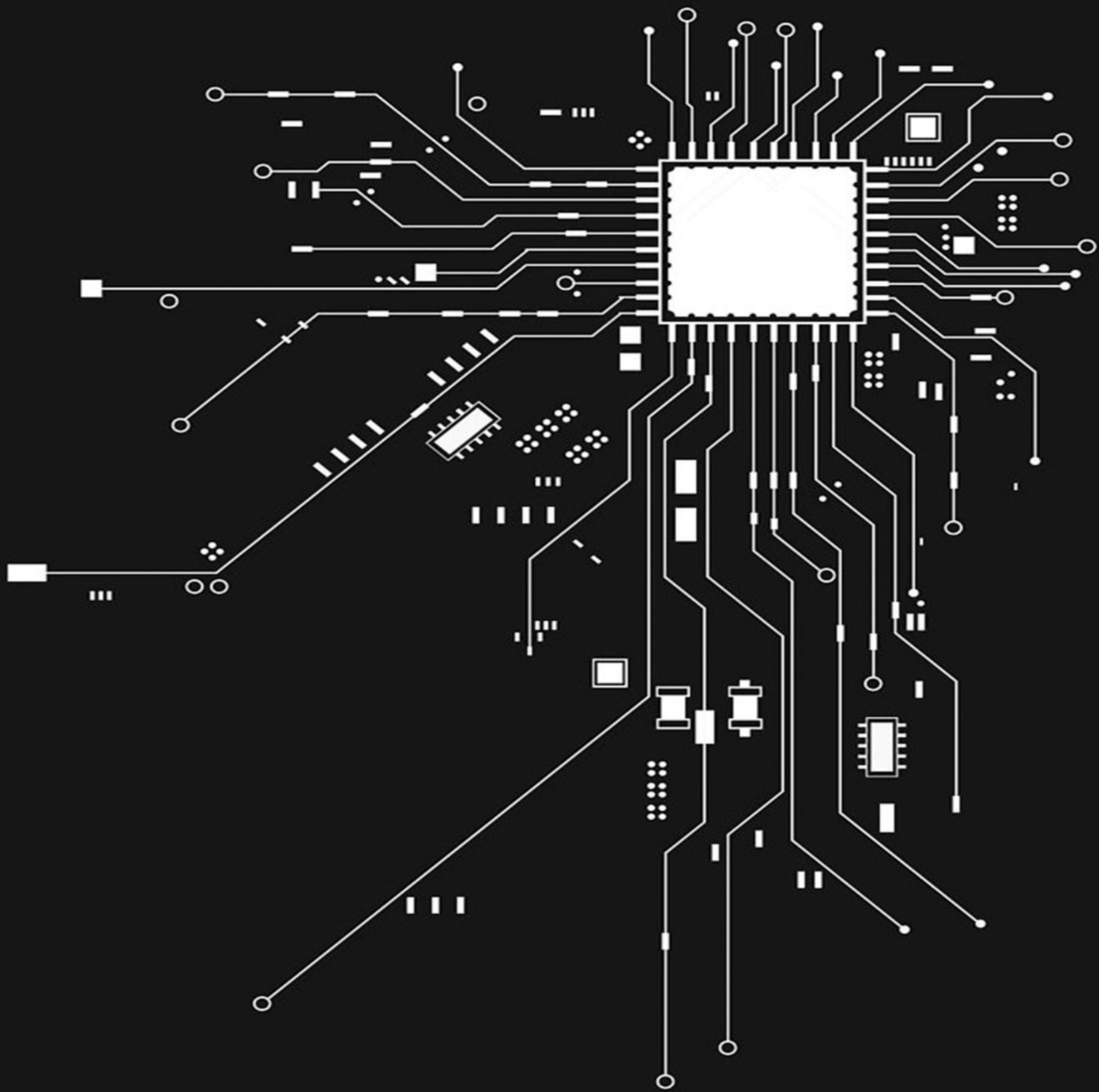
Nas- U4



-Shriya Arunkumar

Unit-4

- Causality
- Stability

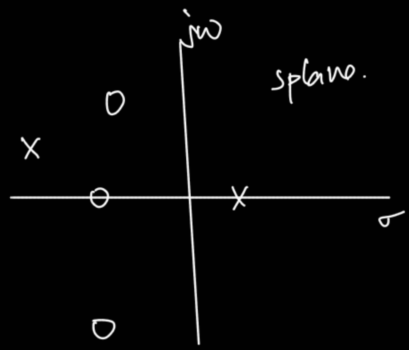
- Hurwitz Polynomials
  - ↳ Properties
  - ↳ Conditions to check
    - Positive real function
      - ↳ Conditions

- Elementary Synthesis Procedure
  - LC Immitance functions
    - Foster form
      - ↳ Form I
      - ↳ Form II
    - Cauer form
      - ↳ Form I
      - ↳ Form II
  - RC and RL functions

→ transfer functions → $H(s) = \dfrac{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots b_0}$
$\longrightarrow R(s) \rightarrow$ response
$\longrightarrow E(s) \rightarrow$ Excitation

synthesis is feasible only for causality and stability.

→ on the s plane → $s = \sigma + j\omega$



splane.

x → pole ↗ roots of denominator func

o → zero. ↳ roots of numerator func

Causality

↳ right sided signal

↳ no output without being excited

↳ response should be zero for $t<0$

$h(t) = u(n)$ = causal ← $e^{-at} u(t)$

$u(-n-1)$ → non causal ← $e^{-a|t|}$

→ Non causal responses can be modified to become causal by shifting the function (using delay)

→ modified signals are called realisable impulse response

Conditions to be realisable

1) $h(t)$ must have Fourier Transform

2) $$\int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega < \infty$$ should be integrable

3) Amplitude function must satisfy.

$$\int_{-\infty}^{\infty} \frac{|\log|H(j\omega)||}{1+\omega^2} d\omega < \infty$$ } Paley Wiener criterion

exception

$|H(j\omega)| = e^{-\omega^2}$

$|\log(H(j\omega))| = \omega^2$

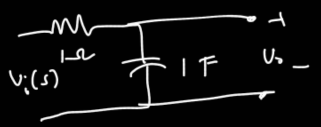
$$\int_{-\infty}^{\infty} \frac{\log(H(j\omega))}{1+\omega^2} = \int_{-\infty}^{\infty} \frac{\omega^2}{1+\omega^2} \rightarrow \text{not finite}$$

×

low pass filter

→ ex of funct satisfying Paley wiener condition

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$



Stability

BIBO stable ⇒ Bounded input provides a bounded output

denominator of a stable function must be a hurwitz polynomial

→ Conditions for Stability

1. $H(s)$ can't have poles in right half of Splane
2. $H(s)$ can't have multiple poles on $(j\omega)$ axis
3. Degree of nu ≤ degree of denom+1
4. final value theorem must hold good.

Properties of Hurwitz Polynomial

→ all coeffs are +ve
→ all intermediate powers of 's' should be present
↳ special cond → only even powers or odd powers
→ all roots should be on -ve side of s-plane (left side)
→ roots on ($j\omega$) axis are simple → shouldn't repeat
→ complex roots should be in pairs $1 \pm 2j$

Procedure to check if P(s) is Hurwitz

→ $P(s) = b_m s^m + b_{m-1} s^{m-1} + \dots b_1 s + b_0$ → denominator

1) Decompose P(s) to M(S) → even polynomial
and N(S) → odd polynomial

2) Synthetic division → $\frac{M(s)}{N(s)}$ or $\frac{N(s)}{M(s)}$

* If Quotient is +ve → P(s) is Hurwitz

* If P(s) is odd or even, → generate P'(s) and get even & odd part for P'(s)

Examples

1) $F(s) = s^4 + 3s^3 + 5s^2 + 3s + 4$ → check if Hurwitz

$F(-s) = s^4 - 3s^3 + 5s^2 - 3s + 4$

$M(s) = \frac{F(s) + F(-s)}{2} = s^4 + 5s^2 + 4$

$N(s) = 3s^3 + 3s$

$\psi(s) = \frac{M(s)}{n(s)} =$

$$
\begin{array}{l}
3s^3+3s\,\big)\, s^4 + 5s^2 + 4 \,\big(\, s \\
\quad -\cancel{s^4} \mp 3s^2 \\
\quad 2s^2+4\,\big)\, \cancel{s^3} + 3s \,\big(\, s/2 \\
\quad\quad -\cancel{s^3} \mp 2s \\
\quad\quad s\,\big)\, \cancel{2s^2}+4 \,\big(\, 2s \\
\quad\quad\quad -\cancel{2s^2} \mp \\
\quad\quad\quad 4\,\big)\, s \,\big(\, s/4 \\
\quad\quad\quad\quad -s \\
\quad\quad\quad\quad 0
\end{array}
$$

→ CFE continued factor expression

$$\psi(s) = \frac{m(s)}{n(s)} = s + \cfrac{1}{s/2 + \cfrac{1}{2s + \cfrac{1}{s/4}}}$$

all quotients are +ve ∴ F(s) is Hurwitz

2) $G(s) = s^3 + 2s^2 + 3s + 6$

$m(s) = 2s^2 + 6 \qquad n(s) = s^3 + 3s$

$$2s^2+6 \overline{\big) \cancel{s^3} + 3s} \big(s/2$$
$$-\cancel{s^3} \mp \cancel{3s}$$
$$\overline{\quad 0 \quad}$$

common factor b/w $m(s)$ & $n(s)$

$\psi(s) = \frac{s}{2}$ $\qquad G(s) = W(s) \times P(s)$

3) $F(s) = s^4 + s^3 + 2s^2 + 3s + 2$

$m(s) = s^4 + 2s^2 + 2 \qquad n(s) = s^3 + 3s$

$$s^3+3s \overline{\big) \cancel{s^4} + 2s^2 + 2} \big(s$$
$$-\cancel{s^4} - 3s^2$$
$$-s^2+2 \overline{\big) \cancel{s^3} + 3s} \big(-s$$
$$\mp \cancel{s^3} \pm 2s$$
$$5s \overline{\big) -\cancel{s^2} + 2} \big(-s/5$$
$$+\cancel{s^2}$$
$$2 \overline{\big) s/5} \big(\tfrac{5}{2}s$$

$$\psi(s) = s + \cfrac{1}{-s + \cfrac{1}{-\frac{s}{5} + \cfrac{1}{\frac{5}{2}s}}}$$

$\rightarrow$ negative quotients $\therefore$ Not Hurwitz

try $F(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$

$G(s) = 6s^4 + 2s^2 \rightarrow$ express as CFE

Positive Real Function

$\rightarrow$ Used to represent impedance or admittance

Conditions for Positive, Real function

① $F(s)$ is real for real s
i.e $s = \sigma$

② $\mathrm{Re}\{F(s)\} \geq 0$

③ $F(s) \in$ Rational

Ex

$F(s) = Ls \qquad L > 0$

$F(s) = R$

$F(s) = \frac{1}{sC} \qquad C > 0$

} Impedance

$Y(s) = \frac{K}{s}$

$Y(s) = K$

$Y(s) = Ks$

} admittance

Properties of PR function.

- Reciprocal of PR func = PRfunc
  - Y(s) & Z(s) are PR
- Sum of PR = PR
- Difference of PR ≠ PR
- PR should be Hurwitz but Hurwitz need not be PR
- $\frac{a s^m + a_n s^{m-1} \cdots a_1 s}{b s^n + b s^{n-1} \cdots b_1 s + b_0}$ $m-n=1$ } highest degree difference ≤ 1; lowest degree difference ≤ 1

Conditions for checking Hurwitz is PR

Hurwitz can be PR if

1. F(s) → no poles on right half of s-plane
2. F(s) → simple poles on (jw) axis with real & +ve Residues.
3. $Re\{F(j\omega)\} \geq 0 \quad \omega \geq 0$
   - $F(s) = \frac{P(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$ → even poly, → odd

$$F(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)} \times \frac{M_2(s) - N_2(s)}{M_2(s) - N_2(s)}$$

$$Re\{F(s)\} = \frac{M_1(s) M_2(s) - N_1(s) N_2(s)}{M_2^2(s) - N_2^2(s)}$$

$s = j\omega$

$$Re\{F(j\omega)\} = M_1(j\omega) M_2(j\omega) - N_1(j\omega) N_2(j\omega)$$

$F(s) = \frac{s+a}{s^2+bs+c}$ → $(b \geq a, \ a,b,c \geq 0)$ → should be satisfied

$F(s) = \frac{s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$ → $a_1 b_1 \geq (\sqrt{a_0} - \sqrt{b_0})^2$

ex(1) Check if $F_1(s) = \frac{s+2}{s^2+3s+2}$ is PR

$F(s) = \frac{s+a}{s^2+bs+c}$ → $b \geq a, \quad a,b,c \geq 0$

$a = 4 \quad b = 3 \quad c = 2 \qquad 3 > 2 \ \therefore$ Is PR

* first check if roots are on left side of plane

ex2 $F_2(s) = \frac{s+1}{s^2+2}$

$a = 1 \quad b = 0 \quad c = 2$

$0 < 1 \quad \therefore$ Not PR

$b < a$

ex3 $F_3(s) = \dfrac{s+4}{s^2+2s+1}$

$a=4 \quad b=2 \quad c=1$ Not PR

ex4 $F(s) = \dfrac{s^2+2s+25}{s^2+5s+16}$

$a_1 b_1 \geq (\sqrt{a_0} - \sqrt{b_0})^2$

$a_1 = 2 \quad a_0 = 25 \quad 10 \geq (5-4)^2$

$b_1 = 5 \quad b_0 = 16 \quad 10 \geq 1$

$\therefore$ is PR

ex5 $F(s) = \dfrac{2s^3+2s^2+3s+2}{s^2+1}$

$M_1(s) = 2s^2+2 \quad N_1(s) = 2s^3+3s$

$$2s^2+2 \,\big)\, 2s^3+3s \,\big(\, s$$
$$-2s^3 - 2s$$
$$s \,\big)\, 2s^2+2 \,\big(\, 2s$$
$$-2s^2$$
$$2 \,\big)\, s \,\big(\, s/2$$
$$-s$$
$$0$$

$\psi(s)$ is Hurwitz

(s^2+1)

$s = \sqrt{-1}$

$s = \pm i$

Elementary Synthesis Procedure

$\hookrightarrow$ impedance function which is PR can be decompose to elements $Z_i(s)$ and those are basically $\rightarrow$ RLC circuits

$\rightarrow$ can be done for admittance parameters also

Analysis

$$Z(s) = \frac{P(s)}{Q(s)} = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}$$

CRITERION-1

$\hookrightarrow$ when $b_0 = 0 \Rightarrow$ there is a pole at $z=0$.

$$Z(s) = Z_1(s) + Z_2(s)$$
$$= \frac{K}{s} + Z_2(s) \Big\} \text{Partial fraction}$$

then you synthesize a circuit for it

CRITERION-2

$\hookrightarrow n-m=1 \Rightarrow$ pole at $s=\infty$

$$Z(s) = Z_1(s) + Z_2(s) = \underbrace{Ls + Z_2(s)}$$

Remove pole by synthetic division $\Rightarrow \dfrac{P(s)}{Q(s)} = Ks + \dfrac{1}{1+\_\_}$

CRITERION 3

↳ $z(s) \to$ poles are complex conjugates

$$z(s) = \frac{2ks}{s^2+\omega_1^2} + Z_2(s)$$

CRITERION 4

↳ $\min(\mathrm{Re}\{Z(j\omega)\}) =$

ex1 $$\frac{7s+2}{2s+4} = \frac{2(4)-7s(2s)}{16-(2s)^2} = \left(\frac{8-14s^2}{16-4s^2}\right) = \frac{7s^2-4}{2s^2-8}$$

ex2 $$\frac{(3s^2+1)(0)-(6s^3+3s)(6s^3+3s)}{0^2-(6s^2+3s)^2} = 1$$

ex1

$$z(s) = \frac{s^2+2s+6}{s(s+3)}$$

criteria 1 (pole at $s=0$)

$$\frac{s^2+2s+6}{s(s+3)} = \frac{A}{s}+\frac{B}{s+3}$$

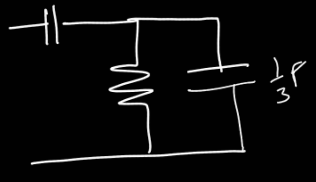
$A = 2$

$B = -3$

$$Z_2(s) = Z(s) - \frac{2}{s} = \frac{s^2+2s+6}{s(s+3)} - \frac{2(s+3)}{s(s+3)}$$

$$Z_2(s) = \frac{s^{\cancel{2}}}{\cancel{s}(s+3)} = \frac{s}{s+3}$$

$$z(s) = \frac{2}{s}+\frac{s}{(s+3)} = \frac{2}{s} + \frac{1}{1+\left(\frac{3}{s}\right)}$$



ex2 $$z(s) = \frac{6s^3+3s^2+3s+1}{6s^3+3s}$$

CR 4

$$z(s) = \min(\mathrm{Re}\{F(s)\}) + Z_2(s)$$

$$\min(\mathrm{Re}\{F(e)\}) = \frac{M_1M_2 - N_1N_2}{M_2^2-N_2^2} = \frac{6s^2+1(0) - (6s^3+\cancel{3s})(6s^3+3s)}{0^2-(6s^3+3s)^{\cancel{2}}}$$

$= 1$

$$z(s) = 1 + Z_2(s)$$

$$Z_2(s) = \frac{6s^3+3s^2+3s+1}{6s^3+3s} - 1$$

$$= \frac{3s^2+1}{6s^3+3s} = \frac{1}{Y(s)}$$

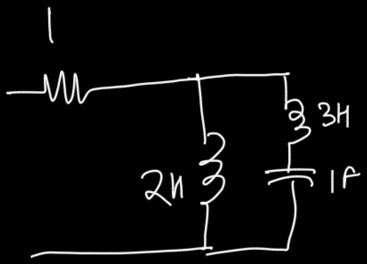
$$Y_2(s) = \frac{6s^3+3s}{3s^2+1}$$ $n-m=1 \Rightarrow s=\infty$ (pole)

$\therefore$ criteria 2

$$3s^2+1 \,)\, \cancel{6s^3}+3s \,(\, 2s$$
$$\cancel{6s^3} - 2s$$
$$s$$

$$\Rightarrow Y_2(s) = 2s + \frac{s}{3s^2+1}$$

$$Z(s) = 1 + \cfrac{1}{2s + \cfrac{1}{3s + 1/s}} \rightarrow$$



Network Synthesis

LC immittance

admittance or impedance

→ LC one port N/W → dissipation less

→ LC N/W aka → reactive networks

$$z(s) = \frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$$

$$P_{avg} = \frac{1}{2} Re\left[z(j\omega)\right] |I|^2$$

for reactive N/w → $P=0$

$$R\left[z(j\omega)\right] = 0 = Ev\left[z(j\omega)\right]$$

where $Ev\left[z(s)\right] = \frac{M_1(s).M_2(s) - N_1(s)N_2(s)}{M_2^2(s)-N_2^2(s)} = 0$

$$\Rightarrow M_1(j\omega).M_2(j\omega) - N_1(j\omega).N_2(j\omega) = 0$$

Cases

ⓐ $M_1 = 0 = N_2 \Longrightarrow z(s) = \frac{N_1(s)}{M_2(s)}$

ⓑ $M_2 = 0 = N_1 \Rightarrow z(s) = \frac{M_1(s)}{N_2(s)}$

∴ we can conclude → LC immittance → ratio of odd to even polynomials or even to odd polynomials

→ E, odd → Both are Hurwitz ⇒ poles, zeros are on Im axis

Properties

1. ratio of $\frac{\text{odd}}{\text{even}}$ or $\frac{\text{even}}{\text{odd}}$ polynomials
2. poles & zero → on Im $(j\omega)$ axis and are simple
3. all co-efficients in the function must be real and positive
4. highest and lowest powers of numerator & denominator must differ by 1
5. If polynomial is even → all even powers; if odd → all odd powers
6. $Re[Z(j\omega)] = 0$ for $\omega \geq 0$
7. as all poles are imaginary, ⇒ residues should be real & +ve
8. LC impedance / admittance can be give by

$$Z(s) = \frac{K(s^2+\omega_1^2)(s^2+\omega_2^2)}{s(s^2+\omega_3^2)(s^2+\omega_4^2)} \ldots\ldots$$

complex conjugates + pole at 0 and ∞.

Partial fractions

$$Z(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2+\omega_2^2} + \frac{2K_4 s}{s^2+\omega_4^2} + \ldots\ldots + K_\infty s$$

all poles are on Im axis

as $s = j\omega$, $Re\{F(j\omega)\} = 0$

$$Z(j\omega) = j\left[-\frac{K_0}{\omega} + \frac{2K_2\omega}{\omega_2^2-\omega^2} + \frac{2K_4\omega}{\omega_4^2-\omega^2} + \ldots + K_\infty\omega\right]$$

$$= jX(\omega)$$ → pure reactance

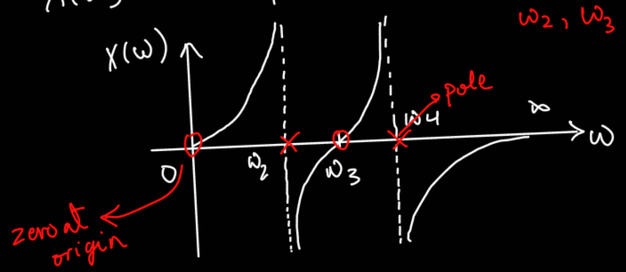
since residues are +ve ⇒ $\frac{dX(\omega)}{d\omega} \geq 0$

9. $$Z(s) = \frac{Ks(s^2+\omega_3^2)}{(s^2+\omega_2^2)(s^2+\omega_4^2)} + \ldots\ldots$$

$s = j\omega$

$$Z(j\omega) = jX(\omega) = j\frac{K\omega(-\omega^2+\omega_3^2)}{(-\omega^2+\omega_2^2)(-\omega^2+\omega_4^2)} \ldots\ldots$$

$X(\omega)$ can be plotted as



$0, \infty$ → external critical frequencies

$\omega_2, \omega_3, \omega_4$ → internal critical frequencies

* poles and zeros alternate

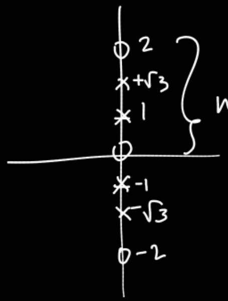
Numerical

1) State if these are driving point immittance of LC N/ws

$$z(s) = \frac{5s(s^2+4)}{(s^2+1)(s^2+3)}$$

poles $= \pm 1, \pm\sqrt{3}$

zeros $= \pm 2, 0$



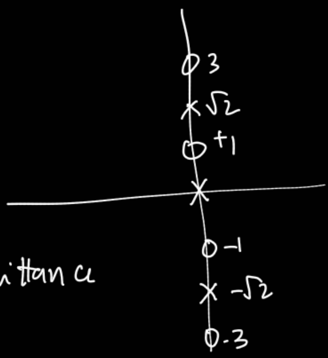
not alternating

$\therefore$ NOT LC immittance

2) $$z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+2)}$$

zeros $= \pm 1, \pm 3$

poles $= 0, \pm\sqrt{2} = 1.414$



alternating, on Im axis $\Rightarrow$ LC immittance func

Synthesis of LC immitance

→ Foster form (Partial Fraction Expansion

→ Cauer form (Continuous Fraction Expansion)

Foster Realisation

↳ advantage → all poles can be removed simultaneously

$$F(s) = \frac{H(s^2+\omega_1^2)(s^2+\omega_3^2)}{s(s^2+\omega_2^2)(s^2+\omega_4^2)} \cdots$$

by PFE

conjugates

$$F(s) = \frac{K_0}{s} + \frac{K_2}{s+j\omega_2} + \frac{K_2^*}{s-j\omega_2} + \cdots + K_\infty s$$

on combining these two take $K = K^*$

$$F(s) = \frac{K_0}{s} + \frac{2K_2}{s^2+\omega_2^2} + \cdots K_\infty s$$

$K_0, K_1, K_2 \ldots K_i, K_\infty$ are residues of $F(s)$ at $0, 1, 2, \ldots j\omega_i$ and ∞ respectively

Residues are → $K_0 = sF(s)\Big|_{s=0}$

$$K_i = \frac{(s^2+\omega_i^2)F(s)}{2s}\Bigg|_{s^2=-\omega_i^2}$$

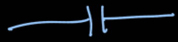
$$K_\infty = \frac{F(s)}{s}\Bigg|_{s\to\infty}$$

Foster I Form → impedance func

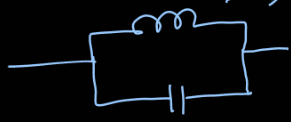
II form → Admittance func

forster I form

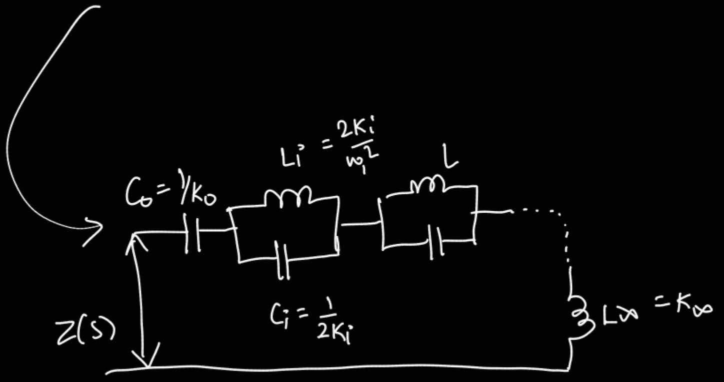
* $F(s) = Z(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2+\omega_2^2} + \dots K_\infty s$
$= Z_1(s) + Z_2(s) + \dots Z_n(s)$ } When summed up → series connection

$\frac{K_0}{s}$ → Capacitor of $\frac{1}{K_0}$ F  $1/K_0 = C_0$

$K_\infty s$ → inductor of K_∞ H  $K_\infty = L_\infty$

$\frac{2K_i s}{s^2+\omega_i^2}$ → L||C →  → $\frac{2K_i}{\omega_i^2} = L_i$; $1/2K_i = C_i$

$\frac{\left(\frac{1}{C_i}\right)s}{s^2 + \frac{1}{L_i C_i}}$



* Pole at origin → C_0
* pole at ∞ → L_∞

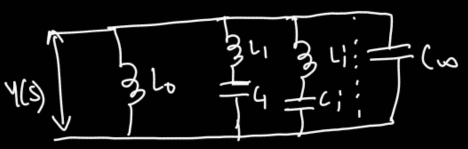
forster II form

$F(s) = Y(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2+\omega_2^2} + \dots + K_\infty s$
$= Y_1(s) + Y_2(s) + \dots + Y_n(s)$ } each term represents impedance
When added → gives parallel connection of each term

$\frac{K_0}{s}$ →  $\frac{1}{K_0}$ H

K_∞ →  K_∞ F

$\frac{2K_i s}{s^2+\omega_i^2}$ →  (series)
$L_i = \frac{1}{2K_i}$ $\frac{2K_i}{\omega_i^2} = C_i$



Cauer form I

↳ successive removal of pole/zero at $s=\infty$

↳ first you have $Z(s)$ → impedance func.

by removing a pole at ∞,

$Z_2(s) = Z(s) - L_1 s$ } criterion 2

after removing the pole at ∞, $Z_2(s)$ has a zero at $s=\infty$

Now invert $Z_2(s)$ to get $Y_2(s)$; $Y_2(s)$ has a pole at ∞.

$Y_3(s) = Y_2(s) - C_2 s$ } criterion 2 for removing pole at ∞

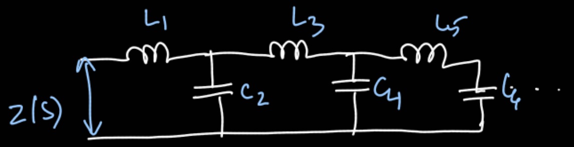
$Y_3(s) \to$ zero at $s=\infty$.

↳ $\frac{1}{Y_3(s)} = Z_3(s) \to$ pole at $s=\infty$

for impedance func like $Z_2(s), Z_4(s) \to$ when pole is removed its replaced by inductor
for admittance func $\to Y_3(s) \to$ when is removed $\to$ put Capacitor

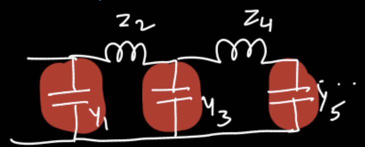
$$Z(s) = L_1 s + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s} \cdots}}$$

Represented in diagram



} Ladder network

* If $Z(s) = \frac{P(s)}{Q(s)} \to$ if degree of num − deg of den $= 1 \Rightarrow$ zero at ∞
↳ when this is satisfied invert $Z(s)$ and continue with $\underline{\underline{Y(s)}}$
↳ when this is given first element is a capacitor ↳ it is removed in admittance func.

↳ 

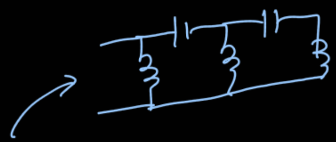
Cauer form II

↳ removal of pole origin.
↳ the lowest deg of Num − lowest deg of denominator $=1$

↳ $\frac{s(s+1)}{s^2+2s+4} \to$ zero at $s=0$

↳ so invert it $\frac{s^2+2s+4}{s(s+1)} \Rightarrow$ pole at $s=0$
↓
remove the pole by division method.
↓
Keep repeating.

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{L_2 s} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cdots}}}$$



* if $Z(s)$ has zero at origin $\to$ then first element is is ─ᴦᴦᴦ─ and the cap.. –
* if $Z(s)$ has pole at origin $\to$ first element is capacitor

ex ① Realise the foster & Cauer forms $\to Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)}$

↳ it is an LC impedance func $\to$ poles or zero at $0, \pm j, \pm j2, \pm j3$

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)}$$

* Numerator degree is greater than deno.
∴ do division first

$$= 4(s^4 + 10s^2 + 9) / s^3 + 4s$$

$$= \frac{4s^4 + 40s^2 + 36}{s^3 + 4s}$$

FOSTER form I

$$s^3+4s \,\overline{)\, 4s^4 + 40s^2 + 36 \,}\; 4s$$
$$\cancel{-4s^4} - 16s^2$$
$$24s^2 + 36$$

$$= 4s + \frac{24s^2+36}{s^3+4s} = 4s + \frac{24s^2+36}{s(s^2+4)}$$ * Now apply PFE

$$= 4s + \frac{K_0}{s} + \frac{K_1}{s+j2} + \frac{K_1^*}{s-j2}$$

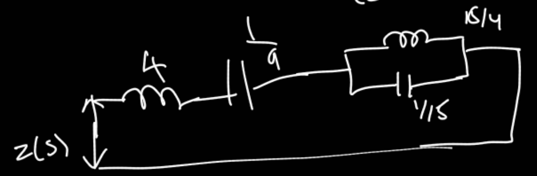
$$= 4s + \frac{K_0}{s} + \frac{2K_1 s}{(s^2+4)}$$

$$K_0 = sZ(s)\Big|_{z=0} = \frac{4(s^2+1)(s^2+9)\cancel{(s)}}{\cancel{s}(s^2+4)} = \frac{\cancel{4}(1)(9)}{\cancel{4}} = \underline{\underline{9}}$$

$$K_1 = \frac{(s^2+4)Z(s)}{2s}\Big|_{s^2=-4} = \frac{\cancel{(s^2+4)}(\overset{2}{\cancel{4}})(s^2+1)(s^2+9)}{s\cancel{(s^2+4)}\cancel{(2s)}} = \frac{\cancel{2}(-3)(+5)}{\cancel{-4}\;2} = \underline{\underline{\frac{15}{2}}}$$

$$Z(s) = 4s + \frac{9}{s} + \frac{15s}{(s^2+4)}$$

$\frac{1}{2K_i} = C$ $\quad \frac{2K_i}{\omega_i^2} =$



exp

$$Y(s) = \frac{s(s^2+4)}{4(s^2+1)(s^2+9)}$$

FOSTER form II

$$\frac{s(s^2+4)}{4(s^2+1)(s^2+9)} =$$

By PFE

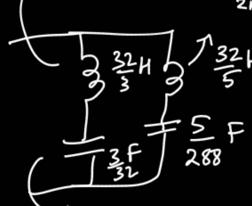
$$Y(s) = \frac{2K_1 s}{s^2+1} + \frac{2K_2 s}{s^2+9}$$

$$K_1 = \frac{(s^2+1)Y(s)}{2s}\Big|_{s^2=-1} = \frac{\cancel{(s^2+1)}}{2\cancel{s}} \cdot \frac{\cancel{s}(s^2+4)}{4\cancel{(s^2+1)}(s^2+9)} = \frac{1}{8}\frac{(3)}{(8)} = \frac{3}{64}$$

$$K_2 = \frac{(s^2+9)Y(s)}{2s}\Big|_{s^2=-9} = \frac{\cancel{(s^2+9)}\,\cancel{s}\cdot s^2+4}{2\cancel{s}\cdot 4(s^2+1)\cancel{(s^2+9)}} = \frac{1}{8}\frac{(-5)}{(-8)} = \frac{5}{64}$$

=

$L = \frac{1}{2K_i} = \frac{\cancel{64}^{32}}{\cancel{2}(3)}$ $\quad L = \frac{1}{2K_i} = \frac{64}{2(5)}$



$\frac{2K_i}{\omega_1^2} = \frac{\cancel{2}(3)}{\cancel{64}_{32}(1)}$ $\quad \frac{\cancel{2}(5)}{\cancel{64}_{32}(9)}$

ex3 $Z(s) = \dfrac{4s^4 + 40s^2 + 36}{s^3 + 4s}$

* deg of Num > deg dem by 1 ⇒ pole at ∞

Do CFE

CAUER form 1

$s^3+4s \,)\, 4s^4 + 40s^2 + 36 \,(\, 4s \Leftarrow Z(s)$

$-\cancel{4s^4} - 16s^2$

$24s^2 + 36 \,)\, s^3 + 4s \,(\, s/24 \leftarrow Y(s)$

$-\cancel{s^3} - \frac{3}{2}s$

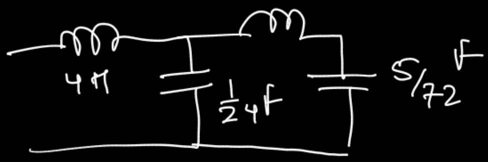
$\frac{5}{2}s \,)\, 24s^2 + 36 \,(\, \frac{48}{5}s \leftarrow$

$-24s^2$

$36 \,)\, \frac{5}{2}s \,(\, \frac{5s}{72}$

$-\frac{5}{2}s$

0



ex4 $Z(s) = \dfrac{4s^4 + 40s^2 + 36}{s^3 + 4s}$

CAUER- form II

* Arrange numerator & denominator in ascending order of power.

$Z(s) = \dfrac{36 + 40s^2 + 4s^4}{4s + s^3}$

*Do CFE

$4s + s^3 \,)\, 36 + 40s^2 + 4s^4 \,(\, \frac{9}{s} \leftarrow Z(s)$

$-\cancel{36} - 9s^2$

$31s^2 + 4s^4 \,)\, 4s + s^3 \,(\, \frac{4}{31s} \leftarrow Y_2(s)$

$-\cancel{4s} - \frac{16}{31}s^3$

$\frac{15}{31}s^3 \,)\, 31s^2 + 4s^4 \,(\, \frac{961}{15s} \leftarrow Z_3$

$-31s^2$

$4s^4 \,)\, \frac{15}{31}s^3 \,(\, \frac{15}{124s} \leftarrow Y_4$

$-\frac{15}{31}s^3$

0

$\frac{48}{31s^2} =$ x

$\frac{31-16}{31} = \frac{15}{31}$

$\frac{31s^2 (31)}{15 s^3}$

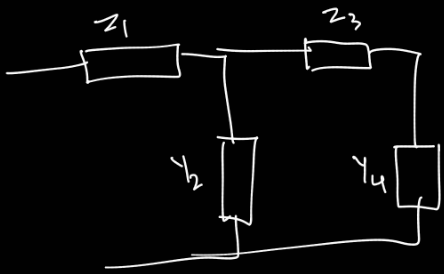
$\frac{15 s^3}{31 \; 4s^4}$

$Z(s) \rightarrow Ks \rightarrow L$

$\frac{1}{Ks} \rightarrow K = C$

$Y(s) \rightarrow Ks \rightarrow C$

$\rightarrow \frac{1}{Ks} \rightarrow L = K$



RC Driving Point Impedances

→ Replace all inductances by resistances in the foster form I

$Z(s) = \dfrac{K_0}{s} + K_\infty + \dfrac{K_1}{s+\sigma_1} + \dfrac{K_2}{s+\sigma_2} + \dots$

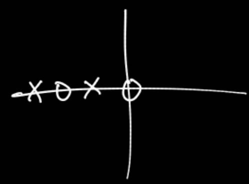
$C_0 = \frac{1}{K_0}$ $\left(\frac{1}{Cs} = \frac{K_0}{s}\right)$ $R_\infty = K_\infty$ $\left(\frac{1}{\frac{s}{K_1} + \frac{\sigma_1}{K}}\right)$ $C_1 = \frac{1}{K_1}$ $R_1 = \frac{K_1}{\sigma_1}$

$C_2 = \frac{1}{K_2}$ $R_2 = \frac{K_2}{\sigma_2}$

$$\boxed{Z(s) = \frac{1/C}{s + \frac{1}{RC}}} \qquad \boxed{Y(s) = \frac{1/L}{s + \frac{R}{L}}}$$

Properties

1) Poles & zeros lie on -ve real axis and are interlacing →



2) $Z_0(0) = \begin{cases} \infty \to C_0 \text{ is present} \\ \sum R_i \to C_0 \text{ is missing} \end{cases} \to Z(0) = \frac{K_0}{s} = Z(0) = \infty$

$Z(\infty) = \begin{cases} R_\infty \to R_\infty \text{ is present} \\ 0, \ R_\infty \text{ is missing} \end{cases}$

$Z(0) > Z(\infty) \to RC$

$Z(\infty) > Z(0) \to RL$

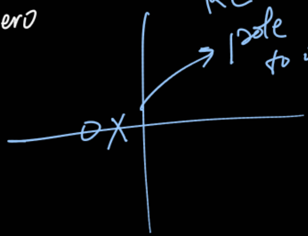
③ Critical frequency ⟹ Singularity σ_1

↳ singularity closest to origin (σ_1) should be a pole

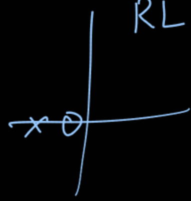
↳ singularity closest to ∞ (σ_n) should be a zero

RC

pole closest to origin



RL



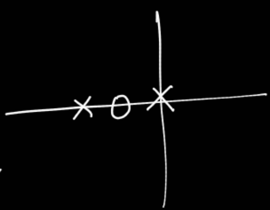
④ Residues of poles must be real & +ve

⑤ $\frac{d}{d\sigma} Z_{RC} < 0$

⑥ $Z_{RC}(\infty) < Z_{RC}(0)$

Q ① Det. which of the following represent RC impedance func.

$Z(s) = \frac{(s+1)(s+4)(s+8)}{s(s+2)(s+6)}$



→ it is a RC imp func because

critical frequency at $s=0$ → is a pole

and nearest critical frequency to $-\infty$ (-8) is a zero

→ Poles & zeros interlace

(ii) $Z(s) = \frac{(s+1)(s+8)}{(s+2)(s+4)}$ Not RC impd → critical frequency closest to origin (-1) is a zero

(iii) $Z(s) = \frac{(s+2)(s+4)}{(s+1)}$ → Not RC

↳ poles & zeros do not interlace

(iv) $Z(s) = \frac{(s+1)(s+2)}{(s+3)}$ Not RC

↳ closest to origin is a zero

Synthesis of RC Impedance & RL Admittance

→ Foster I → RC impedance

→ Foster II → RC admittance

* If Numerator degree ≥ Denominator → divide till num < denominator and then apply PFE to get series Resistance & parallel impedance
* Quotient is $Z(\infty)$ → C for foster I, L for foster II

ex 1 $$H(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{3s^2+18s+24}{s^2+3s}$$

$$s^2+3s \overline{)\; \cancel{3s^2}+18s+24 \;} (3$$
$$\underline{-\cancel{3s^2}-9s}$$
$$9s+24$$

$$H(s) = 3 + \frac{9s+24}{s(s+3)}$$

$$\frac{9s+24}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$s=0 \quad A = 8$

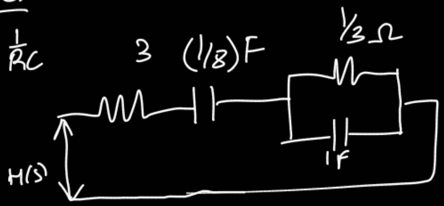
$s=-3 \quad B = 1$

$$H(s) = 3 + \frac{8}{s} + \frac{1}{s+3}$$

$$\frac{(1/C)}{s+\frac{1}{RC}}$$

$\frac{1}{RC} = 3 \quad R = \frac{1}{3}$

⇔



If H(s) is realised for RL admittance

↳ treat H(s) as Y(s)

$$H(s) = \frac{8}{s} + 3 + \frac{1}{s+3}$$

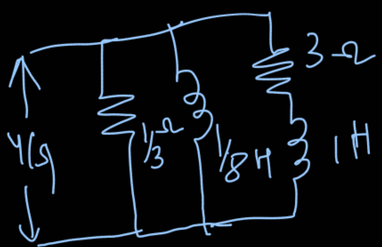
$$\frac{1/L}{s+\frac{R}{L}}$$

$R = 3$

$L = 1$

$\frac{1}{L} = 8$

$L = 1/8\ H$



→ Cauer I → numerator in descending order

→ apply CFE

ex $$H(s) = \frac{3(s+2)(s+4)}{s(s+3)} = \frac{3s^2+18s+24}{s^2+3s}$$

$$s^2+3s \,\overline{)\, 3s^2+18s+24}\ (3 \leftarrow 2$$
$$-3s^2-9s$$
$$9s+24\,\overline{)\, s^2+3s}\ \left(\frac{s}{9}\right.$$
$$-s^2-\frac{8s}{3}$$
$$\frac{s}{3}\,\overline{)\, 9s+24}\ (27$$
$$-9s$$
$$24\,\overline{)\, \frac{s}{3}}\ \left(\frac{s}{72}\right.$$
$$-\frac{s}{3}$$
$$0$$

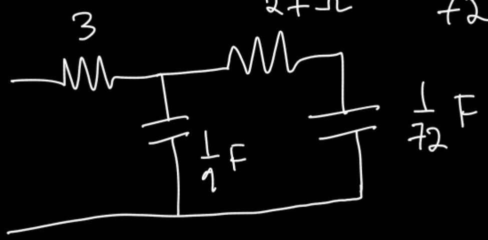
$\frac{s^2}{9s}$ $\quad \frac{24}{9} \cdot 8 \quad \frac{9s(3)}{s} \quad \frac{s}{3}(24)$

If H(s) is impedance function then

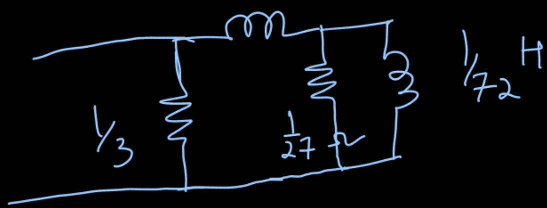
$$3+\cfrac{1}{\cfrac{s}{9}+\cfrac{1}{\left(27+\cfrac{1}{\cfrac{s}{72}}\right)}}$$

$72 = \frac{1}{L}$

$L = \frac{1}{72}$



If H(s) is admittance function RL func



* If H(s) represents RL impedance or RC admittance then the poles & zeros are -ve so we can't use foster so use $\frac{H(s)}{s} = \frac{K_0}{s} + K_\infty + \frac{K_1}{s+\sigma_1} \cdots\cdots$

ex $H(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$

by PFE of $H(s) = 2 - \frac{1/2}{s+2} - \frac{15/2}{s+6}$ ✗

$1(2)(-1)$

so $\frac{H(s)}{s} = \frac{2(s+1)(s+3)}{s(s+2)(s+6)} = 2(s+1)(s+3) = A(s+2)(s+6) + B(s)(s+6) + C(s)(s+2)$

$s=-2$ $\quad B = \frac{\cancel{2}\,\frac{1}{4}}{\cancel{-8}}\ \frac{1}{4}$

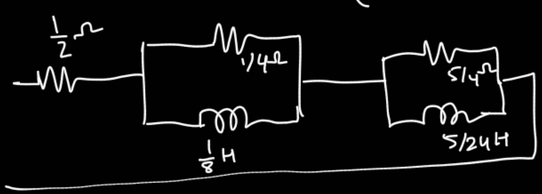
$s=0$ $\quad A = \frac{6}{12} = \frac{1}{2}$

$s=-6$ $\quad C = \frac{\cancel{2}(-5)(\cancel{-3})}{-6(-4)} = 5/4$

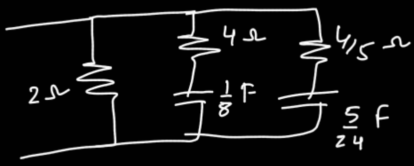
$$\frac{H(s)}{s} = \frac{1/2}{s} + \frac{1/4}{(s+2)} + \frac{5/4}{s+6}$$

$$H(s) = \frac{1}{2} - \frac{3/4}{(s+2)} + \frac{55/4}{s+6}$$ } represents an impedance function

If H(s) is impedance func.



If H(s) is admittance



Ex3 $H(s) = \dfrac{2(s+1)(s+3)}{(s+2)(s+6)}$ using Cauer form II

Cauer II → $\dfrac{6+8s+2s^2}{12+8s+s^2}$

CFE

$12+8s+s^2 \,)\, 6+8s+2s^2 \,(\, 1/2$

$-\ 6+4s+s^2/2$

$4s+\frac{3}{2}s^2 \,)\, 12+8s+s^2 \,(\, 3/s$

$-\ 12+\frac{9s}{2}$

$\frac{7s}{2}+s^2 \,)\, 4s+\frac{3}{2}s^2 \,(\, 8/7$

$-\ 4s+8/7\, s^2$

$\frac{5}{14}s^2 \,)\, \frac{7s}{2}+s^2 \,(\, 49/5s$

$-\ \frac{7s}{2}$

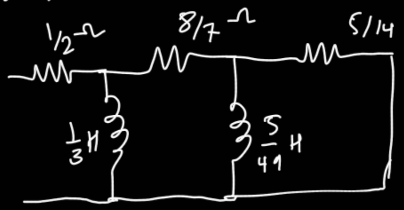
$s^2 \,)\, \frac{5}{14}s^2 \,(\, 5/14$

$\underline{0}$

$4s = \frac{7s}{2} \times x$

8

If H(s) is an impedance func



If H(s) is an admittance func