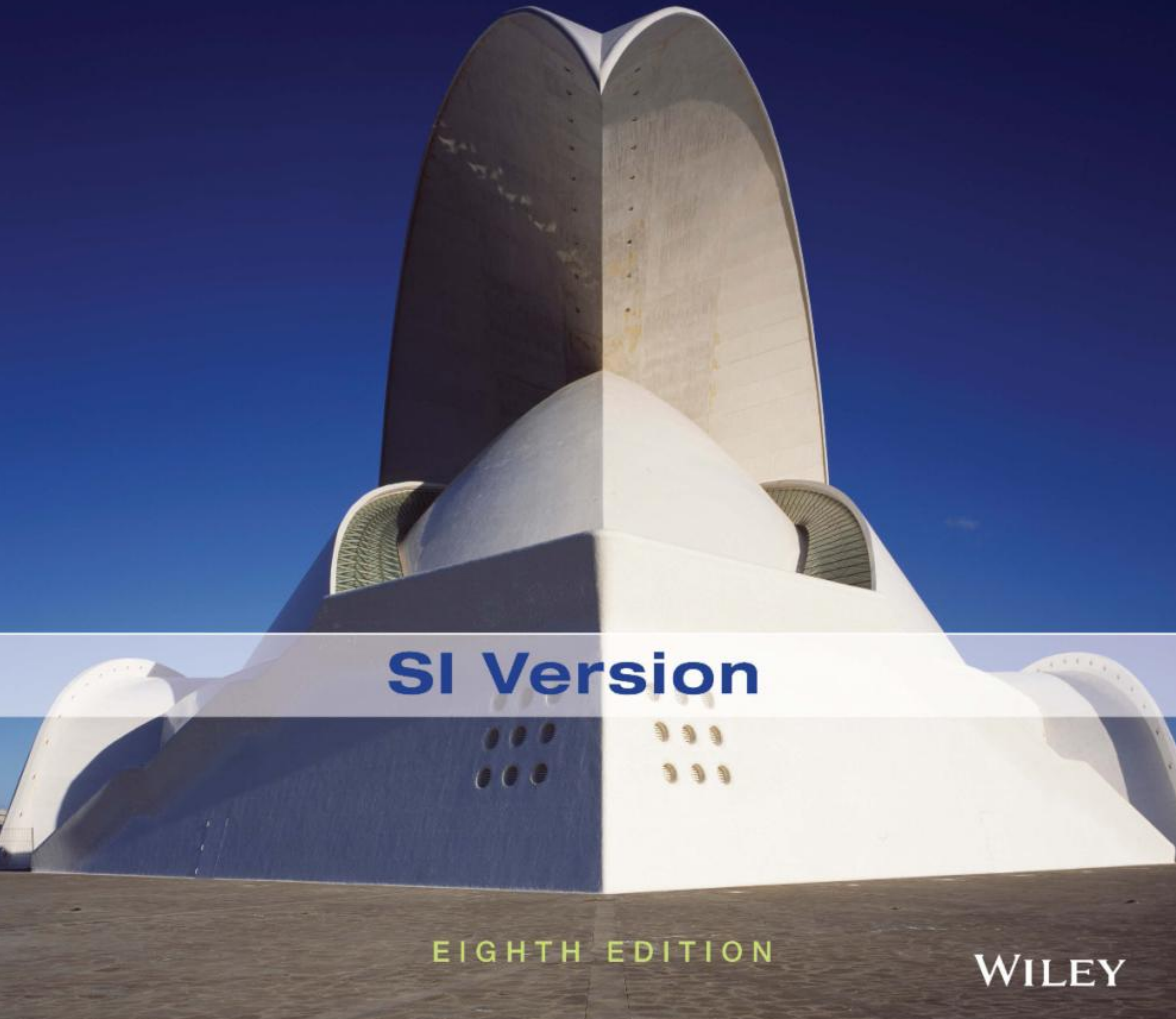
ENGINEERING MECHANICS
STATICS

J. L. MERIAM • L. G. KRAIGE • J. N. BOLTON



SI Version

EIGHTH EDITION

WILEY

Conversion Factors

U.S. Customary Units to SI Units

To convert from	To	Multiply by
(Acceleration)		
foot/second^2 (ft/sec^2)	meter/second^2 (m/s^2)	3.048×10^{-1}*
inch/second^2 (in./sec^2)	meter/second^2 (m/s^2)	2.54×10^{-2}*
(Area)		
foot^2 (ft^2)	meter^2 (m^2)	9.2903×10^{-2}
inch^2 (in.^{2})	meter^2 (m^2)	6.4516×10^{-4}*
(Density)		
pound mass/inch^3 (lbm/in.^{3})	kilogram/meter^3 (kg/m^3)	2.7680×10^4
pound mass/foot^3 (lbm/ft^3)	kilogram/meter^3 (kg/m^3)	1.6018×10
(Force)		
kip (1000 lb)	newton (N)	4.4482×10^3
pound force (lb)	newton (N)	4.4482
(Length)		
foot (ft)	meter (m)	3.048×10^{-1}*
inch (in.)	meter (m)	2.54×10^{-2}*
mile (mi), (U.S. statute)	meter (m)	1.6093×10^3
mile (mi), (international nautical)	meter (m)	1.852×10^3*
(Mass)		
pound mass (lbm)	kilogram (kg)	4.5359×10^{-1}
slug (lb-sec^2/ft)	kilogram (kg)	1.4594×10
ton (2000 lbm)	kilogram (kg)	9.0718×10^2
(Moment of force)		
pound-foot (lb-ft)	newton-meter (N·m)	1.3558
pound-inch (lb-in.)	newton-meter (N·m)	0.1129 8
(Moment of inertia, area)		
inch^4	meter^4 (m^4)	41.623×10^{-8}
(Moment of inertia, mass)		
pound-foot-second^2 (lb-ft-sec^2)	kilogram-meter^2 (kg·m^2)	1.3558
(Momentum, linear)		
pound-second (lb-sec)	kilogram-meter/second (kg·m/s)	4.4482
(Momentum, angular)		
pound-foot-second (lb-ft-sec)	newton-meter-second (kg·m^2/s)	1.3558
(Power)		
foot-pound/minute (ft-lb/min)	watt (W)	2.2597×10^{-2}
horsepower (550 ft-lb/sec)	watt (W)	7.4570×10^2
(Pressure, stress)		
atmosphere (std)(14.7 lb/in.^{2})	newton/meter^2 (N/m^2 or Pa)	1.0133×10^5
pound/foot^2 (lb/ft^2)	newton/meter^2 (N/m^2 or Pa)	4.7880×10
pound/inch^2 (lb/in.^{2} or psi)	newton/meter^2 (N/m^2 or Pa)	6.8948×10^3
(Spring constant)		
pound/inch (lb/in.)	newton/meter (N/m)	1.7513×10^2
(Velocity)		
foot/second (ft/sec)	meter/second (m/s)	3.048×10^{-1}*
knot (nautical mi/hr)	meter/second (m/s)	5.1444×10^{-1}
mile/hour (mi/hr)	meter/second (m/s)	4.4704×10^{-1}*
mile/hour (mi/hr)	kilometer/hour (km/h)	1.6093
(Volume)		
foot^3 (ft^3)	meter^3 (m^3)	2.8317×10^{-2}
inch^3 (in.^{3})	meter^3 (m^3)	1.6387×10^{-5}
(Work, Energy)		
British thermal unit (BTU)	joule (J)	1.0551×10^3
foot-pound force (ft-lb)	joule (J)	1.3558
kilowatt-hour (kw-h)	joule (J)	3.60×10^6*

*Exact value

SI Units Used in Mechanics

Quantity	Unit	SI Symbol
(Base Units)		
Length	meter*	m
Mass	kilogram	kg
Time	second	s
(Derived Units)		
Acceleration, linear	meter/second^2	m/s^2
Acceleration, angular	radian/second^2	rad/s^2
Area	meter^2	m^2
Density	kilogram/meter^3	kg/m^3
Force	newton	N (= kg·m/s^2)
Frequency	hertz	Hz (= 1/s)
Impulse, linear	newton-second	N·s
Impulse, angular	newton-meter-second	N·m·s
Moment of force	newton-meter	N·m
Moment of inertia, area	meter^4	m^4
Moment of inertia, mass	kilogram-meter^2	kg·m^2
Momentum, linear	kilogram-meter/second	kg·m/s (= N·s)
Momentum, angular	kilogram-meter^2/second	kg·m^2/s (= N·m·s)
Power	watt	W (= J/s = N·m/s)
Pressure, stress	pascal	Pa (= N/m^2)
Product of inertia, area	meter^4	m^4
Product of inertia, mass	kilogram-meter^2	kg·m^2
Spring constant	newton/meter	N/m
Velocity, linear	meter/second	m/s
Velocity, angular	radian/second	rad/s
Volume	meter^3	m^3
Work, energy	joule	J (= N·m)
(Supplementary and Other Acceptable Units)		
Distance (navigation)	nautical mile	(= 1.852 km)
Mass	ton (metric)	t (= 1000 kg)
Plane angle	degrees (decimal)	°
Plane angle	radian	—
Speed	knot	(1.852 km/h)
Time	day	d
Time	hour	h
Time	minute	min

*Also spelled *metre*.

SI Unit Prefixes

Multiplication Factor	Prefix	Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto	h
10 = 10	deka	da
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p

Selected Rules for Writing Metric Quantities

1. (a) Use prefixes to keep numerical values generally between 0.1 and 1000.
   (b) Use of the prefixes hecto, deka, deci, and centi should generally be avoided except for certain areas or volumes where the numbers would be awkward otherwise.
   (c) Use prefixes only in the numerator of unit combinations. The one exception is the base unit kilogram. (*Example:* write kN/m not N/mm; J/kg not mJ/g)
   (d) Avoid double prefixes. (*Example:* write GN not kMN)
2. Unit designations
   (a) Use a dot for multiplication of units. (*Example:* write N·m not Nm)
   (b) Avoid ambiguous double solidus. (*Example:* write N/m^2 not N/m/m)
   (c) Exponents refer to entire unit. (*Example:* mm^2 means (mm)^2)
3. Number grouping
   Use a space rather than a comma to separate numbers in groups of three, counting from the decimal point in both directions. (*Example:* 4 607 321.048 72) Space may be omitted for numbers of four digits. (*Example:* 4296 or 0.0476)

Engineering Mechanics

Volume 1
Statics

SI Version

Eighth Edition

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L.G. KRAIGE
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WILEY

On the cover: The Auditorio de Tenerife "Adán Martin" is located in Santa Cruz de Tenerife, the capital of the Canary Islands, Spain. It was designed by architect Santiago Calatrava Valls and was opened in 2003.

Cover Photo: Chris Gascolgne/VIEW Pictures/Newscom. Depiction on cover used by permission of Auditorio de Tenerife.

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Foreword

This series of textbooks was begun in 1951 by the late Dr. James L. Meriam. At that time, the books represented a revolutionary transformation in undergraduate mechanics education. They became the definitive textbooks for the decades that followed as well as models for other engineering mechanics texts that have subsequently appeared. Published under slightly different titles prior to the 1978 First Editions, this textbook series has always been characterized by logical organization, clear and rigorous presentation of the theory, instructive sample problems, and a rich collection of real-life problems, all with a high standard of illustration. In addition to the U.S. versions, the books have appeared in SI versions and have been translated into many foreign languages. These textbooks collectively represent an international standard for undergraduate texts in mechanics.

The innovations and contributions of Dr. Meriam (1917–2000) to the field of engineering mechanics cannot be overstated. He was one of the premier engineering educators of the second half of the twentieth century. Dr. Meriam earned the B.E., M.Eng., and Ph.D. degrees from Yale University. He had early industrial experience with Pratt and Whitney Aircraft and the General Electric Company. During the Second World War he served in the U.S. Coast Guard. He was a member of the faculty of the University of California—Berkeley, Dean of Engineering at Duke University, a faculty member at the California Polytechnic State University, and visiting professor at the University of California—Santa Barbara, finally retiring in 1990. Professor Meriam always placed great emphasis on teaching, and this trait was recognized by his students wherever he taught. He was the recipient of several teaching awards, including the Benjamin Garver Lamme Award, which is the highest annual national award of the American Society of Engineering Education (ASEE).

Dr. L. Glenn Kraige, coauthor of the *Engineering Mechanics* series since the early 1980s, has also made significant contributions to mechanics education. Dr. Kraige earned his B.S., M.S., and Ph.D. degrees at the University of Virginia, principally in aerospace engineering, and he is Professor Emeritus of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University. During the mid-1970s, I had the singular pleasure of chairing Professor Kraige's graduate committee and take particular pride in the fact that he was the first of my fifty Ph.D. graduates. Professor Kraige was invited by Professor Meriam to team with him, thereby ensuring that the Meriam legacy of textbook authorship excellence would be carried forward to future generations of engineers.

In addition to his widely recognized research and publications in the field of spacecraft dynamics, Professor Kraige has devoted his attention to the teaching of mechanics at both introductory and advanced levels. His outstanding teaching has been widely recognized and has earned him teaching awards at the departmental, college, university, state, regional, and national levels. These awards include the Outstanding Educator Award from the State Council of Higher Education for the Commonwealth of Virginia. In 1996, the

Mechanics Division of ASEE bestowed upon him the Archie Higdon Distinguished Educator Award. The Carnegie Foundation for the Advancement of Teaching and the Council for Advancement and Support of Education awarded him the distinction of Virginia Professor of the Year for 1997. In his teaching, Professor Kraige stresses the development of analytical capabilities along with the strengthening of physical insight and engineering judgment. Since the early 1980s, he has worked on personal-computer software designed to enhance the teaching/learning process in statics, dynamics, strength of materials, and higher-level areas of dynamics and vibrations.

Welcomed as a new coauthor for this edition is Dr. Jeffrey N. Bolton, Assistant Professor of Mechanical Engineering Technology at Bluefield State College. Dr. Bolton earned his B.S., M.S., and Ph.D. in Engineering Mechanics from Virginia Polytechnic Institute and State University. His research interests include automatic balancing of six-degree-of-freedom elastically-mounted rotors. He has a wealth of teaching experience, including at Virginia Tech, where he was the 2010 recipient of the Sporn Teaching Award for Engineering Subjects, which is primarily chosen by students. In 2014, Professor Bolton received the Outstanding Faculty Award from Bluefield State College. He has the unusual ability to set high levels of rigor and achievement in the classroom while establishing a high degree of rapport with his students. In addition to maintaining time-tested traditions for future generations of students, Dr. Bolton will bring effective application of technology to this textbook series.

The Eighth Edition of *Engineering Mechanics* continues the same high standards set by previous editions and adds new features of help and interest to students. It contains a vast collection of interesting and instructive problems. The faculty and students privileged to teach or study from the Meriam/Kraige/Bolton *Engineering Mechanics* series will benefit from several decades of investment by three highly accomplished educators. Following the pattern of the previous editions, this textbook stresses the application of theory to actual engineering situations, and at this important task it remains the best.

John L. Junkins
Distinguished Professor of Aerospace Engineering
Holder of the Royce E. Wisebaker '39 Chair in Engineering Innovation
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College Station, Texas

Preface

Engineering mechanics is both a foundation and a framework for most of the branches of engineering. Many of the topics in such areas as civil, mechanical, aerospace, and agricultural engineering, and of course engineering mechanics itself, are based upon the subjects of statics and dynamics. Even in a discipline such as electrical engineering, practitioners, in the course of considering the electrical components of a robotic device or a manufacturing process, may find themselves first having to deal with the mechanics involved.

Thus, the engineering mechanics sequence is critical to the engineering curriculum. Not only is this sequence needed in itself, but courses in engineering mechanics also serve to solidify the student's understanding of other important subjects, including applied mathematics, physics, and graphics. In addition, these courses serve as excellent settings in which to strengthen problem-solving abilities.

Philosophy

The primary purpose of the study of engineering mechanics is to develop the capacity to predict the effects of force and motion while carrying out the creative design functions of engineering. This capacity requires more than a mere knowledge of the physical and mathematical principles of mechanics; also required is the ability to visualize physical configurations in terms of real materials, actual constraints, and the practical limitations which govern the behavior of machines and structures. One of the primary objectives in a mechanics course is to help the student develop this ability to visualize, which is so vital to problem formulation. Indeed, the construction of a meaningful mathematical model is often a more important experience than its solution. Maximum progress is made when the principles and their limitations are learned together within the context of engineering application.

There is a frequent tendency in the presentation of mechanics to use problems mainly as a vehicle to illustrate theory rather than to develop theory for the purpose of solving problems. When the first view is allowed to predominate, problems tend to become overly idealized and unrelated to engineering with the result that the exercise becomes dull, academic, and uninteresting. This approach deprives the student of valuable experience in formulating problems and thus of discovering the need for and meaning of theory. The second view provides by far the stronger motive for learning theory and leads to a better balance between theory and application. The crucial role played by interest and purpose in providing the strongest possible motive for learning cannot be overemphasized.

Furthermore, as mechanics educators, we should stress the understanding that, at best, theory can only approximate the real world of mechanics rather than the view that the real world approximates the theory. This difference in philosophy is indeed basic and distinguishes the *engineering* of mechanics from the *science* of mechanics.

Over the past several decades, several unfortunate tendencies have occurred in engineering education. First, emphasis on the geometric and physical meanings of prerequisite mathematics appears to have diminished. Second, there has been a significant reduction and even elimination of instruction in graphics, which in the past enhanced the visualization and representation of mechanics problems. Third, in advancing the mathematical level of our treatment of mechanics, there has been a tendency to allow the notational manipulation of vector operations to mask or replace geometric visualization. Mechanics is inherently a subject which depends on geometric and physical perception, and we should increase our efforts to develop this ability.

A special note on the use of computers is in order. The experience of formulating problems, where reason and judgment are developed, is vastly more important for the student than is the manipulative exercise in carrying out the solution. For this reason, computer usage must be carefully controlled. At present, constructing free-body diagrams and formulating governing equations are best done with pencil and paper. On the other hand, there are instances in which the *solution* to the governing equations can best be carried out and displayed using the computer. Computer-oriented problems should be genuine in the sense that there is a condition of design or criticality to be found, rather than "make-work" problems in which some parameter is varied for no apparent reason other than to force artificial use of the computer. These thoughts have been kept in mind during the design of the computer-oriented problems in the Eighth Edition. To conserve adequate time for problem formulation, it is suggested that the student be assigned only a limited number of the computer-oriented problems.

As with previous editions, this Eighth Edition of *Engineering Mechanics* is written with the foregoing philosophy in mind. It is intended primarily for the first engineering course in mechanics, generally taught in the second year of study. *Engineering Mechanics* is written in a style which is both concise and friendly. The major emphasis is on basic principles and methods rather than on a multitude of special cases. Strong effort has been made to show both the cohesiveness of the relatively few fundamental ideas and the great variety of problems which these few ideas will solve.

Pedagogical Features

The basic structure of this textbook consists of an article which rigorously treats the particular subject matter at hand, followed by one or more Sample Problems, followed by a group of Problems. There is a Chapter Review at the end of each chapter which summarizes the main points in that chapter, followed by a Review Problem set.

Problems

The 89 Sample Problems appear on specially colored pages by themselves. The solutions to typical statics problems are presented in detail. In addition, explanatory and cautionary notes (Helpful Hints) in blue type are number-keyed to the main presentation.

There are 1060 homework exercises, of which more than 50 percent are new to the Eighth Edition. The problem sets are divided into *Introductory Problems* and *Representative Problems*. The first section consists of simple, uncomplicated problems designed to help students gain confidence with the new topic, while most of the problems in the second section are of average difficulty and length. The problems are generally arranged in order of increasing difficulty. More difficult exercises appear near the end of the *Representative Problems* and are marked with the triangular symbol ▶. *Computer-Oriented Problems*, marked with an asterisk, appear throughout the problems and also in a special section at the conclusion of the *Review Problems* at the end of each chapter. The answers to all problems have been provided in a special section near the end of the textbook.

SI units are used throughout the book, except in a limited number of introductory areas in which U.S. units are mentioned for purposes of completeness and contrast with SI units.

A notable feature of the Eighth Edition, as with all previous editions, is the wealth of interesting and important problems which apply to engineering design. Whether directly identified as such or not, virtually all of the problems deal with principles and procedures inherent in the design and analysis of engineering structures and mechanical systems.

Illustrations

In order to bring the greatest possible degree of realism and clarity to the illustrations, this textbook series continues to be produced in full color. It is important to note that color is used consistently for the identification of certain quantities:

- *red* for forces and moments
- *green* for velocity and acceleration arrows
- *orange dashes* for selected trajectories of moving points

Subdued colors are used for those parts of an illustration which are not central to the problem at hand. Whenever possible, mechanisms or objects which commonly have a certain color will be portrayed in that color. All of the fundamental elements of technical illustration which have been an essential part of this *Engineering Mechanics* series of textbooks have been retained. The authors wish to restate the conviction that a high standard of illustration is critical to any written work in the field of mechanics.

Special Features

We have retained the following hallmark features of previous editions:

- All theory portions are constantly reexamined in order to maximize rigor, clarity, readability, and level of friendliness.
- Key Concepts areas within the theory presentation are specially marked and highlighted.
- The Chapter Reviews are highlighted and feature itemized summaries.
- Approximately 50 percent of the homework problems are new to this Eighth Edition. All new problems have been independently solved in order to ensure a high degree of accuracy.
- All Sample Problems are printed on specially colored pages for quick identification.
- Within-the-chapter photographs are provided in order to provide additional connection to actual situations in which statics has played a major role.

Organization

In Chapter 1, the fundamental concepts necessary for the study of mechanics are established.

In Chapter 2, the properties of forces, moments, couples, and resultants are developed so that the student may proceed directly to the equilibrium of nonconcurrent force systems in Chapter 3 without unnecessarily belaboring the relatively trivial problem of the equilibrium of concurrent forces acting on a particle.

In both Chapters 2 and 3, analysis of two-dimensional problems is presented in Section A before three-dimensional problems are treated in Section B. With this arrangement, the instructor may cover all of Chapter 2 before beginning Chapter 3 on equilibrium, or the instructor may cover the two chapters in the order 2A, 3A, 2B, 3B. The latter order treats force systems and equilibrium in two dimensions and then treats these topics in three dimensions.

Application of equilibrium principles to simple trusses and to frames and machines is presented in Chapter 4 with primary attention given to two-dimensional systems. A sufficient number of three-dimensional examples are included, however, to enable students to exercise more general vector tools of analysis.

The concepts and categories of distributed forces are introduced at the beginning of Chapter 5, with the balance of the chapter divided into two main sections. Section A treats centroids and mass centers; detailed examples are presented to help students master early applications of calculus to physical and geometrical problems. Section B includes the special topics of beams, flexible cables, and fluid forces, which may be omitted without loss of continuity of basic concepts.

Chapter 6 on friction is divided into Section A on the phenomenon of dry friction and Section B on selected machine applications. Although Section B may be omitted if time is limited, this material does provide a valuable experience for the student in dealing with both concentrated and distributed friction forces.

Chapter 7 presents a consolidated introduction to virtual work with applications limited to single-degree-of-freedom systems. Special emphasis is placed on the advantage of the virtual-work and energy method for interconnected systems and stability determination. Virtual work provides an excellent opportunity to convince the student of the power of mathematical analysis in mechanics.

Moments and products of inertia of areas are presented in Appendix A. This topic helps to bridge the subjects of statics and solid mechanics. Appendix C contains a summary review of selected topics of elementary mathematics as well as several numerical techniques which the student should be prepared to use in computer-solved problems. Useful tables of physical constants, centroids, and moments of inertia are contained in Appendix D.

Supplements

The following items have been prepared to complement this textbook:

Instructor's Manual

Prepared by the authors and independently checked, fully worked solutions to all problems in the text are available to faculty by contacting their local Wiley representative.

Instructor Lecture Resources

The following resources are available online at *www.wiley.com/college/meriam*. There may be additional resources not listed.

WileyPlus: A complete online learning system to help prepare and present lectures, assign and manage homework, keep track of student progress, and customize your course content and delivery. See the description at the back of the book for more information, and the website for a demonstration. Talk to your Wiley representative for details on setting up your WileyPlus course.

Lecture software specifically designed to aid the lecturer, especially in larger classrooms. Written by the author and incorporating figures from the textbooks, this software is based on the Macromedia Flash platform. Major use of animation, concise review of the theory, and numerous sample problems make this tool extremely useful for student self-review of the material.

All ***figures*** in the text are available in electronic format for use in creating lecture presentations.

All ***Sample Problems*** are available as electronic files for display and discussion in the classroom.

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Special recognition is due Dr. A. L. Hale, formerly of Bell Telephone Laboratories, for his continuing contribution in the form of invaluable suggestions and accurate checking of the manuscript. Dr. Hale has rendered similar service for all previous versions of this entire series of mechanics books, dating back to the 1950s. He reviews all aspects of the books, including all old and new text and figures. Dr. Hale carries out an independent solution to each new homework exercise and provides the authors with suggestions and needed corrections to the solutions which appear in the *Instructor's Manual*. Dr. Hale is well known for being extremely accurate in his work, and his fine knowledge of the English language is a great asset which aids every user of this textbook.

We would like to thank the faculty members of the Department of Engineering Science and Mechanics at VPI&SU who regularly offer constructive suggestions. These include Saad A. Ragab, Norman E. Dowling, Michael W. Hyer, and J. Wallace Grant. Scott L. Hendricks has been particularly effective and accurate in his extensive review of the manuscript and preparation of WileyPlus materials.

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We are extremely pleased to participate in extending the time duration of this textbook series well past the sixty-five-year mark. In the interest of providing you with the best possible educational materials over future years, we encourage and welcome all comments and suggestions.

L. Glenn Kraige

Blacksburg, Virginia

Princeton, West Virginia

CONTENTS

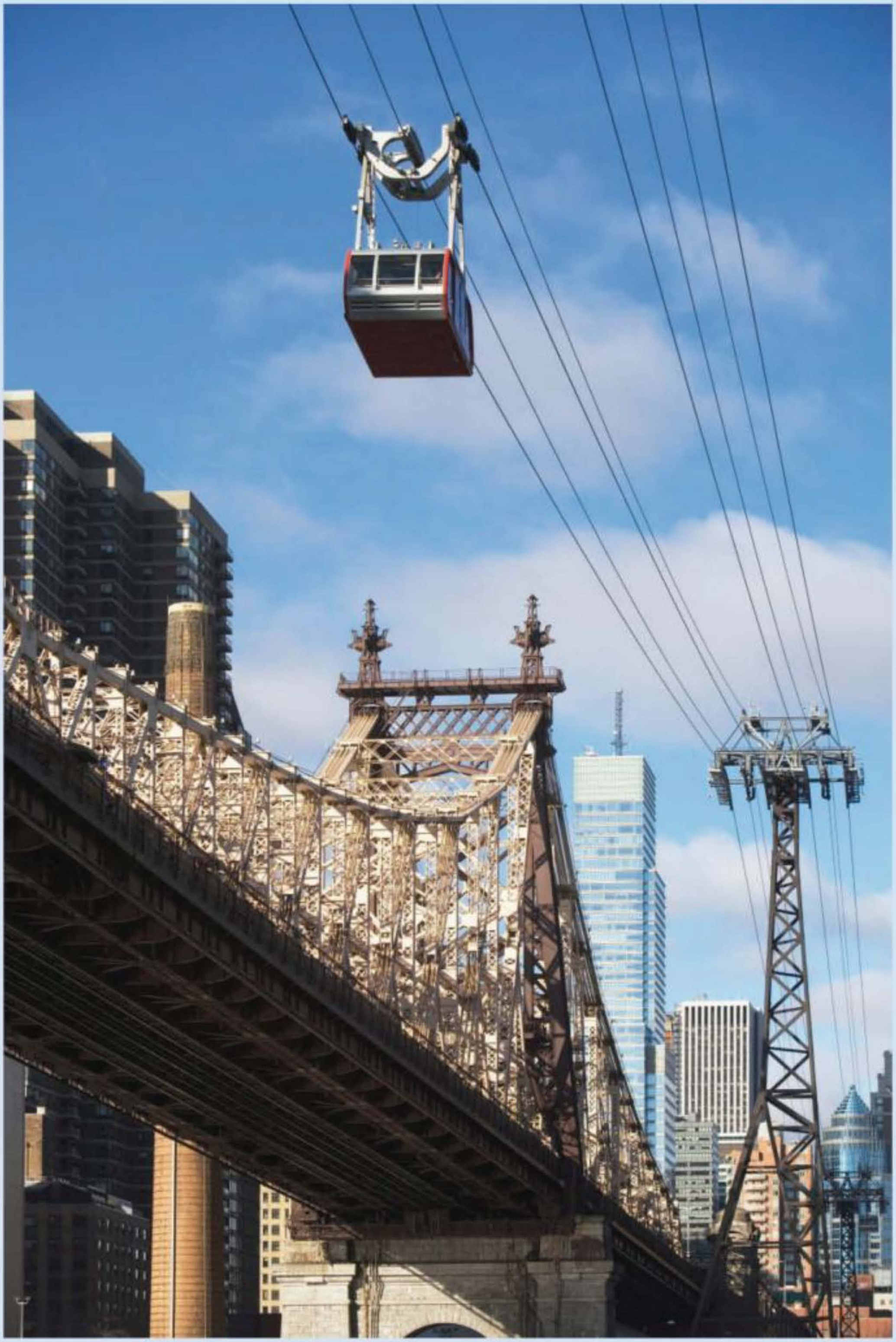


Image Source/Ditto/AGE fotostock

Structures which support large forces must be designed with the principles of mechanics foremost in mind. In this view of New York, one can see a variety of such structures.

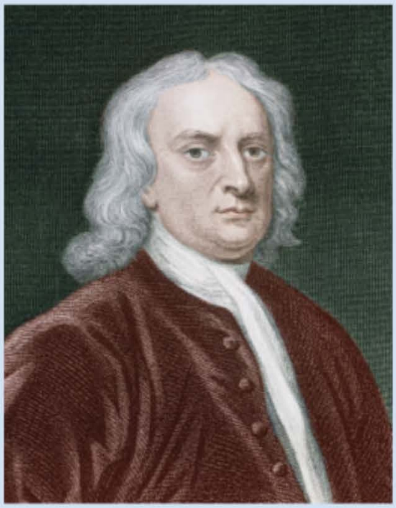
1 Introduction to Statics

CHAPTER OUTLINE

1/1 Mechanics

Mechanics is the physical science which deals with the effects of forces on objects. No other subject plays a greater role in engineering analysis than mechanics. Although the principles of mechanics are few, they have wide application in engineering. The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and apparatus, and molecular, atomic, and subatomic behavior. A thorough understanding of this subject is an essential prerequisite for work in these and many other fields.

Mechanics is the oldest of the physical sciences. The early history of this subject is synonymous with the very beginnings of engineering. The earliest recorded writings in mechanics are those of Archimedes (287–212 B.C.) on the principle of the lever and the principle of buoyancy. Substantial progress came later with the formulation of the laws of vector combination of forces by Stevinus (1548–1620), who also formulated most of the principles of statics. The first investigation of a dynamics problem is credited to Galileo (1564–1642) for his experiments with falling stones. The accurate formulation of the laws of motion, as well as the law of gravitation, was made by Newton (1642–1727), who



Sir Isaac Newton

also conceived the idea of the infinitesimal in mathematical analysis. Substantial contributions to the development of mechanics were also made by da Vinci, Varignon, Euler, D'Alembert, Lagrange, Laplace, and others.

In this book we will be concerned with both the development of the principles of mechanics and their application. The principles of mechanics as a science are rigorously expressed by mathematics, and thus mathematics plays an important role in the application of these principles to the solution of practical problems.

The subject of mechanics is logically divided into two parts: ***statics***, which concerns the equilibrium of bodies under action of forces, and ***dynamics***, which concerns the motion of bodies. *Engineering Mechanics* is divided into these two parts, *Vol. 1 Statics* and *Vol. 2 Dynamics*.

1/2 Basic Concepts

The following concepts and definitions are basic to the study of mechanics, and they should be understood at the outset.

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity, and its properties are discussed in detail in Chapter 2.

A ***particle*** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, which normally follows statics in the curriculum.

1/3 Scalars and Vectors

We use two kinds of quantities in mechanics—scalars and vectors. *Scalar quantities* are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy, and mass. *Vector quantities*, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition as described later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Speed is a scalar. It is the magnitude of velocity, which is a vector. Thus velocity is specified by a direction as well as a speed.

Vectors representing physical quantities can be classified as free, sliding, or fixed.

A ***free vector*** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

A ***sliding vector*** has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole,* and thus it is a sliding vector.

A ***fixed vector*** is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

Conventions for Equations and Diagrams

A vector quantity $\mathbf{V}$ is represented by a line segment, Fig. 1/1, having the direction of the vector and having an arrowhead to indicate the sense. The length of the directed line segment represents to some convenient scale the magnitude $|\mathbf{V}|$ of the vector, which is printed with lightface italic type V. For example, we may choose a scale such that an arrow one inch long represents a force of twenty pounds.

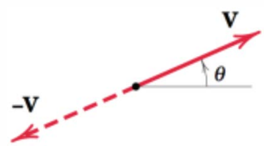


Figure 1/1

In scalar equations, and frequently on diagrams where only the magnitude of a vector is labeled, the symbol will appear in lightface italic type. Boldface type is used for vector quantities whenever the directional aspect of the vector is a part of its mathematical representation. When writing vector equations, *always* be certain to preserve the mathematical distinction between vectors and scalars. In handwritten work, use a distinguishing mark for each vector quantity, such as an underline, $\underline{V}$, or an arrow over the symbol, $\vec{V}$, to take the place of boldface type in print.

Working with Vectors

The direction of the vector $\mathbf{V}$ may be measured by an angle θ from some known reference direction as shown in Fig. 1/1. The negative of $\mathbf{V}$

*This is the *principle of transmissibility*, which is discussed in Art. 2/2.

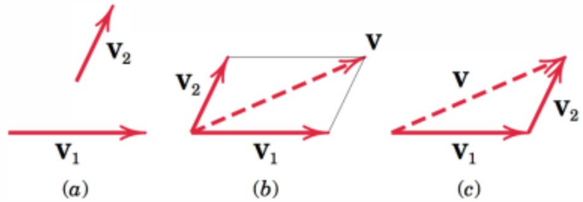


Figure 1/2

is a vector $-\mathbf{V}$ having the same magnitude as $\mathbf{V}$ but directed in the sense opposite to $\mathbf{V}$, as shown in Fig. 1/1.

Vectors must obey the parallelogram law of combination. This law states that two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, treated as free vectors, Fig. 1/2*a*, may be replaced by their equivalent vector $\mathbf{V}$, which is the diagonal of the parallelogram formed by $\mathbf{V}_1$ and $\mathbf{V}_2$ as its two sides, as shown in Fig. 1/2*b*. This combination is called the *vector sum* and is represented by the vector equation

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

where the plus sign, when used with the vector quantities (in boldface type), means *vector* and not *scalar* addition. The scalar sum of the magnitudes of the two vectors is written in the usual way as $V_1 + V_2$. The geometry of the parallelogram shows that $V \neq V_1 + V_2$.

The two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, again treated as free vectors, may also be added head-to-tail by the triangle law, as shown in Fig. 1/2*c*, to obtain the identical vector sum $\mathbf{V}$. We see from the diagram that the order of addition of the vectors does not affect their sum, so that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$.

The difference $\mathbf{V}_1 - \mathbf{V}_2$ between the two vectors is easily obtained by adding $-\mathbf{V}_2$ to $\mathbf{V}_1$ as shown in Fig. 1/3, where either the triangle or parallelogram procedure may be used. The difference $\mathbf{V}'$ between the two vectors is expressed by the vector equation

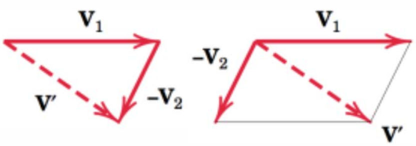


Figure 1/3

$$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$$

where the minus sign denotes *vector subtraction*.

Any two or more vectors whose sum equals a certain vector $\mathbf{V}$ are said to be the *components* of that vector. Thus, the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ in Fig. 1/4*a* are the components of $\mathbf{V}$ in the directions 1 and 2, respectively. It is usually most convenient to deal with vector components which are mutually perpendicular; these are called *rectangular components*. The

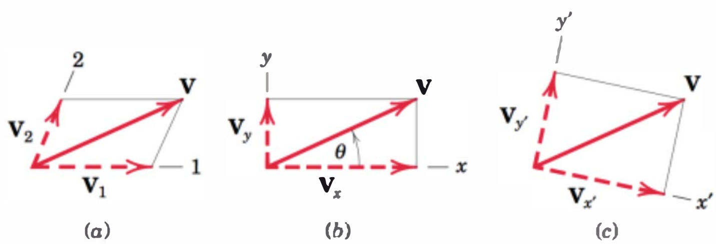


Figure 1/4

vectors $\mathbf{V}_x$ and $\mathbf{V}_y$ in Fig. 1/4*b* are the x- and y-components, respectively, of $\mathbf{V}$. Likewise, in Fig. 1/4*c*, $\mathbf{V}_{x'}$ and $\mathbf{V}_{y'}$ are the x'- and y'-components of $\mathbf{V}$. When expressed in rectangular components, the direction of the vector with respect to, say, the x-axis is clearly specified by the angle θ, where

$$\theta = \tan^{-1}\frac{V_y}{V_x}$$

A vector $\mathbf{V}$ may be expressed mathematically by multiplying its magnitude V by a vector $\mathbf{n}$ whose magnitude is one and whose direction coincides with that of $\mathbf{V}$. The vector $\mathbf{n}$ is called a *unit vector*. Thus,

$$\mathbf{V} = V\mathbf{n}$$

In this way both the magnitude and direction of the vector are conveniently contained in one mathematical expression. In many problems, particularly three-dimensional ones, it is convenient to express the rectangular components of $\mathbf{V}$, Fig. 1/5, in terms of unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, which are vectors in the x-, y-, and z-directions, respectively, with unit magnitudes. Because the vector $\mathbf{V}$ is the vector sum of the components in the x-, y-, and z-directions, we can express $\mathbf{V}$ as follows:

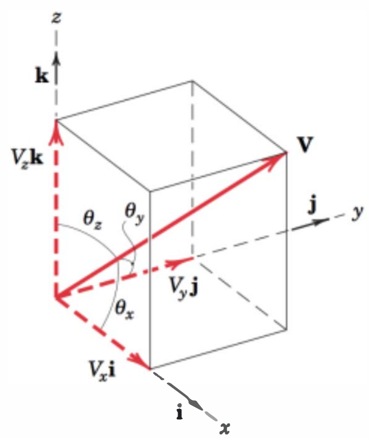


Figure 1/5

$$\boxed{\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}}$$

We now make use of the *direction cosines* l, m, and n of $\mathbf{V}$, which are defined by

$$l = \cos\theta_x \qquad m = \cos\theta_y \qquad n = \cos\theta_z$$

Thus, we may write the magnitudes of the components of $\mathbf{V}$ as

$$\boxed{V_x = lV \qquad V_y = mV \qquad V_z = nV}$$

where, from the Pythagorean theorem,

$$\boxed{V^2 = V_x{}^2 + V_y{}^2 + V_z{}^2}$$

Note that this relation implies that $l^2 + m^2 + n^2 = 1$.

1/4 Newton's Laws

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.* Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

*Newton's original formulations may be found in the translation of his *Principia* (1687) revised by F. Cajori, University of California Press, 1934.

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

The correctness of these laws has been verified by innumerable accurate physical measurements. Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m, it may be stated as

$$\boxed{\mathbf{F} = m\mathbf{a}} \tag{1/1}$$

where $\mathbf{F}$ is the vector sum of forces acting on the particle and $\mathbf{a}$ is the resulting acceleration. This equation is a *vector* equation because the direction of $\mathbf{F}$ must agree with the direction of $\mathbf{a}$, and the magnitudes of $\mathbf{F}$ and $m\mathbf{a}$ must be equal.

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at rest or is moving with a uniform velocity. The first law adds nothing new to the description of motion but is included here because it was part of Newton's classical statements.

The third law is basic to our understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on the desk by the pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source, and holds at every instant of time during which the forces are applied. Lack of careful attention to this basic law is the cause of frequent error by the beginner.

In the analysis of bodies under the action of forces, it is absolutely necessary to be clear about which force of each action–reaction pair is being considered. It is necessary first of all to *isolate* the body under consideration and then to consider only the one force of the pair which acts *on* the body in question.

1/5 Units

In mechanics we use four fundamental quantities called *dimensions*. These are length, mass, force, and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law, Eq. 1/1. Although there are a number of different systems of units, only the two systems most commonly used in science and technology will be used in this text. The four fundamental dimensions and their units and symbols in the two systems are summarized in the following table.

QUANTITY	DIMENSIONAL SYMBOL	SI UNITS			U.S. CUSTOMARY UNITS		
			UNIT	SYMBOL		UNIT	SYMBOL
Mass	M	Base units	kilogram	kg		slug	—
Length	L	Base units	meter	m	Base units	foot	ft
Time	T	Base units	second	s	Base units	second	sec
Force	F		newton	N	Base units	pound	lb

SI Units

The International System of Units, abbreviated SI (from the French, Système International d'Unités), is accepted in the United States and throughout the world, and is a modern version of the metric system. By international agreement, SI units will in time replace other systems. As shown in the table, in SI, the units kilogram (kg) for mass, meter (m) for length, and second (s) for time are selected as the base units, and the newton (N) for force is derived from the preceding three by Eq. 1/1. Thus, force (N) = mass (kg) × acceleration (m/s^2) or

$$\mathrm{N} = \mathrm{kg \cdot m/s^2}$$

Thus, 1 newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2.

Consider a body of mass m which is allowed to fall freely near the surface of the earth. With only the force of gravitation acting on the body, it falls with an acceleration g toward the center of the earth. This gravitational force is the *weight* W of the body and is found from Eq. 1/1:

$$W\,(\mathrm{N}) = m\,(\mathrm{kg}) \times g\,(\mathrm{m/s^2})$$

U.S. Customary Units

The U.S. customary, or British system of units, also called the foot-pound-second (FPS) system, has been the common system in business and industry in English-speaking countries. Although this system will in time be replaced by SI units, for many more years engineers must be able to work with both SI units and FPS units.

As shown in the table, in the U.S. or FPS system, the units of feet (ft) for length, seconds (sec) for time, and pounds (lb) for force are selected as base units, and the slug for mass is derived from Eq. 1/1. Thus, force (lb) = mass (slugs) × acceleration (ft/sec^2), or

$$\mathrm{slug} = \frac{\mathrm{lb\text{-}sec^2}}{\mathrm{ft}}$$

Therefore, 1 slug is the mass which is given an acceleration of 1 ft/sec^2 when acted on by a force of 1 lb. If W is the gravitational force or weight and g is the acceleration due to gravity, Eq. 1/1 gives

$$m\ (\mathrm{slugs}) = \frac{W\ (\mathrm{lb})}{g\,(\mathrm{ft/sec^2})}$$

Note that seconds is abbreviated as *s* in SI units, and as *sec* in FPS units.

In U.S. units the pound is also used on occasion as a unit of mass, especially to specify thermal properties of liquids and gases. When distinction between the two units is necessary, the force unit is frequently written as lbf and the mass unit as lbm. In this book we use almost exclusively the force unit, which is written simply as lb. Other common units of force in the U.S. system are the *kilopound* (kip), which equals 1000 lb, and the *ton*, which equals 2000 lb.

The International System of Units (SI) is termed an *absolute* system because the measurement of the base quantity mass is independent of its environment. On the other hand, the U.S. system (FPS) is termed a *gravitational* system because its base quantity force is defined as the gravitational attraction (weight) acting on a standard mass under specified conditions (sea level and 45° latitude). A standard pound is also the force required to give a one-pound mass an acceleration of 32.1740 ft/sec^2.

In SI units the kilogram is used *exclusively* as a unit of mass—*never* force. In the MKS (meter, kilogram, second) gravitational system, which has been used for many years in non-English-speaking countries, the kilogram, like the pound, has been used both as a unit of force and as a unit of mass.

Primary Standards

Primary standards for the measurements of mass, length, and time have been established by international agreement and are as follows:

Omikron/Science Source

The standard kilogram

Mass. The kilogram is defined as the mass of a specific platinum–iridium cylinder which is kept at the International Bureau of Weights and Measures near Paris, France. An accurate copy of this cylinder is kept in the United States at the National Institute of Standards and Technology (NIST), formerly the National Bureau of Standards, and serves as the standard of mass for the United States.

Length. The meter, originally defined as one ten-millionth of the distance from the pole to the equator along the meridian through Paris, was later defined as the length of a specific platinum–iridium bar kept at the International Bureau of Weights and Measures. The difficulty of accessing the bar and reproducing accurate measurements prompted the adoption of a more accurate and reproducible standard of length for the meter, which is now defined as 1 650 763.73 wavelengths of a specific radiation of the krypton-86 atom.

Time. The second was originally defined as the fraction 1/(86 400) of the mean solar day. However, irregularities in the earth's rotation led to difficulties with this definition, and a more accurate and reproducible standard has been adopted. The second is now defined as the duration of 9 192 631 770 periods of the radiation of a specific state of the cesium-133 atom.

For most engineering work, and for our purpose in studying mechanics, the accuracy of these standards is considerably beyond our needs. The standard value for gravitational acceleration g is its value at sea level and at a 45° latitude. In the two systems these values are

SI units	$g = 9.806\ 65\ \text{m/s}^2$
U.S. units	$g = 32.1740\ \text{ft/sec}^2$

The approximate values of 9.81 m/s^2 and 32.2 ft/sec^2, respectively, are sufficiently accurate for the vast majority of engineering calculations.

Unit Conversions

The characteristics of SI units are shown inside the front cover of this book, along with the numerical conversions between U.S. customary and SI units. In addition, charts giving the approximate conversions between selected quantities in the two systems appear inside the back cover for convenient reference. Although these charts are useful for obtaining a feel for the relative size of SI and U·S. units, in time engineers will find it essential to think directly in terms of SI units without converting from U·S. units. In statics we are primarily concerned with the units of length and force, with mass needed only when we compute gravitational force, as explained previously.

Figure 1/6 depicts examples of force, mass, and length in the two systems of units, to aid in visualizing their relative magnitudes.

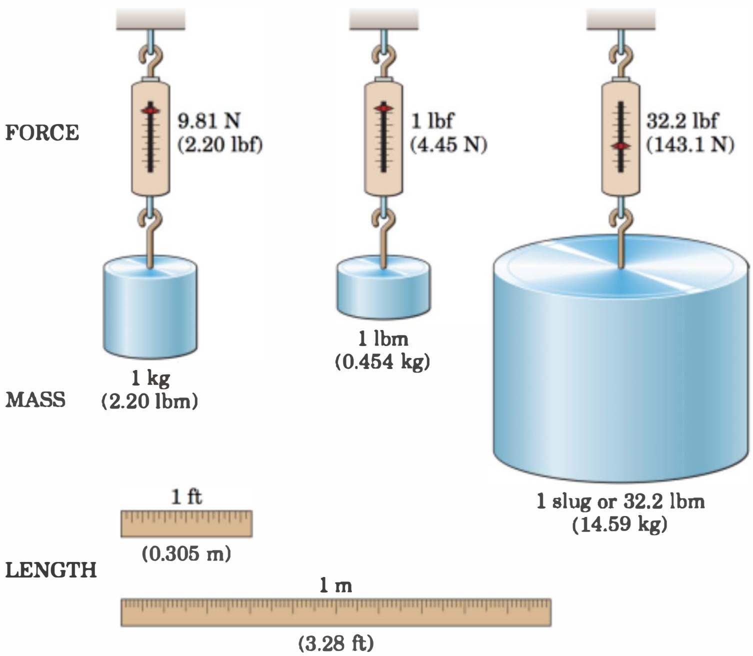


Figure 1/6

1/6 Law of Gravitation

In statics as well as dynamics we often need to compute the weight of a body, which is the gravitational force acting on it. This computation depends on the *law of gravitation*, which was also formulated by Newton. The law of gravitation is expressed by the equation

$$\boxed{F = G\frac{m_1 m_2}{r^2}} \tag{1/2}$$

where F = the mutual force of attraction between two particles

G = a universal constant known as the *constant of gravitation*

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

The mutual forces F obey the law of action and reaction, since they are equal and opposite and are directed along the line joining the centers of the particles, as shown in Fig. 1/7. By experiment the gravitational constant is found to be $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.

Gravitational Attraction of the Earth

NASA Marshall Space Flight Center (NASA-MSFC)

The gravitational force which the earth exerts on the moon (foreground) is a key factor in the motion of the moon.

Gravitational forces exist between every pair of bodies. On the surface of the earth the only gravitational force of appreciable magnitude is the force due to the attraction of the earth. For example, each of two iron spheres 100 mm in diameter is attracted to the earth with a gravitational force of 37.1 N, which is its weight. On the other hand, the force of mutual attraction between the spheres if they are just touching is 0.000 000 095 1 N. This force is clearly negligible compared with the earth's attraction of 37.1 N. Consequently the gravitational attraction of the earth is the only gravitational force we need to consider for most engineering applications on the earth's surface.

The gravitational attraction of the earth on a body (its weight) exists whether the body is at rest or in motion. Because this attraction is a force, the weight of a body should be expressed in newtons (N) in SI units and in pounds (lb) in U.S. customary units. Unfortunately, in common practice the mass unit kilogram (kg) has been frequently used as a measure of weight. This usage should disappear in time as SI units become more widely used, because in SI units the kilogram is used exclusively for mass and the newton is used for force, including weight.

For a body of mass m near the surface of the earth, the gravitational attraction F on the body is specified by Eq. 1/2. We usually denote the

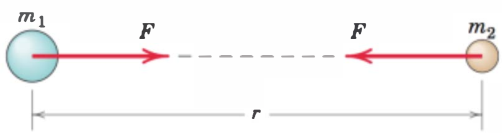


Figure 1/7

magnitude of this gravitational force or weight with the symbol W. Because the body falls with an acceleration g, Eq. 1/1 gives

$$\boxed{W = mg} \tag{1/3}$$

The weight W will be in newtons (N) when the mass m is in kilograms (kg) and the acceleration of gravity g is in meters per second squared (m/s^2). In U.S. customary units, the weight W will be in pounds (lb) when m is in slugs and g is in feet per second squared. The standard values for g of 9.81 m/s^2 and 32.2 ft/sec^2 will be sufficiently accurate for our calculations in statics.

The true weight (gravitational attraction) and the apparent weight (as measured by a spring scale) are slightly different. The difference, which is due to the rotation of the earth, is quite small and will be neglected. This effect will be discussed in *Vol. 2 Dynamics*.

1/7 Accuracy, Limits, and Approximations

The number of significant figures in an answer should be no greater than the number of figures justified by the accuracy of the given data. For example, suppose the 24-mm side of a square bar was measured to the nearest millimeter, so we know the side length to two significant figures. Squaring the side length gives an area of 576 mm^2. However, according to our rule, we should write the area as 580 mm^2, using only two significant figures.

When calculations involve small differences in large quantities, greater accuracy in the data is required to achieve a given accuracy in the results. Thus, for example, it is necessary to know the numbers 4.2503 and 4.2391 to an accuracy of five significant figures to express their difference 0.0112 to three-figure accuracy. It is often difficult in lengthy computations to know at the outset how many significant figures are needed in the original data to ensure a certain accuracy in the answer. Accuracy to three significant figures is considered satisfactory for most engineering calculations.

In this text, answers will generally be shown to three significant figures unless the answer begins with the digit 1, in which case the answer will be shown to four significant figures. For purposes of calculation, consider all data given in this book to be exact.

Differentials

The *order* of differential quantities frequently causes misunderstanding in the derivation of equations. Higher-order differentials may always be neglected compared with lower-order differentials when the mathematical limit is approached. For example, the element of volume ΔV of a right circular cone of altitude h and base radius r may be taken to be a circular slice a distance x from the vertex and of thickness Δx. The expression for the volume of the element is

$$\Delta V = \frac{\pi r^2}{h^2}\left[x^2\,\Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3\right]$$

Note that, when passing to the limit in going from ΔV to dV and from Δx to dx, the terms containing $(\Delta x)^2$ and $(\Delta x)^3$ drop out, leaving merely

$$dV = \frac{\pi r^2}{h^2} x^2\, dx$$

which gives an exact expression when integrated.

Small-Angle Approximations

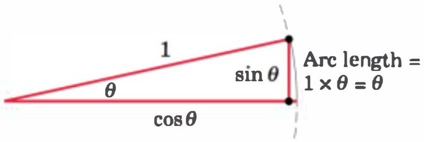


Figure 1/8

When dealing with small angles, we can usually make use of simplifying approximations. Consider the right triangle of Fig. 1/8 where the angle θ, expressed in radians, is relatively small. If the hypotenuse is unity, we see from the geometry of the figure that the arc length $1 \times \theta$ and $\sin\theta$ are very nearly the same. Also, $\cos\theta$ is close to unity. Furthermore, $\sin\theta$ and $\tan\theta$ have almost the same values. Thus, for small angles we may write

$$\sin\theta \cong \tan\theta \cong \theta \qquad \cos\theta \cong 1$$

provided that the angles are expressed in radians. These approximations may be obtained by retaining only the first terms in the series expansions for these three functions. As an example of these approximations, for an angle of 1°

$$1° = 0.017\,453 \text{ rad} \qquad \tan 1° = 0.017\,455$$

$$\sin 1° = 0.017\,452 \qquad \cos 1° = 0.999\,848$$

If a more accurate approximation is desired, the first two terms may be retained, and they are

$$\sin\theta \cong \theta - \theta^3/6 \qquad \tan\theta \cong \theta + \theta^3/3 \qquad \cos\theta \cong 1 - \theta^2/2$$

where the angles must be expressed in radians. (To convert degrees to radians, multiply the angle in degrees by $\pi/180°$.) The error in replacing the sine by the angle for 1° (0.0175 rad) is only 0.005 percent. For 5° (0.0873 rad) the error is 0.13 percent, and for 10° (0.1745 rad), the error is still only 0.51 percent. As the angle θ approaches zero, the following relations are true in the mathematical limit:

$$\sin d\theta = \tan d\theta = d\theta \qquad \cos d\theta = 1$$

where the differential angle $d\theta$ must be expressed in radians.

1/8 Problem Solving in Statics

We study statics to obtain a quantitative description of forces which act on engineering structures in equilibrium. Mathematics establishes the relations between the various quantities involved and enables us to predict effects from these relations. We use a dual thought process in solving statics problems: We think about both the physical situation and the corresponding mathematical description. In the analysis of every problem, we make a transition between the physical and the mathematical. One of the most important goals for the student is to develop the ability to make this transition freely.

Making Appropriate Assumptions

We should recognize that the mathematical formulation of a physical problem represents an ideal description, or *model*, which approximates but never quite matches the actual physical situation. When we construct an idealized mathematical model for a given engineering problem, certain approximations will always be involved. Some of these approximations may be mathematical, whereas others will be physical.

For instance, it is often necessary to neglect small distances, angles, or forces compared with large distances, angles, or forces. Suppose a force is distributed over a small area of the body on which it acts. We may consider it to be a concentrated force if the dimensions of the area involved are small compared with other pertinent dimensions.

We may neglect the weight of a steel cable if the tension in the cable is many times greater than its total weight. However, if we must calculate the deflection or sag of a suspended cable under the action of its weight, we may not ignore the cable weight.

Thus, what we may assume depends on what information is desired and on the accuracy required. We must be constantly alert to the various assumptions called for in the formulation of real problems. The ability to understand and make use of the appropriate assumptions in the formulation and solution of engineering problems is certainly one of the most important characteristics of a successful engineer. One of the major aims of this book is to provide many opportunities to develop this ability through the formulation and analysis of many practical problems involving the principles of statics.

Using Graphics

Graphics is an important analytical tool for three reasons:

1. We use graphics to represent a physical system on paper with a sketch or diagram. Representing a problem geometrically helps us with its physical interpretation, especially when we must visualize three-dimensional problems.
2. We can often obtain a graphical solution to problems more easily than with a direct mathematical solution. Graphical solutions are both a practical way to obtain results and an aid in our thought processes. Because graphics represents the physical situation and its mathematical expression simultaneously, graphics helps us make the transition between the two.
3. Charts or graphs are valuable aids for representing results in a form which is easy to understand.

Key Concepts

FORMULATING PROBLEMS AND OBTAINING SOLUTIONS

In statics, as in all engineering problems, we need to use a precise and logical method for formulating problems and obtaining their solutions. We formulate each problem and develop its solution through the following sequence of steps.

1. Formulate the problem:
   (*a*) State the given data.
   (*b*) State the desired result.
   (*c*) State your assumptions and approximations.
2. Develop the solution:
   (*a*) Draw any diagrams you need to understand the relationships.
   (*b*) State the governing principles to be applied to your solution.
   (*c*) Make your calculations.
   (*d*) Ensure that your calculations are consistent with the accuracy justified by the data.
   (*e*) Be sure that you have used consistent units throughout your calculations.
   (*f*) Ensure that your answers are reasonable in terms of magnitudes, directions, common sense, etc.
   (*g*) Draw conclusions.

Keeping your work neat and orderly will help your thought process and enable others to understand your work. The discipline of doing orderly work will help you develop skill in formulation and analysis. Problems which seem complicated at first often become clear when you approach them with logic and discipline.

The Free-Body Diagram

The subject of statics is based on surprisingly few fundamental concepts and involves mainly the application of these basic relations to a variety of situations. In this application the *method* of analysis is all important. In solving a problem, it is essential that the laws which apply be carefully fixed in mind and that we apply these principles literally and exactly. In applying the principles of mechanics to analyze forces acting on a body, it is essential that we *isolate* the body in question from all other bodies so that a complete and accurate account of all forces acting on this body can be taken. This *isolation* should exist mentally and should be represented on paper. The diagram of such an isolated body with the representation of *all* external forces acting *on* it is called a *free-body diagram*.

The free-body-diagram method is the key to the understanding of mechanics. This is so because the *isolation* of a body is the tool by which *cause* and *effect* are clearly separated and by which our attention is clearly focused on the literal application of a principle of mechanics. The technique of drawing free-body diagrams is covered in Chapter 3, where they are first used.

Numerical Values versus Symbols

In applying the laws of statics, we may use numerical values to represent quantities, or we may use algebraic symbols and leave the answer as a formula. When numerical values are used, the magnitude of each quantity expressed in its particular units is evident at each stage of the calculation. This is useful when we need to know the magnitude of each term.

The symbolic solution, however, has several advantages over the numerical solution. First, the use of symbols helps to focus our attention on the connection between the physical situation and its related mathematical description. Second, we can use a symbolic solution repeatedly for obtaining answers to the same type of problem, but having different units or numerical values. Third, a symbolic solution enables us to make a dimensional check at every step, which is more difficult to do when numerical values are used. In any equation representing a physical situation, the dimensions of every term on both sides of the equation must be the same. This property is called *dimensional homogeneity*.

Thus, facility with both numerical and symbolic forms of solution is essential.

Solution Methods

Solutions to the problems of statics may be obtained in one or more of the following ways.

1. Obtain mathematical solutions by hand, using either algebraic symbols or numerical values. We can solve most problems this way.
2. Obtain graphical solutions for certain problems.
3. Solve problems by computer. This is useful when a large number of equations must be solved, when a parameter variation must be studied, or when an intractable equation must be solved.

Many problems can be solved with two or more of these methods. The method utilized depends partly on the engineer's preference and partly on the type of problem to be solved. The choice of the most expedient method of solution is an important aspect of the experience to be gained from the problem work. There are a number of problems in *Vol. 1 Statics* which are designated as *Computer-Oriented Problems*. These problems appear at the end of the Review Problem sets and are selected to illustrate the type of problem for which solution by computer offers a distinct advantage.

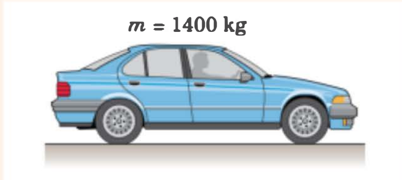
1/9 Chapter Review

This chapter has introduced the concepts, definitions, and units used in statics, and has given an overview of the procedure used to formulate and solve problems in statics. Now that you have finished this chapter, you should be able to do the following:

1. Express vectors in terms of unit vectors and perpendicular components, and perform vector addition and subtraction.
2. State Newton's laws of motion.
3. Perform calculations using SI and U.S. customary units, using appropriate accuracy.
4. Express the law of gravitation and calculate the weight of an object.
5. Apply simplifications based on differential and small-angle approximations.
6. Describe the methodology used to formulate and solve statics problems.

Sample Problem 1/1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.



Solution. From relationship 1/3, we have

① $$W = mg = 1400(9.81) = 13\ 730 \text{ N} \qquad \textit{Ans.}$$

From the table of conversion factors inside the front cover of the textbook, we see that 1 slug is equal to 14.594 kg. Thus, the mass of the car in slugs is

② $$m = 1400 \text{ kg}\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right] = 95.9 \text{ slugs} \qquad \textit{Ans.}$$

Finally, its weight in pounds is

③ $$W = mg = (95.9)(32.2) = 3090 \text{ lb} \qquad \textit{Ans.}$$

As another route to the last result, we can convert from kg to lbm. Again using the table inside the front cover, we have

$$m = 1400 \text{ kg}\left[\frac{1 \text{ lbm}}{0.45359 \text{ kg}}\right] = 3090 \text{ lbm}$$

The weight in pounds associated with the mass of 3090 lbm is 3090 lb, as calculated above. We recall that 1 lbm is the amount of mass which under standard conditions has a weight of 1 lb of force. We rarely refer to the U.S. mass unit lbm in this textbook series, but rather use the slug for mass. The sole use of slug, rather than the unnecessary use of two units for mass, will prove to be powerful and simple—especially in dynamics.

Helpful Hints

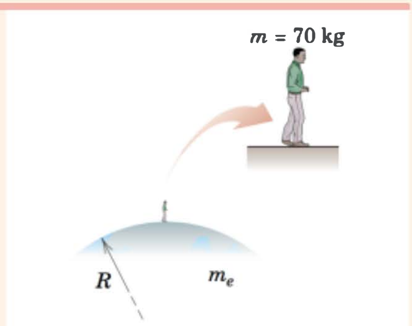
① Our calculator indicates a result of 13 734 N. Using the rules of significant-figure display used in this textbook, we round the written result to four significant figures, or 13 730 N. Had the number begun with any digit other than 1, we would have rounded to three significant figures.

② A good practice with unit conversion is to multiply by a factor such as $\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right]$, which has a value of 1, because the numerator and the denominator are equivalent. Make sure that cancellation of the units leaves the units desired; here the units of kg cancel, leaving the desired units of slug.

③ Note that we are using a previously calculated result (95.9 slugs). We must be sure that when a calculated number is needed in subsequent calculations, it is retained in the calculator to its full accuracy, (95.929834 . . .), until it is needed. This may require storing it in a register upon its initial calculation and recalling it later. We must not merely punch 95.9 into our calculator and proceed to multiply by 32.2—this practice will result in loss of numerical accuracy. Some individuals like to place a small indication of the storage register used in the right margin of the work paper, directly beside the number stored.

Sample Problem 1/2

Use Newton's law of universal gravitation to calculate the weight of a 70-kg person standing on the surface of the earth. Then repeat the calculation by using $W = mg$ and compare your two results. Use Table D/2 as needed.



Solution. The two results are

① $$W = \frac{Gm_e m}{R^2} = \frac{(6.673\cdot 10^{-11})(5.976\cdot 10^{24})(70)}{[6371\cdot 10^3]^2} = 688 \text{ N} \qquad \textit{Ans.}$$

$$W = mg = 70(9.81) = 687 \text{ N} \qquad \textit{Ans.}$$

The discrepancy is due to the fact that Newton's universal gravitational law does not take into account the rotation of the earth. On the other hand, the value g = 9.81 m/s^2 used in the second equation does account for the earth's rotation. Note that had we used the more accurate value g = 9.80665 m/s^2 (which likewise accounts for the earth's rotation) in the second equation, the discrepancy would have been larger (686 N would have been the result).

Helpful Hint

① The effective distance between the mass centers of the two bodies involved is the radius of the earth.

Sample Problem 1/3

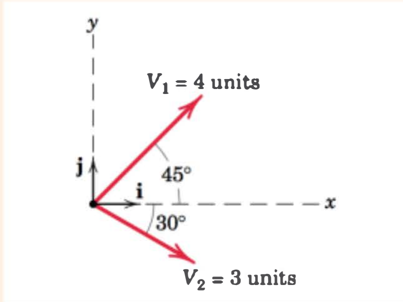
For the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ shown in the figure,

(*a*) determine the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$

(*b*) determine the angle α between $\mathbf{S}$ and the positive x-axis

(*c*) write $\mathbf{S}$ as a vector in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ and then write a unit vector $\mathbf{n}$ along the vector sum $\mathbf{S}$

(*d*) determine the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$



Solution. (*a*) We construct to scale the parallelogram shown in Fig. *a* for adding $\mathbf{V}_1$ and $\mathbf{V}_2$. Using the law of cosines, we have

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$

① (*b*) Using the law of sines for the lower triangle, we have

$$\frac{\sin 105°}{5.59} = \frac{\sin(\alpha + 30°)}{4}$$

$$\sin(\alpha + 30°) = 0.692$$

$$(\alpha + 30°) = 43.8° \qquad \alpha = 13.76° \qquad \textit{Ans.}$$

(*c*) With knowledge of both S and α, we can write the vector $\mathbf{S}$ as

$$\mathbf{S} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= 5.59[\mathbf{i}\cos 13.76° + \mathbf{j}\sin 13.76°] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

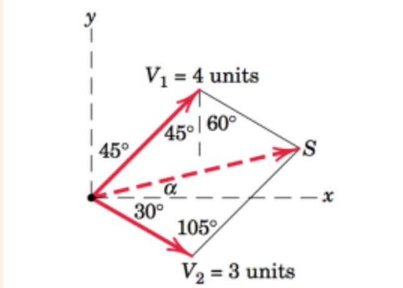
② Then $$\mathbf{n} = \frac{\mathbf{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j} \qquad \textit{Ans.}$$

(*d*) The vector difference $\mathbf{D}$ is

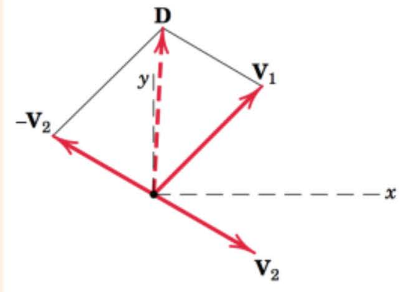
$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) - 3(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

The vector $\mathbf{D}$ is shown in Fig. *b* as $\mathbf{D} = \mathbf{V}_1 + (-\mathbf{V}_2)$.



(*a*)



(*b*)

Helpful Hints

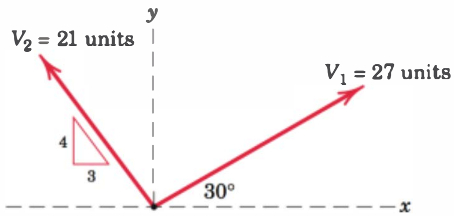
① You will frequently use the laws of cosines and sines in mechanics. See Art. C/6 of Appendix C for a review of these important geometric principles.

② A unit vector may always be formed by dividing a vector by its magnitude. Note that a unit vector is dimensionless.

PROBLEMS

1/1 Determine the angles made by the vector $\mathbf{V} = -36\mathbf{i} + 15\mathbf{j}$ with the positive x- and y-axes. Write the unit vector $\mathbf{n}$ in the direction of $\mathbf{V}$.

1/2 Determine the magnitude of the vector sum $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ and the angle θ_x which $\mathbf{V}$ makes with the positive x-axis. Complete both graphical and algebraic solutions.



Problem 1/2

1/3 For the given vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ of Prob. 1/2, determine the magnitude of the vector difference $\mathbf{V}' = \mathbf{V}_2 - \mathbf{V}_1$ and the angle θ_x which $\mathbf{V}'$ makes with the positive x-axis. Complete both graphical and algebraic solutions.

1/4 A force is specified by the vector $\mathbf{F} = 160\mathbf{i} + 80\mathbf{j} - 120\mathbf{k}$ N. Calculate the angles made by $\mathbf{F}$ with the positive x-, y-, and z-axes.

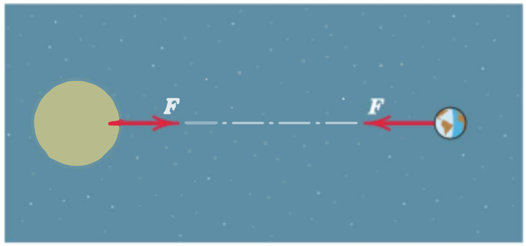
1/5 What is the mass in both slugs and kilograms of a 1000-lb beam?

1/6 From the gravitational law calculate the weight W (gravitational force with respect to the earth) of an 85-kg man in a spacecraft traveling in a circular orbit 250 km above the earth's surface. Express W in both newtons and pounds.

1/7 Determine the weight in newtons of a woman whose weight in pounds is 125. Also, find her mass in slugs and in kilograms. Determine your own weight in newtons.

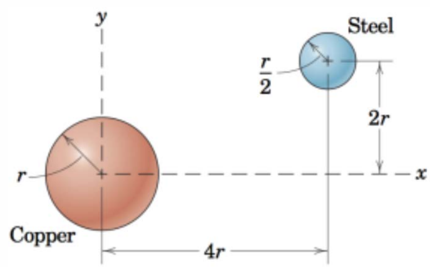
1/8 Suppose that two nondimensional quantities are exactly $A = 8.67$ and $B = 1.429$. Using the rules for significant figures as stated in this chapter, express the four quantities $(A + B)$, $(A - B)$, (AB), and (A/B).

1/9 Compute the magnitude F of the force which the sun exerts on the earth. Perform the calculation first in pounds and then convert your result to newtons. Refer to Table D/2 in Appendix D for necessary physical quantities.



Problem 1/9

1/10 Determine the small gravitational force $\mathbf{F}$ which the copper sphere exerts on the steel sphere. Both spheres are homogeneous, and the value of r is 50 mm. Express your result as a vector.



Problem 1/10

1/11 Evaluate the expression $E = 3 \sin^2\theta \tan\theta \cos\theta$ for $\theta = 2°$. Then use the small-angle assumptions and repeat the calculation.

1/12 A general expression is given by $Q = kmbc/t^2$, where k is a dimensionless constant, m is mass, b and c are lengths, and t is time. Determine both the SI and U.S. units of Q, being sure to use the base units in each system.

06photo/Shutterstock

The properties of force systems must be thoroughly understood by the engineers who design structures such as these overhead cranes.

2 FORCE SYSTEMS

CHAPTER OUTLINE

2/1 INTRODUCTION

In this and the following chapters, we study the effects of forces which act on engineering structures and mechanisms. The experience gained here will help you in the study of mechanics and in other subjects such as stress analysis, design of structures and machines, and fluid flow. This chapter lays the foundation for a basic understanding not only of statics but also of the entire subject of mechanics, and you should master this material thoroughly.

2/2 FORCE

Before dealing with a group or *system* of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in Chapter 1 as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

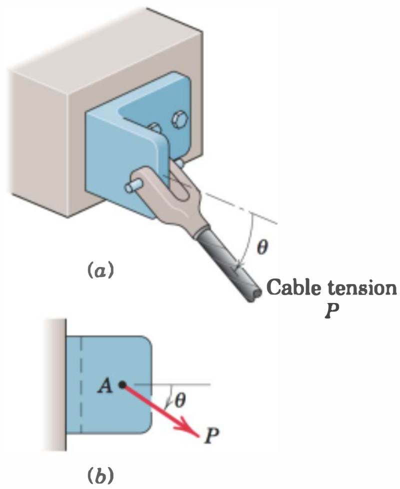


Figure 2/1

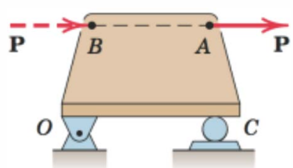
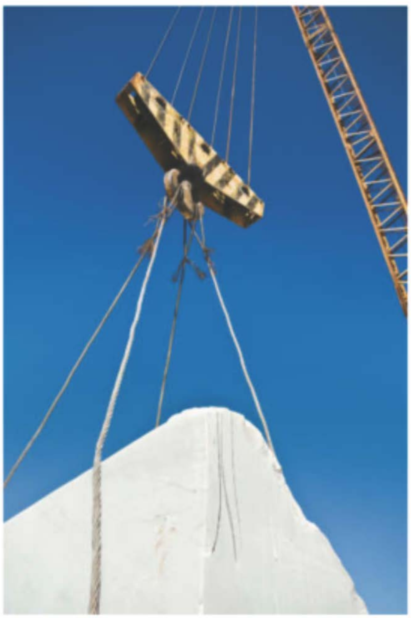


Figure 2/2



The forces associated with this lifting rig must be carefully identified, classified, and analyzed in order to provide a safe and effective working environment.

The action of the cable tension on the bracket in Fig. 2/1*a* is represented in the side view, Fig. 2/1*b*, by the force vector **P** of magnitude P. The effect of this action on the bracket depends on P, the angle θ, and the location of the point of application A. Changing any one of these three specifications will alter the effect on the bracket, such as the force in one of the bolts which secure the bracket to the base, or the internal force and deformation in the material of the bracket at any point. Thus, the complete specification of the action of a force must include its *magnitude*, *direction*, and *point of application*, and therefore we must treat it as a fixed vector.

External and Internal Effects

We can separate the action of a force on a body into two effects, *external* and *internal*. For the bracket of Fig. 2/1 the effects of **P** external to the bracket are the reactive forces (not shown) exerted on the bracket by the foundation and bolts because of the action of **P**. Forces external to a body can be either *applied* forces or *reactive* forces. The effects of **P** internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The relation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

Principle of Transmissibility

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Fig. 2/2 may be applied at A or at B or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at O and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*. Because this book deals essentially with the mechanics of rigid bodies, we will treat almost all forces as sliding vectors for the rigid body on which they act.

Force Classification

Forces are classified as either *contact* or *body* forces. A contact force is produced by direct physical contact; an example is the force exerted on a body by a supporting surface. On the other hand, a body force is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight.

Forces may be further classified as either *concentrated* or *distributed*. Every contact force is actually applied over a finite area and is therefore really a distributed force. However, when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an *area*, as in the case of mechanical contact, over a *volume* when a body force such as weight is acting, or over a *line*, as in the case of the weight of a suspended cable.

The *weight* of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity. The position of the center of gravity is frequently obvious if the body is symmetric. If the position is not obvious, then a separate calculation, explained in Chapter 5, will be necessary to locate the center of gravity.

We can measure a force either by comparison with other known forces, using a mechanical balance, or by the calibrated movement of an elastic element. All such comparisons or calibrations have as their basis a primary standard. The standard unit of force in SI units is the newton (N) and in the U.S. customary system is the pound (lb), as defined in Art. 1/5.

Action and Reaction

According to Newton's third law, the *action* of a force is always accompanied by an *equal* and *opposite reaction*. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first *isolate* the body in question and then identify the force exerted *on* that body (not the force exerted *by* the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be *concurrent at a point* if their lines of action intersect at that point. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Fig. 2/3*a* have a common point of application and are concurrent at the point A. Thus, they can be added using the parallelogram law in their common plane to obtain their sum or *resultant* $\mathbf{R}$, as shown in Fig. 2/3*a*. The resultant lies in the same plane as $\mathbf{F}_1$ and $\mathbf{F}_2$.

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 2/3*b*. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum $\mathbf{R}$ at the point of concurrency A, as shown in Fig. 2/3*b*. We can replace $\mathbf{F}_1$ and $\mathbf{F}_2$ with the resultant $\mathbf{R}$ without altering the external effects on the body upon which they act.

We can also use the triangle law to obtain $\mathbf{R}$, but we need to move the line of action of one of the forces, as shown in Fig. 2/3*c*. If we add the same two forces as shown in Fig. 2/3*d*, we correctly preserve the magnitude and direction of $\mathbf{R}$, but we lose the correct line of action, because $\mathbf{R}$ obtained in this way does not pass through A. Therefore this type of combination should be avoided.

We can express the sum of the two forces mathematically by the vector equation

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

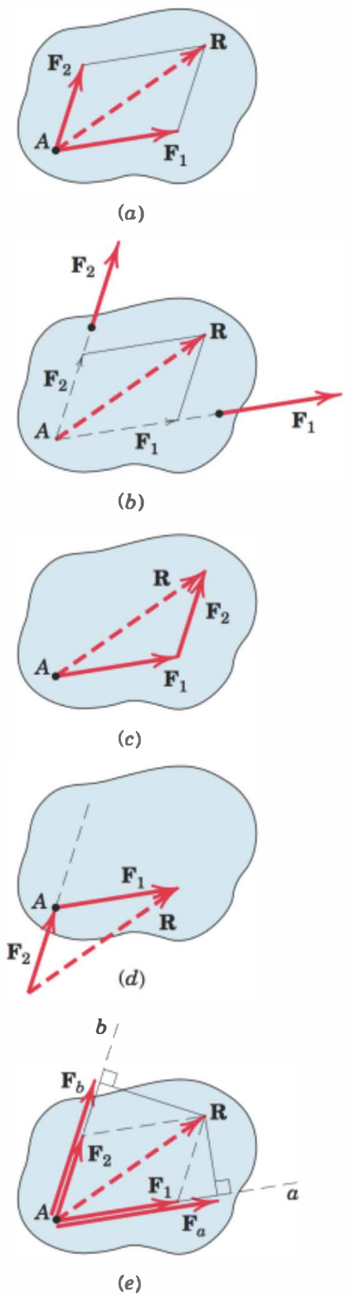


Figure 2/3

Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its *vector components* in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force $\mathbf{R}$ in Fig. 2/3*a* may be replaced by, or *resolved* into, two vector components $\mathbf{F}_1$ and $\mathbf{F}_2$ with the specified directions by completing the parallelogram as shown to obtain the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular* projections onto the same axes. Figure 2/3*e* shows the perpendicular projections $\mathbf{F}_a$ and $\mathbf{F}_b$ of the given force $\mathbf{R}$ onto axes a and b, which are parallel to the vector components $\mathbf{F}_1$ and $\mathbf{F}_2$ of Fig. 2/3*a*. Figure 2/3*e* shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections $\mathbf{F}_a$ and $\mathbf{F}_b$ is not the vector $\mathbf{R}$, because the parallelogram law of vector addition must be used to form the sum. The components and projections of $\mathbf{R}$ are equal only when the axes a and b are perpendicular.

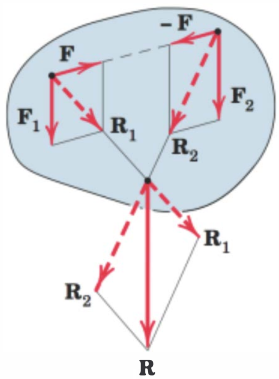


Figure 2/4

A Special Case of Vector Addition

To obtain the resultant when the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are parallel as in Fig. 2/4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces $\mathbf{F}$ and $-\mathbf{F}$ of convenient magnitude, which taken together produce no external effect on the body. Adding $\mathbf{F}_1$ and $\mathbf{F}$ to produce $\mathbf{R}_1$, and combining with the sum $\mathbf{R}_2$ of $\mathbf{F}_2$ and $-\mathbf{F}$ yield the resultant $\mathbf{R}$, which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

It is usually helpful to master the analysis of force systems in two dimensions before undertaking three-dimensional analysis. Thus the remainder of Chapter 2 is subdivided into these two categories.

SECTION A TWO-DIMENSIONAL FORCE SYSTEMS

2/3 Rectangular Components

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector $\mathbf{F}$ of Fig. 2/5 may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \tag{2/1}$$

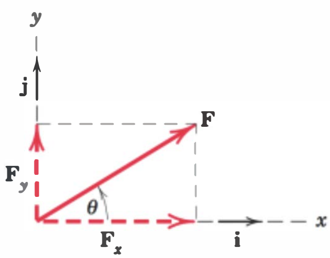


Figure 2/5

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of $\mathbf{F}$ in the x- and y-directions. Each of the two vector components may be written as a scalar times the

*Perpendicular projections are also called *orthogonal* projections.

appropriate unit vector. In terms of the unit vectors **i** and **j** of Fig. 2/5, $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2/2}$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 2/5, the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$\boxed{\begin{aligned} F_x &= F\cos\theta \qquad & F &= \sqrt{F_x^{\,2} + F_y^{\,2}} \\ F_y &= F\sin\theta \qquad & \theta &= \tan^{-1}\frac{F_y}{F_x} \end{aligned}} \tag{2/3}$$

Conventions for Describing Vector Components

We express the magnitude of a vector with lightface italic type in print; that is, $|\mathbf{F}|$ is indicated by F, a quantity which is always *nonnegative*. However, the scalar components, also denoted by lightface italic type, will include sign information. See Sample Problems 2/1 and 2/3 for numerical examples which involve both positive and negative scalar components.

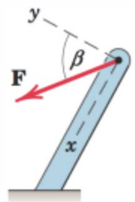
When both a force and its vector components appear in a diagram, it is desirable to show the vector components of the force with dashed lines, as in Fig. 2/5, and show the force with a solid line, or vice versa. With either of these conventions it will always be clear that a force and its components are being represented, and not three separate forces, as would be implied by three solid-line vectors.

Actual problems do not come with reference axes, so their assignment is a matter of arbitrary convenience, and the choice is frequently up to the student. The logical choice is usually indicated by the way in which the geometry of the problem is specified. When the principal dimensions of a body are given in the horizontal and vertical directions, for example, you would typically assign reference axes in these directions.

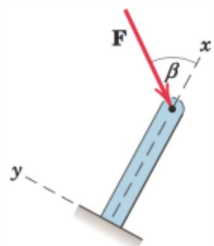
Determining the Components of a Force

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinates need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Figure 2/6 suggests a few typical examples of vector resolution in two dimensions.

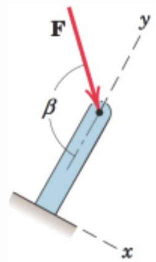
Memorization of Eqs. 2/3 is not a substitute for understanding the parallelogram law and for correctly projecting a vector onto a reference axis. A neatly drawn sketch always helps to clarify the geometry and avoid error.



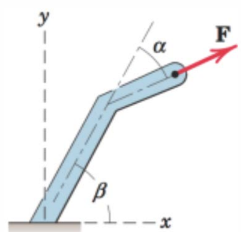
$F_x = F\sin\beta$
$F_y = F\cos\beta$



$F_x = -F\cos\beta$
$F_y = -F\sin\beta$



$F_x = F\sin(\pi - \beta)$
$F_y = -F\cos(\pi - \beta)$



$F_x = F\cos(\beta - \alpha)$
$F_y = F\sin(\beta - \alpha)$

Figure 2/6

Rectangular components are convenient for finding the sum or resultant $\mathbf{R}$ of two forces which are concurrent. Consider two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ which are originally concurrent at a point O. Figure 2/7 shows the line of action of $\mathbf{F}_2$ shifted from O to the tip of $\mathbf{F}_1$ according to the triangle rule of Fig. 2/3. In adding the force vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

from which we conclude that

$$\begin{aligned} R_x &= F_{1_x} + F_{2_x} = \Sigma F_x \\ R_y &= F_{1_y} + F_{2_y} = \Sigma F_y \end{aligned} \tag{2/4}$$

The term ΣF_x means "the algebraic sum of the x scalar components". For the example shown in Fig. 2/7, note that the scalar component F_{2_y} would be negative.

Vince Streano/Photographer's Choice/Getty Images

The structural elements in the foreground transmit concentrated forces to the brackets at both ends.

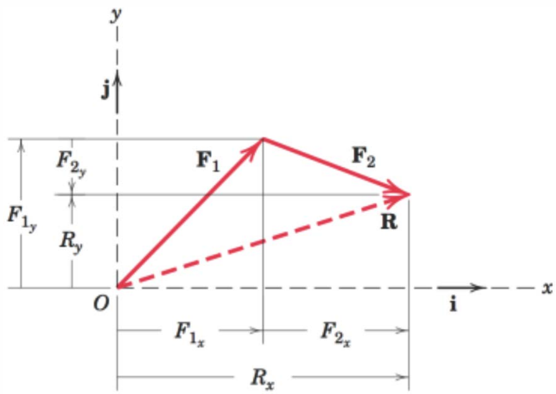
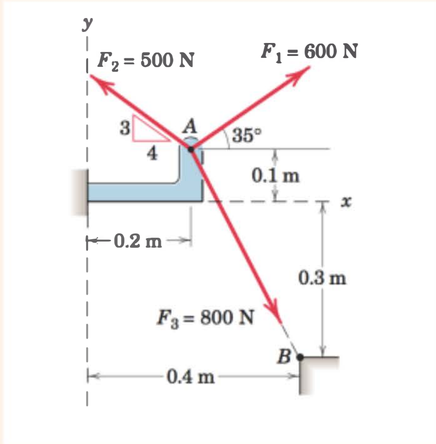


Figure 2/7

Sample Problem 2/1

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. a, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

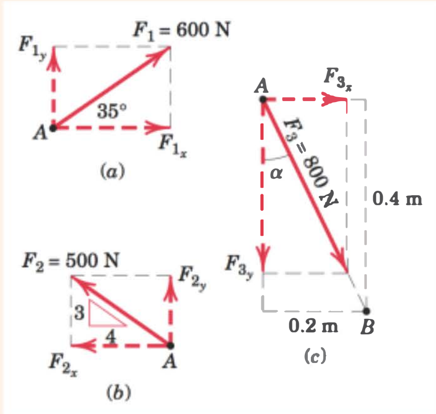
The scalar components of $\mathbf{F}_2$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$

Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.



$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

① Then, $\quad F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358 \text{ N} \qquad \textit{Ans.}$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$

Alternatively, the scalar components of $\mathbf{F}_3$ can be obtained by writing $\mathbf{F}_3$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB. Thus,

② $$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800\,[0.447\mathbf{i} - 0.894\mathbf{j}]$$

$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

The required scalar components are then

$$F_{3_x} = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -716 \text{ N} \qquad \textit{Ans.}$$

which agree with our previous results.

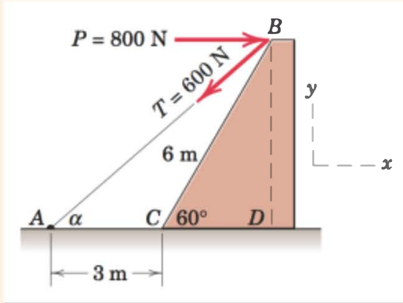
Helpful Hints

① You should carefully examine the geometry of each component determination problem and not rely on the blind use of such formulas as $F_x = F\cos\theta$ and $F_y = F\sin\theta$.

② A unit vector can be formed by dividing *any* vector, such as the geometric position vector $\overrightarrow{AB}$, by its length or magnitude. Here we use the overarrow to denote the vector which runs from A to B and the overbar to determine the distance between A and B.

Sample Problem 2/2

Combine the two forces **P** and **T**, which act on the fixed structure at B, into a single equivalent force **R**.

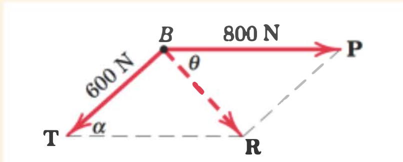


① ***Graphical solution.*** The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. *a*. The scale used here is 1 cm = 800 N; a scale of 1 cm = 200 N would be more suitable for regular-size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallelogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525 \text{ N} \qquad \theta = 49^\circ \qquad \textit{Ans.}$$



(*a*)

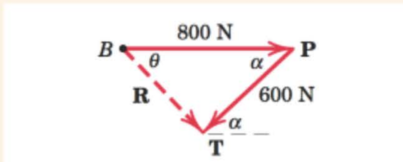
② ***Geometric solution.*** The triangle for the vector addition of **T** and **P** is shown in Fig. *b*. The angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9^\circ = 274\,300$$

$$R = 524 \text{ N} \qquad \textit{Ans.}$$

From the law of sines, we may determine the angle θ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9^\circ} \qquad \sin\theta = 0.750 \qquad \theta = 48.6^\circ \qquad \textit{Ans.}$$



(*b*)

Algebraic solution. By using the x-y coordinate system on the given figure, we may write

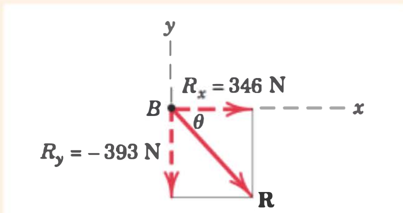
$$R_x = \Sigma F_x = 800 - 600\cos 40.9^\circ = 346 \text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9^\circ = -393 \text{ N}$$

The magnitude and direction of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ N} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6^\circ \qquad \textit{Ans.}$$



(*c*)

The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ N} \qquad \textit{Ans.}$$

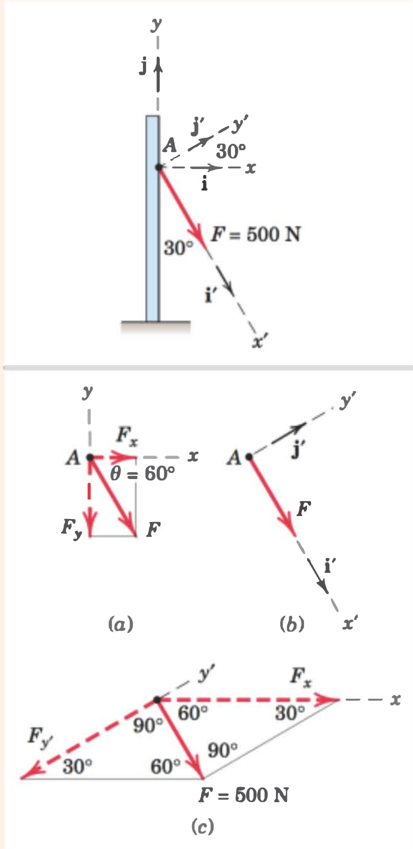
Helpful Hints

① Note the repositioning of **P** to permit parallelogram addition at B.

② Note the repositioning of **T** so as to preserve the correct line of action of the resultant **R**.

Sample Problem 2/3

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the x'- and y'-axes. (3) Determine the scalar components of **F** along the x- and y'-axes.



Solution. ***Part (1).*** From Fig. *a* we may write **F** as

$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

$$= (500\cos 60°)\mathbf{i} - (500\sin 60°)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.

Part (2). From Fig. *b* we may write **F** as $\mathbf{F} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

Part (3). The components of **F** in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,

① $$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000\text{ N}$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866\text{ N}$$

The required scalar components are then

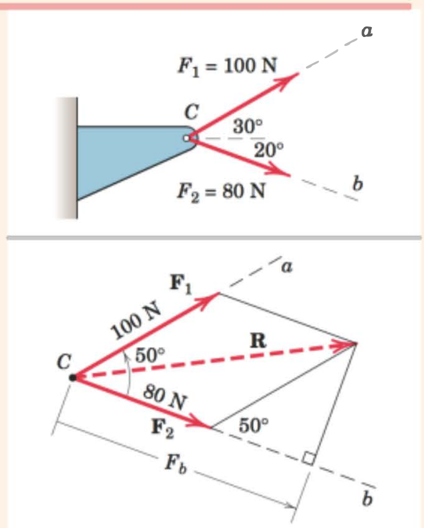
$$F_x = 1000\text{ N} \qquad F_{y'} = -866\text{ N} \qquad \textit{Ans.}$$

Helpful Hint

① Obtain F_x and $F_{y'}$ graphically and compare your results with the calculated values.

Sample Problem 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant **R** onto the b-axis.



Solution. The parallelogram addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130° \qquad R = 163.4\text{ N}$$

The figure also shows the orthogonal projection $\mathbf{F}_b$ of **R** onto the b-axis. Its length is

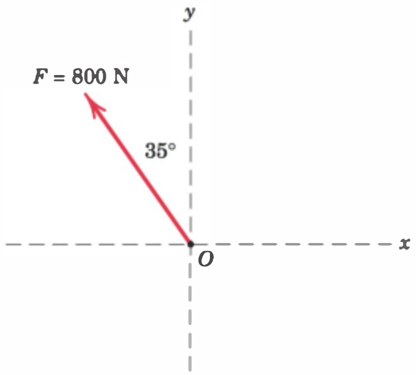
$$F_b = 80 + 100\cos 50° = 144.3\text{ N} \qquad \textit{Ans.}$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of **R** would have been equal.

PROBLEMS

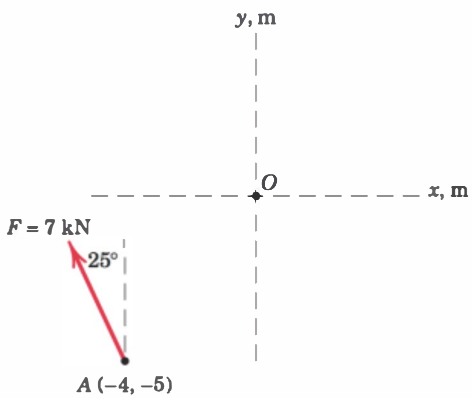
Introductory Problems

2/1 The force **F** has a magnitude of 800 N. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify the x and y scalar components of **F**.



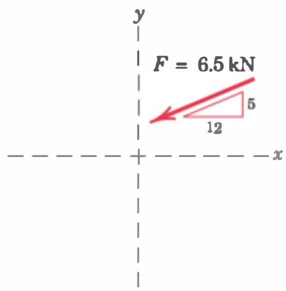
Problem 2/1

2/2 The force **F** has a magnitude of 7 kN and acts at the location indicated. Express **F** as a vector in terms of the unit vectors **i** and **j**. Next, determine the x and y scalar components of **F**.



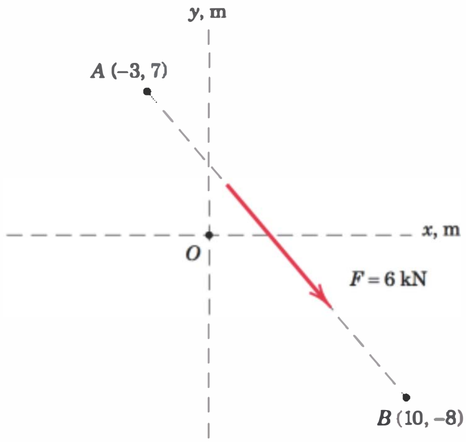
Problem 2/2

2/3 The slope of the 6.5-kN force **F** is specified as shown in the figure. Express **F** as a vector in terms of the unit vectors **i** and **j**.



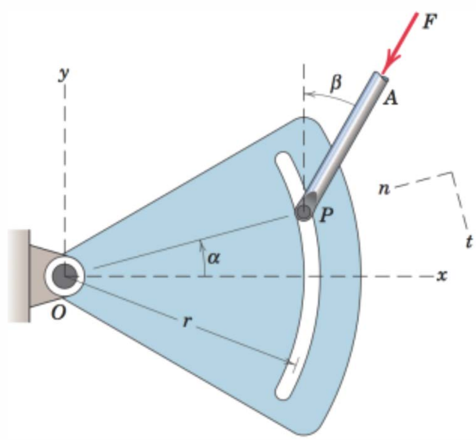
Problem 2/3

2/4 The force **F** has a magnitude of 6 kN and has the indicated line of action. Write the unit vector **n** associated with **F** and use **n** to determine the x and y scalar components of **F**.



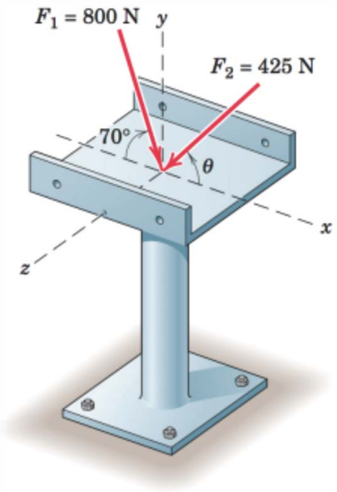
Problem 2/4

2/5 The control rod AP exerts a force **F** on the sector as shown. Determine both the x-y and the n-t components of the force.



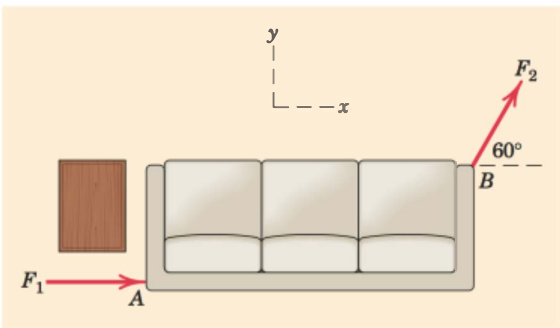
Problem 2/5

2/6 Two forces are applied to the construction bracket as shown. Determine the angle θ which makes the resultant of the two forces vertical. Determine the magnitude R of the resultant.



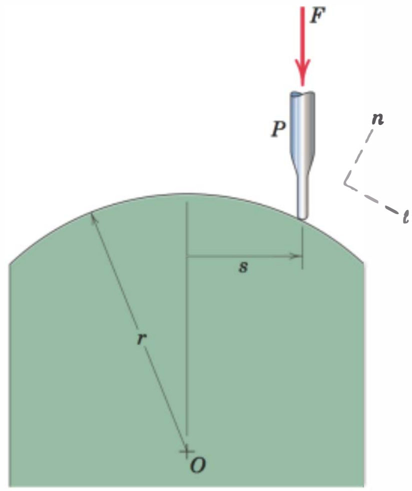
Problem 2/6

2/7 Two individuals are attempting to relocate a sofa by applying forces in the indicated directions. If $F_1 = 500$ N and $F_2 = 350$ N, determine the vector expression for the resultant $\mathbf{R}$ of the two forces. Then determine the magnitude of the resultant and the angle which it makes with the positive x-axis.



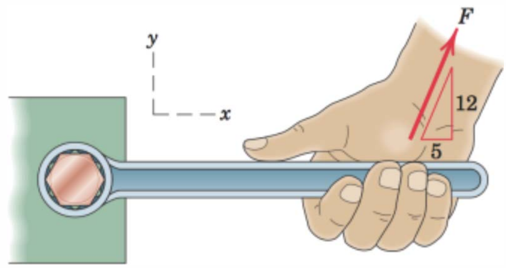
Problem 2/7

2/8 A small probe P is gently forced against the circular surface with a vertical force $\mathbf{F}$ as shown. Determine the n- and t-components of this force as functions of the horizontal position s.



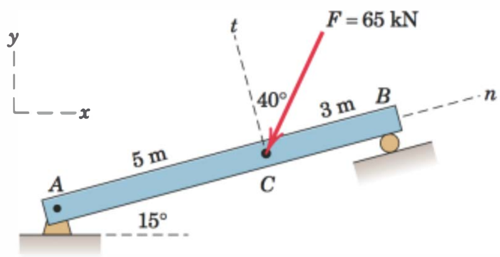
Problem 2/8

2/9 The y-component of the force $\mathbf{F}$ which a person exerts on the handle of the box wrench is known to be 320 N. Determine the x-component and the magnitude of $\mathbf{F}$.



Problem 2/9

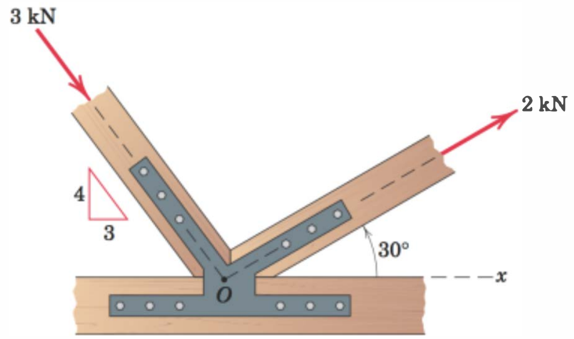
2/10 Determine the x-y and n-t components of the 65-kN force F acting on the simply-supported beam.



Problem 2/10

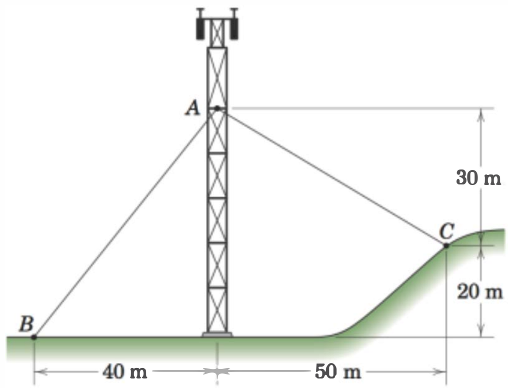
Representative Problems

2/11 The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.



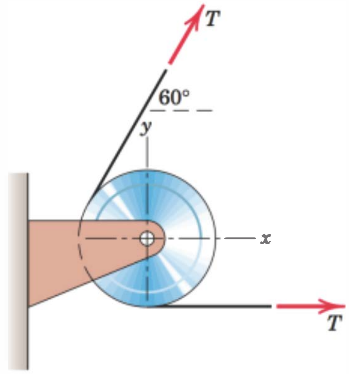
Problem 2/11

2/12 The guy cables AB and AC are attached to the top of the transmission tower. The tension in cable AB is 8 kN. Determine the required tension T in cable AC such that the net effect of the two cable tensions is a downward force at point A. Determine the magnitude R of this downward force.



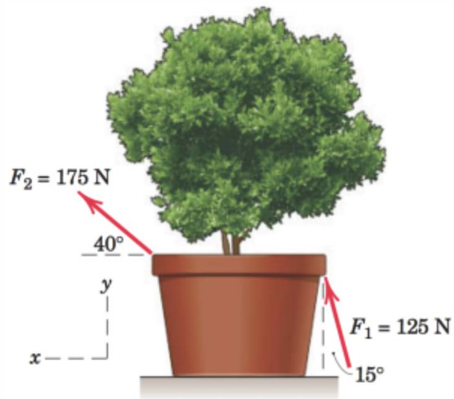
Problem 2/12

2/13 If the equal tensions T in the pulley cable are 400 N, express in vector notation the force $\mathbf{R}$ exerted on the pulley by the two tensions. Determine the magnitude of $\mathbf{R}$.



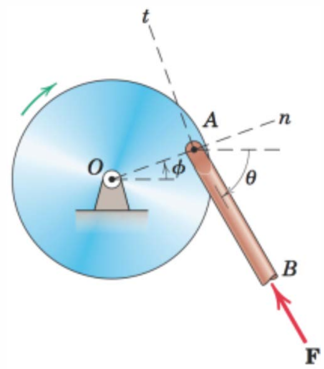
Problem 2/13

2/14 Two people exert the forces shown on the potted shrub. Determine the vector expression for the resultant $\mathbf{R}$ of the forces and determine the angle which the resultant makes with the positive y-axis.



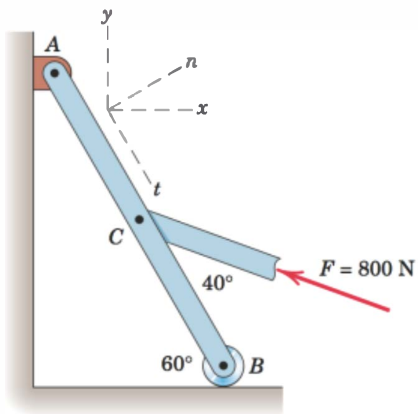
Problem 2/14

2/15 A compressive force $\mathbf{F}$ is transmitted via the coupler arm AB to disk OA. Develop the general expression for the n- and t-components of $\mathbf{F}$ as they act on the disk. Evaluate your expressions for (*a*) $F = 500$ N, $\theta = 60°$, $\phi = 20°$ and (*b*) $F = 800$ N, $\theta = 45°$, $\phi = 150°$.



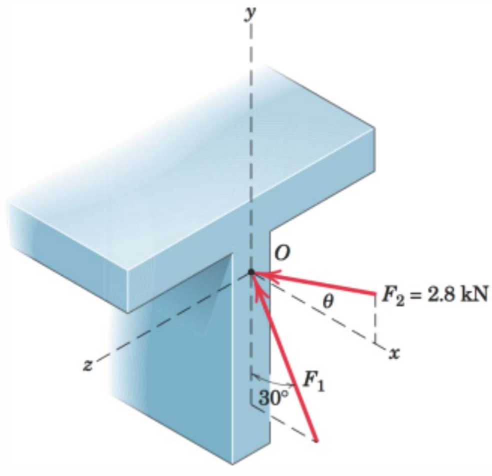
Problem 2/15

2/16 A force **F** of magnitude 800 N is applied to point C of the bar AB as shown. Determine both the x-y and the n-t components of **F**.



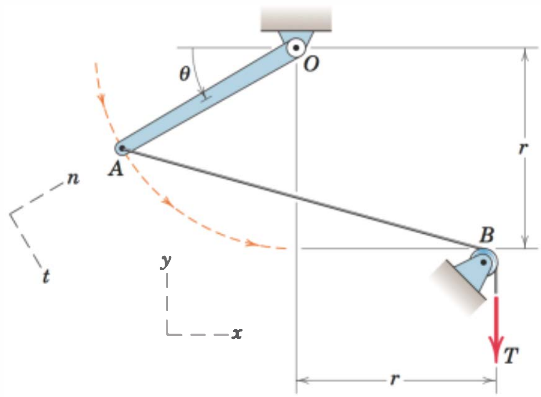
Problem 2/16

2/17 The two forces shown act in the x-y plane of the T-beam cross section. If it is known that the resultant **R** of the two forces has a magnitude of 3.5 kN and a line of action that lies 15° above the negative x-axis, determine the magnitude of $\mathbf{F}_1$ and the inclination θ of $\mathbf{F}_2$.



Problem 2/17

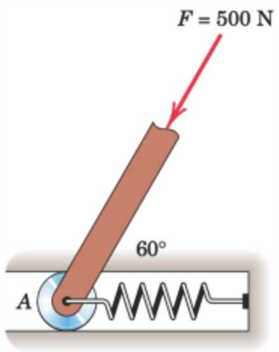
2/18 Determine the x- and y-components of the tension T which is applied to point A of the bar OA. Neglect the effects of the small pulley at B. Assume that r and θ are known.



Problem 2/18

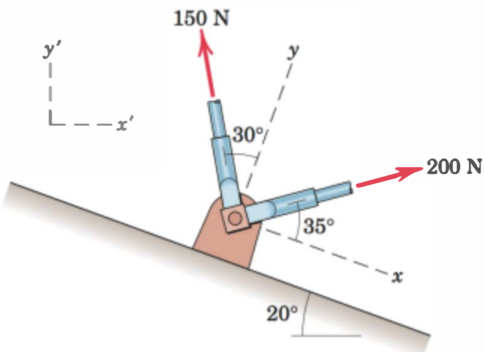
2/19 Refer to the mechanism of the previous problem. Develop general expressions for the n- and t-components of the tension T applied to point A. Then evaluate your expressions for $T = 100$ N and $\theta = 35°$.

2/20 Determine the magnitude F_s of the tensile spring force in order that the resultant of $\mathbf{F}_s$ and $\mathbf{F}$ is a vertical force. Determine the magnitude R of this vertical resultant force.



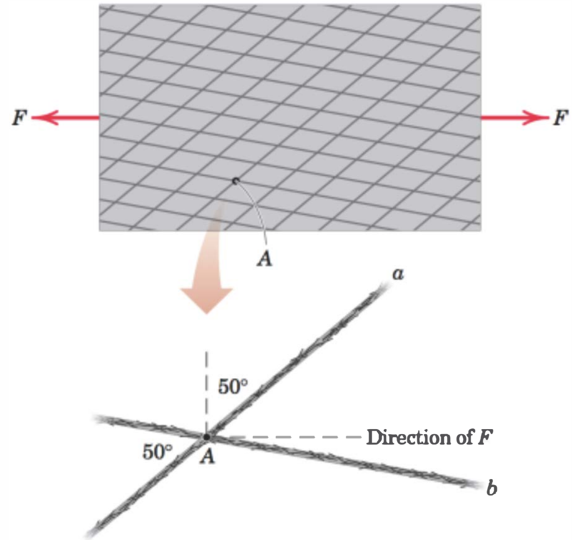
Problem 2/20

2/21 Determine the resultant $\mathbf{R}$ of the two forces applied to the bracket. Write $\mathbf{R}$ in terms of unit vectors along the x- and y-axes shown.



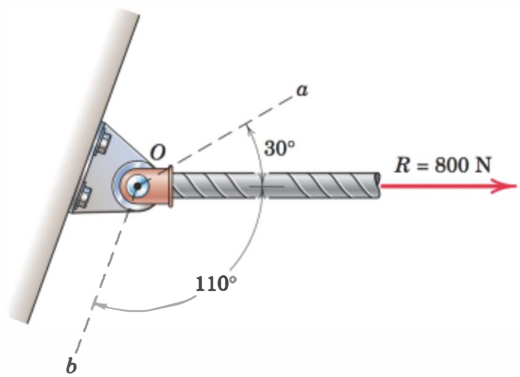
Problem 2/21

2/22 A sheet of an experimental composite is subjected to a simple tension test to determine its strength along a particular direction. The composite is reinforced by the Kevlar fibers shown, and a close-up showing the direction of the applied tension force $\mathbf{F}$ in relation to the fiber directions at point A is shown. If the magnitude of $\mathbf{F}$ is 2.5 kN, determine the components F_a and F_b of the force $\mathbf{F}$ along the oblique axes a and b. Also determine the projections P_a and P_b of $\mathbf{F}$ onto the a-b axes.



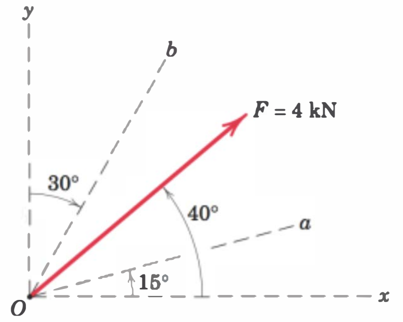
Problem 2/22

2/23 Determine the scalar components R_a and R_b of the force $\mathbf{R}$ along the nonrectangular axes a and b. Also determine the orthogonal projection P_a of $\mathbf{R}$ onto axis a.



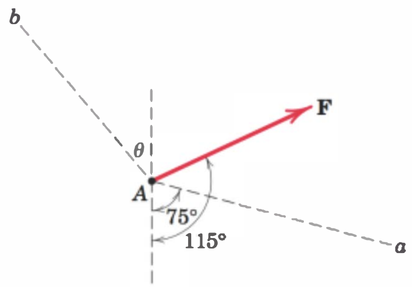
Problem 2/23

2/24 Determine the components F_a and F_b of the 4-kN force along the oblique axes a and b. Determine the projections P_a and P_b of $\mathbf{F}$ onto the a- and b-axes.



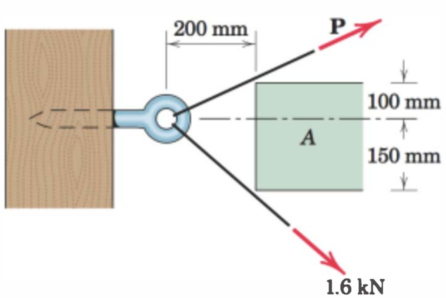
Problem 2/24

2/25 If the projection P_a and component F_b of the force **F** along oblique axes a and b are both 325 N, determine the magnitude F and the orientation θ of the b-axis.



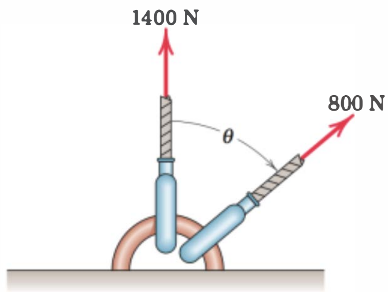
Problem 2/25

2/26 It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction A prevents direct access, so that two forces, one 1.6 kN and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find T.



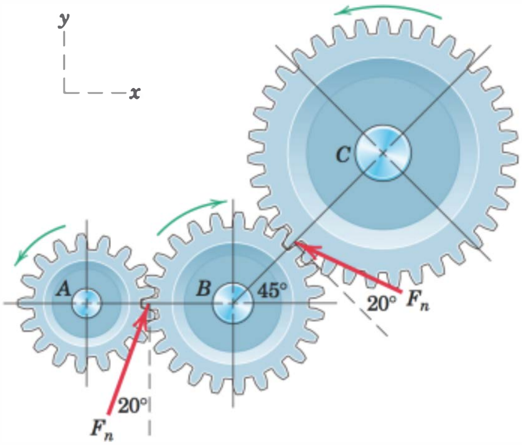
Problem 2/26

2/27 At what angle θ must the 800-N force be applied in order that the resultant **R** of the two forces have a magnitude of 2000 N? For this condition, determine the angle β between **R** and the vertical.



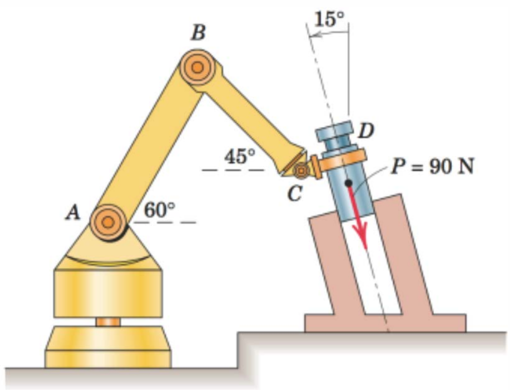
Problem 2/27

2/28 Power is to be transferred from the pinion A to the output gear C inside a mechanical drive. Because of output motion requirements and space limitations, an idler gear B is introduced as shown. A force analysis has determined that the total contact force between each pair of meshing teeth has a magnitude $F_n = 5500$ N, and these forces are shown acting on idler gear B. Determine the magnitude of the resultant **R** of the two contact forces acting on the idler gear. Complete both a graphical and a vector solution.



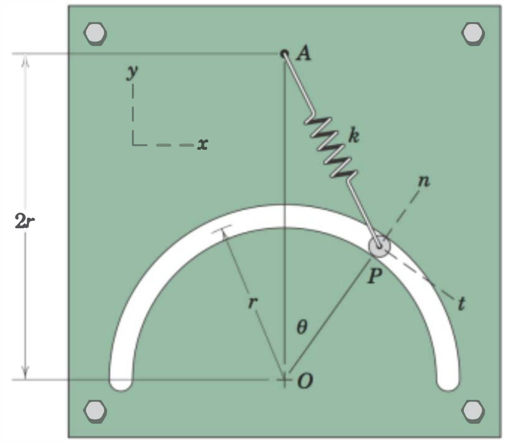
Problem 2/28

2/29 To insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force P on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts *on* the robot along axes (*a*) parallel and perpendicular to the arm AB, and (*b*) parallel and perpendicular to the arm BC.



Problem 2/29

▶2/30 The unstretched length of the spring is r. When pin P is in an arbitrary position θ, determine the x- and y-components of the force which the spring exerts on the pin. Evaluate your general expressions for $r = 400$ mm, $k = 1.4$ kN/m, and $\theta = 40°$. (*Note:* The force in a spring is given by $F = k\delta$, where δ is the extension from the unstretched length.)



Problem 2/30

2/4 Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.

As a familiar example of the concept of moment, consider the pipe wrench of Fig. 2/8*a*. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

Moment about a Point

Figure 2/8*b* shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm* d, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$\boxed{M = Fd} \qquad \textbf{(2/5)}$$

The moment is a vector **M** perpendicular to the plane of the body. The sense of **M** depends on the direction in which **F** tends to rotate the body. The right-hand rule, Fig. 2/8*c*, is used to identify this sense. We represent the moment of **F** about O-O as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.

The moment **M** obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters (N·m), and in the U.S. customary system are pound-feet (lb-ft).

When dealing with forces which all act in a given plane, we customarily speak of the moment *about a point*. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force **F** about point A in Fig. 2/8*d* has the magnitude $M = Fd$ and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (−) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig. 2/8*d*, the moment of **F** about point A (or about the z-axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

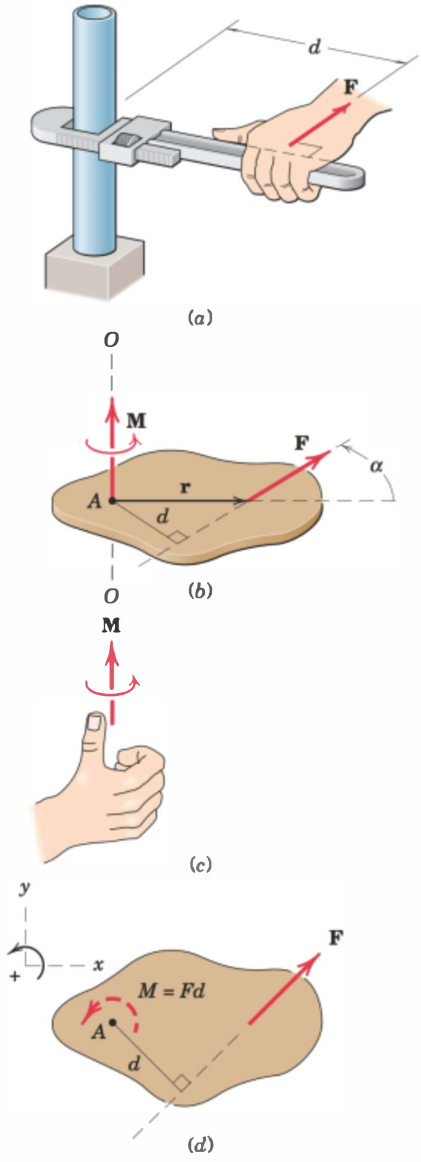


Figure 2/8

The Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of **F** about point *A* of Fig. 2/8*b* may be represented by the cross-product expression

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}} \tag{2/6}$$

where **r** is a position vector which runs from the moment reference point *A* to *any* point on the line of action of **F**. The magnitude of this expression is given by*

$$M = Fr \sin \alpha = Fd \tag{2/7}$$

which agrees with the moment magnitude as given by Eq. 2/5. Note that the moment arm $d = r \sin \alpha$ does not depend on the particular point on the line of action of **F** to which the vector **r** is directed. We establish the direction and sense of **M** by applying the right-hand rule to the sequence $\mathbf{r} \times \mathbf{F}$. If the fingers of the right hand are curled in the direction of rotation from the positive sense of **r** to the positive sense of **F**, then the thumb points in the positive sense of **M**.

We must maintain the sequence $\mathbf{r} \times \mathbf{F}$, because the sequence $\mathbf{F} \times \mathbf{r}$ would produce a vector with a sense opposite to that of the correct moment. As was the case with the scalar approach, the moment **M** may be thought of as the moment about point *A* or as the moment about the line *O-O* which passes through point *A* and is perpendicular to the plane containing the vectors **r** and **F**. When we evaluate the moment of a force about a given point, the choice between using the vector cross product or the scalar expression depends on how the geometry of the problem is specified. If we know or can easily determine the perpendicular distance between the line of action of the force and the moment center, then the scalar approach is generally simpler. If, however, **F** and **r** are not perpendicular and are easily expressible in vector notation, then the cross-product expression is often preferable.

In Section B of this chapter, we will see how the vector formulation of the moment of a force is especially useful for determining the moment of a force about a point in three-dimensional situations.

Varignon's Theorem

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

*See item 7 in Art. C/7 of Appendix C for additional information concerning the cross product.

To prove this theorem, consider the force **R** acting in the plane of the body shown in Fig. 2/9*a*. The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point *O* is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$, we may write

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

Using the distributive law for cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q} \qquad \textbf{(2/8)}$$

which says that the moment of **R** about *O* equals the sum of the moments about *O* of its components **P** and **Q**. This proves the theorem.

Varignon's theorem need not be restricted to the case of two components, but it applies equally well to three or more. Thus we could have used any number of concurrent components of **R** in the foregoing proof.*

Figure 2/9*b* illustrates the usefulness of Varignon's theorem. The moment of **R** about point *O* is *Rd*. However, if *d* is more difficult to determine than *p* and *q*, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

where we take the clockwise moment sense to be positive.

Sample Problem 2/5 shows how Varignon's theorem can help us to calculate moments.

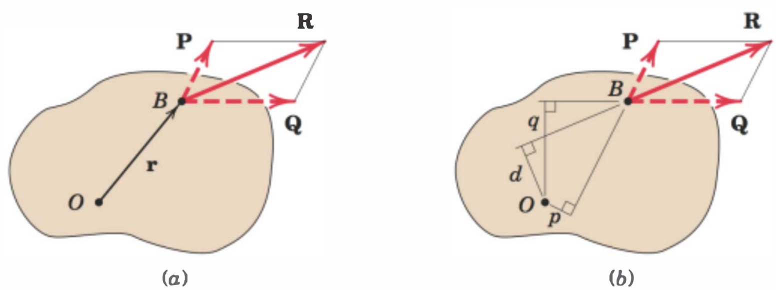
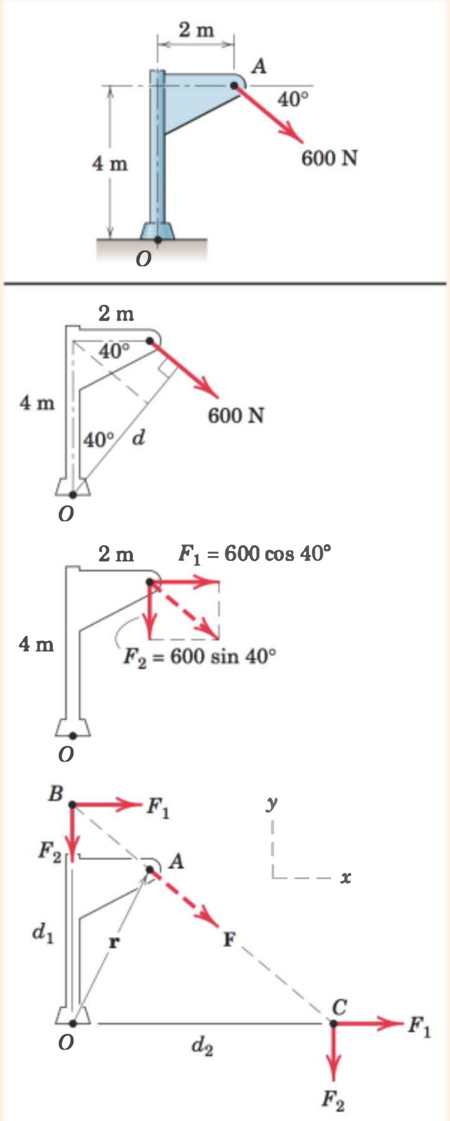


Figure 2/9

*As originally stated, Varignon's theorem was limited to the case of two concurrent components of a given force. See *The Science of Mechanics*, by Ernst Mach, originally published in 1883.

Sample Problem 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.



Solution. ***(I)*** The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

① By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

② $$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

③ ***(IV)*** Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

④ $$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°)$$

$$= -2610\mathbf{k} \text{ N}\cdot\text{m}$$

The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

$$M_O = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Helpful Hints

① The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.

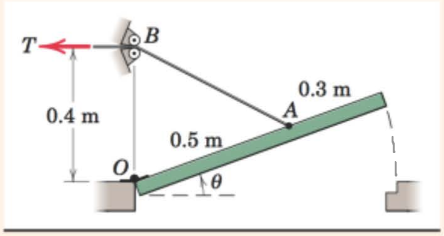
② This procedure is frequently the shortest approach.

③ The fact that points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.

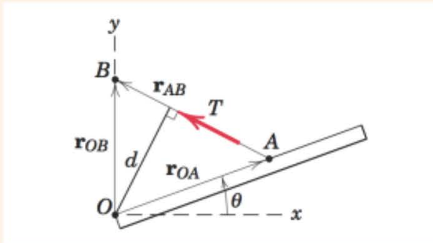
④ Alternative choices for the position vector $\mathbf{r}$ are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

Sample Problem 2/6

The trap door OA is raised by the cable AB, which passes over the small frictionless guide pulleys at B. The tension everywhere in the cable is T, and this tension applied at A causes a moment M_O about the hinge at O. Plot the quantity M_O/T as a function of the door elevation angle θ over the range $0 \le \theta \le 90°$ and note minimum and maximum values. What is the physical significance of this ratio?



Solution. We begin by constructing a figure which shows the tension force **T** acting directly on the door, which is shown in an arbitrary angular position θ. It should be clear that the direction of **T** will vary as θ varies. In order to deal with this variation, we write a unit vector $\mathbf{n}_{AB}$ which "aims" **T**:



① $$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\mathbf{r}_{OB} - \mathbf{r}_{OA}}{r_{AB}}$$

Using the x-y coordinates of our figure, we can write

② $$\mathbf{r}_{OB} = 0.4\mathbf{j} \text{ m and } \mathbf{r}_{OA} = 0.5(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \text{ m}$$

So

$$\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} = 0.4\mathbf{j} - (0.5)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$$
$$= -0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j} \text{ m}$$

and

$$r_{AB} = \sqrt{(0.5\cos\theta)^2 + (0.4 - 0.5\sin\theta)^2}$$
$$= \sqrt{0.41 - 0.4\sin\theta} \text{ m}$$

The desired unit vector is

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}$$

Our tension vector can now be written as

$$\mathbf{T} = T\mathbf{n}_{AB} = T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$

③ The moment of **T** about point O, as a vector, is $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}$, where $\mathbf{r}_{OB} = 0.4\mathbf{j}$ m, or

$$\mathbf{M}_O = 0.4\mathbf{j} \times T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$
$$= \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}\mathbf{k}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}$$

and the requested ratio is

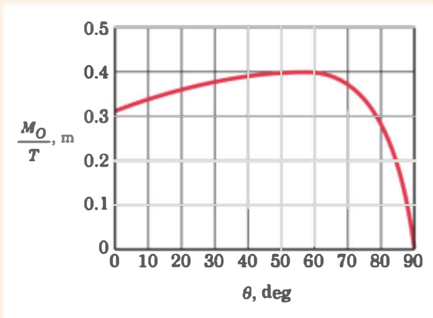
$$\frac{M_O}{T} = \frac{0.2\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}} \qquad \textit{Ans.}$$

which is plotted in the accompanying graph. The expression M_O/T is the moment arm d (in meters) which runs from O to the line of action of **T**. It has a maximum value of 0.4 m at $\theta = 53.1°$ (at which point **T** is horizontal) and a minimum value of 0 at $\theta = 90°$ (at which point **T** is vertical). The expression is valid even if T varies.

This sample problem treats moments in two-dimensional force systems, and it also points out the advantages of carrying out a solution for an arbitrary position, so that behavior over a range of positions can be examined.

Helpful Hints

① Recall that any unit vector can be written as a vector divided by its magnitude. In this case the vector in the numerator is a position vector.



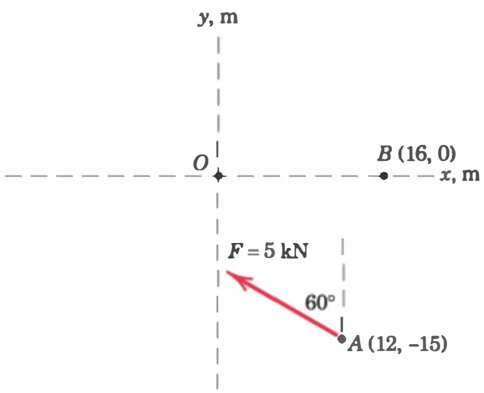
② Recall that any vector may be written as a magnitude times an "aiming" unit vector.

③ In the expression $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, the position vector $\mathbf{r}$ runs *from* the moment center *to* any point on the line of action of **F**. Here, $\mathbf{r}_{OB}$ is more convenient than $\mathbf{r}_{OA}$.

PROBLEMS

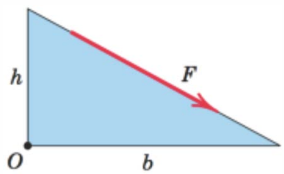
Introductory Problems

2/31 Determine the moments of the 5-kN force about point O and about point B.



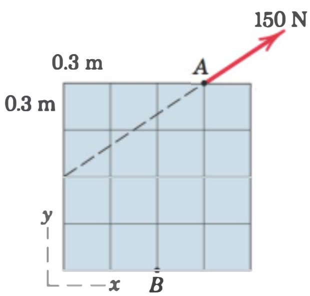
Problem 2/31

2/32 The force of magnitude F acts along the edge of the triangular plate. Determine the moment of **F** about point O.



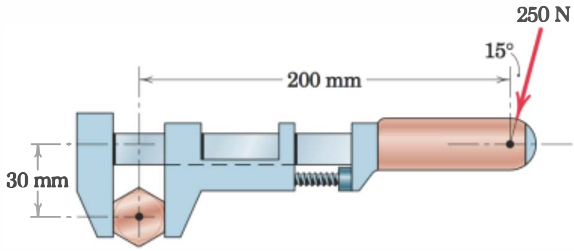
Problem 2/32

2/33 The rectangular plate is made up of 0.3-m squares as shown. A 150-N force is applied at point A in the direction shown. Calculate the moment M_B of the force about point B by at least two different methods.



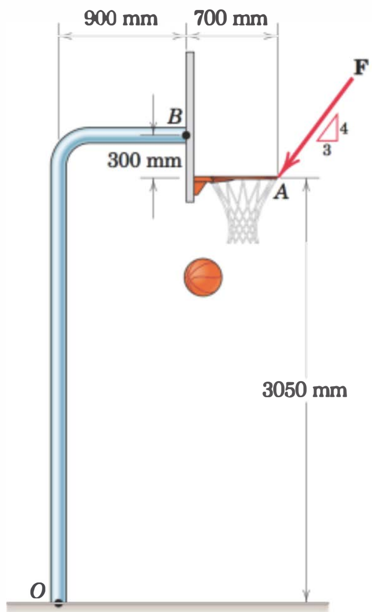
Problem 2/33

2/34 Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



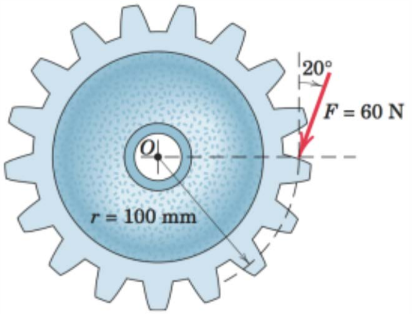
Problem 2/34

2/35 An experimental device imparts a force of magnitude $F = 225$ N to the front edge of the rim at A to simulate the effect of a slam dunk. Determine the moments of the force F about point O and about point B. Finally, locate, from the base at O, a point C on the ground where the force imparts zero moment.



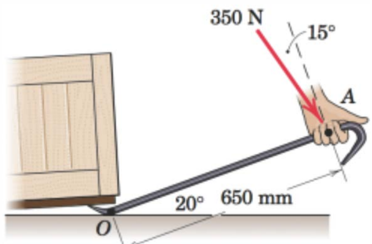
Problem 2/35

2/36 A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.



Problem 2/36

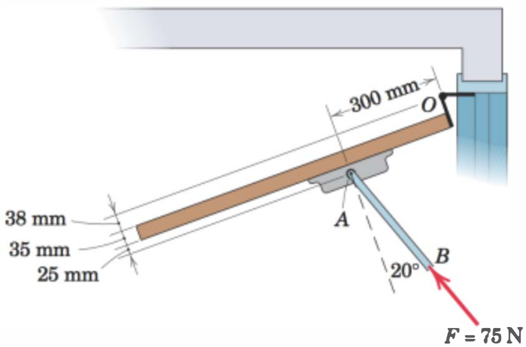
2/37 A man uses a crowbar to lift the corner of a hot tub for maintenance purposes. Determine the moment made by the 350-N force about point O. Neglect the small thickness of the crowbar.



Problem 2/37

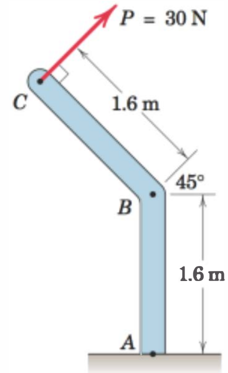
Representative Problems

2/38 An overhead view of a door is shown. If the compressive force F acting in the coupler arm of the hydraulic door closer is 75 N with the orientation shown, determine the moment of this force about the hinge axis O.



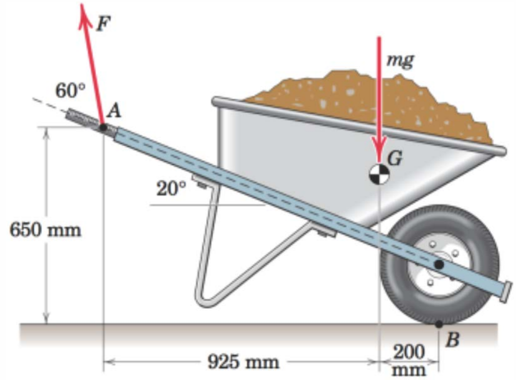
Problem 2/38

2/39 The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.



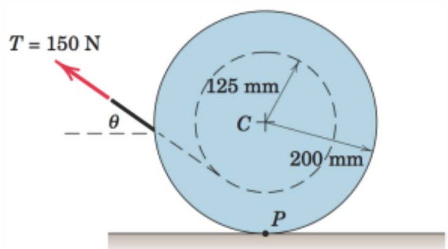
Problem 2/39

2/40 A man exerts a force F on the handle of the stationary wheelbarrow at A. The mass of the wheelbarrow along with its load of dirt is 85 kg with center of mass at G. For the configuration shown, what force F must the man apply at A to make the net moment about the tire contact point B equal to zero?



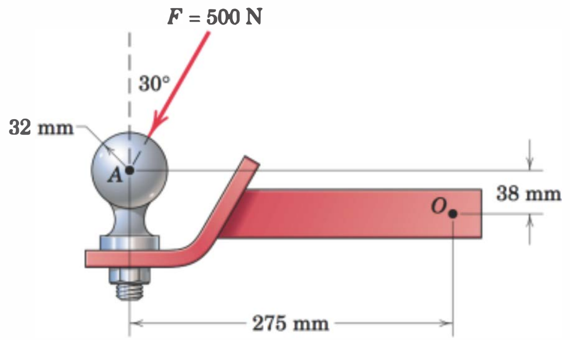
Problem 2/40

2/41 A 150-N pull T is applied to a cord, which is wound securely around the inner hub of the drum. Determine the moment of T about the drum center C. At what angle θ should T be applied so that the moment about the contact point P is zero?



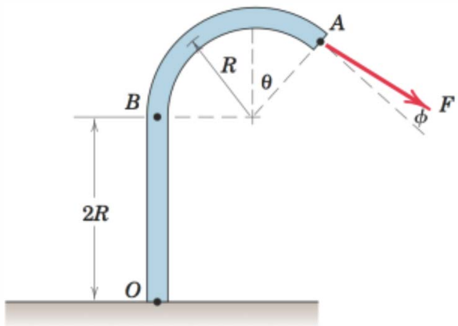
Problem 2/41

2/42 As a trailer is towed in the forward direction, the force $F = 500$ N is applied as shown to the ball of the trailer hitch. Determine the moment of this force about point O.



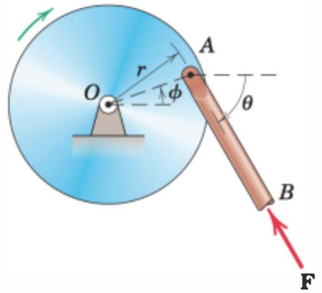
Problem 2/42

2/43 Determine the general expressions for the moments of F about (*a*) point B and (*b*) point O. Evaluate your expressions for $F = 750$ N, $R = 2.4$ m, $\theta = 30°$, and $\phi = 15°$.



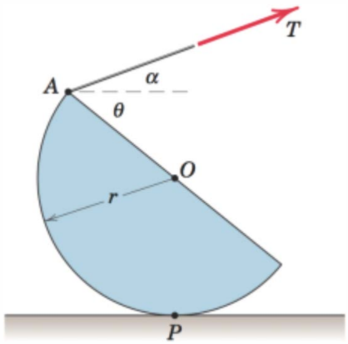
Problem 2/43

2/44 The mechanism of Prob. 2/15 is repeated here. Develop a general expression for the moment M_O of the force acting on the coupler arm AB about the center O of the disk. Evaluate your expression for (*a*) $F = 500$ N, $\theta = 60°$, $\phi = 20°$, and (*b*) $F = 800$ N, $\theta = 45°$, $\phi = 150°$. Assume a value of $r = 0.4$ m for both cases.



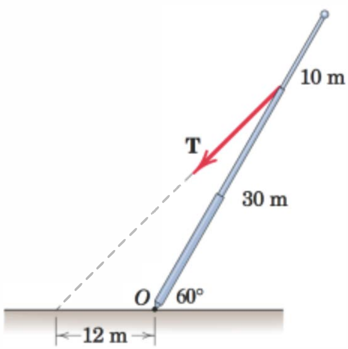
Problem 2/44

2/45 Determine the moments of the tension T about point P and about point O.



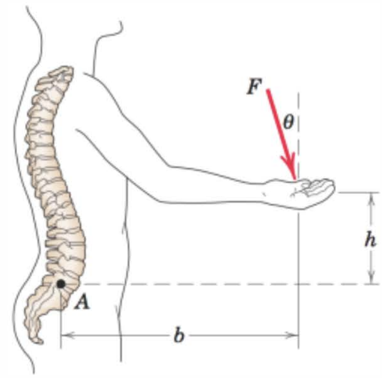
Problem 2/45

2/46 In raising the pole from the position shown, the tension T in the cable must supply a moment about O of 72 kN·m. Determine T.



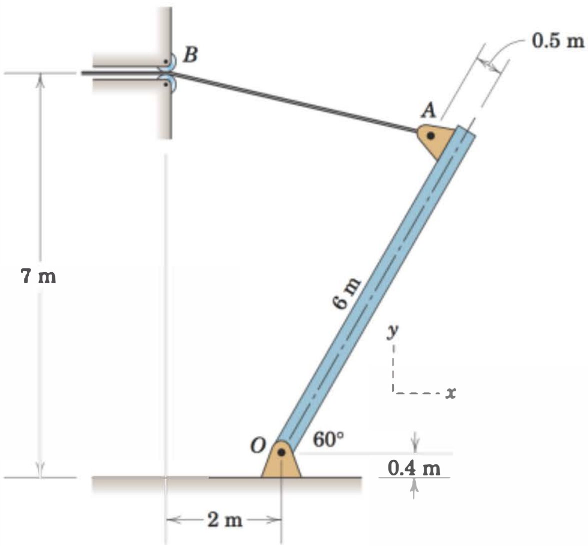
Problem 2/46

2/47 The lower lumbar region A of the spine is the part of the spinal column most susceptible to abuse while resisting excessive bending caused by the moment about A of a force F. For given values of F, b, and h, determine the angle θ which causes the most severe bending strain.



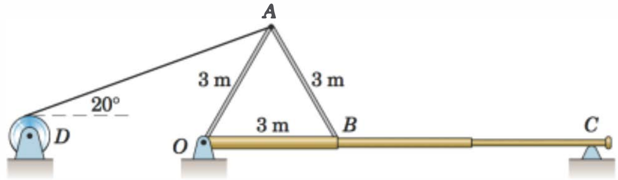
Problem 2/47

2/48 A gate is held in the position shown by cable AB. If the tension in the cable is 6.75 kN, determine the moment $\mathbf{M}_O$ of the tension (as applied to point A) about the pivot point O of the gate.



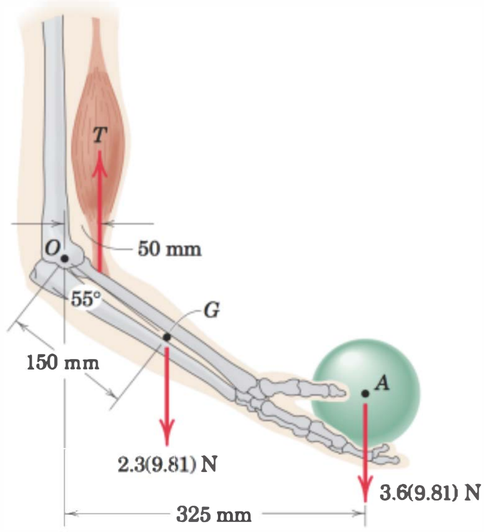
Problem 2/48

2/49 In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 3.2 kN is developed in the hoisting cable by the power winch D. Calculate the moment M_O of this tension about the hinge point O.



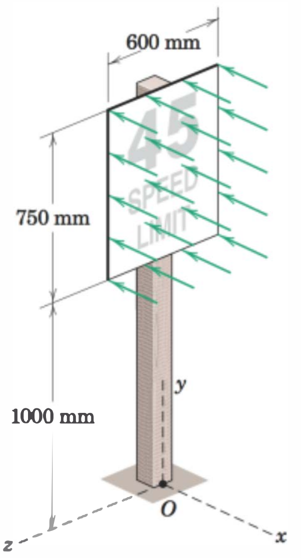
Problem 2/49

2/50 Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with center of mass at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the sphere. What must the biceps tension force be so that the overall moment about O is zero?



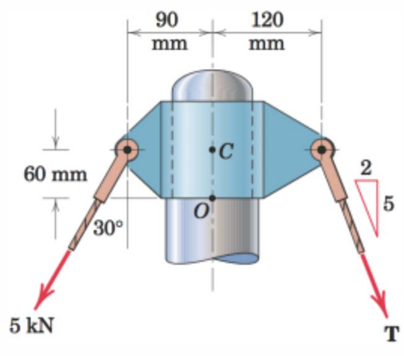
Problem 2/50

2/51 As the result of a wind blowing normal to the plane of the rectangular sign, a uniform pressure of 175 N/m² is exerted in the direction shown in the figure. Determine the moment of the resulting force about point O. Express your result as a vector using the coordinates shown.



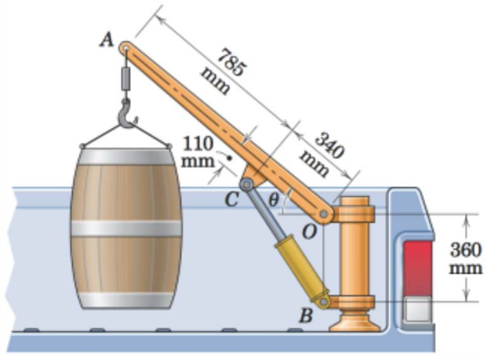
Problem 2/51

2/52 The masthead fitting supports the two forces shown. Determine the magnitude of $\mathbf{T}$ which will cause no bending of the mast (zero moment) at point O.



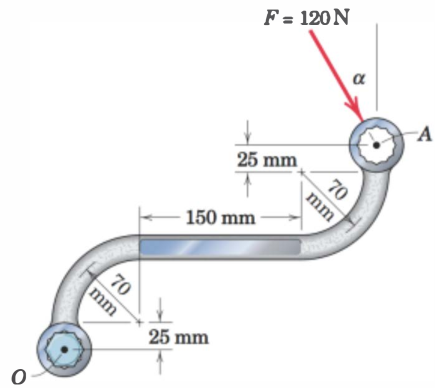
Problem 2/52

2/53 The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is $\theta = 40°$, the force in the hydraulic cylinder BC is 4.5 kN, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



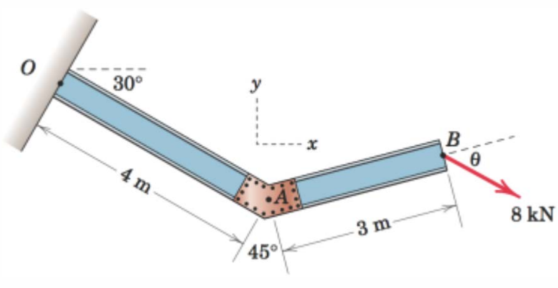
Problem 2/53

2/54 The 120-N force is applied as shown to one end of the curved wrench. If $\alpha = 30°$, calculate the moment of F about the center O of the bolt. Determine the value of α which would maximize the moment about O; state the value of this maximum moment.



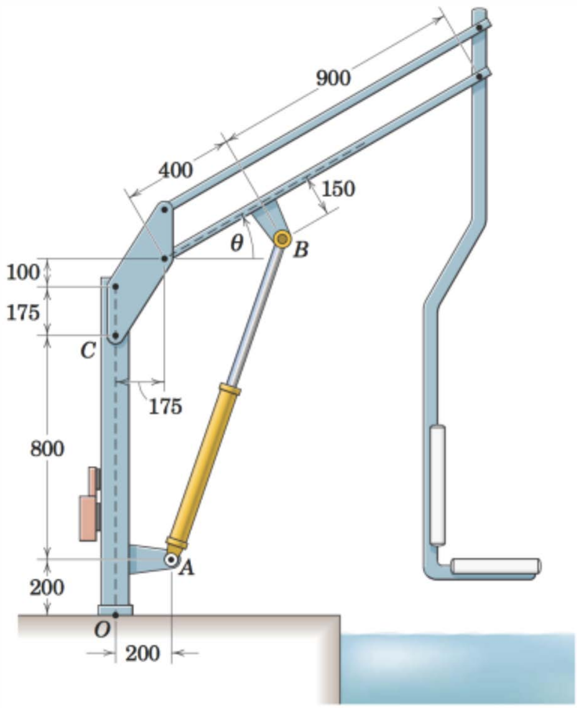
Problem 2/54

2/55 The bent cantilever beam is acted upon by an 8-kN force at B. If the angle $\theta = 35°$, determine (*a*) the moment $\mathbf{M}_O$ of the force about point O and (*b*) the moment $\mathbf{M}_A$ of the force about point A. What value(s) of θ ($0 < \theta < 360°$) will result in the maximum possible moment about point O, and what is the magnitude of the moment at those orientations?



Problem 2/55

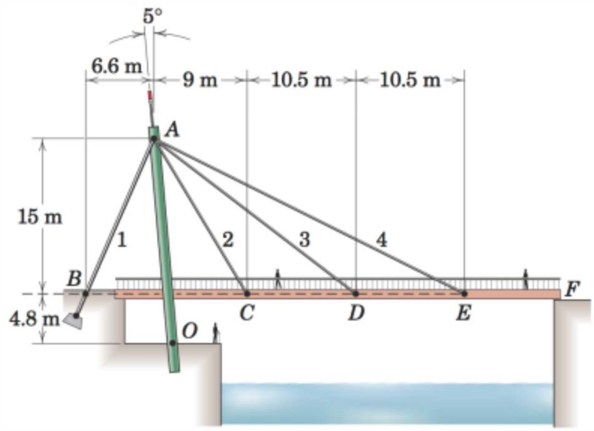
2/56 The mechanism shown is used to lower disabled persons into a whirlpool tub for therapeutic treatment. In the unloaded configuration, the weight of the boom and hanging chair induces a compressive force of 575 N in hydraulic cylinder AB. (Compressive means that the force which cylinder AB exerts on point B is directed from A toward B.) If $\theta = 30°$, determine the moment of this cylinder force acting on pin B about (a) point O and (b) point C.



Dimensions in millimeters

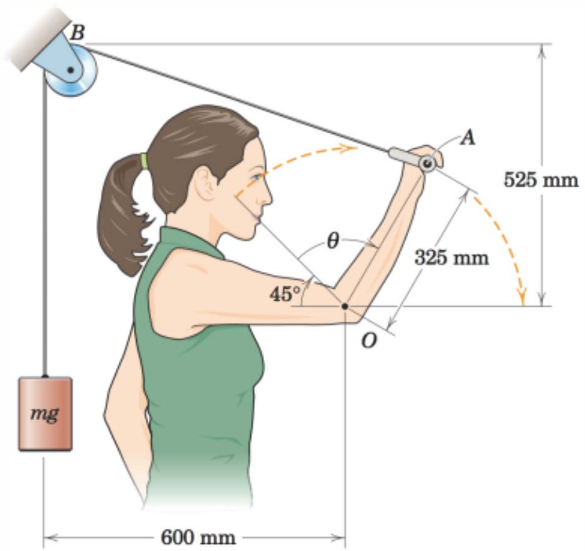
Problem 2/56

2/57 The asymmetrical support arrangement is chosen for a pedestrian bridge because conditions at the right end F do not permit a support tower and anchorages. During a test, the tensions in cables 2, 3, and 4 are all adjusted to the same value T. If the combined moment of all four cable tensions about point O is to be zero, what should be the value T_1 of the tension in cable 1? Determine the corresponding value of the compression force P at O resulting from the four tensions applied at A. Neglect the weight of the tower.



Problem 2/57

***2/58** The woman maintains a slow steady motion over the indicated 135° range as she exercises her triceps muscle. For this condition, the tension in the cable can be assumed to be constant at $mg = 50$ N. Determine and plot the moment M of the cable tension as applied at A about the elbow joint O over the range $0 \leq \theta \leq 135°$. Find the maximum value of M and the value of θ for which it occurs.



Problem 2/58

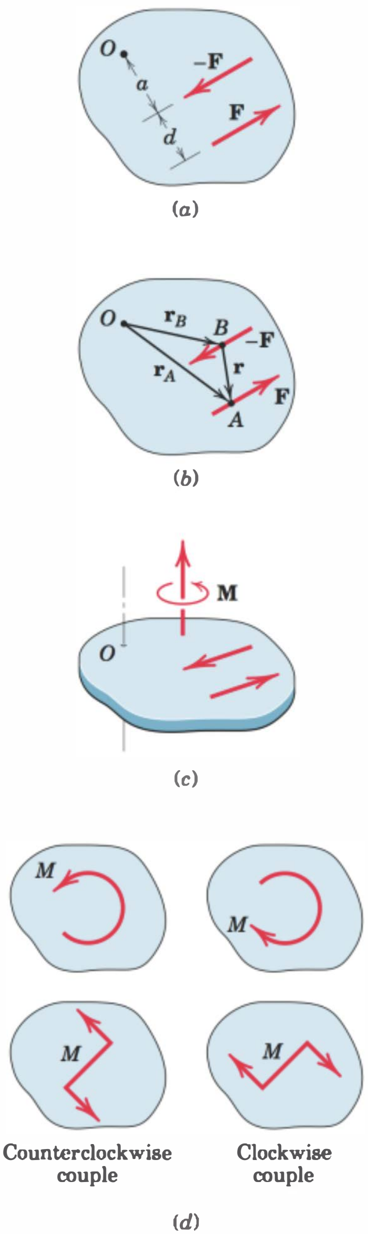


Figure 2/10

2/5 Couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces **F** and −**F** a distance d apart, as shown in Fig. 2/10*a*. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O. It follows that the moment of a couple has the same value for all moment centers.

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross-product notation of Eq. 2/6, the combined moment about point O of the forces forming the couple of Fig. 2/10*b* is

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

where $\mathbf{r}_A$ and $\mathbf{r}_B$ are position vectors which run from point O to arbitrary points A and B on the lines of action of **F** and −**F**, respectively. Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, we can express **M** as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Here again, the moment expression contains no reference to the moment center O and, therefore, is the same for all moment centers. Thus, we may represent **M** by a free vector, as shown in Fig. 2/10*c*, where the direction of **M** is normal to the plane of the couple and the sense of **M** is established by the right-hand rule.

Because the couple vector **M** is always perpendicular to the plane of the forces which constitute the couple, in two-dimensional analysis we can represent the sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Fig. 2/10*d*. Later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2/11

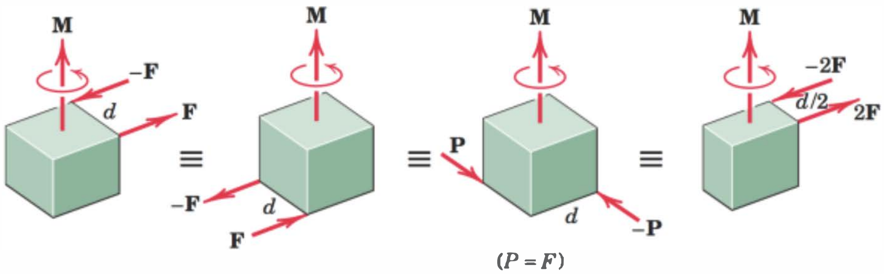


Figure 2/11

shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 2/12, where the given force **F** acting at point A is replaced by an equal force **F** at some point B and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces **F** and $-$**F** are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. 2/12 is referred to as a *force–couple system*.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force–couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

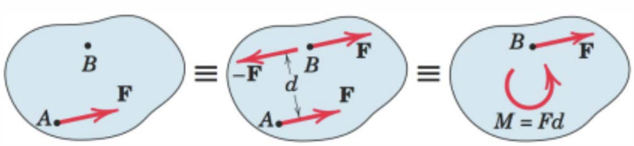
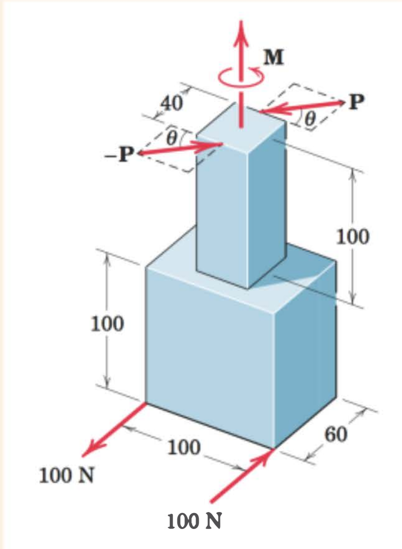


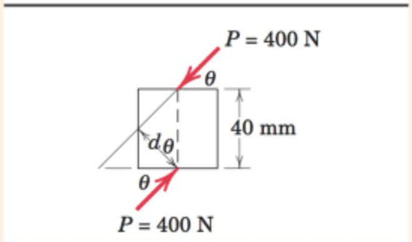
Figure 2/12

Sample Problem 2/7

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters



Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10 \text{ N}\cdot\text{m}$

The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040) \cos \theta$$

① Equating the two expressions gives

$$10 = (400)(0.040) \cos \theta$$

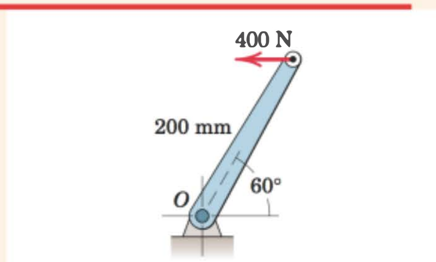
$$\theta = \cos^{-1} \frac{10}{16} = 51.3° \qquad \textit{Ans.}$$

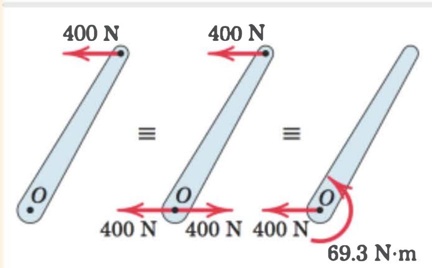
Helpful Hint

① Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

Sample Problem 2/8

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.





Solution. We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

$[M = Fd]$ $\qquad M = 400(0.200 \sin 60°) = 69.3 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$

① Thus, the original force is equivalent to the 400-N force at O and the 69.3-N·m couple as shown in the third of the three equivalent figures.

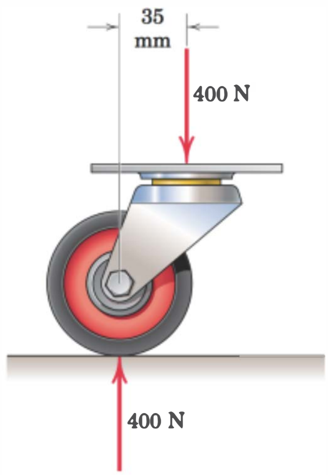
Helpful Hint

① The reverse of this problem is often encountered, namely, the replacement of a force and a couple by a single force. Proceeding in reverse is the same as replacing the couple by two forces, one of which is equal and opposite to the 400-N force at O. The moment arm to the second force would be $M/F = 69.3/400 = 0.1732$ m, which is 0.2 sin 60°, thus determining the line of action of the single resultant force of 400 N.

PROBLEMS

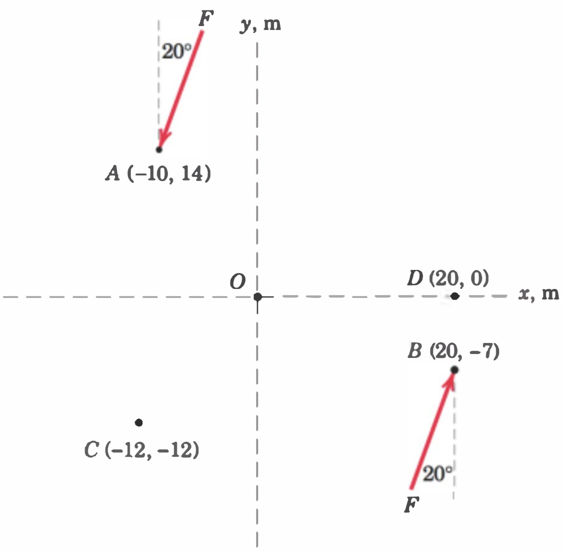
Introductory Problems

2/59 The caster unit is subjected to the pair of 400-N forces shown. Determine the moment associated with these forces.



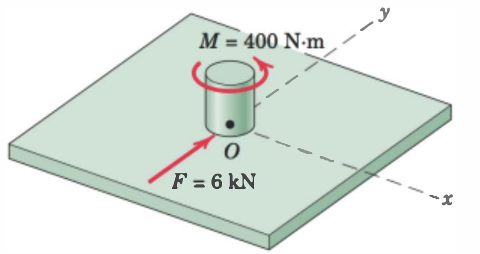
Problem 2/59

2/60 For $F = 300$ N, compute the combined moment of the two forces about (*a*) point O, (*b*) point C, and (*c*) point D.



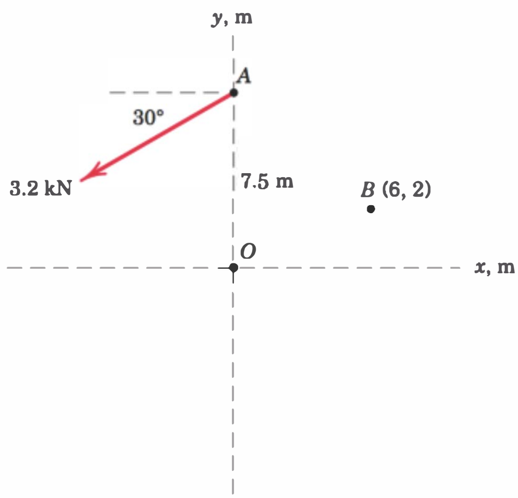
Problem 2/60

2/61 The indicated force–couple system is applied to a small shaft at the center of the plate. Replace this system by a single force and specify the coordinate of the point on the x-axis through which the line of action of this resultant force passes.



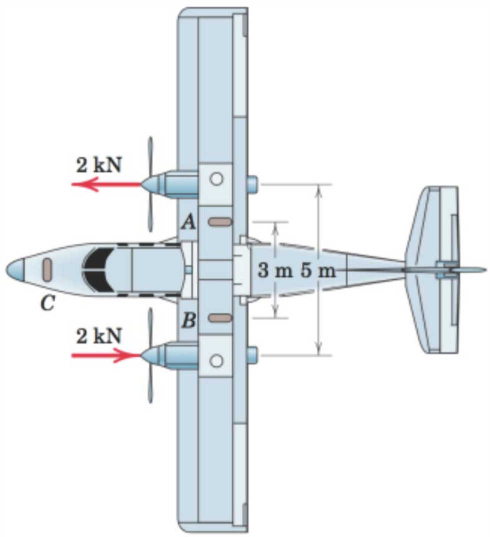
Problem 2/61

2/62 Replace the 3.2-kN force by an equivalent force–couple system at (*a*) point O and (*b*) point B. Record your answers in vector format.



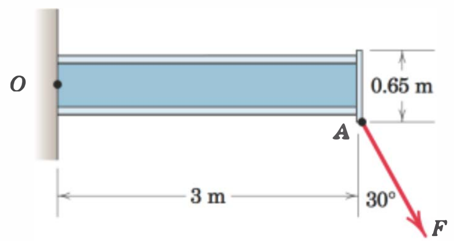
Problem 2/62

2/63 As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel C, which is turned 90° and unbraked.



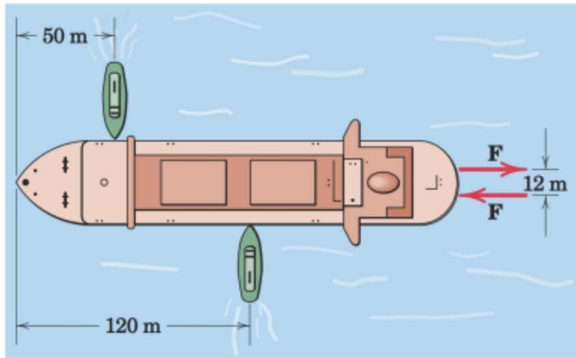
Problem 2/63

2/64 The cantilevered W530 × 150 beam shown is subjected to an 8-kN force F applied by means of a welded plate at A. Determine the equivalent force–couple system at the centroid of the beam cross section at the cantilever O.



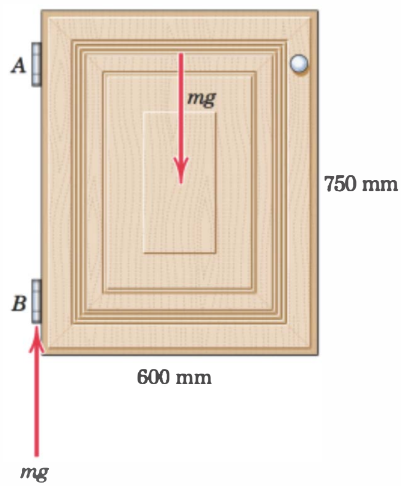
Problem 2/64

2/65 Each propeller of the twin-screw ship develops a full-speed thrust of 300 kN. In maneuvering the ship, one propeller is turning full speed ahead and the other full speed in reverse. What thrust P must each tug exert on the ship to counteract the effect of the ship's propellers?



Problem 2/65

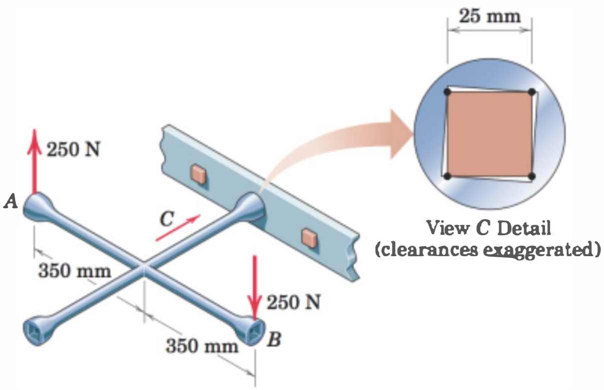
2/66 The upper hinge A of the uniform cabinet door has malfunctioned, causing the entire weight mg of the 5-kg door to be carried by the lower hinge B. Determine the couple associated with these two forces. You may neglect the slight offset from the edge of the cabinet door to the hinge centerline.



Problem 2/66

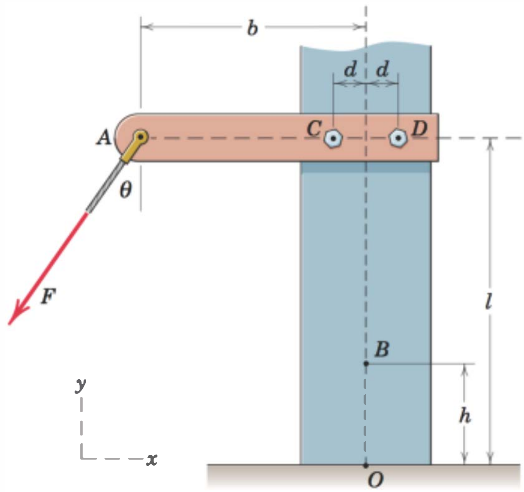
Representative Problems

2/67 A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.



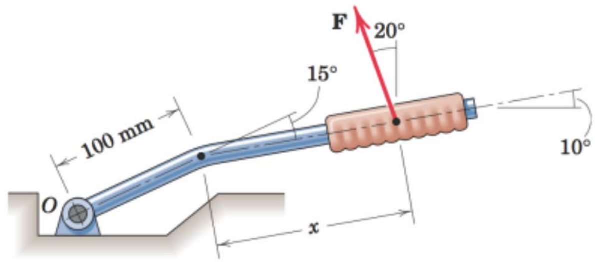
Problem 2/67

2/68 The force F is applied at the end of arm ACD, which is mounted to a vertical post. Replace this single force F by an equivalent force–couple system at B. Next, redistribute this force and couple by replacing it with two forces acting in the same direction as F, one at C and the other at D, and determine the forces supported by the two hex-bolts. Use values of $F = 425$ N, $\theta = 30°$, $b = 1.9$ m, $d = 0.2$ m, $h = 0.8$ m, and $l = 2.75$ m.



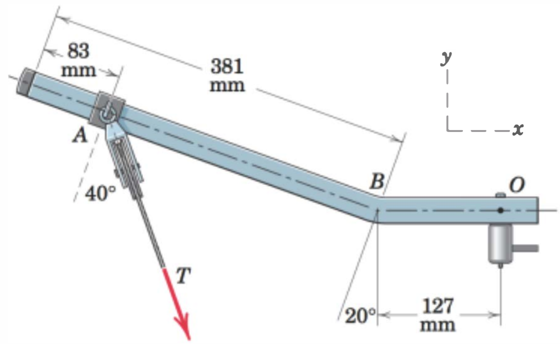
Problem 2/68

2/69 A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point O.



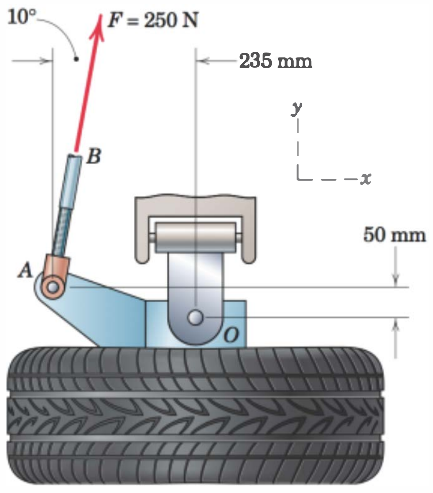
Problem 2/69

2/70 An overhead view of a portion of an exercise machine is shown. If the tension in the cable is $T = 780$ N, determine the equivalent force–couple system at (a) point B and at (b) point O. Record your answers in vector format.



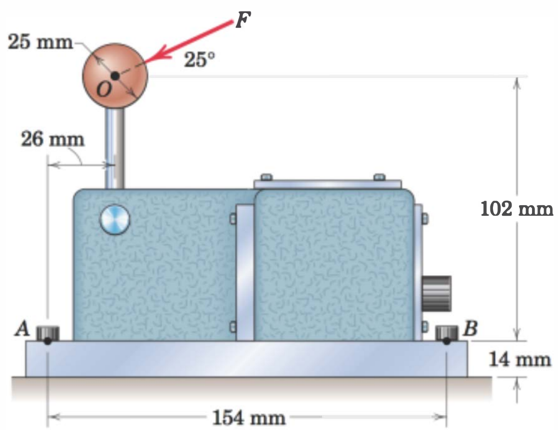
Problem 2/70

2/71 The tie-rod AB exerts the 250-N force on the steering knuckle AO as shown. Replace this force by an equivalent force–couple system at O.



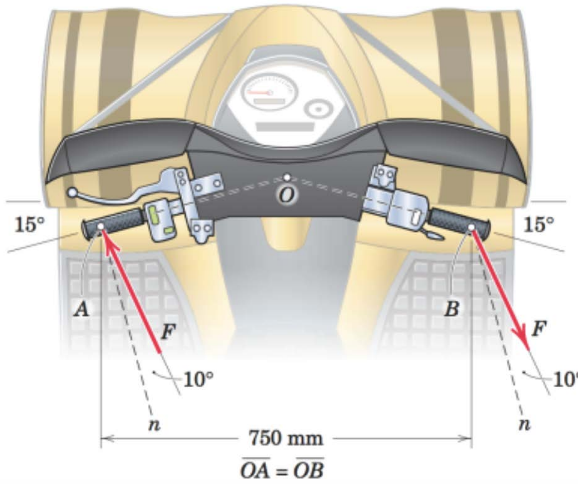
Problem 2/71

2/72 The 20-N force F is applied to the handle of the directional control valve as shown. Compute the equivalent force–couple system at point B.



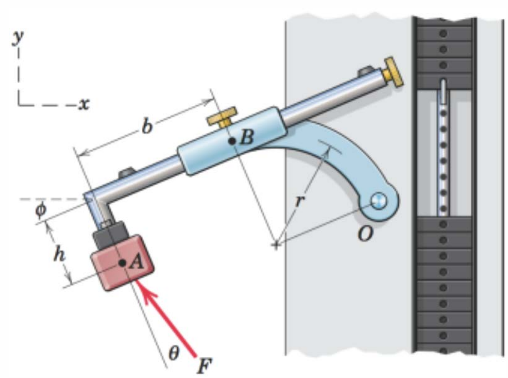
Problem 2/72

2/73 An overhead view of the handlebars on an all-terrain vehicle is shown. If the indicated forces have a magnitude of $F = 150$ N, determine the moment created by the two forces about the vertical steering axis through point O. Both n-axes are perpendicular to the left handlebar. Treat the problem as two-dimensional.



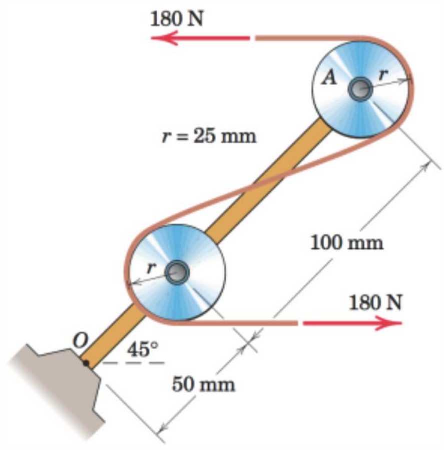
Problem 2/73

2/74 The force F is applied to the leg-extension exercise machine as shown. Determine the equivalent force–couple system at point O. Use values of $F = 520$ N, $b = 450$ mm, $h = 215$ mm, $r = 325$ mm, $\theta = 15°$, and $\phi = 10°$.



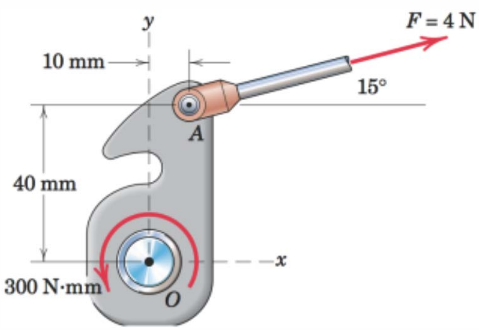
Problem 2/74

2/75 The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.



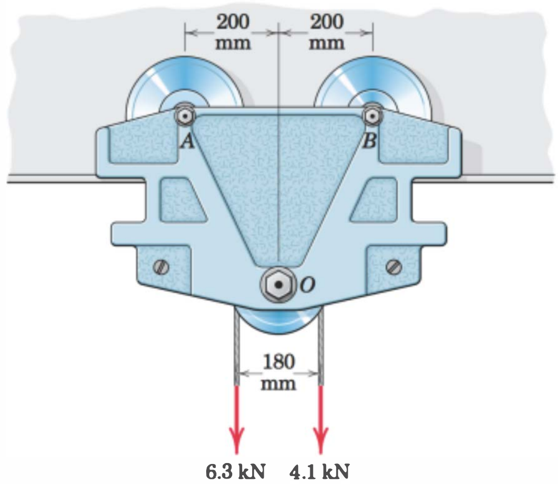
Problem 2/75

2/76 The device shown is a part of an automobile seat-back-release mechanism. The part is subjected to the 4-N force exerted at A and a 300-N·mm restoring moment exerted by a hidden torsional spring. Determine the y-intercept of the line of action of the single equivalent force.



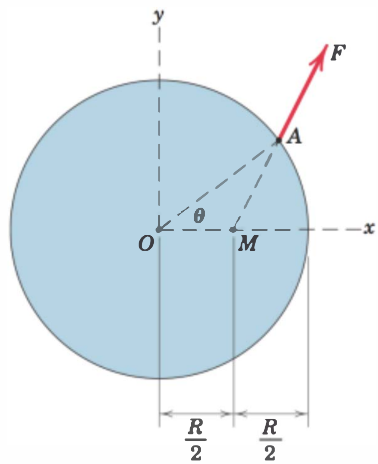
Problem 2/76

2/77 Replace the two cable tensions which act on the pulley at O of the beam trolley by two parallel forces which act at the track-wheel connections A and B.



Problem 2/77

2/78 The force F acts along line MA, where M is the midpoint of the radius along the x-axis. Determine the equivalent force–couple system at O if $\theta = 40°$.



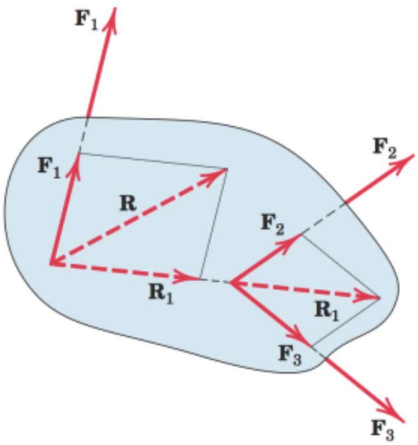
Problem 2/78

2/6 Resultants

The properties of force, moment, and couple were developed in the previous four articles. Now we are ready to describe the resultant action of a group or *system* of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action. The *resultant* of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

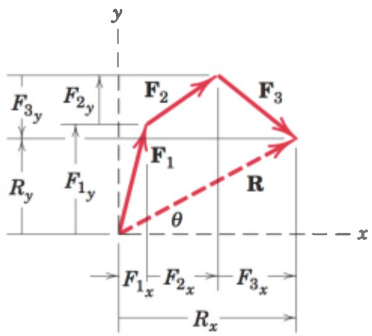
Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

The most common type of force system occurs when the forces all act in a single plane, say, the *x-y* plane, as illustrated by the system of three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ in Fig. 2/13*a*. We obtain the magnitude and direction of the resultant force $\mathbf{R}$ by forming the *force polygon* shown in part *b* of the figure, where the forces are added head-to-tail in any sequence. Thus, for any system of coplanar forces we may write



(*a*)

$$
\begin{gathered}
\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F} \\
R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\
\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}
\end{gathered}
\tag{2/9}
$$



(*b*)

Figure 2/13

Graphically, the correct line of action of $\mathbf{R}$ may be obtained by preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part *a* of the figure for the case of three forces where the sum $\mathbf{R}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ is added to $\mathbf{F}_1$ to obtain $\mathbf{R}$. The principle of transmissibility has been used in this process.

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. 2/14*a* and *b*, where M_1, M_2, and M_3 are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point O.
2. Add all forces at O to form the resultant force $\mathbf{R}$, and add all couples to form the resultant couple M_O. We now have the single force–couple system, as shown in Fig. 2/14*c*.
3. In Fig. 2/14*d*, find the line of action of $\mathbf{R}$ by requiring $\mathbf{R}$ to have a moment of M_O about point O. Note that the force systems of Figs. 2/14*a* and 2/14*d* are equivalent, and that $\Sigma(Fd)$ in Fig. 2/14*a* is equal to Rd in Fig. 2/14*d*.

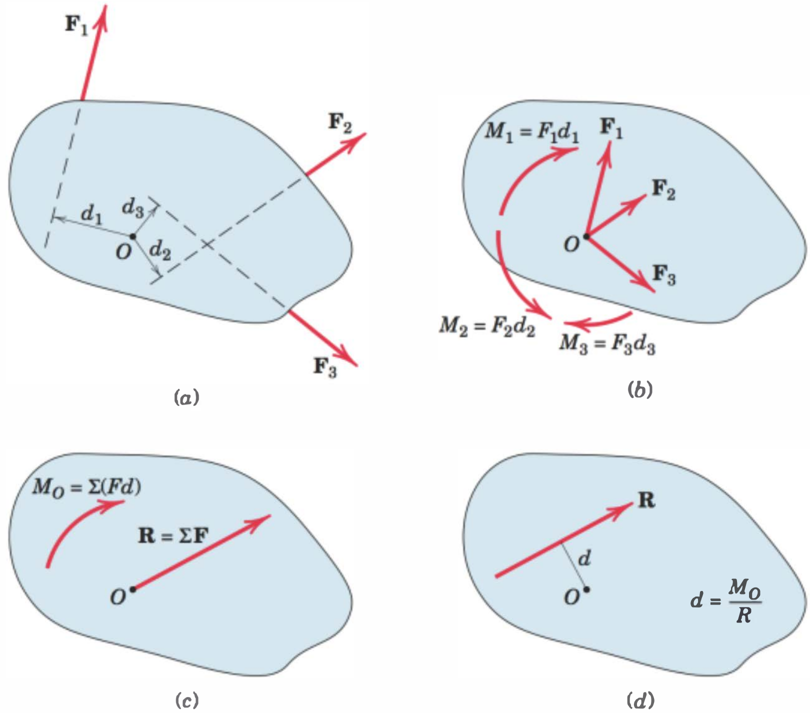


Figure 2/14

Principle of Moments

This process is summarized in equation form by

$$\boxed{\begin{aligned} \mathbf{R} &= \Sigma\mathbf{F} \\ M_O &= \Sigma M = \Sigma(Fd) \\ Rd &= M_O \end{aligned}} \qquad \textbf{(2/10)}$$

The first two of Eqs. 2/10 reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of $\mathbf{R}$, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of *nonconcurrent* force systems; we call this extension the *principle of moments*.

For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣM_O about that point is zero. Thus, the line of action of the resultant $\mathbf{R} = \Sigma\mathbf{F}$, determined by the first of Eqs. 2/10, passes through point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force $\mathbf{R}$ for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 2/15, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3d$.

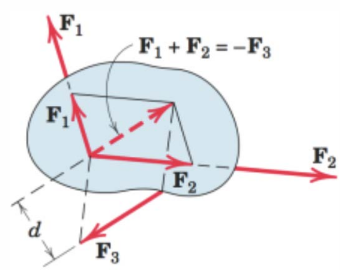
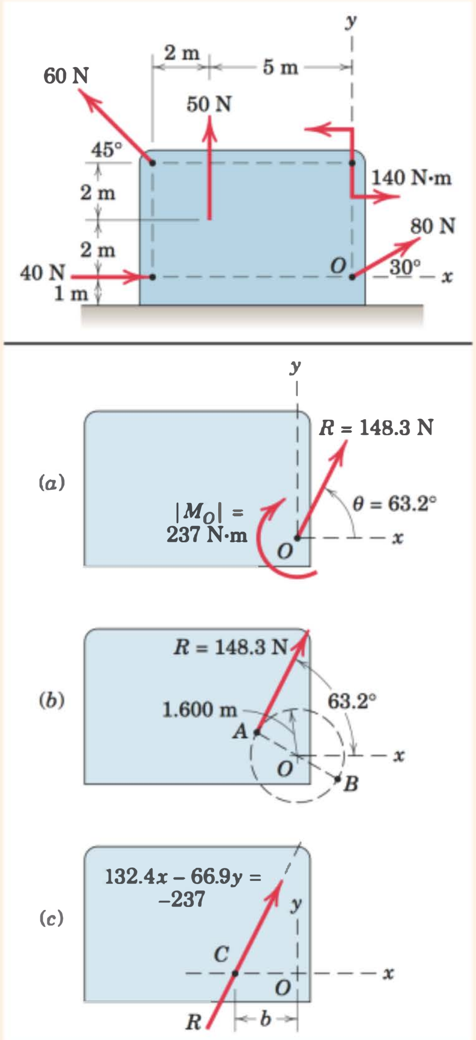


Figure 2/15

Sample Problem 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x] \qquad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y] \qquad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}] \qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right] \qquad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*

① $[M_O = \Sigma(Fd)] \qquad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$\qquad = -237 \text{ N·m}$

The force–couple system consisting of $\mathbf{R}$ and M_O is shown in Fig. *a*.

We now determine the final line of action of $\mathbf{R}$ such that $\mathbf{R}$ alone represents the original system.

$[Rd = |M_O|] \qquad 148.3d = 237 \qquad d = 1.600 \text{ m}$ *Ans.*

Hence, the resultant $\mathbf{R}$ may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.600-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. *a*, dictate the final placement of $\mathbf{R}$. Had M_O been counterclockwise, the correct line of action of $\mathbf{R}$ would have been the tangent at point B.

The resultant $\mathbf{R}$ may also be located by determining its intercept distance b to point C on the x-axis, Fig. *c*. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the y-intercept could have been obtained by noting that the moment about O would be due to R_x only.

A more formal approach in determining the final line of action of $\mathbf{R}$ is to use the vector expression

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is a position vector running from point O to any point on the line of action of $\mathbf{R}$. Substituting the vector expressions for $\mathbf{r}$, $\mathbf{R}$, and $\mathbf{M}_O$ and carrying out the cross product result in

$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

Thus, the desired line of action, Fig. *c*, is given by

$$132.4x - 66.9y = -237$$

② By setting $y = 0$, we obtain $x = -1.792$ m, which agrees with our earlier calculation of the distance b.

Helpful Hints

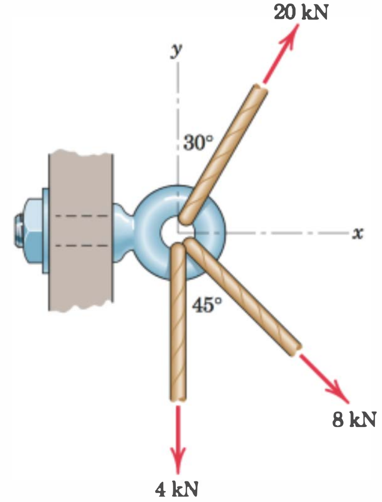
① We note that the choice of point O as a moment center eliminates any moments due to the two forces which pass through O. Had the clockwise sign convention been adopted, M_O would have been $+237$ N·m, with the plus sign indicating a sense which agrees with the sign convention. Either sign convention, of course, leads to the conclusion of a clockwise moment M_O.

② Note that the vector approach yields sign information automatically, whereas the scalar approach is more physically oriented. You should master both methods.

PROBLEMS

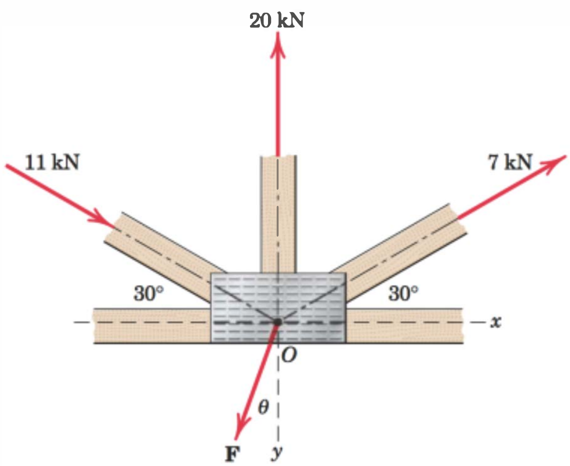
Introductory Problems

2/79 Determine the resultant **R** of the three tension forces acting on the eye bolt. Find the magnitude of **R** and the angle θ_x which **R** makes with the positive x-axis.



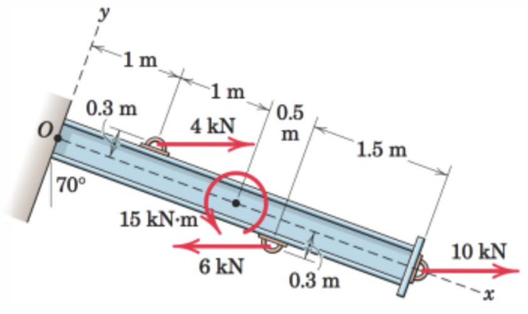
Problem 2/79

2/80 Determine the force magnitude F and direction θ (measured clockwise from the positive y-axis) that will cause the resultant **R** of the four applied forces to be directed to the right with a magnitude of 9 kN.



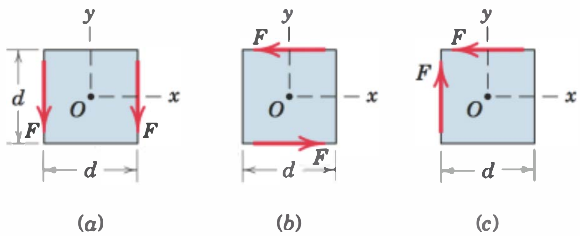
Problem 2/80

2/81 Replace the three horizontal forces and applied couple with an equivalent force–couple system at O by specifying the resultant **R** and couple M_O. Next, determine the equation for the line of action of the stand-alone resultant force **R**.



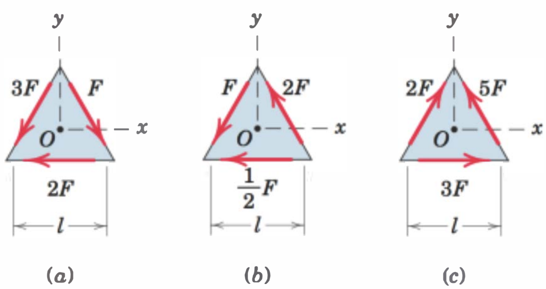
Problem 2/81

2/82 Determine the equivalent force–couple system at the center O for each of the three cases of forces being applied along the edges of a square plate of side d.



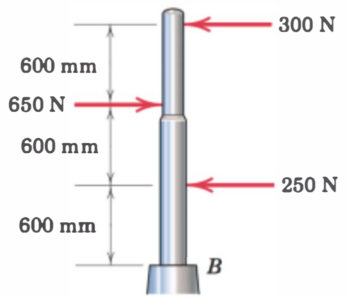
Problem 2/82

2/83 Determine the equivalent force–couple system at O for each of the three cases of forces applied along the edges of an equilateral triangle of side l. Where possible, replace this force–couple system with a single force and specify the location along the y-axis through which the single force acts. Note that the location of O in each case is at the centroid of the triangle. See Table D/3 in Appendix D for the centroid location of a triangle.



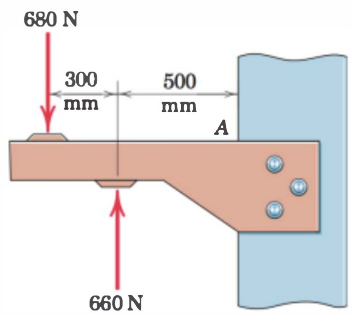
Problem 2/83

2/84 Determine the height h above the base B at which the resultant of the three forces acts.



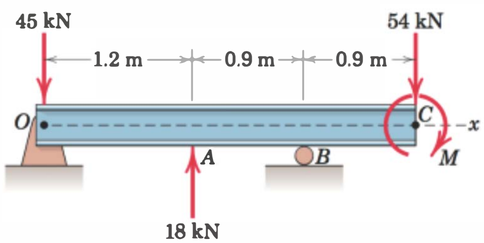
Problem 2/84

2/85 Where does the resultant of the two forces act?



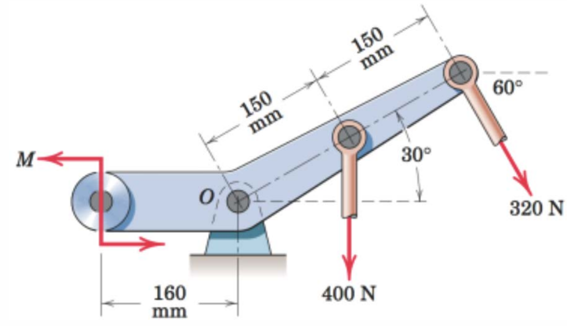
Problem 2/85

2/86 If the resultant of the loads shown passes through point B, determine the equivalent force–couple system at O.



Problem 2/86

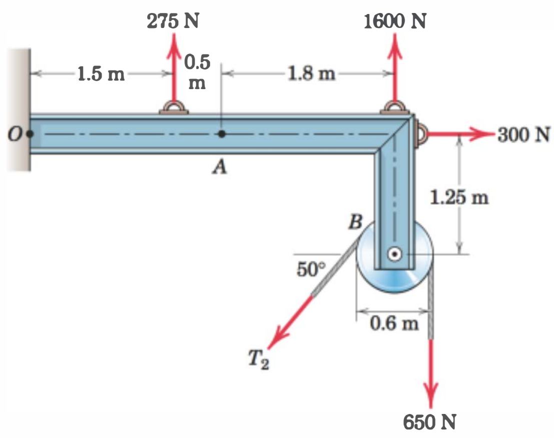
2/87 If the resultant of the two forces and couple M passes through point O, determine M.



Problem 2/87

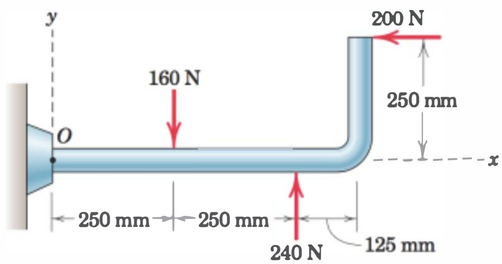
Representative Problems

2/88 If the resultant of the forces shown passes through point A, determine the magnitude of the unknown tension T_2 which acts on the braked pulley.



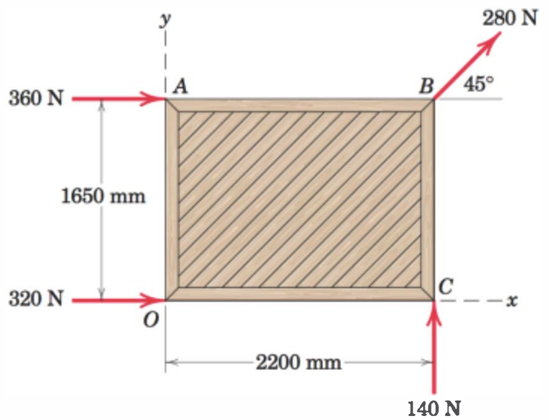
Problem 2/88

2/89 Replace the three forces acting on the bent pipe by a single equivalent force **R**. Specify the distance x from point O to the point on the x-axis through which the line of action of **R** passes.



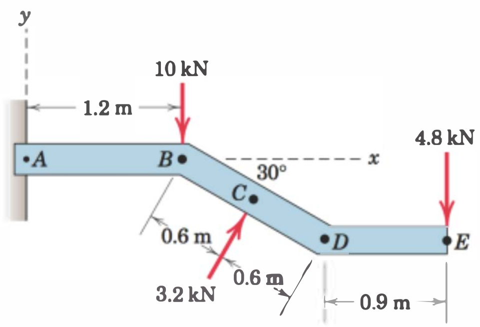
Problem 2/89

2/90 Four people are attempting to move a stage platform across the floor. If they exert the horizontal forces shown, determine (*a*) the equivalent force–couple system at *O* and (*b*) the points on the *x*- and *y*-axes through which the line of action of the single resultant force **R** passes.



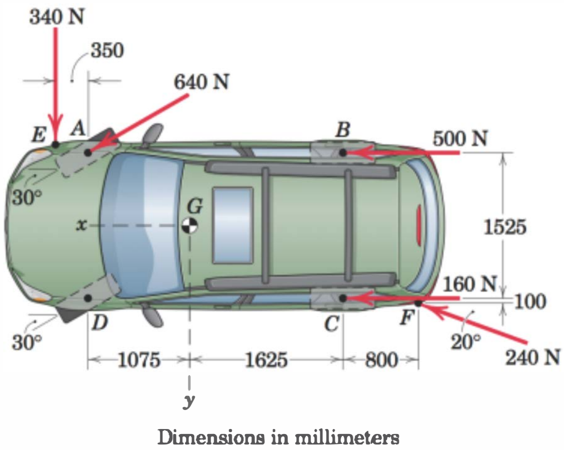
Problem 2/90

2/91 Replace the three forces which act on the bent bar by a force–couple system at the support point *A*. Then determine the *x*-intercept of the line of action of the stand-alone resultant force **R**.



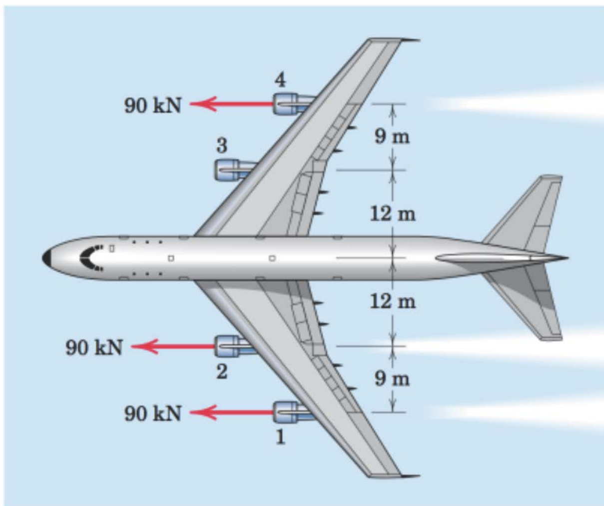
Problem 2/91

2/92 Uneven terrain conditions cause the left front wheel of the all-wheel-drive vehicle to lose traction with the ground. If the driver causes the traction forces shown to be generated by the other three wheels while his two friends exert the indicated forces on the vehicle periphery at points *E* and *F*, determine the resultant of this system and the *x*- and *y*-intercepts of its line of action. Note that the front and rear tracks of the vehicle are equivalent; that is, $AD = BC$. Treat this as a two-dimensional problem and realize that *G* lies on the car centerline.



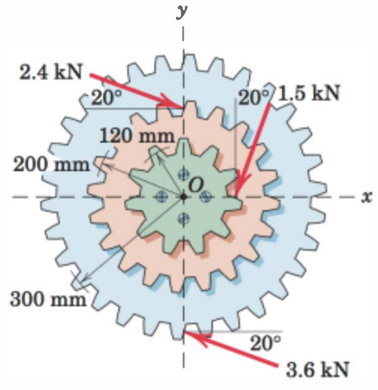
Problem 2/92

2/93 A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two-dimensional problem.



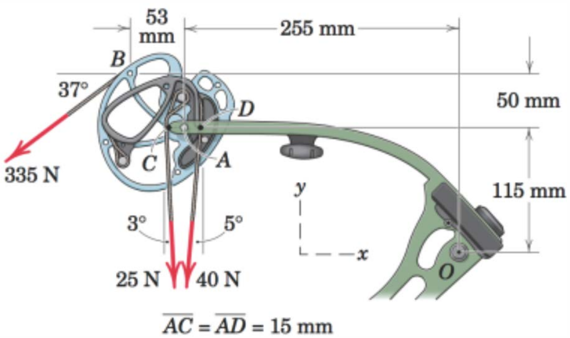
Problem 2/93

2/94 Determine the x- and y-axis intercepts of the line of action of the resultant of the three loads applied to the gearset.



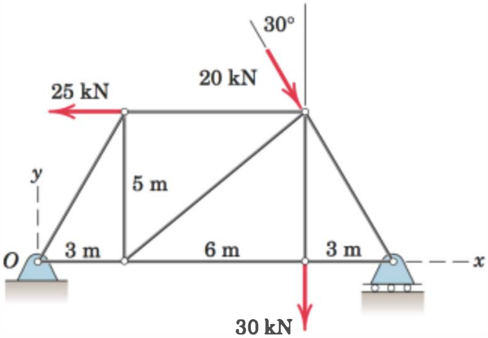
Problem 2/94

2/95 Replace the three cable tensions acting on the upper portion of the compound bow with an equivalent force–couple system at O.



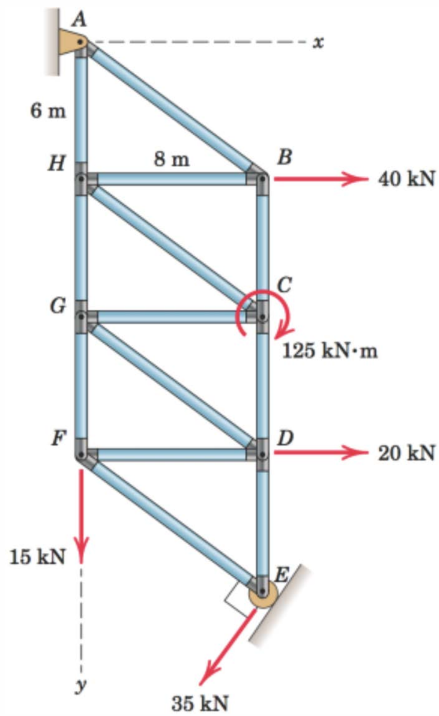
Problem 2/95

2/96 Determine the resultant $\mathbf{R}$ of the three forces acting on the simple truss. Specify the points on the x- and y-axes through which $\mathbf{R}$ must pass.



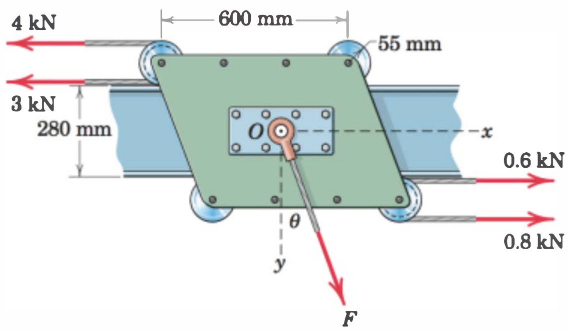
Problem 2/96

2/97 For the truss loaded as shown, determine the equation for the line of action of the stand-alone resultant $\mathbf{R}$ and state the coordinates of the points on the x- and y-axes through which the line of action passes. All triangles are 3-4-5.



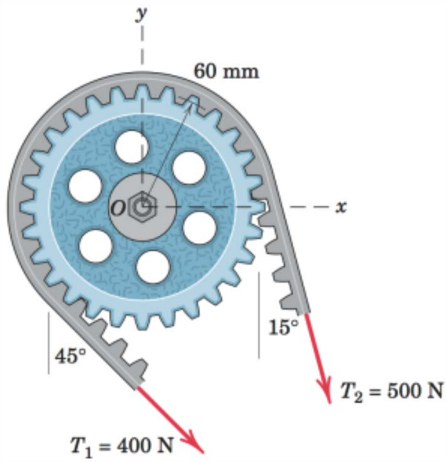
Problem 2/97

2/98 Five forces are applied to the beam trolley as shown. Determine the coordinates of the point on the y-axis through which the stand-alone resultant $\mathbf{R}$ must pass if $F = 5$ kN and $\theta = 30°$.



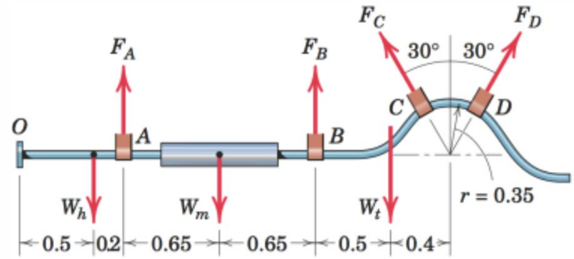
Problem 2/98

2/99 As part of a design test, the camshaft–drive sprocket is fixed, and then the two forces shown are applied to a length of belt wrapped around the sprocket. Find the resultant of this system of two forces and determine where its line of action intersects both the x- and y-axes.



Problem 2/99

2/100 An exhaust system for a pickup truck is shown in the figure. The weights W_h, W_m, and W_t of the headpipe, muffler, and tailpipe are 10, 100, and 50 N, respectively, and act at the indicated points. If the exhaust-pipe hanger at point A is adjusted so that its tension F_A is 50 N, determine the required forces in the hangers at points B, C, and D so that the force–couple system at point O is zero. Why is a zero force–couple system at O desirable?



Dimensions in meters

Problem 2/100

SECTION B THREE-DIMENSIONAL FORCE SYSTEMS

2/7 Rectangular Components

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point O in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z, where

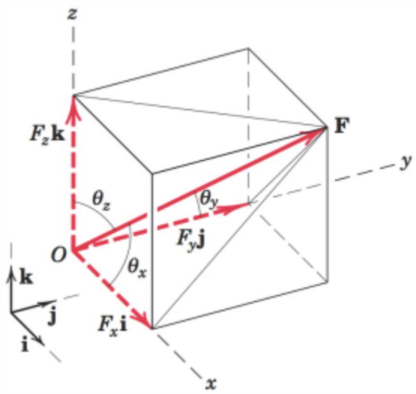


Figure 2/16

$$\begin{aligned} F_x &= F\cos\theta_x & F &= \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}} \\ F_y &= F\cos\theta_y & \mathbf{F} &= F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \\ F_z &= F\cos\theta_z & \mathbf{F} &= F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z) \end{aligned} \qquad \textbf{(2/11)}$$

The unit vectors **i**, **j**, and **k** are in the x-, y-, and z-directions, respectively. Using the direction cosines of **F**, which are $l = \cos\theta_x$, $m = \cos\theta_y$, and $n = \cos\theta_z$, where $l^2 + m^2 + n^2 = 1$, we may write the force as

$$\boxed{\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})} \qquad \textbf{(2/12)}$$

We may regard the right-side expression of Eq. 2/12 as the force magnitude F times a unit vector $\mathbf{n}_F$ which characterizes the direction of **F**, or

$$\mathbf{F} = F\mathbf{n}_F \qquad \textbf{(2/12a)}$$

It is clear from Eqs. 2/12 and 2/12a that $\mathbf{n}_F = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$, which shows that the scalar components of the unit vector $\mathbf{n}_F$ are the direction cosines of the line of action of **F**.

In solving three-dimensional problems, one must usually find the x, y, and z scalar components of a force. In most cases, the direction of a force is described (a) by two points on the line of action of the force or (b) by two angles which orient the line of action.

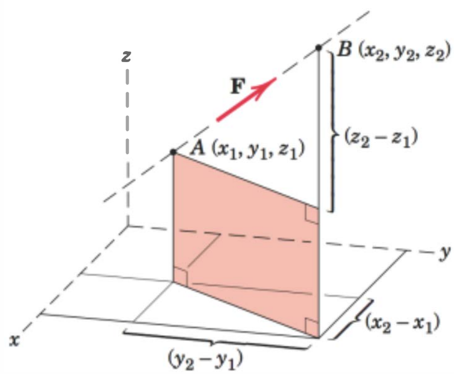


Figure 2/17

(a) Specification by two points on the line of action of the force. If the coordinates of points A and B of Fig. 2/17 are known, the force **F** may be written as

$$\mathbf{F} = F\mathbf{n}_F = F\frac{\overrightarrow{AB}}{\overline{AB}} = F\frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Thus the x, y, and z scalar components of **F** are the scalar coefficients of the unit vectors **i**, **j**, and **k**, respectively.

(b) Specification by two angles which orient the line of action of the force. Consider the geometry of Fig. 2/18. We assume that the angles θ and ϕ are known. First resolve **F** into horizontal and vertical components.

$$F_{xy} = F \cos \phi$$
$$F_z = F \sin \phi$$

Then resolve the horizontal component F_{xy} into x- and y-components.

$$F_x = F_{xy} \cos \theta = F \cos \phi \cos \theta$$
$$F_y = F_{xy} \sin \theta = F \cos \phi \sin \theta$$

The quantities F_x, F_y, and F_z are the desired scalar components of **F**.

The choice of orientation of the coordinate system is arbitrary, with convenience being the primary consideration. However, we must use a right-handed set of axes in our three-dimensional work to be consistent with the right-hand-rule definition of the cross product. When we rotate from the x- to the y-axis through the 90° angle, the positive direction for the z-axis in a right-handed system is that of the advancement of a right-handed screw rotated in the same sense. This is equivalent to the right-hand rule.

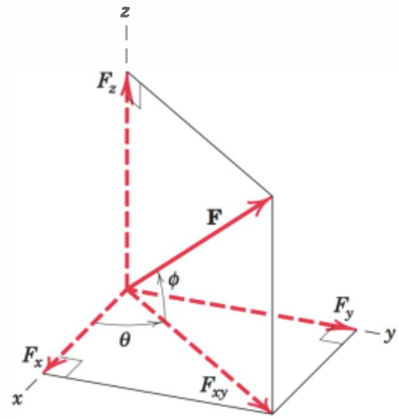


Figure 2/18

Dot Product

We can express the rectangular components of a force **F** (or any other vector) with the aid of the vector operation known as the *dot* or *scalar product* (see item 6 in Art. C/7 of Appendix C). The dot product of two vectors **P** and **Q**, Fig. 2/19a, is defined as the product of their magnitudes times the cosine of the angle α between them. It is written as

$$\mathbf{P} \cdot \mathbf{Q} = PQ \cos \alpha$$

We can view this product either as the orthogonal projection $P \cos \alpha$ of **P** in the direction of **Q** multiplied by Q, or as the orthogonal projection $Q \cos \alpha$ of **Q** in the direction of **P** multiplied by P. In either case the dot product of the two vectors is a scalar quantity. Thus, for instance, we can express the scalar component $F_x = F \cos \theta_x$ of the force **F** in Fig. 2/16 as $F_x = \mathbf{F} \cdot \mathbf{i}$, where **i** is the unit vector in the x-direction.

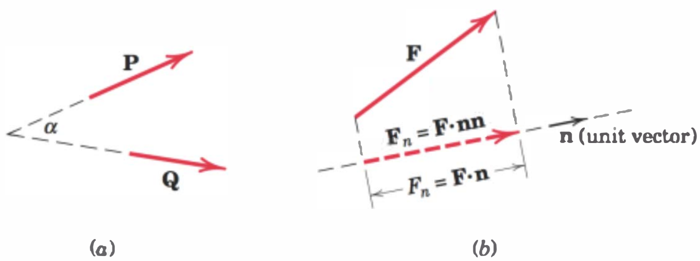


Figure 2/19

In more general terms, if $\mathbf{n}$ is a unit vector in a specified direction, the projection of $\mathbf{F}$ in the $\mathbf{n}$-direction, Fig. 2/19*b*, has the magnitude $F_n = \mathbf{F}\cdot\mathbf{n}$. If we want to express the projection in the $\mathbf{n}$-direction as a vector quantity, then we multiply its scalar component, expressed by $\mathbf{F}\cdot\mathbf{n}$, by the unit vector $\mathbf{n}$ to give $\mathbf{F}_n = (\mathbf{F}\cdot\mathbf{n})\mathbf{n}$. We may write this as $\mathbf{F}_n = \mathbf{F}\cdot\mathbf{nn}$ without ambiguity because the term $\mathbf{nn}$ is not defined, and so the complete expression cannot be misinterpreted as $\mathbf{F}\cdot(\mathbf{nn})$.

If the direction cosines of $\mathbf{n}$ are α, β, and γ, then we may write $\mathbf{n}$ in vector component form like any other vector as

$$\mathbf{n} = \alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$$

where in this case its magnitude is unity. If the direction cosines of $\mathbf{F}$ with respect to reference axes x-y-z are l, m, and n, then the projection of $\mathbf{F}$ in the $\mathbf{n}$-direction becomes

$$\begin{aligned} F_n = \mathbf{F}\cdot\mathbf{n} &= F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})\cdot(\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}) \\ &= F(l\alpha + m\beta + n\gamma) \end{aligned}$$

because

$$\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1$$

and

$$\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{i} = \mathbf{i}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{j} = 0$$

The latter two sets of equations are true because $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ have unit length and are mutually perpendicular.

Angle between Two Vectors

If the angle between the force $\mathbf{F}$ and the direction specified by the unit vector $\mathbf{n}$ is θ, then from the dot-product definition we have $\mathbf{F}\cdot\mathbf{n} = Fn\cos\theta = F\cos\theta$, where $|\mathbf{n}| = n = 1$. Thus, the angle between $\mathbf{F}$ and $\mathbf{n}$ is given by

$$\theta = \cos^{-1}\frac{\mathbf{F}\cdot\mathbf{n}}{F} \qquad \textbf{(2/13)}$$

In general, the angle between any two vectors $\mathbf{P}$ and $\mathbf{Q}$ is

$$\theta = \cos^{-1}\frac{\mathbf{P}\cdot\mathbf{Q}}{PQ} \qquad \textbf{(2/13}\boldsymbol{a}\textbf{)}$$

If a force $\mathbf{F}$ is perpendicular to a line whose direction is specified by the unit vector $\mathbf{n}$, then $\cos\theta = 0$, and $\mathbf{F}\cdot\mathbf{n} = 0$. Note that this relationship does not mean that either $\mathbf{F}$ or $\mathbf{n}$ is zero, as would be the case with scalar multiplication where $(A)(B) = 0$ requires that either A or B (or both) be zero.

The dot-product relationship applies to nonintersecting vectors as well as to intersecting vectors. Thus, the dot product of the nonintersecting vectors $\mathbf{P}$ and $\mathbf{Q}$ in Fig. 2/20 is Q times the projection of $\mathbf{P}'$ on $\mathbf{Q}$, or $P'Q\cos\alpha = PQ\cos\alpha$ because $\mathbf{P}'$ and $\mathbf{P}$ are the same when treated as free vectors.

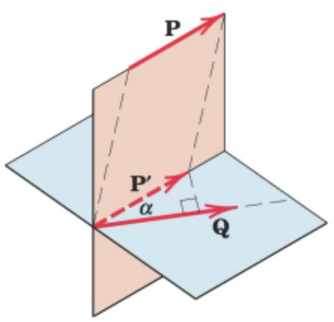
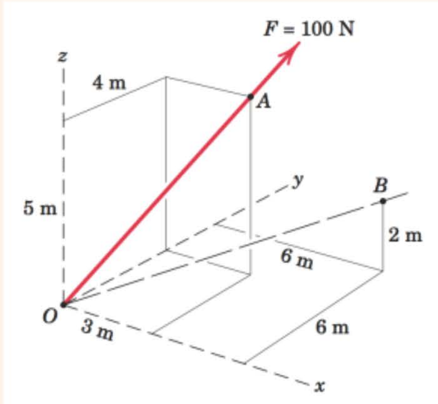


Figure 2/20

Sample Problem 2/10

A force **F** with a magnitude of 100 N is applied at the origin O of the axes x-y-z as shown. The line of action of **F** passes through a point A whose coordinates are 3 m, 4 m, and 5 m. Determine (*a*) the x, y, and z scalar components of **F**, (*b*) the projection F_{xy} of **F** on the x-y plane, and (*c*) the projection F_{OB} of **F** along the line OB.



Solution. ***Part (a).*** We begin by writing the force vector **F** as its magnitude F times a unit vector $\mathbf{n}_{OA}$.

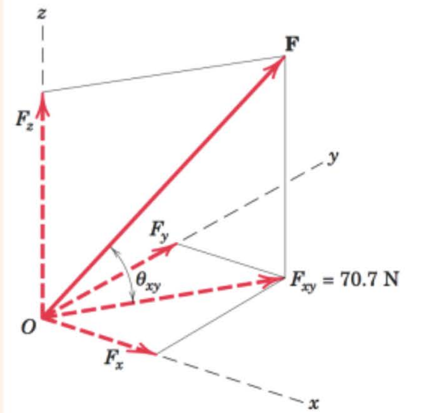
$$\mathbf{F} = F\mathbf{n}_{OA} = F\frac{\overrightarrow{OA}}{OA} = 100\left[\frac{3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}}{\sqrt{3^2 + 4^2 + 5^2}}\right]$$

$$= 100[0.424\mathbf{i} + 0.566\mathbf{j} + 0.707\mathbf{k}]$$

$$= 42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k} \text{ N}$$

The desired scalar components are thus

① $$F_x = 42.4 \text{ N} \qquad F_y = 56.6 \text{ N} \qquad F_z = 70.7 \text{ N} \qquad \textit{Ans.}$$



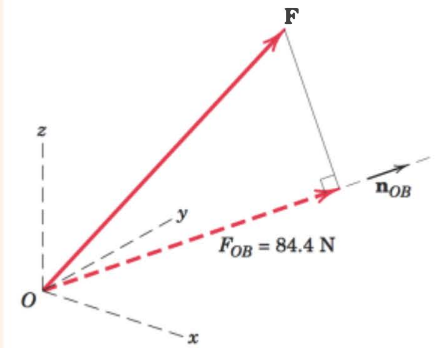
Part (b). The cosine of the angle θ_{xy} between **F** and the x-y plane is

$$\cos\theta_{xy} = \frac{\sqrt{3^2 + 4^2}}{\sqrt{3^2 + 4^2 + 5^2}} = 0.707$$

so that $F_{xy} = F\cos\theta_{xy} = 100(0.707) = 70.7$ N *Ans.*

Part (c). The unit vector $\mathbf{n}_{OB}$ along OB is

$$\mathbf{n}_{OB} = \frac{\overrightarrow{OB}}{OB} = \frac{6\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}}{\sqrt{6^2 + 6^2 + 2^2}} = 0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k}$$



The scalar projection of **F** on OB is

② $$F_{OB} = \mathbf{F}\cdot\mathbf{n}_{OB} = (42.4\mathbf{i} + 56.6\mathbf{j} + 70.7\mathbf{k})\cdot(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k})$$

$$= (42.4)(0.688) + (56.6)(0.688) + (70.7)(0.229)$$

$$= 84.4 \text{ N} \qquad \textit{Ans.}$$

If we wish to express the projection as a vector, we write

$$\mathbf{F}_{OB} = \mathbf{F}\cdot\mathbf{n}_{OB}\mathbf{n}_{OB}$$

$$= 84.4(0.688\mathbf{i} + 0.688\mathbf{j} + 0.229\mathbf{k})$$

$$= 58.1\mathbf{i} + 58.1\mathbf{j} + 19.35\mathbf{k} \text{ N}$$

Helpful Hints

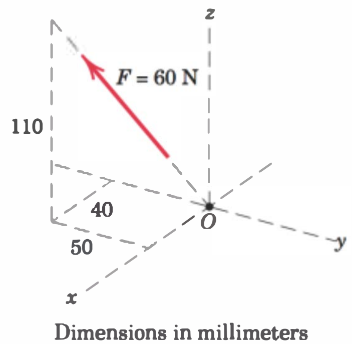
① In this example all scalar components are positive. Be prepared for the case where a direction cosine, and hence the scalar component, are negative.

② The dot product automatically finds the projection or scalar component of **F** along line OB as shown.

PROBLEMS

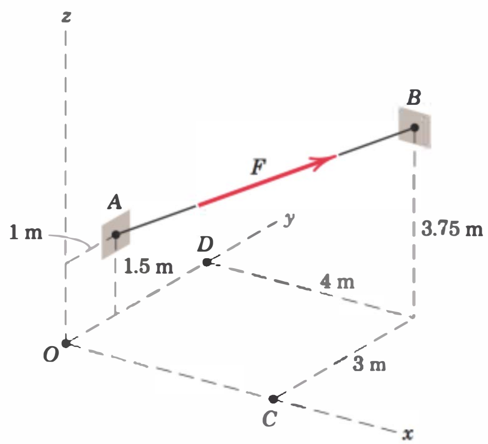
Introductory Problems

2/101 Express **F** as a vector in terms of the unit vectors **i**, **j**, and **k**. Determine the angle between **F** and the y-axis.



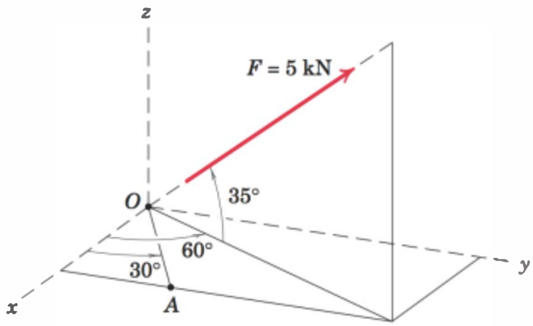
Problem 2/101

2/102 Cable AB exerts a force of magnitude $F = 6$ kN on point A. Express **F** as a vector in terms of the unit vectors **i**, **j**, and **k**. Determine the angle between **F** and the x-axis.



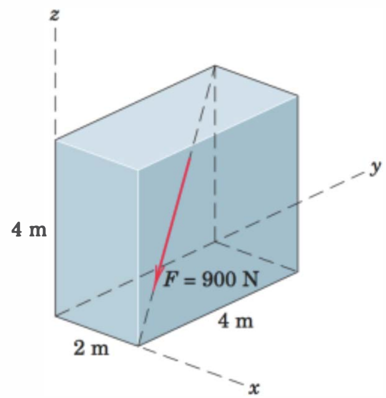
Problem 2/102

2/103 Express the 5-kN force **F** as a vector in terms of the unit vectors **i**, **j**, and **k**. Determine the projections of **F** onto the x-axis and onto the line OA.



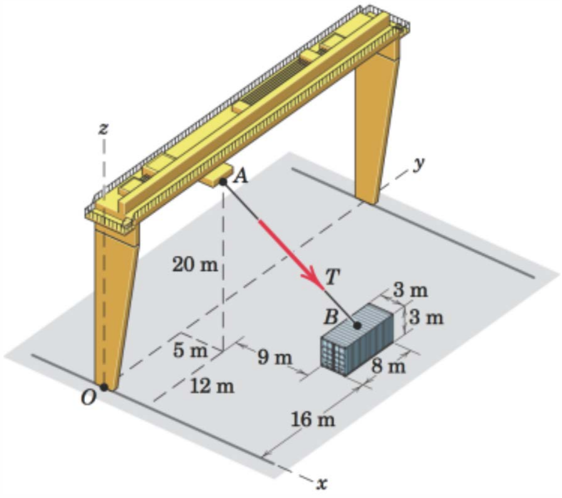
Problem 2/103

2/104 The force **F** has a magnitude of 900 N and acts along the diagonal of the parallelepiped as shown. Express **F** in terms of its magnitude times the appropriate unit vector and determine its x-, y-, and z-components.



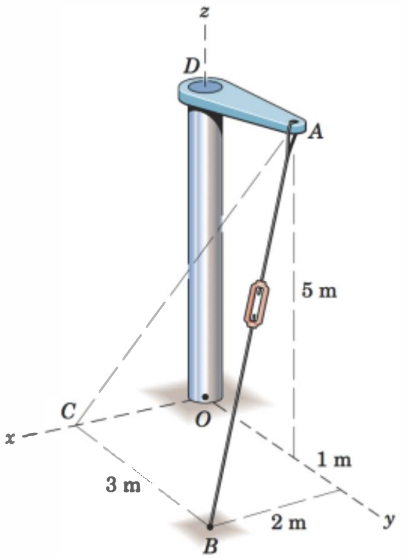
Problem 2/104

2/105 If the tension in the gantry-crane hoisting cable is $T = 14$ kN, determine the unit vector **n** in the direction of **T** and use **n** to determine the scalar components of **T**. Point B is located at the center of the container top.



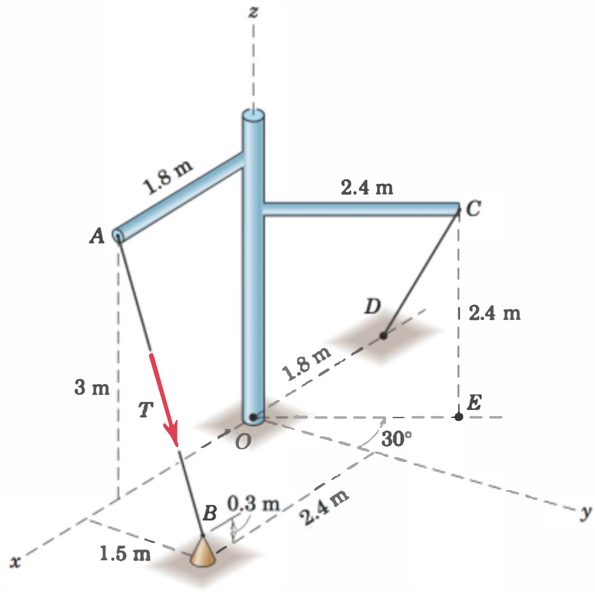
Problem 2/105

2/106 The turnbuckle is tightened until the tension in the cable AB equals 2.4 kN. Determine the vector expression for the tension $\mathbf{T}$ as a force acting on member AD. Also find the magnitude of the projection of $\mathbf{T}$ along the line AC.



Problem 2/106

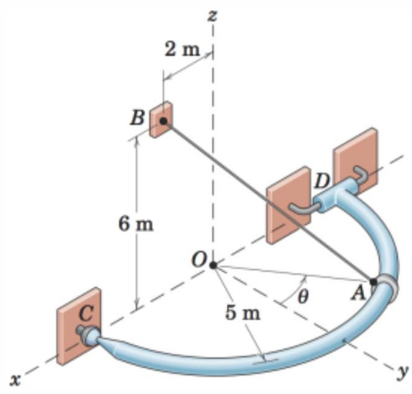
2/107 If the tension in cable AB is 8 kN, determine the angles which it makes with the x-, y-, and z-axes as it acts on point A of the structure.



Problem 2/107

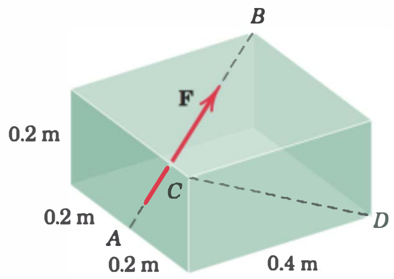
Representative Problems

2/108 The tension in the supporting cable AB is $T = 425$ N. Write this tension as a vector (*a*) as it acts on point A and (*b*) as it acts on point B. Assume a value of $\theta = 30°$.



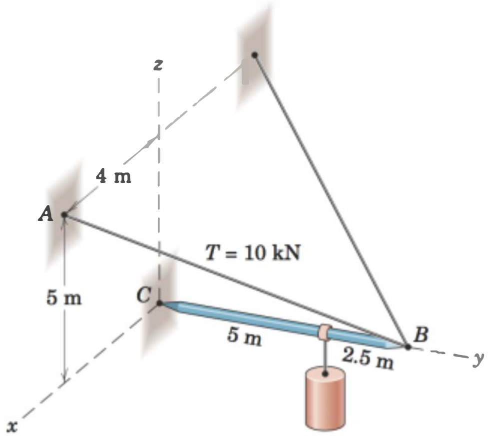
Problem 2/108

2/109 The force $\mathbf{F}$ has a magnitude of 2 kN and is directed from A to B. Calculate the projection F_{CD} of $\mathbf{F}$ onto line CD and determine the angle θ between $\mathbf{F}$ and CD.



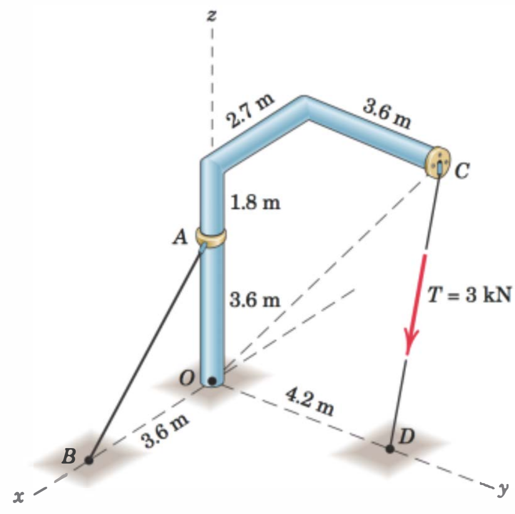
Problem 2/109

2/110 The tension in the supporting cable AB is 10 kN. Write the force which the cable exerts on the boom BC as a vector $\mathbf{T}$. Determine the angles θ_x, θ_y, and θ_z which the line of action of $\mathbf{T}$ forms with the positive x-, y-, and z-axes.



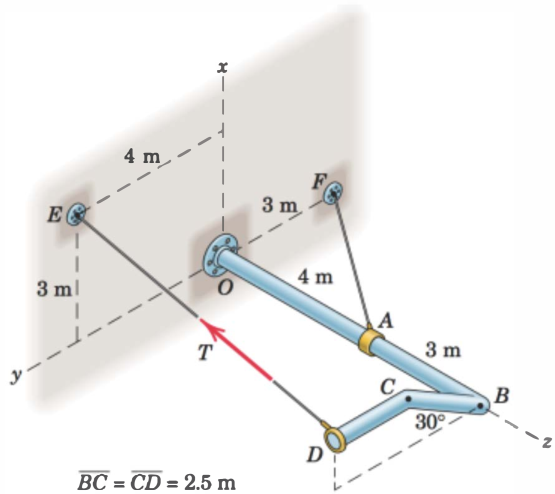
Problem 2/110

2/111 If the tension in cable CD is $T = 3$ kN, determine the magnitude of the projection of $\mathbf{T}$ onto line CO.



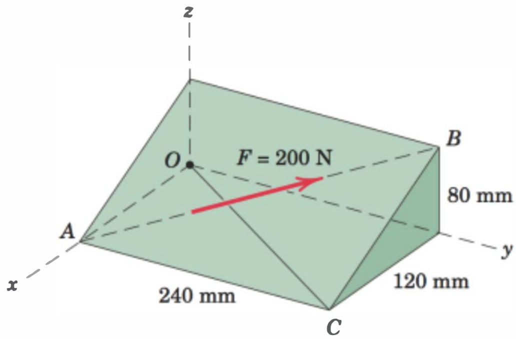
Problem 2/111

2/112 If the tension in cable DE is $T = 575$ N, determine (*a*) the scalar projection of $\mathbf{T}$ onto line EO and (*b*) the vector expression for the projection of $\mathbf{T}$ onto line EO.



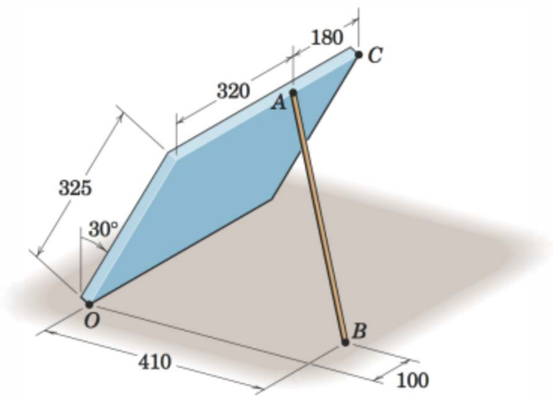
Problem 2/112

2/113 Determine the angle θ between the 200-N force and line OC.



Problem 2/113

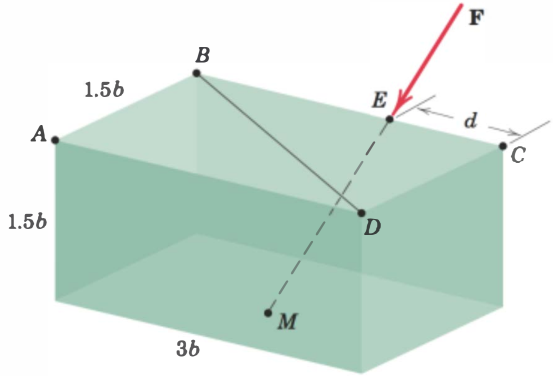
2/114 Compression member AB is used to hold up the 325 × 500-mm rectangular plate. If the compressive force in the member is 320 N for the position shown, determine the magnitude of the projection of this force (as it acts at point A) along diagonal OC.



Dimensions in millimeters

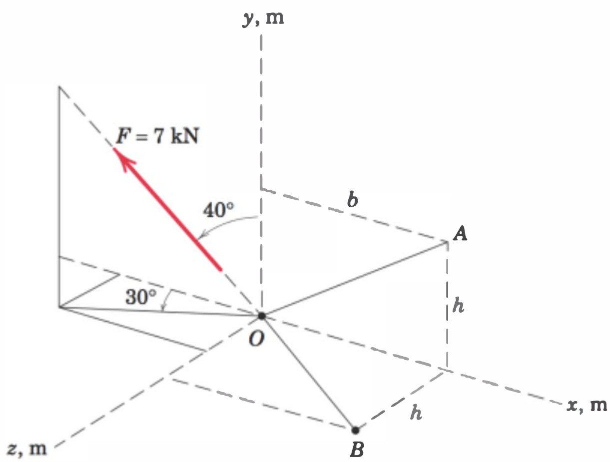
Problem 2/114

2/115 Determine a general expression for the scalar projection of **F** onto line BD. Point M is located at the center of the bottom face of the parallelepiped. Evaluate your expression for $d = b/2$ and $d = 5b/2$.



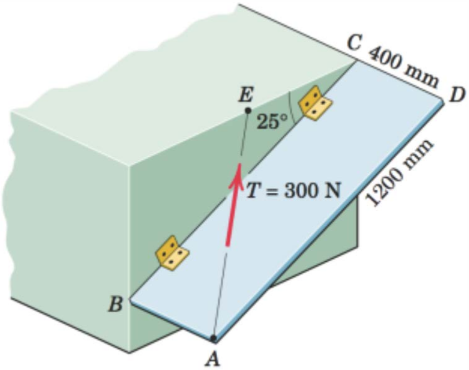
Problem 2/115

2/116 If the scalar projection of **F** onto line OA is 0, determine the scalar projection of **F** onto line OB. Use a value of $b = 2$ m.



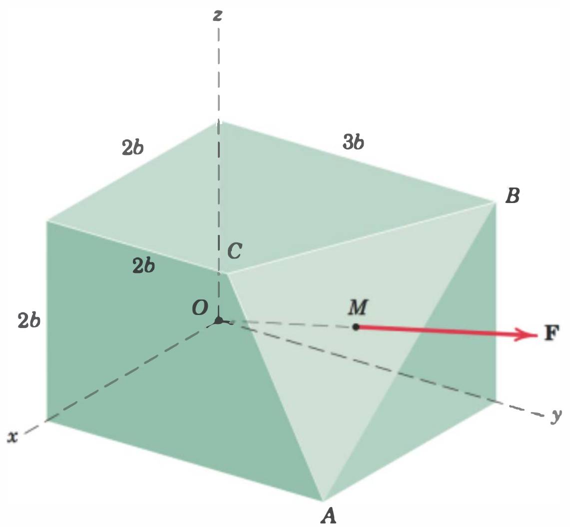
Problem 2/116

2/117 The rectangular plate is supported by hinges along its side BC and by the cable AE. If the cable tension is 300 N, determine the projection onto line BC of the force exerted on the plate by the cable. Note that E is the midpoint of the horizontal upper edge of the structural support.



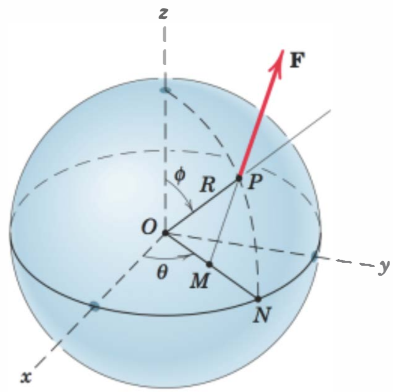
Problem 2/117

▶2/118 Express the force **F** in terms of the unit vectors **i**, **j**, and **k**. Point M is located at the centroid of the triangle ABC formed by "chopping off" the corner of the parallelepiped. (See Table D/3 in Appendix D for the centroid location of a triangle.)



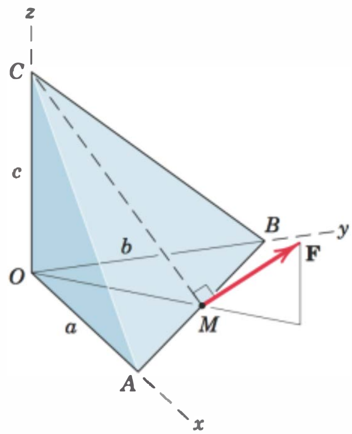
Problem 2/118

▶2/119 A force **F** is applied to the surface of the sphere as shown. The angles θ and ϕ locate point P, and point M is the midpoint of ON. Express **F** in vector form, using the given x-, y-, and z-coordinates.



Problem 2/119

▶2/120 Determine the x-, y-, and z-components of force **F** which acts on the tetrahedron as shown. The quantities a, b, c, and F are known, and M is the midpoint of edge AB.



Problem 2/120

2/8 Moment and Couple

In two-dimensional analyses it is often convenient to determine a moment magnitude by scalar multiplication using the moment-arm rule. In three dimensions, however, the determination of the perpendicular distance between a point or line and the line of action of the force can be a tedious computation. A vector approach with cross-product multiplication then becomes advantageous.

Moments in Three Dimensions

Consider a force $\mathbf{F}$ with a given line of action acting on a body, Fig. 2/21*a*, and any point O not on this line. Point O and the line of $\mathbf{F}$ establish a plane A. The moment $\mathbf{M}_O$ of $\mathbf{F}$ about an axis through O normal to the plane has the magnitude $M_O = Fd$, where d is the perpendicular distance from O to the line of $\mathbf{F}$. This moment is also referred to as the moment of $\mathbf{F}$ about the *point* O.

The vector $\mathbf{M}_O$ is normal to the plane and is directed along the axis through O. We can describe both the magnitude and the direction of $\mathbf{M}_O$ by the vector cross-product relation introduced in Art. 2/4. (Refer to item 7 in Art. C/7 of Appendix C.) The vector $\mathbf{r}$ runs from O to *any* point on the line of action of $\mathbf{F}$. As described in Art. 2/4, the cross product of $\mathbf{r}$ and $\mathbf{F}$ is written $\mathbf{r} \times \mathbf{F}$ and has the magnitude $(r \sin \alpha)F$, which is the same as Fd, the magnitude of $\mathbf{M}_O$.

The correct direction and sense of the moment are established by the right-hand rule, described previously in Arts. 2/4 and 2/5. Thus, with $\mathbf{r}$ and $\mathbf{F}$ treated as free vectors emanating from O, Fig. 2/21*b*, the thumb points in the direction of $\mathbf{M}_O$ if the fingers of the right hand curl in the direction of rotation from $\mathbf{r}$ to $\mathbf{F}$ through the angle α. Therefore, we may write the moment of $\mathbf{F}$ about the axis through O as

$$\boxed{\mathbf{M}_O = \mathbf{r} \times \mathbf{F}} \qquad \textbf{(2/14)}$$

The order $\mathbf{r} \times \mathbf{F}$ of the vectors *must* be maintained because $\mathbf{F} \times \mathbf{r}$ would produce a vector with a sense opposite to that of $\mathbf{M}_O$; that is, $\mathbf{F} \times \mathbf{r} = -\mathbf{M}_O$.

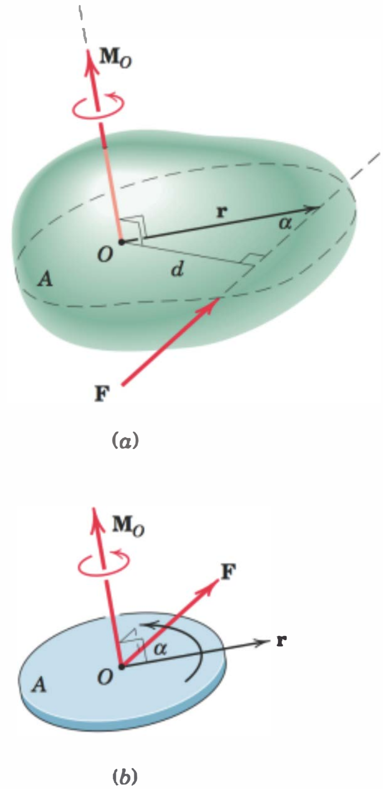


Figure 2/21

Evaluating the Cross Product

The cross-product expression for $\mathbf{M}_O$ may be written in the determinant form

$$\boxed{\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}} \qquad \textbf{(2/15)}$$

(Refer to item 7 in Art. C/7 of Appendix C if you are not already familiar with the determinant representation of the cross product.) Note the symmetry and order of the terms, and note that a *right-handed* coordinate system must be used. Expansion of the determinant gives

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

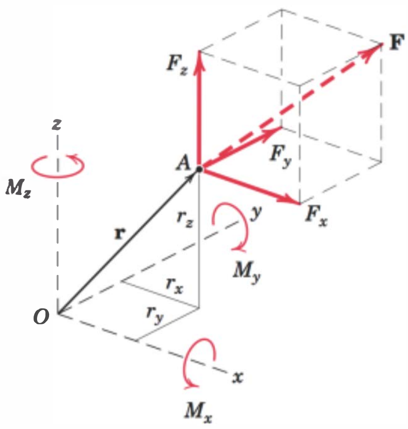


Figure 2/22

To gain more confidence in the cross-product relationship, examine the three components of the moment of a force about a point as obtained from Fig. 2/22. This figure shows the three components of a force **F** acting at a point A located relative to O by the vector **r**. The scalar magnitudes of the moments of these forces about the positive x-, y-, and z-axes through O can be obtained from the moment-arm rule, and are

$$M_x = r_y F_z - r_z F_y \qquad M_y = r_z F_x - r_x F_z \qquad M_z = r_x F_y - r_y F_x$$

which agree with the respective terms in the determinant expansion for the cross product $\mathbf{r} \times \mathbf{F}$.

Moment about an Arbitrary Axis

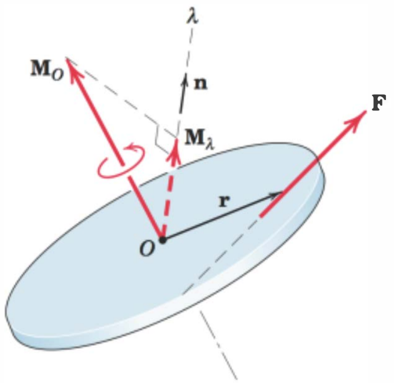


Figure 2/23

We can now obtain an expression for the moment $\mathbf{M}_\lambda$ of **F** about *any* axis λ through O, as shown in Fig. 2/23. If **n** is a unit vector in the λ-direction, then we can use the dot-product expression for the component of a vector as described in Art. 2/7 to obtain $\mathbf{M}_O \cdot \mathbf{n}$, the component of $\mathbf{M}_O$ in the direction of λ. This scalar is the magnitude of the moment $\mathbf{M}_\lambda$ of **F** about λ.

To obtain the vector expression for the moment $\mathbf{M}_\lambda$ of **F** about λ, multiply the magnitude by the directional unit vector **n** to obtain

$$\boxed{\mathbf{M}_\lambda = (\mathbf{r} \times \mathbf{F} \cdot \mathbf{n})\mathbf{n}} \qquad \textbf{(2/16)}$$

where $\mathbf{r} \times \mathbf{F}$ replaces $\mathbf{M}_O$. The expression $\mathbf{r} \times \mathbf{F} \cdot \mathbf{n}$ is known as a *triple scalar product* (see item 8 in Art. C/7, Appendix C). It need not be written $(\mathbf{r} \times \mathbf{F}) \cdot \mathbf{n}$ because a cross product cannot be formed by a vector and a scalar. Thus, the association $\mathbf{r} \times (\mathbf{F} \cdot \mathbf{n})$ would have no meaning.

The triple scalar product may be represented by the determinant

$$\boxed{|\mathbf{M}_\lambda| = M_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ \alpha & \beta & \gamma \end{vmatrix}} \qquad \textbf{(2/17)}$$

where α, β, γ are the direction cosines of the unit vector **n**.

Varignon's Theorem in Three Dimensions

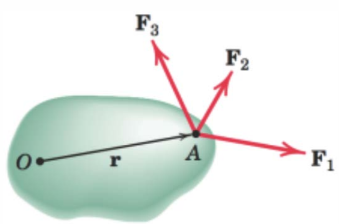


Figure 2/24

In Art. 2/4 we introduced Varignon's theorem in two dimensions. The theorem is easily extended to three dimensions. Figure 2/24 shows a system of concurrent forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots$. The sum of the moments about O of these forces is

$$\begin{aligned} \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \cdots &= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots) \\ &= \mathbf{r} \times \Sigma\mathbf{F} \end{aligned}$$

where we have used the distributive law for cross products. Using the symbol $\mathbf{M}_O$ to represent the sum of the moments on the left side of the above equation, we have

$$\boxed{\mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F}) = \mathbf{r} \times \mathbf{R}} \qquad \textbf{(2/18)}$$

This equation states that the sum of the moments of a system of concurrent forces about a given point equals the moment of their sum about the same point. As mentioned in Art. 2/4, this principle has many applications in mechanics.

Couples in Three Dimensions

The concept of the couple was introduced in Art. 2/5 and is easily extended to three dimensions. Figure 2/25 shows two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ acting on a body. The vector $\mathbf{r}$ runs from *any* point B on the line of action of $-\mathbf{F}$ to *any* point A on the line of action of $\mathbf{F}$. Points A and B are located by position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ from *any* point O. The combined moment of the two forces about O is

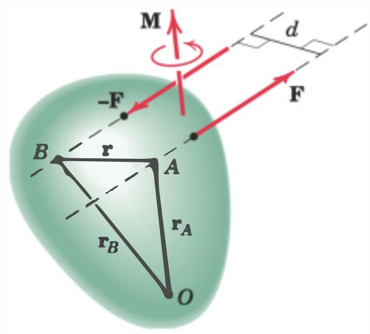


Figure 2/25

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

However, $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, so that all reference to the moment center O disappears, and the moment of the couple becomes

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}} \qquad \textbf{(2/19)}$$

Thus, the moment of a couple is the *same about all points*. The magnitude of $\mathbf{M}$ is $M = Fd$, where d is the perpendicular distance between the lines of action of the two forces, as described in Art. 2/5.

The moment of a couple is a *free vector*, whereas the moment of a force about a point (which is also the moment about a defined axis through the point) is a *sliding vector* whose direction is along the axis through the point. As in the case of two dimensions, a couple tends to produce a pure rotation of the body about an axis normal to the plane of the forces which constitute the couple.

Couple vectors obey all of the rules which govern vector quantities. Thus, in Fig. 2/26 the couple vector $\mathbf{M}_1$ due to $\mathbf{F}_1$ and $-\mathbf{F}_1$ may be added

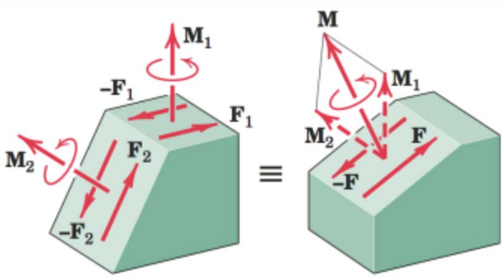
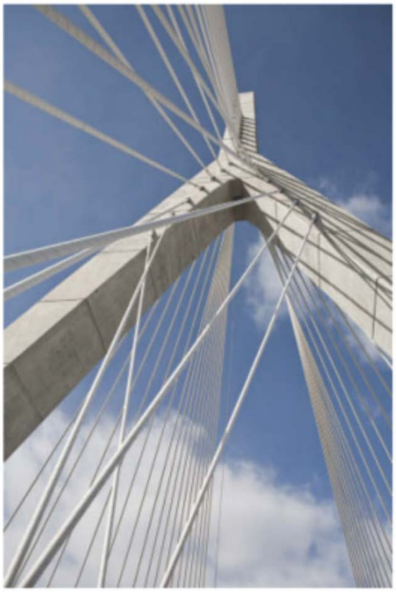


Figure 2/26



Bluemoon/Inmagine.com

The three-dimensionality of the cable system on the Leonard P. Zakim Bunker Hill Bridge is evident in this view.

as shown to the couple vector $\mathbf{M}_2$ due to $\mathbf{F}_2$ and $-\mathbf{F}_2$ to produce the couple $\mathbf{M}$, which, in turn, can be produced by $\mathbf{F}$ and $-\mathbf{F}$.

In Art. 2/5 we learned how to replace a force by its equivalent force–couple system. You should also be able to carry out this replacement in three dimensions. The procedure is represented in Fig. 2/27, where the force $\mathbf{F}$ acting on a rigid body at point A is replaced by an equal force at point B and the couple $\mathbf{M} = \mathbf{r} \times \mathbf{F}$. By adding the equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ at B, we obtain the couple composed of $-\mathbf{F}$ and the original $\mathbf{F}$. Thus, we see that the couple vector is simply the moment of the original force about the point to which the force is being moved. We emphasize that $\mathbf{r}$ is a vector which runs from B to *any* point on the line of action of the original force passing through A.

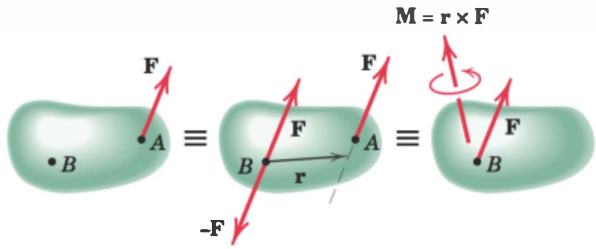
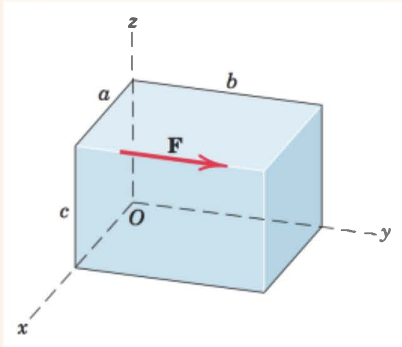
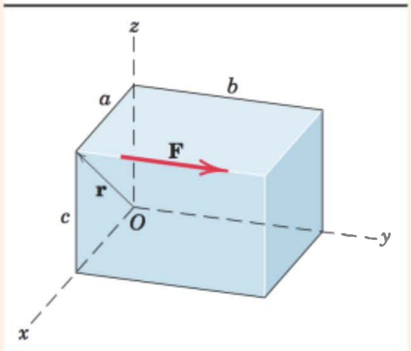


Figure 2/27

Sample Problem 2/11

Determine the moment of force **F** about point O (*a*) by inspection and (*b*) by the formal cross-product definition $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$.





Solution. (*a*) Because **F** is parallel to the y-axis, **F** has no moment about that axis. It should be clear that the moment arm from the x-axis to the line of action of **F** is c and that the moment of **F** about the x-axis is negative. Similarly, the moment arm from the z-axis to the line of action of **F** is a, and the moment of **F** about the z-axis is positive. So we have

$$\mathbf{M}_O = -cF\mathbf{i} + aF\mathbf{k} = F(-c\mathbf{i} + a\mathbf{k}) \qquad \textit{Ans.}$$

(*b*) Formally,

① $$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (a\mathbf{i} + c\mathbf{k}) \times F\mathbf{j} = aF\mathbf{k} - cF\mathbf{i}$$

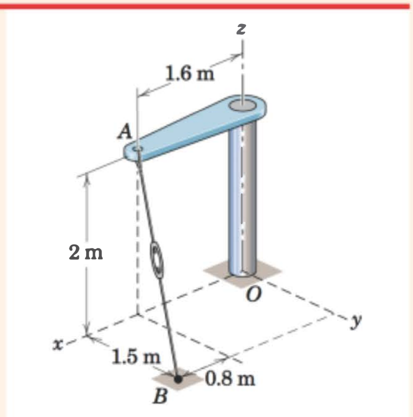
$$= F(-c\mathbf{i} + a\mathbf{k}) \qquad \textit{Ans.}$$

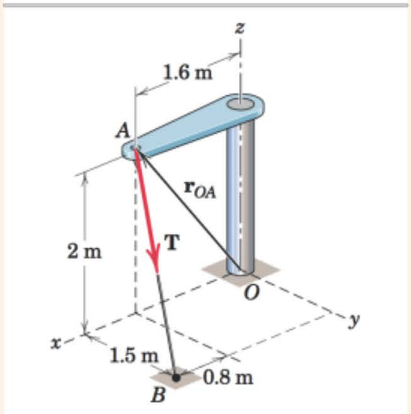
Helpful Hint

① Again we stress that **r** runs *from* the moment center *to* the line of action of **F**. Another permissible, but less convenient, position vector is $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

Sample Problem 2/12

The turnbuckle is tightened until the tension in cable AB is 2.4 kN. Determine the moment about point O of the cable force acting on point A and the magnitude of this moment.





Solution. We begin by writing the described force as a vector.

$$\mathbf{T} = T\mathbf{n}_{AB} = 2.4\left[\frac{0.8\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k}}{\sqrt{0.8^2 + 1.5^2 + 2^2}}\right]$$

$$= 0.731\mathbf{i} + 1.371\mathbf{j} - 1.829\mathbf{k} \text{ kN}$$

The moment of this force about point O is

① $$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T} = (1.6\mathbf{i} + 2\mathbf{k}) \times (0.731\mathbf{i} + 1.371\mathbf{j} - 1.829\mathbf{k})$$

$$= -2.74\mathbf{i} + 4.39\mathbf{j} + 2.19\mathbf{k} \text{ kN·m} \qquad \textit{Ans.}$$

This vector has a magnitude

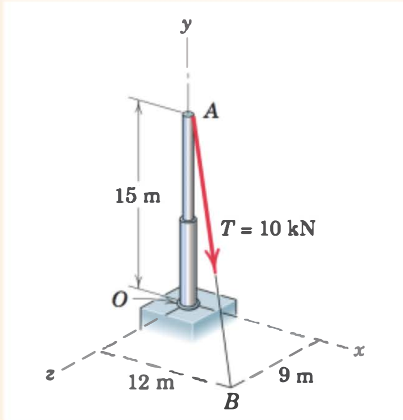
$$M_O = \sqrt{2.74^2 + 4.39^2 + 2.19^2} = 5.62 \text{ kN·m} \qquad \textit{Ans.}$$

Helpful Hint

① The student should verify by inspection the signs of the moment components.

Sample Problem 2/13

A tension $\mathbf{T}$ of magnitude 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground at B. Determine the moment M_z of $\mathbf{T}$ about the z-axis passing through the base O.



Solution (a). The required moment may be obtained by finding the component along the z-axis of the moment $\mathbf{M}_O$ of $\mathbf{T}$ about point O. The vector $\mathbf{M}_O$ is normal to the plane defined by $\mathbf{T}$ and point O, as shown in the accompanying figure. In the use of Eq. 2/14 to find $\mathbf{M}_O$, the vector $\mathbf{r}$ is any vector from point O to
① the line of action of $\mathbf{T}$. The simplest choice is the vector from O to A, which is written as $\mathbf{r} = 15\mathbf{j}$ m. The vector expression for $\mathbf{T}$ is

$$\mathbf{T} = T\mathbf{n}_{AB} = 10\left[\frac{12\mathbf{i} - 15\mathbf{j} + 9\mathbf{k}}{\sqrt{(12)^2 + (-15)^2 + (9)^2}}\right]$$

$$= 10(0.566\mathbf{i} - 0.707\mathbf{j} + 0.424\mathbf{k}) \text{ kN}$$

From Eq. 2/14,

$$[\mathbf{M}_O = \mathbf{r} \times \mathbf{F}] \qquad \mathbf{M}_O = 15\mathbf{j} \times 10(0.566\mathbf{i} - 0.707\mathbf{j} + 0.424\mathbf{k})$$

$$= 150(-0.566\mathbf{k} + 0.424\mathbf{i}) \text{ kN}\cdot\text{m}$$

The value M_z of the desired moment is the scalar component of $\mathbf{M}_O$ in the z-direction or $M_z = \mathbf{M}_O \cdot \mathbf{k}$. Therefore,

$$M_z = 150(-0.566\mathbf{k} + 0.424\mathbf{i})\cdot\mathbf{k} = -84.9 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

② The minus sign indicates that the vector $\mathbf{M}_z$ is in the negative z-direction. Expressed as a vector, the moment is $\mathbf{M}_z = -84.9\mathbf{k}$ kN·m.

Solution (b). The force of magnitude T is resolved into components T_z and T_{xy} in the x-y plane. Since T_z is parallel to the z-axis, it can exert no moment about
③ this axis. The moment M_z is, then, due only to T_{xy} and is $M_z = T_{xy}d$, where d is the perpendicular distance from T_{xy} to O. The cosine of the angle between T and T_{xy} is $\sqrt{15^2 + 12^2}/\sqrt{15^2 + 12^2 + 9^2} = 0.906$, and therefore,

$$T_{xy} = 10(0.906) = 9.06 \text{ kN}$$

The moment arm d equals $\overline{OA}$ multiplied by the sine of the angle between T_{xy} and OA, or

$$d = 15\frac{12}{\sqrt{12^2 + 15^2}} = 9.37 \text{ m}$$

Hence, the moment of $\mathbf{T}$ about the z-axis has the magnitude

$$M_z = 9.06(9.37) = 84.9 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

and is clockwise when viewed in the x-y plane.

Solution (c). The component T_{xy} is further resolved into its components T_x and T_y. It is clear that T_y exerts no moment about the z-axis since it passes through it, so that the required moment is due to T_x alone. The direction cosine of $\mathbf{T}$ with respect to the x-axis is $12/\sqrt{9^2 + 12^2 + 15^2} = 0.566$ so that $T_x = 10(0.566) = 5.66$ kN. Thus,

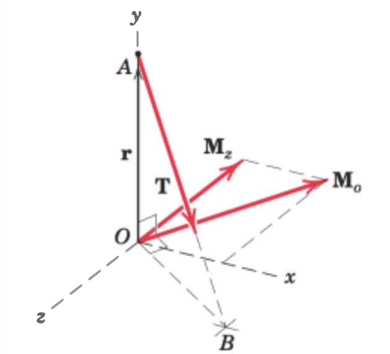
$$M_z = 5.66(15) = 84.9 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

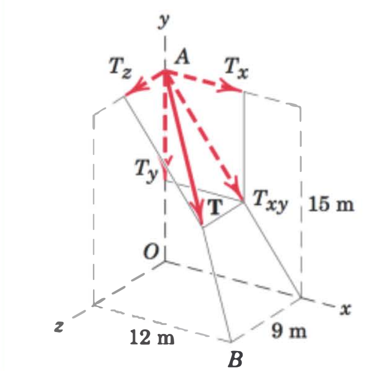
Helpful Hints

① We could also use the vector from O to B for $\mathbf{r}$ and obtain the same result, but using vector OA is simpler.

② It is always helpful to accompany your vector operations with a sketch of the vectors so as to retain a clear picture of the geometry of the problem.

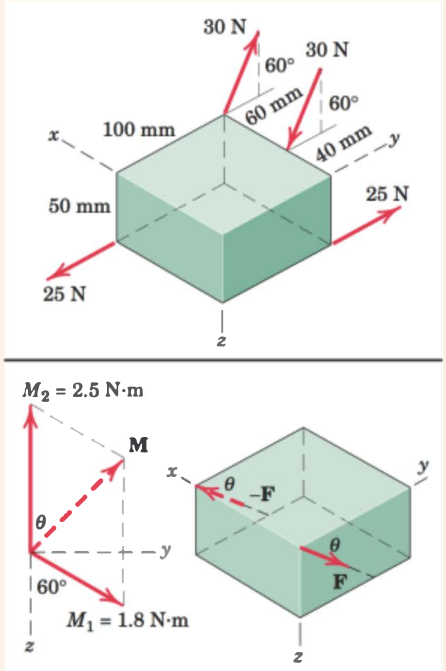
③ Sketch the x-y view of the problem and show d.





Sample Problem 2/14

Determine the magnitude and direction of the couple **M** which will replace the two given couples and still produce the same external effect on the block. Specify the two forces **F** and −**F**, applied in the two faces of the block parallel to the *y-z* plane, which may replace the four given forces. The 30-N forces act parallel to the *y-z* plane.



Solution. The couple due to the 30-N forces has the magnitude $M_1 = 30(0.06) = 1.80$ N·m. The direction of $\mathbf{M}_1$ is normal to the plane defined by the two forces, and the sense, shown in the figure, is established by the right-hand convention. The couple due to the 25-N forces has the magnitude $M_2 = 25(0.10) = 2.50$ N·m with the direction and sense shown in the same figure. The two couple vectors combine to give the components

$$M_y = 1.80 \sin 60° = 1.559 \text{ N·m}$$

$$M_z = -2.50 + 1.80 \cos 60° = -1.600 \text{ N·m}$$

① Thus, $$M = \sqrt{(1.559)^2 + (-1.600)^2} = 2.23 \text{ N·m} \qquad \textit{Ans.}$$

with $$\theta = \tan^{-1}\frac{1.559}{1.600} = \tan^{-1} 0.974 = 44.3° \qquad \textit{Ans.}$$

The forces **F** and −**F** lie in a plane normal to the couple **M**, and their moment arm as seen from the right-hand figure is 100 mm. Thus, each force has the magnitude

$[M = Fd]$ $$F = \frac{2.23}{0.10} = 22.3 \text{ N} \qquad \textit{Ans.}$$

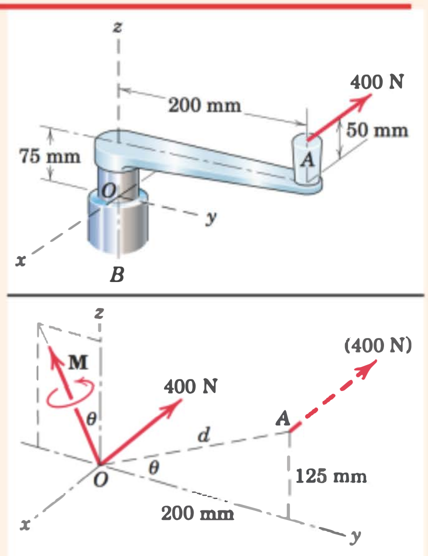
and the direction $\theta = 44.3°$.

Helpful Hint

① Bear in mind that the couple vectors are *free vectors* and therefore have no unique lines of action.

Sample Problem 2/15

A force of 400 N is applied at *A* to the handle of the control lever which is attached to the fixed shaft *OB*. In determining the effect of the force on the shaft at a cross section such as that at *O*, we may replace the force by an equivalent force at *O* and a couple. Describe this couple as a vector **M**.



Solution. The couple may be expressed in vector notation as $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, where $\mathbf{r} = \overrightarrow{OA} = 0.2\mathbf{j} + 0.125\mathbf{k}$ m and $\mathbf{F} = -400\mathbf{i}$ N. Thus,

$$\mathbf{M} = (0.2\mathbf{j} + 0.125\mathbf{k}) \times (-400\mathbf{i}) = -50\mathbf{j} + 80\mathbf{k} \text{ N·m}$$

Alternatively we see that moving the 400-N force through a distance $d = \sqrt{0.125^2 + 0.2^2} = 0.236$ m to a parallel position through *O* requires the addition of a couple **M** whose magnitude is

$$M = Fd = 400(0.236) = 94.3 \text{ N·m} \qquad \textit{Ans.}$$

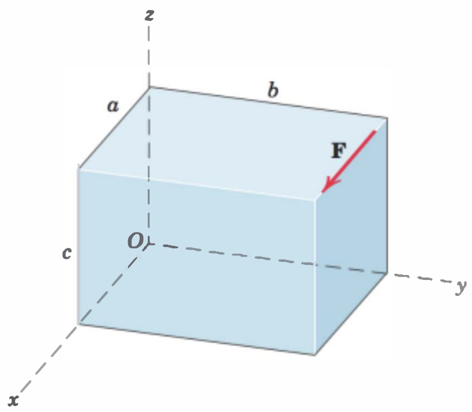
The couple vector is perpendicular to the plane in which the force is shifted, and its sense is that of the moment of the given force about *O*. The direction of **M** in the *y-z* plane is given by

$$\theta = \tan^{-1}\frac{125}{200} = 32.0° \qquad \textit{Ans.}$$

PROBLEMS

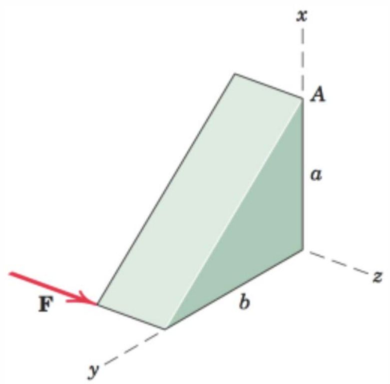
Introductory Problems

2/121 Determine the moment of force **F** about point O.



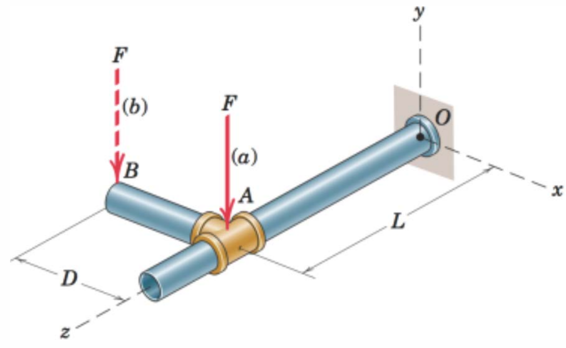
Problem 2/121

2/122 Determine the moment of force **F** about point A.



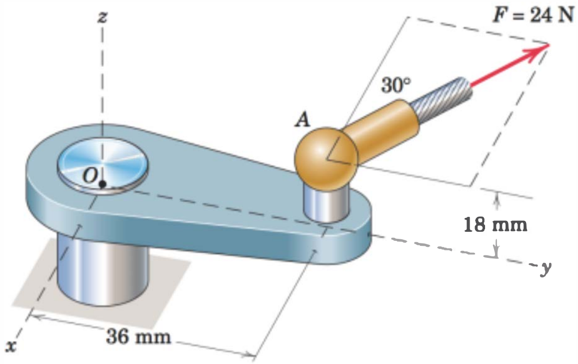
Problem 2/122

2/123 Determine the moment about O of the force of magnitude F for the case (a) when the force **F** is applied at A and for the case (b) when **F** is applied at B.



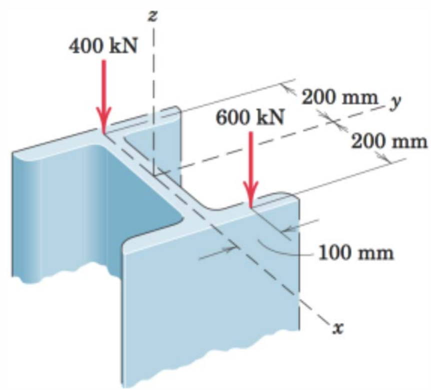
Problem 2/123

2/124 The 24-N force is applied at point A of the crank assembly. Determine the moment of this force about point O.



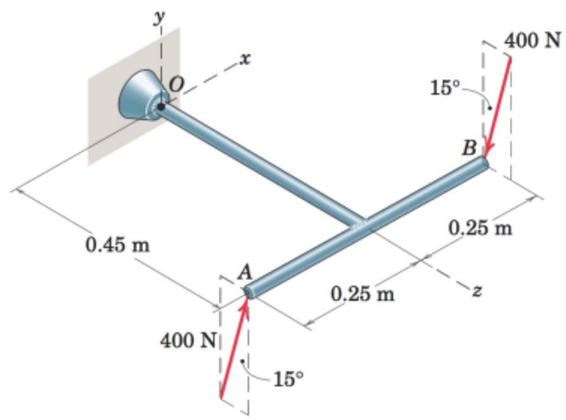
Problem 2/124

2/125 The steel H-beam is being designed as a column to support the two vertical forces shown. Replace these forces by a single equivalent force along the vertical centerline of the column and a couple **M**.



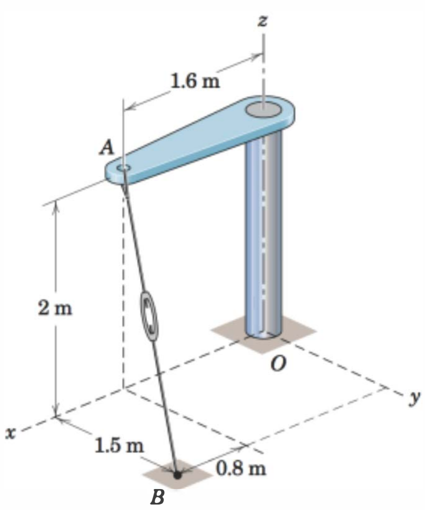
Problem 2/125

2/126 Determine the moment associated with the pair of 400-N forces applied to the T-shaped structure.



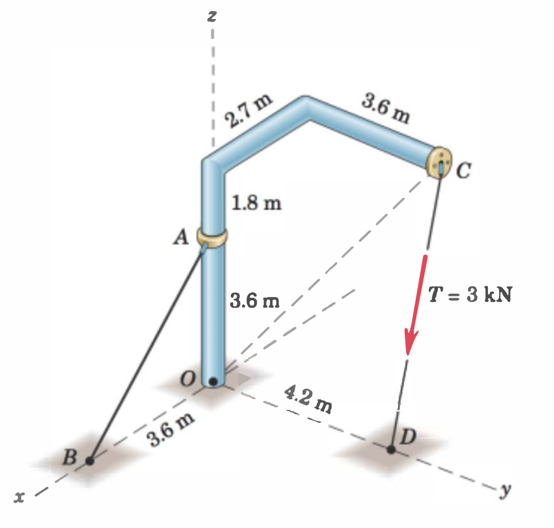
Problem 2/126

2/127 The turnbuckle is tightened until the tension in cable AB is 1.2 kN. Calculate the magnitude of the moment about point O of the force acting on point A.



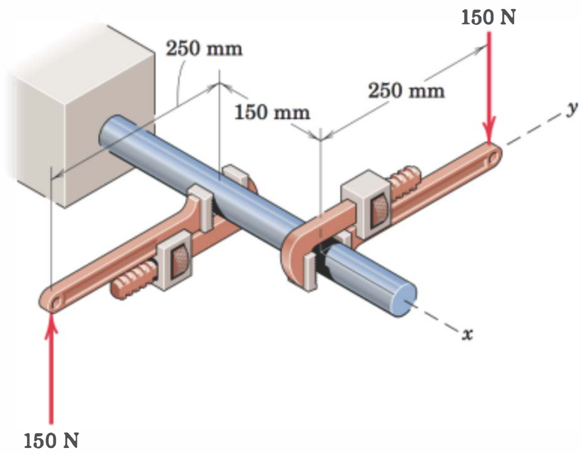
Problem 2/127

2/128 The system of Prob. 2/111 is repeated here, and the tension in cable CD is $T = 3$ kN. Consider the force which this cable exerts on point C and determine its moment about point O.



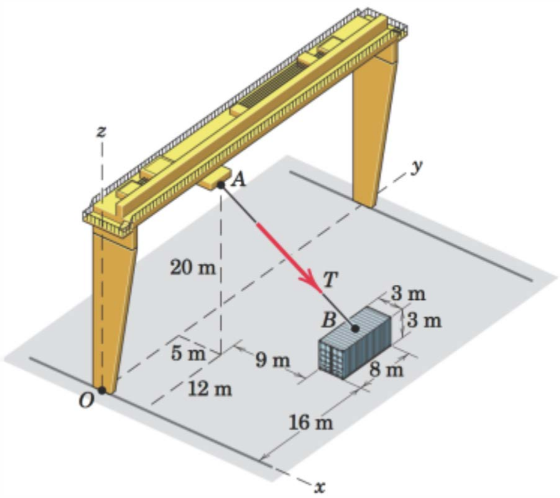
Problem 2/128

2/129 The two forces acting on the handles of the pipe wrenches constitute a couple **M**. Express the couple as a vector.



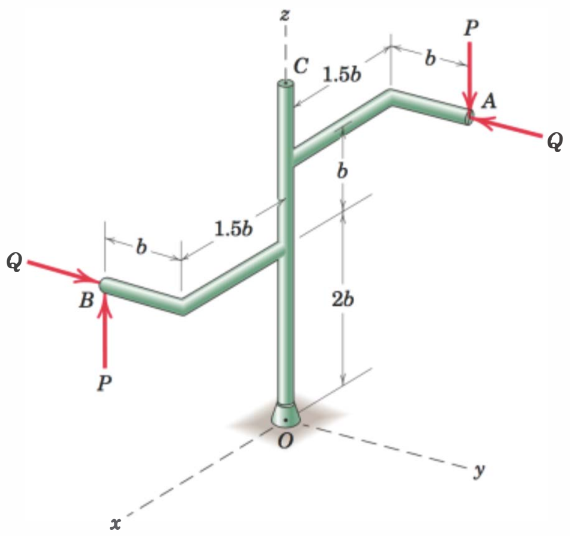
Problem 2/129

2/130 The gantry crane of Prob. 2/105 is repeated here, and the tension in cable *AB* is 14 kN. Replace this force as it acts on point *A* by an equivalent force–couple system at *O*. Point *B* is located at the center of the container top.



Problem 2/130

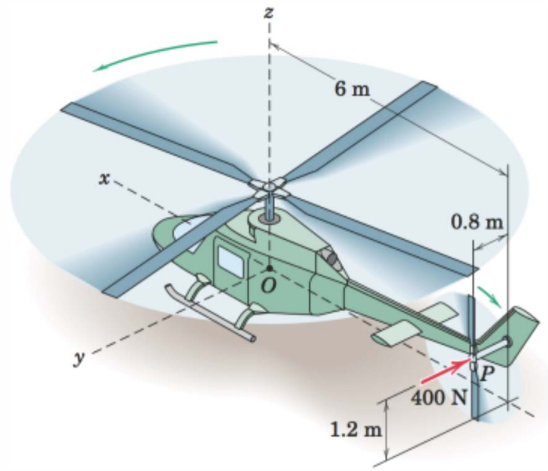
2/131 Determine the combined moment made by the two pairs of forces about point *O* and about point *C*. Use the values $P = 4$ kN, $Q = 7.5$ kN, and $b = 3$ m.



Problem 2/131

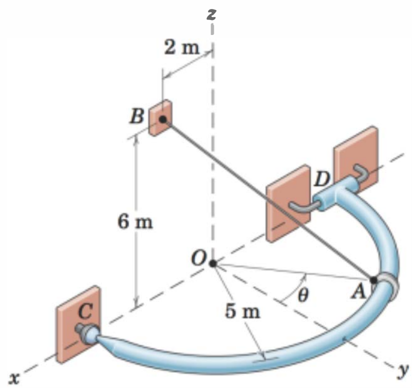
Representative Problems

2/132 A helicopter is shown here with certain three-dimensional geometry given. During a ground test, a 400-N aerodynamic force is applied to the tail rotor at *P* as shown. Determine the moment of this force about point *O* of the airframe.



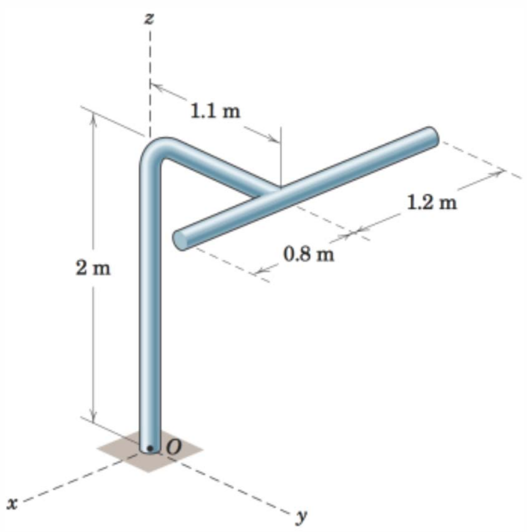
Problem 2/132

2/133 The system of Prob. 2/108 is repeated here, and the tension in the supporting cable *AB* is 425 N. Determine the magnitude of the moment which this force, as it acts at point *A*, makes about the *x*-axis. Use a value of $\theta = 30°$.



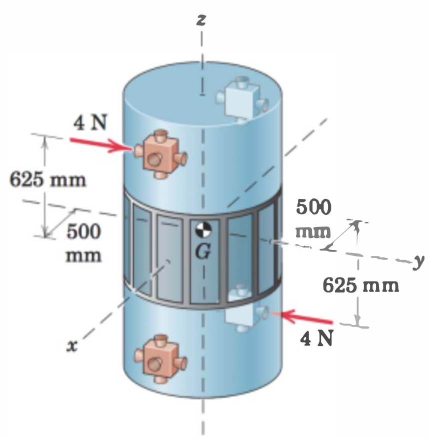
Problem 2/133

2/134 The structure shown is constructed of circular rod which has a mass of 7 kg per meter of length. Determine the moment $\mathbf{M}_O$ about O caused by the weight of the structure. Find the magnitude of $\mathbf{M}_O$.



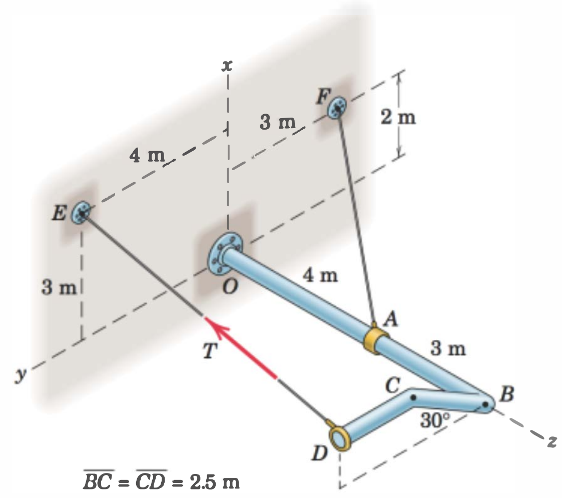
Problem 2/134

2/135 Two 4-N thrusters on the nonrotating satellite are simultaneously fired as shown. Compute the moment associated with this couple and state about which satellite axes rotations will begin to occur.



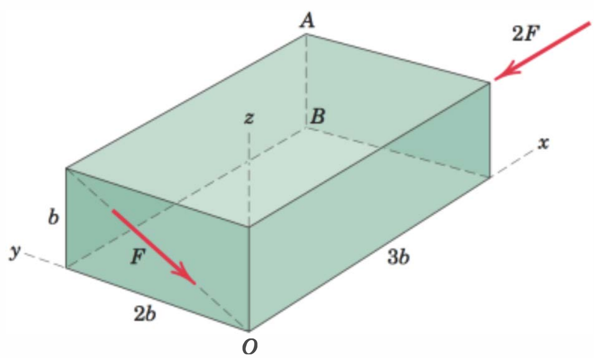
Problem 2/135

2/136 If the tension in cable DE is 575 N, determine the moments of this tensile force (as it acts at point D) about point O and about line OF.



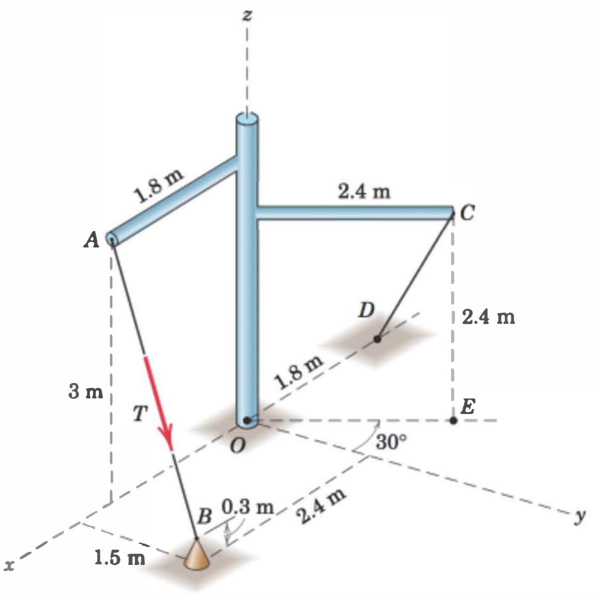
Problem 2/136

2/137 Determine the moment of each individual force about (*a*) point A and (*b*) point B.



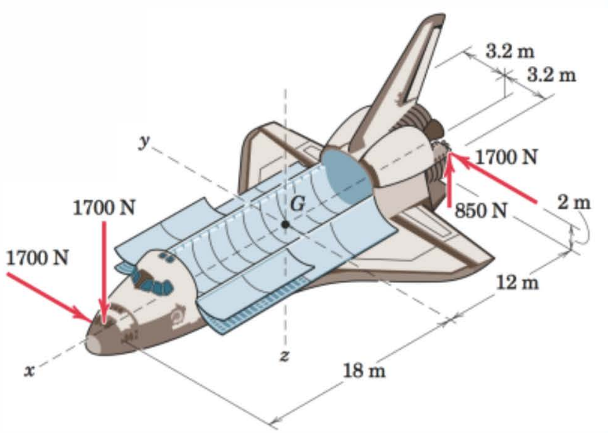
Problem 2/137

2/138 The system of Prob. 2/107 is repeated here, and the tension in cable AB is 8 kN. Consider the force which this cable exerts on point A and determine the equivalent force–couple system at point O.



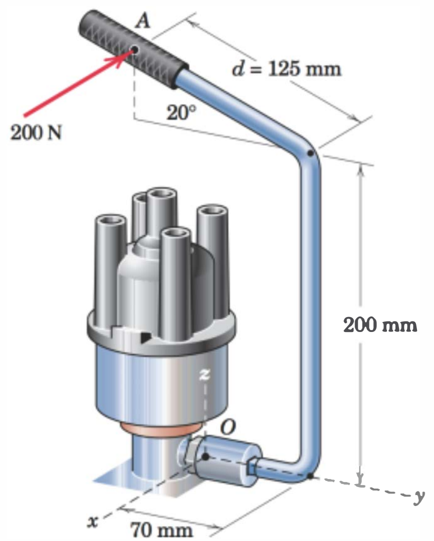
Problem 2/138

2/139 A space shuttle orbiter is subjected to thrusts from five of the engines of its reaction control system. Four of the thrusts are shown in the figure; the fifth is an 850-N upward thrust at the right rear, symmetric to the 850-N thrust shown on the left rear. Compute the moment of these forces about point G and show that the forces have the same moment about all points.



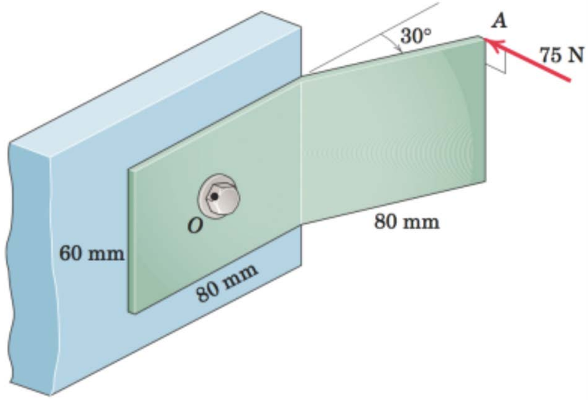
Problem 2/139

2/140 The specialty wrench shown in the figure is designed for access to the hold-down bolt on certain automobile distributors. For the configuration shown where the wrench lies in a vertical plane and a horizontal 200-N force is applied at A perpendicular to the handle, calculate the moment $\mathbf{M}_O$ applied to the bolt at O. For what value of the distance d would the z-component of $\mathbf{M}_O$ be zero?



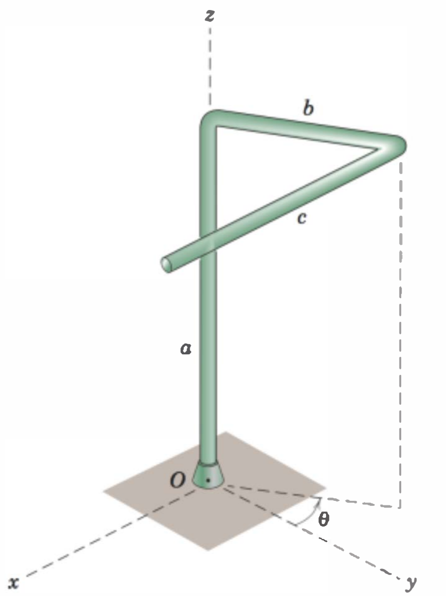
Problem 2/140

2/141 The 75-N force acts perpendicular to the bent portion of the wall bracket shown. Determine the magnitude of the moment made by this force about point O, which is at the center of the 60 × 80-mm portion of the plate in contact with the wall.



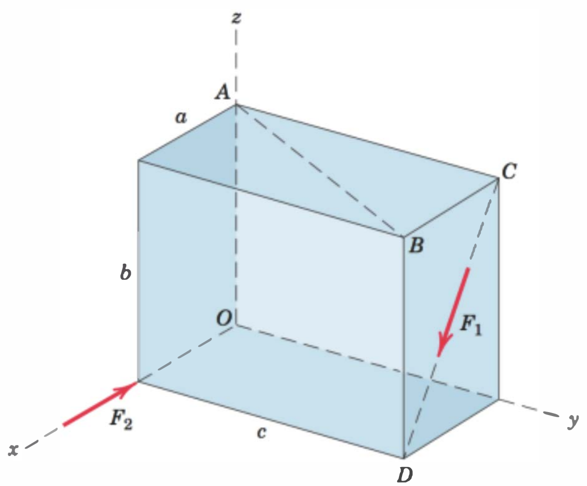
Problem 2/141

2/142 The body is composed of a slender uniform rod bent into the shape shown and having a mass ρ per unit length. Determine the expression for the moment of the weight of the structure about the base O. Evaluate your result for the values $a = 5$ m, $b = 2.5$ m, $c = 4$ m, $\theta = 30°$, and $\rho = 24$ kg/m. What value of c would render the moment about the y-axis equal to zero?



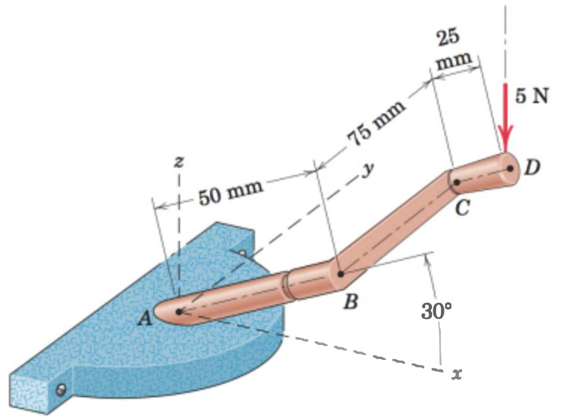
Problem 2/142

2/143 If $F_1 = 450$ N and the magnitude of the moment of both forces about line AB is 30 N·m, determine the magnitude of $\mathbf{F}_2$. Use the values $a = 200$ mm, $b = 400$ mm, and $c = 500$ mm.



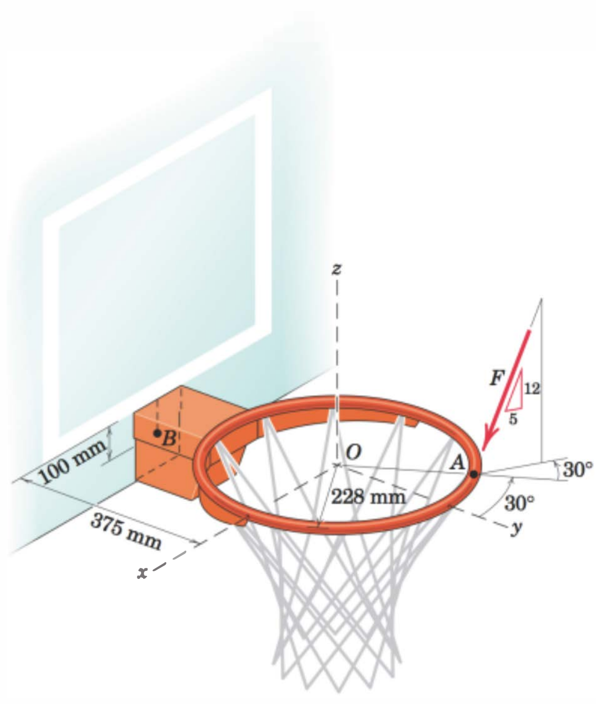
Problem 2/143

2/144 A 5-N vertical force is applied to the knob of the window-opener mechanism when the crank BC is horizontal. Determine the moment of the force about point A and about line AB.



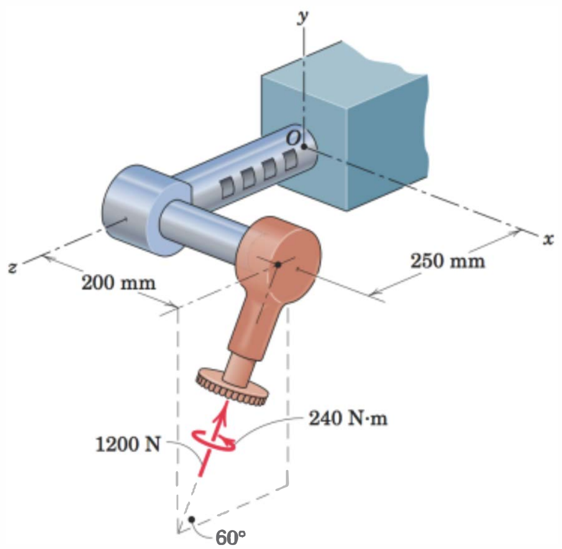
Problem 2/144

2/145 A basketball player applies a force $F = 275$ N to the rim at A. Determine the equivalent force–couple system at point B, which is at the center of the rim mounting bracket on the backboard.



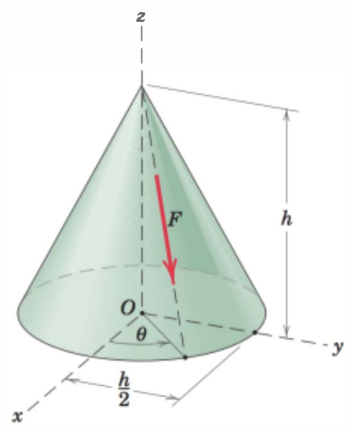
Problem 2/145

2/146 The special-purpose milling cutter is subjected to the force of 1200 N and a couple of 240 N·m as shown. Determine the moment of this system about point O.



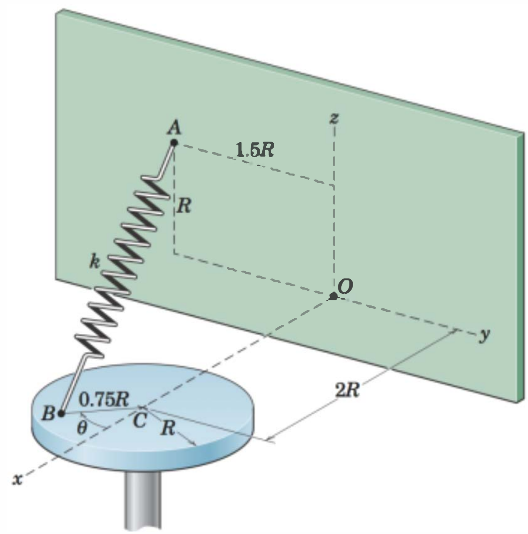
Problem 2/146

2/147 The force F acts along an element of the right circular cone as shown. Determine the equivalent force–couple system at point O.



Problem 2/147

***2/148** The spring of stiffness k and unstretched length $1.5R$ is attached to the disk at a radial distance of $0.75R$ from the center C. Considering the tension in the spring to act on point A, plot the moment which the spring tension creates about each of the three coordinate axes at O during one revolution of the disk ($0 \leq \theta \leq 360°$). Determine the maximum magnitude attained by each moment component along with the corresponding angle of rotation θ at which it occurs. Finally, determine the overall maximum magnitude for the moment of the spring tension about O along with the corresponding angle of rotation θ.



Problem 2/148

2/9 Resultants

In Art. 2/6 we defined the resultant as the simplest force combination which can replace a given system of forces without altering the external effect on the rigid body on which the forces act. We found the magnitude and direction of the resultant force for the two-dimensional force system by a vector summation of forces, Eq. 2/9, and we located the line of action of the resultant force by applying the principle of moments, Eq. 2/10. These same principles can be extended to three dimensions.

In the previous article we showed that a force could be moved to a parallel position by adding a corresponding couple. Thus, for the system of forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ . . . acting on a rigid body in Fig. 2/28*a*, we may move each of them in turn to the arbitrary point O, provided we also introduce a couple for each force transferred. Thus, for example, we may move force $\mathbf{F}_1$ to O, provided we introduce the couple $\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1$, where $\mathbf{r}_1$ is a vector from O to any point on the line of action of $\mathbf{F}_1$. When all forces are shifted to O in this manner, we have a system of concurrent forces at O and a system of couple vectors, as represented in part *b* of the figure. The concurrent forces may then be added vectorially to produce a resultant force $\mathbf{R}$, and the couples may also be added to produce a resultant couple $\mathbf{M}$, Fig. 2/28*c*. The general force system, then, is reduced to

$$\begin{aligned} \mathbf{R} &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F} \\ \mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F}) \end{aligned} \tag{2/20}$$

The couple vectors are shown through point O, but because they are free vectors, they may be represented in any parallel positions. The magnitudes of the resultants and their components are

$$\begin{gathered} R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z \\ R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2} \\ \mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \qquad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \qquad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z \\ M = \sqrt{M_x^2 + M_y^2 + M_z^2} \end{gathered} \tag{2/21}$$

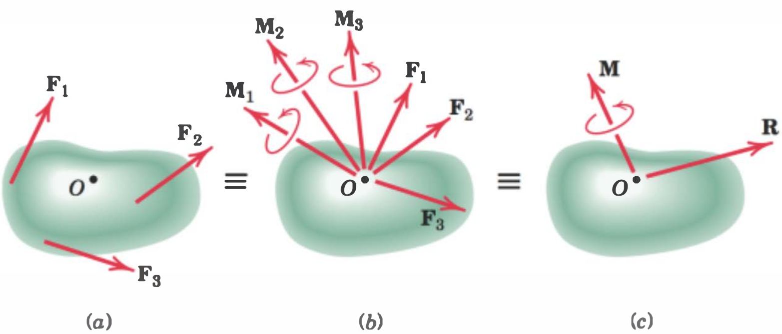


Figure 2/28

The cables of this cable-stayed bridge exert a three-dimensional system of concentrated forces on the bridge tower.

The point O selected as the point of concurrency for the forces is arbitrary, and the magnitude and direction of $\mathbf{M}$ depend on the particular point O selected. The magnitude and direction of $\mathbf{R}$, however, are the same no matter which point is selected.

In general, any system of forces may be replaced by its resultant force $\mathbf{R}$ and the resultant couple $\mathbf{M}$. In dynamics we usually select the mass center as the reference point. The change in the linear motion of the body is determined by the resultant force, and the change in the angular motion of the body is determined by the resultant couple. In statics, the body is in *complete equilibrium* when the resultant force $\mathbf{R}$ is zero and the resultant couple $\mathbf{M}$ is also zero. Thus, the determination of resultants is essential in both statics and dynamics.

We now examine the resultants for several special force systems.

Concurrent Forces. When forces are concurrent at a point, only the first of Eqs. 2/20 needs to be used because there are no moments about the point of concurrency.

Parallel Forces. For a system of parallel forces not all in the same plane, the magnitude of the parallel resultant force $\mathbf{R}$ is simply the magnitude of the algebraic sum of the given forces. The position of its line of action is obtained from the principle of moments by requiring that $\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$. Here $\mathbf{r}$ is a position vector extending from the force–couple reference point O to the final line of action of $\mathbf{R}$, and $\mathbf{M}_O$ is the sum of the moments of the individual forces about O. See Sample Problem 2/17 for an example of parallel-force systems.

Coplanar Forces. Article 2/6 was devoted to this force system.

Wrench Resultant. When the resultant couple vector $\mathbf{M}$ is parallel to the resultant force $\mathbf{R}$, as shown in Fig. 2/29, the resultant is called a *wrench*. By definition a wrench is positive if the couple and force vectors point in the same direction and negative if they point in opposite directions. A common example of a positive wrench is found with the application of a screwdriver, to drive a right-handed screw. Any general force system may be represented by a wrench applied along a unique line of action. This reduction is illustrated in Fig. 2/30, where part a of the figure represents, for the general force system, the resultant force $\mathbf{R}$ acting at some point O and the corresponding resultant couple $\mathbf{M}$. Although $\mathbf{M}$ is a free vector, for convenience we represent it as acting through O.

In part b of the figure, $\mathbf{M}$ is resolved into components $\mathbf{M}_1$ along the direction of $\mathbf{R}$ and $\mathbf{M}_2$ normal to $\mathbf{R}$. In part c of the figure, the couple $\mathbf{M}_2$ is replaced by its equivalent of two forces $\mathbf{R}$ and $-\mathbf{R}$ separated by a distance

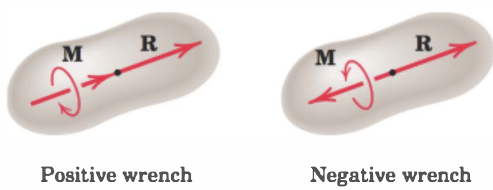


Figure 2/29

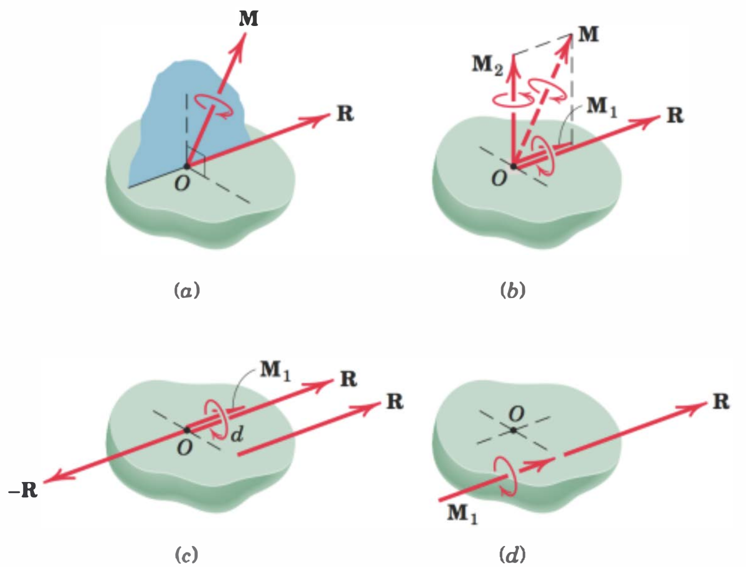
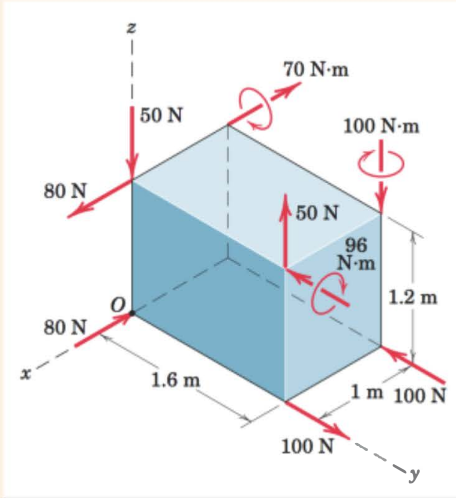


Figure 2/30

$d = M_2/R$ with $-\mathbf{R}$ applied at O to cancel the original $\mathbf{R}$. This step leaves the resultant $\mathbf{R}$, which acts along a new and unique line of action, and the parallel couple $\mathbf{M}_1$, which is a free vector, as shown in part *d* of the figure. Thus, the resultants of the original general force system have been transformed into a wrench (positive in this illustration) with its unique axis defined by the new position of $\mathbf{R}$.

We see from Fig. 2/30 that the axis of the wrench resultant lies in a plane through O normal to the plane defined by $\mathbf{R}$ and $\mathbf{M}$. The wrench is the simplest form in which the resultant of a general force system may be expressed. This form of the resultant, however, has limited application, because it is usually more convenient to use as the reference point some point O such as the mass center of the body or another convenient origin of coordinates not on the wrench axis.

Sample Problem 2/16



Determine the resultant of the force and couple system which acts on the rectangular solid.

Solution. We choose point O as a convenient reference point for the initial step of reducing the given forces to a force–couple system. The resultant force is

① $$\mathbf{R} = \Sigma\mathbf{F} = (80 - 80)\mathbf{i} + (100 - 100)\mathbf{j} + (50 - 50)\mathbf{k} = \mathbf{0}\ \text{N}$$

The sum of the moments about O is

② $$\mathbf{M}_O = [50(1.6) - 70]\mathbf{i} + [80(1.2) - 96]\mathbf{j} + [100(1) - 100]\mathbf{k}$$
$$= 10\mathbf{i}\ \text{N}\cdot\text{m}$$

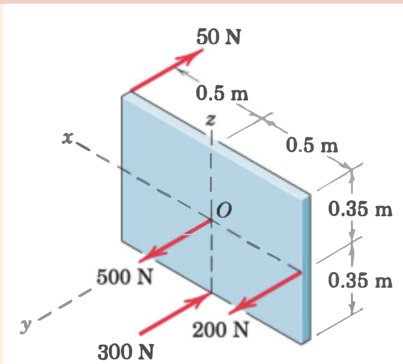
Hence, the resultant consists of a couple, which of course may be applied at any point on the body or the body extended.

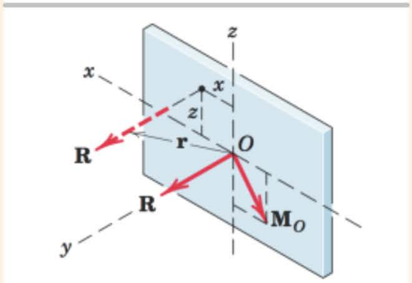
Helpful Hints

① Since the force summation is zero, we conclude that the resultant, if it exists, must be a couple.

② The moments associated with the force pairs are easily obtained by using the $M = Fd$ rule and assigning the unit-vector direction by inspection. In many three-dimensional problems, this may be simpler than the $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ approach.

Sample Problem 2/17





Determine the resultant of the system of parallel forces which act on the plate. Solve with a vector approach.

Solution. Transfer of all forces to point O results in the force–couple system

$$\mathbf{R} = \Sigma\mathbf{F} = (200 + 500 - 300 - 50)\mathbf{j} = 350\mathbf{j}\ \text{N}$$

$$\mathbf{M}_O = [50(0.35) - 300(0.35)]\mathbf{i} + [-50(0.50) - 200(0.50)]\mathbf{k}$$
$$= -87.5\mathbf{i} - 125\mathbf{k}\ \text{N}\cdot\text{m}$$

The placement of $\mathbf{R}$ so that it alone represents the above force–couple system is determined by the principle of moments in vector form

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$
$$(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times 350\mathbf{j} = -87.5\mathbf{i} - 125\mathbf{k}$$
$$350x\mathbf{k} - 350z\mathbf{i} = -87.5\mathbf{i} - 125\mathbf{k}$$

From the one vector equation we may obtain the two scalar equations

$$350x = -125 \quad \text{and} \quad -350z = -87.5$$

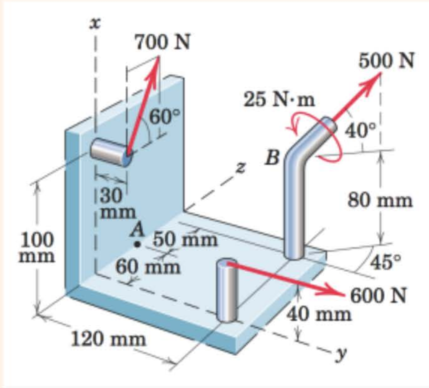
① Hence, $x = -0.357$ m and $z = 0.250$ m are the coordinates through which the line of action of $\mathbf{R}$ must pass. The value of y may, of course, be any value, as permitted by the principle of transmissibility. Thus, as expected, the variable y drops out of the above vector analysis.

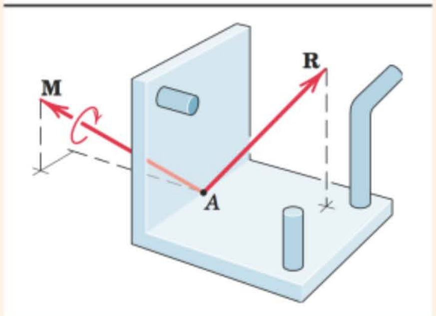
Helpful Hint

① You should also carry out a scalar solution to this problem.

Sample Problem 2/18

Replace the two forces and the negative wrench by a single force **R** applied at *A* and the corresponding couple **M**.





Solution. The resultant force has the components

$[R_x = \Sigma F_x]$ $\quad R_x = 500 \sin 40° + 700 \sin 60° = 928 \text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 600 + 500 \cos 40° \cos 45° = 871 \text{ N}$

$[R_z = \Sigma F_z]$ $\quad R_z = 700 \cos 60° + 500 \cos 40° \sin 45° = 621 \text{ N}$

Thus,
$$\mathbf{R} = 928\mathbf{i} + 871\mathbf{j} + 621\mathbf{k} \text{ N}$$

and
$$R = \sqrt{(928)^2 + (871)^2 + (621)^2} = 1416 \text{ N} \qquad \textit{Ans.}$$

The couple to be added as a result of moving the 500-N force is

① $[\mathbf{M} = \mathbf{r} \times \mathbf{F}]$ $\quad \mathbf{M}_{500} = (0.08\mathbf{i} + 0.12\mathbf{j} + 0.05\mathbf{k}) \times 500(\mathbf{i} \sin 40° + \mathbf{j} \cos 40° \cos 45° + \mathbf{k} \cos 40° \sin 45°)$

where **r** is the vector from *A* to *B*.

The term-by-term, or determinant, expansion gives

$$\mathbf{M}_{500} = 18.95\mathbf{i} - 5.59\mathbf{j} - 16.90\mathbf{k} \text{ N·m}$$

② The moment of the 600-N force about *A* is written by inspection of its *x*- and *z*-components, which gives

$$\mathbf{M}_{600} = (600)(0.060)\mathbf{i} + (600)(0.040)\mathbf{k}$$
$$= 36.0\mathbf{i} + 24.0\mathbf{k} \text{ N·m}$$

The moment of the 700-N force about *A* is easily obtained from the moments of the *x*- and *z*-components of the force. The result becomes

$$\mathbf{M}_{700} = (700 \cos 60°)(0.030)\mathbf{i} - [(700 \sin 60°)(0.060) + (700 \cos 60°)(0.100)]\mathbf{j} - (700 \sin 60°)(0.030)\mathbf{k}$$
$$= 10.5\mathbf{i} - 71.4\mathbf{j} - 18.19\mathbf{k} \text{ N·m}$$

③ Also, the couple of the given wrench may be written

$$\mathbf{M}' = 25.0(-\mathbf{i} \sin 40° - \mathbf{j} \cos 40° \cos 45° - \mathbf{k} \cos 40° \sin 45°)$$
$$= -16.07\mathbf{i} - 13.54\mathbf{j} - 13.54\mathbf{k} \text{ N·m}$$

Therefore, the resultant couple on adding together the **i**-, **j**-, and **k**-terms of the four **M**'s is

④ $$\mathbf{M} = 49.4\mathbf{i} - 90.5\mathbf{j} - 24.6\mathbf{k} \text{ N·m}$$

and
$$M = \sqrt{(49.4)^2 + (90.5)^2 + (24.6)^2} = 106.0 \text{ N·m} \qquad \textit{Ans.}$$

Helpful Hints

① *Suggestion:* Check the cross-product results by evaluating the moments about *A* of the components of the 500-N force directly from the sketch.

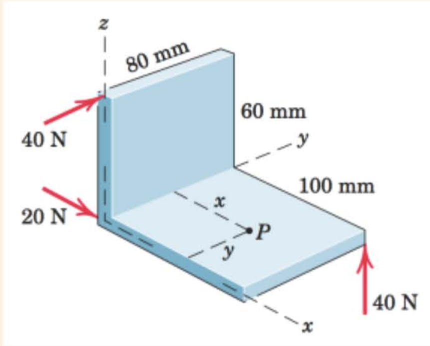
② For the 600-N and 700-N forces it is easier to obtain the components of their moments about the coordinate directions through *A* by inspection of the figure than it is to set up the cross-product relations.

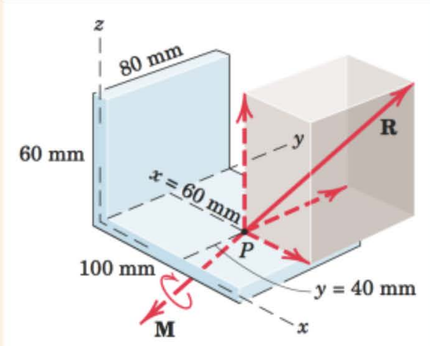
③ The 25-N·m couple vector of the *wrench* points in the direction opposite to that of the 500-N force, and we must resolve it into its *x*-, *y*-, and *z*-components to be added to the other couple-vector components.

④ Although the resultant couple vector **M** in the sketch of the resultants is shown through *A*, we recognize that a couple vector is a free vector and therefore has no specified line of action.

Sample Problem 2/19

Determine the wrench resultant of the three forces acting on the bracket. Calculate the coordinates of the point P in the x-y plane through which the resultant force of the wrench acts. Also find the magnitude of the couple $\mathbf{M}$ of the wrench.





Solution. The direction cosines of the couple $\mathbf{M}$ of the wrench must be the
① same as those of the resultant force $\mathbf{R}$, assuming that the wrench is positive. The resultant force is

$$\mathbf{R} = 20\mathbf{i} + 40\mathbf{j} + 40\mathbf{k}\text{ N} \qquad R = \sqrt{(20)^2 + (40)^2 + (40)^2} = 60\text{ N}$$

and its direction cosines are

$$\cos\theta_x = 20/60 = 1/3 \qquad \cos\theta_y = 40/60 = 2/3 \qquad \cos\theta_z = 40/60 = 2/3$$

The moment of the wrench couple must equal the sum of the moments of the given forces about point P through which $\mathbf{R}$ passes. The moments about P of the three forces are

$$(\mathbf{M})_{R_1} = 20y\mathbf{k}\text{ N}\cdot\text{mm}$$

$$(\mathbf{M})_{R_2} = -40(60)\mathbf{i} - 40x\mathbf{k}\text{ N}\cdot\text{mm}$$

$$(\mathbf{M})_{R_3} = 40(80 - y)\mathbf{i} - 40(100 - x)\mathbf{j}\text{ N}\cdot\text{mm}$$

and the total moment is

$$\mathbf{M} = (800 - 40y)\mathbf{i} + (-4000 + 40x)\mathbf{j} + (-40x + 20y)\mathbf{k}\text{ N}\cdot\text{mm}$$

The direction cosines of $\mathbf{M}$ are

$$\cos\theta_x = (800 - 40y)/M$$

$$\cos\theta_y = (-4000 + 40x)/M$$

$$\cos\theta_z = (-40x + 20y)/M$$

where M is the magnitude of $\mathbf{M}$. Equating the direction cosines of $\mathbf{R}$ and $\mathbf{M}$ gives

$$800 - 40y = \frac{M}{3}$$

$$-4000 + 40x = \frac{2M}{3}$$

$$-40x + 20y = \frac{2M}{3}$$

Solution of the three equations gives

$$M = -2400\text{ N}\cdot\text{mm} \qquad x = 60\text{ mm} \qquad y = 40\text{ mm} \qquad \textit{Ans.}$$

We see that M turned out to be negative, which means that the couple vector is pointing in the direction opposite to $\mathbf{R}$, which makes the wrench negative.

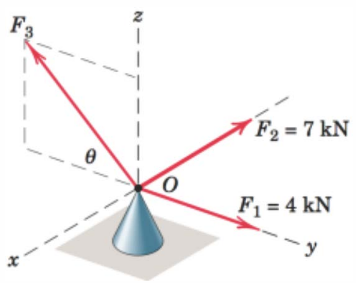
Helpful Hint

① We assume initially that the wrench is positive. If M turns out to be negative, then the direction of the couple vector is opposite to that of the resultant force.

PROBLEMS

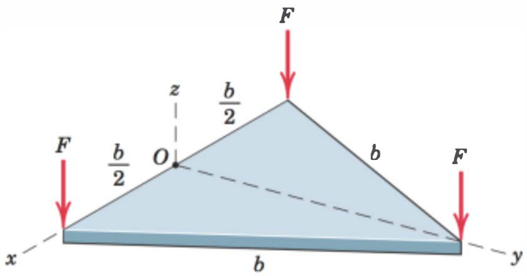
Introductory Problems

2/149 Three forces act at point O. If it is known that the y-component of the resultant $\mathbf{R}$ is -5 kN and that the z-component is 6 kN, determine F_3, θ, and R.



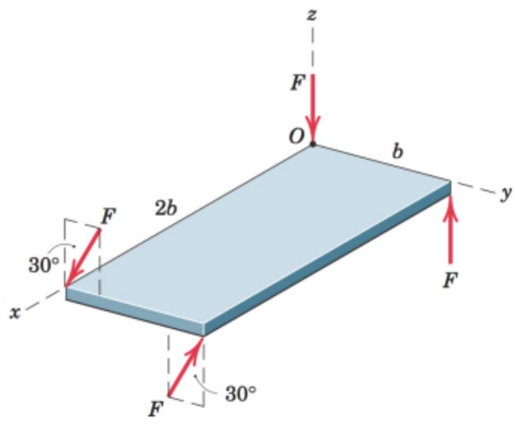
Problem 2/149

2/150 Three equal forces are exerted on the equilateral plate as shown. Reduce the force system to an equivalent force–couple system at point O. Show that $\mathbf{R}$ is perpendicular to $\mathbf{M}_O$.



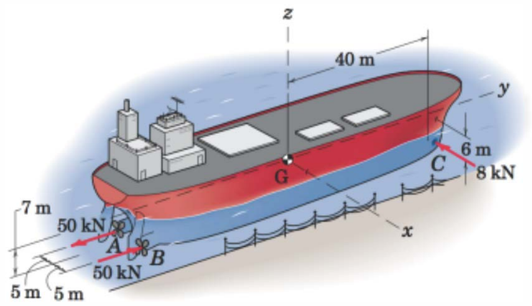
Problem 2/150

2/151 The thin rectangular plate is subjected to the four forces shown. Determine the equivalent force–couple system at O. What is the resultant of the system?



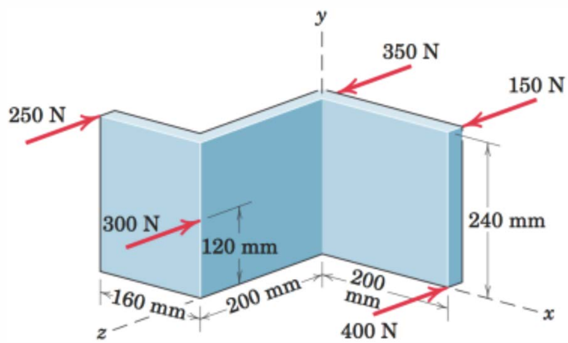
Problem 2/151

2/152 An oil tanker moves away from its docked position under the action of reverse thrust from screw A, forward thrust from screw B, and side thrust from the bow thruster C. Determine the equivalent force–couple system at the mass center G.



Problem 2/152

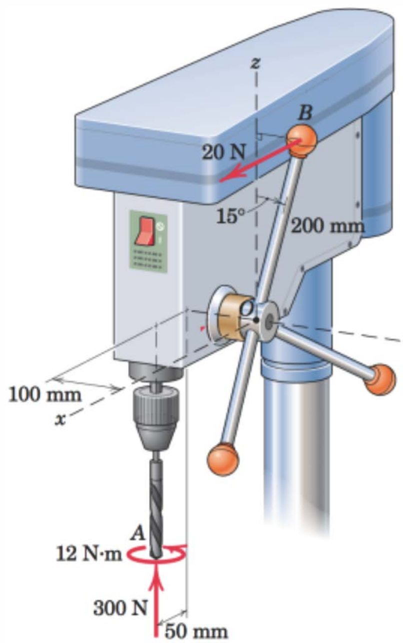
2/153 Determine the x- and y-coordinates of a point through which the resultant of the parallel forces passes.



Problem 2/153

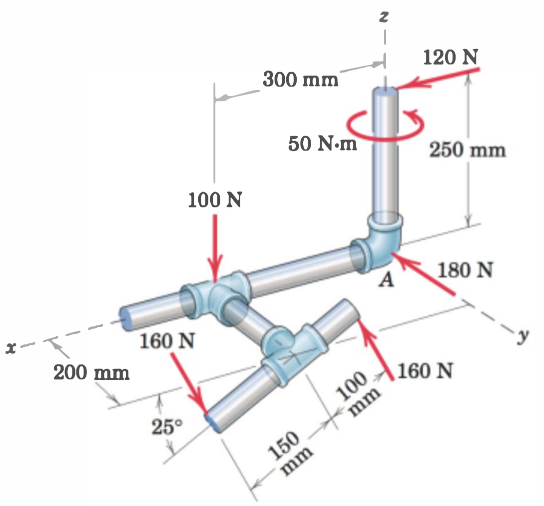
Representative Problems

2/154 The two forces and one couple act on the elements of a drill press as shown. Determine the equivalent force–couple system at point O.



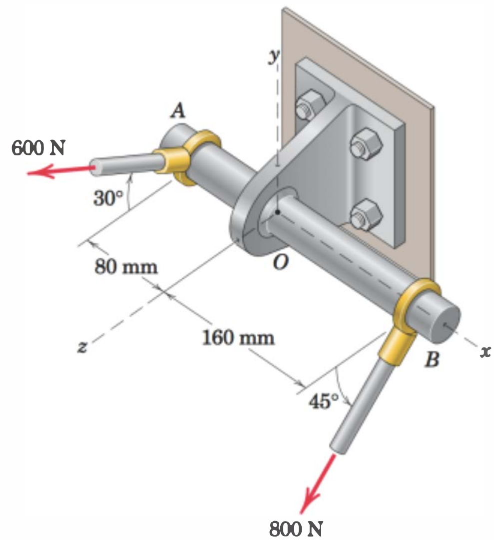
Problem 2/154

2/155 Represent the resultant of the force system acting on the pipe assembly by a single force **R** at A and a couple **M**.



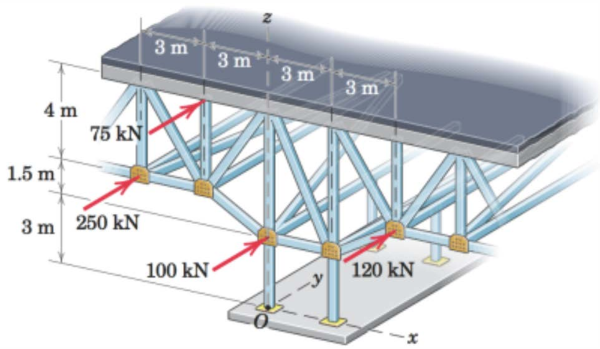
Problem 2/155

2/156 Determine the force–couple system at O which is equivalent to the two forces applied to the shaft AOB. Is **R** perpendicular to $\mathbf{M}_O$?



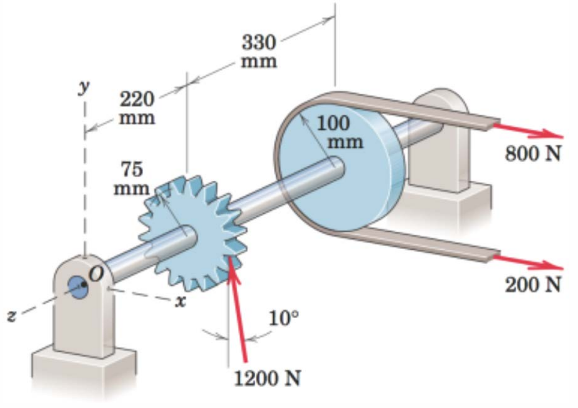
Problem 2/156

2/157 The portion of a bridge truss is subjected to several loads. For the loading shown, determine the location in the x-z plane through which the resultant passes.



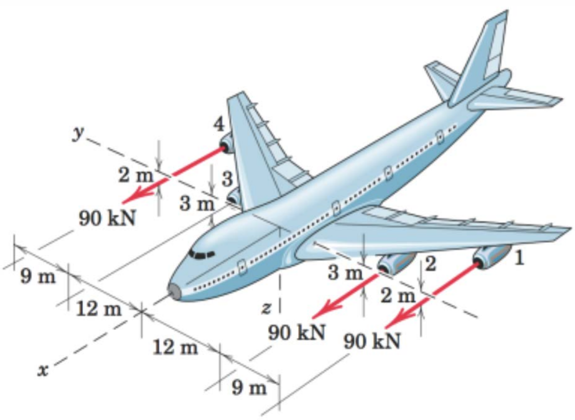
Problem 2/157

2/158 The pulley and gear are subjected to the loads shown. For these forces, determine the equivalent force–couple system at point O.



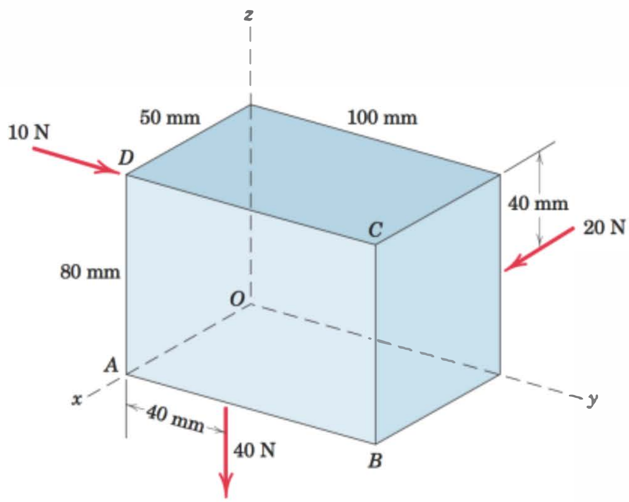
Problem 2/158

2/159 The commercial airliner of Prob. 2/93 is redrawn here with three-dimensional information supplied. If engine 3 suddenly fails, determine the resultant of the three remaining engine thrust vectors, each of which has a magnitude of 90 kN. Specify the y- and z-coordinates of the point through which the line of action of the resultant passes. This information would be critical to the design criteria of performance with engine failure.



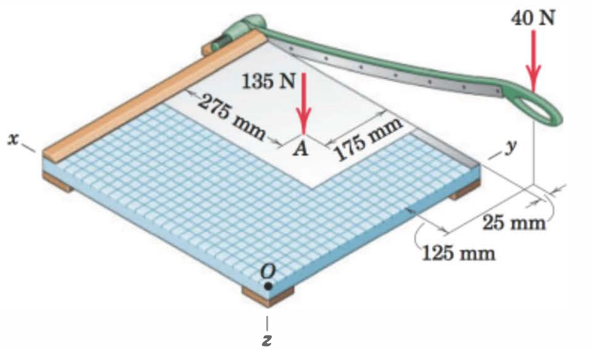
Problem 2/159

2/160 Replace the three forces acting on the rectangular solid with a wrench. Specify the magnitude of the moment M associated with the wrench and state whether it acts in a positive or negative sense. Specify the coordinates of the point P in plane $ABCD$ through which the line of action of the wrench passes. Illustrate the wrench moment and resultant in an appropriate sketch.



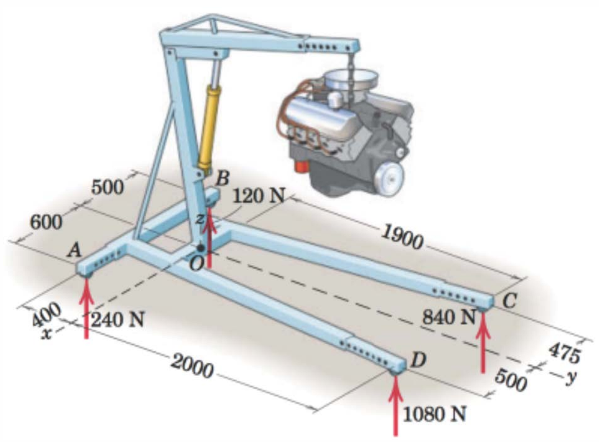
Problem 2/160

2/161 While cutting a piece of paper, a person exerts the two indicated forces on a paper cutter. Reduce the two forces to an equivalent force–couple system at corner O and then specify the coordinates of the point P in the x-y plane through which the resultant of the two forces passes. The cutting surface is 600 mm × 600 mm.



Problem 2/161

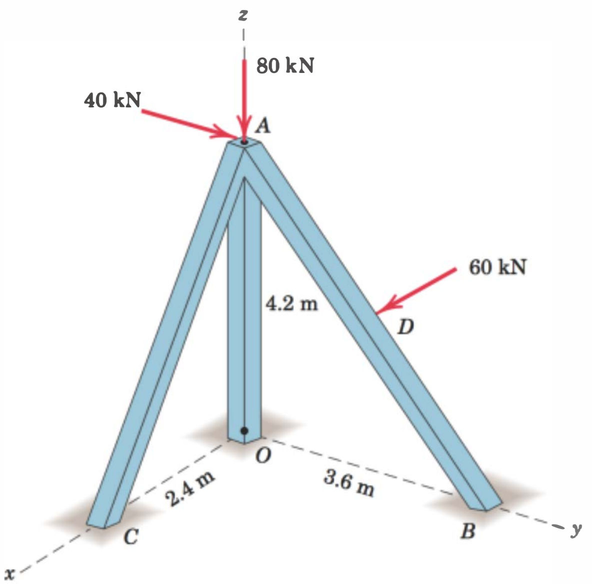
2/162 The floor exerts the four indicated forces on the wheels of an engine hoist. Determine the location in the x-y plane at which the resultant of the forces acts.



Dimensions in millimeters

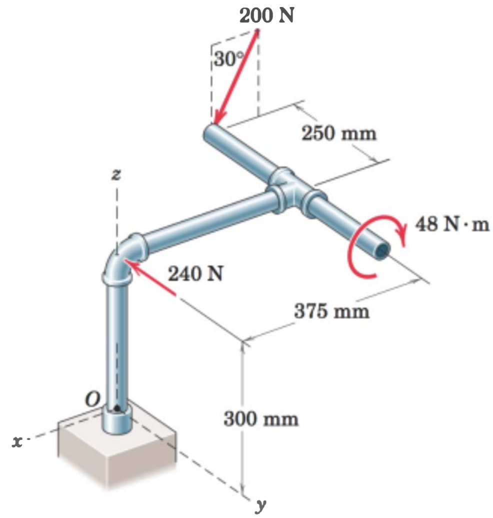
Problem 2/162

2/163 Replace the three forces acting on the structural support with a wrench. Specify the point P in the x-y plane through which the line of action of the wrench passes. Note that the 60-kN force is applied at the midpoint of member AB and lies parallel to the x-direction. Illustrate the wrench moment and resultant in an appropriate sketch.



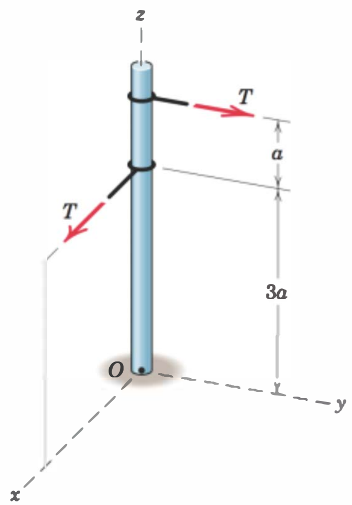
Problem 2/163

2/164 Replace the two forces and one couple acting on the rigid pipe frame by their equivalent resultant force $\mathbf{R}$ acting at point O and a couple $\mathbf{M}_O$.



Problem 2/164

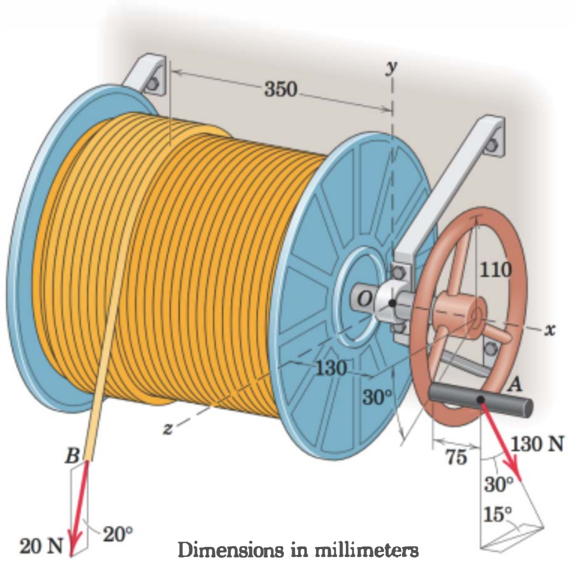
2/165 Replace the two forces acting on the pole by a wrench. Write the moment $\mathbf{M}$ associated with the wrench as a vector and specify the coordinates of the point P in the y-z plane through which the line of action of the wrench passes.



Problem 2/165

2/166 For the system of Prob. 2/154, write the moment $\mathbf{M}$ of the wrench resultant of the two forces and couple and specify the coordinates of the point P in the y-z plane through which the line of action of the wrench passes.

2/167 Replace the two forces which act on the pneumatic-hose reel by an equivalent force–couple system at O. The 20-N force which results from the weight of excess hose being wound up lies in a plane parallel to the y-z plane and loses contact with the hose reel at a radius of 160 mm.



Problem 2/167

2/168 For the system of forces in Prob. 2/167, determine the coordinates of the point P in the x-z plane through which the line of action of the resultant of the system passes. Illustrate the resultant in an appropriate sketch.

2/10 Chapter Review

In Chapter 2 we have established the properties of forces, moments, and couples, and the correct procedures for representing their effects. Mastery of this material is essential for our study of equilibrium in the chapters which follow. Failure to correctly use the procedures of Chapter 2 is a common cause of errors in applying the principles of equilibrium. When difficulties arise, you should refer to this chapter to be sure that the forces, moments, and couples are correctly represented.

Forces

There is frequent need to represent forces as vectors, to resolve a single force into components along desired directions, and to combine two or more concurrent forces into an equivalent resultant force. Specifically, you should be able to:

1. Resolve a given force vector into its components along given directions, and express the vector in terms of the unit vectors along a given set of axes.
2. Express a force as a vector when given its magnitude and information about its line of action. This information may be in the form of two points along the line of action or angles which orient the line of action.
3. Use the dot product to compute the projection of a vector onto a specified line and the angle between two vectors.
4. Compute the resultant of two or more forces concurrent at a point.

Moments

The tendency of a force to rotate a body about an axis is described by a moment (or torque), which is a vector quantity. We have seen that finding the moment of a force is often facilitated by combining the moments of the components of the force. When working with moment vectors you should be able to:

1. Determine a moment by using the moment-arm rule.
2. Use the vector cross product to compute a moment vector in terms of a force vector and a position vector locating the line of action of the force.
3. Utilize Varignon's theorem to simplify the calculation of moments, in both scalar and vector forms.
4. Use the triple scalar product to compute the moment of a force vector about a given axis through a given point.

Couples

A couple is the combined moment of two equal, opposite, and noncollinear forces. The unique effect of a couple is to produce a pure twist or rotation regardless of where the forces are located. The couple is useful in replacing a force acting at a point by a force–couple system at a different point. To solve problems involving couples you should be able to:

1. Compute the moment of a couple, given the couple forces and either their separation distance or any position vectors locating their lines of action.
2. Replace a given force by an equivalent force–couple system, and vice versa.

Resultants

We can reduce an arbitrary system of forces and couples to a single resultant force applied at an arbitrary point, and a corresponding resultant couple. We can further combine this resultant force and couple into a wrench to give a single resultant force along a unique line of action, together with a parallel couple vector. To solve problems involving resultants you should be able to:

1. Compute the magnitude, direction, and line of action of the resultant of a system of coplanar forces if that resultant is a force; otherwise, compute the moment of the resultant couple.
2. Apply the principle of moments to simplify the calculation of the moment of a system of coplanar forces about a given point.
3. Replace a given general force system by a wrench along a specific line of action.

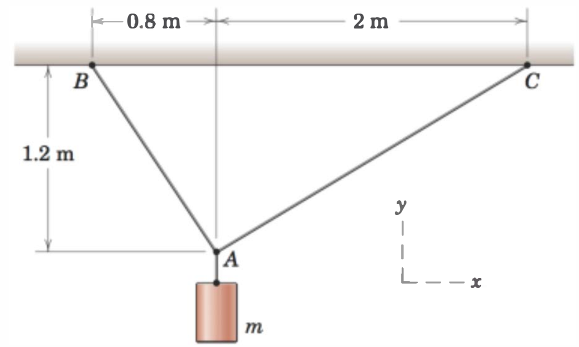
Equilibrium

You will use the preceding concepts and methods when you study equilibrium in the following chapters. Let us summarize the concept of equilibrium:

1. When the resultant force on a body is zero ($\Sigma \mathbf{F} = \mathbf{0}$), the body is in *translational* equilibrium. This means that its center of mass is either at rest or moving in a straight line with constant velocity.
2. In addition, if the resultant couple is zero ($\Sigma \mathbf{M} = \mathbf{0}$), the body is in *rotational* equilibrium, either having no rotational motion or rotating with a constant angular velocity.
3. When both resultants are zero, the body is in *complete* equilibrium.

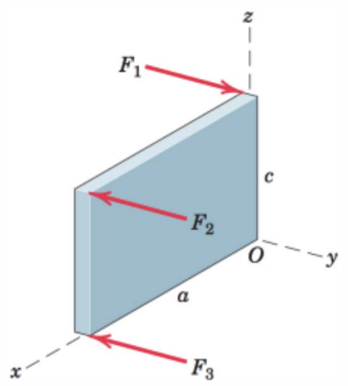
REVIEW PROBLEMS

2/169 Using the principles of equilibrium to be developed in Chapter 3, you will soon be able to verify that the tension in cable AB is 85.8% of the weight of the cylinder of mass m, while the tension in cable AC is 55.5% of the suspended weight. Write each tension force acting on point A as a vector if the mass m is 60 kg.



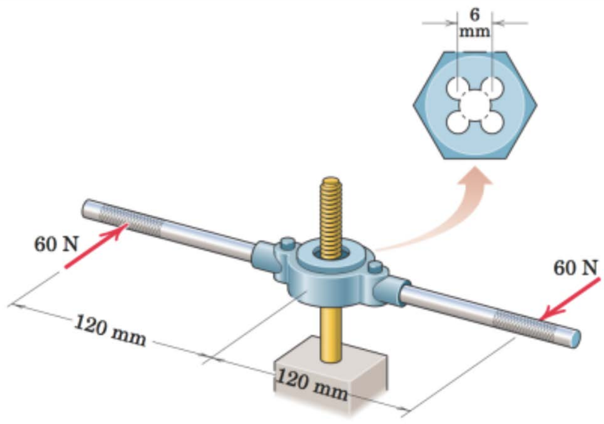
Problem 2/169

2/170 The three forces act perpendicular to the rectangular plate as shown. Determine the moments $\mathbf{M}_1$ of $\mathbf{F}_1$, $\mathbf{M}_2$ of $\mathbf{F}_2$, and $\mathbf{M}_3$ of $\mathbf{F}_3$, all about point O.



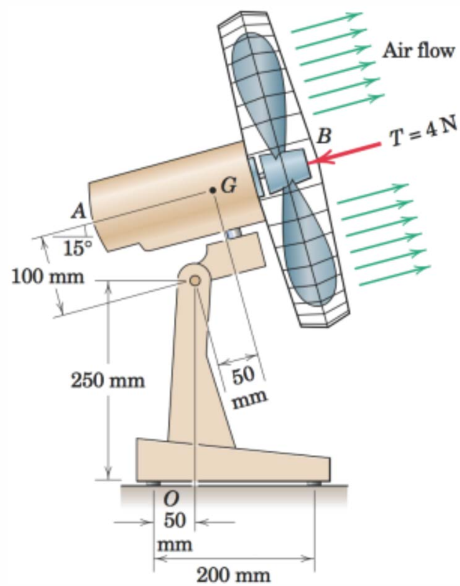
Problem 2/170

2/171 A die is being used to cut threads on a rod. If 60-N forces are applied as shown, determine the magnitude F of the equal forces exerted on the 6-mm rod by each of the four cutting surfaces so that their external effect on the rod is equivalent to that of the two 60-N forces.



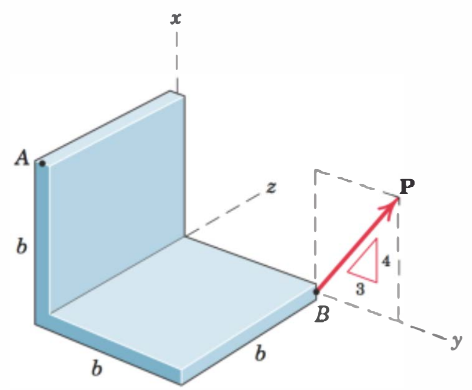
Problem 2/171

2/172 The blades of the portable fan generate a 4-N thrust $\mathbf{T}$ as shown. Compute the moment M_O of this force about the rear support point O. For comparison, determine the moment about O due to the weight of the motor–fan unit AB, whose weight of 40 N acts at G.



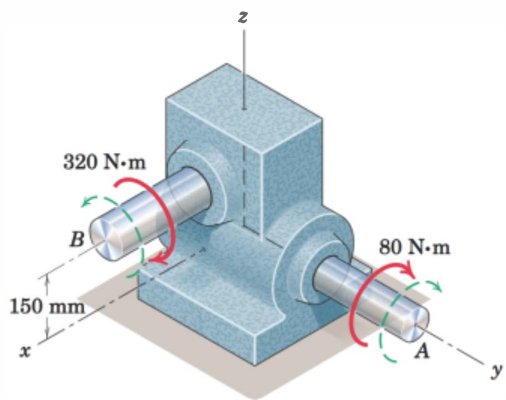
Problem 2/172

2/173 Determine the moment of the force **P** about point A.



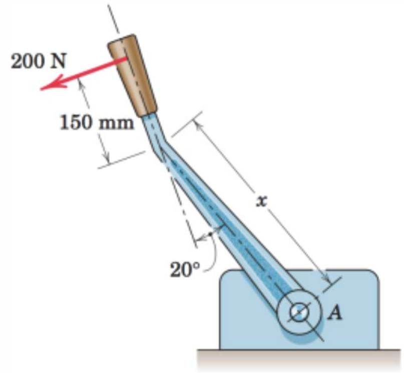
Problem 2/173

2/174 The directions of rotation of the input shaft A and output shaft B of the worm-gear reducer are indicated by the curved dashed arrows. An input torque (couple) of 80 N·m is applied to shaft A in the direction of rotation. The output shaft B supplies a torque of 320 N·m to the machine which it drives (not shown). The shaft of the driven machine exerts an equal and opposite reacting torque on the output shaft of the reducer. Determine the resultant **M** of the two couples which act on the reducer unit and calculate the direction cosine of **M** with respect to the x-axis.



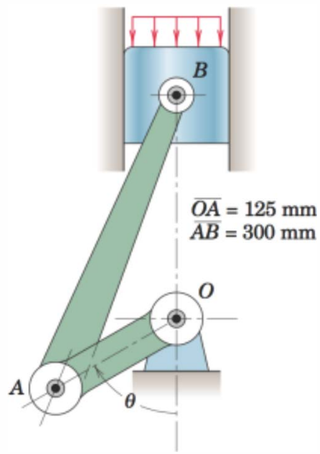
Problem 2/174

2/175 The control lever is subjected to a clockwise couple of 80 N·m exerted by its shaft at A and is designed to operate with a 200-N pull as shown. If the resultant of the couple and the force passes through A, determine the proper dimension x of the lever.



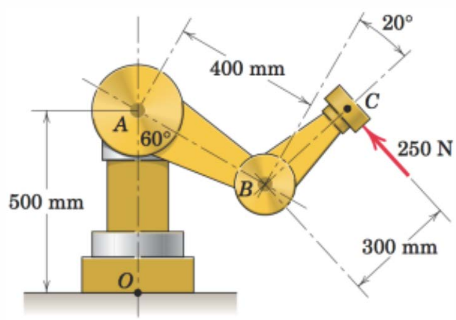
Problem 2/175

2/176 For the angular position $\theta = 60°$ of the crank OA, the gas pressure on the piston induces a compressive force P in the connecting rod along its centerline AB. If this force produces a moment of 720 N·m about the crank axis O, calculate P.



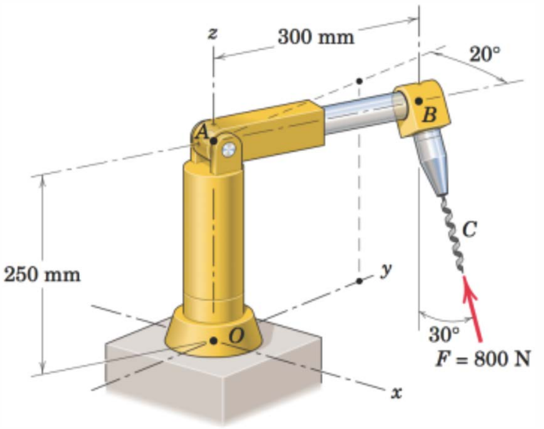
Problem 2/176

2/177 Calculate the moment M_O of the 250-N force about the base point O of the robot.



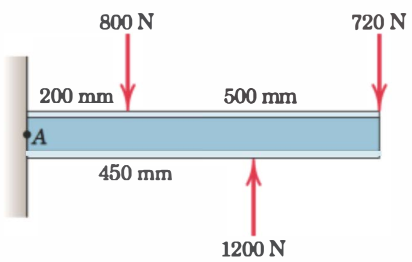
Problem 2/177

2/178 During a drilling operation, the small robotic device is subjected to an 800-N force at point C as shown. Replace this force by an equivalent force–couple system at point O.



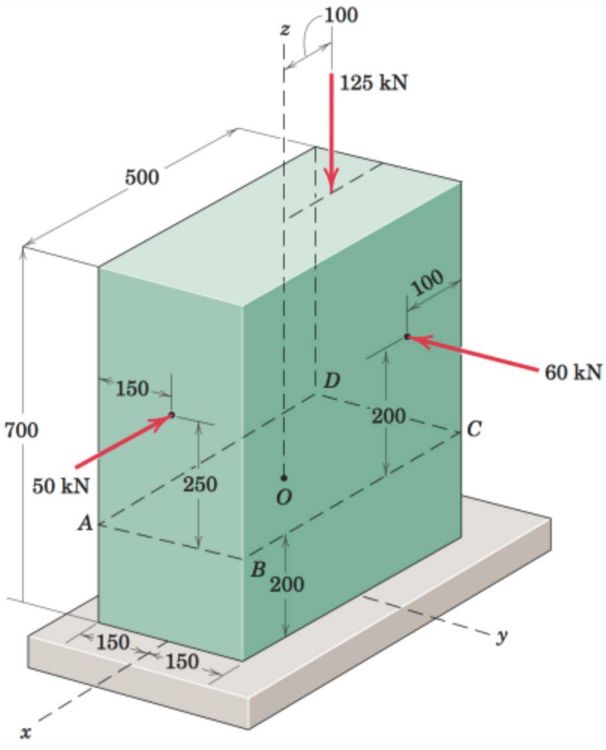
Problem 2/178

2/179 Reduce the given loading system to a force–couple system at point A. Then determine the distance x to the right of point A at which the resultant of the three forces acts.



Problem 2/179

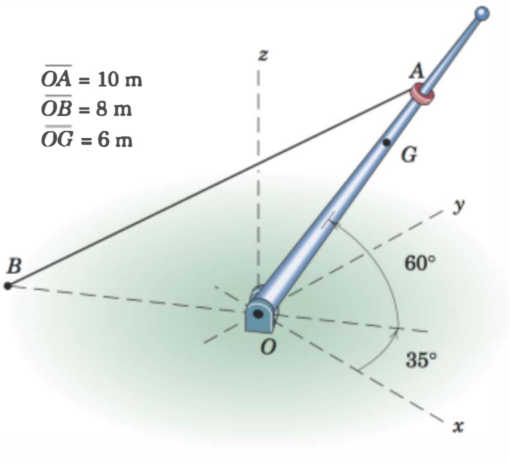
2/180 The $300 \times 500 \times 700$-mm column is subjected to the indicated forces. Replace the given loads by an equivalent force–couple system at point O, which lies at the center of plane $ABCD$. Show the components of the resultant force and couple on an appropriate sketch.



Dimensions in millimeters

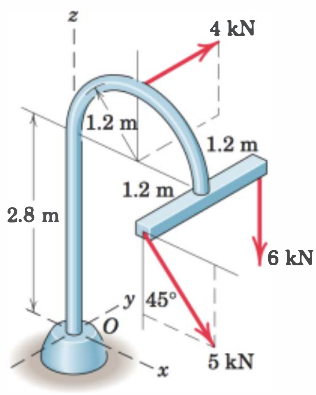
Problem 2/180

2/181 When the pole OA is in the position shown, the tension in cable AB is 3 kN. (*a*) Write the tension force exerted on the small collar at point A as a vector using the coordinates shown. (*b*) Determine the moment of this force about point O and state the moments about the x-, y-, and z-axes. (*c*) Determine the projection of this tension force onto line AO.



Problem 2/181

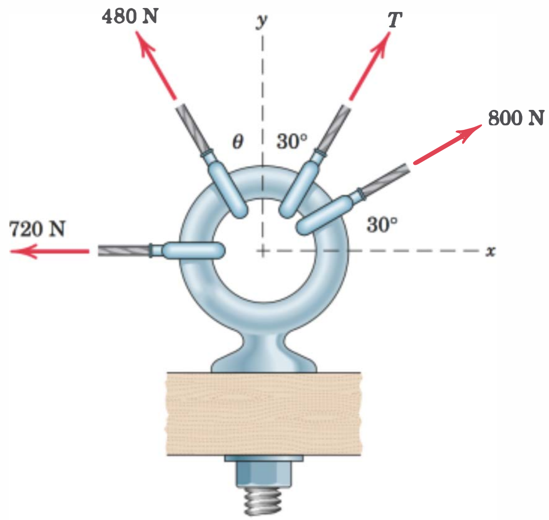
2/182 The combined action of the three forces on the base at O may be obtained by establishing their resultant through O. Determine the magnitudes of **R** and the accompanying couple **M**.



Problem 2/182

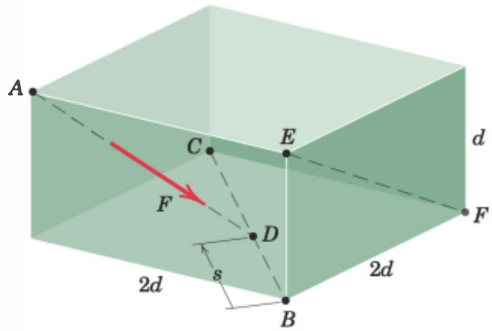
*Computer-Oriented Problems

***2/183** Four forces are exerted on the eyebolt as shown. If the net effect on the bolt is a direct pull of 1200 N in the y-direction, determine the necessary values of T and θ.



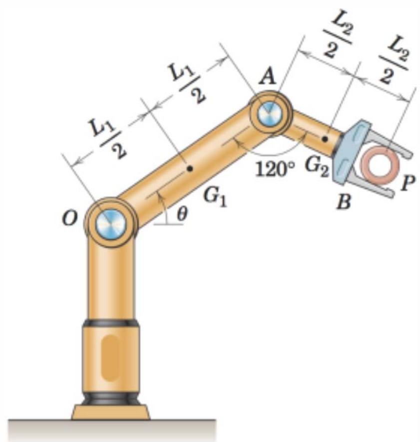
Problem 2/183

***2/184** The force **F** is directed from A toward D and D is allowed to move from B to C as measured by the variable s. Consider the projection of **F** onto line EF as a function of s. In particular, determine and plot the fraction n of the magnitude F which is projected as a function of s/d. Note that s/d varies from 0 to $2\sqrt{2}$.



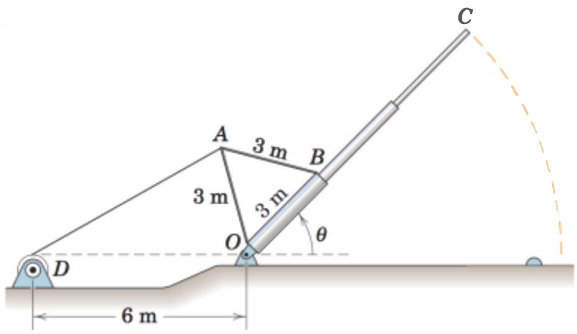
Problem 2/184

***2/185** With the cylindrical part P of weight 1500 N in its grip, the robotic arm pivots about O through the range $-45° \leq \theta \leq 45°$ with the angle at A locked at 120°. Determine and plot (as a function of θ) the moment at O due to the combined effects of the weight of part P, the 600-N weight of member OA (mass center at G_1), and the 250-N weight of member AB (mass center at G_2). The end grip is included as a part of member AB. The lengths L_1 and L_2 are 900 mm and 600 mm, respectively. What is the maximum value of M_O, and at what value of θ does this maximum occur?



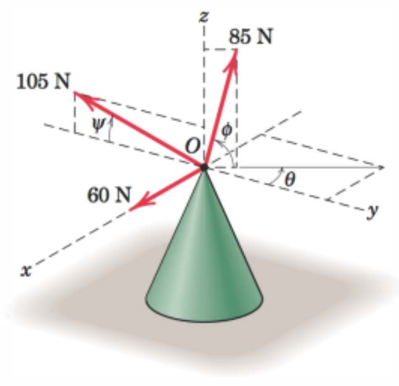
Problem 2/185

***2/186** A flagpole with attached light triangular frame is shown here for an arbitrary position during its raising. The 75-N tension in the erecting cable remains constant. Determine and plot the moment about the pivot O of the 75-N force for the range $0 \leq \theta \leq 90°$. Determine the maximum value of this moment and the elevation angle at which it occurs; comment on the physical significance of the latter. The effects of the diameter of the drum at D may be neglected.



Problem 2/186

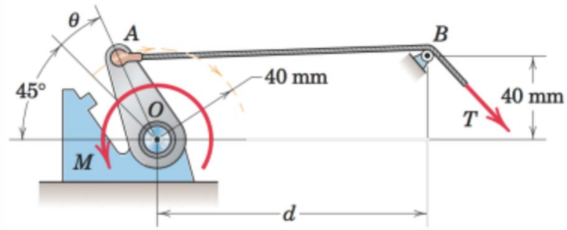
***2/187** Plot the magnitude of the resultant **R** of the three forces as a function of θ for $0 \leq \theta \leq 360°$ and determine the value of θ which makes the magnitude R of the resultant of the three loads (*a*) a maximum and (*b*) a minimum. Record the magnitude of the resultant in each case. Use values of $\phi = 75°$ and $\psi = 20°$.



Problem 2/187

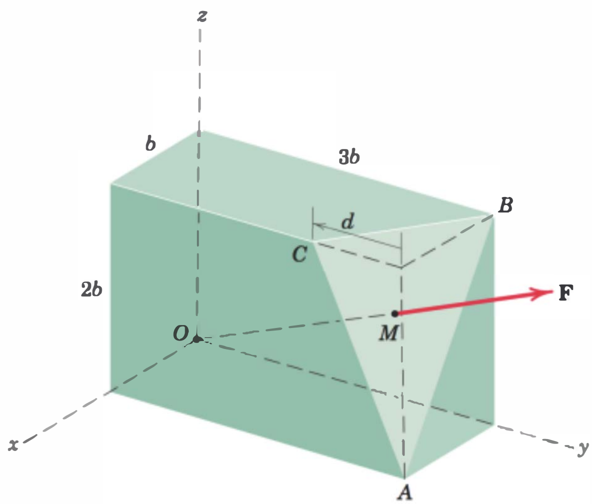
***2/188** For the previous problem, determine the combination of angles θ and ϕ which makes the magnitude R of the resultant **R** of the three loads (*a*) a maximum and (*b*) a minimum. Record the magnitude of the resultant in each case and show a plot of R as a function of both θ and ϕ. The angle ψ is fixed at 20°.

***2/189** The throttle-control lever OA rotates in the range $0 \leq \theta \leq 90°$. An internal torsional return spring exerts a restoring moment about O given by $M = K(\theta + \pi/4)$, where $K = 500$ N·mm/rad and θ is in radians. Determine and plot as a function of θ the tension T required to make the net moment about O zero. Use the two values $d = 60$ mm and $d = 160$ mm and comment on the relative design merits. The effects of the radius of the pulley at B are negligible.



Problem 2/189

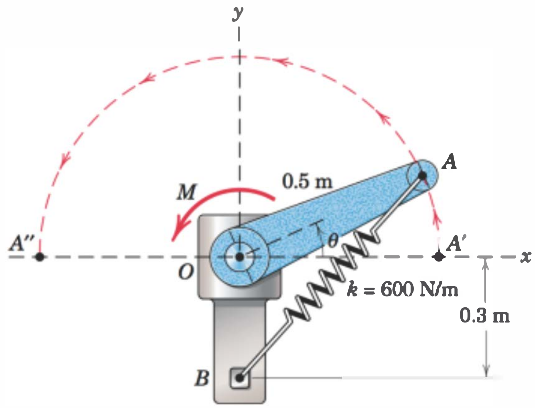
***2/190** For the rectangular parallelepiped shown, develop an expression for the scalar projection F_{BC} of **F** onto line BC. Point M is located at the centroid of the triangle ABC formed by "chopping off" the corner of the parallelepiped. Plot your expression over the range $0 \leq d \leq 3b$, and determine the maximum value of F_{BC} along with the corresponding value of d. See Table D/3 in Appendix D for the centroid location of a triangle.



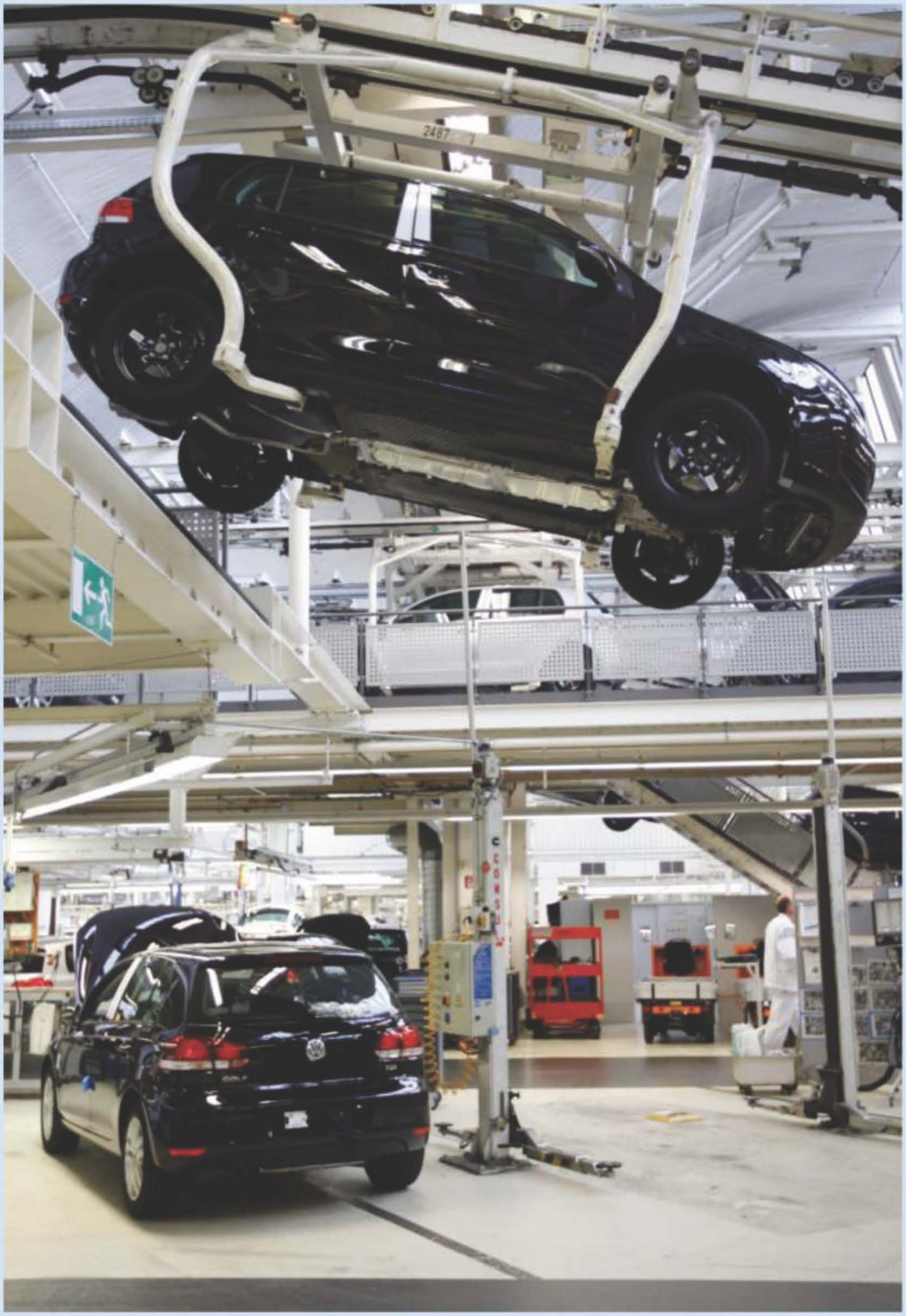
Problem 2/190

***2/191** Consider the rectangular parallelepiped of Prob. 2/190. Develop the scalar expression for the moment M_{BC} which the force **F** makes about line BC of the "chopped off" corner ABC. Point M is located at the centroid of the triangle ABC formed by "chopping off" the corner of the parallelepiped. Plot your expression over the range $0 \leq d \leq 3b$, and determine the maximum value of M_{BC} along with the corresponding value of d.

***2/192** A motor attached to the shaft at O causes the arm OA to rotate over the range $0 \leq \theta \leq 180°$. The unstretched length of the spring is 0.65 m, and it can support both tension and compression. If the net moment about O must be zero, determine and plot the required motor torque M as a function of θ.



Problem 2/192



Adam Berry/Bloomberg via Getty Images

In many applications of mechanics, the sum of the forces acting on a body is zero or near zero, and a state of equilibrium is assumed to exist. This apparatus is designed to hold a car body in equilibrium for a considerable range of orientations during vehicle production. Even though there is motion, it is slow and steady with minimal acceleration, so that the assumption of equilibrium is justified during the design of the mechanism.

3 Equilibrium

Chapter Outline

3/1 Introduction

Statics deals primarily with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures. This chapter on equilibrium, therefore, constitutes the most important part of statics, and the procedures developed here form the basis for solving problems in both statics and dynamics. We will make continual use of the concepts developed in Chapter 2 involving forces, moments, couples, and resultants as we apply the principles of equilibrium.

When a body is in equilibrium, the resultant of *all* forces acting on it is zero. Thus, the resultant force $\mathbf{R}$ and the resultant couple $\mathbf{M}$ are both zero, and we have the equilibrium equations

$$\boxed{\mathbf{R} = \Sigma\mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma\mathbf{M} = \mathbf{0}} \tag{3/1}$$

These requirements are both necessary and sufficient conditions for equilibrium.

All physical bodies are three-dimensional, but we can treat many of them as two-dimensional when the forces to which they are subjected act in a single plane or can be projected onto a single plane. When this simplification is not possible, the problem must be treated as three-dimensional. We will follow the arrangement used in Chapter 2 and discuss in Section A the equilibrium of bodies subjected to two-dimensional force systems and in Section B the equilibrium of bodies subjected to three-dimensional force systems.

SECTION A EQUILIBRIUM IN TWO DIMENSIONS

3/2 System Isolation and the Free-Body Diagram

Before we apply Eqs. 3/1, we must define unambiguously the particular body or mechanical system to be analyzed and represent clearly and completely *all* forces acting *on* the body. Omission of a force which acts *on* the body in question, or inclusion of a force which does not act *on* the body, will give erroneous results.

A *mechanical system* is defined as a body or group of bodies which can be conceptually isolated from all other bodies. A system may be a single body or a combination of connected bodies. The bodies may be rigid or nonrigid. The system may also be an identifiable fluid mass, either liquid or gas, or a combination of fluids and solids. In statics we study primarily forces which act on rigid bodies at rest, although we also study forces acting on fluids in equilibrium.

Once we decide which body or combination of bodies to analyze, we then treat this body or combination as a single body *isolated* from all surrounding bodies. This isolation is accomplished by means of the ***free-body diagram***, which is a diagrammatic representation of the isolated system treated as a single body. The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. If appreciable body forces are present, such as gravitational or magnetic attraction, then these forces must also be shown on the free-body diagram of the isolated system. Only after such a diagram has been carefully drawn should the equilibrium equations be written. Because of its critical importance, we emphasize here that

> **the free-body diagram is the most important single step in the solution of problems in mechanics.**

Before attempting to draw a free-body diagram, we must recall the basic characteristics of force. These characteristics were described in Art. 2/2, with primary attention focused on the vector properties of force. Forces can be applied either by direct physical contact or by remote action. Forces can be either internal or external to the system under consideration. Application of force is accompanied by reactive force, and both applied and reactive forces may be either concentrated or distributed. The principle of transmissibility permits the treatment of force as a sliding vector as far as its external effects on a rigid body are concerned.

We will now use these force characteristics to develop conceptual models of isolated mechanical systems. These models enable us to write the appropriate equations of equilibrium, which can then be analyzed.

Modeling the Action of Forces

Figure 3/1 shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted *on* the body to be *isolated*, *by* the body to be *removed*. Newton's third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted *on* the body in question *by* a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.

In Fig. 3/1, Example 1 depicts the action of a flexible cable, belt, rope, or chain on the body to which it is attached. Because of its flexibility, a rope or cable is unable to offer any resistance to bending,

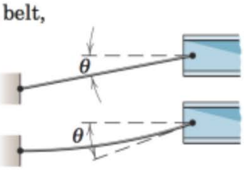
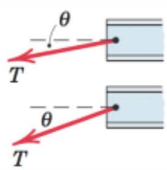
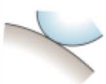
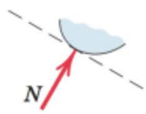
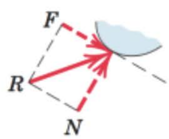
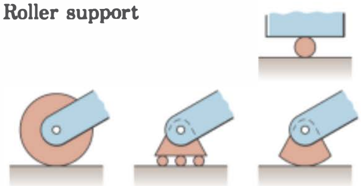
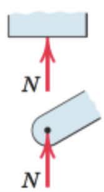
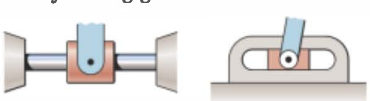
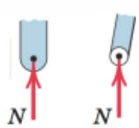
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS			
Type of Contact and Force Origin		Action on Body to Be Isolated	
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible			Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces			Contact force is compressive and is normal to the surface.
3. Rough surfaces			Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support			Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide			Collar or slider free to move along smooth guides; can support force normal to guide only.

Figure 3/1

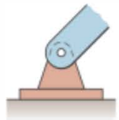
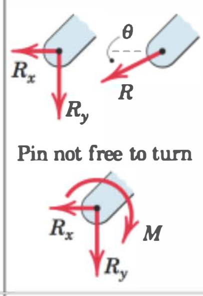
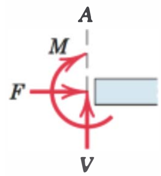
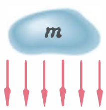
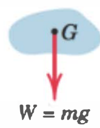
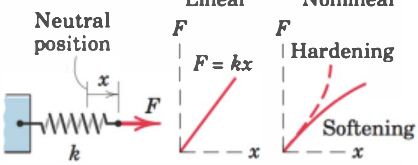
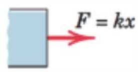
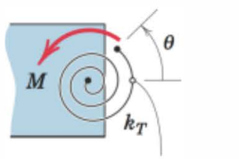
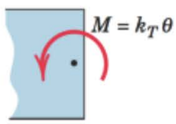
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection 		A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.
9. Spring action 		Spring force is tensile if the spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.
10. Torsional spring action 		For a linear torsional spring, the applied moment M is proportional to the angular deflection θ from the neutral position. The stiffness k_T is the moment required to deform the spring one radian.

Figure 3/1, continued

shear, or compression and therefore exerts only a tension force in a direction tangent to the cable at its point of attachment. The force exerted *by* the cable *on* the body to which it is attached is always *away* from the body. When the tension T is large compared with the weight of the cable, we may assume that the cable forms a straight line. When the cable weight is not negligible compared with its tension, the sag of the cable becomes important, and the tension in the cable changes direction and magnitude along its length.

When the smooth surfaces of two bodies are in contact, as in Example 2, the force exerted by one on the other is *normal* to the tangent to the surfaces and is compressive. Although no actual surfaces are perfectly smooth, we can assume this to be so for practical purposes in many instances.

When mating surfaces of contacting bodies are rough, as in Example 3, the force of contact is not necessarily normal to the tangent to the surfaces, but may be resolved into a *tangential* or *frictional component* F and a *normal component* N.

Example 4 illustrates a number of forms of mechanical support which effectively eliminate tangential friction forces. In these cases the net reaction is normal to the supporting surface.

Example 5 shows the action of a smooth guide on the body it supports. There cannot be any resistance parallel to the guide.

Example 6 illustrates the action of a pin connection. Such a connection can support force in any direction normal to the axis of the pin. We usually represent this action in terms of two rectangular components. The correct sense of these components in a specific problem depends on how the member is loaded. When not otherwise initially known, the sense is arbitrarily assigned and the equilibrium equations are then written. If the solution of these equations yields a positive algebraic sign for the force component, the assigned sense is correct. A negative sign indicates the sense is opposite to that initially assigned.

If the joint is free to turn about the pin, the connection can support only the force R. If the joint is not free to turn, the connection can also support a resisting couple M. The sense of M is arbitrarily shown here, but the true sense depends on how the member is loaded.

Example 7 shows the resultants of the rather complex distribution of force over the cross section of a slender bar or beam at a built-in or fixed support. The sense of the reactions F and V and the bending couple M in a given problem depends, of course, on how the member is loaded.

One of the most common forces is that due to gravitational attraction, Example 8. This force affects all elements of mass in a body and is, therefore, distributed throughout it. The resultant of the gravitational forces on all elements is the weight $W = mg$ of the body, which passes through the center of gravity G and is directed toward the center of the earth for earthbound structures. The location of G is frequently obvious from the geometry of the body, particularly where there is symmetry. When the location is not readily apparent, it must be determined by experiment or calculations.

Similar remarks apply to the remote action of magnetic and electric forces. These forces of remote action have the same overall effect on a rigid body as forces of equal magnitude and direction applied by direct external contact.

Example 9 illustrates the action of a *linear* elastic spring and of a *nonlinear* spring with either hardening or softening characteristics. The force exerted by a linear spring, in tension or compression,

Another type of car-lifting apparatus to be considered along with that in the chapter-opening photograph.

is given by $F = kx$, where k is the *stiffness* or *modulus* of the spring and x is its deformation measured from the neutral or undeformed position.

In Example 10 we see the action of a torsional (or clockwork) spring. Shown is a linear version; as suggested in Example 9 for extension springs, nonlinear torsional springs also exist.

The representations in Fig. 3/1 are *not* free-body diagrams, but are merely elements used to construct free-body diagrams. Study these ten conditions and identify them in the problem work so that you can draw the correct free-body diagrams.

Key Concepts

CONSTRUCTION OF FREE-BODY DIAGRAMS

The full procedure for drawing a free-body diagram which isolates a body or system consists of the following steps.

Step 1. Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

Step 2. Next isolate the chosen system by drawing a diagram which represents its *complete external boundary*. This boundary defines the isolation of the system from *all* other attracting or contacting bodies, which are considered removed. This step is often the most crucial of all. Make certain that you have *completely isolated* the system before proceeding with the next step.

Step 3. Identify all forces which act *on* the isolated system as applied *by* the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system. Make a systematic traverse of the entire boundary to identify all contact forces. Include body forces such as weights, where appreciable. Represent all known forces by vector arrows, each with its proper magnitude, direction, and sense indicated. Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. If the sense of the vector is also unknown, you must arbitrarily assign a sense. The subsequent calculations with the equilibrium equations will yield a positive quantity if the correct sense was assumed and a negative quantity if the incorrect sense was assumed. It is necessary to be *consistent* with the assigned characteristics of unknown forces throughout all of the calculations. If you are consistent, the solution of the equilibrium equations will reveal the correct senses.

Step 4. Show the choice of coordinate axes directly on the diagram. Pertinent dimensions may also be represented for convenience. Note, however, that the free-body diagram serves the purpose of focusing attention on the action of the external forces, and therefore the diagram should not be cluttered with excessive extraneous information. Clearly distinguish force arrows from arrows representing quantities other than forces. For this purpose a colored pencil may be used.

Completion of the foregoing four steps will produce a correct free-body diagram to use in applying the governing equations, both in statics and in dynamics. Be careful not to omit from the free-body diagram certain forces which may not appear at first glance to be needed in the calculations. It is only through *complete* isolation and a systematic representation of *all* external forces that a reliable accounting of the effects of all applied and reactive forces can be made. Very often a force which at first glance may not appear to influence a desired result does indeed have an influence. Thus, the only safe procedure is to include on the free-body diagram all forces whose magnitudes are not obviously negligible.

Examples of Free-Body Diagrams

Figure 3/2 gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes

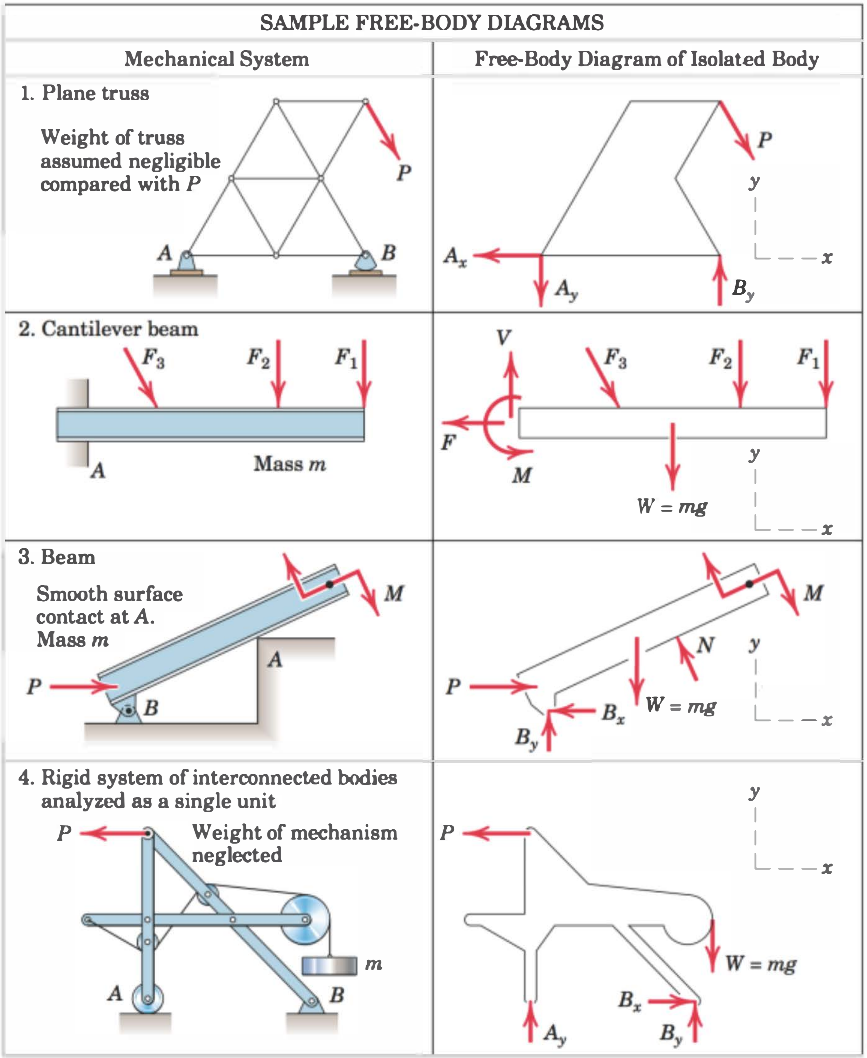


Figure 3/2

are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown. The characteristics of the various types of contact forces illustrated in Fig. 3/1 are used in the four examples as they apply.

In Example 1 the truss is composed of structural elements which, taken all together, constitute a rigid framework. Thus, we may remove the entire truss from its supporting foundation and treat it as a single rigid body. In addition to the applied external load P, the free-body diagram must include the reactions on the truss at A and B. The rocker at B can support a vertical force only, and this force is transmitted to the structure at B (Example 4 of Fig. 3/1). The pin connection at A (Example 6 of Fig. 3/1) is capable of supplying both a horizontal and a vertical force component to the truss. If the total weight of the truss members is appreciable compared with P and the forces at A and B, then the weights of the members must be included on the free-body diagram as external forces.

In this relatively simple example it is clear that the vertical component A_y must be directed down to prevent the truss from rotating clockwise about B. Also, the horizontal component A_x will be to the left to keep the truss from moving to the right under the influence of the horizontal component of P. Thus, in constructing the free-body diagram for this simple truss, we can easily perceive the correct sense of each of the components of force exerted *on* the truss *by* the foundation at A and can, therefore, represent its correct physical sense on the diagram. When the correct physical sense of a force or its component is not easily recognized by direct observation, it must be assigned arbitrarily, and the correctness of or error in the assignment is determined by the algebraic sign of its calculated value.

In Example 2 the cantilever beam is secured to the wall and subjected to three applied loads. When we isolate that part of the beam to the right of the section at A, we must include the reactive forces applied *to* the beam *by* the wall. The resultants of these reactive forces are shown acting on the section of the beam (Example 7 of Fig. 3/1). A vertical force V to counteract the excess of downward applied force is shown, and a tension F to balance the excess of applied force to the right must also be included. Then, to prevent the beam from rotating about A, a counterclockwise couple M is also required. The weight mg of the beam must be represented through the mass center (Example 8 of Fig. 3/1).

In the free-body diagram of Example 2, we have represented the somewhat complex system of forces which actually act on the cut section of the beam by the equivalent force–couple system in which the force is broken down into its vertical component V (shear force) and its horizontal component F (tensile force). The couple M is the bending moment in the beam. The free-body diagram is now complete and shows the beam in equilibrium under the action of six forces and one couple.

In Example 3 the weight $W = mg$ is shown acting through the center of mass of the beam, whose location is assumed known (Example 8 of Fig. 3/1). The force exerted by the corner A on the beam is normal to the smooth surface of the beam (Example 2 of Fig. 3/1). To perceive this action more clearly, visualize an enlargement of the contact point A, which would appear somewhat rounded, and consider the force exerted by this rounded corner on the straight surface of the beam, which is assumed to be smooth. If the contacting surfaces at the corner were not smooth, a tangential frictional component of force could exist. In addition to the applied force P and couple M, there is the pin connection at B, which exerts both an x- and a y-component of force on the beam. The positive senses of these components are assigned arbitrarily.

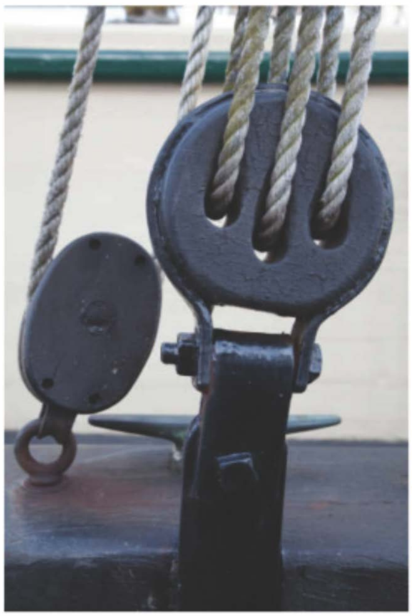
In Example 4 the free-body diagram of the entire isolated mechanism contains three unknown forces if the loads mg and P are known. Any one of many internal configurations for securing the cable leading from the mass m would be possible without affecting the external response of the mechanism as a whole, and this fact is brought out by the free-body diagram. This hypothetical example is used to show that the forces internal to a rigid assembly of members do not influence the values of the external reactions.

We use the free-body diagram in writing the equilibrium equations, which are discussed in the next article. When these equations are solved, some of the calculated force magnitudes may be zero. This would indicate that the assumed force does not exist. In Example 1 of Fig. 3/2, any of the reactions A_x, A_y, or B_y can be zero for specific values of the truss geometry and of the magnitude, direction, and sense of the applied load P. A zero reaction force is often difficult to identify by inspection, but can be determined by solving the equilibrium equations.

Similar comments apply to calculated force magnitudes which are negative. Such a result indicates that the actual sense is the opposite of the assumed sense. The assumed positive senses of B_x and B_y in Example 3 and B_y in Example 4 are shown on the free-body diagrams. The correctness of these assumptions is proved or disproved according to whether the algebraic signs of the computed forces are plus or minus when the calculations are carried out in an actual problem.

The isolation of the mechanical system under consideration is a crucial step in the formulation of the mathematical model. The most important aspect to the correct construction of the all-important free-body diagram is the clear-cut and unambiguous decision as to what is included and what is excluded. This decision becomes unambiguous only when the boundary of the free-body diagram represents a complete traverse of the body or system of bodies to be isolated, starting at some arbitrary point on the boundary and returning to that same point. The system within this closed boundary is the isolated free body, and all contact forces and all body forces transmitted to the system across the boundary must be accounted for.

The following exercises provide practice with drawing free-body diagrams. This practice is helpful before using such diagrams in the application of the principles of force equilibrium in the next article.

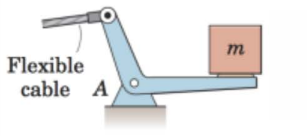
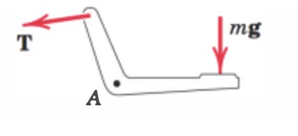
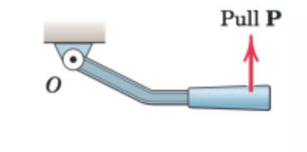
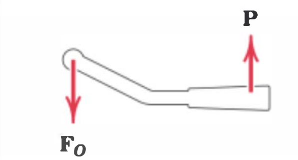
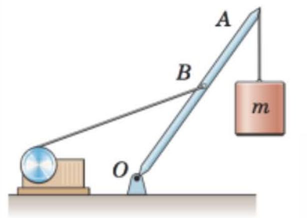
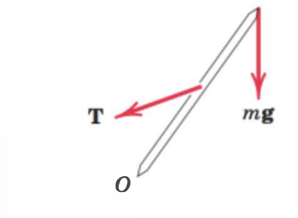
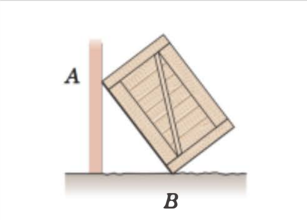
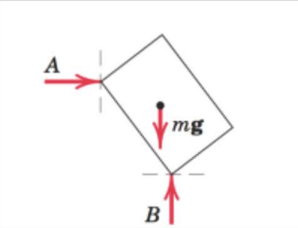
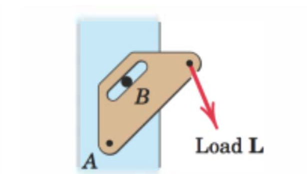
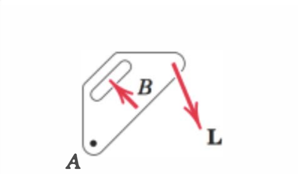


Purestock/Inmagine.com

Complex pulley systems are easily handled with a systematic equilibrium analysis.

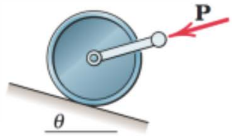
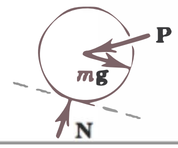
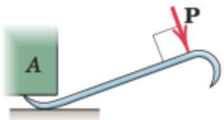
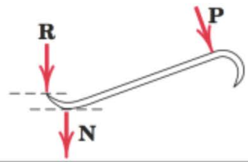
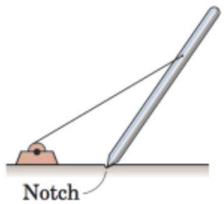
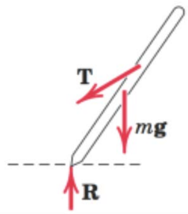
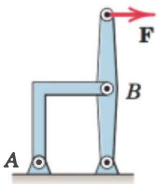
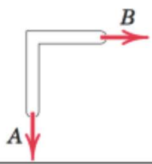
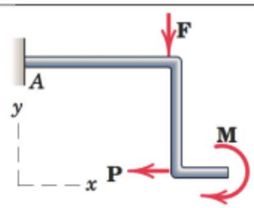
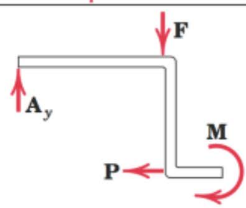
FREE-BODY DIAGRAM EXERCISES

3/A In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and an *incomplete* free-body diagram (FBD) of the isolated body is shown on the right. Add whatever forces are necessary in each case to form a complete free-body diagram. The weights of the bodies are negligible unless otherwise indicated. Dimensions and numerical values are omitted for simplicity.

	Body	Incomplete FBD
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

Problem 3/A

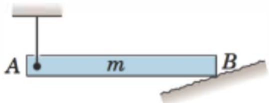
3/B In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and either a *wrong* or an *incomplete* free-body diagram (FBD) is shown on the right. Make whatever changes or additions are necessary in each case to form a correct and complete free-body diagram. The weights of the bodies are negligible unless otherwise indicated. Dimensions and numerical values are omitted for simplicity.

	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

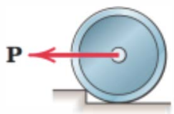
Problem 3/B

3/C Draw a complete and correct free-body diagram of each of the bodies designated in the statements. The weights of the bodies are significant only if the mass is stated. All forces, known and unknown, should be labeled. (*Note:* The sense of some reaction components cannot always be determined without numerical calculation.)

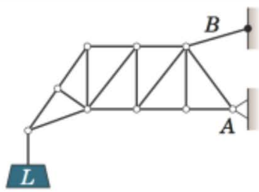
1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.



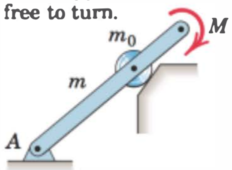
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



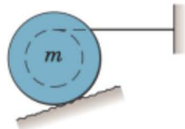
3. Loaded truss supported by pin joint at A and by cable at B.



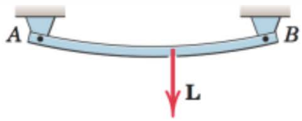
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



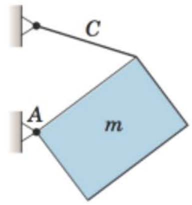
5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.



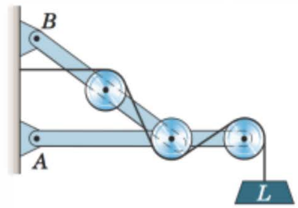
6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.



7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.



Problem 3/C

3/3 Equilibrium Conditions

In Art. 3/1 we defined equilibrium as the condition in which the resultant of all forces and moments acting on a body is zero. Stated in another way, a body is in equilibrium if all forces and moments applied to it are in balance. These requirements are contained in the vector equations of equilibrium, Eqs. 3/1, which in two dimensions may be written in scalar form as

$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0} \tag{3/2}$$

The third equation represents the zero sum of the moments of all forces about any point O on or off the body. Equations 3/2 are the necessary and sufficient conditions for complete equilibrium in two dimensions. They are necessary conditions because, if they are not satisfied, there can be no force or moment balance. They are sufficient because once they are satisfied, there can be no imbalance, and equilibrium is assured.

The equations relating force and acceleration for rigid-body motion are developed in *Vol. 2 Dynamics* from Newton's second law of motion. These equations show that the acceleration of the mass center of a body is proportional to the resultant force $\Sigma\mathbf{F}$ acting on the body. Consequently, if a body moves with constant velocity (zero acceleration), the resultant force on it must be zero, and the body may be treated as in a state of translational equilibrium.

For complete equilibrium in two dimensions, all three of Eqs. 3/2 must hold. However, these conditions are independent requirements, and one may hold without another. Take, for example, a body which slides along a horizontal surface with increasing velocity under the action of applied forces. The force–equilibrium equations will be satisfied in the vertical direction where the acceleration is zero, but not in the horizontal direction. Also, a body, such as a flywheel, which rotates about its fixed mass center with increasing angular speed is not in rotational equilibrium, but the two force–equilibrium equations will be satisfied.

Categories of Equilibrium

Applications of Eqs. 3/2 fall naturally into a number of categories which are easily identified. The categories of force systems acting on bodies in two-dimensional equilibrium are summarized in Fig. 3/3 and are explained further as follows.

Category 1, equilibrium of collinear forces, clearly requires only the one force equation in the direction of the forces (x-direction), since all other equations are automatically satisfied.

Category 2, equilibrium of forces which lie in a plane (x-y plane) and are concurrent at a point O, requires the two force equations only, since the moment sum about O, that is, about a z-axis through O, is necessarily zero. Included in this category is the case of the equilibrium of a particle.

Category 3, equilibrium of parallel forces in a plane, requires the one force equation in the direction of the forces (x-direction) and one moment equation about an axis (z-axis) normal to the plane of the forces.

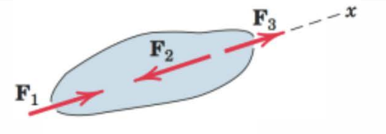
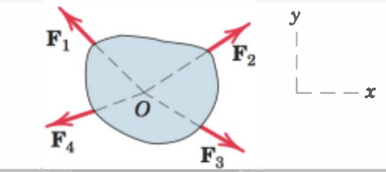
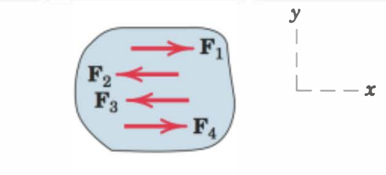
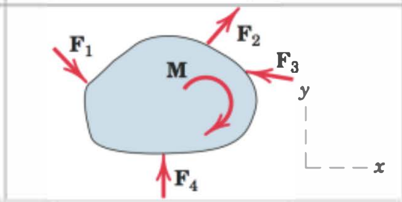
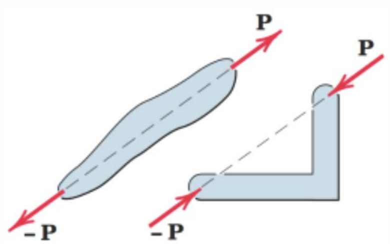
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Figure 3/3

Category 4, equilibrium of a general system of forces in a plane (x-y), requires the two force equations in the plane and one moment equation about an axis (z-axis) normal to the plane.

Two- and Three-Force Members



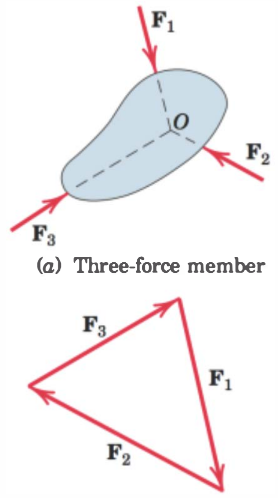
Two-force members

Figure 3/4

You should be alert to two frequently occurring equilibrium situations. The first situation is the equilibrium of a body under the action of two forces only. Two examples are shown in Fig. 3/4, and we see that for such a *two-force member* to be in equilibrium, the forces must be *equal*, *opposite*, and *collinear*. The shape of the member does not affect this simple requirement. In the illustrations cited, we consider the weights of the members to be negligible compared with the applied forces.

The second situation is a *three-force member*, which is a body under the action of three forces, Fig. 3/5a. We see that equilibrium requires the lines of action of the three forces to be *concurrent*. If they were not concurrent, then one of the forces would exert a resultant moment about the point of intersection of the other two, which would violate the requirement of zero moment about every point. The only exception occurs when the three forces are parallel. In this case we may consider the point of concurrency to be at infinity.

The principle of the concurrency of three forces in equilibrium is of considerable use in carrying out a graphical solution of the force equations. In this case the polygon of forces is drawn and made to close, as shown in Fig. 3/5*b*. Frequently, a body in equilibrium under the action of more than three forces may be reduced to a three-force member by a combination of two or more of the known forces.



(*a*) Three-force member

(*b*) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

Figure 3/5

Alternative Equilibrium Equations

In addition to Eqs. 3/2, there are two other ways to express the general conditions for the equilibrium of forces in two dimensions. The first way is illustrated in Fig. 3/6, parts (*a*) and (*b*). For the body shown in Fig. 3/6*a*, if $\Sigma M_A = 0$, then the resultant, if it still exists, cannot be a couple, but must be a force **R** passing through *A*. If now the equation $\Sigma F_x = 0$ holds, where the *x*-direction is arbitrary, it follows from Fig. 3/6*b* that the resultant force **R**, if it still exists, not only must pass through *A*, but also must be perpendicular to the *x*-direction as shown. Now, if $\Sigma M_B = 0$, where *B* is any point such that the line *AB* is not perpendicular to the *x*-direction, we see that **R** must be zero, and thus the body is in equilibrium. Therefore, an alternative set of equilibrium equations is

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

where the two points *A* and *B* must not lie on a line perpendicular to the *x*-direction.

A third formulation of the equilibrium conditions may be made for a coplanar force system. This is illustrated in Fig. 3/6, parts (*c*) and (*d*). Again, if $\Sigma M_A = 0$ for any body such as that shown in Fig. 3/6*c*, the resultant, if any, must be a force **R** through *A*. In addition, if $\Sigma M_B = 0$, the resultant, if one still exists, must pass through *B* as shown in Fig. 3/6*d*. Such a force cannot exist, however, if $\Sigma M_C = 0$, where *C* is not

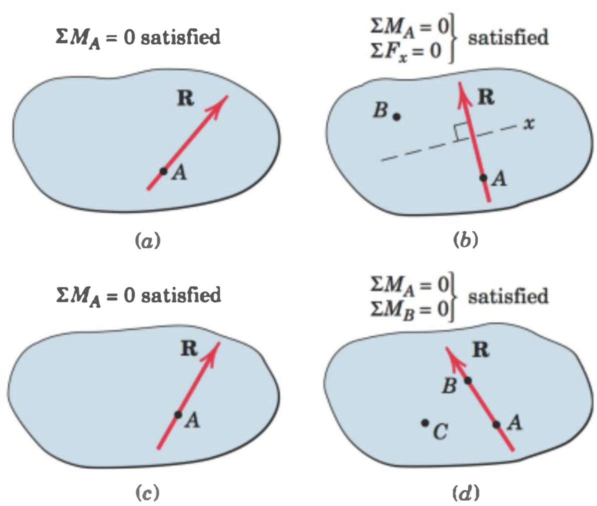


Figure 3/6

collinear with A and B. Thus, we may write the equations of equilibrium as

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

where A, B, and C are any three points not on the same straight line.

When equilibrium equations are written which are not independent, redundant information is obtained, and a correct solution of the equations will yield $0 = 0$. For example, for a general problem in two dimensions with three unknowns, three moment equations written about three points which lie on the same straight line are not independent. Such equations will contain duplicated information, and solution of two of them can at best determine two of the unknowns, with the third equation merely verifying the identity $0 = 0$.

Constraints and Statical Determinacy

The equilibrium equations developed in this article are both necessary and sufficient conditions to establish the equilibrium of a body. However, they do not necessarily provide all the information required to calculate all the unknown forces which may act on a body in equilibrium. Whether the equations are adequate to determine all the unknowns depends on the characteristics of the constraints against possible movement of the body provided by its supports. By *constraint* we mean the restriction of movement.

In Example 4 of Fig. 3/1 the roller, ball, and rocker provide constraint normal to the surface of contact, but none tangent to the surface. Thus, a tangential force cannot be supported. For the collar and slider of Example 5, constraint exists only normal to the guide. In Example 6 the fixed-pin connection provides constraint in both directions, but offers no resistance to rotation about the pin unless the pin is not free to turn. The fixed support of Example 7, however, offers constraint against rotation as well as lateral movement.

If the rocker which supports the truss of Example 1 in Fig. 3/2 were replaced by a pin joint, as at A, there would be one additional constraint beyond those required to support an equilibrium configuration with no freedom of movement. The three scalar conditions of equilibrium, Eqs. 3/2, would not provide sufficient information to determine all four unknowns, since A_x and B_x could not be solved for separately; only their sum could be determined. These two components of force would be dependent on the deformation of the members of the truss as influenced by their corresponding stiffness properties. The horizontal reactions A_x and B_x would also depend on any initial deformation required to fit the dimensions of the structure to those of the foundation between A and B. Thus, we cannot determine A_x and B_x by a rigid-body analysis.

Again referring to Fig. 3/2, we see that if the pin B in Example 3 were not free to turn, the support could transmit a couple to the beam through the pin. Therefore, there would be four unknown supporting reactions acting on the beam—namely, the force at A, the two components of force at B, and the couple at B. Consequently the three independent scalar equations of equilibrium would not provide enough information to compute all four unknowns.

A rigid body, or rigid combination of elements treated as a single body, which possesses more external supports or constraints than are necessary to maintain an equilibrium position is called *statically indeterminate*. Supports which can be removed without destroying the equilibrium condition of the body are said to be *redundant*. The number of redundant supporting elements present corresponds to the *degree of statical indeterminacy* and equals the total number of unknown external forces, minus the number of available independent equations of equilibrium. On the other hand, bodies which are supported by the minimum number of constraints necessary to ensure an equilibrium configuration are called *statically determinate*, and for such bodies the equilibrium equations are sufficient to determine the unknown external forces.

The problems on equilibrium in this article and throughout *Vol. 1 Statics* are generally restricted to statically determinate bodies where the constraints are just sufficient to ensure a stable equilibrium configuration and where the unknown supporting forces can be completely determined by the available independent equations of equilibrium.

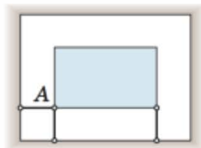
We must be aware of the nature of the constraints before we attempt to solve an equilibrium problem. A body can be recognized as statically indeterminate when there are more unknown external reactions than there are available independent equilibrium equations for the force system involved. It is always well to count the number of unknown variables on a given body and to be certain that an equal number of independent equations can be written; otherwise, effort might be wasted in attempting an impossible solution with the aid of the equilibrium equations only. The unknown variables may be forces, couples, distances, or angles.

Adequacy of Constraints

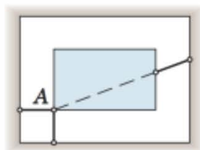
In discussing the relationship between constraints and equilibrium, we should look further at the question of the adequacy of constraints. The existence of three constraints for a two-dimensional problem does not always guarantee a stable equilibrium configuration. Figure 3/7 shows four different types of constraints. In part *a* of the figure, point *A* of the rigid body is fixed by the two links and cannot move, and the third link prevents any rotation about *A*. Thus, this body is *completely fixed* with three *adequate* (*proper*) *constraints*.

In part *b* of the figure, the third link is positioned so that the force transmitted by it passes through point *A* where the other two constraint forces act. Thus, this configuration of constraints can offer no initial resistance to rotation about *A*, which would occur when external loads were applied to the body. We conclude, therefore, that this body is *incompletely fixed* under *partial constraints*.

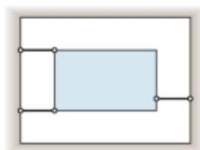
The configuration in part *c* of the figure gives us a similar condition of incomplete fixity because the three parallel links could offer no initial resistance to a small vertical movement of the body as a result



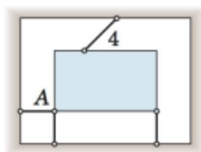
(*a*) Complete fixity
Adequate constraints



(*b*) Incomplete fixity
Partial constraints



(*c*) Incomplete fixity
Partial constraints



(*d*) Excessive fixity
Redundant constraint

Figure 3/7

of external loads applied to it in this direction. The constraints in these two examples are often termed *improper*.

In part *d* of Fig. 3/7 we have a condition of complete fixity, with link 4 acting as a fourth constraint which is unnecessary to maintain a fixed position. Link 4, then, is a *redundant constraint*, and the body is statically indeterminate.

As in the four examples of Fig. 3/7, it is generally possible by direct observation to conclude whether the constraints on a body in two-dimensional equilibrium are adequate (proper), partial (improper), or redundant. As indicated previously, the vast majority of problems in this book are statically determinate with adequate (proper) constraints.

Key Concepts

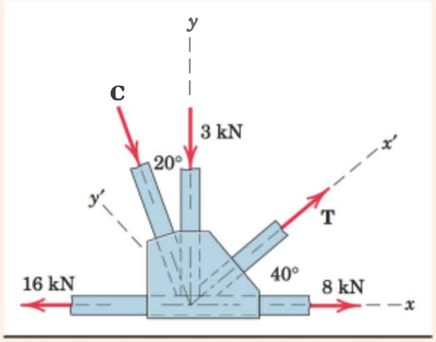
APPROACH TO SOLVING PROBLEMS

The sample problems at the end of this article illustrate the application of free-body diagrams and the equations of equilibrium to typical statics problems. These solutions should be studied thoroughly. In the problem work of this chapter and throughout mechanics, it is important to develop a logical and systematic approach which includes the following steps:

1. Identify clearly the quantities which are known and unknown.
2. Make an unambiguous choice of the body (or system of connected bodies treated as a single body) to be isolated and draw its complete free-body diagram, labeling all external known and unknown but identifiable forces and couples which act on it.
3. Choose a convenient set of reference axes, always using right-handed axes when vector cross products are employed. Choose moment centers with a view to simplifying the calculations. Generally the best choice is one through which as many unknown forces pass as possible. Simultaneous solutions of equilibrium equations are frequently necessary, but can be minimized or avoided by a careful choice of reference axes and moment centers.
4. Identify and state the applicable force and moment principles or equations which govern the equilibrium conditions of the problem. In the following sample problems these relations are shown in brackets and precede each major calculation.
5. Match the number of independent equations with the number of unknowns in each problem.
6. Carry out the solution and check the results. In many problems engineering judgment can be developed by first making a reasonable guess or estimate of the result prior to the calculation and then comparing the estimate with the calculated value.

Sample Problem 3/1

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution. The given sketch constitutes the free-body diagram of the isolated
① section of the joint in question and shows the five forces which are in equilibrium.

Solution I (scalar algebra). For the x-y axes as shown we have

$$[\Sigma F_x = 0] \qquad 8 + T\cos 40° + C\sin 20° - 16 = 0$$

$$0.766T + 0.342C = 8 \qquad (a)$$

$$[\Sigma F_y = 0] \qquad T\sin 40° - C\cos 20° - 3 = 0$$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

Solution II (scalar algebra). To avoid a simultaneous solution, we may use axes
② x'-y' with the first summation in the y'-direction to eliminate reference to T. Thus,

$$[\Sigma F_{y'} = 0] \qquad -C\cos 20° - 3\cos 40° - 8\sin 40° + 16\sin 40° = 0$$

$$C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

$$[\Sigma F_{x'} = 0] \qquad T + 8\cos 40° - 16\cos 40° - 3\sin 40° - 3.03\sin 20° = 0$$

$$T = 9.09 \text{ kN} \qquad \textit{Ans.}$$

Solution III (vector algebra). With unit vectors **i** and **j** in the x- and y-directions, the zero summation of forces for equilibrium yields the vector equation

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad 8\mathbf{i} + (T\cos 40°)\mathbf{i} + (T\sin 40°)\mathbf{j} - 3\mathbf{j} + (C\sin 20°)\mathbf{i} - (C\cos 20°)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

Equating the coefficients of the **i**- and **j**-terms to zero gives

$$8 + T\cos 40° + C\sin 20° - 16 = 0$$

$$T\sin 40° - 3 - C\cos 20° = 0$$

which are the same, of course, as Eqs. (a) and (b), which we solved above.

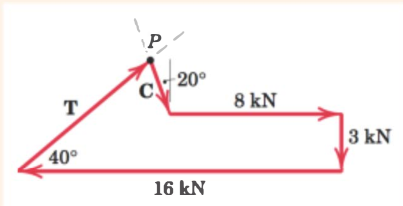
Solution IV (geometric). The polygon representing the zero vector sum of the five forces is shown. Equations (a) and (b) are seen immediately to give the projections of the vectors onto the x- and y-directions. Similarly, projections onto the x'- and y'-directions give the alternative equations in Solution II.

A graphical solution is easily obtained. The known vectors are laid off head-
③ to-tail to some convenient scale, and the directions of **T** and **C** are then drawn to close the polygon. The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of **T** and **C** directly from the drawing to whatever degree of accuracy we incorporate in the construction.

Helpful Hints

① Since this is a problem of concurrent forces, no moment equation is necessary.

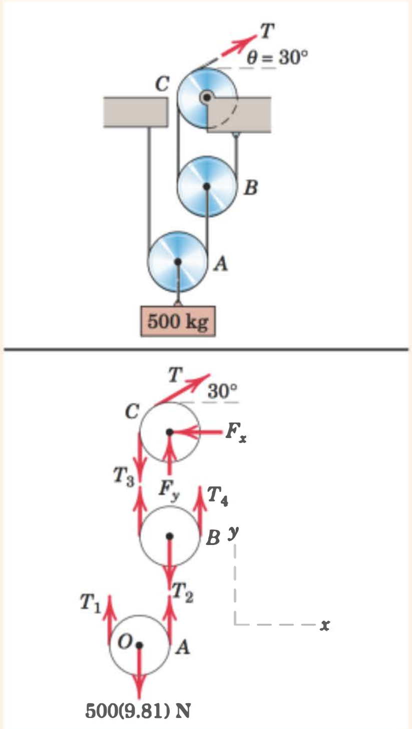
② The selection of reference axes to facilitate computation is always an important consideration. Alternatively in this example we could take a set of axes along and normal to the direction of **C** and employ a force summation normal to **C** to eliminate it.



③ The known vectors may be added in any order desired, but they must be added before the unknown vectors.

Sample Problem 3/2

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force. With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

① $[\Sigma M_O = 0]$ $\qquad T_1 r - T_2 r = 0 \qquad T_1 = T_2$

$[\Sigma F_y = 0]$ $\qquad T_1 + T_2 - 500(9.81) = 0 \qquad 2T_1 = 500(9.81) \qquad T_1 = T_2 = 2450 \text{ N}$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

$$T = T_3 \qquad \text{or} \qquad T = 1226 \text{ N}$$ *Ans.*

Equilibrium of the pulley in the x- and y-directions requires

$[\Sigma F_x = 0]$ $\qquad 1226 \cos 30° - F_x = 0 \qquad F_x = 1062 \text{ N}$

$[\Sigma F_y = 0]$ $\qquad F_y + 1226 \sin 30° - 1226 = 0 \qquad F_y = 613 \text{ N}$

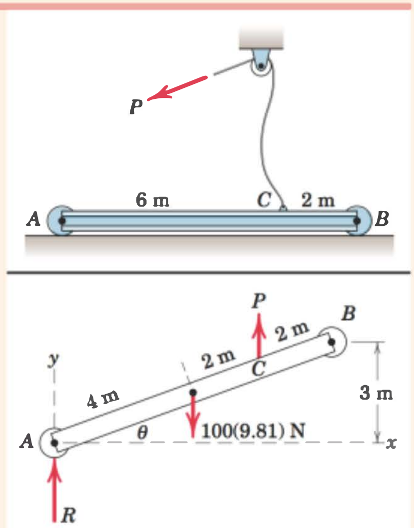
$[F = \sqrt{F_x{}^2 + F_y{}^2}]$ $\qquad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N}$ *Ans.*

Helpful Hint

① Clearly the radius r does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Sample Problem 3/3

The uniform 100-kg I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at C, it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A, and the angle θ made by the beam with the horizontal in the elevated position.



Solution. In constructing the free-body diagram, we note that the reaction on the roller at A and the weight are vertical forces. Consequently, in the absence of other horizontal forces, P must also be vertical. From Sample Problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C.

Moment equilibrium about A eliminates force R and gives

① $[\Sigma M_A = 0]$ $\qquad P(6 \cos \theta) - 981(4 \cos \theta) = 0 \qquad P = 654 \text{ N}$ *Ans.*

Equilibrium of vertical forces requires

$[\Sigma F_y = 0]$ $\qquad 654 + R - 981 = 0 \qquad R = 327 \text{ N}$ *Ans.*

The angle θ depends only on the specified geometry and is

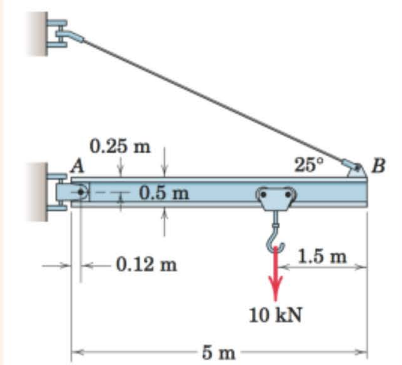
$$\sin \theta = 3/8 \qquad \theta = 22.0°$$ *Ans.*

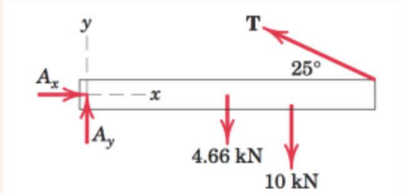
Helpful Hint

① Clearly the equilibrium of this parallel force system is independent of θ.

Sample Problem 3/4

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.





Free-body diagram

Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T, which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equation. ① In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of $\mathbf{T}$ than it is to compute the perpendicular distance from $\mathbf{T}$ to A. Hence, with the counterclockwise sense as positive we write

② $[\Sigma M_A = 0]$ $$(T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

from which $T = 19.61 \text{ kN}$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

$[\Sigma F_x = 0]$ $A_x - 19.61 \cos 25° = 0$ $A_x = 17.77 \text{ kN}$

$[\Sigma F_y = 0]$ $A_y + 19.61 \sin 25° - 4.66 - 10 = 0$ $A_y = 6.37 \text{ kN}$

③ $[A = \sqrt{A_x^2 + A_y^2}]$ $A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$ *Ans.*

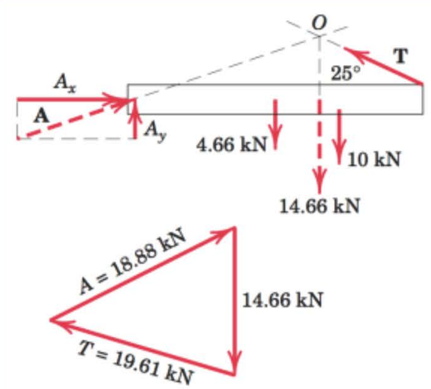
Graphical solution. The principle that three forces in equilibrium must be concurrent is utilized for a graphical solution by combining the two known vertical forces of 4.66 and 10 kN into a single 14.66-kN force, located as shown on the modified free-body diagram of the beam in the lower figure. The position of this resultant load may easily be determined graphically or algebraically. The intersection of the 14.66-kN force with the line of action of the unknown tension $\mathbf{T}$ defines the point of concurrency O through which the pin reaction $\mathbf{A}$ must pass. The unknown magnitudes of $\mathbf{T}$ and $\mathbf{A}$ may now be found by adding the forces head-to-tail to form the closed equilibrium polygon of forces, thus satisfying their zero vector sum. After the known vertical load is laid off to a convenient scale, as shown in the lower part of the figure, a line representing the given direction of the tension $\mathbf{T}$ is drawn through the tip of the 14.66-kN vector. Likewise a line representing the direction of the pin reaction $\mathbf{A}$, determined from the concurrency established with the free-body diagram, is drawn through the tail of the 14.66-kN vector. The intersection of the lines representing vectors $\mathbf{T}$ and $\mathbf{A}$ establishes the magnitudes T and A necessary to make the vector sum of the forces equal to zero. These magnitudes are scaled from the diagram. The x- and y-components of $\mathbf{A}$ may be constructed on the force polygon if desired.

Helpful Hints

① The justification for this step is Varignon's theorem, explained in Art. 2/4. Be prepared to take full advantage of this principle frequently.

② The calculation of moments in two-dimensional problems is generally handled more simply by scalar algebra than by the vector cross product $\mathbf{r} \times \mathbf{F}$. In three dimensions, as we will see later, the reverse is often the case.

③ The direction of the force at A could be easily calculated if desired. However, in designing the pin A or in checking its strength, it is only the magnitude of the force that matters.

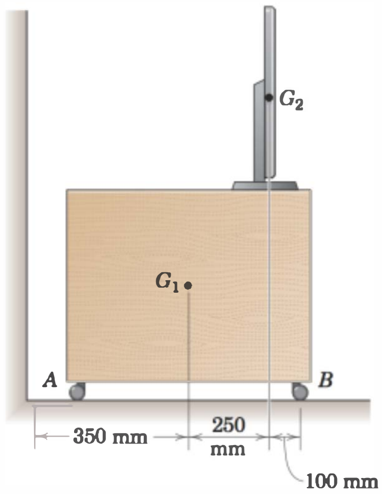


Graphical solution

PROBLEMS

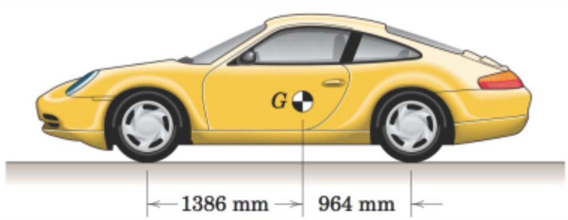
Introductory Problems

3/1 In the side view of a 25-kg flat-screen television resting on a 40-kg cabinet, the respective centers of mass are labeled G_2 and G_1. Assume symmetry into the paper and calculate the normal reaction force at each of the four casters.



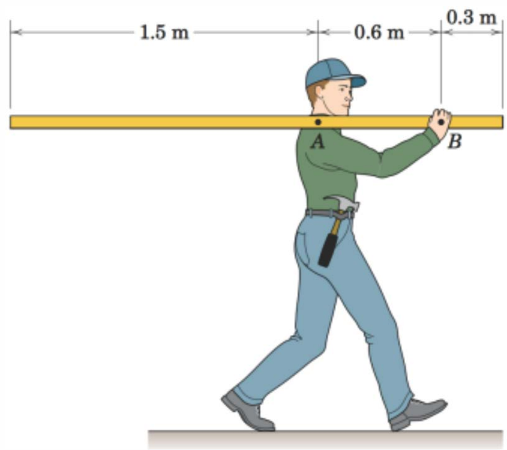
Problem 3/1

3/2 The mass center G of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.



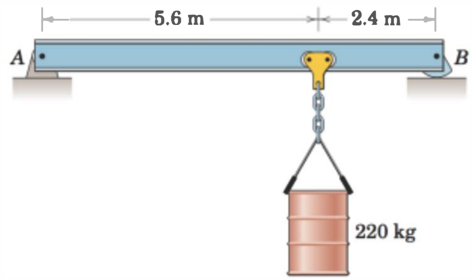
Problem 3/2

3/3 A carpenter carries a 6-kg uniform board as shown. What downward force does he feel on his shoulder at A?



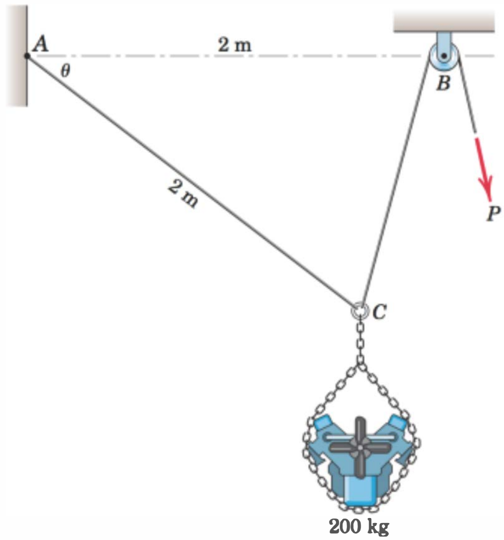
Problem 3/3

3/4 The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



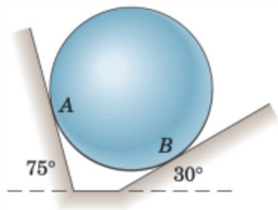
Problem 3/4

3/5 Determine the force P required to maintain the 200-kg engine in the position for which $\theta = 30°$. The diameter of the pulley at B is negligible.



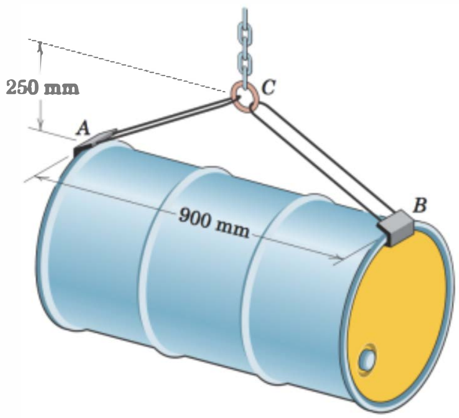
Problem 3/5

3/6 The 20-kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at A and B.



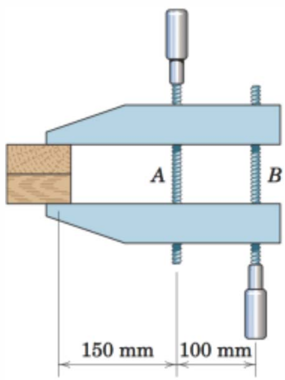
Problem 3/6

3/7 The 275-kg drum is being hoisted by the lifting device which hooks over the end lips of the drum. Determine the tension T in each of the equal-length rods which form the two U-shaped members of the device.



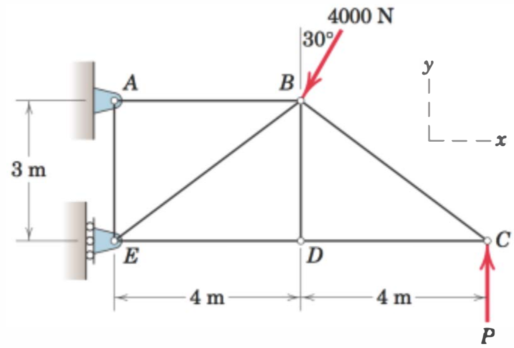
Problem 3/7

3/8 If the screw B of the wood clamp is tightened so that the two blocks are under a compression of 500 N, determine the force in screw A. (*Note:* The force supported by each screw may be taken in the direction of the screw.)



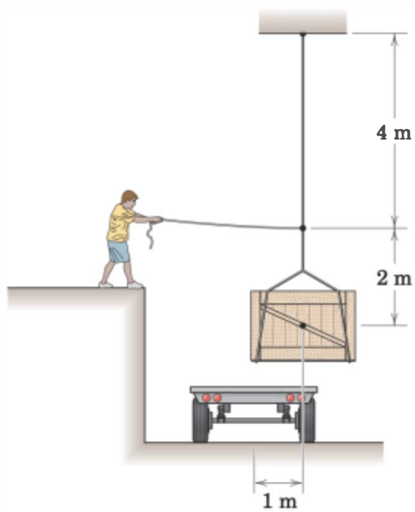
Problem 3/8

3/9 Determine the reactions at A and E if $P = 500$ N. What is the maximum value which P may have for static equilibrium? Neglect the weight of the structure compared with the applied loads.



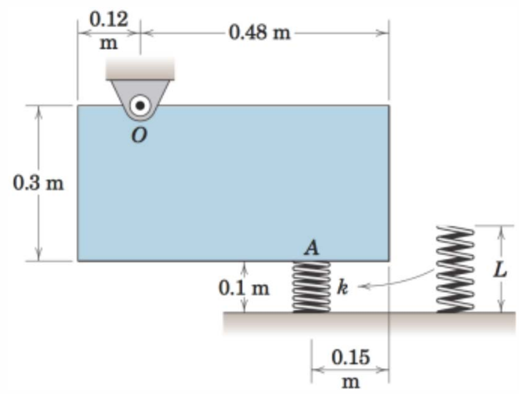
Problem 3/9

3/10 What horizontal force P must a worker exert on the rope to position the 50-kg crate directly over the trailer?



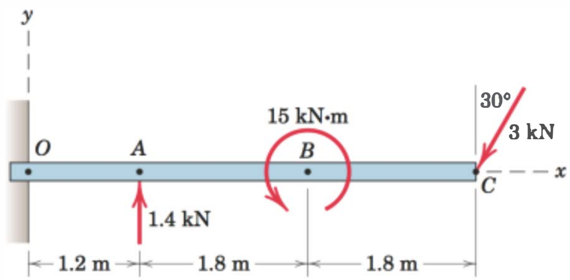
Problem 3/10

3/11 The 20-kg uniform rectangular plate is supported by an ideal pivot at O and a spring which must be compressed prior to being slipped into place at point A. If the modulus of the spring is $k = 2$ kN/m, what must be its undeformed length L?



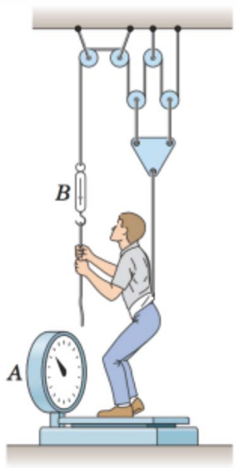
Problem 3/11

3/12 The 500-kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point O. The x-y plane is vertical.



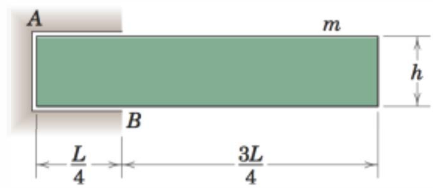
Problem 3/12

3/13 A former student of mechanics wishes to weigh himself but has access only to a scale A with capacity limited to 400 N and a small 80-N spring dynamometer B. With the rig shown he discovers that when he exerts a pull on the rope so that B registers 76 N, the scale A reads 268 N. What are his correct weight W and mass m?



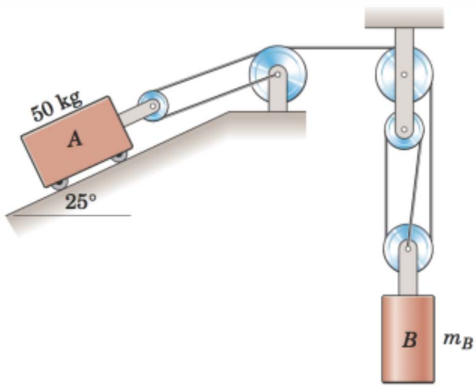
Problem 3/13

3/14 The uniform rectangular body of mass m is placed into a fixed opening with slight clearances as shown. Determine the forces at the contact points A and B. Do your results depend on the height h?



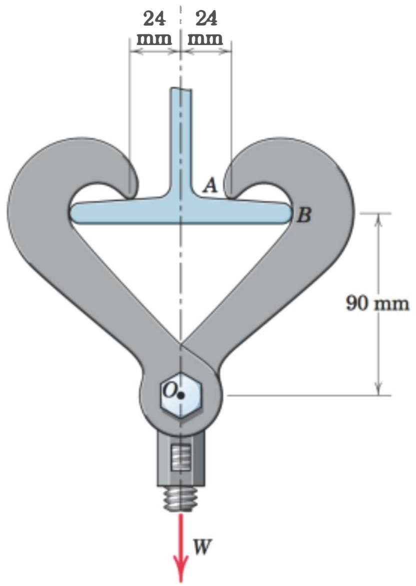
Problem 3/14

3/15 What mass m_B will cause the system to be in equilibrium? Neglect all friction, and state any other assumptions.



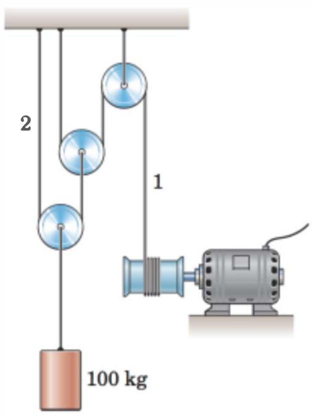
Problem 3/15

3/16 The pair of hooks is designed for the hanging of loads from horizontal I-beams. If the load $W = 5$ kN, estimate the contact forces at A and B. Neglect all friction.



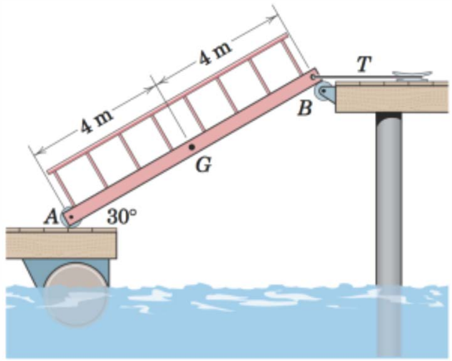
Problem 3/16

3/17 The winch takes in cable at the constant rate of 200 mm/s. If the cylinder mass is 100 kg, determine the tension in cable 1. Neglect all friction.



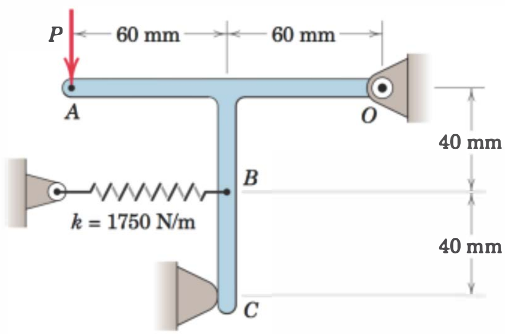
Problem 3/17

3/18 To accommodate the rise and fall of the tide, a walkway from a pier to a float is supported by two rollers as shown. If the mass center of the 300-kg walkway is at G, calculate the tension T in the horizontal cable which is attached to the cleat and find the force under the roller at A.



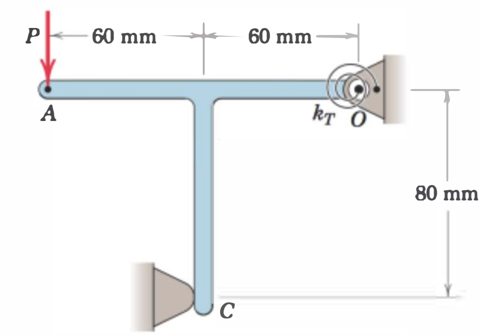
Problem 3/18

3/19 When the 0.05-kg body is in the position shown, the linear spring is stretched 10 mm. Determine the force P required to break contact at C. Complete solutions for (*a*) including the effect of the weight and (*b*) neglecting the weight.



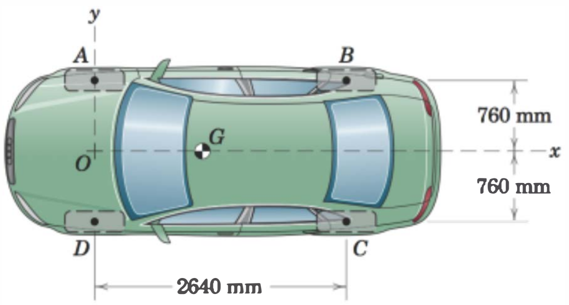
Problem 3/19

3/20 When the 0.05-kg body is in the position shown, the torsional spring at O is pretensioned so as to exert a 0.75-N·m clockwise moment on the body. Determine the force P required to break contact at C. Complete solutions for (*a*) including the effect of the weight and (*b*) neglecting the weight.



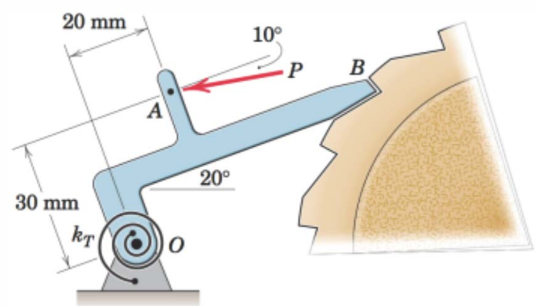
Problem 3/20

3/21 When on level ground, the car is placed on four individual scales—one under each tire. The scale readings are 4450 N at each front wheel and 2950 N at each rear wheel. Determine the x-coordinate of the mass center G and the mass of the car.



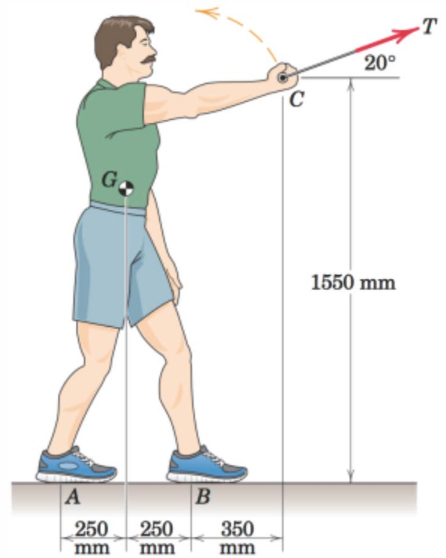
Problem 3/21

3/22 Determine the magnitude P of the force required to rotate the release pawl OB counterclockwise from its locked position. The torsional spring constant is $k_T = 3.4$ N·m/rad and the pawl end of the spring has been deflected 25° counterclockwise from the neutral position in the configuration shown. Neglect any forces at the contact point B.



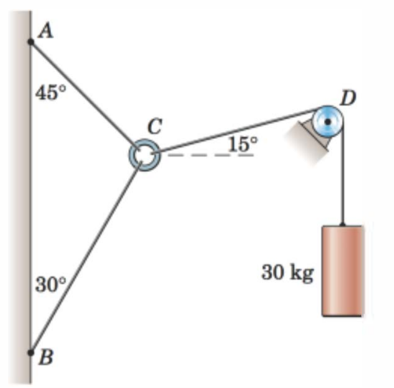
Problem 3/22

3/23 The 80-kg exerciser is beginning to execute some slow, steady bicep curls. As the tension $T = 65$ N is developed against an exercise machine (not shown), determine the normal reaction forces at the feet A and B. Friction is sufficient to prevent slipping, and the exerciser maintains the position shown with center of mass at G.



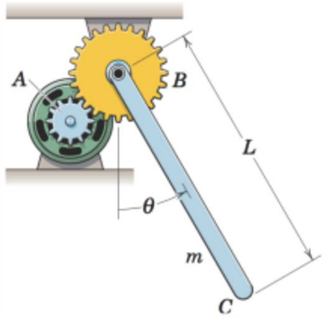
Problem 3/23

3/24 Three cables are joined at the junction ring C. Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.



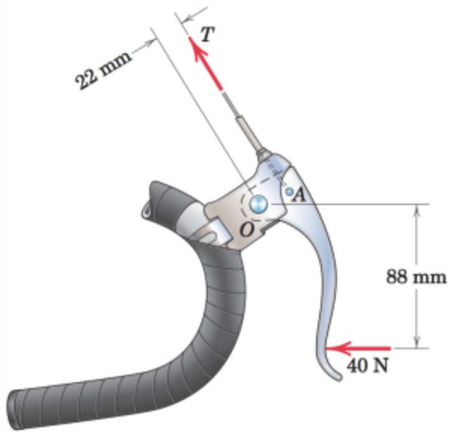
Problem 3/24

3/25 Determine the moment M which the motor must exert in order to position the uniform slender bar of mass m and length L in the arbitrary position θ. The ratio of the radius of the gear wheel B attached to the bar to that of the gear wheel A attached to the motor shaft is 2.



Problem 3/25

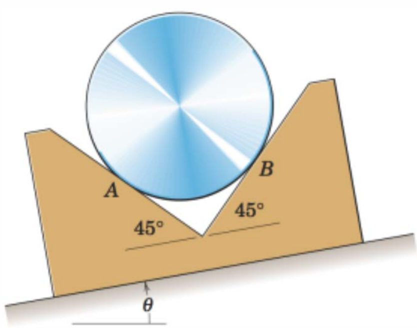
3/26 A bicyclist applies a 40-N force to the brake lever of her bicycle as shown. Determine the corresponding tension T transmitted to the brake cable. Neglect friction at the pivot O.



Problem 3/26

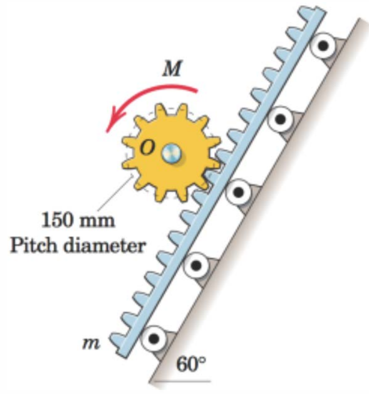
Representative Problems

3/27 Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder.



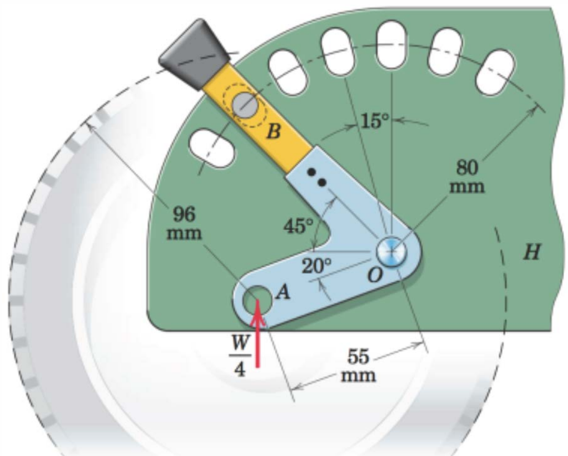
Problem 3/27

3/28 The rack has a mass $m = 75$ kg. What moment M must be exerted on the gear wheel by the motor in order to lower the rack at a slow steady speed down the 60° incline? Neglect all friction. The fixed motor which drives the gear wheel via the shaft at O is not shown.



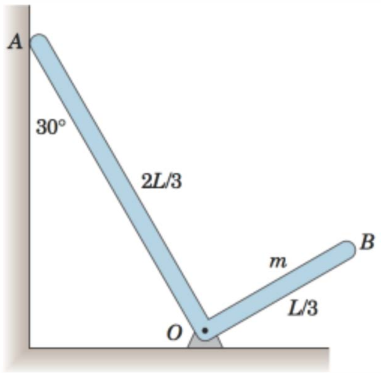
Problem 3/28

3/29 The elements of a wheel-height adjuster for a lawn mower are shown. The wheel (partial outline shown dashed for clarity) bolts through the hole at A, which goes through the bracket but not the housing H. A pin fixed to the back of the bracket at B fits into one of the seven elongated holes of the housing. For the position shown, determine the force at the pin B and the magnitude of the reaction at the pivot O. The wheel supports a force of magnitude $W/4$, where W is the weight of the entire mower.



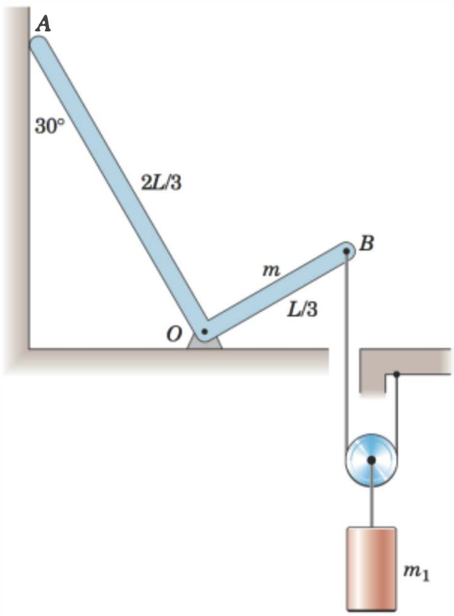
Problem 3/29

3/30 The right-angle uniform slender bar *AOB* has mass *m*. If friction at the pivot *O* is neglected, determine the magnitude of the normal force at *A* and the magnitude of the pin reaction at *O*.



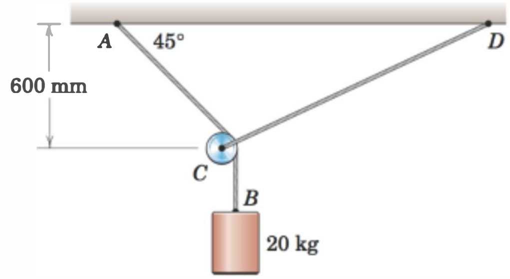
Problem 3/30

3/31 Determine the minimum cylinder mass m_1 required to cause loss of contact at *A*.



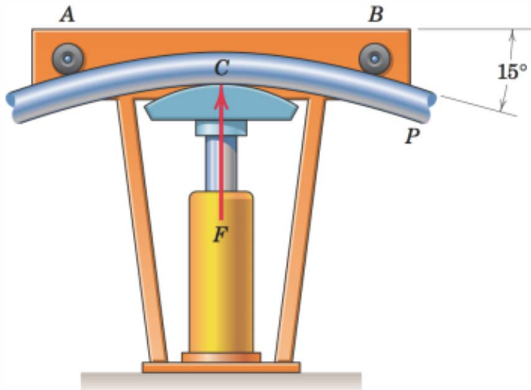
Problem 3/31

3/32 Cable *AB* passes over the small ideal pulley *C* without a change in its tension. What length of cable *CD* is required for static equilibrium in the position shown? What is the tension *T* in cable *CD*?



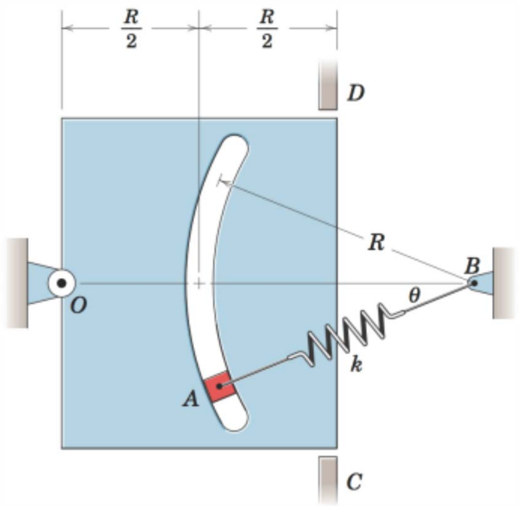
Problem 3/32

3/33 A pipe *P* is being bent by the pipe bender as shown. If the hydraulic cylinder applies a force of magnitude $F = 24$ kN to the pipe at *C*, determine the magnitude of the roller reactions at *A* and *B*.



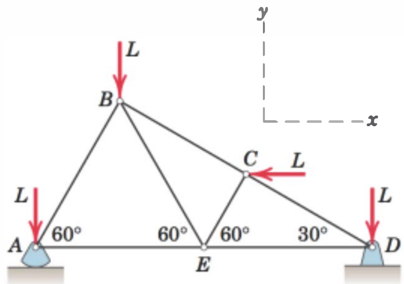
Problem 3/33

3/34 The small slider *A* is moved along the circular slot by a mechanism attached to the back side of the rectangular plate. For the slider position $\theta = 20°$ shown, determine the normal forces exerted at the small stops *C* and *D*. The unstretched length of the spring of constant $k = 1.6$ kN/m is $R/3$. The value of *R* is 25 mm, and the plate lies in a horizontal plane. Neglect all friction.



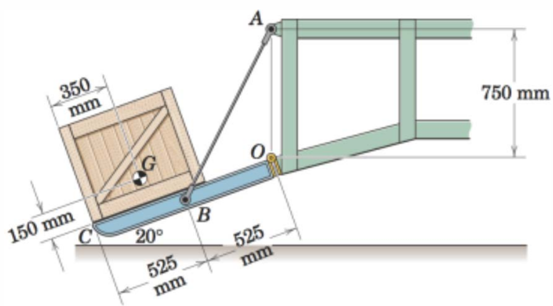
Problem 3/34

3/35 The asymmetric simple truss is loaded as shown. Determine the reactions at A and D. Neglect the weight of the structure compared with the applied loads. Is knowledge of the size of the structure necessary?



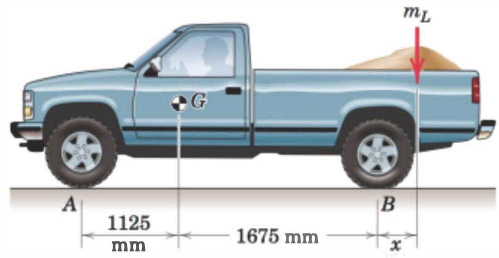
Problem 3/35

3/36 The tailgate OBC is attached to the rear of a trailer via hinges at O and two restraining cables AB. The 55-kg tailgate is 100 mm thick with center of mass at B, which is at midthickness. The crate is centered between the two cables and has a mass of 90 kg with center of mass at G. Determine the tension T in each cable.



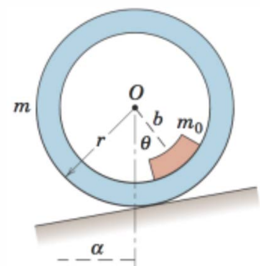
Problem 3/36

3/37 The indicated location of the center of mass of the 1600-kg pickup truck is for the unladen condition. If a load whose center of mass is $x = 400$ mm behind the rear axle is added to the truck, determine the load mass m_L for which the normal forces under the front and rear wheels are equal.



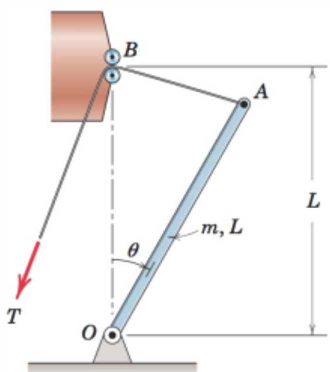
Problem 3/37

3/38 A uniform ring of mass m and radius r carries an eccentric mass m_0 at a radius b and is in an equilibrium position on the incline, which makes an angle α with the horizontal. If the contacting surfaces are rough enough to prevent slipping, write the expression for the angle θ which defines the equilibrium position.



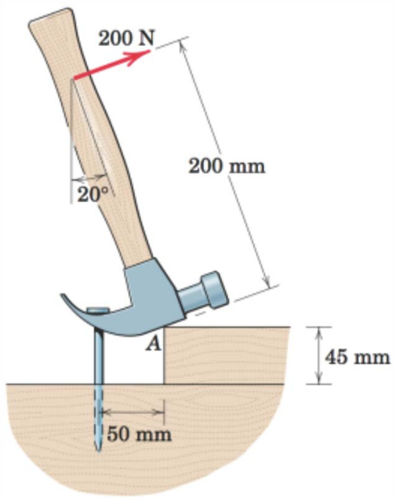
Problem 3/38

3/39 Determine the force T required to hold the uniform bar of mass m and length L in an arbitrary angular position θ. Plot your result over the range $0 \leq \theta \leq 90°$, and state the value of T for $\theta = 40°$.



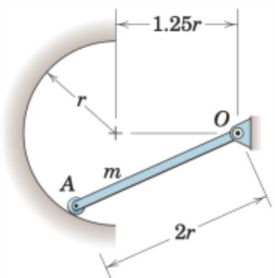
Problem 3/39

3/40 A block placed under the head of the claw hammer as shown greatly facilitates the extraction of the nail. If a 200-N pull on the handle is required to pull the nail, calculate the tension T in the nail and the magnitude A of the force exerted by the hammer head on the block. The contacting surfaces at A are sufficiently rough to prevent slipping.



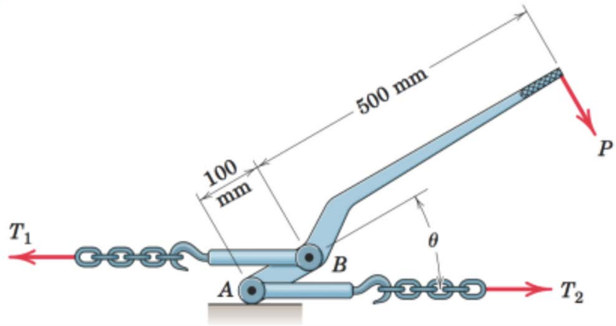
Problem 3/40

3/41 The uniform slender bar of length $2r$ and mass m rests against the circular surface as shown. Determine the normal force at the small roller A and the magnitude of the ideal pivot reaction at O.



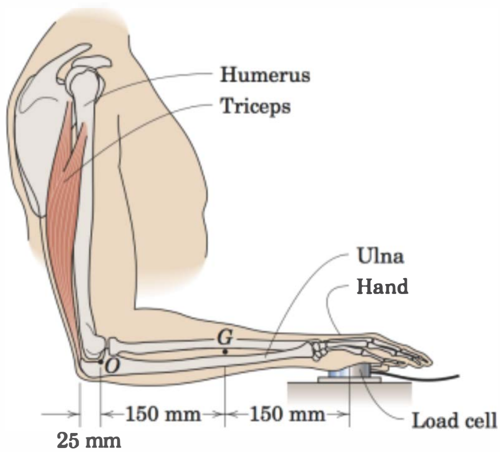
Problem 3/41

3/42 The chain binder is used to secure loads of logs, lumber, pipe, and the like. If the tension T_1 is 2 kN when $\theta = 30°$, determine the force P required on the lever and the corresponding tension T_2 for this position. Assume that the surface under A is perfectly smooth.



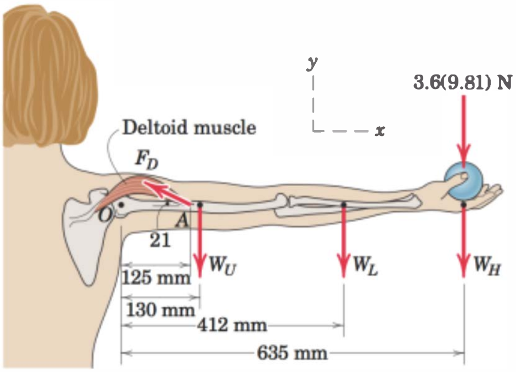
Problem 3/42

3/43 In a procedure to evaluate the strength of the triceps muscle, a person pushes down on a load cell with the palm of his hand as indicated in the figure. If the load-cell reading is 160 N, determine the vertical tensile force F generated by the triceps muscle. The mass of the lower arm is 1.5 kg with mass center at G. State any assumptions.



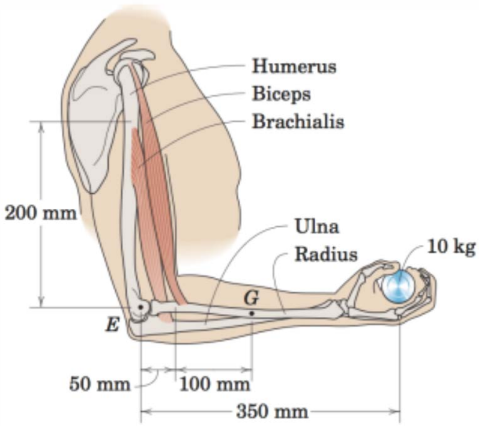
Problem 3/43

3/44 A woman is holding a 3.6-kg sphere in her hand with the entire arm held horizontally as shown in the figure. A tensile force in the deltoid muscle prevents the arm from rotating about the shoulder joint O; this force acts at the 21° angle shown. Determine the force exerted by the deltoid muscle on the upper arm at A and the x- and y-components of the force reaction at the shoulder joint O. The mass of the upper arm is $m_U = 1.9$ kg, the mass of the lower arm is $m_L = 1.1$ kg, and the mass of the hand is $m_H = 0.4$ kg; all the corresponding weights act at the locations shown in the figure.



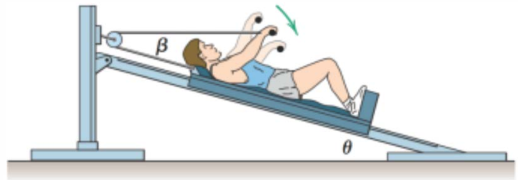
Problem 3/44

3/45 A person is performing slow arm curls with a 10-kg weight as indicated in the figure. The brachialis muscle group (consisting of the biceps and brachialis muscles) is the major factor in this exercise. Determine the magnitude F of the brachialis-muscle-group force and the magnitude E of the elbow joint reaction at point E for the forearm position shown in the figure. Take the dimensions shown to locate the effective points of application of the two muscle groups; these points are 200 mm directly above E and 50 mm directly to the right of E. Include the effect of the 1.5-kg forearm mass with mass center at point G. State any assumptions.



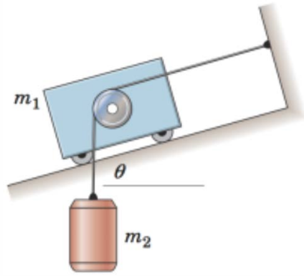
Problem 3/45

3/46 The exercise machine is designed with a lightweight cart which is mounted on small rollers so that it is free to move along the inclined ramp. Two cables are attached to the cart—one for each hand. If the hands are together so that the cables are parallel and if each cable lies essentially in a vertical plane, determine the force P which each hand must exert on its cable in order to maintain an equilibrium position. The mass of the person is 70 kg, the ramp angle θ is 15°, and the angle β is 18°. In addition, calculate the force R which the ramp exerts on the cart.



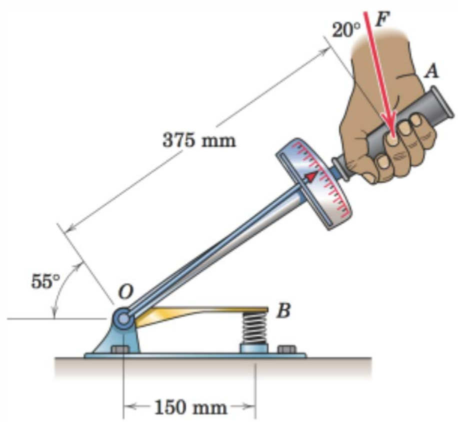
Problem 3/46

3/47 For a given value m_1 for the cart mass, determine the value m_2 for the cylinder mass which results in equilibrium of the system. Neglect all friction. Evaluate your expression for $\theta = 15°$, 45°, and 60°.



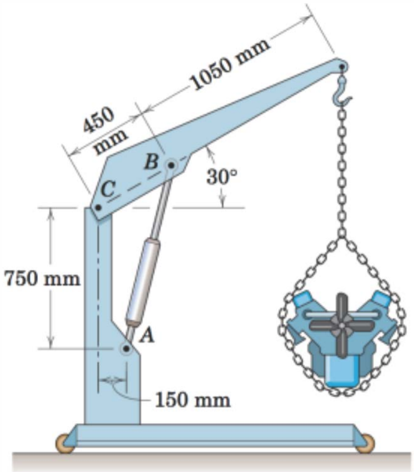
Problem 3/47

3/48 The device shown is used to test automobile-engine valve springs. The torque wrench is directly connected to arm OB. The specification for the automotive intake-valve spring is that 370 N of force should reduce its length from 50 mm (unstressed length) to 42 mm. What is the corresponding reading M on the torque wrench, and what force F exerted on the torque-wrench handle is required to produce this reading? Neglect the small effects of changes in the angular position of arm OB.



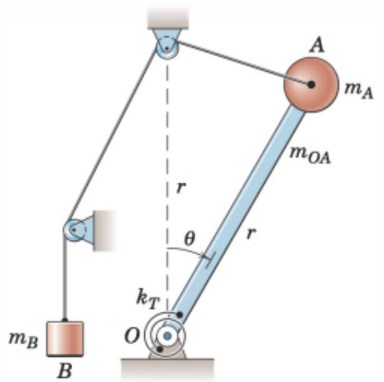
Problem 3/48

3/49 The portable floor crane in the automotive shop is lifting a 100-kg engine. For the position shown compute the magnitude of the force supported by the pin at C and the oil pressure p against the 80-mm-diameter piston of the hydraulic-cylinder unit AB.



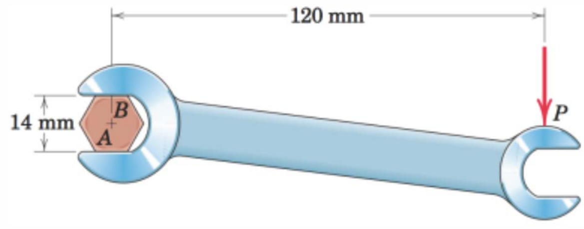
Problem 3/49

***3/50** The torsional spring of constant $k_T = 50$ N·m/rad is undeformed when $\theta = 0$. Determine the value(s) of θ over the range $0 \le \theta \le 180°$ for which equilibrium exists. Use the values $m_A = 10$ kg, $m_B = 1$ kg, $m_{OA} = 5$ kg, and $r = 0.8$ m. Assume that OA is a uniform slender rod with a particle A (negligible size) at its end, and neglect the effects of the small ideal rollers.



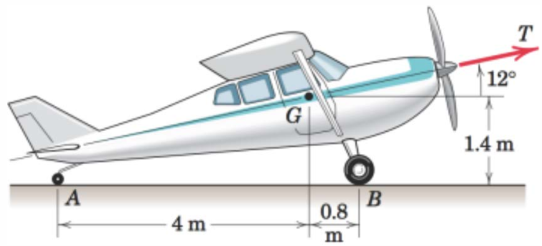
Problem 3/50

3/51 A torque (moment) of 24 N·m is required to turn the bolt about its axis. Determine P and the forces between the smooth hardened jaws of the wrench and the corners A and B of the hexagonal head. Assume that the wrench fits easily on the bolt so that contact is made at corners A and B only.



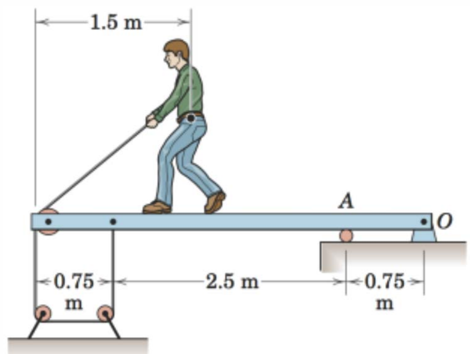
Problem 3/51

3/52 During an engine test on the ground, a propeller thrust $T = 3000$ N is generated on the 1800-kg airplane with mass center at G. The main wheels at B are locked and do not skid; the small tail wheel at A has no brake. Compute the percent change n in the normal forces at A and B as compared with their "engine-off" values.



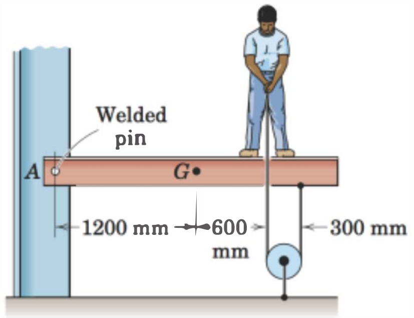
Problem 3/52

3/53 To test the deflection of the uniform 100-kg beam the 50-kg boy exerts a pull of 150 N on the rope rigged as shown. Compute the force supported by the pin at the hinge O.



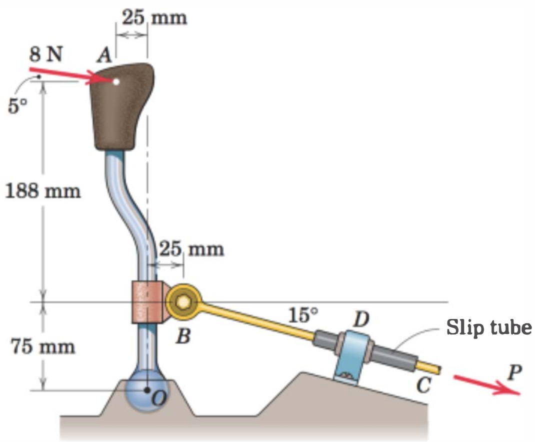
Problem 3/53

3/54 The pin A, which connects the 200-kg steel beam with center of mass at G to the vertical column, is welded both to the beam and to the column. To test the weld, the 80-kg man loads the beam by exerting a 300-N force on the rope which passes through a hole in the beam as shown. Calculate the torque (couple) M supported by the pin.



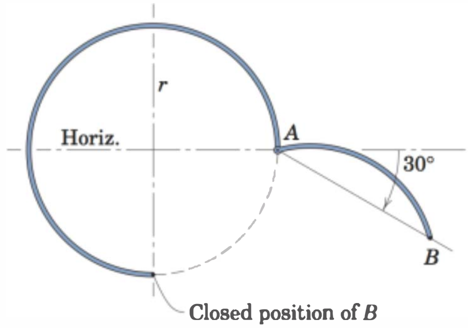
Problem 3/54

3/55 A portion of the shifter mechanism for a manual car transmission is shown in the figure. For the 8-N force exerted on the shift knob, determine the corresponding force P exerted by the shift link BC on the transmission (not shown). Neglect friction in the ball-and-socket joint at O, in the joint at B, and in the slip tube near support D. Note that a soft rubber bushing at D allows the slip tube to self-align with link BC.



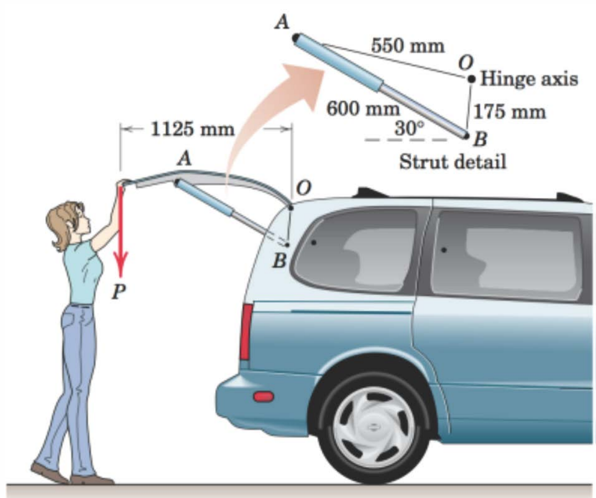
Problem 3/55

3/56 The cargo door for an airplane of circular fuselage section consists of the uniform quarter-circular segment AB of mass m. A detent in the hinge at A holds the door open in the position shown. Determine the moment exerted by the hinge on the door.



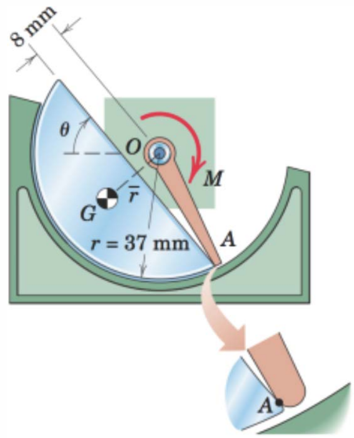
Problem 3/56

3/57 It is desired that a person be able to begin closing the van hatch from the open position shown with a 40-N vertical force P. As a design exercise, determine the necessary force in each of the two hydraulic struts AB. The mass center of the 40-kg door is 37.5 mm directly below point A. Treat the problem as two-dimensional.



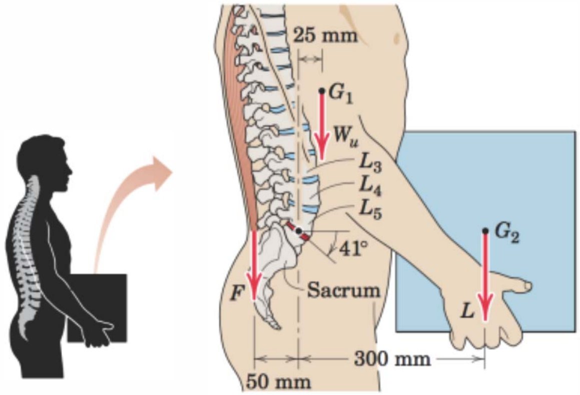
Problem 3/57

3/58 Certain elements of an in-refrigerator ice-cube maker are shown in the figure. (A "cube" has the form of a cylindrical segment!) Once the cube freezes and a small heater (not shown) forms a thin film of water between the cube and supporting surface, a motor rotates the ejector arm OA to remove the cube. If there are eight cubes and eight arms, determine the required torque M as a function of θ. The mass of eight cubes is 0.25 kg, and the center-of-mass distance $\bar{r} = 0.55r$. Neglect friction, and assume that the resultant of the distributed normal force acting on the cube passes through point O.



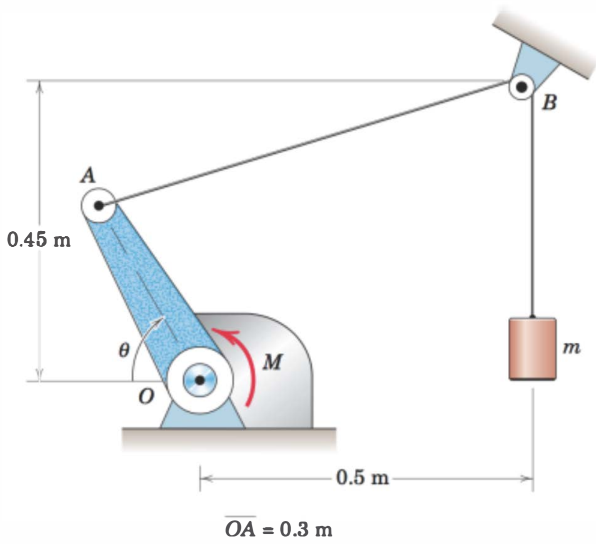
Problem 3/58

▶3/59 The lumbar portion of the human spine supports the entire weight of the upper torso and the force load imposed on it. We consider here the disk (shaded red) between the lowest vertebra of the lumbar region (L_5) and the uppermost vertebra of the sacrum region. (*a*) For the case $L = 0$, determine the compressive force C and the shear force S supported by this disk in terms of the body weight W. The weight W_u of the upper torso (above the disk in question) is 68% of the total body weight W and acts at G_1. The vertical force F which the rectus muscles of the back exert on the upper torso acts as shown in the figure. (*b*) Repeat for the case when the person holds a weight of magnitude $L = W/3$ as shown. State any assumptions.



Problem 3/59

***3/60** Determine and plot the moment M which much be applied to the crank OA in order to hold the cylinder of mass $m = 5$ kg in equilibrium. Neglect the effects of the mass of OA and friction and consider the range $0 \leq \theta \leq 180°$. State the maximum and minimum values of the absolute value of M and the values of θ for which these extremes occur, and physically justify these results.



Problem 3/60

SECTION B EQUILIBRIUM IN THREE DIMENSIONS

3/4 Equilibrium Conditions

We now extend our principles and methods developed for two-dimensional equilibrium to the case of three-dimensional equilibrium. In Art. 3/1 the general conditions for the equilibrium of a body were stated in Eqs. 3/1, which require that the resultant force and resultant couple on a body in equilibrium be zero. These two vector equations of equilibrium and their scalar components may be written as

$$\Sigma\mathbf{F} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases}$$
$$\Sigma\mathbf{M} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma M_x = 0 \\ \Sigma M_y = 0 \\ \Sigma M_z = 0 \end{cases} \tag{3/3}$$

The first three scalar equations state that there is no resultant force acting on a body in equilibrium in any of the three coordinate directions. The second three scalar equations express the further equilibrium requirement that there be no resultant moment acting on the body about any of the coordinate axes or about axes parallel to the coordinate axes. These six equations are both necessary and sufficient conditions for complete equilibrium. The reference axes may be chosen arbitrarily as a matter of convenience, the only restriction being that a right-handed coordinate system should be chosen when vector notation is used.

The six scalar relationships of Eqs. 3/3 are independent conditions because any of them can be valid without the others. For example, for a car which accelerates on a straight and level road in the x-direction, Newton's second law tells us that the resultant force on the car equals its mass times its acceleration. Thus $\Sigma F_x \neq 0$, but the remaining two force–equilibrium equations are satisfied because all other acceleration components are zero. Similarly, if the flywheel of the engine of the accelerating car is rotating with increasing angular speed about the x-axis, it is not in rotational equilibrium about this axis. Thus, for the flywheel alone, $\Sigma M_x \neq 0$ along with $\Sigma F_x \neq 0$, but the remaining four equilibrium equations for the flywheel would be satisfied for its mass-center axes.

In applying the vector form of Eqs. 3/3, we first express each of the forces in terms of the coordinate unit vectors **i**, **j**, and **k**. For the first equation, $\Sigma\mathbf{F} = \mathbf{0}$, the vector sum will be zero only if the coefficients of **i**, **j**, and **k** in the expression are, respectively, zero. These three sums, when each is set equal to zero, yield precisely the three scalar equations of equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$.

For the second equation, $\Sigma\mathbf{M} = \mathbf{0}$, where the moment sum may be taken about any convenient point O, we express the moment of each force as the cross product $\mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector from O to any point on the line of action of the force $\mathbf{F}$. Thus $\Sigma\mathbf{M} = \Sigma(\mathbf{r} \times \mathbf{F}) = \mathbf{0}$.

When the coefficients of $\mathbf{i}, \mathbf{j}$, and $\mathbf{k}$ in the resulting moment equation are set equal to zero, respectively, we obtain the three scalar moment equations $\Sigma M_x = 0$, $\Sigma M_y = 0$, and $\Sigma M_z = 0$.

Free-Body Diagrams

The summations in Eqs. 3/3 include the effects of *all* forces on the body under consideration. We learned in the previous article that the free-body diagram is the only reliable method for disclosing all forces and moments which should be included in our equilibrium equations. In three dimensions the free-body diagram serves the same essential purpose as it does in two dimensions and should *always* be drawn. We have our choice either of drawing a pictorial view of the isolated body with all external forces represented or of drawing the orthogonal projections of the free-body diagram. Both representations are illustrated in the sample problems at the end of this article.

The correct representation of forces on the free-body diagram requires knowledge of the characteristics of contacting surfaces. These characteristics were described in Fig. 3/1 for two-dimensional problems, and their extension to three-dimensional problems is represented in Fig. 3/8 for the most common situations of force transmission. The representations in both Figs. 3/1 and 3/8 will be used in three-dimensional analysis.

The essential purpose of the free-body diagram is to develop a reliable picture of the physical action of all forces (and couples if any) acting on a body. So it is helpful to represent the forces in their correct physical sense whenever possible. In this way, the free-body diagram becomes a closer model to the actual physical problem than it would be if the forces were arbitrarily assigned or always assigned in the same mathematical sense as that of the assigned coordinate axis.

For example, in part 4 of Fig. 3/8, the correct sense of the unknowns R_x and R_y may be known or perceived to be in the sense opposite to those of the assigned coordinate axes. Similar conditions apply to the sense of couple vectors, parts 5 and 6, where their sense by the right-hand rule may be assigned opposite to that of the respective coordinate direction. By this time, you should recognize that a negative answer for an unknown force or couple vector merely indicates that its physical action is in the sense opposite to that assigned on the free-body diagram. Frequently, of course, the correct physical sense is not known initially, so that an arbitrary assignment on the free-body diagram becomes necessary.

Categories of Equilibrium

Application of Eqs. 3/3 falls into four categories which we identify with the aid of Fig. 3/9. These categories differ in the number and type (force or moment) of independent equilibrium equations required to solve the problem.

Category 1, equilibrium of forces all concurrent at point O, requires all three force equations, but no moment equations because the moment of the forces about any axis through O is zero.

Category 2, equilibrium of forces which are concurrent with a line, requires all equations except the moment equation about that line, which is automatically satisfied.

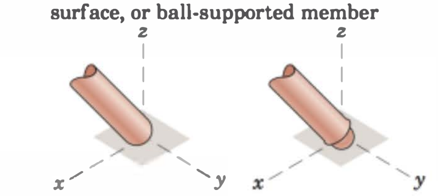
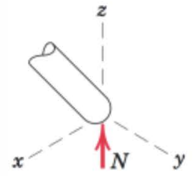
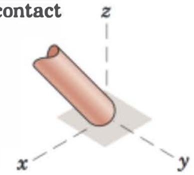
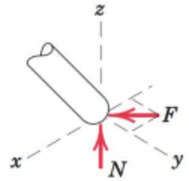
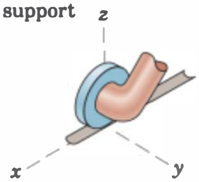
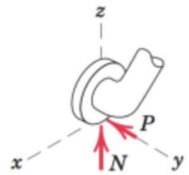
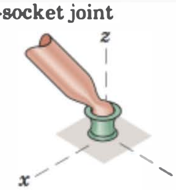
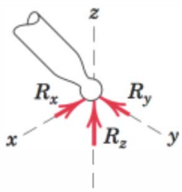
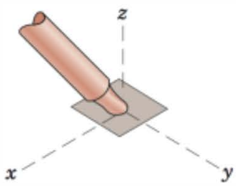
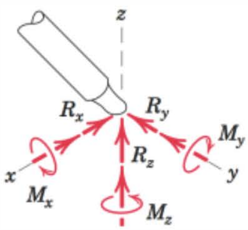
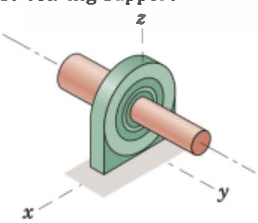
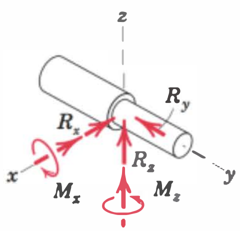
MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS		
Type of Contact and Force Origin	Action on Body to Be Isolated	
1. Member in contact with smooth surface, or ball-supported member 		Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 		The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 		A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint 		A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 		In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 		Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

Figure 3/8

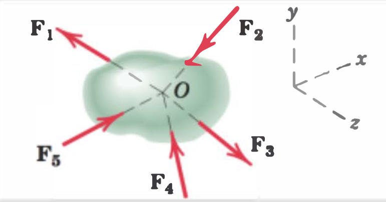
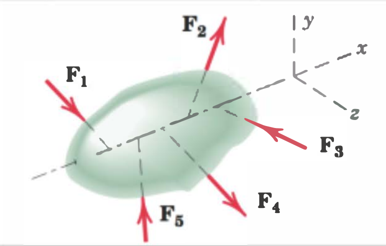
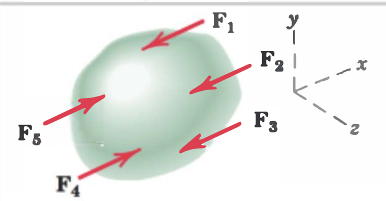
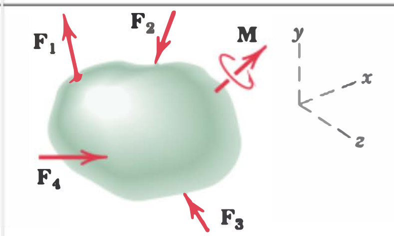
CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	**Free-Body Diagram**	**Independent Equations**
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_x = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_z = 0$ $\quad \Sigma M_z = 0$

Figure 3/9

Category 3, equilibrium of parallel forces, requires only one force equation, the one in the direction of the forces (x-direction as shown), and two moment equations about the axes (y and z) which are normal to the direction of the forces.

Category 4, equilibrium of a general system of forces, requires all three force equations and all three moment equations.

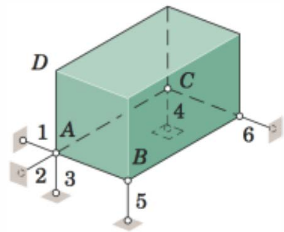
The observations contained in these statements are generally quite evident when a given problem is being solved.

Constraints and Statical Determinacy

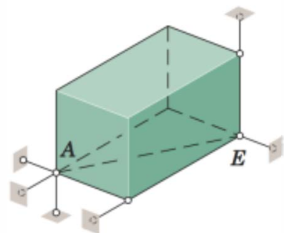
The six scalar relations of Eqs. 3/3, although necessary and sufficient conditions to establish equilibrium, do not necessarily provide all of the information required to calculate the unknown forces acting in a three-dimensional equilibrium situation. Again, as we found with two dimensions, the question of adequacy of information is decided by the characteristics of the constraints provided by the supports. An analytical criterion for determining the adequacy of constraints is available, but it is beyond the scope of this treatment.* In Fig. 3/10, however, we cite four examples of constraint conditions to alert the reader to the problem.

Part *a* of Fig. 3/10 shows a rigid body whose corner point *A* is completely fixed by the links 1, 2, and 3. Links 4, 5, and 6 prevent rotations about the axes of links 1, 2, and 3, respectively, so that the body is *completely fixed* and the constraints are said to be *adequate*. Part *b* of the figure shows the same number of constraints, but we see that they provide no resistance to a moment which might be applied about axis *AE*. Here the body is *incompletely fixed* and only *partially constrained*.

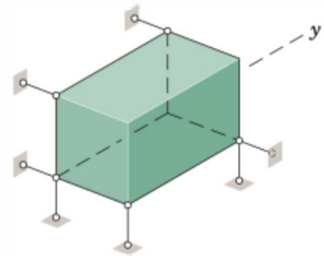
Similarly, in Fig. 3/10*c* the constraints provide no resistance to an unbalanced force in the *y*-direction, so here also is a case of incomplete fixity with partial constraints. In Fig. 3/10*d*, if a seventh constraining link were imposed on a system of six constraints placed properly for complete fixity, more supports would be provided than would be necessary to establish the equilibrium position, and link 7 would be *redundant*. The body would then be *statically indeterminate* with such a seventh link in place. With only a few exceptions, the supporting constraints for rigid bodies in equilibrium in this book are adequate, and the bodies are statically determinate.



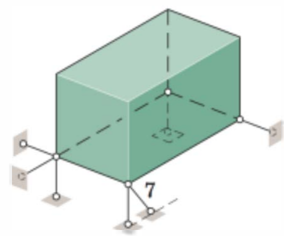
(*a*) Complete fixity
Adequate constraints



(*b*) Incomplete fixity
Partial constraints



(*c*) Incomplete fixity
Partial constraints



(*d*) Excessive fixity
Redundant constraints

Figure 3/10

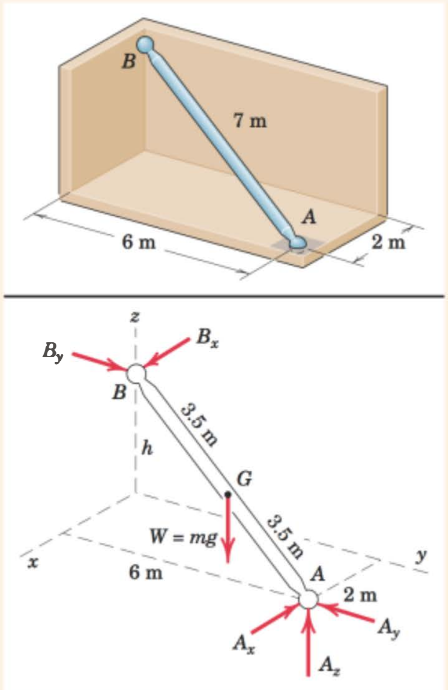
neiromobile/iStockphoto

The three-dimensional equilibrium of the cell-phone tower must be carefully analyzed so that excessive net horizontal force applied by the cable system is avoided.

*See the first author's *Statics, 2nd Edition SI Version*, 1975, Art. 16.

Sample Problem 3/5

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A in the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.



Solution. The free-body diagram of the shaft is first drawn where the contact forces acting on the shaft at B are shown normal to the wall surfaces. In addition to the weight $W = mg = 200(9.81) = 1962$ N, the force exerted by the floor on the ball joint at A is represented by its x-, y-, and z-components. These compo-
① nents are shown in their correct physical sense, as should be evident from the requirement that A be held in place. The vertical position of B is found from $7 = \sqrt{2^2 + 6^2 + h^2}$, $h = 3$ m. Right-handed coordinate axes are assigned as shown.

Vector solution. We will use A as a moment center to eliminate reference to the forces at A. The position vectors needed to compute the moments about A are

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k} \text{ m} \qquad \text{and} \qquad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} \text{ m}$$

where the mass center G is located halfway between A and B.

The vector moment equation gives

$$[\Sigma \mathbf{M}_A = \mathbf{0}] \qquad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$$

$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = 0$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of **i**, **j**, and **k** to zero and solving give

② $$B_x = 654 \text{ N} \qquad \text{and} \qquad B_y = 1962 \text{ N} \qquad \textit{Ans.}$$

The forces at A are easily determined by

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$$

and $$A_x = 654 \text{ N} \qquad A_y = 1962 \text{ N} \qquad A_z = 1962 \text{ N}$$

Finally, $$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850 \text{ N} \qquad \textit{Ans.}$$

Scalar solution. Evaluating the scalar moment equations about axes through A parallel, respectively, to the x- and y-axes, gives

$$[\Sigma M_{A_x} = 0] \qquad 1962(3) - 3B_y = 0 \qquad B_y = 1962 \text{ N}$$

③ $$[\Sigma M_{A_y} = 0] \qquad -1962(1) + 3B_x = 0 \qquad B_x = 654 \text{ N}$$

The force equations give, simply,

$$[\Sigma F_x = 0] \qquad -A_x + 654 = 0 \qquad A_x = 654 \text{ N}$$

$$[\Sigma F_y = 0] \qquad -A_y + 1962 = 0 \qquad A_y = 1962 \text{ N}$$

$$[\Sigma F_z = 0] \qquad A_z - 1962 = 0 \qquad A_z = 1962 \text{ N}$$

Helpful Hints

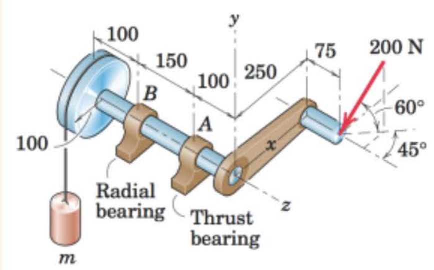
① We could, of course, assign all of the unknown components of force in the positive mathematical sense, in which case A_x and A_y would turn out to be negative upon computation. The free-body diagram describes the physical situation, so it is generally preferable to show the forces in their correct physical senses wherever possible.

② Note that the third equation $-2B_y + 6B_x = 0$ merely checks the results of the first two equations. This result could be anticipated from the fact that an equilibrium system of forces concurrent with a line requires only two moment equations (Category 2 under *Categories of Equilibrium*).

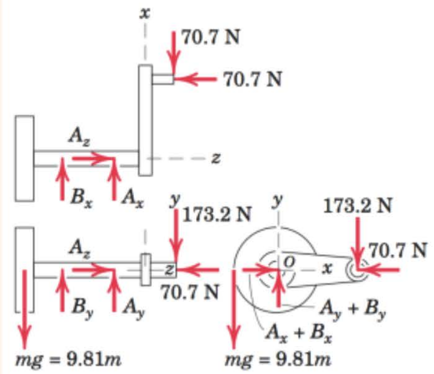
③ We observe that a moment sum about an axis through A parallel to the z-axis merely gives us $6B_x - 2B_y = 0$, which serves only as a check as noted previously. Alternatively we could have first obtained A_z from $\Sigma F_z = 0$ and then taken our moment equations about axes through B to obtain A_x and A_y.

Sample Problem 3/6

A 200-N force is applied to the handle of the hoist in the direction shown. The bearing A supports the thrust (force in the direction of the shaft axis), while bearing B supports only radial load (load normal to the shaft axis). Determine the mass m which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.



Dimensions in millimeters



Solution. The system is clearly three-dimensional with no lines or planes of symmetry, and therefore the problem must be analyzed as a general space system of forces. A scalar solution is used here to illustrate this approach, although a solution using vector notation would also be satisfactory. The free-body diagram of the shaft, lever, and drum considered a single body could be shown by a ① space view if desired, but is represented here by its three orthogonal projections.

The 200-N force is resolved into its three components, and each of the three views shows two of these components. The correct directions of A_x and B_x may be seen by inspection by observing that the line of action of the resultant of the two 70.7-N forces passes between A and B. The correct sense of the forces A_y and B_y cannot be determined until the magnitudes of the moments are obtained, so they are arbitrarily assigned. The x-y projection of the bearing forces is shown in terms of the sums of the unknown x- and y-components. The addition of A_z and the weight $W = mg$ completes the free-body diagrams. It should be noted that the three views represent three two-dimensional problems related by the corresponding components of the forces.

② From the x-y projection:

$[\Sigma M_O = 0]$ $\quad 100(9.81m) - 250(173.2) = 0 \qquad m = 44.1 \text{ kg}$ *Ans.*

From the x-z projection:

$[\Sigma M_A = 0]$ $\quad 150B_x + 175(70.7) - 250(70.7) = 0 \qquad B_x = 35.4 \text{ N}$

$[\Sigma F_x = 0]$ $\quad A_x + 35.4 - 70.7 = 0 \qquad A_x = 35.4 \text{ N}$

③ The y-z view gives

$[\Sigma M_A = 0]$ $\quad 150B_y + 175(173.2) - 250(44.1)(9.81) = 0 \qquad B_y = 520 \text{ N}$

$[\Sigma F_y = 0]$ $\quad A_y + 520 - 173.2 - (44.1)(9.81) = 0 \qquad A_y = 86.8 \text{ N}$

$[\Sigma F_z = 0]$ $\quad A_z = 70.7 \text{ N}$

The total radial forces on the bearings become

$[A_r = \sqrt{A_x^2 + A_y^2}]$ $\quad A_r = \sqrt{(35.4)^2 + (86.8)^2} = 93.5 \text{ N}$ *Ans.*

④ $[B = \sqrt{B_x^2 + B_y^2}]$ $\quad B = \sqrt{(35.4)^2 + (520)^2} = 521 \text{ N}$ *Ans.*

Helpful Hints

① If the standard three views of orthographic projection are not entirely familiar, then review and practice them. Visualize the three views as the images of the body projected onto the front, top, and end surfaces of a clear plastic box placed over and aligned with the body.

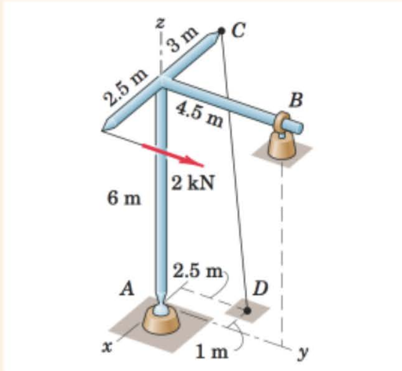
② We could have started with the x-z projection rather than with the x-y projection.

③ The y-z view could have followed immediately after the x-y view since the determination of A_y and B_y may be made after m is found.

④ Without the assumption of zero moment supported by each bearing about a line normal to the shaft axis, the problem would be statically indeterminate.

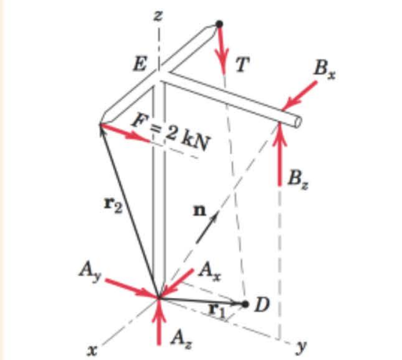
Sample Problem 3/7

The welded tubular frame is secured to the horizontal x-y plane by a ball-and-socket joint at A and receives support from the loose-fitting ring at B. Under the action of the 2-kN load, rotation about a line from A to B is prevented by the cable CD, and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension T in the cable, the reaction at the ring, and the reaction components at A.



Solution. The system is clearly three-dimensional with no lines or planes of symmetry, and therefore the problem must be analyzed as a general space system of forces. The free-body diagram is drawn, where the ring reaction is shown in terms of its two components. All unknowns except $\mathbf{T}$ may be eliminated by a moment sum about the line AB. The direction of AB is specified by the unit

① vector $\mathbf{n} = \dfrac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k})$. The moment of $\mathbf{T}$ about AB is the component in the direction of AB of the vector moment about the point A and equals $\mathbf{r}_1 \times \mathbf{T}\cdot\mathbf{n}$. Similarly the moment of the applied load F about AB is $\mathbf{r}_2 \times \mathbf{F}\cdot\mathbf{n}$. With $\overline{CD} = \sqrt{46.2}$ m, the vector expressions for $\mathbf{T}$, $\mathbf{F}$, $\mathbf{r}_1$, and $\mathbf{r}_2$ are

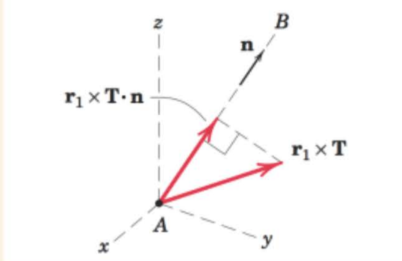


$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j}\ \text{kN}$$

② $$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j}\ \text{m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k}\ \text{m}$$

The moment equation now becomes

$$[\Sigma M_{AB} = 0] \quad (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k})\cdot\tfrac{1}{5}(3\mathbf{j} + 4\mathbf{k})$$
$$+ (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j})\cdot\tfrac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = 0$$



Completion of the vector operations gives

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83\ \text{kN} \qquad \textit{Ans.}$$

and the components of T become

$$T_x = 0.833\ \text{kN} \qquad T_y = 1.042\ \text{kN} \qquad T_z = -2.50\ \text{kN}$$

We may find the remaining unknowns by moment and force summations as follows:

$$[\Sigma M_z = 0] \quad 2(2.5) - 4.5B_x - 1.042(3) = 0 \qquad B_x = 0.417\ \text{kN} \qquad \textit{Ans.}$$

$$[\Sigma M_x = 0] \quad 4.5B_z - 2(6) - 1.042(6) = 0 \qquad B_z = 4.06\ \text{kN} \qquad \textit{Ans.}$$

$$[\Sigma F_x = 0] \quad A_x + 0.417 + 0.833 = 0 \qquad A_x = -1.250\ \text{kN} \qquad \textit{Ans.}$$

③ $$[\Sigma F_y = 0] \quad A_y + 2 + 1.042 = 0 \qquad A_y = -3.04\ \text{kN} \qquad \textit{Ans.}$$

$$[\Sigma F_z = 0] \quad A_z + 4.06 - 2.50 = 0 \qquad A_z = -1.556\ \text{kN} \qquad \textit{Ans.}$$

Helpful Hints

① The advantage of using vector notation in this problem is the freedom to take moments directly about any axis. In this problem this freedom permits the choice of an axis that eliminates five of the unknowns.

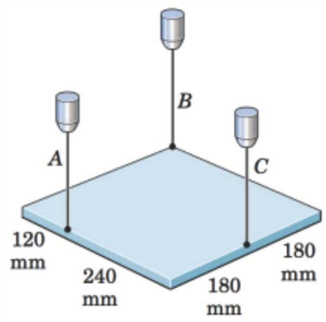
② Recall that the vector $\mathbf{r}$ in the expression $\mathbf{r} \times \mathbf{F}$ for the moment of a force is a vector from the moment center to *any* point on the line of action of the force. Instead of $\mathbf{r}_1$, an equally simple choice would be the vector $\overrightarrow{AC}$.

③ The negative signs associated with the A-components indicate that they are in the opposite direction to those shown on the free-body diagram.

PROBLEMS

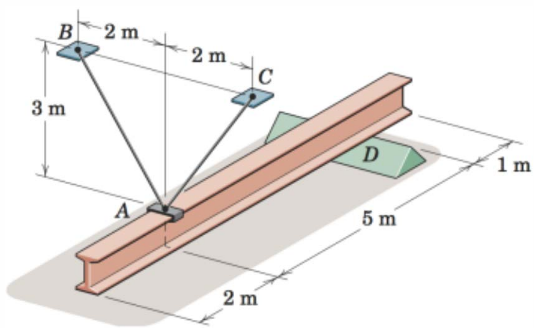
Introductory Problems

3/61 A uniform steel plate 360 mm square with a mass of 15 kg is suspended in the horizontal plane by the three vertical wires as shown. Calculate the tension in each wire.



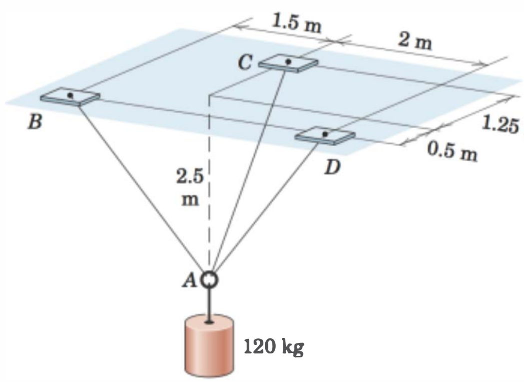
Problem 3/61

3/62 The uniform I-beam has a mass of 60 kg per meter of its length. Determine the tension in the two supporting cables and the reaction at D.



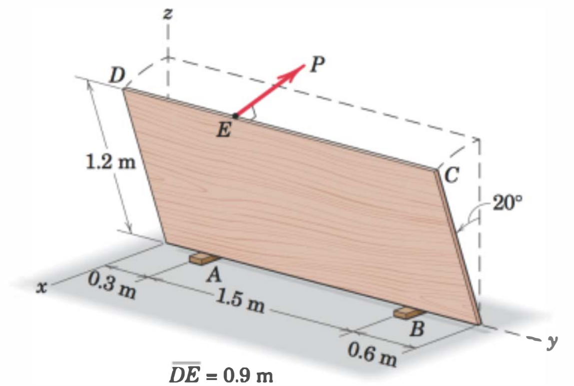
Problem 3/62

3/63 Determine the tensions in cables AB, AC, and AD.



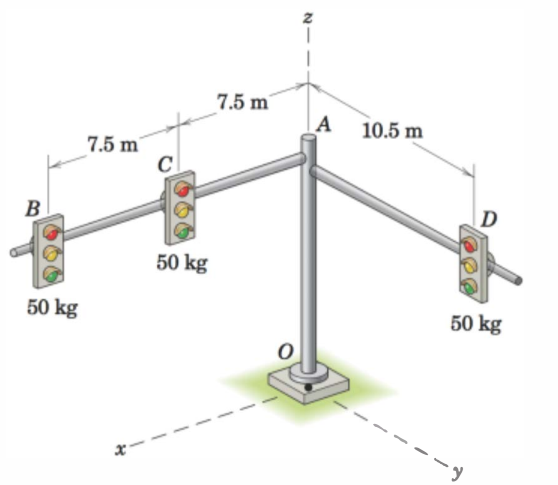
Problem 3/63

3/64 A 36-kg sheet of plywood rests on two small wooden blocks as shown. It is allowed to lean 20° from the vertical under the action of a force P which is perpendicular to the sheet. Friction at all surfaces of blocks A and B is sufficient to prevent slipping. Determine the magnitude P and the vertical reaction forces at A and B.



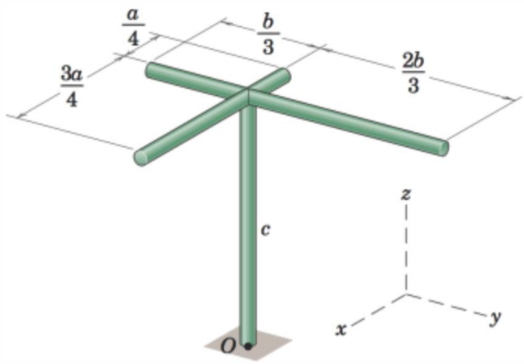
Problem 3/64

3/65 The vertical and horizontal poles at the traffic-light assembly are erected first. Determine the additional force and moment reactions at the base O caused by the addition of the three 50-kg traffic signals B, C, and D. Report your answers as a force magnitude and a moment magnitude.



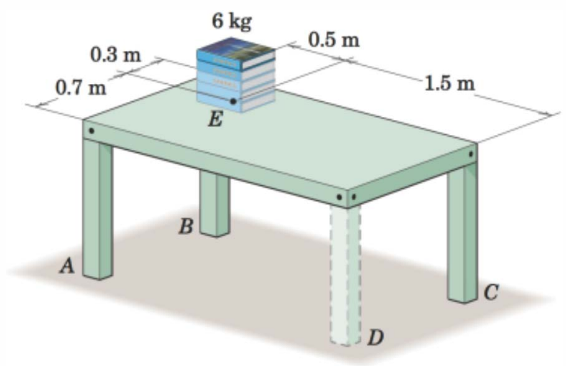
Problem 3/65

3/66 The body is constructed of uniform slender rod which has mass ρ per unit length. Determine the magnitudes of the force and moment reactions at the built-in support O.



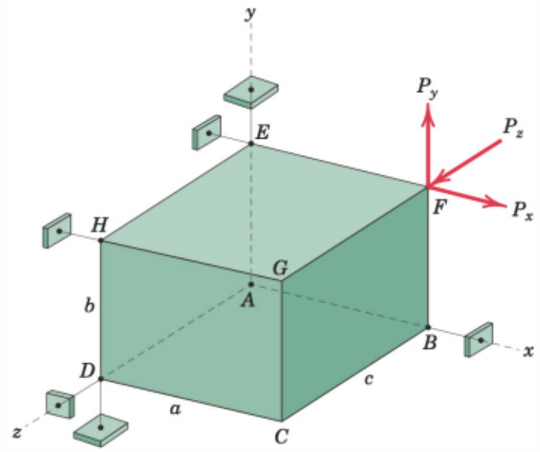
Problem 3/66

3/67 In order to make an adjustment, engineering students remove leg D from a laboratory worktable. To ensure that the table remains stable, they place a 6-kg stack of statics textbooks centered at point E of the tabletop as shown. Determine the normal reaction force at each leg A, B, and C. The uniform tabletop has a mass of 40 kg, and each leg has a mass of 5 kg.



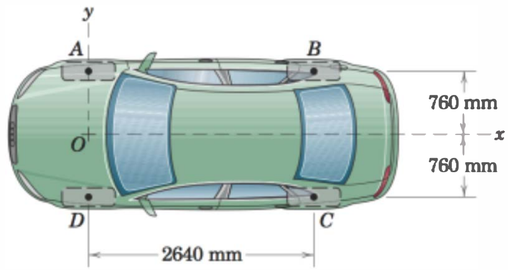
Problem 3/67

3/68 The rectangular solid is loaded by a force which has been resolved into three given components P_x, P_y, and P_z acting at corner F. Determine the force in each supporting link. State what type of physical support could be used in place of the ideal links shown at points D and E.



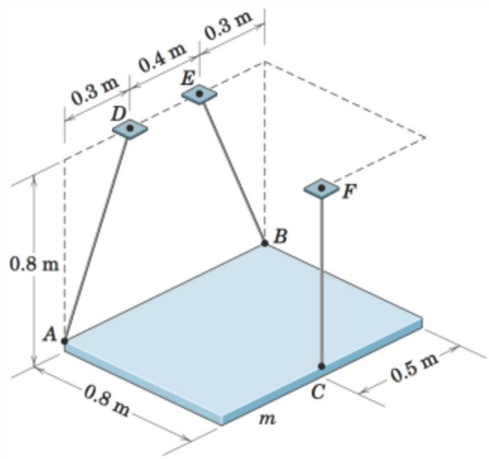
Problem 3/68

3/69 When on level ground, the car is placed on four individual scales—one under each tire. The four scale readings are 4300 N at A, 2900 N at B, 3000 N at C, and 4600 N at D. Determine the x- and y-coordinates of the mass center G and the mass of the car.



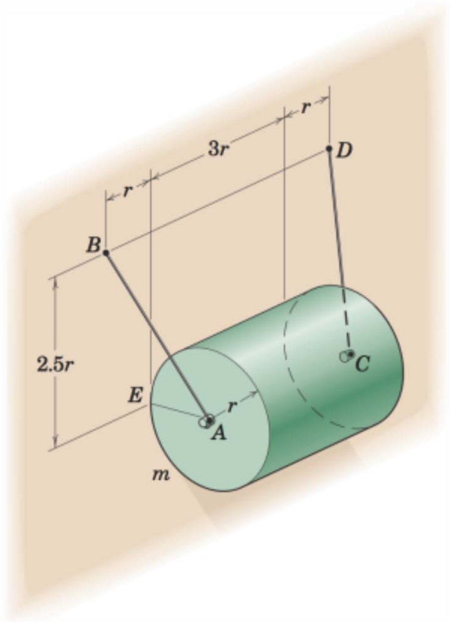
Problem 3/69

3/70 The uniform rectangular plate of mass m is suspended by three cables. Determine the tension in each cable.



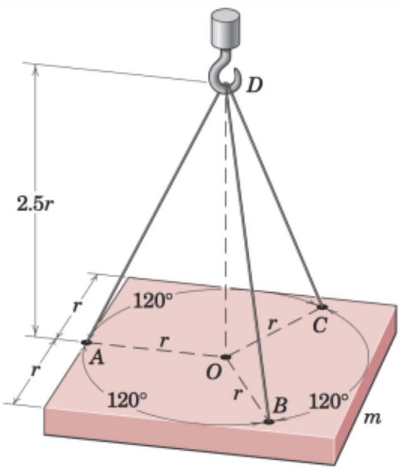
Problem 3/70

3/71 A uniform right-circular cylinder of mass m is supported by two cables and a vertical wall as shown. Determine the tension in each cable and the normal force exerted by the wall. Neglect friction.



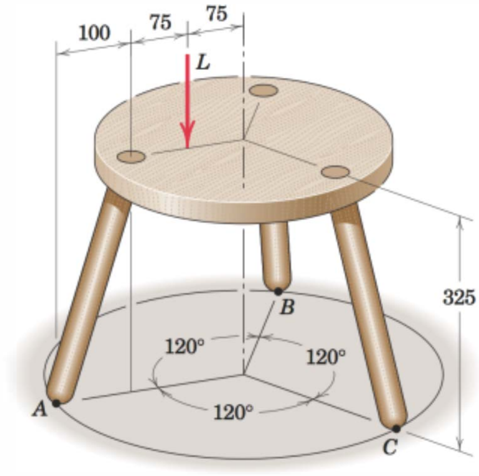
Problem 3/71

3/72 The uniform square plate is suspended by three equal-length cables as shown. Determine the tension in each cable.



Problem 3/72

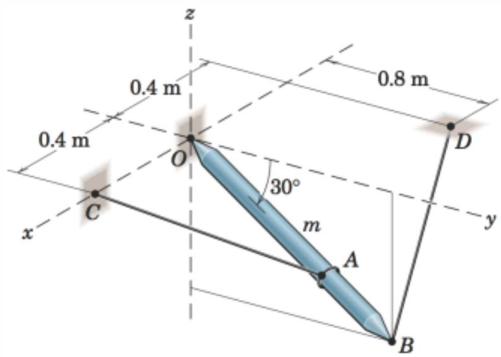
3/73 A three-legged stool is subjected to the load L as shown. Determine the vertical force reaction under each leg. Neglect the weight of the stool.



Dimensions in millimeters

Problem 3/73

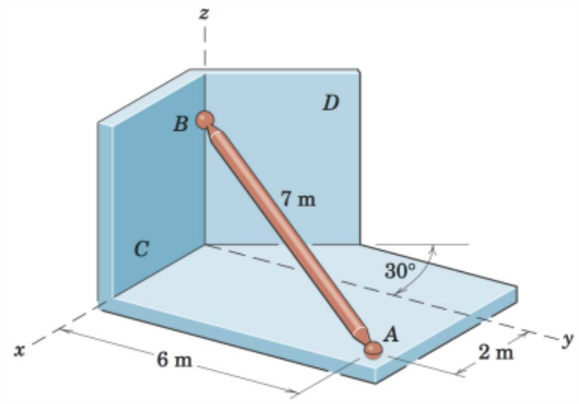
3/74 The uniform slender rod of mass m is suspended by a ball-and-socket joint at O and two cables. Determine the force reactions at O and the tension in each cable.



$\overline{OB} = 1.2$ m, $\overline{OA} = 0.9$ m

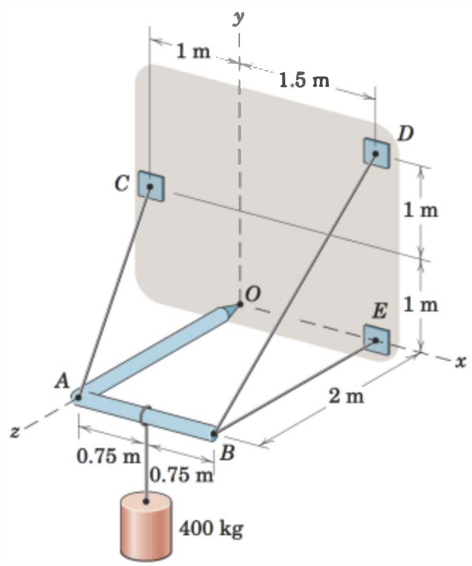
Problem 3/74

3/75 One of the vertical walls supporting end B of the 200-kg uniform shaft of Sample Problem 3/5 is turned through a 30° angle as shown here. End A is still supported by the ball-and-socket connection in the horizontal x-y plane. Calculate the magnitudes of the forces $\mathbf{P}$ and $\mathbf{R}$ exerted on the ball end B of the shaft by the vertical walls C and D, respectively.



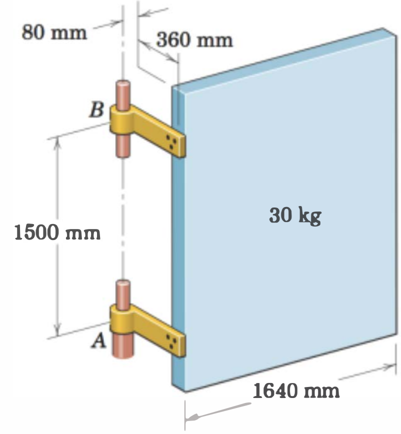
Problem 3/75

3/76 The light right-angle boom which supports the 400-kg cylinder is supported by three cables and a ball-and-socket joint at O attached to the vertical x-y surface. Determine the reactions at O and the cable tensions.



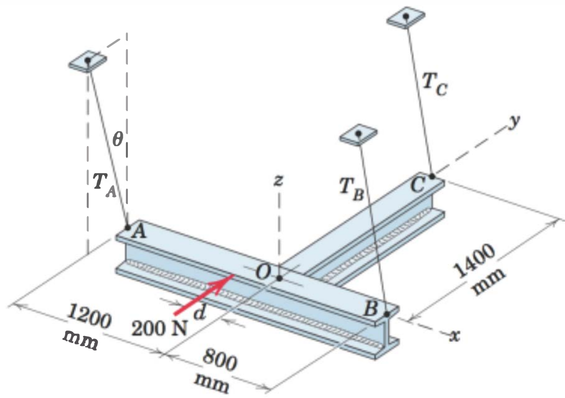
Problem 3/76

3/77 The mass center of the 30-kg door is in the center of the panel. If the weight of the door is supported entirely by the lower hinge A, calculate the magnitude of the total force supported by the hinge at B.



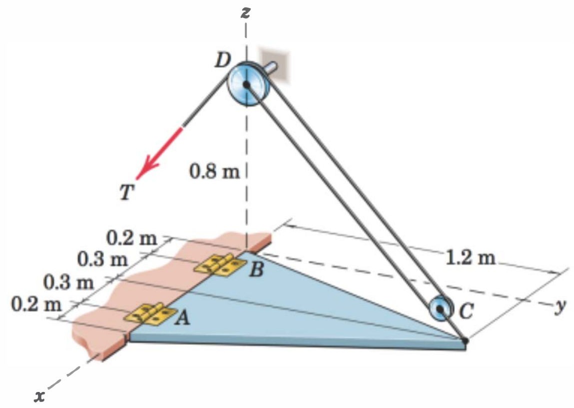
Problem 3/77

3/78 The two I-beams are welded together and are initially supported by the three cables of equal length hanging vertically from supports directly above A, B, and C. When applied with the appropriate offset d, the 200-N force causes the system to assume the new equilibrium configuration shown. All three cables are inclined at the same angle θ from the vertical, in planes parallel to the y-z plane. Determine this deflection θ and the proper offset d. Beams AB and OC have masses of 72 kg and 50 kg, respectively. The mass center of beam OC has a y-coordinate of 725 mm.



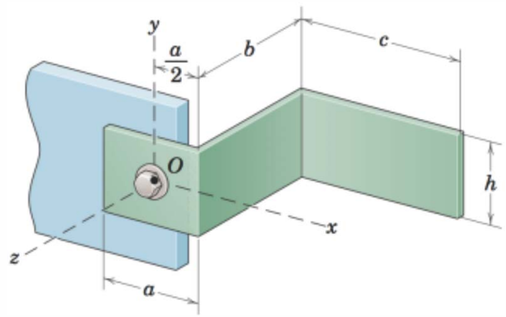
Problem 3/78

3/79 The 50-kg uniform triangular plate is supported by two small hinges A and B and the cable system shown. For the horizontal position of the plate, determine all hinge reactions and the tension T in the cable. Hinge A can resist axial thrust, but hinge B cannot. See Table D/3 in Appendix D for the mass-center location of a triangular plate.



Problem 3/79

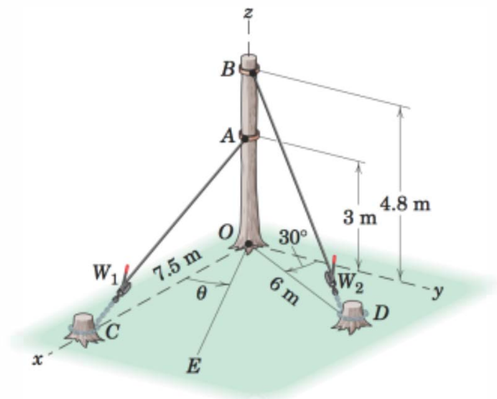
3/80 The large bracket is constructed of heavy plate which has a mass ρ per unit area. Determine the force and moment reactions at the support bolt at O.



Problem 3/80

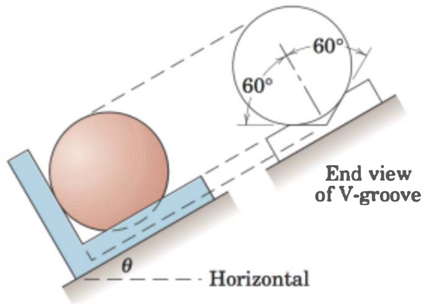
Representative Problems

3/81 The 360-kg tree trunk is known to have insect damage near point O, so the winch arrangement shown is used to fell the tree with no cutting. If winch W_1 is tightened to 900 N and winch W_2 to 1350 N, determine the force and moment reactions at O. If the tree ultimately falls at this point because of the moment at O, determine the angle θ which characterizes the line of impact OE. Assume that the base of the tree is equally strong in all directions.



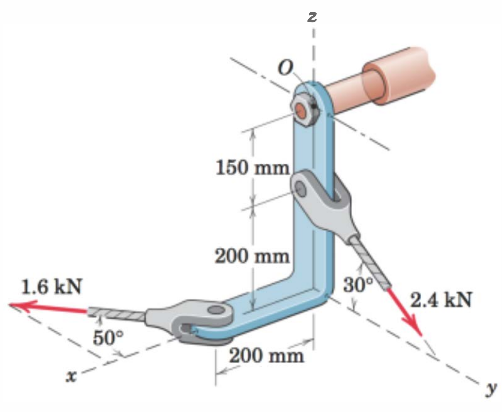
Problem 3/81

3/82 The smooth homogeneous sphere rests in the 120° groove and bears against the end plate, which is normal to the direction of the groove. Determine the angle θ, measured from the horizontal, for which the reaction on each side of the groove equals the force supported by the end plate.



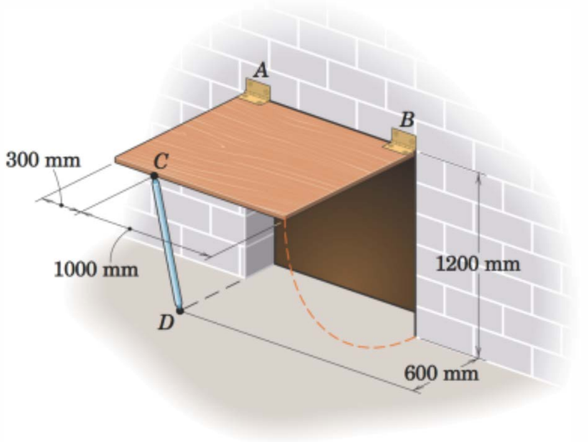
Problem 3/82

3/83 Determine the magnitudes of the force **R** and couple **M** exerted by the nut and bolt on the loaded bracket at O to maintain equilibrium.



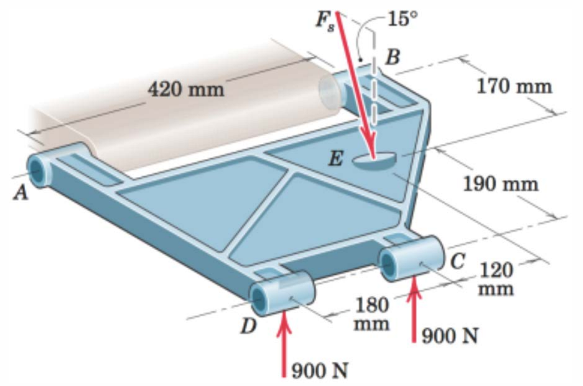
Problem 3/83

3/84 The 25-kg rectangular access door is held in the 90° open position by the single prop CD. Determine the force F in the prop and the magnitude of the force normal to the hinge axis AB in each of the small hinges A and B.



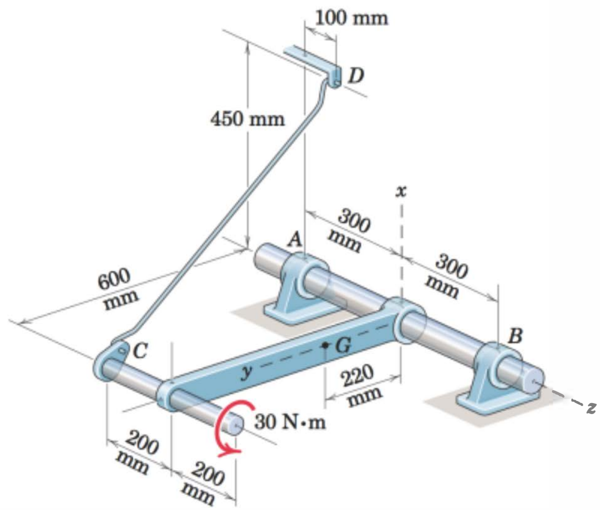
Problem 3/84

3/85 As part of a check on its design, a lower A-arm (part of an automobile suspension) is supported by bearings at A and B and subjected to the pair of 900-N forces at C and D. The suspension spring, not shown for clarity, exerts a force F_s at E as shown, where E is in plane $ABCD$. Determine the magnitude F_s of the spring force and the magnitudes F_A and F_B of the bearing forces at A and B which are perpendicular to the hinge axis AB.



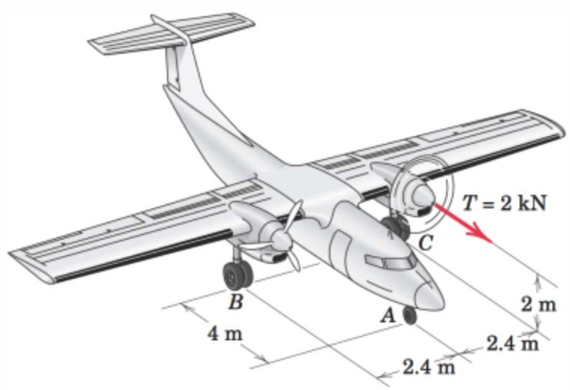
Problem 3/85

3/86 The shaft, lever, and handle are welded together and constitute a single rigid body. Their combined mass is 28 kg with mass center at G. The assembly is mounted in bearings A and B, and rotation is prevented by link CD. Determine the forces exerted on the shaft by bearings A and B while the 30-N·m couple is applied to the handle as shown. Would these forces change if the couple were applied to the shaft AB rather than to the handle?



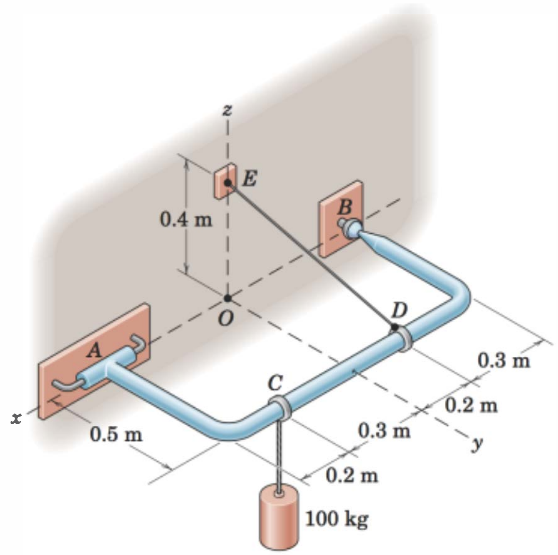
Problem 3/86

3/87 During a test, the left engine of the twin-engine airplane is revved up and a 2-kN thrust is generated. The main wheels at B and C are braked in order to prevent motion. Determine the change (compared with the nominal values with both engines off) in the normal reaction forces at A, B, and C.



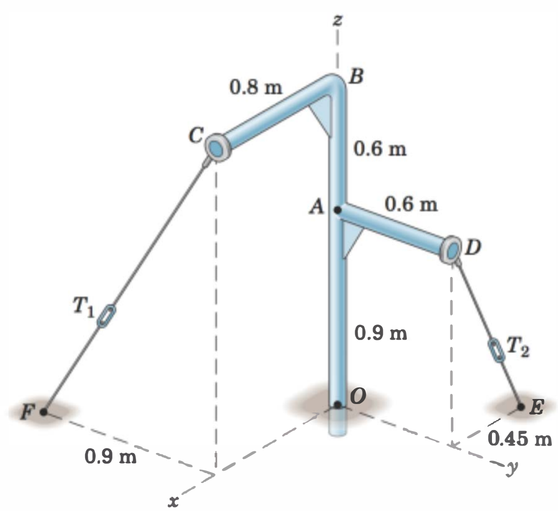
Problem 3/87

3/88 The bent rod $ACDB$ is supported by a sleeve at A and a ball-and-socket joint at B. Determine the components of the reactions and the tension in the cable. Neglect the mass of the rod.



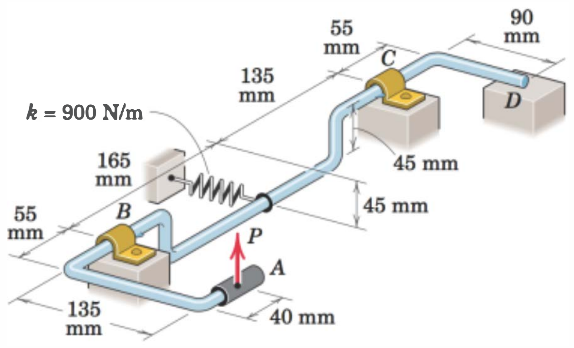
Problem 3/88

3/89 Turnbuckle T_1 is tightened to a tension of 750 N and turnbuckle T_2 is tightened to 500 N. Determine the components of the corresponding force and moment reactions at the built-in support at O. Neglect the weight of the structure.



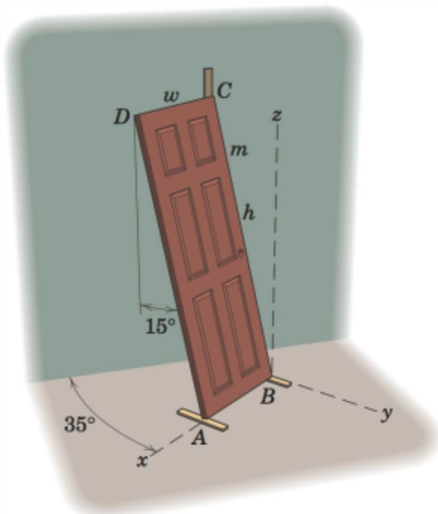
Problem 3/89

3/90 The spring of modulus $k = 900$ N/m is stretched a distance $\delta = 60$ mm when the mechanism is in the position shown. Calculate the force P_{min} required to initiate rotation about the hinge axis BC, and determine the corresponding magnitudes of the bearing forces which are perpendicular to BC. What is the normal reaction force at D if $P = P_{min}/2$?



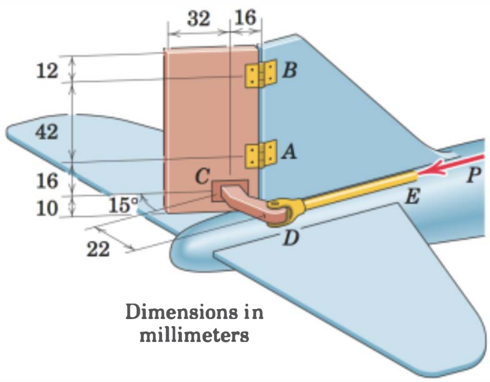
Problem 3/90

3/91 A homogeneous door of mass m, height h, and width w is leaned against a wall for painting. Small wooden strips are placed beneath corners A, B, and C. There is negligible friction at C, but friction at A and B is sufficient to prevent slipping. Determine the y- and z-components of the force reactions at A and B and the force normal to the wall at C.



Problem 3/91

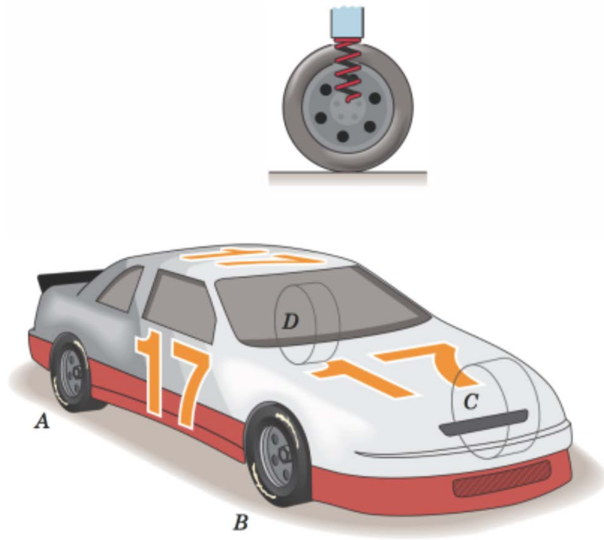
3/92 Consider the rudder assembly of a radio-controlled model airplane. For the 15° position shown in the figure, the net pressure acting on the left side of the rectangular rudder area is $p = 4(10^{-5})$ N/mm^2. Determine the required force P in the control rod DE and the horizontal components of the reactions at hinges A and B which are parallel to the rudder surface. Assume the aerodynamic pressure to be uniform.



Problem 3/92

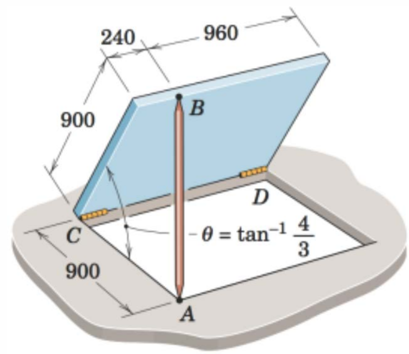
3/93 The upper ends of the vertical coil springs in the stock racecar can be moved up and down by means of a screw mechanism not shown. This adjustment permits a change in the downward force at each wheel as an optimum handling setup is sought. Initially, scales indicate the normal forces to be 3600 N, 3600 N, 4500 N, and 4500 N at A, B, C, and D, respectively. If the top of the right rear spring at A is lowered so that the scale at A reads an additional 450 N, determine the corresponding changes in the normal forces at B, C, and D. Neglect the effects of the small attitude changes (pitch and roll angles) caused by the spring adjustment. The front wheels are the same distance apart as the rear wheels.

Simplified spring detail



Problem 3/93

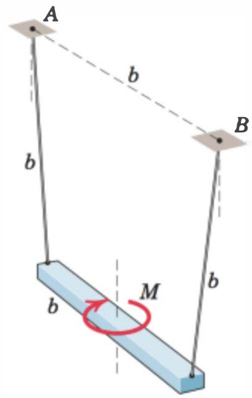
3/94 The uniform 900- by 1200-mm trap door has a mass of 200 kg and is propped open by the light strut AB at the angle $\theta = \tan^{-1}(4/3)$. Calculate the compression F_B in the strut and the force supported by the hinge D normal to the hinge axis. Assume that the hinges act at the extreme ends of the lower edge.



Dimensions in millimeters

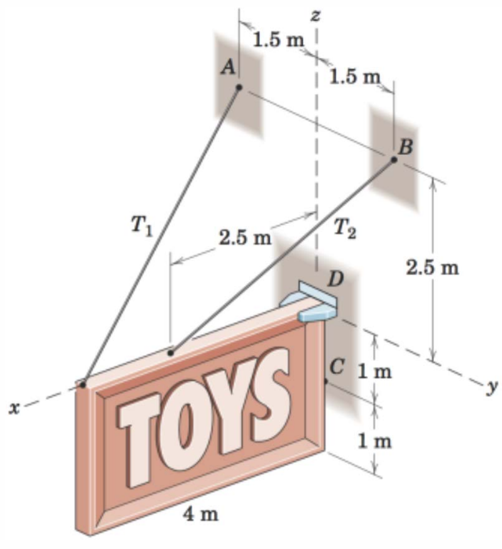
Problem 3/94

3/95 A uniform bar of length b and mass m is suspended at its ends by two wires, each of length b, from points A and B in the horizontal plane a distance b apart. A couple M is applied to the bar, causing it to rotate about a vertical axis to the equilibrium position shown. Derive an expression for the height h which it rises from its original equilibrium position where it hangs freely with no applied moment. What value of M is required to raise the bar the maximum amount b?



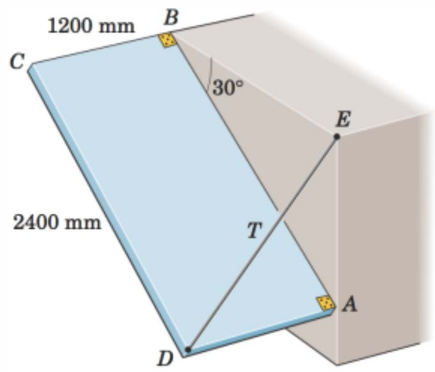
Problem 3/95

3/96 A rectangular sign over a store has a mass of 100 kg, with the center of mass in the center of the rectangle. The support against the wall at point C may be treated as a ball-and-socket joint. At corner D support is provided in the y-direction only. Calculate the tensions T_1 and T_2 in the supporting wires, the total force supported at C, and the lateral force R supported at D.



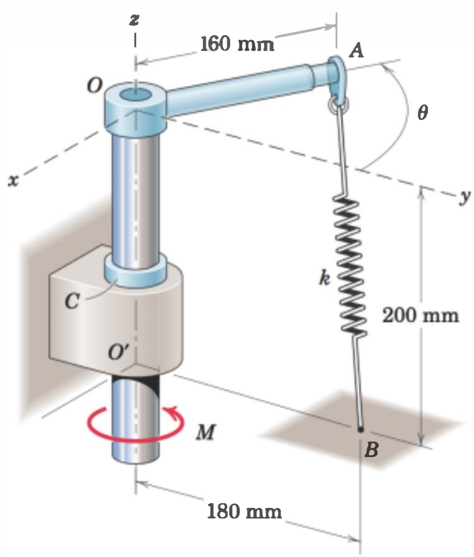
Problem 3/96

▶**3/97** The uniform rectangular panel $ABCD$ has a mass of 40 kg and is hinged at its corners A and B to the fixed vertical surface. A wire from E to D keeps edges BC and AD horizontal. Hinge A can support thrust along the hinge axis AB, whereas hinge B supports force normal to the hinge axis only. Find the tension T in the wire and the magnitude B of the force supported by hinge B.



Problem 3/97

***3/98** Determine and plot the moment M required to rotate arm OA over the range $0 \leq \theta \leq 180°$. Find the maximum value of M and the angle θ at which it occurs. The collar C fastened to the shaft prevents downward motion of the shaft in its bearing. Determine and plot the magnitude of the distributed vertical force supported by this collar over the same range of θ. The spring constant $k = 200$ N/m, and the spring is unstretched when $\theta = 0$. Neglect the mass of the structure and any effects of mechanical interference.



Problem 3/98

3/5 Chapter Review

In Chapter 3 we have applied our knowledge of the properties of forces, moments, and couples studied in Chapter 2 to solve problems involving rigid bodies in equilibrium. Complete equilibrium of a body requires that the vector resultant of all forces acting on it be zero ($\Sigma\mathbf{F} = \mathbf{0}$) and the vector resultant of all moments on the body about a point (or axis) also be zero ($\Sigma\mathbf{M} = \mathbf{0}$). We are guided in all of our solutions by these two requirements, which are easily comprehended physically.

Frequently, it is not the theory but its application which presents difficulty. The crucial steps in applying our principles of equilibrium should be quite familiar by now. They are:

1. Make an unequivocal decision as to which system (a body or collection of bodies) in equilibrium is to be analyzed.
2. Isolate the system in question from all contacting bodies by drawing its *free-body diagram* showing *all* forces and couples acting *on* the isolated system from external sources.
3. Observe the principle of action and reaction (Newton's third law) when assigning the sense of each force.
4. Label all forces and couples, known and unknown.
5. Choose and label reference axes, always choosing a right-handed set when vector notation is used (which is usually the case for three-dimensional analysis).
6. Check the adequacy of the constraints (supports) and match the number of unknowns with the number of available independent equations of equilibrium.

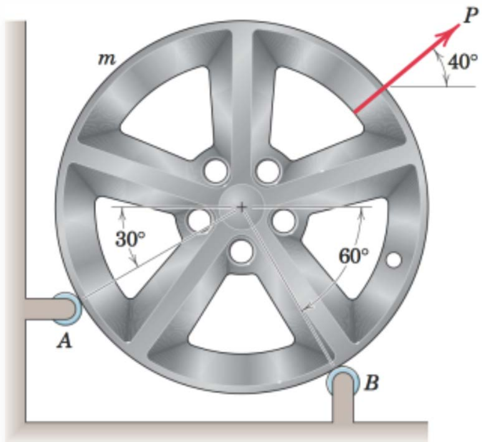
When solving an equilibrium problem, we should first check to see that the body is statically determinate. If there are more supports than are necessary to hold the body in place, the body is statically indeterminate, and the equations of equilibrium by themselves will not enable us to solve for all of the external reactions. In applying the equations of equilibrium, we choose scalar algebra, vector algebra, or graphical analysis according to both preference and experience; vector algebra is particularly useful for many three-dimensional problems.

The algebra of a solution can be simplified by the choice of a moment axis which eliminates as many unknowns as possible or by the choice of a direction for a force summation which avoids reference to certain unknowns. A few moments of thought to take advantage of these simplifications can save appreciable time and effort.

The principles and methods covered in Chapters 2 and 3 constitute the most basic part of statics. They lay the foundation for what follows not only in statics but in dynamics as well.

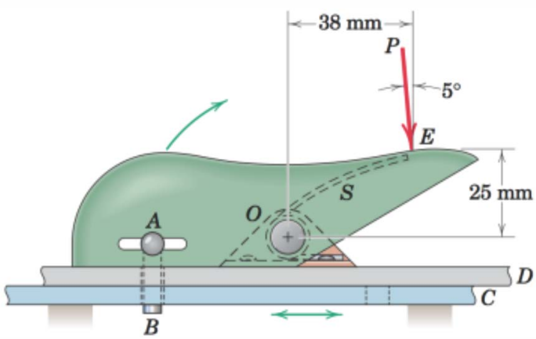
REVIEW PROBLEMS

3/99 The rack for storing automobile wheels consists of two parallel rods A and B. Determine the magnitude of the force P required to begin extracting the wheel. The mass of the wheel is m. Neglect all friction.



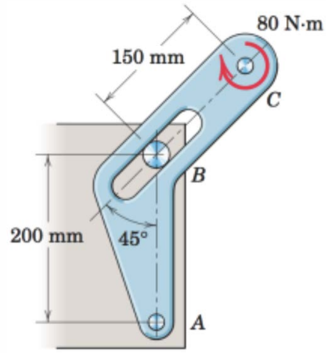
Problem 3/99

3/100 The positioning device locks the sliding panel C into place relative to the fixed panel D, to which the device is attached. Pressing at E rotates the moving part of the device clockwise about O, retracting the pin AB from the hole in panel C. If a force $P = 40$ N is required to begin clockwise rotation of the device, determine the moment M about point O exerted by the internal coiled spring S.



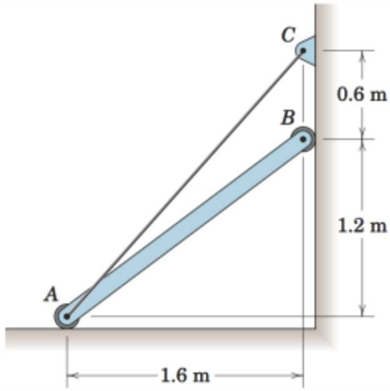
Problem 3/100

3/101 The light bracket ABC is freely hinged at A and is constrained by the fixed pin in the smooth slot at B. Calculate the magnitude R of the force supported by the pin at A under the action of the 80-N·m applied couple.



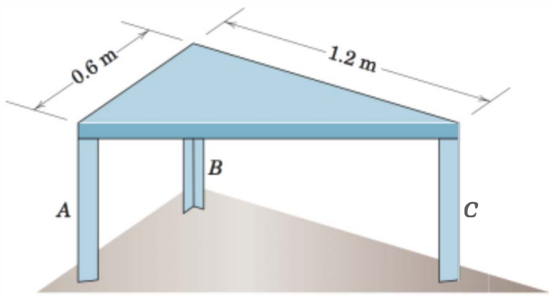
Problem 3/101

3/102 The uniform 30-kg bar with end rollers is supported by the horizontal and vertical surfaces and by the wire AC. Calculate the tension T in the wire and the reactions against the rollers at A and at B.



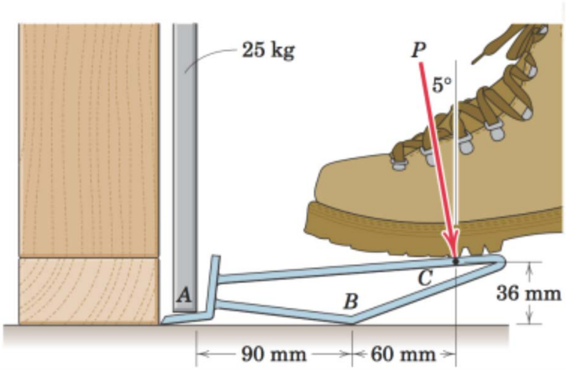
Problem 3/102

3/103 The mass of the uniform right-triangular tabletop is 30 kg, and that of each of the vertical legs is 2 kg. Determine the normal reaction force exerted by the floor on each leg. The mass center of a right-triangular body can be obtained from Table D/3 in Appendix D.



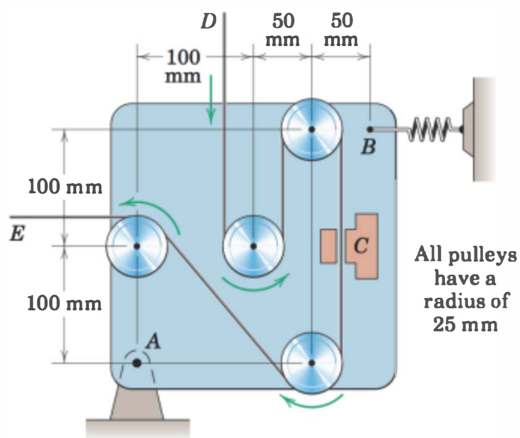
Problem 3/103

3/104 The device shown in the figure is useful for lifting drywall panels into position prior to fastening to the stud wall. Estimate the magnitude P of the force required to lift the 25-kg panel. State any assumptions.



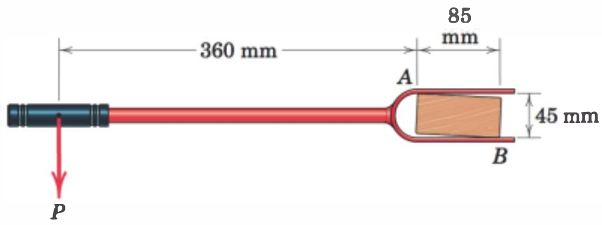
Problem 3/104

3/105 Magnetic tape under a tension of 10 N at D passes around the guide pulleys and through the erasing head at C at constant speed. As a result of a small amount of friction in the bearings of the pulleys, the tape at E is under a tension of 11 N. Determine the tension T in the supporting spring at B. The plate lies in a horizontal plane and is mounted on a precision needle bearing at A.



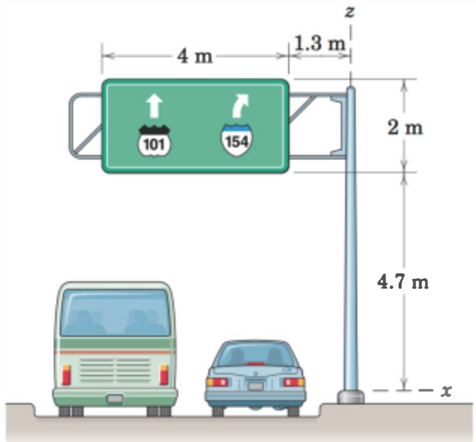
Problem 3/105

3/106 The tool shown is used for straightening twisted members as wooden framing is completed. If the force $P = 150$ N is applied to the handle as shown, determine the normal forces applied to the installed stud at points A and B. Ignore friction.



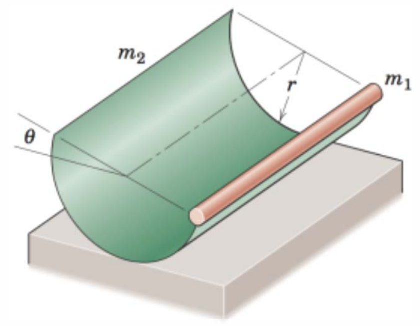
Problem 3/106

3/107 A freeway sign measuring 4 m by 2 m is supported by the single mast as shown. The sign, supporting framework, and mast together have a mass of 300 kg, with center of mass 3.3 m away from the vertical centerline of the mast. When the sign is subjected to the direct blast of a 125-km/h wind, an average pressure difference of 700 Pa is developed between the front and back sides of the sign, with the resultant of the wind-pressure forces acting at the center of the sign. Determine the magnitudes of the force and moment reactions at the base of the mast. Such results would be instrumental in the design of the base.



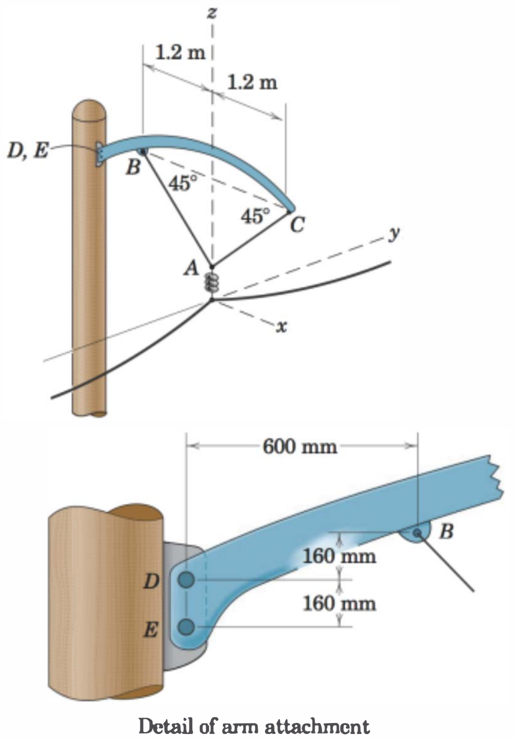
Problem 3/107

3/108 A slender rod of mass m_1 is welded to the horizontal edge of a uniform semicylindrical shell of mass m_2. Determine an expression for the angle θ with the horizontal made by the diameter of the shell through m_1. (Consult Table D/3 in Appendix D to locate the center of mass of the semicircular section.)



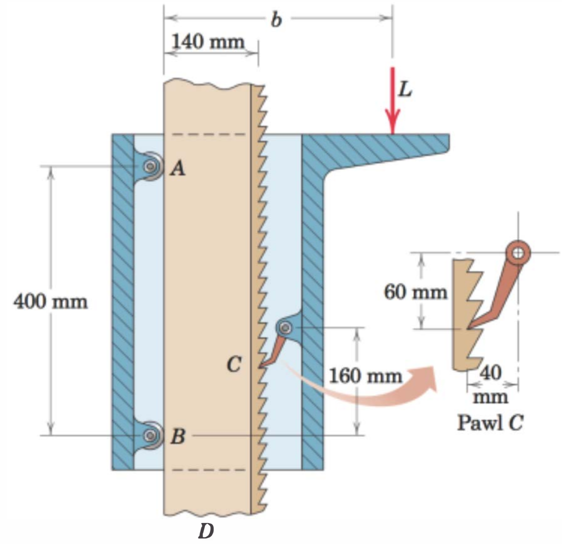
Problem 3/108

3/109 The curved arm BC and attached cables AB and AC support a power line which lies in the vertical y-z plane. The tangents to the power line at the insulator below A make 15° angles with the horizontal y-axis. If the tension in the power line at the insulator is 1.3 kN, calculate the total force supported by the bolt at D on the pole bracket. The weight of the arm BC can be neglected compared with the other forces, and it can be assumed that the bolt at E supports horizontal force only.



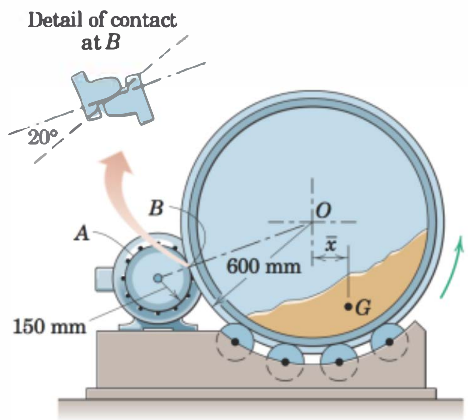
Problem 3/109

3/110 The device shown in section can support the load L at various heights by resetting the pawl C in another tooth at the desired height on the fixed vertical column D. Determine the distance b at which the load should be positioned in order for the two rollers A and B to support equal forces. The weight of the device is negligible compared with L.



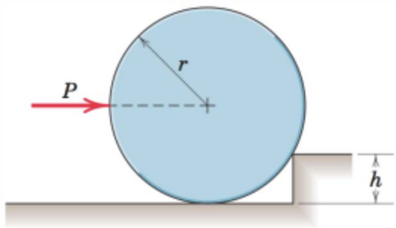
Problem 3/110

3/111 A large symmetrical drum for drying sand is operated by the geared motor drive shown. If the mass of the sand is 750 kg and an average gear-tooth force of 2.6 kN is supplied by the motor pinion A to the drum gear normal to the contacting surfaces at B, calculate the average offset $\bar{x}$ of the center of mass G of the sand from the vertical centerline. Neglect all friction in the supporting rollers.



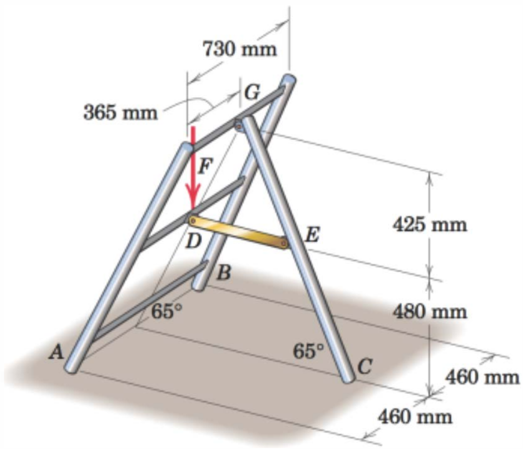
Problem 3/111

3/112 Determine the force P required to begin rolling the uniform cylinder of mass m over the obstruction of height h.



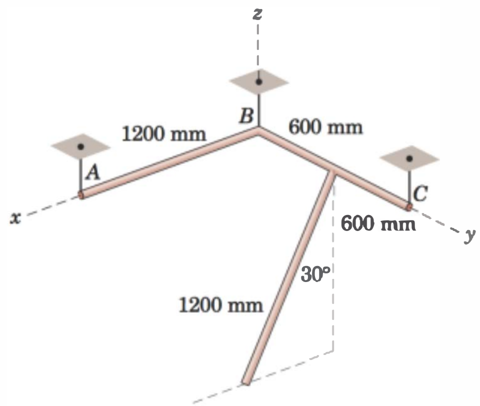
Problem 3/112

3/113 The small tripod-like stepladder is useful for supporting one end of a walking board. If F denotes the magnitude of the downward load from such a board (not shown), determine the reaction at each of the three feet A, B, and C. Neglect friction.



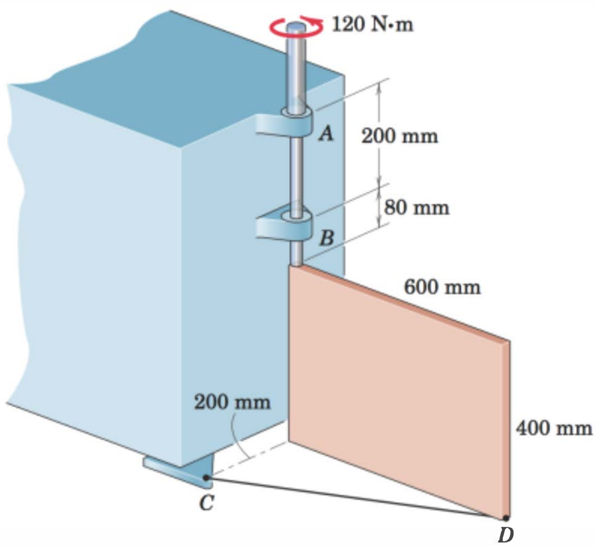
Problem 3/113

3/114 Each of the three uniform 1200-mm bars has a mass of 20 kg. The bars are welded together into the configuration shown and suspended by three vertical wires. Bars AB and BC lie in the horizontal x-y plane, and the third bar lies in a plane parallel to the x-z plane. Compute the tension in each wire.



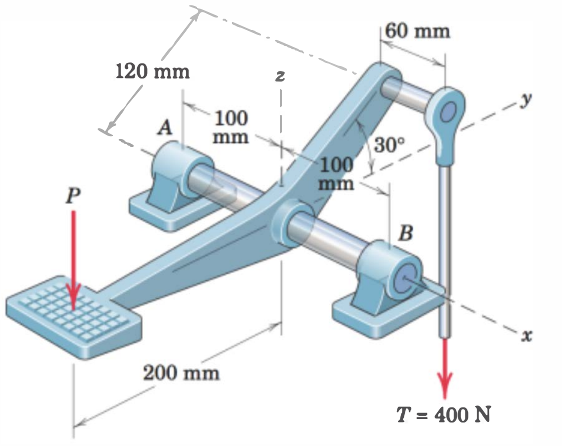
Problem 3/114

3/115 The uniform 15-kg plate is welded to the vertical shaft, which is supported by bearings A and B. Calculate the magnitude of the force supported by bearing B during application of the 120-N·m couple to the shaft. The cable from C to D prevents the plate and shaft from turning, and the weight of the assembly is carried entirely by bearing A.



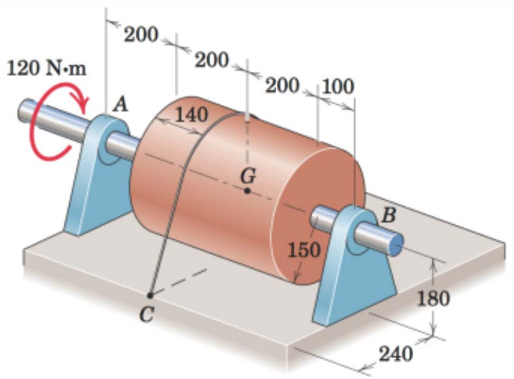
Problem 3/115

3/116 A vertical force P on the foot pedal of the bell crank is required to produce a tension T of 400 N in the vertical control rod. Determine the corresponding bearing reactions at A and B.



Problem 3/116

3/117 The drum and shaft are welded together and have a mass of 50 kg with mass center at G. The shaft is subjected to a torque (couple) of 120 N·m and the drum is prevented from rotating by the cord wrapped securely around it and attached to point C. Calculate the magnitudes of the forces supported by bearings A and B.

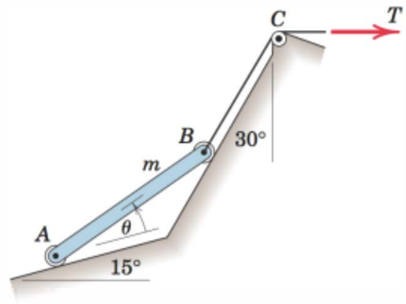


Dimensions in millimeters

Problem 3/117

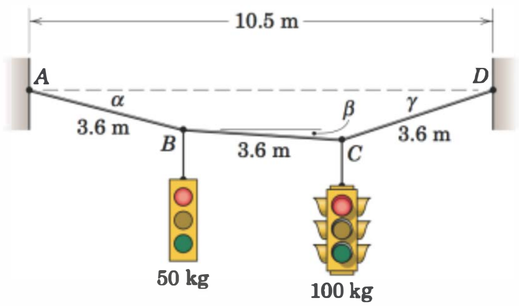
Computer-Oriented Problems

***3/118** Determine and plot the tension ratio T/mg required to hold the uniform slender bar in equilibrium for any angle θ from just above zero to just under 45°. The bar AB of mass m is uniform.



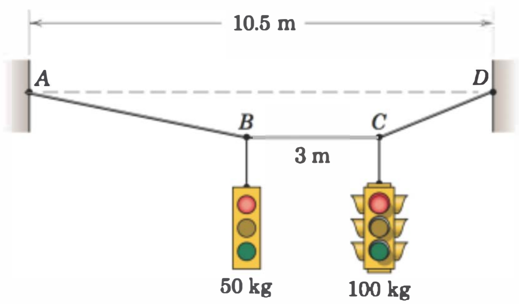
Problem 3/118

***3/119** Two traffic signals are attached to the 10.8-m support cable at equal intervals as shown. Determine the equilibrium configuration angles α, β, and γ, as well as the tension in each cable segment.



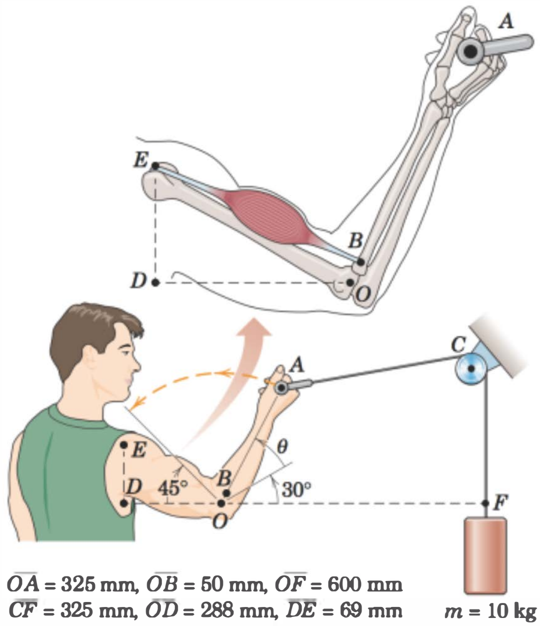
Problem 3/119

***3/120** The two traffic signals of Prob. 3/119 are now repositioned so that segment BC of the 10.8-m support cable is 3 m in length and is horizontal. Specify the necessary lengths AB and CD and the tensions in all three cable segments.



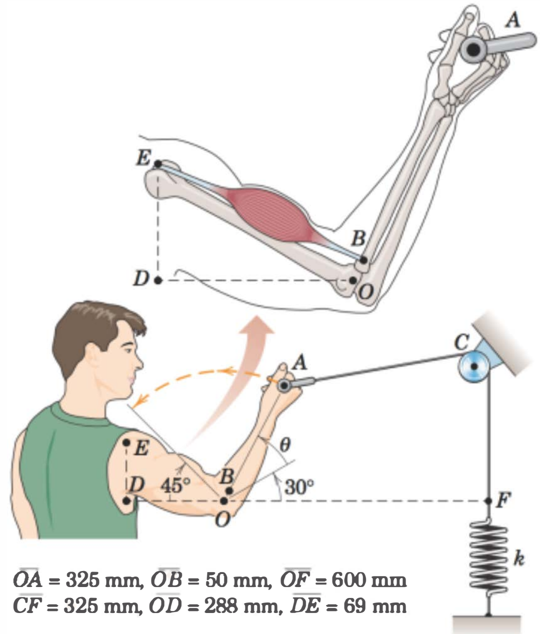
Problem 3/120

***3/121** In executing the biceps-curl exercise, the man holds his shoulder and upper arm stationary and rotates the lower arm OA through the range $0 \leq \theta \leq 105°$. The detailed drawing shows the effective origin and insertion points for the biceps muscle group. Determine and plot the tension T_B in this muscle group over the specified range. State the value of T_B for $\theta = 90°$. Neglect the weight of the forearm, and assume slow, steady motion.



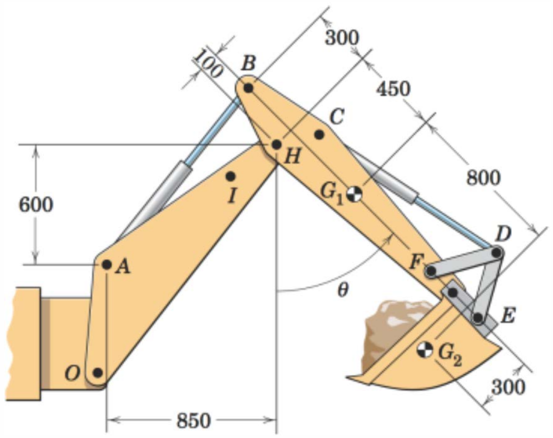
Problem 3/121

***3/122** All the conditions of Prob. 3/121 are repeated here, except the mass m is replaced by a spring of constant $k = 350$ N/m. The spring is unstretched when $\theta = 0$. Determine and plot the tension T_B in the biceps muscle group over the range $0 \leq \theta \leq 105°$, and state the maximum value of T_B and the angle θ at which it occurs.



Problem 3/122

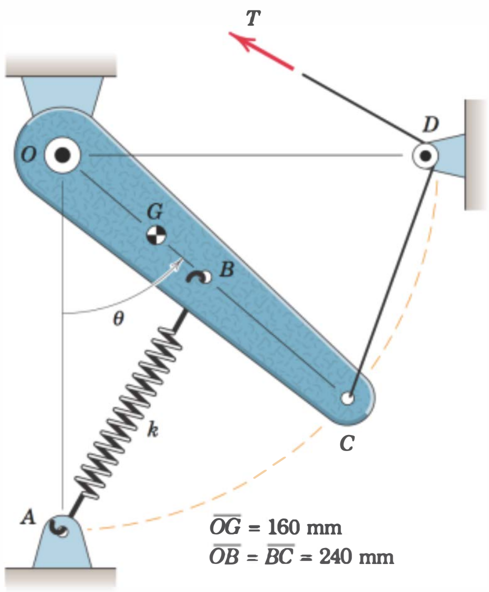
***3/123** The basic features of a small backhoe are shown in the illustration. Member BE (complete with hydraulic cylinder CD and bucket-control links DF and DE) has a mass of 200 kg with mass center at G_1. The bucket and its load of clay have a mass of 140 kg with mass center at G_2. To disclose the operational design characteristics of the backhoe, determine and plot the force T in the hydraulic cylinder AB as a function of the angular position θ of member BE over the range $0 \leq \theta \leq 90°$. For what value of θ is the force T equal to zero? Member OH is fixed for this exercise; note that its controlling hydraulic cylinder (hidden) extends from near point O to pin I. Similarly, the bucket-control hydraulic cylinder CD is held at a fixed length.



Dimensions in millimeters

Problem 3/123

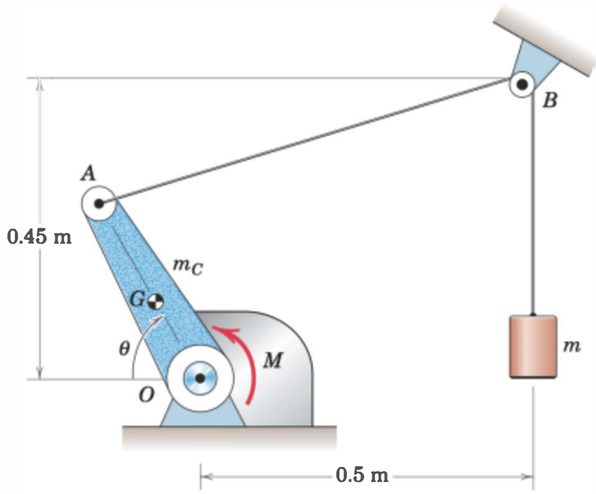
***3/124** The mass center of the 1.5-kg link OC is located at G, and the spring of constant $k = 25$ N/m is unstretched when $\theta = 0$. Plot the tension T required for static equilibrium over the range $0 \le \theta \le 90°$ and state the values of T for $\theta = 45°$ and $\theta = 90°$.



$\overline{OG} = 160$ mm
$\overline{OB} = \overline{BC} = 240$ mm

Problem 3/124

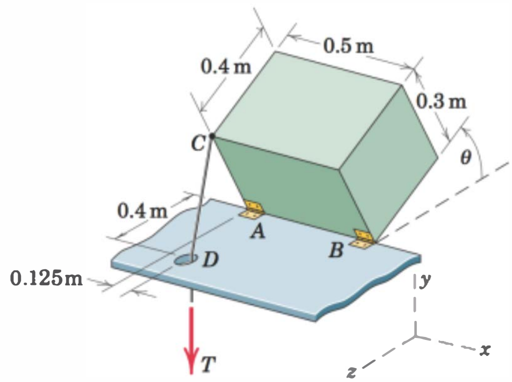
***3/125** The system of Prob. 3/60 is repeated here, but now the crank OA has mass $m_C = 3$ kg with mass center at G. Determine and plot the moment M which must be applied to the crank in order to hold the cylinder of mass $m = 5$ kg in equilibrium. Use the range $0 \le \theta \le 180°$ and neglect all friction. State the value of θ for which $M = 0$.



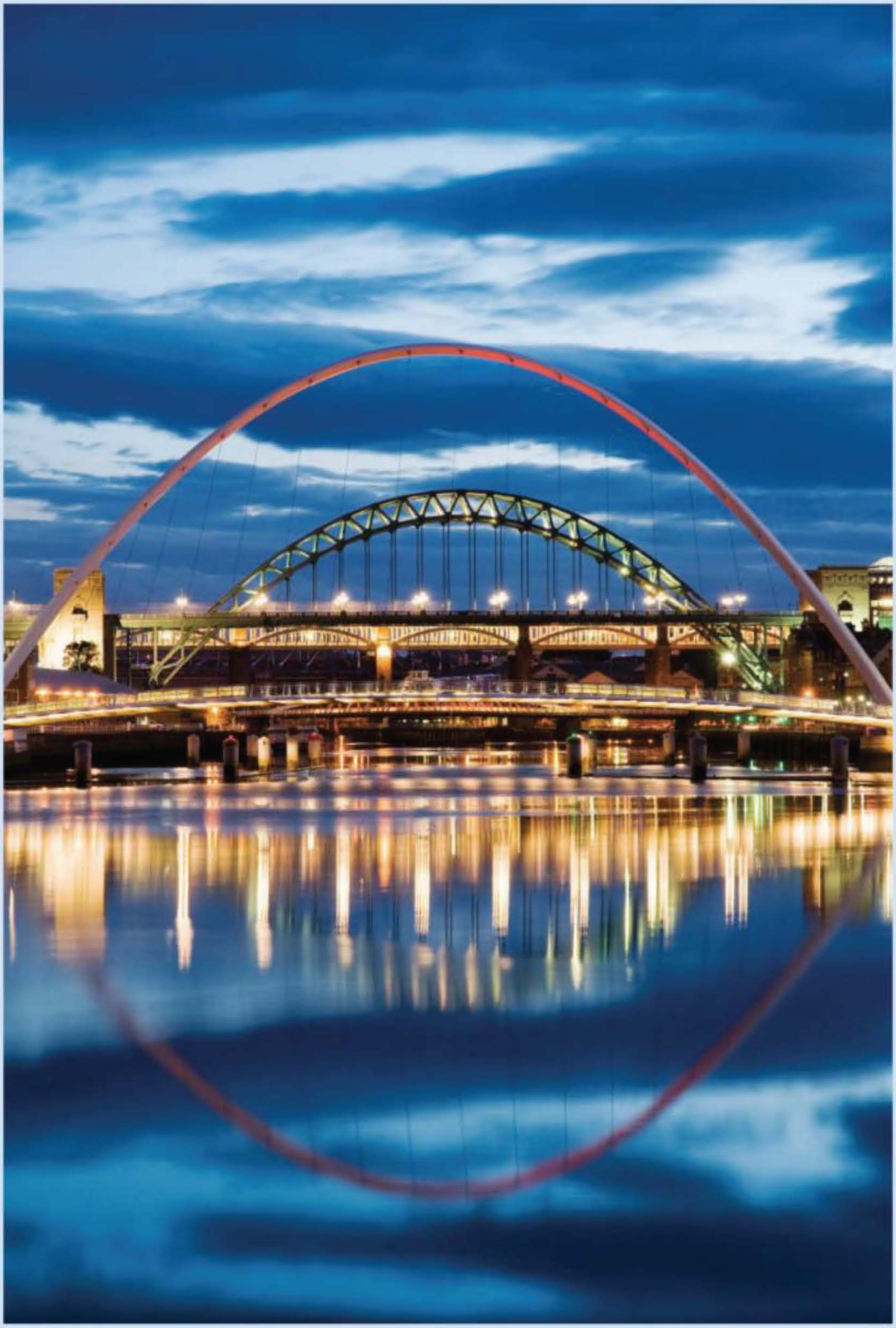
$\overline{OA} = 0.3$ m, $\overline{OG} = 0.12$ m

Problem 3/125

***3/126** The 125-kg homogeneous rectangular solid is held in the arbitrary position shown by the tension T in the cable. Determine and plot the following quantities as functions of θ over the range $0 \le \theta \le 60°$: T, A_y, A_z, B_x, B_y, and B_z. The hinge at A cannot exert an axial thrust. Assume all hinge force components to be in the positive coordinate directions. Friction at D is negligible.



Problem 3/126



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This view of the River Tyne in the United Kingdom shows a variety of structures, including the Gateshead Millennium Bridge (closest to the camera). This award-winning bridge can rotate about a horizontal axis along its span to allow ships to pass underneath.

4 STRUCTURES

CHAPTER OUTLINE

4/1 INTRODUCTION

In Chapter 3 we studied the equilibrium of a single rigid body or a system of connected members treated as a single rigid body. We first drew a free-body diagram of the body showing all forces external to the isolated body, and then we applied the force and moment equations of equilibrium. In Chapter 4 we focus on the determination of the forces internal to a structure—that is, forces of action and reaction between the connected members. An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it. To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free-body diagrams of individual members or combinations of members. This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.

In Chapter 4 we analyze the internal forces acting in several types of structures—namely, trusses, frames, and machines. In this treatment we consider only *statically determinate* structures, which do not have more supporting constraints than are necessary to maintain an equilibrium configuration. Thus, as we have already seen, the equations of equilibrium are adequate to determine all unknown reactions.

The analysis of trusses, frames and machines, and beams under concentrated loads constitutes a straightforward application of the material developed in the previous two chapters. The basic procedure developed in Chapter 3 for isolating a body by constructing a correct free-body diagram is essential for the analysis of statically determinate structures.

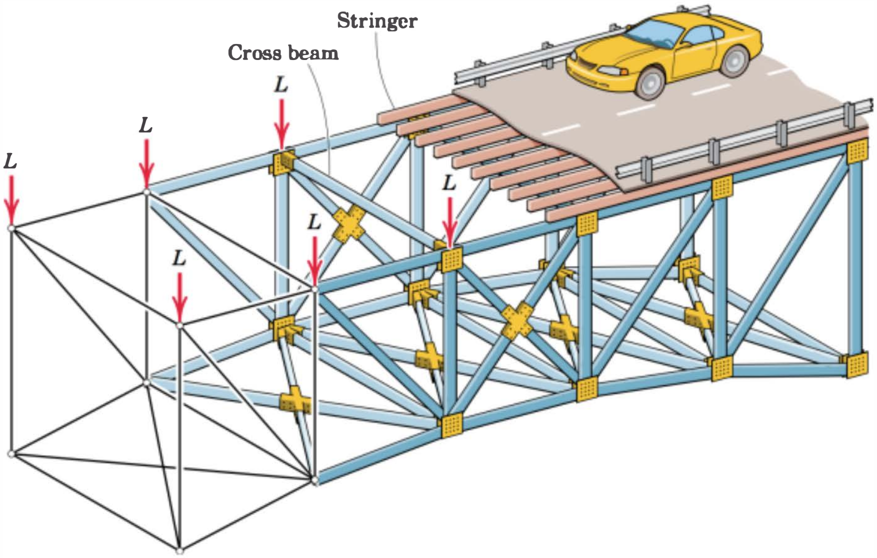

Figure 4/1

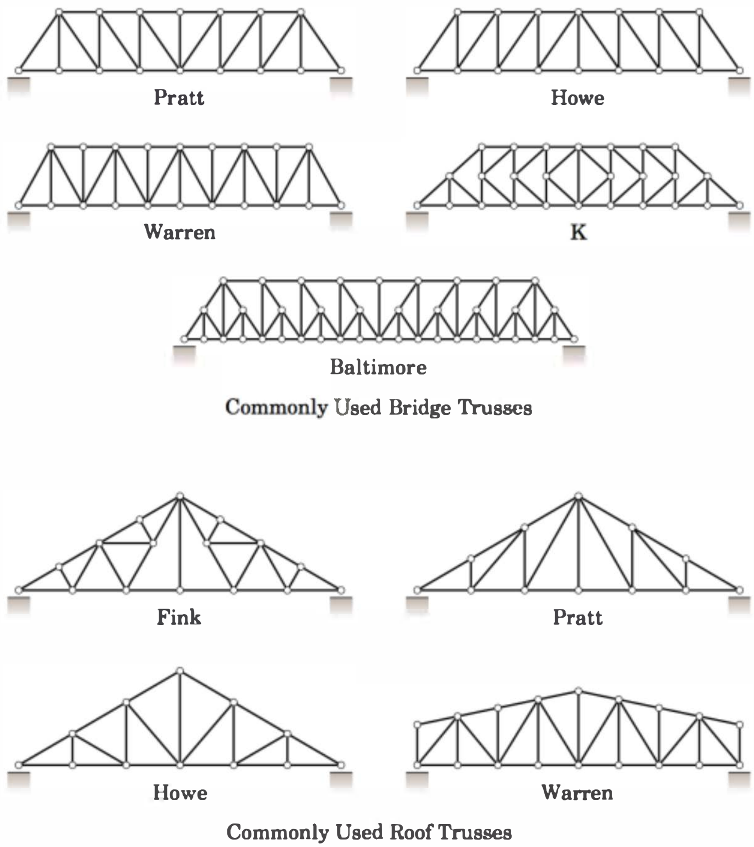

Figure 4/2

4/2 Plane Trusses

A framework composed of members joined at their ends to form a rigid structure is called a *truss*. Bridges, roof supports, derricks, and other such structures are common examples of trusses. Structural members commonly used are I-beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins. When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*.

For bridges and similar structures, plane trusses are commonly utilized in pairs with one truss assembly placed on each side of the structure. A section of a typical bridge structure is shown in Fig. 4/1. The combined weight of the roadway and vehicles is transferred to the longitudinal stringers, then to the cross beams, and finally, with the weights of the stringers and cross beams accounted for, to the upper joints of the two plane trusses which form the vertical sides of the structure. A simplified model of the truss structure is indicated at the left side of the illustration; the forces L represent the joint loadings.

Several examples of commonly used trusses which can be analyzed as plane trusses are shown in schematic form in Fig. 4/2.

Simple Trusses

The basic element of a plane truss is the triangle. Three bars joined by pins at their ends, Fig. 4/3a, constitute a rigid frame. The term *rigid* is used to mean noncollapsible and also to mean that deformation of the members due to induced internal strains is negligible. On the other hand, four or more bars pin-jointed to form a polygon of as many sides constitute a nonrigid frame. We can make the nonrigid frame in Fig. 4/3b rigid, or stable, by adding a diagonal bar joining A and D or B and C and thereby forming two triangles. We can extend the structure by adding additional units of two end-connected bars, such as DE and CE or AF and DF, Fig. 4/3c, which are pinned to two fixed joints. In this way the entire structure will remain rigid.

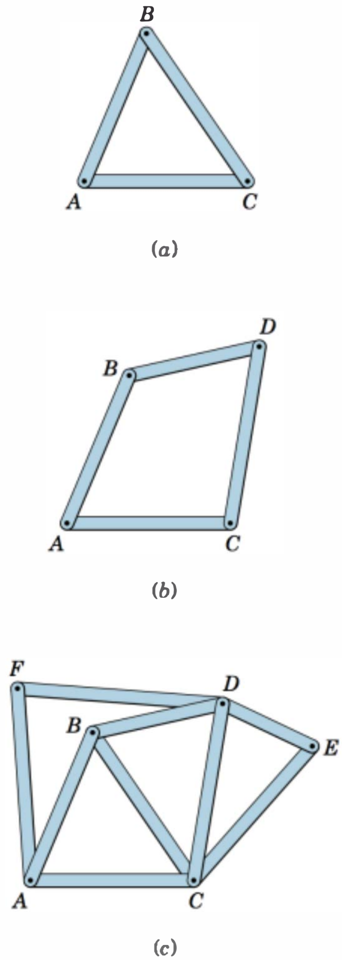

Figure 4/3

Structures built from a basic triangle in the manner described are known as *simple trusses*. When more members are present than are needed to prevent collapse, the truss is statically indeterminate. A statically indeterminate truss cannot be analyzed by the equations of equilibrium alone. Additional members or supports which are not necessary for maintaining the equilibrium configuration are called *redundant*.

To design a truss, we must first determine the forces in the various members and then select appropriate sizes and structural shapes to withstand the forces. Several assumptions are made in the force analysis of simple trusses. First, we assume all members to be *two-force members*. A two-force member is one in equilibrium under the action of two forces only, as defined in general terms with Fig. 3/4 in Art. 3/3. Each member of a truss is normally a straight link joining the two points of application of force. The two forces are applied at the ends of the member and are necessarily equal, opposite, and *collinear* for equilibrium.

The member may be in tension or compression, as shown in Fig. 4/4. When we represent the equilibrium of a portion of a two-force member, the tension T or compression C acting on the cut section is the same

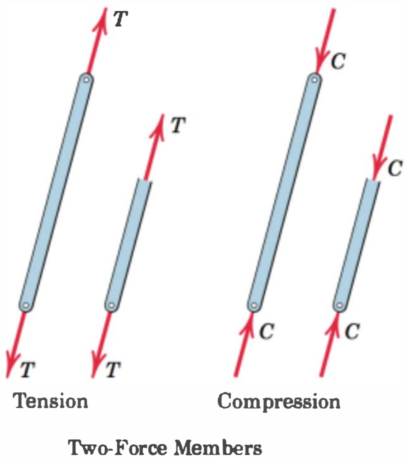

Figure 4/4

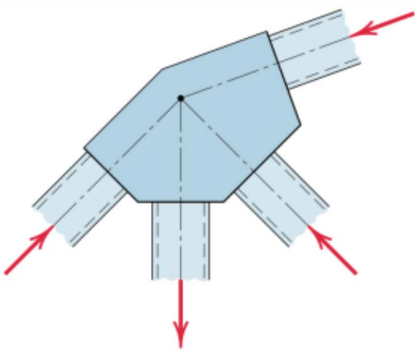

Figure 4/5

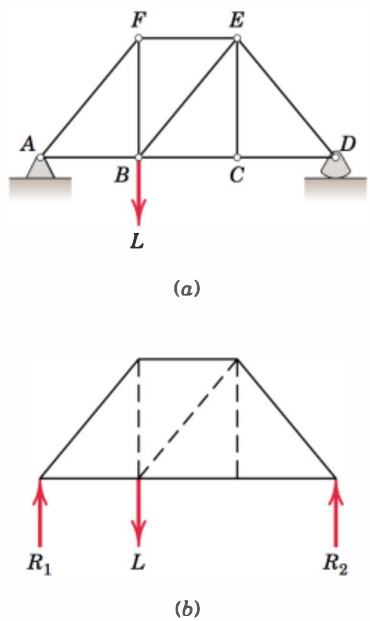

Figure 4/6

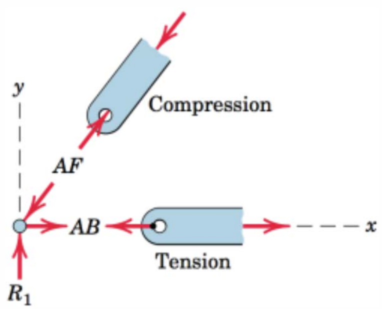

Figure 4/7

for all sections. We assume here that the weight of the member is small compared with the force it supports. If it is not, or if we must account for the small effect of the weight, we can replace the weight W of the member by two forces, each $W/2$ if the member is uniform, with one force acting at each end of the member. These forces, in effect, are treated as loads externally applied to the pin connections. Accounting for the weight of a member in this way gives the correct result for the average tension or compression along the member but will not account for the effect of bending of the member.

Truss Connections and Supports

When welded or riveted connections are used to join structural members, we may usually assume that the connection is a pin joint if the centerlines of the members are concurrent at the joint as in Fig. 4/5.

We also assume in the analysis of simple trusses that all external forces are applied at the pin connections. This condition is satisfied in most trusses. In bridge trusses the deck is usually laid on cross beams which are supported at the joints, as shown in Fig. 4/1.

For large trusses, a roller, rocker, or some kind of slip joint is used at one of the supports to provide for expansion and contraction due to temperature changes and for deformation from applied loads. Trusses and frames in which no such provision is made are statically indeterminate, as explained in Art. 3/3. Figure 3/1 shows examples of such joints.

Two methods for the force analysis of simple trusses will be given. Each method will be explained for the simple truss shown in Fig. 4/6*a*. The free-body diagram of the truss as a whole is shown in Fig. 4/6*b*. The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. Then the force analysis of the remainder of the truss is performed.

4/3 Method of Joints

This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint. The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

We begin the analysis with any joint where at least one known load exists and where not more than two unknown forces are present. The solution may be started with the pin at the left end. Its free-body diagram is shown in Fig. 4/7. With the joints indicated by letters, we usually designate the force in each member by the two letters defining the ends of the member. The proper directions of the forces should be evident by inspection for this simple case. The free-body diagrams of portions of members AF and AB are also shown to clearly indicate the mechanism of the action and reaction. The member AB actually makes contact on the left side of the pin, although the force AB is drawn from the right side and is shown acting away from the pin. Thus, if we consistently draw the force arrows on the *same* side of the pin as the member, then tension (such as AB) will always be indicated by an arrow *away* from the pin, and compression (such as AF) will always be indicated by an arrow *toward* the pin. The magnitude of AF is obtained from the equation $\Sigma F_y = 0$ and AB is then found from $\Sigma F_x = 0$.

Joint F may be analyzed next, since it now contains only two unknowns, EF and BF. Proceeding to the next joint having no more than two unknowns, we subsequently analyze joints B, C, E, and D in that order. Figure 4/8 shows the free-body diagram of each joint and its corresponding force polygon, which represents graphically the two equilibrium conditions $\Sigma F_x = 0$ and $\Sigma F_y = 0$. The numbers indicate the order in which the joints are analyzed. We note that, when joint D is finally reached, the computed reaction R_2 must be in equilibrium with the forces in members CD and ED, which were determined previously from the two neighboring joints. This requirement provides a check on the correctness of our work. Note that isolation of joint C shows that the force in CE is zero when the equation $\Sigma F_y = 0$ is applied. The force in

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This New York City bridge structure suggests that members of a simple truss need not be straight.

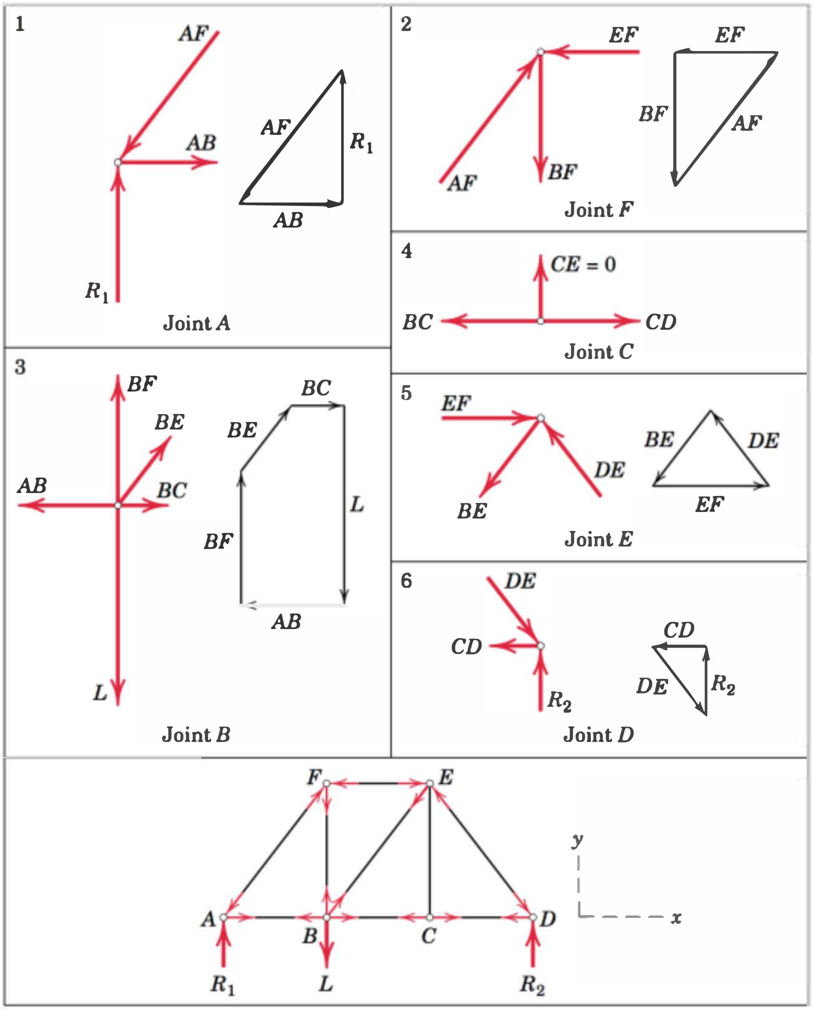


Figure 4/8

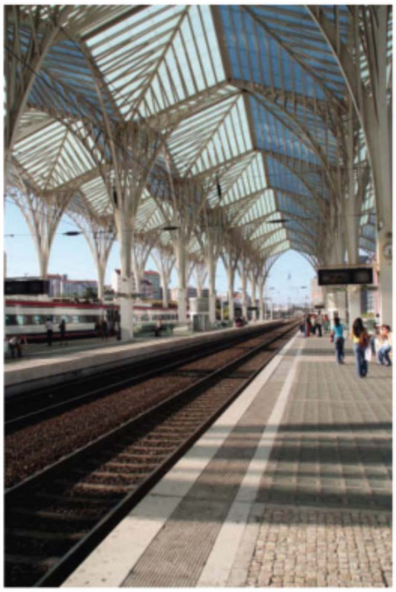
this member would not be zero, of course, if an external vertical load were applied at C.

It is often convenient to indicate the tension T and compression C of the various members directly on the original truss diagram by drawing arrows away from the pins for tension and toward the pins for compression. This designation is illustrated at the bottom of Fig. 4/8.

Sometimes we cannot initially assign the correct direction of one or both of the unknown forces acting on a given pin. If so, we may make an arbitrary assignment. A negative computed force value indicates that the initially assumed direction is incorrect.

Internal and External Redundancy

If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute *external* redundancy. If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute *internal* redundancy and the truss is again statically indeterminate.



An interesting array of trusses at the Lisbon Oriente Station in Portugal.

For a truss which is statically determinate externally, there is a definite relation between the number of its members and the number of its joints necessary for internal stability without redundancy. Because we can specify the equilibrium of each joint by two scalar force equations, there are in all $2j$ such equations for a truss with j joints. For the entire truss composed of m two-force members and having the maximum of three unknown support reactions, there are in all $m + 3$ unknowns (m tension or compression forces and three reactions). Thus, for any plane truss, the equation $m + 3 = 2j$ will be satisfied if the truss is statically determinate internally.

A *simple* plane truss, formed by starting with a triangle and adding two new members to locate each new joint with respect to the existing structure, satisfies the relation automatically. The condition holds for the initial triangle, where $m = j = 3$, and m increases by 2 for each added joint while j increases by 1. Some other (nonsimple) statically determinate trusses, such as the K-truss in Fig. 4/2, are arranged differently, but can be seen to satisfy the same relation.

This equation is a necessary condition for stability but it is not a sufficient condition, since one or more of the m members can be arranged in such a way as not to contribute to a stable configuration of the entire truss. If $m + 3 > 2j$, there are more members than independent equations, and the truss is statically indeterminate internally with redundant members present. If $m + 3 < 2j$, there is a deficiency of internal members, and the truss is unstable and will collapse under load.

Special Conditions

We often encounter several special conditions in the analysis of trusses. When two collinear members are under compression, as indicated in Fig. 4/9a, it is necessary to add a third member to maintain

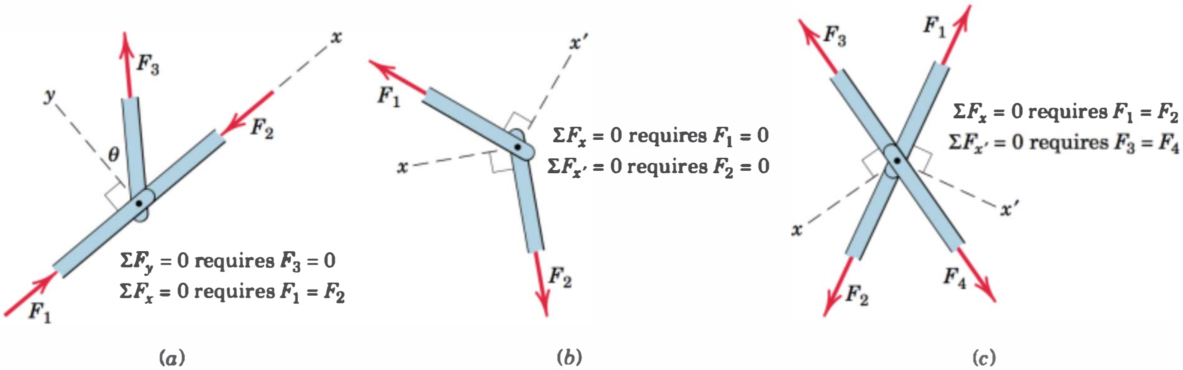


Figure 4/9

alignment of the two members and prevent buckling. We see from a force summation in the y-direction that the force F_3 in the third member must be zero and from the x-direction that $F_1 = F_2$. This conclusion holds regardless of the angle θ and holds also if the collinear members are in tension. If an external force with a component in the y-direction were applied to the joint, then F_3 would no longer be zero.

When two noncollinear members are joined as shown in Fig. 4/9b, then in the absence of an externally applied load at this joint, the forces in both members must be zero, as we can see from the two force summations.

When two pairs of collinear members are joined as shown in Fig. 4/9c, the forces in each pair must be equal and opposite. This conclusion follows from the force summations indicated in the figure.

Truss panels are frequently cross-braced as shown in Fig. 4/10a. Such a panel is statically indeterminate if each brace can support either tension or compression. However, when the braces are flexible members incapable of supporting compression, as are cables, then only the tension member acts and we can disregard the other member. It is usually evident from the asymmetry of the loading how the panel will deflect. If the deflection is as indicated in Fig. 4/10b, then member AB should be retained and CD disregarded. When this choice cannot be made by inspection, we may arbitrarily select the member to be retained. If the assumed tension turns out to be positive upon calculation, then the choice was correct. If the assumed tension force turns out to be negative, then the opposite member must be retained and the calculation redone.

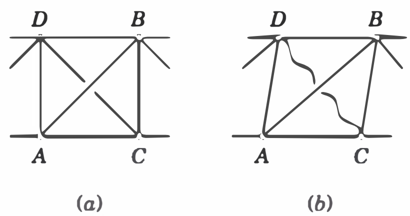


Figure 4/10

We can avoid simultaneous solution of the equilibrium equations for two unknown forces at a joint by a careful choice of reference axes. Thus, for the joint indicated schematically in Fig. 4/11 where L is known and F_1 and F_2 are unknown, a force summation in the x-direction eliminates reference to F_1 and a force summation in the x'-direction eliminates reference to F_2. When the angles involved are not easily found, then a simultaneous solution of the equations using one set of reference directions for both unknowns may be preferable.

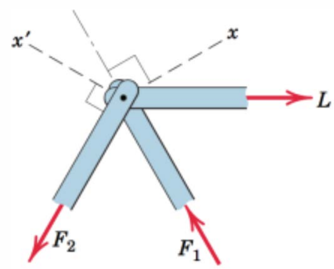
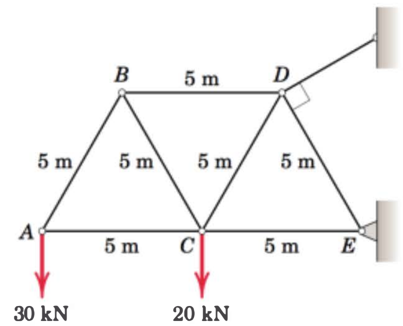


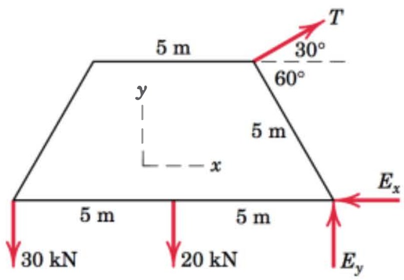
Figure 4/11

Sample Problem 4/1

Compute the force in each member of the loaded cantilever truss by the method of joints.



Solution. If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give



$[\Sigma M_E = 0] \qquad 5T - 20(5) - 30(10) = 0 \qquad T = 80 \text{ kN}$

$[\Sigma F_x = 0] \qquad 80 \cos 30° - E_x = 0 \qquad E_x = 69.3 \text{ kN}$

$[\Sigma F_y = 0] \qquad 80 \sin 30° + E_y - 20 - 30 = 0 \qquad E_y = 10 \text{ kN}$

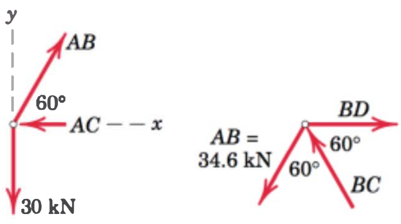
Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0] \qquad 0.866AB - 30 = 0 \qquad AB = 34.6 \text{ kN } T \qquad$ *Ans.*

$[\Sigma F_x = 0] \qquad AC - 0.5(34.6) = 0 \qquad AC = 17.32 \text{ kN } C \qquad$ *Ans.*

① where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from



Joint A Joint B

$[\Sigma F_y = 0] \qquad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6 \text{ kN } C \qquad$ *Ans.*

$[\Sigma F_x = 0] \qquad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6 \text{ kN } T \qquad$ *Ans.*

Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0] \qquad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T \qquad$ *Ans.*

$[\Sigma F_x = 0] \qquad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C \qquad$ *Ans.*

Finally, from joint E there results

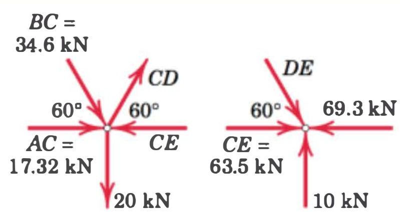
$[\Sigma F_y = 0] \qquad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C \qquad$ *Ans.*

and the equation $\Sigma F_x = 0$ checks.

Note that the weights of the truss members have been neglected in comparison with the external loads.

Helpful Hint

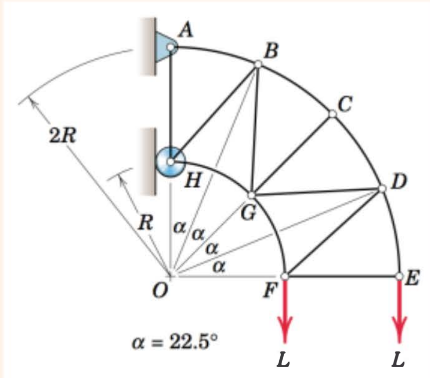
① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).



Joint C Joint E

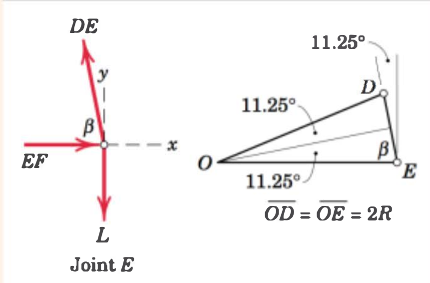
Sample Problem 4/2

The simple truss shown supports the two loads, each of magnitude L. Determine the forces in members DE, DF, DG, and CD.



Solution. First of all, we note that the curved members of this simple truss are all two-force members, so that the effect of each curved member within the truss is the same as that of a straight member.

We can begin with joint E because there are only two unknown member forces acting there. With reference to the free-body diagram and accompanying geometry for joint E, we note that $\beta = 180° - 11.25° - 90° = 78.8°$.



① $[\Sigma F_y = 0]$ $\quad DE \sin 78.8° - L = 0 \qquad DE = 1.020L\ T$ *Ans.*

$[\Sigma F_x = 0]$ $\quad EF - DE \cos 78.8° = 0 \qquad EF = 0.1989L\ C$

We must now move to joint F, as there are still three unknown members at joint D. From the geometric diagram,

$$\gamma = \tan^{-1}\left[\frac{2R \sin 22.5°}{2R \cos 22.5° - R}\right] = 42.1°$$

From the free-body diagram of joint F,

$[\Sigma F_x = 0]$ $\quad -GF \cos 67.5° + DF \cos 42.1° - 0.1989L = 0$

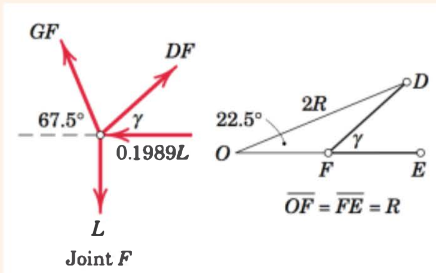
$[\Sigma F_y = 0]$ $\quad GF \sin 67.5° + DF \sin 42.1° - L = 0$

Simultaneous solution of these two equations yields

$GF = 0.646L\ T \qquad DF = 0.601L\ T$ *Ans.*

Helpful Hint

① Rather than calculate and use the angle $\beta = 78.8°$ in the force equations, we could have used the 11.25° angle directly.



For member DG, we move to the free-body diagram of joint D and the accompanying geometry.

$$\delta = \tan^{-1}\left[\frac{2R \cos 22.5° - 2R \cos 45°}{2R \sin 45° - 2R \sin 22.5°}\right] = 33.8°$$

$$\epsilon = \tan^{-1}\left[\frac{2R \sin 22.5° - R \sin 45°}{2R \cos 22.5° - R \cos 45°}\right] = 2.92°$$

Then from joint D:

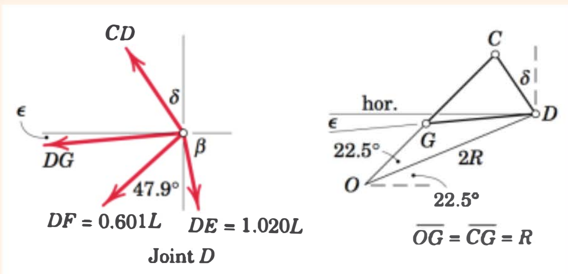
$[\Sigma F_x = 0]$ $-DG \cos 2.92° - CD \sin 33.8° - 0.601L \sin 47.9° + 1.020L \cos 78.8° = 0$

$[\Sigma F_y = 0]$ $-DG \sin 2.92° + CD \cos 33.8° - 0.601L \cos 47.9° - 1.020L \sin 78.8° = 0$

The simultaneous solution is

$CD = 1.617L\ T \qquad DG = -1.147L$ or $DG = 1.147L\ C$ *Ans.*

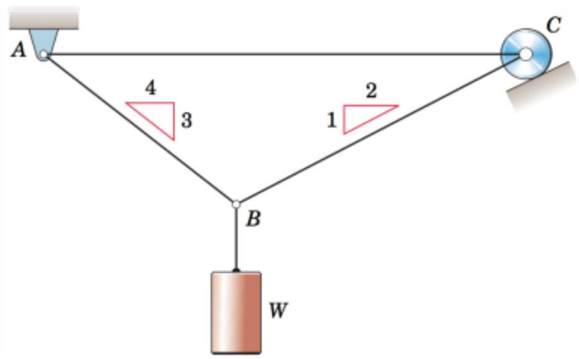
Note that ϵ is shown exaggerated in the accompanying figures.



PROBLEMS

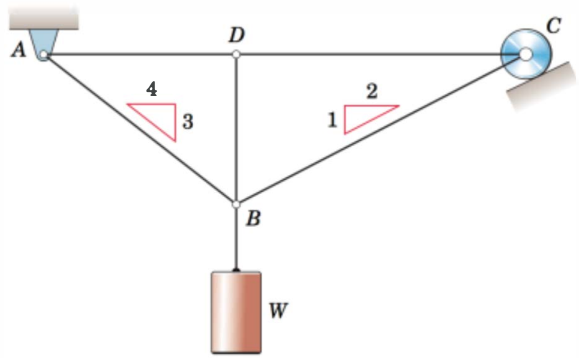
Introductory Problems

4/1 Determine the force in each member of the loaded truss as a result of the hanging weight W.



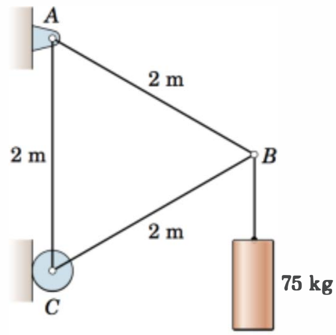
Problem 4/1

4/2 The truss of the previous problem is modified by adding the vertical support member BD. Determine the force in each member of the modified truss as a result of the hanging weight W.



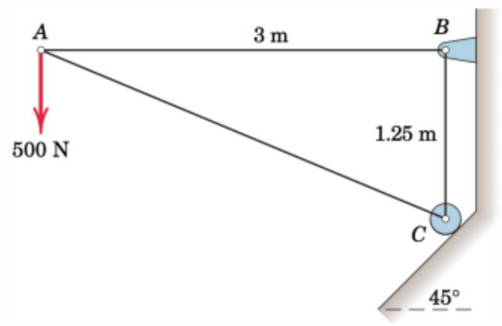
Problem 4/2

4/3 Determine the force in each member of the simple equilateral truss.



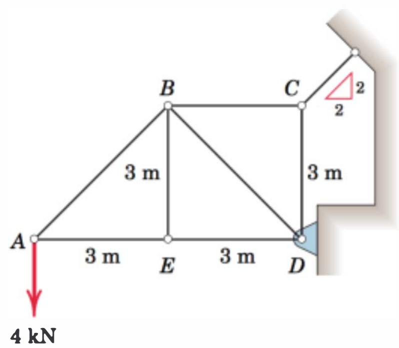
Problem 4/3

4/4 Determine the force in each member of the loaded truss. Discuss the effects of varying the angle of the 45° support surface at C.



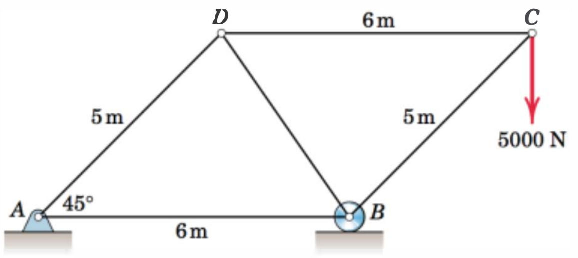
Problem 4/4

4/5 Calculate the forces in members BE and BD of the loaded truss.



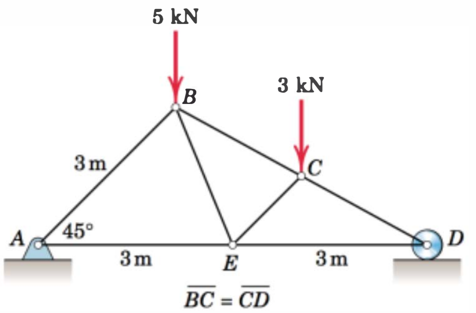
Problem 4/5

4/6 Determine the force in each member of the loaded truss.



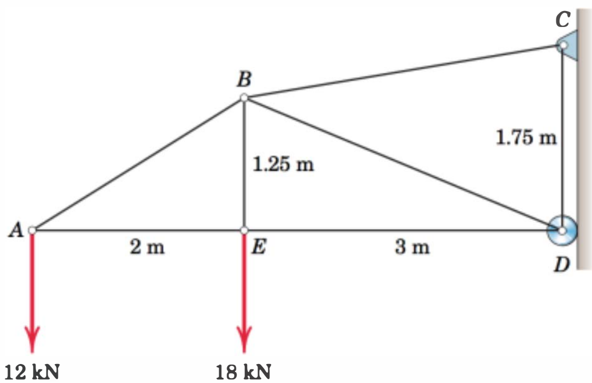
Problem 4/6

4/7 Determine the forces in members *BE* and *CE* of the loaded truss.



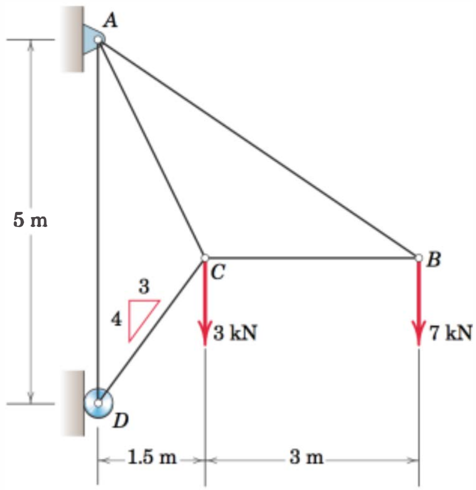
Problem 4/7

4/8 Determine the force in each member of the loaded truss.



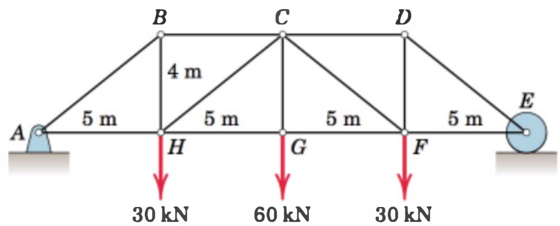
Problem 4/8

4/9 Determine the force in each member of the loaded truss.



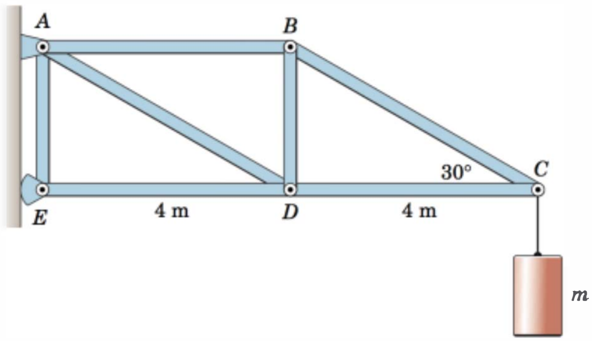
Problem 4/9

4/10 Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.



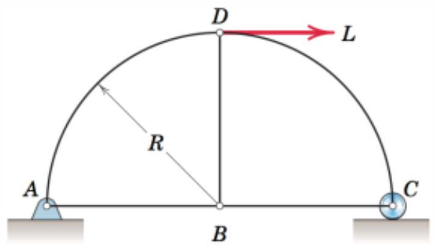
Problem 4/10

4/11 If the maximum tensile force in any of the truss members must be limited to 24 kN, and the maximum compressive force must be limited to 35 kN, determine the largest permissible mass *m* which may be supported by the truss.



Problem 4/11

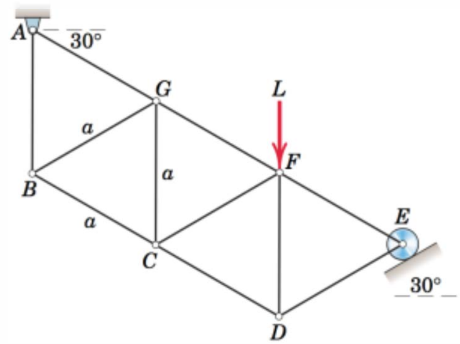
4/12 Determine the forces in members AB, BC, and BD of the loaded truss.



Problem 4/12

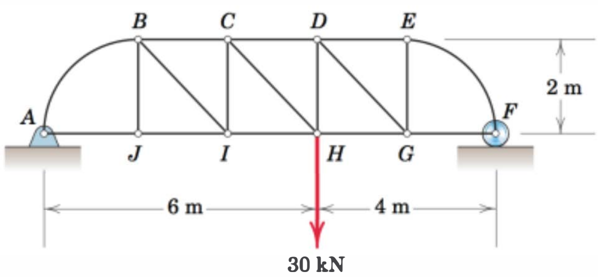
Representative Problems

4/13 The truss is composed of equilateral triangles of sides a and is loaded and supported as shown. Determine the forces in members EF, DE, and DF.



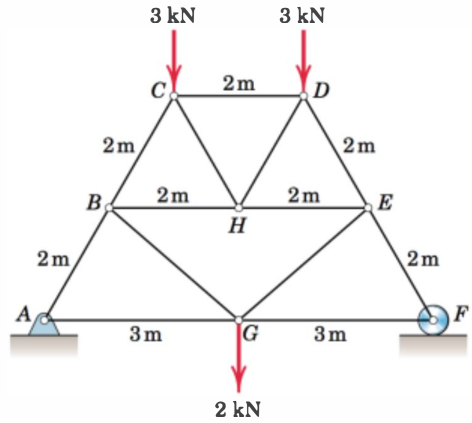
Problem 4/13

4/14 Determine the forces in members BJ, BI, CI, CH, DG, DH, and EG of the loaded truss. All triangles are 45°-45°-90°.



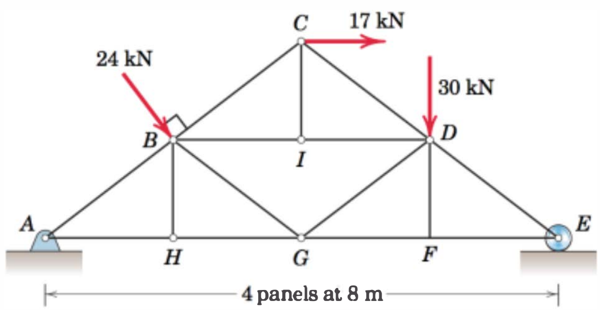
Problem 4/14

4/15 Determine the forces in members BC and BG of the loaded truss.



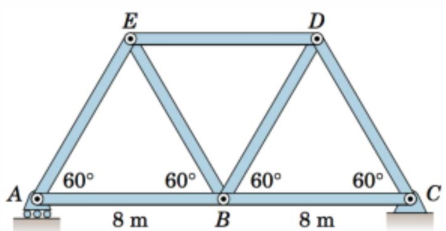
Problem 4/15

4/16 Determine the force in each member of the loaded truss. All triangles are 3-4-5.



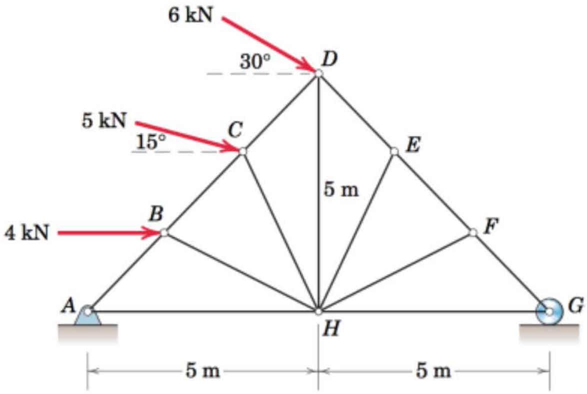
Problem 4/16

4/17 Each member of the truss is a uniform 8-m bar with a mass of 400 kg. Calculate the average tension or compression in each member due to the weights of the members.



Problem 4/17

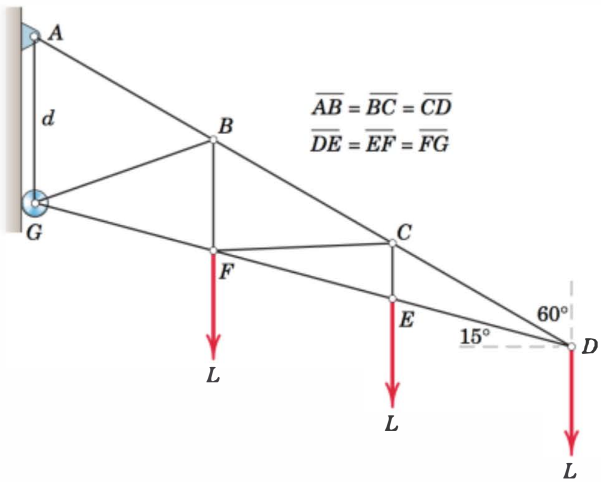
4/18 Determine the force in each member of the loaded Palladian truss.



$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE} = \overline{EF} = \overline{FG}$

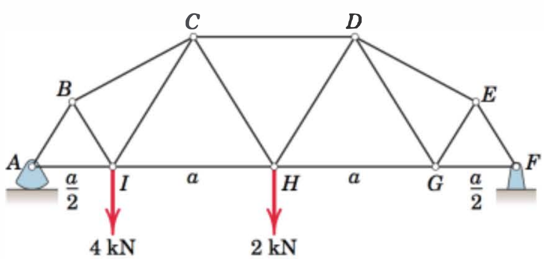
Problem 4/18

4/19 Determine the forces in members BG and BF of the loaded truss.



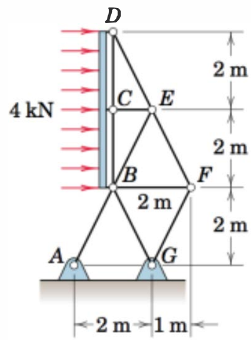
Problem 4/19

4/20 Determine the forces in members BI, CI, and HI for the loaded truss. All angles are 30°, 60°, or 90°.



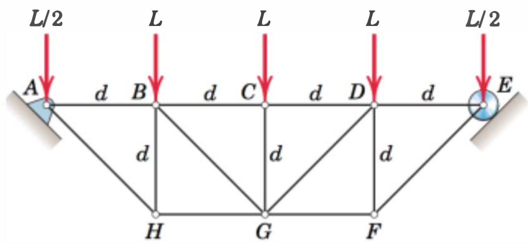
Problem 4/20

4/21 The signboard truss is designed to support a horizontal wind load of 4 kN. A separate analysis shows that $\frac{5}{8}$ of this force is transmitted to the center connection at C and the rest is equally divided between D and B. Calculate the forces in members BE and BC.



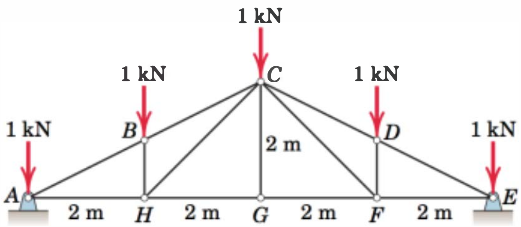
Problem 4/21

4/22 Determine the forces in members AB, CG, and DE of the loaded truss.



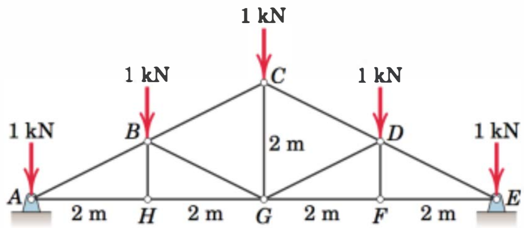
Problem 4/22

4/23 A snow load transfers the forces shown to the upper joints of a Pratt roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members.



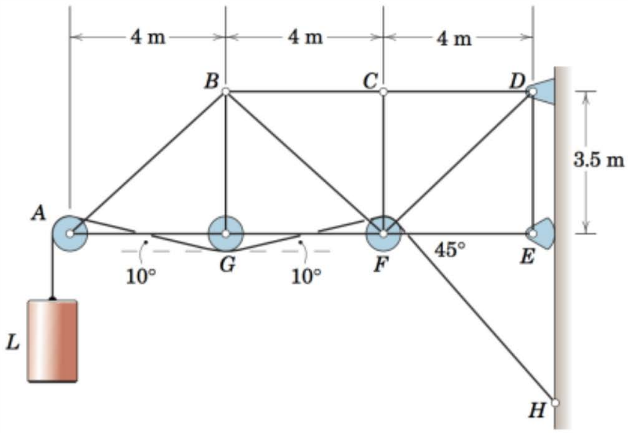
Problem 4/23

4/24 The loading of Prob. 4/23 is shown applied to a Howe roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members. Compare with the results of Prob. 4/23.



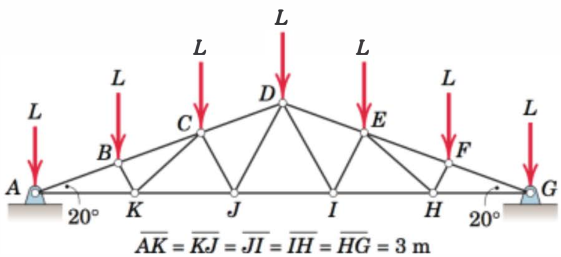
Problem 4/24

4/25 Determine the force in each member of the loaded truss.



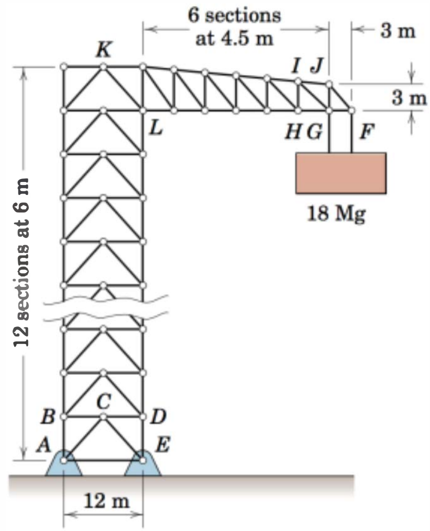
Problem 4/25

4/26 Determine the forces in members *EH* and *EI* of the double Fink truss. Neglect any horizontal reactions at the supports and note that joints *E* and *F* divide $\overline{DG}$ into thirds.



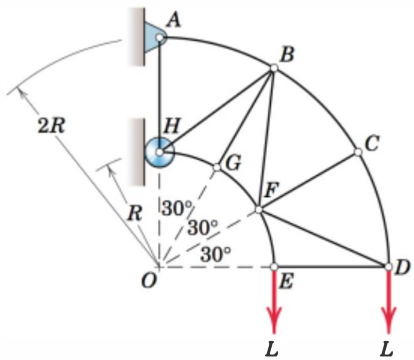
Problem 4/26

4/27 The 72-m structure is used to provide various support services to launch vehicles prior to liftoff. In a test, an 18-Mg mass is suspended from joints *F* and *G*, with its weight equally divided between the two joints. Determine the forces in members *GJ* and *GI*. What would be your path of joint analysis for members in the vertical tower, such as *AB* or *KL*?



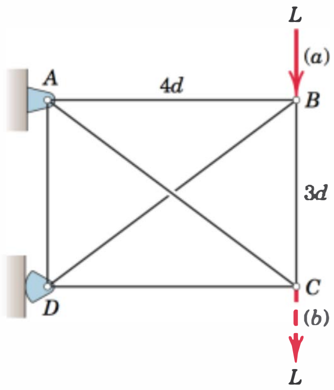
Problem 4/27

4/28 Determine the force in member *BF* of the loaded truss.



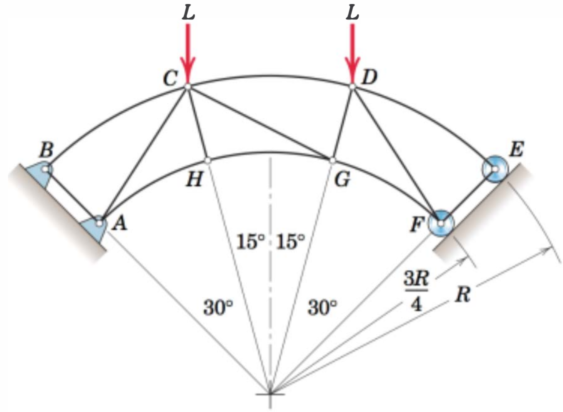
Problem 4/28

4/29 The rectangular frame is composed of four perimeter two-force members and two cables AC and BD which are incapable of supporting compression. Determine the forces in all members due to the load L in position (*a*) and then in position (*b*).



Problem 4/29

▶4/30 Determine the force in member CG of the loaded truss. Assume that the four external reactions at A, B, E, and F are equal in magnitude and are directed perpendicular to the local supporting surface.



Problem 4/30

4/4 Method of Sections

When analyzing plane trusses by the method of joints, we need only two of the three equilibrium equations because the procedures involve concurrent forces at each joint. We can take advantage of the third or moment equation of equilibrium by selecting an entire section of the truss for the free body in equilibrium under the action of a nonconcurrent system of forces. This *method of sections* has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has cut that member. Thus, it is not necessary to proceed with the calculation from joint to joint until the member in question has been reached. In choosing a section of the truss, we note that, in general, not more than three members whose forces are unknown should be cut, since there are only three available independent equilibrium relations.

Illustration of the Method

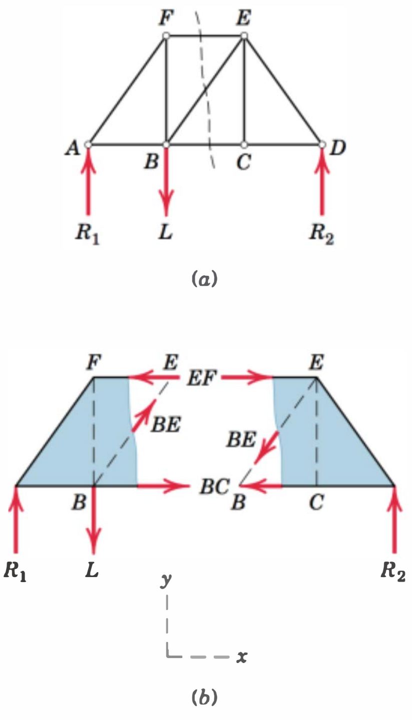


Figure 4/12

The method of sections will now be illustrated for the truss in Fig. 4/6, which was used in the explanation of the method of joints. The truss is shown again in Fig. 4/12*a* for ready reference. The external reactions are first computed as with the method of joints, by considering the truss as a whole.

Let us determine the force in the member *BE*, for example. An imaginary section, indicated by the dashed line, is passed through the truss, cutting it into two parts, Fig. 4/12*b*. This section has cut three members whose forces are initially unknown. In order for the portion of the truss on each side of the section to remain in equilibrium, it is necessary to apply to each cut member the force which was exerted on it by the member cut away. For simple trusses composed of straight two-force members, these forces, either tensile or compressive, will always be in the directions of the respective members. The left-hand section is in equilibrium under the action of the applied load L, the end reaction R_1, and the three forces exerted on the cut members by the right-hand section which has been removed.

We can usually draw the forces with their proper senses by a visual approximation of the equilibrium requirements. Thus, in balancing the moments about point B for the left-hand section, the force EF is clearly to the left, which makes it compressive, because it acts toward the cut section of member EF. The load L is greater than the reaction R_1, so that the force BE must be up and to the right to supply the needed upward component for vertical equilibrium. Force BE is therefore tensile, since it acts away from the cut section.

With the approximate magnitudes of R_1 and L in mind, we see that the balance of moments about point E requires that BC be to the right. A casual glance at the truss should lead to the same conclusion when it is realized that the lower horizontal member will stretch under the tension caused by bending. The equation of moments about joint B eliminates three forces from the relation, and EF can be determined directly. The force BE is calculated from the equilibrium equation for the y-direction. Finally, we determine BC by balancing moments about point E. In this way each of the three unknowns has been determined independently of the other two.

The right-hand section of the truss, Fig. 4/12*b*, is in equilibrium under the action of R_2 and the same three forces in the cut members applied in the directions opposite to those for the left section. The proper sense for the horizontal forces can easily be seen from the balance of moments about points B and E.

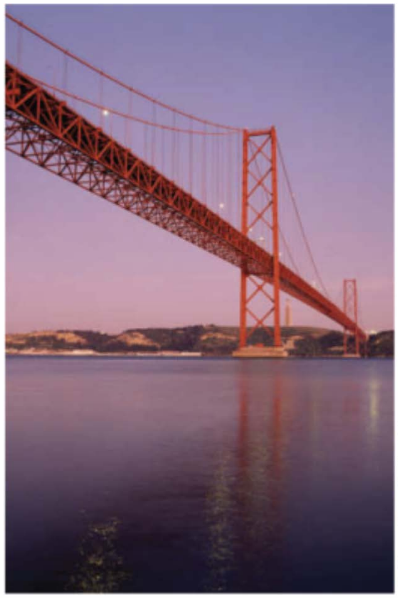
Additional Considerations

It is essential to understand that in the method of sections an entire portion of the truss is considered a single body in equilibrium. Thus, the forces in members internal to the section are not involved in the analysis of the section as a whole. To clarify the free body and the forces acting externally on it, the cutting section is preferably passed through the members and not the joints. We may use either portion of a truss for the calculations, but the one involving the smaller number of forces will usually yield the simpler solution.

In some cases the methods of sections and joints can be combined for an efficient solution. For example, suppose we wish to find the force in a central member of a large truss. Furthermore, suppose that it is not possible to pass a section through this member without passing through at least four unknown members. It may be possible to determine the forces in nearby members by the method of sections and then progress to the unknown member by the method of joints. Such a combination of the two methods may be more expedient than exclusive use of either method.

The moment equations are used to great advantage in the method of sections. One should choose a moment center, either on or off the section, through which as many unknown forces as possible pass.

It is not always possible to assign the proper sense of an unknown force when the free-body diagram of a section is initially drawn. Once an arbitrary assignment is made, a positive answer will verify the assumed sense, and a negative result will indicate that the force is in the sense opposite to that assumed. An alternative notation preferred by some is to assign all unknown forces arbitrarily as positive in the tension direction (away from the section) and let the algebraic sign of the answer distinguish between tension and compression. Thus, a plus sign would signify tension and a minus sign compression. On the other hand, the advantage of assigning forces in their correct sense on the free-body diagram of a section wherever possible is that doing so emphasizes the physical action of the forces more directly. This practice is the one which is preferred here.



Many simple trusses are periodic in that there are repeated and identical structural sections.