Mechanics - Statics

Theory Syllabus

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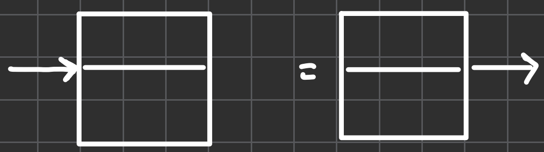
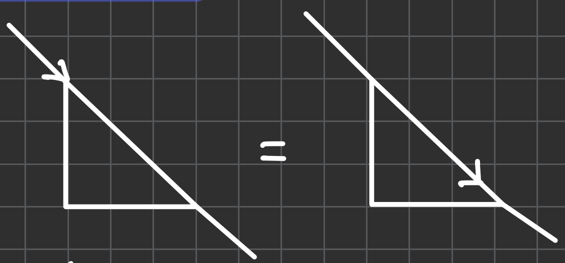
Force Systems

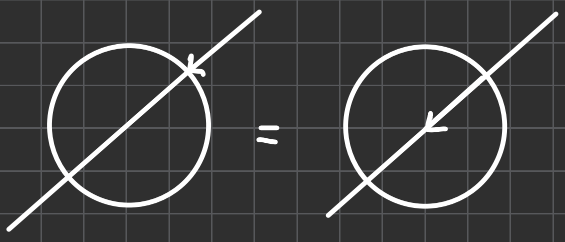
- An action of one body on another
  (or)
- An action which tends to cause acceleration of another body
- Vector quantity (Depends on Magnitude & Direction)
- **Characteristics**
  - Magnitude
  - Point of Application
  - Direction
  - Line of Action

Principle of Transmisibility

- A force may be applied at any point on its given line of action w/o altering resultant effects of force external to the rigid body on which it acts







Moment

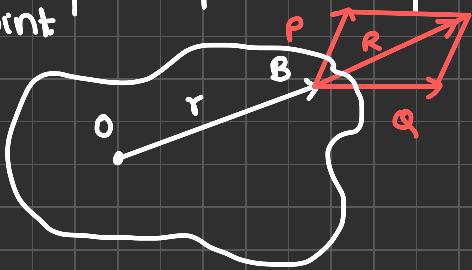
- Moment is the cross product of the position vector which runs from reference point to line of action of the force and Force.

$$\vec{M} = \vec{r} \times \vec{F}$$
$$= rF\sin\theta \qquad (d = r\sin\theta)$$
$$= Fd$$

Varignon's Theorem (Principle of Moments)

- Moment of Force about any point is equal to sum of moments of components of force about the same point

$$M_0 = r \times R = r \times P + r \times Q$$



Note : Clockwise is taken −ve

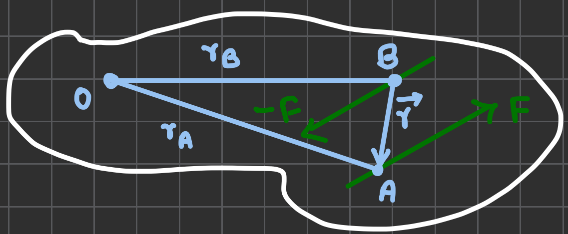
Anti-Clockwise is taken +ve

Couple

- Moment produced by 2 equal, opposite & non-collinear force
- Magnitude of Couple $M = F \cdot d$

Characteristics

- Algebraic sum of forces, having couple is zero
- Algebraic sum of moment of forces, constituting couple, about any point is same & equal to couple
- Couple can't be balanced by single force. Must have 2 equal & opposite forces.
- Any number of coplanar forces can be reduced to a single couple

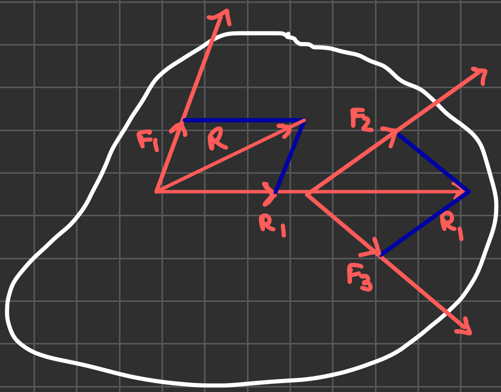


$$M = r_A F + r_B(-F)$$

$$M = F(r_A - r_B)$$

Resultant

- It is the simplest combination which can replace original forces without altering external effect on rigid body to which forces are applied



$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots + \vec{F_n}$$
$$= \Sigma \vec{F_i}$$

$R_x = \Sigma F_x$ & $R_y = \Sigma F_y$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta = \frac{R_y}{R_x}$$

Equilibrium

- The condition in which resultant of all forces acting on the body is zero.

So, $R = \Sigma F = 0$ & $M = 0$

System Isolation

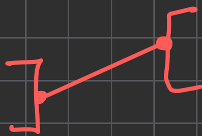
- A mechanical system can be studied conceptually by isolating from all other bodies & finding the resultant forces acting on the system.

Free Body Diagram

- Diagrammatic representation of isolated system treated as single body which shows all forces applied to system by mechanical contact with other bodies & other forces are ignored.

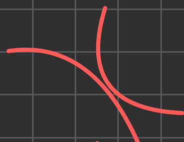
- Model of Action

1) Flexible Cable/Belt (Negligible mass)



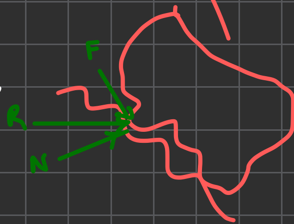
→ Force exerted by cable is always Tension away from the body along cable's direction

2) Smooth Surface



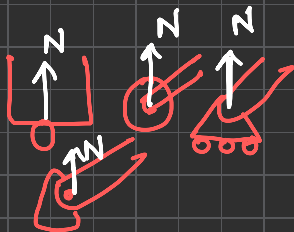
→ Contact force is compressive & normal to surface

3) Rough Surface



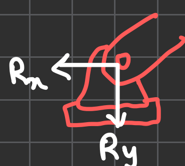
→ They can support a tangential component F & Normal force N of resultant force R

4) Roller Support



→ Roller/Rocker/Ball transmit compressive force normal to supporting force

5) Pin Connection



→ Its capable of supporting a force in any direction in plane normal to pin axis

→ Can show force in either R_x, R_y components, but pin not free can also show couple

6) Built-in/Fixed Support →

→ Capable of supporting axial force F, transverse force V & couple M to prevent rotation

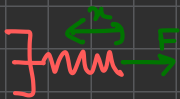
7) Torsional Spring Action

→ Linear torsional spring

$M \propto \theta$ (angle of deflection)

Stiffness K_T required to deform spring

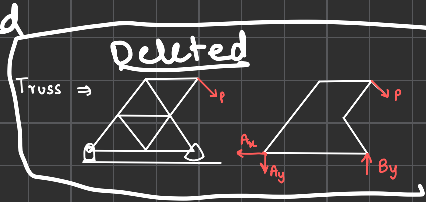
8) Spring Action



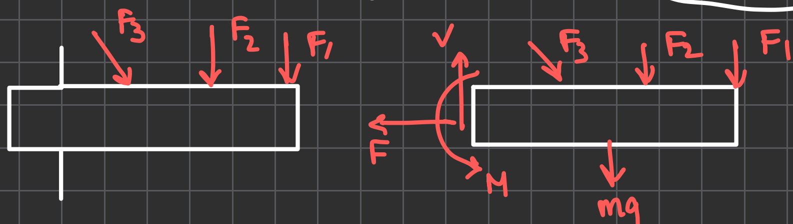
$F = Kx$

→ Spring force is tensile if stretched & compressive if compressed

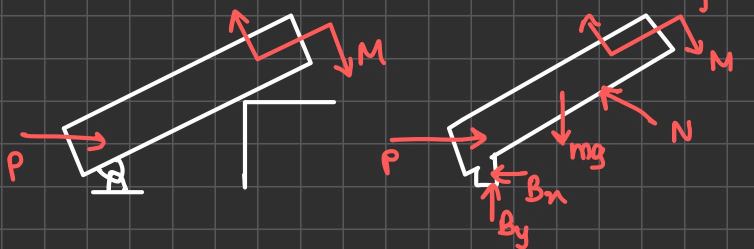
→ For linear elastic spring, stiffness k is required to deform spring



Cantilever



Beam

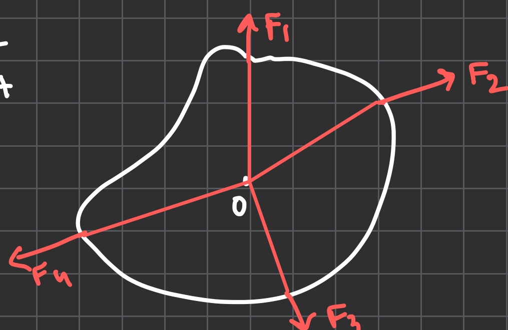


Equilibrium in 2D ☆

- Collinear

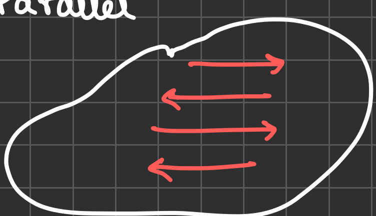
$\Sigma F_x = 0$

- Concurrent at a point



$\Sigma F_x = 0$
$\Sigma F_y = 0$

- Parallel



$\Sigma F_x = 0$
$\Sigma M_z = 0$

Centroid

- Centroid is the geometric centre of object

Dependent on uniform density

Centre of Mass

- The point at which whole of mass of the body or all masses of system of particle appear to be concentrated

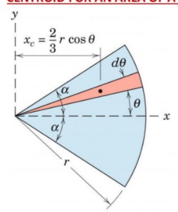
Centre of Gravity

- The point where total weight of object acts.

Not Dependent on uniform density

Centroid of Circular Section

CENTROID FOR AN AREA OF A CIRCULAR SECTOR:



The area may also be covered by swinging a triangle of differential area about the vertex and through the total angle of the sector. This triangle, shown in the illustration, has an area dA = (r/2)(r dΘ), where higher-order terms are neglected. From the centroid of the triangular element of area is two-thirds of its altitude from its vertex, so that the x-coordinate to the centroid of the element is

$$x_c = \frac{2r\cos\theta}{3}$$

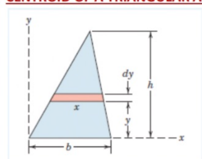
$$\left[A\,\bar{x} = \int x_c \, dA\right] \qquad (r^2\alpha)\bar{x} = \int_{-\alpha}^{\alpha} \left(\frac{2r\cos\theta}{3}\right)\left(\frac{1}{2} r^2 \, d\theta\right)$$

$$(r^2\alpha)\,\bar{x} = \frac{2}{3} r^3 \sin\alpha$$

$$\bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$$

Centroid of Triangle

CENTROID OF A TRIANGULAR AREA:



The *x*-axis is taken to coincide with the base. A differential strip of Area$dA = x\,dy$ is chosen. By similar triangles $\frac{x}{(h-y)} = \frac{b}{h}$

Applying the Equation $\bar{y} = \frac{\int y_c \, dA}{A}$ gives $[A\,\bar{y} = \int y_c dA]$

$$\frac{bh}{2}\,\bar{y} = \int_0^h y \frac{b(h-y)}{h} dy = \frac{bh^2}{6}$$

$$\bar{y} = \frac{h}{3}$$

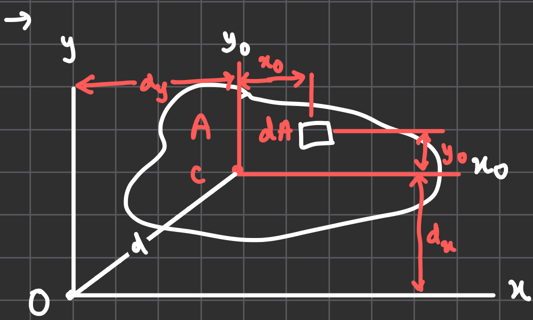
With respect to base of the triangle.

Moment of Inertia

- Parallel Axis Theorem

→ It explains relation b/w MOI of an object about 2 parallel axes and allows for calculation of MOI of rigid body about an axis parallel to known axis

→



$$dI_x = (y_0 + d_x)^2 \, dA$$

$$I_x = \int y_0^2 \, dA + 2d_x \int y_0 \, dA + d_x^2 \int dA$$

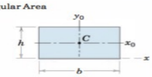
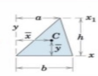
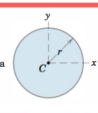
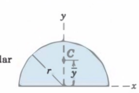
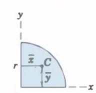
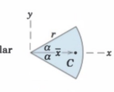
$$I_x = \bar{I}_x + A d_x^2 \longrightarrow (1)$$

Similarly, $\quad I_y = \bar{I}_y + A d_y^2 \longrightarrow (2)$

We know, $\quad I_z = I_x + I_y$

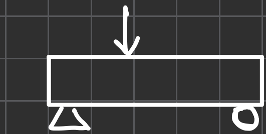
So $(1) + (2)$

$$I_z = \bar{I}_z + A d^2$$

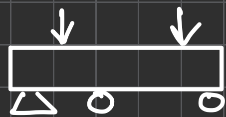
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Circular Area 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \frac{2}{3}\frac{r \sin\alpha}{\alpha}$	$I_x = \frac{r^4}{4}(\alpha - \frac{1}{2}\sin 2\alpha)$ $I_y = \frac{r^4}{4}(\alpha + \frac{1}{2}\sin 2\alpha)$ $I_z = \frac{1}{2}r^4\alpha$

Beams

- Horizontal structural member used to support loads which offer resistance to bending



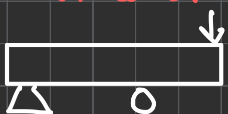
Simple



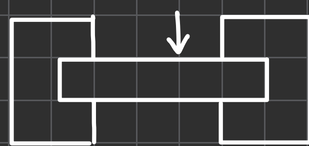
Continuous

Cantilever

End Supported Cantilever



Combination



Fixed

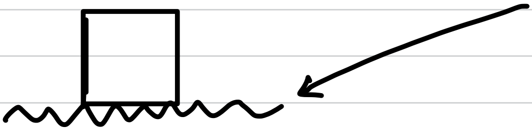
Statically Determinable

↳ Beam supported so that external support r/ns can be calculated by method of statics alone

Statically Indeterminable

↳ Beams which have more supports than needed to provide equilibrium

Friction



Tangential force which occur only when in contact with real surfaces opposite to direction of applied force

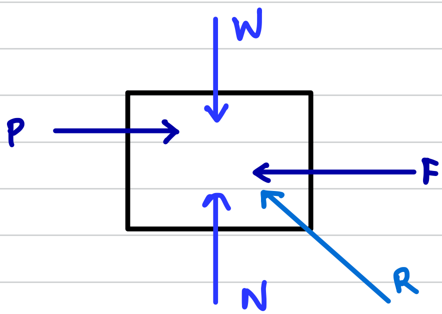
- Types of friction :

i) Dry ⇒ 2 solid's unlubricated surface

ii) Fluid ⇒ Adjacent layer is liquid/gas moving at diff. velocities

iii) Internal ⇒ Solids which have cyclic Loading. (Especially low limit of elasticity ones)

Dry Friction Mechanism



$$R = \sqrt{F^2 + N^2}$$
$$W = N$$
$$P = F$$

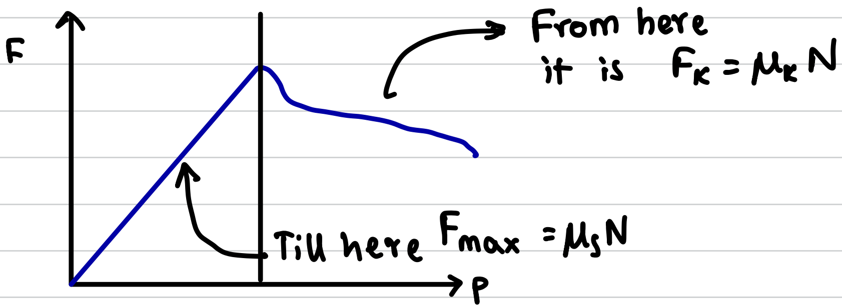
→ Just for this case

- As P increases, F increases but till come certain limit only

$$F_m = \mu_s N$$

μ_s : Coefficient of Static Friction
μ_K : Coefficient of Kinetic Friction

Basically, it increases till certain limit after that μ_K is used uniformly



So, i) When $P = 0$ $\Rightarrow$ Body doesn't move, Equilibrium valid

ii) When $P < F_m$ $\Rightarrow$ Body doesn't move, Equilibrium valid

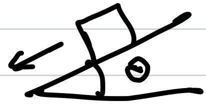
iii) When $P = F_m$ $\Rightarrow$ Body impends to move, Equilibrium valid

$$\tan\phi = F_m/N = \mu_s$$

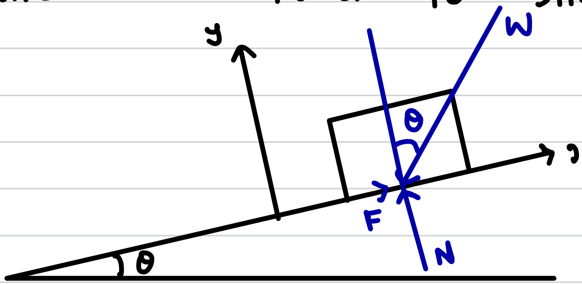
iv) When $P > F_m$ $\Rightarrow$ Body starts to move, Equilibrium invalid

$$\tan\phi = F_K/N = \mu_K$$

Angle of Repose



- The angle at which a body resting on the plane will tend to slide down plane



In x-axis, $\sum F_x = 0$

$$W\sin\theta = F = \mu_s N$$

In y-axis, $\sum F_y = 0$

$$N = W\cos\theta$$

$$\mu_s N = W\sin\theta$$

$$\mu_s (\cancel{W}\cos\theta) = \cancel{W}\sin\theta$$

$$\mu_s = \tan\theta \Rightarrow \theta = \tan^{-1}(\mu_s)$$

↳ Angle of Repose