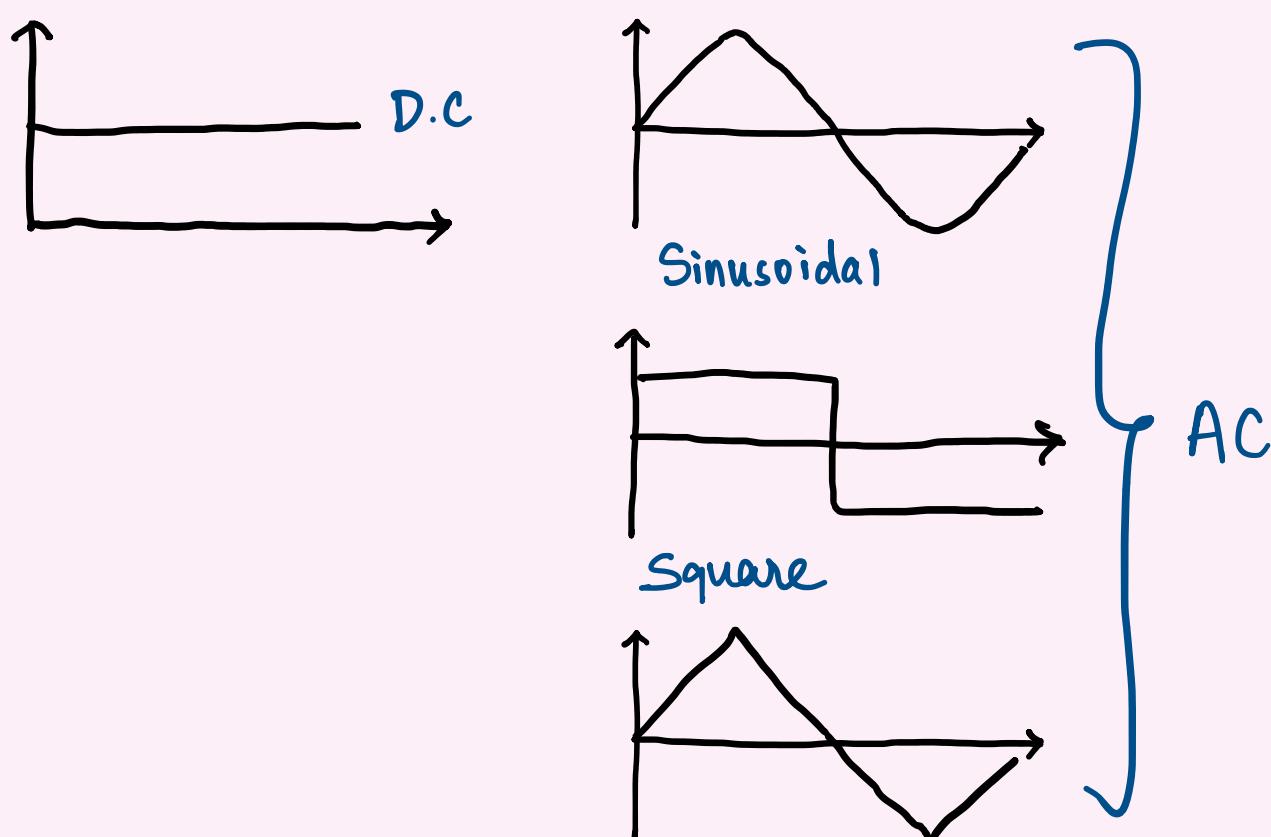
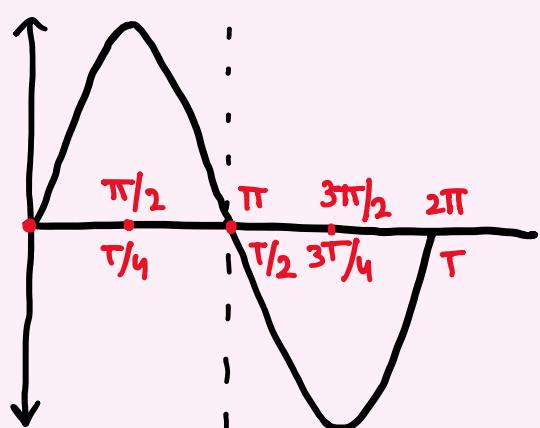


1. Single Phase System



Terminology

- Periodic Waveform: One which repeats itself after certain time interval
- Time Period : Time taken to complete 1 cycle of a periodic waveform
- Frequency : No. of cycles completed in 1 second
- Pure AC waveform: Waveform in which +ve area = -ve area



Average value = 0

Time	Angle
T	2π
$\frac{T}{2}$	π
1	$\frac{2\pi}{T}$
t	$\frac{2\pi t}{T}$

Q. For a sinusoidal function, freq = 50 Hz, Find

- i) Half time period ii) Angular Frequency

A. $T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s} = 20 \text{ ms}$

i) $\frac{T}{2} = 10 \text{ ms}$ ii) $\omega = 2\pi f = 100\pi = 314.59 \text{ rad/sec}$

Q. Max value of sinusoidal AC of frequency 50Hz is 25A.

Write eq for instantaneous expression of current. Determine at 3ms & 14ms

A. $\omega = 2\pi f = 100\pi \text{ rad/s}$

$i(t) = 25 \sin(100\pi t) \text{ A}$

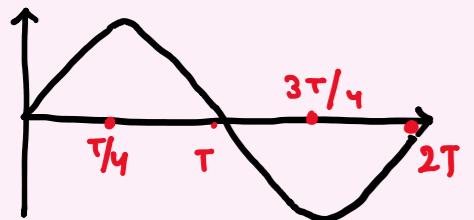
i) $i(3\text{ms}) = 25 \sin(3 \times 10^{-3} \times 100\pi) = 20.22 \text{ A}$

ii) $i(14\text{ms}) = 25 \sin(14 \times 10^{-3} \times 100\pi) = -23.71 \text{ A}$

2. Average & RMS values of sine wave

Average value of sine wave

$$\rightarrow F_{avg} = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{T} \int_0^T A \sin \omega t dt = \frac{A}{T} \left(-\frac{\cos \omega t}{\omega} \right)_0^T = 0$$



Root Mean Square Value of sine wave

→ Energy consumed by resistor during the period is

$$E_{AC} = \int_0^T P(t) dt = \int_0^T \frac{[v(t)]^2}{R} dt$$

$$E_{DC} = \frac{V^2 T}{R}$$

$$E_{AC} = E_{DC}, \text{ Hence, } \int_0^T \frac{[v(t)]^2}{R} dt = \frac{V^2 T}{R}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T [V_m \sin \omega t]^2 dt} = \sqrt{\frac{V_m^2}{T} \int_0^T \sin^2 \omega t dt} \\ = \sqrt{\frac{V_m^2}{T} \cdot \frac{T}{2}} = \frac{V_m}{\sqrt{2}}$$

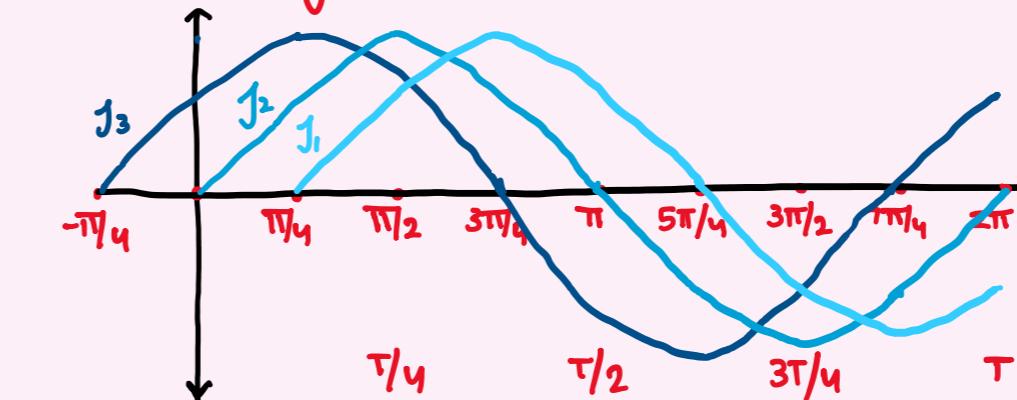
Power & Energy Consumed using RMS

$$\rightarrow P_{av} = \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T \frac{[v(t)]^2}{R} dt = \frac{1}{T} \int_0^T \frac{[V_{rms}]^2}{R} dt = \frac{V_{rms}^2}{R}$$

$$P = I^2 R$$

$$\rightarrow E = Pt = \frac{V^2 t}{R} = (I^2 R)t$$

Phase Lag & Phase Lead



$J_1(t) = A \sin(\omega t - \pi/4)$ lags by $\pi/4$ rad

$J_2(t) = A \sin \omega t$ represents reference line

$J_3(t) = A \sin(\omega t + \pi/4)$ leads by $\pi/4$ rad

Q. Write an eq. to represent sine waves of 50 Hz freq.

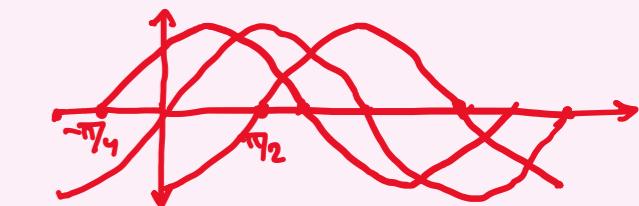
i) RMS = 10A, Starting at 5ms

ii) Peak = 20A, starting at -2.5ms

$$A. \omega = 2\pi f = 100\pi$$

$$i) i_1(t) = 10\sqrt{2} \sin(100\pi t - (100\pi \times 0.005)) \\ = 10\sqrt{2} \sin(100\pi t - \pi/2)$$

$$ii) i_2(t) = 20 \sin(100\pi t + (100\pi \times 2.5 \times 10^{-3})) \\ = 20 \sin(100\pi t + \pi/4)$$



$$T = \frac{1}{f} = \frac{1}{50} = 20 \text{ ms} = 2\pi$$

$$5 \text{ ms} = \pi/2$$

$$-2.5 \text{ ms} = -\pi/4$$

3. Phasor & Phasor Diagram

→ Representation of a sinusoidal function by a crankshaft

Q. $j_1(t) = 100 \sin(100\pi t)$

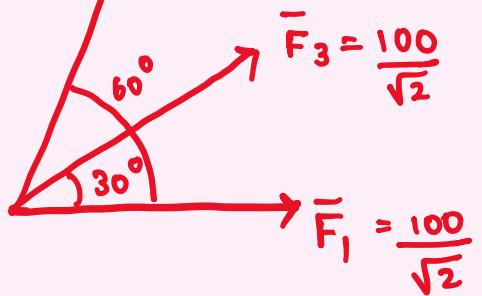
$$j_2(t) = 200 \sin(100\pi t + 60^\circ)$$

$$j_3(t) = 100 \cos(100\pi t - 60^\circ) = 100 \sin(100\pi t - 60^\circ + 90^\circ)$$

$$\bar{F}_2 = \frac{200}{\sqrt{2}}$$

$$= 100 \sin(100\pi t + 30^\circ)$$

A.



$$j_1(t) = \frac{100}{\sqrt{2}} \angle 0^\circ$$

$$j_2(t) = \frac{200}{\sqrt{2}} \angle 60^\circ$$

$$j_3(t) = \frac{100}{\sqrt{2}} \angle 30^\circ$$

Q. There are 3 conducting wires connected to form a junction. The currents flowing into the junction in two wires are $i_1 = 10 \sin 314t$ A and $i_2 = 15 \cos(314t - 45^\circ)$ A. What is the current leaving the junction in the third wire? What is its value at $t=0$?

A

$$i_1 + i_2 = i_3$$

$$i_1 = 10 \sin 314t \quad i_2 = 15 \sin(314t - 45^\circ + 90^\circ) \\ = 15 \sin(314t + 45^\circ)$$

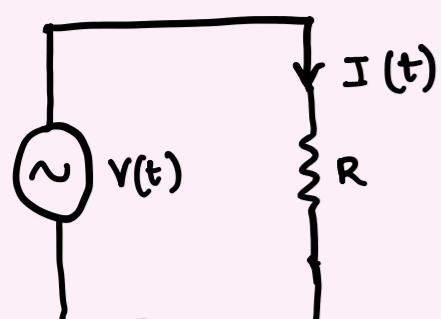
$$\sin(A+B) = \sin A \sin B - \cos A \cos B$$

$$i_1 = \frac{10}{\sqrt{2}} \angle 0^\circ \quad i_2 = \frac{15}{\sqrt{2}} \angle 45^\circ$$

$$i_3 = 16.38 \angle 27.23^\circ \\ = 23.16 \sin(314t + 27.23^\circ)$$

4. Analysis of single phase AC circuits

Pure Resistor Load

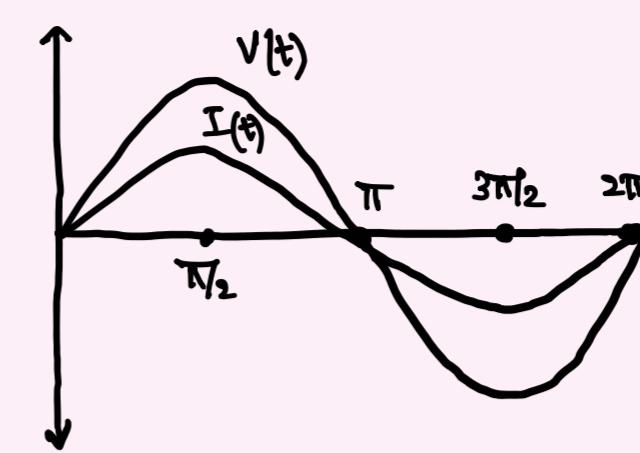


$$V(t) = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

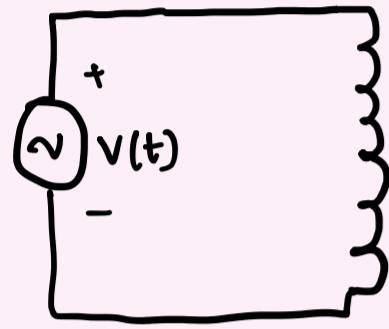
$$I(t) = \frac{V(t)}{R} = I_m \sin \omega t = \frac{I_m}{\sqrt{2}} \angle 0^\circ$$

$$Z = \frac{\bar{V}}{\bar{I}} = R \angle 0^\circ = R \angle 0^\circ$$

$$\frac{V_m}{\sqrt{2}} \quad \frac{I_m}{\sqrt{2}}$$



Pure Inductor



$$V(t) = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

$$I(t) = \frac{1}{L} \int V(t) dt = \frac{1}{L} \int V_m \sin \omega t dt$$

$$i = -\frac{V_m}{WL} \cos \omega t$$

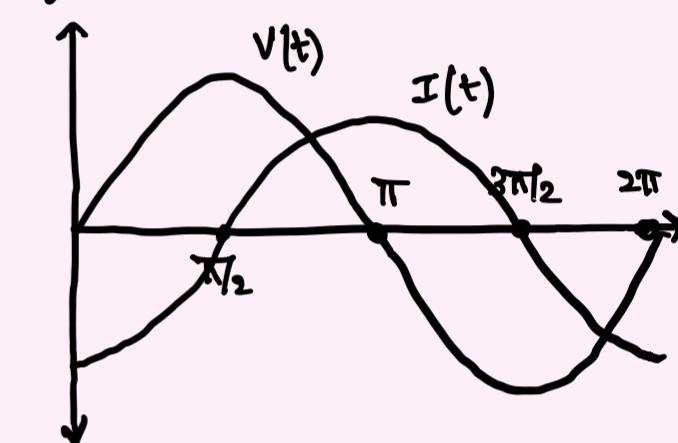
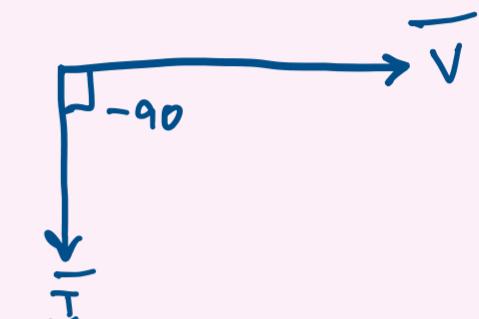
$$I_m = \frac{V_m}{WL}$$

$$i = -I_m \cos \omega t$$

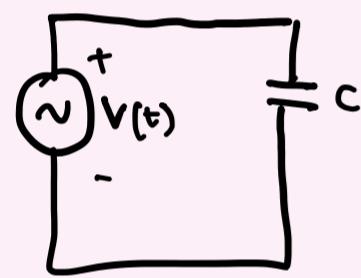
$$= I_m \sin(\omega t - 90^\circ) = \frac{I_m}{\sqrt{2}} \angle -90^\circ$$

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m}{I_m} \angle -90^\circ$$

$$= \frac{X_m}{I_m} \angle -90^\circ = \omega L \angle -90^\circ = j X_L$$



Pure Conductor



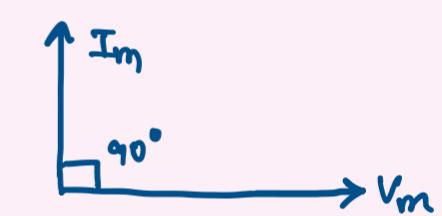
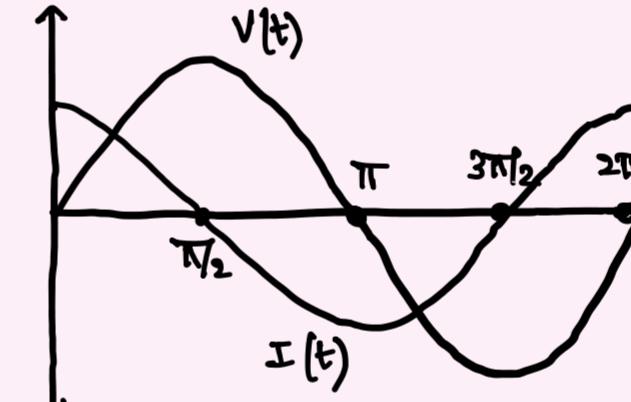
$$V(t) = V_m \sin \omega t$$

$$I(t) = \frac{d(V(t))}{dt} = wC \cdot V_m \cos \omega t$$

$$= I_m \sin(\omega t + 90^\circ)$$

$$= \frac{I_m}{\sqrt{2}} \angle 90^\circ$$

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V_m}{I_m} \angle 90^\circ = \frac{X_m}{I_m} \angle 90^\circ = \frac{1}{wC} \angle 90^\circ = -j X_C$$



R

$$V = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

$$I = I_m \sin \omega t = \frac{I_m}{\sqrt{2}} \angle 0^\circ$$

$$Z = R$$

L

$$V = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

$$I = I_m \sin(\omega t - 90^\circ) = \frac{I_m}{\sqrt{2}} \angle -90^\circ$$

$$Z = \omega L \angle -90^\circ = j X_L$$

C

$$V = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

$$I = I_m \sin(\omega t + 90^\circ) = \frac{I_m}{\sqrt{2}} \angle 90^\circ$$

$$Z = \frac{1}{wC} \angle 90^\circ = -j X_C$$

Q. A Capacitor of Capacitance $100\mu F$ is connected across an AC voltage source $100\sin(100\pi t)$ V. Determine

i) Capacitive Reactance

ii) Impedance

iii) Instantaneous expression for the current Also, draw the phasor diagram.

$$A \quad C = 100 \times 10^{-6} F \quad V(t) = 100 \sin(100\pi t)$$

$$i) \quad X_C = \frac{1}{wC} = \frac{1}{100\pi \times 100 \times 10^{-6}} = 31.83 \Omega$$

$$ii) \quad Z = -j X_C = -j 31.83$$

$$iii) \quad I_m = V_m \cdot wC = 100 \times 100\pi \times 100 \times 10^{-6} = \pi$$

$$I(t) = \pi (100\pi t + 90^\circ)$$

$$\frac{1}{Z} = \frac{3.14}{\sqrt{2}}$$

$$\bar{V} = \frac{100}{\sqrt{2}}$$

$$L = 100mH$$

$$V(t) = 100 \sin(100\pi t)$$

$$A. \quad L = 100 \times 10^{-3} H$$

$$i) \quad X_L = \omega L = 100\pi \times 100 \times 10^{-3} = 0.314 \Omega$$

$$ii) \quad Z = j X_L = j 0.314 \Omega$$

$$iii) \quad I_m = \frac{V_m}{wL} = \frac{100}{100\pi \times 10^{-3}} = \frac{10}{\pi} = 3.18$$

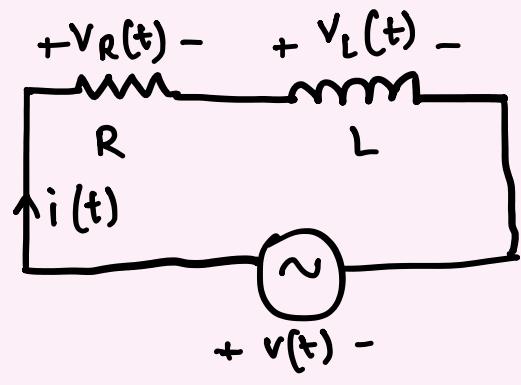
$$I(t) = 3.18 (100\pi t - 90^\circ)$$

$$\bar{V} = \frac{100}{\sqrt{2}}$$

$$\bar{I} = \frac{3.18}{\sqrt{2}}$$

5. Analysis of series RL & RC circuits

R-L Circuit



$$v(t) = v_R(t) + v_L(t)$$

$$= \bar{V}_R + \bar{V}_L$$

$$\bar{V}_R = \bar{I}R$$

$$\bar{V}_L = \bar{I} \times jX_L$$

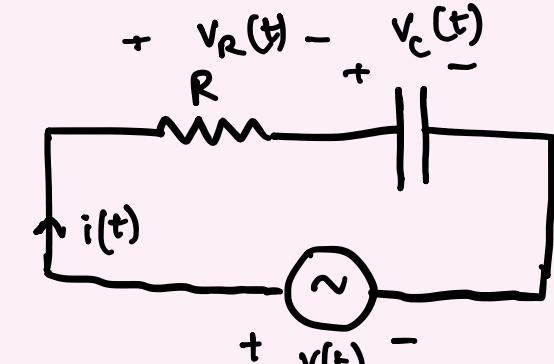
$$v(t) = \bar{I}(R + jX_L)$$

$$Z_T = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\bar{V}_L}{\bar{V}_R}\right) = \tan^{-1}\left(\frac{\bar{V}_L}{\bar{V}_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$$

If ϕ is +ve, voltage leads current

R-C Circuit



$$v(t) = v_R(t) + v_C(t)$$

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$\bar{V}_R = \bar{I} \times R$$

$$\bar{V}_C = \bar{I} \times (-jX_C)$$

$$\bar{V} = \bar{I} \times (R - jX_C)$$

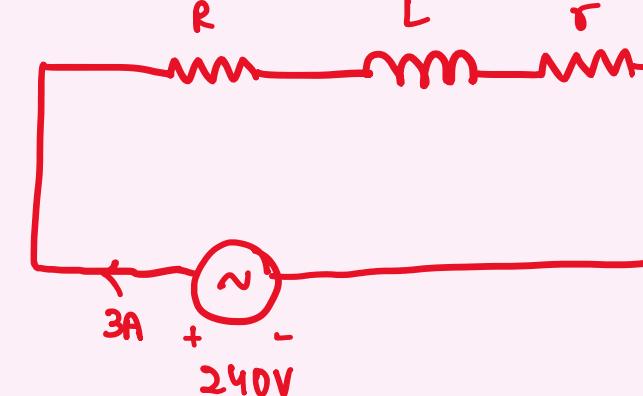
$$Z_T = R - jX_C = \sqrt{R^2 + X_C^2} \angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\phi = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

If ϕ is -ve, voltage lags current

Q. When a resistor and an inductor in series are connected to a 240V supply, a current of 3A flows lagging 37 degrees behind the supply voltage, while the voltage across the inductor is 171V. Find the resistance of the resistor, and the resistance and reactance of the inductor. Find the power factor of the circuit.

A.



$$V(t) = 240 \sin(\omega t)$$

$$I(t) = 3 \sin(\omega t - 37)$$

$$Z_T = \frac{240}{\sqrt{2}} \angle 0^\circ = 80 \angle 37^\circ = 63.89 + j48.14$$

$$Z_T = R + (r + jX_L)$$

$$\Rightarrow R + r = 63.89 \Omega$$

$$X_L = 48.14 \Omega$$

$$|Z| = \sqrt{r^2 + X_L^2}$$

$$57^2 = r^2 + X_L^2$$

$$\sqrt{57^2 - 48.14^2} = r$$

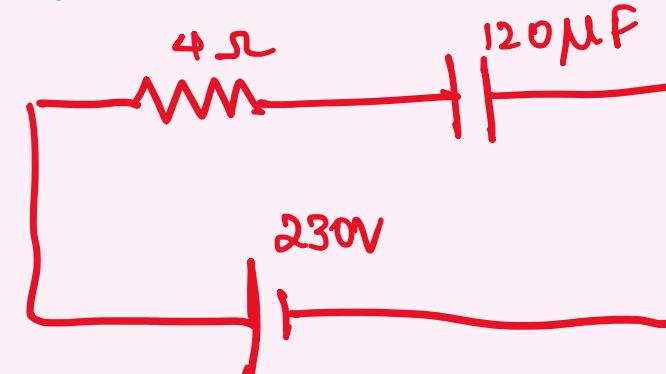
$$r = 30.52 ; R = 33.37$$

$$\text{Power factor} = \cos \phi = \cos \left(\tan^{-1} \left(\frac{X_L}{R+r} \right) \right) = 0.798 \text{ lag}$$

Q. $R = 4\Omega$ $C = 120\mu F$. $V = 230V$, $f = 50Hz$

Calculate Current & Phasor Diagram

A.



$$V = 230 \angle 0^\circ$$

$$X_C = \frac{1}{\omega C} = \frac{10^6}{100 \times 120} = \frac{1000}{12} = 26.52 \Omega$$

$$Z = R - jX_C = 4 - j26.52$$

$$Z = \frac{V}{I} \Rightarrow I = \frac{230}{4 - j26.52} = 8.57 \angle 81.42^\circ A$$

$$\bar{I} = \frac{8.57}{\sqrt{2}} \angle 81.4^\circ$$

Q. 3 coils A, B, C in series. $I = 3A$. $V_A = 12V$, $V_B = 6V$, $V_C = 9V$ on direct current & $V_A = 15V$, $V_B = 9V$, $V_C = 12V$ on alternating current. Find

i) internal parameters

ii) Power dissipated when AC flows

iii) Applied voltage across it

iv) Draw Phasor diagram

v) Power factor

$$A. \quad R_A = \frac{12}{3} = 4\Omega \quad Z_A = \frac{15}{3} = 5\Omega \quad \begin{cases} Z_A^2 = R_A^2 + X_{LA}^2 \\ X_{LA} = 3 \end{cases}$$

$$R_B = \frac{6}{3} = 2\Omega \quad Z_B = \frac{9}{3} = 3\Omega \quad \begin{cases} Z_B^2 = R_B^2 + X_{LB}^2 \\ X_{LB} = \sqrt{5} \end{cases}$$

$$R_C = \frac{9}{3} = 3\Omega \quad Z_C = \frac{12}{3} = 4\Omega \quad \begin{cases} Z_C^2 = R_C^2 + X_{LC}^2 \\ X_{LC} = \sqrt{7} \end{cases}$$

$$R_T = 9\Omega \quad Z = R + jX_L \quad X_{LT} = 7.88\Omega$$

$$Z = 9 + 7.88j \quad = 11.96 \angle 41.2^\circ$$

$$ii) P_A = I^2 Z_A = 45W$$

$$P_B = I^2 Z_B = 27W$$

$$P_C = I^2 Z_C = 36W$$

$$iii) V = I |Z|$$

$$= 3 \times 11.96$$

$$= 35.88$$

$$iv) \bar{V} = \frac{35.88}{\sqrt{2}}$$

$$\bar{I} = \frac{3}{\sqrt{2}}$$

$$v) PF = \cos(41.2^\circ)$$

$$= 0.75 \text{ lag}$$

6. Active, Reactive & Apparent powers

Active Power (P)

- The average power that is generated to produce heat energy
- Measured in Watts (W)

Reactive Power (Q)

- AC Power which is not consumed but circulates b/w element and source without any loss of energy in form of heat
- Measured in Volt-Amperes Reactive (VAR)

Apparent Power (S)

- $\sqrt{P^2+Q^2}$
- Measured in Volt-Amperes (VA)

General AC Circuit

$$\begin{aligned}
 \rightarrow P &= \frac{1}{T} \int_0^T v(t) * i(t) dt \\
 &= \frac{1}{T} \int_0^T V_m I_m \sin(\omega t) \sin(\omega t + \phi) dt \\
 &= \frac{1}{T} \int_0^T V_m I_m \sin(\omega t) (\sin(\omega t) \cos(\phi) + \sin(\phi) \cos(\omega t)) dt \\
 &= \frac{1}{T} \int_0^T V_m I_m (\sin^2(\omega t) \cos(\phi) + \sin(\phi) \sin(\omega t) \cos(\omega t)) dt \\
 &= \frac{1}{T} \int_0^T V_m I_m \left(\frac{1 - \cos(2\omega t)}{2} \cos(\phi) + \sin(\phi) \cdot \frac{\sin(2\omega t)}{2} \right) dt \\
 &= \frac{V_m I_m}{T} \left[\cos(\phi) \left(\frac{\omega t}{2} - \frac{\sin(2\omega t)}{4} \right) + \sin(\phi) \frac{\cos(2\omega t)}{4} \right]_0^T \\
 &= \frac{V_m I_m}{T} \left[\cos(\phi) \left[\frac{T}{2} - 0 \right] + 0 \right] \\
 &= \frac{V_m I_m}{2} \times \cos(\phi) \times \frac{T}{T} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos(\phi) = VI \cos(\phi)
 \end{aligned}$$

Similarly

$$Q = VI \sin(\phi)$$

$$S = VI$$

Element	Phase angle ϕ	P	Q	S
R	0	VI	0	VI
L	90°	0	VI	VI
C	-90°	0	-VI	VI
Series RL	$\tan^{-1}\left(\frac{x_L}{R}\right)$	$VI \cos(\phi)$	$VI \sin(\phi)$	VI
Series RC	$-\tan^{-1}\left(\frac{x_C}{R}\right)$	$VI \cos(\phi)$	$VI \sin(\phi)$	VI

→ Inductive reactive power \Rightarrow +ve

→ Capacitive reactive power \Rightarrow -ve

Power Factor

$$= \frac{P}{S} = \cos(\phi)$$

$$= \frac{R}{|Z_T|}$$

→ P.F of purely resistive circuit = 1

P.F of purely inductor circuit = 0 lag

P.F of purely conductive circuit = 0 lead

Q.

A series RL circuit is connected to a sinusoidal voltage source $v(t) = 100\sin(\omega t)$ V. It draws a current of $10\sin(\omega t - 60^\circ)$ A. Determine

- i) Active, Reactive and Apparent Powers.
- ii) Power factor of the circuit.

A.

$$\begin{aligned}
 \phi &= \angle V - \angle I \\
 &= 0 - (-60^\circ) = 60^\circ
 \end{aligned}$$

$$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

$$i) P = VI \cos(\phi) = \frac{100 \times 10}{2} \times \cos 60^\circ = 250 \text{ W}$$

$$Q = VI \sin(\phi) = \frac{100 \times 10}{2} \times \sin 60^\circ = 433 \text{ VAR}$$

$$S = VI = 500 \text{ VA}$$

$$ii) P.F = \cos(\phi) = \cos 60^\circ = \frac{1}{2}$$

Q. $P = 25 \text{ W}$

$$\begin{aligned}
 I &= 0.4 \\
 V &= 230 \text{ V}
 \end{aligned}$$

$$\text{Find } C \quad f = 50 \text{ Hz}$$

$$A. P = VI \cos(\phi)$$

$$\phi = 74.23^\circ$$

$$Z = \frac{230 \angle 0}{0.4 \angle 74.23} = \underbrace{156.27}_{R} - \underbrace{553.35i}_{X_C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$553.35 = \frac{1}{100C} \Rightarrow C = 5.75 \mu\text{F}$$

Q. Find

i) P.F

ii) Reactive Power

iii) Magnitude of Supply Voltage

Also, redraw by taking supply voltage as reference, mentioning all voltages & current.

$$A. V_T = V_L \angle 36^\circ + V_C \angle -90^\circ = 8.09 \angle -0.86$$

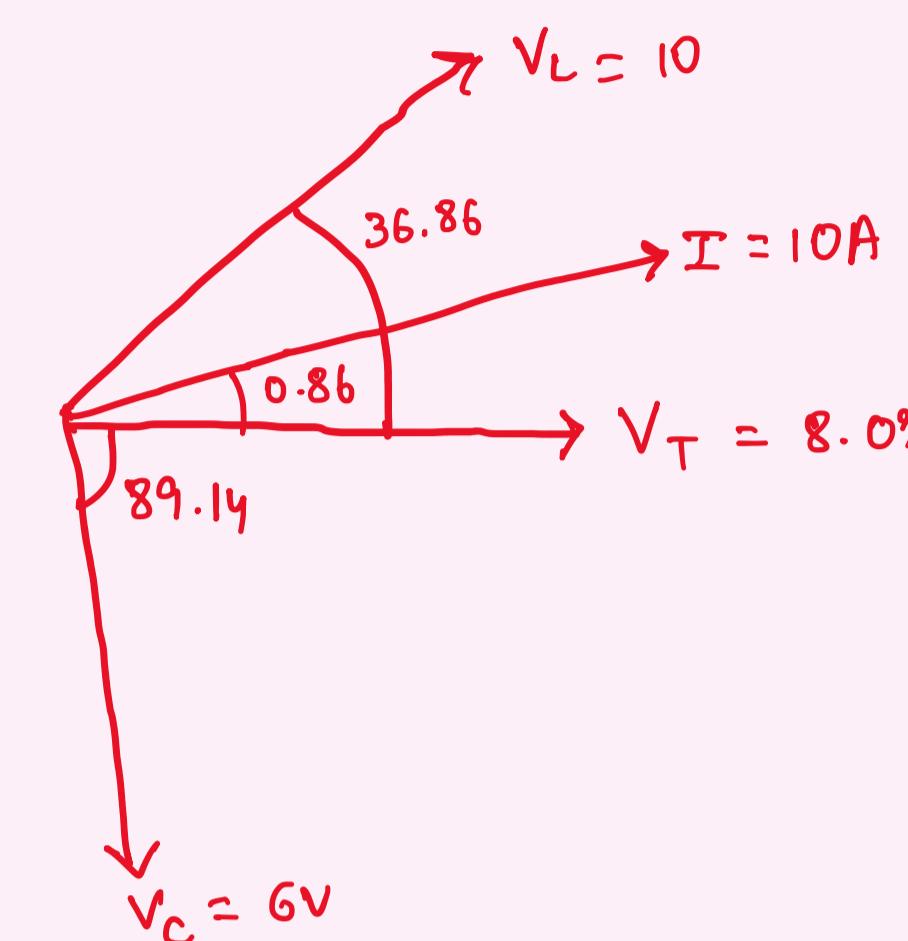
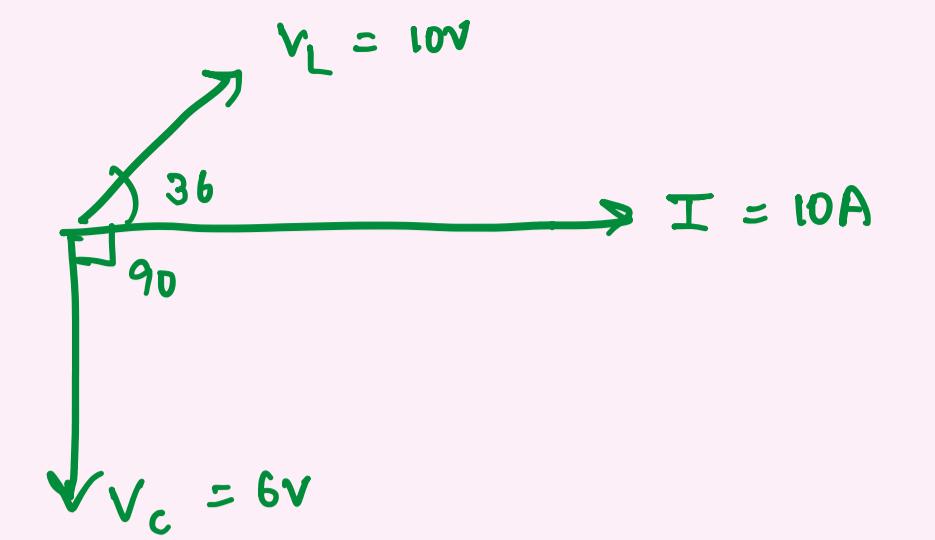
$$I_T = 10 \text{ A}$$

$$\phi = \angle V_T - \angle I_T = -0.86$$

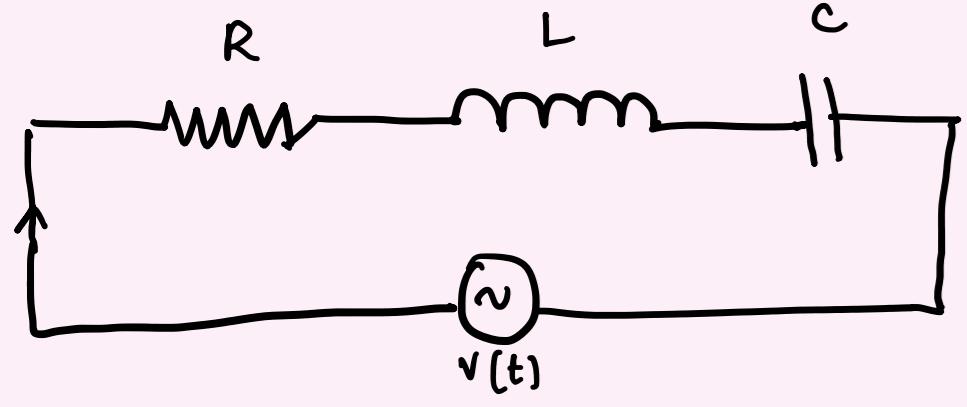
$$i) P.F = \cos(\phi) = 0.99$$

$$ii) Q = VI \sin(\phi) = -1.214 \text{ VAR}$$

$$iii) V_T = 8.09$$



7. Analysis of RLC circuits



$$V(t) = V_R(t) + V_L(t) + V_C(t)$$

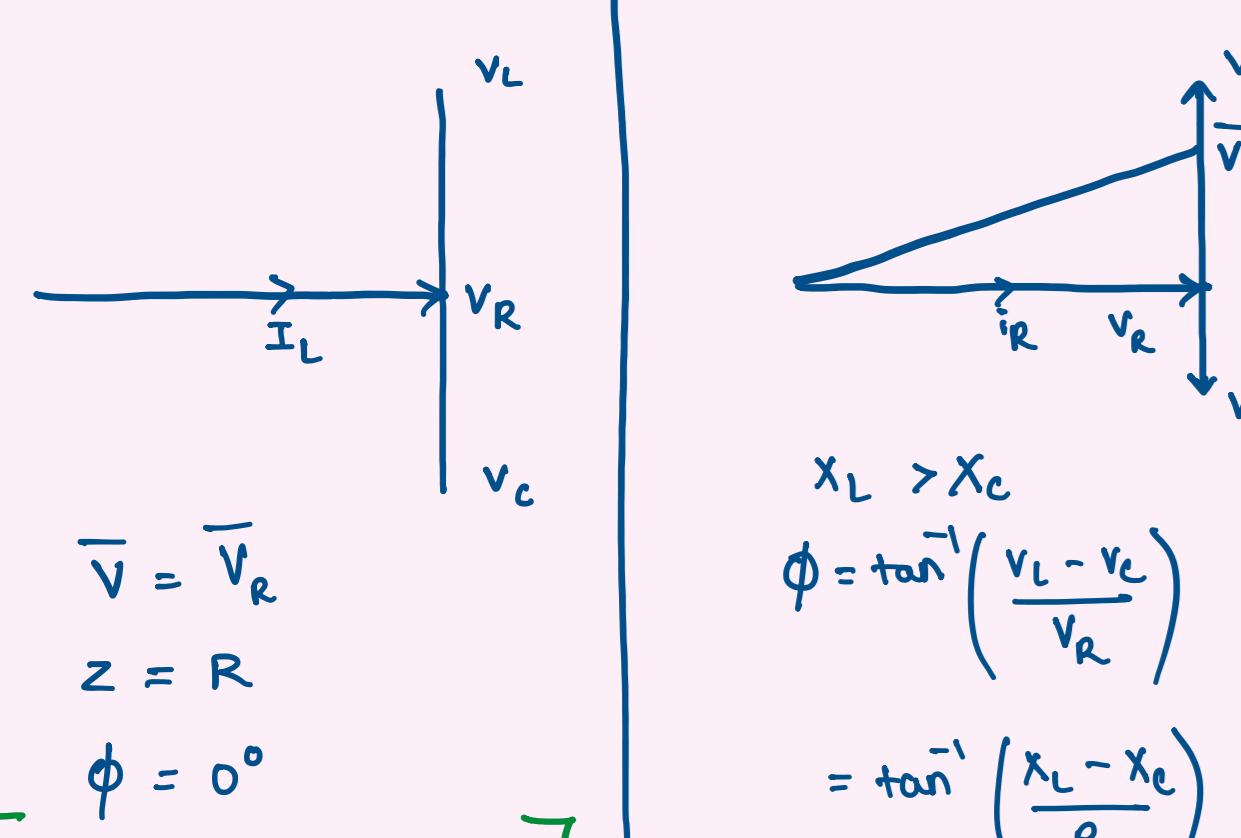
$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

$$= \bar{I}R + \bar{I}(jX_L) - \bar{I}(jX_C)$$

$$\bar{V} = \bar{I}(R + jX_L - jX_C)$$

$$Z_T = \frac{\bar{V}}{\bar{I}} = R + jX_L - X_C = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

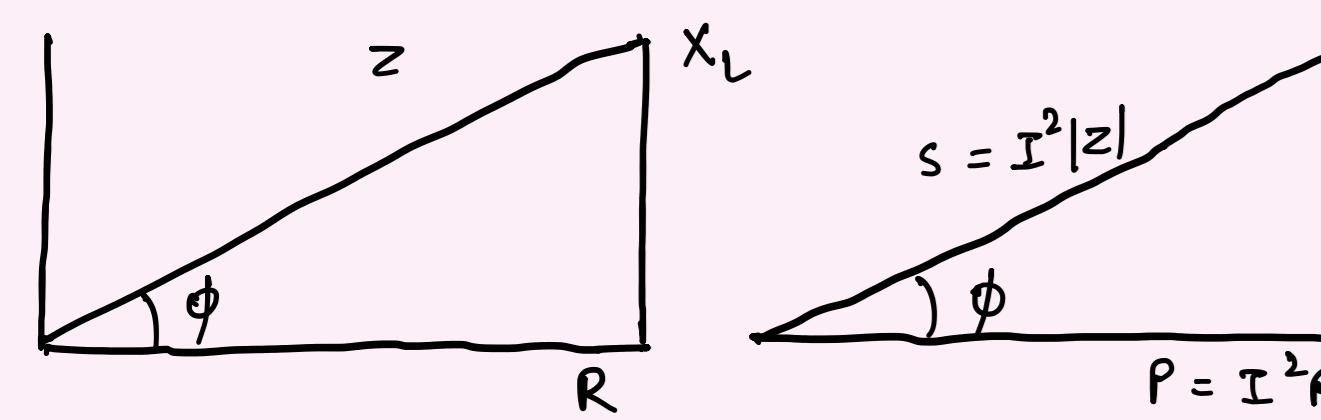
i) $X_L = X_C$



[Series Resonance]

RL Circuit

$$Z = R + jX_L = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$$

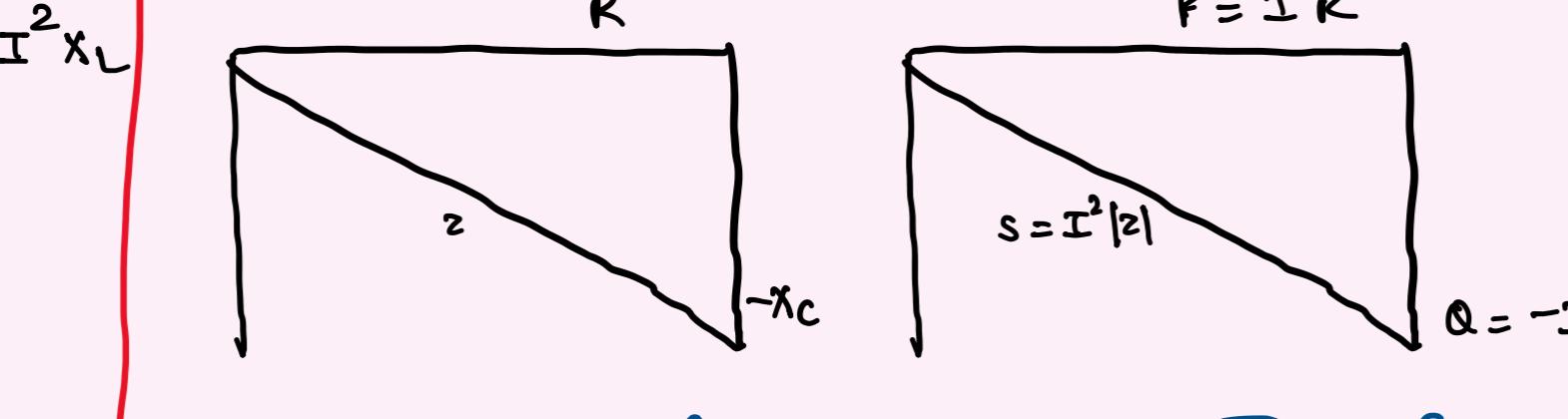


$$P = VI \cos \phi = I|z| \cdot \frac{IR}{|z|} = I^2 R$$

$$Q = VI \sin \phi = I|z| \cdot \frac{IX_L}{|z|} = I^2 X_L$$

$$S = VI = I|z| \cdot I = I^2 |z|$$

$$Z = R - jX_C = \sqrt{R^2 + X_C^2} \angle \tan^{-1}\left(\frac{X_C}{R}\right)$$



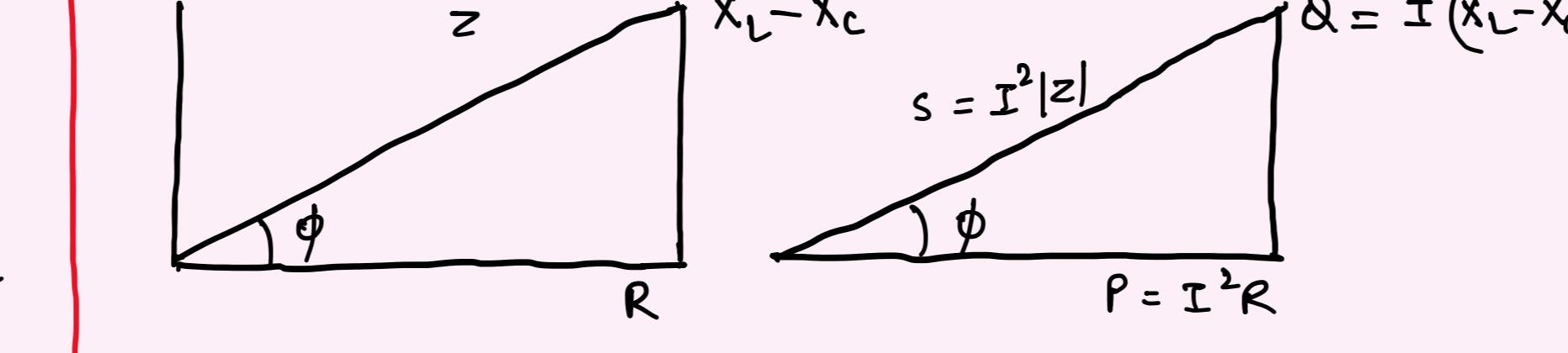
$$P = VI \cos \phi = I|z| \cdot \frac{IR}{|z|} = I^2 R$$

$$Q = VI \sin \phi = I|z| \cdot \frac{-IX_C}{|z|} = -I^2 X_C$$

$$S = VI = I|z| \cdot I = I^2 |z|$$

RLC Circuit ($X_L > X_C$)

$$Z = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$



$$P = VI \cos \phi = I|z| \cdot \frac{IR}{|z|} = I^2 R$$

$$Q = VI \sin \phi = I|z| \cdot \frac{IX_L}{|z|} = I^2 (X_L - X_C)$$

$$S = VI = I|z| \cdot I = I^2 |z|$$

Q. $P = 400W, V = 200V, f = 50Hz$

$R = 4\Omega$, Behaves like LR

i) PF ii) L if $C = 1mF$ iii) Extra C if resonance

$$A. P = I^2 R$$

$$\frac{400}{4} = I^2 \Rightarrow I = 10A$$

$$i) VI \cos \phi = 400 \Rightarrow \cos \phi = \frac{400}{200 \times 10} = \frac{1}{5} = 0.2 \text{ lag}$$

$$\phi = 78.46^\circ$$

$$ii) |z| = \frac{V \angle 78.46}{I} = 4 + 19.59j$$

$$X_L - X_C = 19.59$$

$$wl = 19.59 + \frac{10^3}{100\pi} = 0.0725 = 72.5mH$$

iii) $X_C = 0$

$$L = \frac{10^3}{100\pi \times 100\pi^2} = 0.01H = 10mH$$

Q. PF = 0.6 of coil, $C = 100\mu F$ if $f = 50Hz$

$V_L = V_C$, find R & L

$$A. \quad \boxed{R \text{---} L \text{---} C \text{---} R}$$

$$PF = \frac{R}{X_C} \Rightarrow 0.6 = \frac{R}{\frac{1}{wC}} \Rightarrow R = 0.6 \times \frac{10^6}{100\pi \times 100} = R = 19.09$$

$$V_L = V_C$$

$$|z| = |z_L| = |z_C|$$

$$\sqrt{R^2 + X_L^2} = X_C$$

$$X_L = (31.83)^2 - (19.09)^2$$

$$X_L = 25.47$$

$$L = \frac{25.47}{100\pi} = 81mH$$

Q. A voltage $v(t) = 100 \sin 314t$ is applied to a series circuit consisting of $0.0318H$ inductance and a capacitor of $63.6\mu F$. Find (i) expression for $i(t)$ (ii) phase angle between voltage and current (iii) power factor (iv) active power consumed

$$A. Z = \sqrt{R^2 + (wL - \frac{1}{wC})^2}$$

$$= \sqrt{100 + (0.0318 \times 100\pi - \frac{10^6}{63.6 \times 100\pi})^2}$$

$$= 41.28\Omega$$

$$i) \phi = -\cos^{-1}\left(\frac{R}{Z}\right) = -75.98$$

$$I(t) = 2.42 \sin(314t + 75.98)$$

$$ii) \phi = -75.98$$

$$iii) \text{PF} = 0.242 \text{ lead}$$

$$iv) P = 29.31 W$$

Q. An emf whose instantaneous value at time t is given by $283(\sin 100\pi t + \frac{\pi}{4})$ Volts is applied to an inductive circuit and the current in the circuit is $5.66(\sin 100\pi t - \frac{\pi}{6})$ Amperes. Determine (i) the frequency of the emf, (ii) the resistance and inductance of the circuit, (iii) the active power absorbed. If series capacitance is added so as to bring the circuit into resonance at this frequency and the above emf is applied to the resonant circuit, Find the corresponding expression for the instantaneous value of the current and also find the value of the series capacitance. Draw the phasor diagram representing the circuit before and at resonance.

$$A. V = 283 \sin(100\pi t + \frac{\pi}{4}) = 283 \sin(100\pi)$$

$$I = 5.66 \sin(100\pi t - \frac{\pi}{6}) = 5.66 \sin(100\pi t - \frac{\pi}{12})$$

$$i) f = 50Hz$$

$$ii) Z = \frac{V}{I} = 12.94 + 48.29j$$

$$X_L = 48.29\Omega$$

$$L = \frac{48.29}{100\pi} = 0.153 H = 153.7mH$$

$$iii) P = VI \cos \phi = \frac{283 \times 5.66 \times \cos 75}{2} = 207.28 W$$

$$iv) X_L = X_C \quad w^2 L = \frac{1}{C} \Rightarrow C = \frac{1}{w^2 L} = \frac{1}{100\pi^2 \times 48.29} = 65.9 \mu F$$

$$v) I = \frac{283}{12.94} \sin(100\pi t + \frac{\pi}{4}) \quad I_{max} = 15.46 A$$

$$= 21.87 \sin(100\pi t + \frac{\pi}{4}) \quad \bar{I} = \frac{15.46}{\sqrt{2}} \quad \bar{V} = \frac{283}{\sqrt{2}}$$

Q. $Q = 3k VAR, V = 500V, f = 50Hz$

PF = 0.8 lag

i) $R = ?$

ii) L if $C = 159.15\mu F$

iii) New PF if extra R of 10Ω added in series

A. $\cos \phi = 0.8$

$\phi = 38.86$

$VI \sin \phi = 3000$

$VI = 500 \times 1.5$

$I = 10A$

$|z| = \frac{500}{10} = 50\Omega$

$= 40 + j29.99$

i) $R = 40\Omega$

ii) $X_L - X_C = 29.99$

$wL = 29.99 + \frac{10^6}{100\pi \times 159.15}$

$= 49.99$

$L = 0.1591 = 159.1mH$

PF = 0.857 lag

$$\begin{aligned} & \text{iii)} R = 40 + 10 = 50\Omega \\ & X_L - X_C = 30.96 \\ & \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \\ & = 30.96 \\ & |z| = \frac{500}{10} = 50\Omega \\ & = 40 + j29.99 \end{aligned}$$

$$\begin{aligned} & i) R = 40\Omega \\ & ii) X_L - X_C = 29.99 \\ & wL = 29.99 + \frac{10^6}{100\pi \times 159.15} \\ & = 49.99 \\ & L = 0.1591 = 159.1mH \end{aligned}$$

Q. $P = 2kW$ connected across $200V, 50Hz$ single phase AC.

$R = 5\Omega$ & Behaves like RC circuit

i) PF ii) Q = ? iii) C if $V = 10mV$ iv) L if in resonance

A. $VI \cos \phi = 2kW$

$R = 5\Omega$

$$P = I^2 R \Rightarrow I^2 = \frac{2000}{5^2} \Rightarrow I = 20A$$

$$\cos \phi = \frac{2000}{20 \times 20} = \frac{1}{2} \Rightarrow \phi = 60^\circ$$

i) PF = 1/2 lead

ii) $Q = VI \sin \phi = 4000 \times \frac{\sqrt{3}}{2} = 2000\sqrt{3} = 3464.1 \text{ VAR}$

iii) $\bar{Z} = \frac{V \angle 60}{I} = 5 - 5\sqrt{3}i$

$X_L - X_C = -5\sqrt{3}$

$X_C = 11.8$

$$\frac{1}{wC} = 11.8 \Rightarrow C = \frac{1}{100\pi \times 11.8} = 269.7 \times 10^{-6}$$

$= 269.7 \mu F$

iv) $X_L' - X_C = 0$

$$wL' = \frac{1}{wC} \Rightarrow L' = \frac{1}{w^2 C} = \frac{10^6}{10^4 \pi^2 \times 269.7} = \frac{100}{\pi^2 \times 269.7} = 37mH$$

$\Delta L = L' - L = 37 - 10 = 27mH$

Q. A non-inductive resistor is connected in series with a coil and capacitor of $25.5\mu F$. The current in the circuit is $0.4A$ and the potential difference across the non-inductive resistor is $20V$, across the coil is $35V$, across the capacitor is $50V$ and across the combination of non-inductive resistor and coil is $45V$. Find the resistance and inductance of the coil. Also find the applied voltage, frequency and the power dissipated in the coil and the whole circuit.

A. $C = 25.5\mu F, I = 0.4A, V_R = 20V, V_L = 35V, V_C = 50V, V_{total} = 45V$

$R = \frac{20}{0.4} = 50\Omega = Z_R$

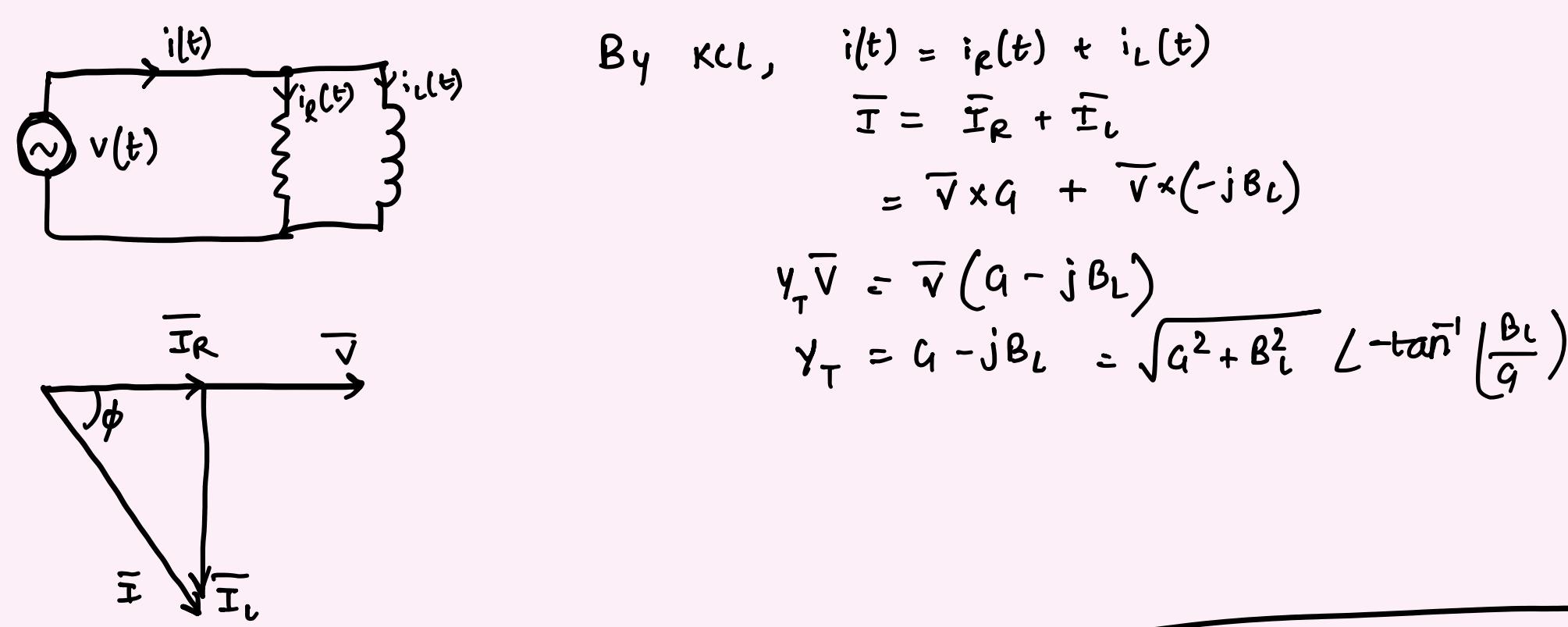
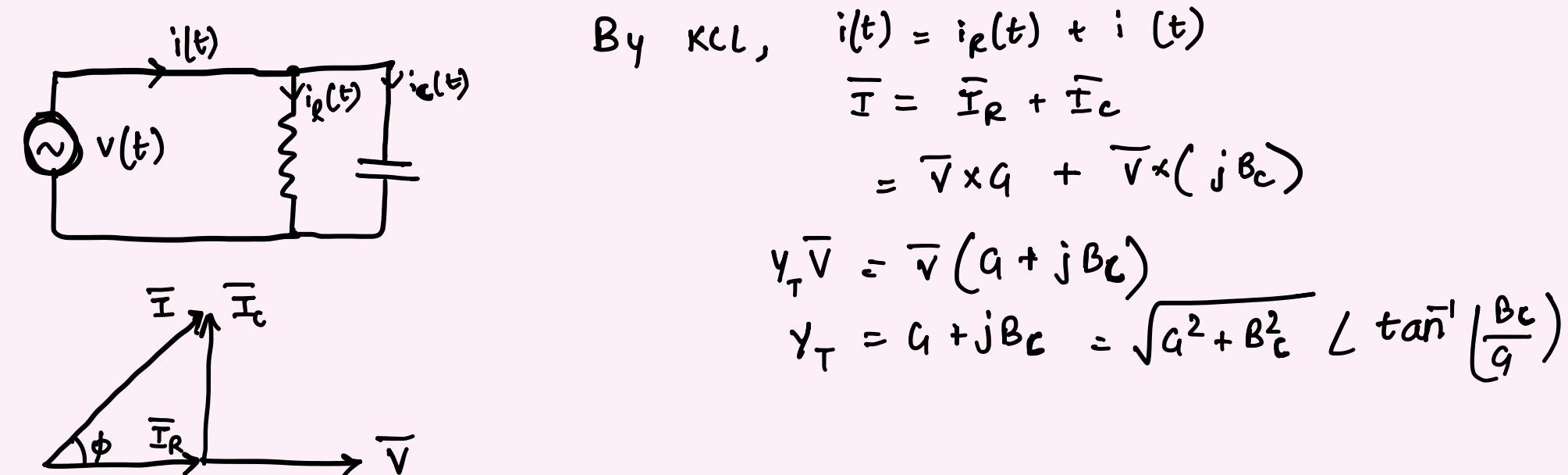
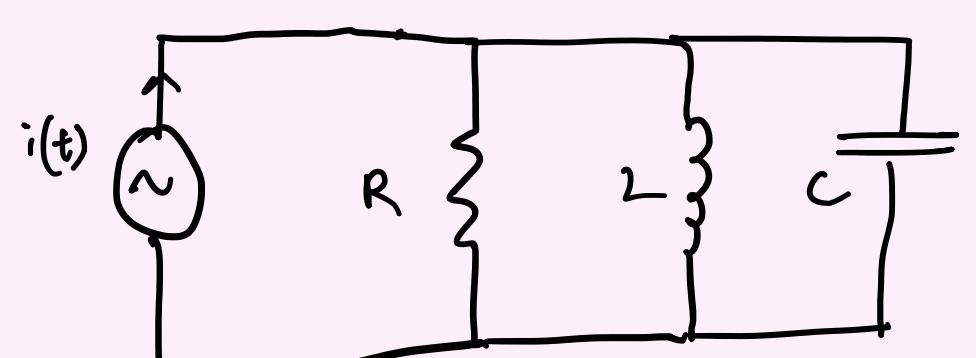
$$X_C = \frac{50}{0.4} = 1$$

Admittance

→ Reciprocal of Impedance

$$Y = \frac{1}{Z} \quad (\text{Siemens / Mho})$$

Element	Z	Y	Remarks
Resistor	R	$\frac{1}{R} = G$	G : conductance
Inductor	jX_L	$\frac{1}{jX_L} = -jB_L$	B_L : inductive susceptance
Conductor	$-jX_C$	$\frac{1}{jX_C} = jB_C$	B_C : capacitive susceptance

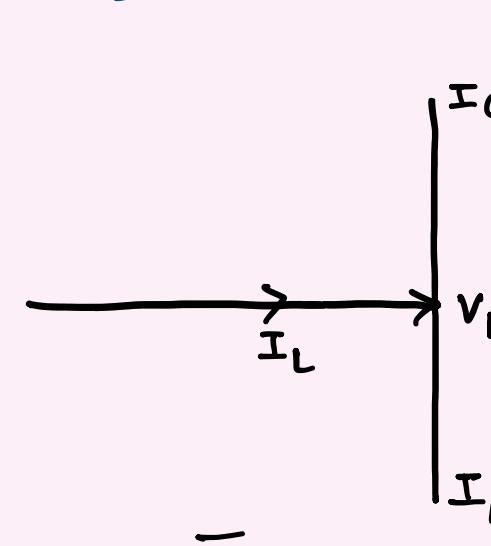
Parallel RL Circuit**Parallel RC Circuit****Parallel RLC Circuit**

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

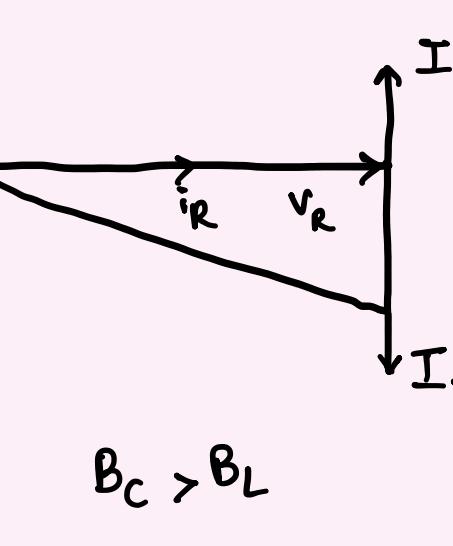
$$\bar{I} = \bar{V} / (G - jB_L + jB_C)$$

$$Y_T = \bar{V} / \bar{I} = G - jB_L + B_C = \sqrt{G^2 + (B_C - B_L)^2} \angle \tan^{-1}(B_C - B_L / G)$$

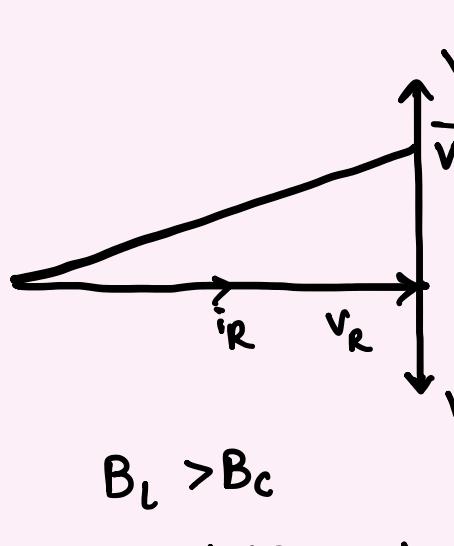
$$i) B_L = B_C$$



$$ii) B_L > B_C$$



$$iii) B_L < B_C$$



Q. The terminal voltage and current for a parallel circuit are $141.4 \sin 2000t$ V and $7.07 \sin(2000t + 36^\circ)$ A. Obtain the simplest two element parallel circuit, which would have the above relationship.

$$A. \bar{V}(t) = 141.4 \sin 2000t$$

$$I(t) = 7.07 \sin(2000t + 36^\circ)$$

$$\bar{V} = \frac{141.4}{\sqrt{2}} \angle 0^\circ$$

$$\bar{I} = \frac{7.07}{\sqrt{2}} \angle 36^\circ$$

$$Y = \frac{\bar{I}}{\bar{V}} = 0.05 \angle 36^\circ S = (0.04 + j0.029) S$$

$$G = 0.04 S ; B_C = 0.029 S$$

$$R = 25 \Omega ; X_C = 34.48 \Omega$$

$$X_C = \frac{1}{\omega C}$$

$$\Rightarrow C = \frac{1}{34.48 \times 2000} = 14.5 \mu F$$

Q. 3 impedances are connected in Π^d . 100V, 50Hz AC Supply where $Z_A = 10 \angle 10^\circ$, $Z_B = -j12.5$, $Z_C = j8$. Find Branch current & supply current. find PF

$$A. \bar{I} = \bar{V} (Y_A + Y_B + Y_C)$$

$$= \bar{V} \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C} \right)$$

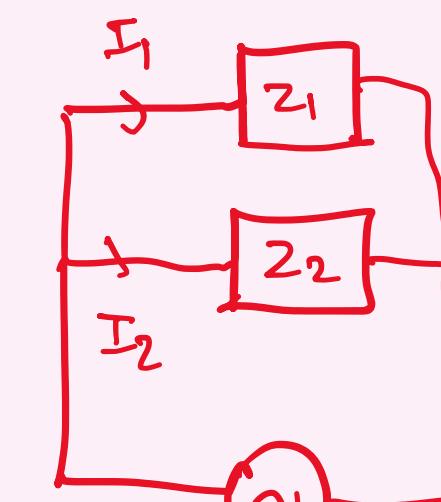
$$= 11.65 \angle -32.34^\circ$$

$$\phi = \bar{V} - \bar{I}Z = 0 - (-32.34) = 32.34^\circ$$

$$PF = \cos \phi = 0.84$$

Q. 2 impedances connected in Π^d $10 - j15$, $6 - j8$. If total current drawn in circuit is 20A. What is Power dissipated in each branch

A.



$$I = I_1 + I_2$$

$$I_1 = \frac{20 \times Z_2}{Z_1 + Z_2} = \frac{20 \times (6 - j8)}{16 - j23} = 7.13 + j0.25$$

$$I_2 = \frac{20 \times Z_1}{Z_1 + Z_2} = \frac{20 \times (10 - j15)}{16 - j23} = 12.87 - j0.25$$

$$P_1 = I_1^2 R = 7.13^2 \times 10 = 508.369$$

$$P_2 = I_2^2 R = 12.87^2 \times 6 = 993.82$$

Q. Parallel RL Circuit. $R = 4 \Omega$, $X_L = 3 \Omega$

Obtain series equivalent such that series circuit draws same current & power at given voltage

$$A. G = \frac{1}{R} = 0.25 S \quad |B_L| = \frac{1}{X_L} = 0.33 S$$

$$Y = \frac{\bar{I}}{\bar{V}} = \sqrt{G^2 + B_L^2} = 0.416$$

$$Y = G - jB_L$$

$$Y = 0.25 - j0.33 S$$

$$Z = \frac{1}{Y} = 1.45 + j1.92 \Omega$$

Q. $Y = 0.05 - j0.08 S$

Find R & X_L if they are in parallel & in series

$$A. i) G = 0.05, B_L = 0.08$$

$$R = \frac{1}{G} = 20 \Omega \quad X_L = \frac{1}{B_L} = 12.5 \Omega$$

$$ii) Y = 0.05 - j0.08$$

$$\frac{1}{Y} = Z = 5.617 + j8.98 \Omega$$

Q. 3 circuit elements $R = 2.5 \Omega$, $X_L = 4 \Omega$, $X_C = 10 \Omega$ are connected in Π^d , Reactance being at 50Hz

i) Determine admittance of each element & input admittance

ii) If circuit is connected across 10V, 50Hz, AC source, determine current in each branch & total input current

$$A. i) G = \frac{1}{R} = \frac{1}{2.5} = 0.4 S ; B_L = \frac{1}{X_L} = 0.25 S ; B_C = \frac{1}{X_C} = 0.1 S$$

$$Y = (G - jB_L + jB_C) = 0.4 - j0.25 + j0.1 = 0.4 - j0.15 S = 0.427 \angle -20.55^\circ$$

$$ii) I = V(Y) = 10(0.427 \angle -20.55^\circ)$$

$$I = 4.27 \angle -20.55^\circ$$

$$I_1 = VY_1 = 10 \times 0.4 = 4 A$$

$$I_2 = VY_2 = 10 \times (-0.25) = 2.5 \angle -90^\circ A$$

$$I_3 = VY_3 = 10 \times (0.1) = 1 \angle 90^\circ A$$

Q. Impedance of 2 element Π^d AC network is $6 + j8 \Omega$. Determine the elements if supply freq is 50Hz

$$A. Z = 6 + j8$$

$$Y = \frac{1}{Z} = \frac{1}{6 + j8}$$

$$Y = 0.1 \angle -53.13^\circ$$

$$Y = \frac{1}{q} - \frac{j}{B_L}$$

$$R = \frac{1}{q} = 16.67 \Omega$$

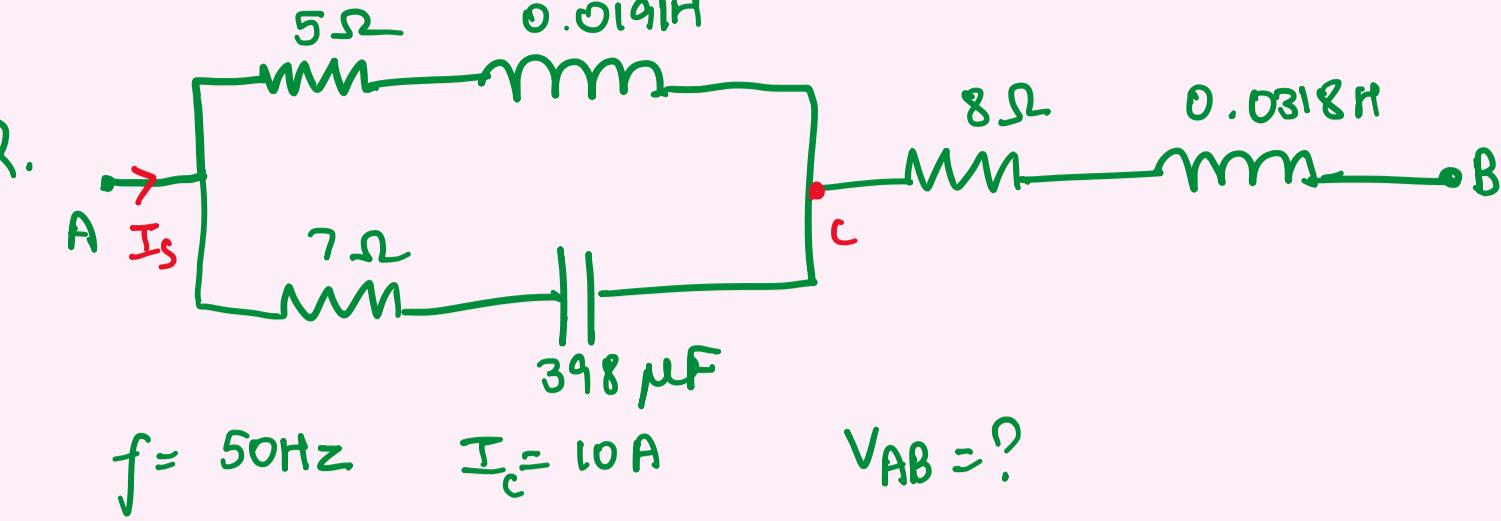
$$X_L = \frac{1}{B_L} = \frac{1}{0.08} = 12.5 \Omega$$

$$X_L = \omega L \Rightarrow L = \frac{12.5}{100\pi} = 39 mH$$

9. Analysis of series & parallel AC circuits

Series-Parallel AC Circuits

Solved using phasor method



$$A. Y_C = I_c(R - jX_C)$$

$$= 10 \left(7 - j \frac{10^6}{2\pi \times 398} \right)$$

$$= 10(7 - 8j)$$

$$V_{AC} = I_L(R + jX_C)$$

$$\frac{70 - 80j}{5 + (2\pi \times 50 \times 0.018\mu F)} = I_L$$

$$I_L = 13.6 \angle -99^\circ$$

$$I_T = I_C + I_L$$

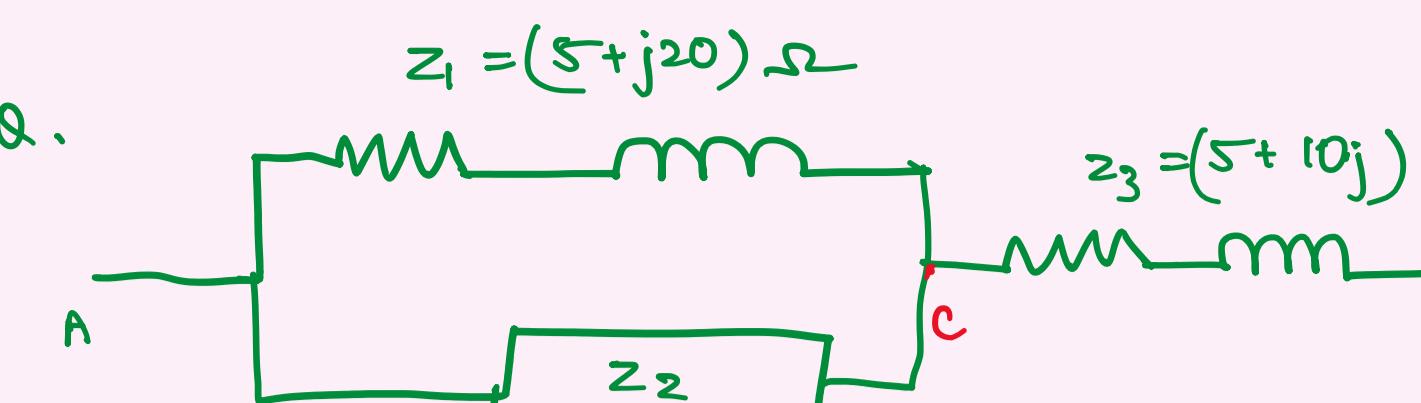
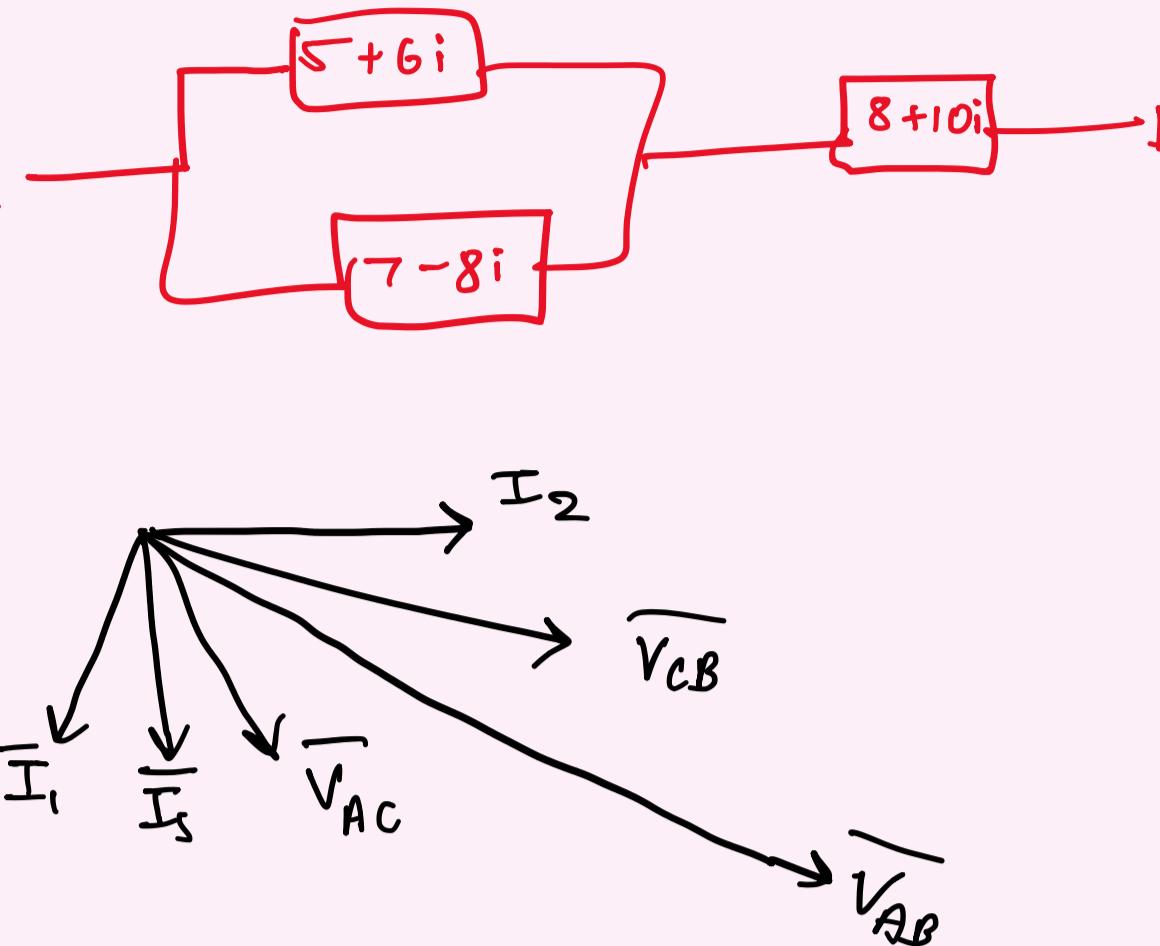
$$= 15.56 \angle -59.6^\circ$$

$$V_{CB} = I_T(8 + 10j)$$

$$V_{CB} = 199.38 \angle -8.28^\circ$$

$$V_{AB} = V_{AC} + V_{CB}$$

$$= 288.57 \angle -22.13^\circ$$



Q. $V = 220V$, $P_{total} = 30.25kW$, $I_{Total} = 20A$, lag
Find I_{Z_1} , I_{Z_2} & phasor diagram

$$A. V_{AB} = 220V$$

$$P = V_{AB} \times I_s \times \cos \phi$$

$$3250 = 220 \times 20 \times \cos \phi$$

$$\cos \phi = 0.74$$

$$\phi = 42.26^\circ$$

$$I_s = 20 \angle -42.26^\circ$$

$$V_{CB} = 20 \angle -42.26 \times (5 + 10j)$$

$$= 223.6 \angle 21.2^\circ$$

$$V_{AC} = V_{AB} - V_{CB}$$

$$= 81.67 \angle -81.9^\circ$$

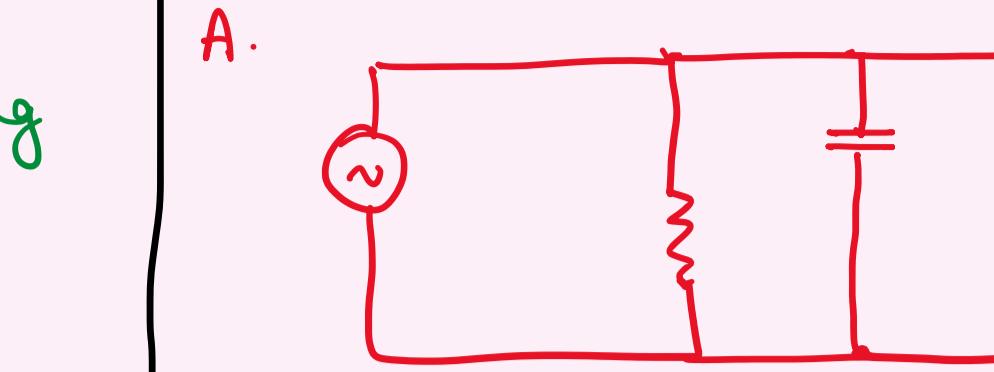
$$I_1 = \frac{V_{AC}}{Z_1} = 3.96 \angle -157.84^\circ$$

$$I_2 = I_T - I_1 = 22 \angle -32.9^\circ$$

$$Z_2 = \frac{V_{AC}}{I_2} = 3.7 \angle -48.9^\circ$$

$$= 2.44 - 2.79j$$

Q. $V = 200V$ applied across R , C & L with 100Ω τ
all are in Π^{st} . $I_T = 2.45A$, component currents
are $1.5, 2, 1.2A$ respectively. Find PF & PF of coil.
Also find P & Q .



$$PF = \cos \phi = \frac{R}{Z}$$

$$I_R = 1.5A$$

$$I_C = 2L0A$$

$$(Z_3) = \frac{V}{I_3} = \frac{200}{1.2} = 166.67$$

$$\cos \phi_L = \frac{\tau}{Z} = \frac{100}{166.67} = 0.6 \text{ lag}$$

$$\phi_L = 53.1^\circ$$

$$I_L = 1.2 \angle -53.1^\circ$$

$$I_s = I_R + I_C + I_L = 2.45 \angle 25.1^\circ$$

$$\Delta \phi = \Delta V - \Delta I = 0 - 25.1 = -25.1$$

$$PF = 0.9$$

$$P = VI \cos \phi$$

$$= 200 \times 2.45 \times \cos(-25.1) = 443.72 W$$

$$Q = VI \sin \phi$$

$$= 200 \times 2.45 \times \sin(-25.1) = -207.85 VAR$$

Q. Load connected across AC consists of heating load of $15kW$, motor load $40kVA$ at 0.6 lag & load of $20kW$ at 0.8 lag. Calculate total power drawn from supply (in kW & kVA) & PF. What would be kVAR rating of capacitor to bring PF to unity & how must capacitor be connected?

$$A. P_1 = 15000W$$

$$Q_1 = 0$$

$$S_1 = 15000VA$$

$$\cos \phi_2 = 0.6 \text{ lag}$$

$$P_T = P_1 + P_2 + P_3 = 59kW$$

$$Q_T = Q_1 + Q_2 + Q_3 = 47kVAR$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = 75.4kVA$$

To make overall PF = 1,
 $Q_{net} = 0$
Hence, capacitor of rating
 $47kVAR$ must be placed in
 Π^g to achieve this

Q. The load taken from AC supply consists of a heating load of $15kW$, a motor load of $40kVA$ at 0.6 lag and a load of $20kW$ at 0.8 lag. Calculate the load from supply in kW & kVA and its power factor. What would be the kVAR rating of a capacitor to bring the power factor to unity and how would the capacitor be connected.

$$A. \begin{array}{l|l|l} P_1 = 15kW & S_2 = 40kVA & P_3 = 20kW \\ Q_1 = 0 & \cos \phi = 0.6 \text{ lag} & \cos \phi = 0.8 \text{ lag} \\ S_1 = 15kVA & P_2 = 24kW & S_3 = 25kVA \\ \cos \phi = 1 & Q_2 = 32kVAR & Q_3 = 15kVAR \end{array}$$

$$P_T = P_1 + P_2 + P_3 = 59kW$$

$$Q_T = Q_1 + Q_2 + Q_3 = 47kVAR$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{59^2 + 47^2} = 75.43kVA$$

To bring PF to 1, Q_T must be = 0
Hence, Capacitor of $47kVAR$ rating in parallel to circuit needs to be connected

Q. The power consumed in the inductive load is $5kW$ at 0.6 lagging power factor. The input voltage is $230V$, $50Hz$. Find the value of the capacitor C which must be placed in parallel, such that the resultant power factor of the input current is 0.8 lagging.

$$P = VI \cos \phi$$

$$2500 = 230 \times I \times 0.71 \Rightarrow I = 15.309A$$

$$\phi = \cos^{-1}(0.71) = 44.76^\circ$$

$$I = 15.309 \angle -44.76^\circ$$

$$I_T \angle -30^\circ = I + I_C \angle 90^\circ$$

$$I_T \angle -30^\circ = 15.309 \angle -44.76^\circ + I_C \cos 90^\circ$$

$$I_T \cos 30^\circ = 15.309 \cos 44.76^\circ$$

$$I_T = 12.5A$$

$$I_C = I_T - I_s$$

$$= 12.55 \angle 30^\circ - 15.309 \angle -44.76^\circ$$

$$= 4.504 \angle 90^\circ$$

$$X_C = \frac{V_C}{I_C} = \frac{230}{4.504}$$

$$= 51.065 \Omega$$

$$C = \frac{1}{2\pi f X_C}$$

$$= 62.33 \mu F$$

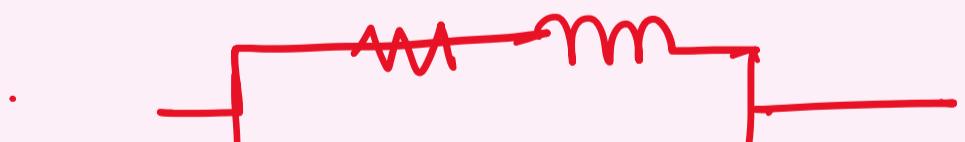
Q. AC circuit has 2 branches A & B in Π^{st} across $230V$, $50Hz$.

A has coil $L = 100mH$, $R = 10\Omega$

B takes load current from supply

Total power drawn from source is $1kW$ & $PF_{overall} = 0.6$ lag.

Determine R & C in branch B



$$X_L = \omega L = 100 \times 10^{-3} \times 100\pi = 31.4\Omega$$

$$R = 10\Omega$$

$$P = VI \cos \phi$$

$$1000 = 230 \times I \times 0.6$$

$$I = 7.24A$$

$$\phi = 53.1^\circ$$

$$I_s \Rightarrow 7.24 \angle -53.13^\circ$$

$$I_2 = \frac{V}{Z_L} = \frac{230}{10 + 31.4} = 6.98 \angle -72.3^\circ$$

$$I_s = I_1 + I_2$$

$$I_1 = I_s - I_2$$

$$= 2.38 \angle 21.1^\circ$$

$$Z_C = \frac{V}{I_1} = \frac{230}{2.38 \angle 21.1^\circ} = 90.18 - 34.78j\Omega$$

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{100\pi \times 34.78} = 9.2 \mu F$$

$$A. \begin{array}{l} P_L = 5000W \\ \cos \phi_L = 0.6 \\ V = 230V \\ f = 50Hz \\ C \text{ such that } \cos \phi_T = 0.8 \end{array}$$

$$P_L = 5000W$$

$$S_1 = \frac{5000}{0.6} = 8333.33VA$$

$$Q_1 = \sqrt{8333.33^2 - 5000^2} = 6666.66VAr$$

$$\phi_1 = \cos^{-1}(0.6) = 53^\circ$$

$$\phi_2 = \cos^{-1}(0.8) = 37^\circ$$

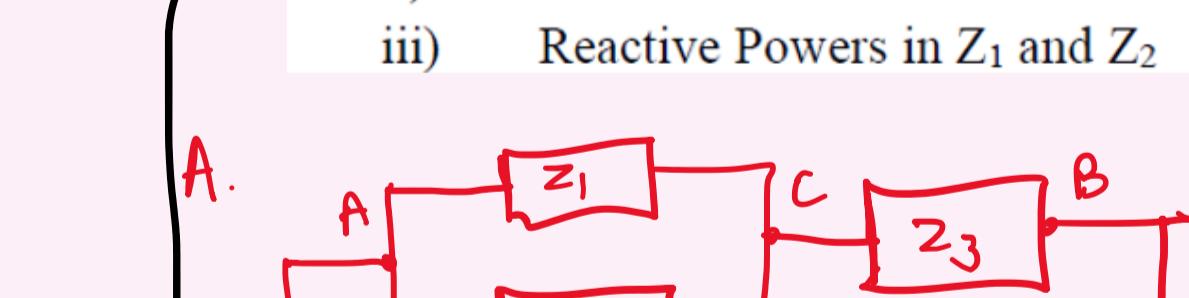
$$Q_2 = P_1 \tan \phi_2 = 5000 \tan 37^\circ = 3767.77VAr$$

$$Q_C = 6666.67 - 3767.77 = 2898.89VAr$$

$$Q_C = \frac{V^2}{X_C} \Rightarrow Q_C = \frac{V^2}{\omega^2 C} \Rightarrow C = \frac{V^2}{\omega^2 Q_C} = 175 \mu F$$

Q. A single-phase AC network consists of impedances Z_1 and Z_2 connected in parallel. This parallel combination is connected in series with another impedance Z_3 . If this network is connected across a $200V$, $50Hz$ AC supply & the supply current is $10A$ at a lagging power factor of 0.6 , determine

- Impedance Z_2 if $Z_1 = (15 + j20)\Omega$ & $Z_3 = (6 + j8)\Omega$
- Branch currents in Z_1 and Z_2
- Reactive Powers in Z_1 and Z_2



$$V = 200V$$

$$I = 10A$$

$$\cos \phi = 0.6 \text{ lag}$$

$$\phi = 53.1^\circ$$

$$I = 10 \angle -53.1^\circ$$

$$i) Z_1 = 15 + 20j$$

$$z_3 = 6 + 8j$$

$$V_{AB} = 200V$$

$$V_{AC} = V_{AB} - V_{BL}$$

$$= 100 \angle -0.03^\circ$$

$$I_{Z_1} = \frac{V_{AC}}{Z_1} = 4 \angle -53.1^\circ$$

$$I_{Z_2} = I - I_{Z_1} = 10 \angle -53.1^\circ - 4 \angle -53.1^\circ$$

$$= 6 \angle -53.1^\circ$$

$$Z_2 = \frac{V_{AC}}{I_{Z_2}} = \frac{100 \angle -0.03^\circ}{6 \angle -53.1^\circ} = 10.01 + 13.32j\Omega$$

$$ii) Z_2 = (10.01 + 13.32j)\Omega$$

10. Improvement of power factor

Importance of P.F

- I^2R losses is reduced
- Reactive power is reduced
- higher PF helps using full capacity of electrical system
- Improves the performance of motor
- PF improved by placing capacitor in 11^{th}
(or)
inductor in series