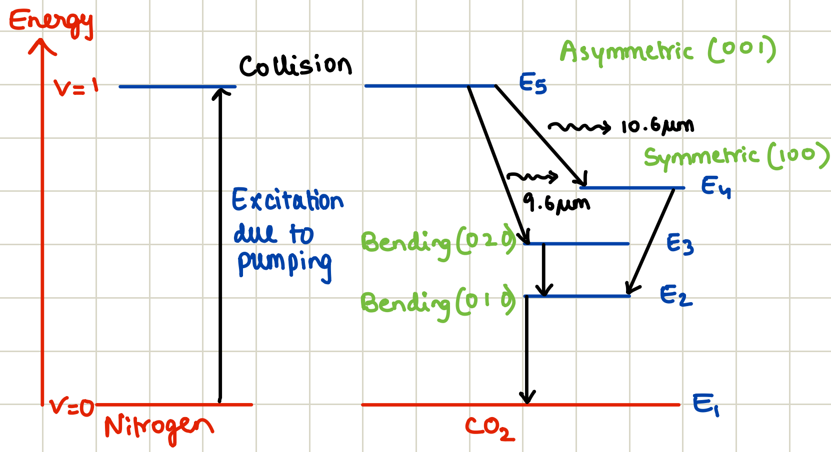
- CO_2 Laser

→ Active Medium – $N_2:CO_2:He$ gas mixture in 1 : 2 : 8 ratio

→ Energy Pump – Electrical Discharge of high voltage DC / RF Source

→ Resonating Cavity – Reflecting Mirrors at ends





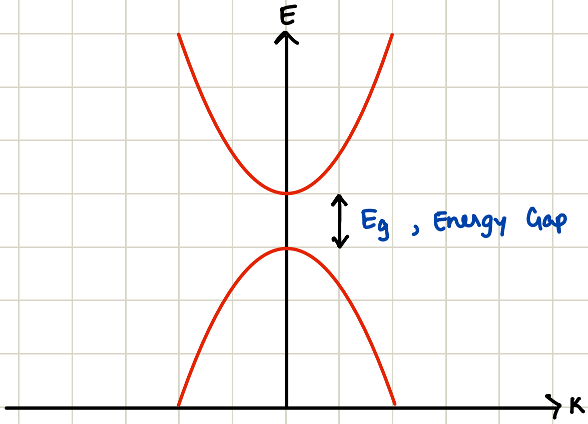
→ 4 level pump

→ Continuous lasing

→ efficiency up to 30% (high)

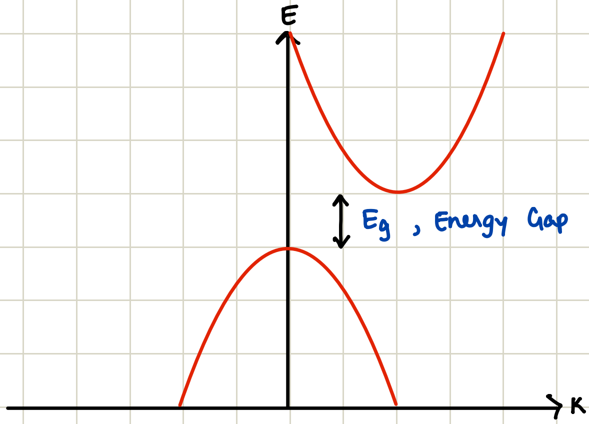
→ high power output

- **Direct Band Gap Semiconductors**



→ e^- can recombine directly with hole
→ recombination leads to light emission
→ Mostly in Compound Semiconductors
ex: GaAs, InP

Indirect Band Gap Semiconductors



→ e^- can't recombine directly with hole
→ recombination leads to heat emission
→ Mostly in Elemental Semiconductors
ex: Si, Ge

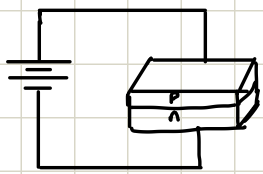
- **LED**

→ Semiconductor diode that emits light when forward bias
→ Made by fabricating direct band gap semiconductors

→ It works under the principle of Injection electroluminescence
→ Recombination of e^--hole is basic mechanism responsible for light

$$\lambda = \frac{hc}{E_g}$$

- **Homo junction Semiconductor laser**



→ Active Medium - Heavily doped direct band gap semiconductor
→ Energy Pump - Forward bias of p-n junction
→ Resonating Cavity - 2 opposite sides are cleaved
→ Working - Pumping ⇒ e^- & holes pumped into junction region in high concⁿ. Upon reaching threshold value of diode current, population inversion is attained. Recombination of e^- & holes in narrow region
→ Limitations - Cavity losses & Inefficient

- **Hetero junction Semiconductor laser**

→ Similar working as homojunction laser
→ Junction b/w 2 diff. bandgap semiconductors

Features of S.C.L

→ Low cost
→ Simple, portable, compact
→ Small
→ Low power operation
→ 40% efficiency
→ Mass producable

- RUBY Laser

→ 3 level pulsated laser

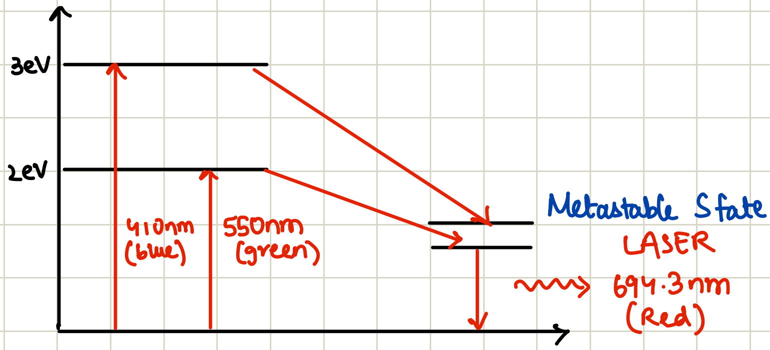
→ Active Medium - Ruby Crystal (Al_2O_3 doped with Cr^{3+})

→ Energy Pump - High power white light flash lamp ($21\ kJm^{-2}$)

→ Resonating Cavity - Ruby rod, silver anti-reflecting coatings

→ Brewster angle ⇒ $\tan\theta = \eta$

$\eta \to$ RI



→ Width of RUBY laser pulse usually in order of milliseconds

→ Al_2O_3 crystal host chromium ion & absorb the pump energy to excite chromium ion through collisions

- Applications of LASERs

1) Frequency Comb

→ Resonant cavity oscillator of L

→ It is a special type of light with spectrum like a comb's teeth

→ It is created by a laser emitting short & fast pulses which creates evenly spaced pattern

→ Used in atomic clocks & GPS systems

2) Holography

→ Technique to create 3D images

→ A laser beam splits into 2, object beam which lights up object & reference beam directed to photographic plate.

Light reflected off object & reference beam meet on the plate creating interference pattern

DIELECTRICS

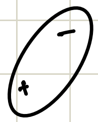
→ They are insulators that respond to external electric field & cause seperation of charges resulting creation of dipoles

→ Net dipole moment created per unit volume is polarisation
↳ measure of how susceptible material is to applied Elec field

Dielectric Material

Polar

→ +ve & −ve charges don't coincide

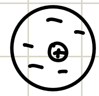


→ posses perm. dipole moment but cuz of randomness, net dipole moment = 0

ex : NH_3, HCl, H_2O etc.,

Non-Polar

→ +ve & −ve charges coincide



→ Symmetric

ex:  , CH_4 etc.,

→ Polarisation in dielectrics

$$E = E_{appl} - E_{depol}$$

Gauss Law,

$$E\,dS = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{\varepsilon_0}\cdot\frac{q}{dS} = \frac{\sigma}{\varepsilon_0}$$

$P = \sigma = E\varepsilon_0$ (w/o dielectric)

$P = \sigma_p = \varepsilon_0 E'$ (w dielectric)

$$E = E_0 - E'$$

$$E = \varepsilon_r E - \frac{\sigma_p}{\varepsilon_0}$$

$$\varepsilon_0 E = \varepsilon_r \varepsilon_0 E - \sigma_p \Rightarrow \varepsilon_0 E = \varepsilon_r \varepsilon_0 E - P$$

$$P = \varepsilon_0 E(\varepsilon_r - 1)$$

$$= \varepsilon_0 E \chi_{Dielec}$$

→ Atomic Polarizability (α_e)

$$\alpha_e \Rightarrow P \propto E \Rightarrow P = \alpha_e E$$

$$P = \varepsilon_0 E \chi_D = \alpha_e E N$$

$$\Rightarrow \alpha_e = \frac{\varepsilon_0 \chi_D}{N}$$

$$\Rightarrow \boxed{N\alpha_e E = P = \varepsilon_0(\varepsilon_r - 1)E}$$

- Components of Electric Field in Dielectric material

$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

E_{loc}: local field; E_0: External field; E_1: Depolarization Field; E_2: Lorentz Field on surface of sphere cavity; E_3: internal field due to other dipoles lying within sphere

- ## Clausius - Mosotti Relation

$$E_{in} = \frac{P}{3\varepsilon_0}$$

$$E_{loc} = E + E_{in}$$
$$= E + \frac{P}{3\varepsilon_0}$$

$$P = N\alpha_e E_{loc} = N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right) = \varepsilon_0(\varepsilon_r - 1)E$$

Then, $$\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha_e}{3\varepsilon_0}$$

Electronic polarisability of Sulphur is 3.28x10^-40 Fm^2. Calculate the dielectric constant of sulphur if the density and atomic weight of sulphur are 2.08x10^3 kg-m^3 and 32 respectively.

A. $$N = \frac{\text{density} \times N_A}{\text{Atomic No}} = \frac{2.08 \times 10^3 \times 10^3 \times N_A}{32} = 3.914 \times 10^{28}$$

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

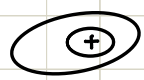
$$\frac{3.914 \times 10^{28} \times 3.28 \times 10^{-40}}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \Rightarrow \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = 0.483$$

$$\varepsilon_r - 1 = 0.483\,\varepsilon_r + 0.966$$

$$\varepsilon_r = \frac{1.966}{1 - 0.483} = 3.8$$

- ## Types of Polarisation

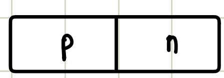
→ **Electronic / Atomic Polarisation**
  - $\alpha_e = 4\pi\varepsilon_0 R^3$
  - non polar dielectrics ⇒ induces dipole
  - temp. independant



→ **Ionic Polarisation**
  - Ionic crystals in $\vec{E}$
  - Independant of Temp
  - Depends on Young's Modulus

→ **Orientational / Molecular Polarization**
  - Polar dielectrics
  - Permanent dipole moment
  - $P = \frac{Np^2E}{3kT}$ ⇒ induced dipole moment

→ **Spatial Charge Polarization**
  - Interface
  - Accumulation of opposite charges



Junction Capacitance

- **Non linear dielectrics** (E & P aren't related linearly)

32 Electro Strictive → 21 Non center of symmetric → 20 Piezo electric classes → 10 Pyro electric classes → Ferroelectric

32 Electro Strictive ↓ 11 Center of Symmetry classes

21 Non center of symmetric ↓ 1 Non-Piezo electric class

20 Piezo electric classes ↓ 10 Non-Pyro electric classes

- **Piezoelectric Effect**

→ When pressure is applied on piezoelectric material, converts mechanical energy to electrical energy to generate electric potential

→ Inverse piezoelectric effect – Can also convert electrical energy to mechanical energy

ex : Quartz

→ Used in transducers, sensors, electric ignitors etc.,

- **Pyroelectric Effect**

→ Ability to generate electric potential when heated/cooled

→ Change in temp modifies position slightly such that polarisation of material changes

ex: Fire alarms, Thermal imaging etc., $P_i = \frac{\partial P_s}{\partial T}$

- **Ferroelectric Materials**

→ Show spontaneous polarisation

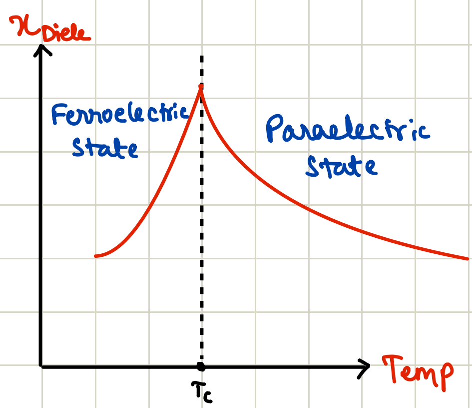
→ non-centro symmetric

→ $\vec{E} \Rightarrow$ P vs E hysteresis loop $\xrightarrow{\text{used in}}$ DRAMs & SRAMs

→ Temp. Dependant

→ DRAMs & SRAMs

ex : $BaTiO_3$

- 

- $BaTiO_3$

→ example of non-centro symmetric ferro-electrical system

→ exhibits phase transition

i) $> 120^\circ C$ ⇒ paraelectric

ii) $55^\circ - 120^\circ C$ ⇒ ferroelectric

iii) $< 5^\circ C$ ⇒ varying ferroelec. behaviour.

Ferro-electric Mateial phase transition