Week 2: Regression with Multiple Input Variables

Contents

Contents

1. Multiple Regression

Multiple Features (Variables)

one feature

Size in feet² (x)	Price ($) in 1000's ($y$)
2104	400
1416	232
1534	315
852	178
...	...

$$f_{w,b}(x) = wx + b$$

Multiple Features (Variables)

Size in feet^2	Number of bedrooms	Number of floors	Age of home in years	Price ($) in $1000's
x_1	x_2	x_3	x_4	
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

Multiple Features (Variables)

Size in feet^2	Number of bedrooms	Number of floors	Age of home in years	Price (\$) in \$1000's
x_1	x_2	x_3	x_4	
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

$j = 1 \ldots 4$

$n = 4$

$i = 2$

x_j = j^{th} feature

n = number of features

$\vec{x}^{(i)}$ = features of i^{th} training example

$x_j^{(i)}$ = value of feature j in i^{th} training example

$\vec{x}^{(2)} = [1416 \;\; 3 \;\; 2 \;\; 40]$

$x_3^{(2)} = 2$

Multiple Features (Variables)

Model:

Previously: $f_{w,b}(x) = wx + b$

$$f_{w,b}(x) = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + b$$

example

$$f_{w,b}(x) = 0.1 x_1 + 4 x_2 + 10 x_3 + -2 x_4 + 80$$

size #bedrooms #floors years base price

Multiple Features (Variables)

$$f_{\vec{w},b}(\vec{x}) = w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$$

$\vec{w} = [w_1 \ w_2 \ w_3 \ldots w_n]$ parameters of the model

b is a number

vector $\vec{x} = [x_1 \ x_2 \ x_3 \ldots x_n]$

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b = w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n + b$$

dot product

multiple linear regression

Vectorization Part 1

Parameters and features

$\vec{w} = [w_1 \quad w_2 \quad w_3]$ $n=3$

b is a number

$\vec{x} = [x_1 \quad x_2 \quad x_3]$

NumPy

linear algebra: count from 1

w[0] w[1] w[2]

```
w = np.array([1.0,2.5,-3.3])
b = 4            x[0] x[1] x[2]
x = np.array([10,20,30])
```

code: count from 0

Without vectorization $n = 100{,}000$

$$f_{\vec{w},b}(\vec{x}) = w_1x_1 + w_2x_2 + w_3x_3 + b$$

```
f = w[0] * x[0] +
    w[1] * x[1] +
    w[2] * x[2] + b
```

Without vectorization

$$f_{\vec{w},b}(\vec{x}) = \left(\sum_{j=1}^{n} w_j x_j\right) + b$$

$\sum_{j=1}^{n} \rightarrow j=1\ldots n$

1, 2, 3

range(0,n) $\rightarrow$ $j = 0 \ldots n-1$

```
f = 0          range(n)
for j in range(0,n):
    f = f + w[j] * x[j]
f = f + b
```

Vectorization

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$

```
f = np.dot(w,x) + b
```

Vectorization Part 2

Without vectorization

```
for j in range(0,16):
    f = f + w[j] * x[j]
```

t_0

`f + w[0] * x[0]`

t_1

`f + w[1] * x[1]`

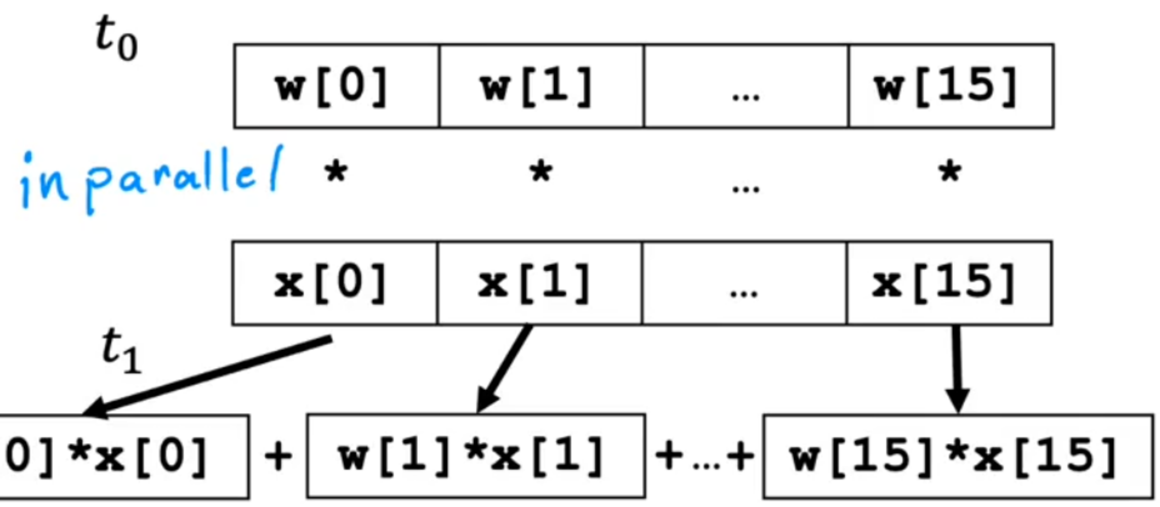
…

t_{15}

`f + w[15] * x[15]`

Vectorization

```
np.dot(w,x)
```



efficient → scale to large datasets

Vectorization Part 2

Gradient descent $\vec{w} = (w_1 \quad w_2 \quad \cdots \quad w_{16})$ $\cancel{b}$ parameters

derivatives $\vec{d} = (d_1 \quad d_2 \quad \cdots \quad d_{16})$

```
w = np.array([0.5, 1.3, … 3.4])
d = np.array([0.3, 0.2, … 0.4])
```

compute $w_j = w_j - \overbrace{0.1}^{\text{learning rate } \alpha} d_j$ for $j = 1 \ldots 16$

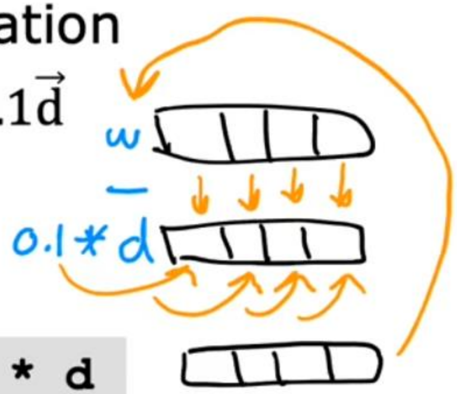
Without vectorization

$$w_1 = w_1 - 0.1 d_1$$
$$w_2 = w_2 - 0.1 d_2$$
$$\vdots$$
$$w_{16} = w_{16} - 0.1 d_{16}$$

```
for j in range(0,16):
    w[j] = w[j] - 0.1 * d[j]
```

With vectorization

$$\vec{w} = \vec{w} - 0.1\vec{d}$$




```
w = w - 0.1 * d
```

Gradient Descent for Multiple Linear Regression

	Previous notation	Vector notation
Parameters	$w_1, \cdots, w_n$ b	vector of length n $\vec{w} = [w_1 \quad \cdots \quad w_n]$ b still a number
Model	$f_{\vec{w},b}(\vec{x}) = w_1x_1 + \cdots + w_nx_n + b$	$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$ dot product
Cost function	$J(\underbrace{w_1, \cdots, w_n, b}_{})$	$J(\vec{w}, b)$

Gradient descent

Previous notation:

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\underbrace{w_1, \cdots, w_n}_{}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\underbrace{w_1, \cdots, w_n}_{}, b)$$

}

Vector notation:

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b)$$

}

Gradient Descent for Multiple Linear Regression

Gradient descent

One feature

repeat {

$$\underbrace{w}_{} = w - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)\underbrace{x^{(i)}}_{}}$$

$$\rightarrow \frac{\partial}{\partial w}J(w,b)$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{w,b}\left(x^{(i)}\right) - y^{(i)}\right)$$

simultaneously update w, b

}

n features $(n \geq 2)$

repeat {

$j=1$

$$\underbrace{w_1}_{} = w_1 - \alpha \boxed{\frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)\underbrace{x_1^{(i)}}_{}}$$

$$\rightarrow \frac{\partial}{\partial w_1}J(\vec{w},b)$$

$\vdots$

$j=n$

$$w_n = w_n - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)x_n^{(i)}$$

$$b = b - \alpha \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)$$

simultaneously update

w_j (for $j = 1, \cdots, n$) and b

}

2. Gradient Descent in Practice

Feature Scaling Part 1

Feature and parameter values

$\widehat{price} = w_1 x_1 + w_2 x_2 + b$

size # bedrooms

x_1: size (feet^2)
range: 300 – 2,000
large

x_2: # bedrooms
range: 0 – 5
small

House: $x_1 = 2000$, $x_2 = 5$, $price = \$500k$ one training example

size of the parameters w_1, w_2?

$w_1 = 50, \quad w_2 = 0.1, \quad b = 50$

$\widehat{price} = \underbrace{50 * 2000}_{100,000K} + \underbrace{0.1 * 5}_{0.5K} + \underbrace{50}_{50K}$

$\widehat{price} = \$100,050.5k = \$100,050,500$

$w_1 = 0.1, \quad w_2 = 50, \quad b = 50$

small large

$\widehat{price} = \underbrace{0.1 * 2000k}_{200K} + \underbrace{50 * 5}_{250K} + \underbrace{50}_{50K}$

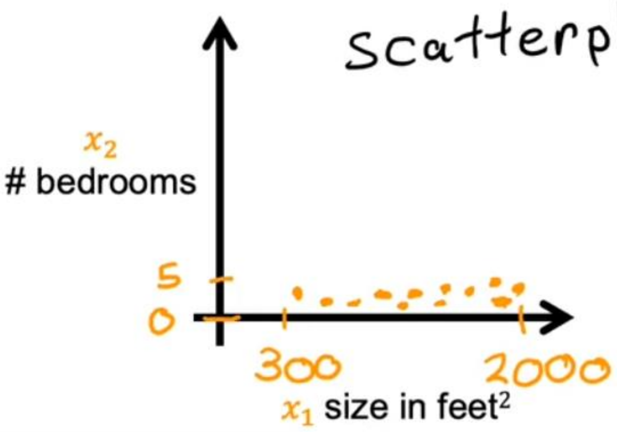
$\widehat{price} = \$500k$ more reasonable

Feature Scaling Part 1

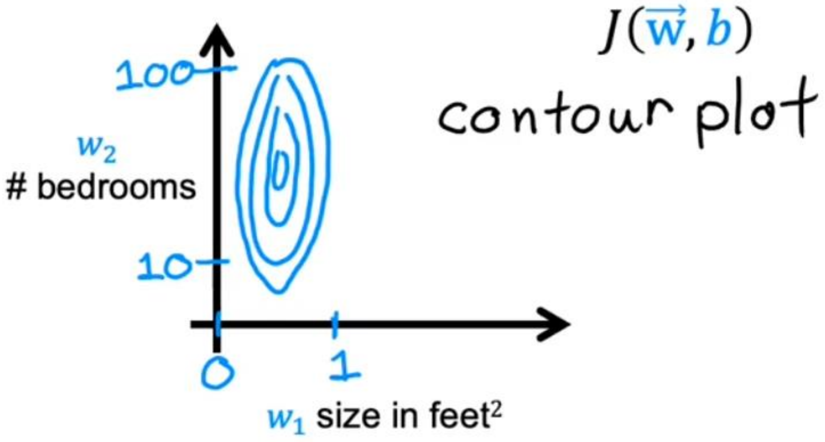
Feature size and parameter size

	size of feature x_j	size of parameter w_j
size in feet^2		
#bedrooms		

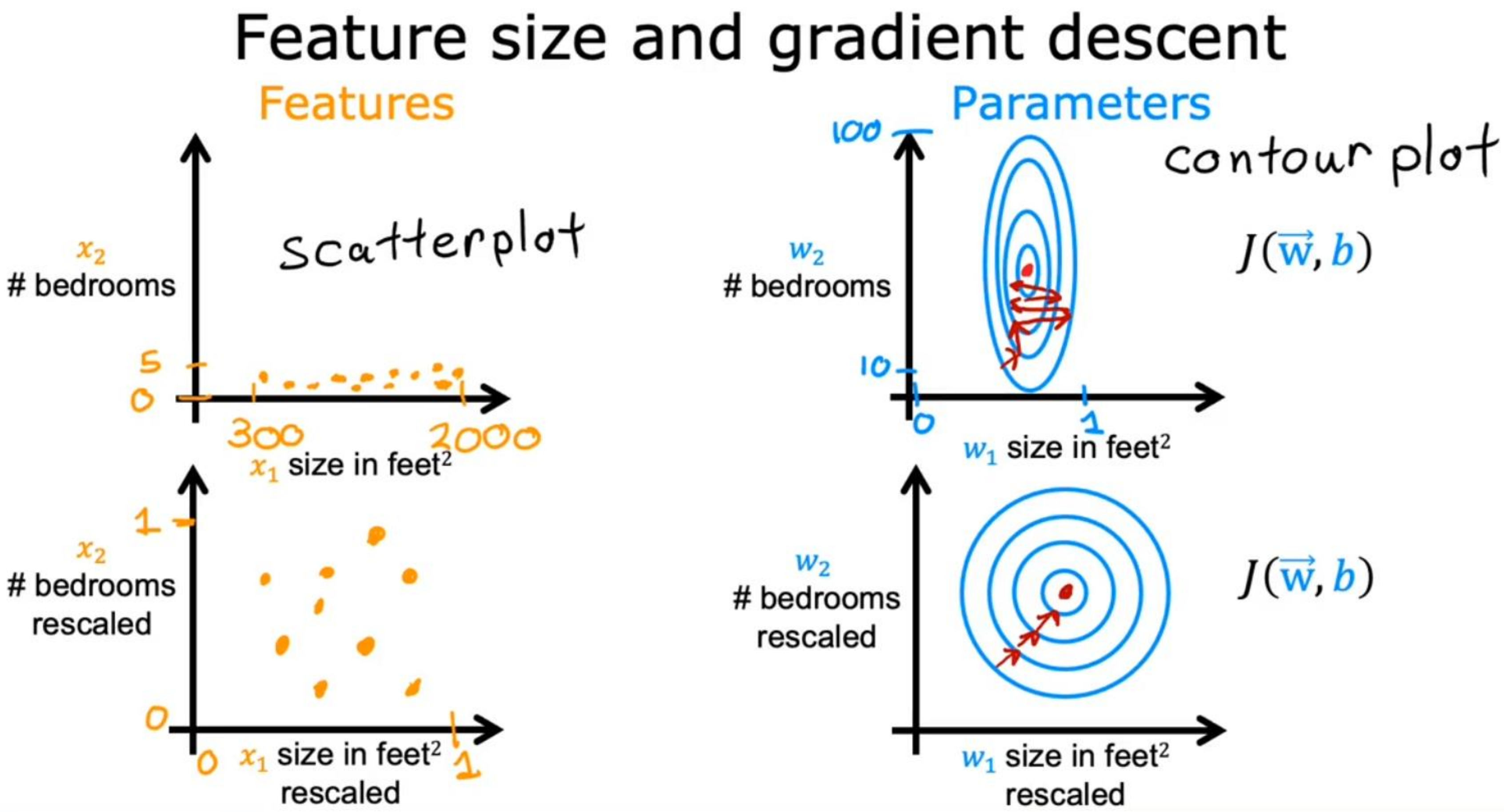
Features



Parameters

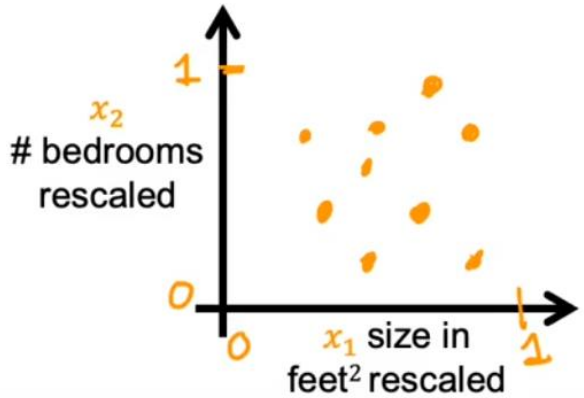


Feature Scaling Part 1



Feature Scaling Part 2

Feature scaling



$300 \le x_1 \le 2000$ $\quad$ $0 \le x_2 \le 5$

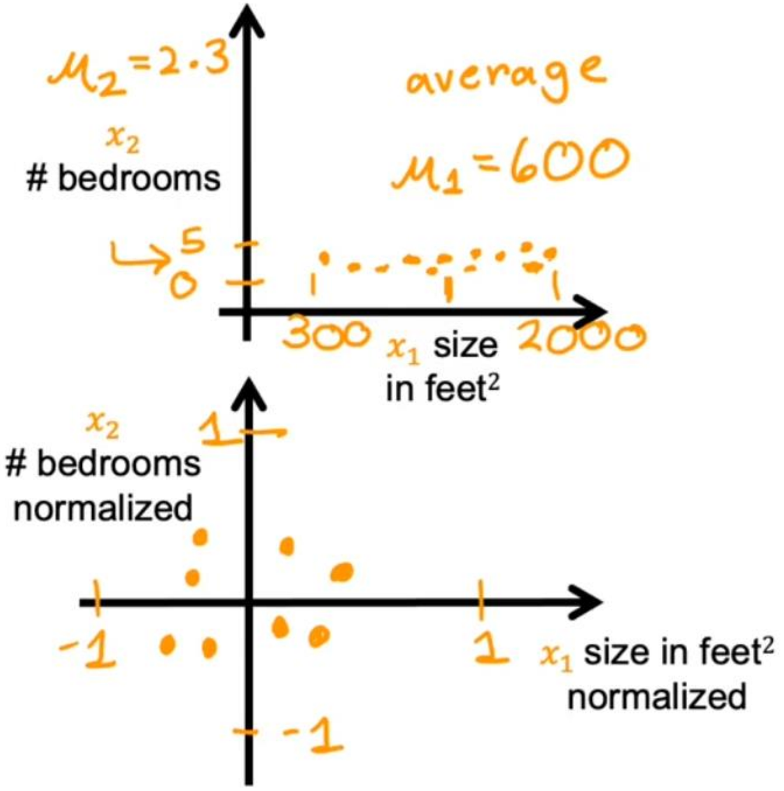
$x_{1,scaled} = \dfrac{x_1}{2000}$ $\quad$ $x_{2,scaled} = \dfrac{x_2}{5}$

max $\quad$ max

$0.15 \le x_{1,scaled} \le 1$ $\quad$ $0 \le x_{2,scaled} \le 1$

Feature Scaling Part 2

Mean normalization



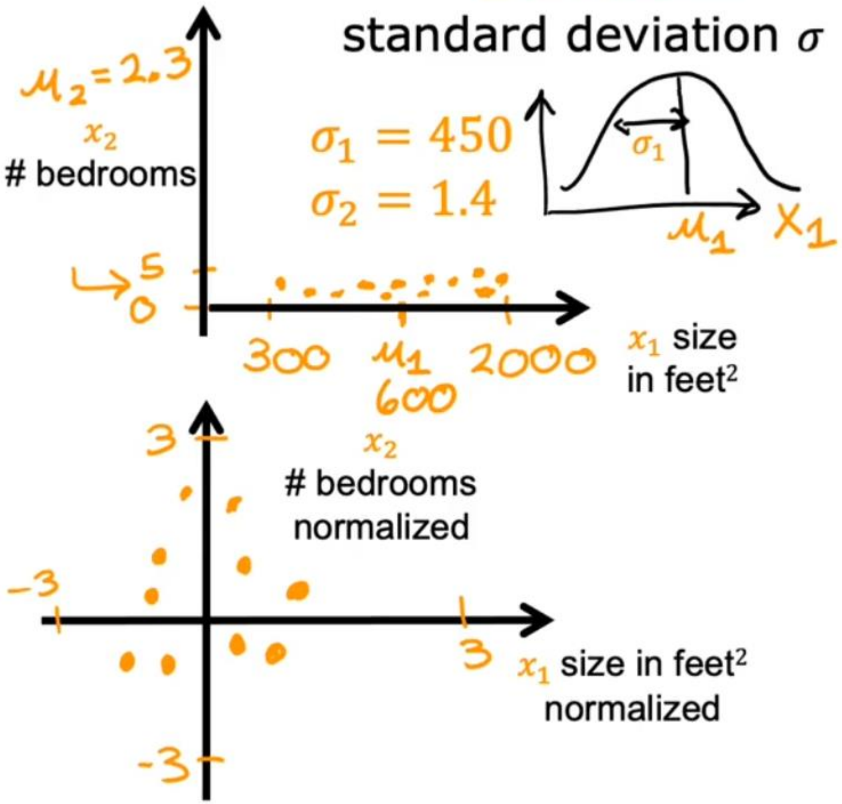
$300 \leq x_1 \leq 2000$ $\quad$ $0 \leq x_2 \leq 5$

$$x_1 = \frac{x_1 - \mu_1}{\underbrace{2000-300}_{max-min}} \qquad x_2 = \frac{x_2 - \mu_2}{\underbrace{5-0}_{max-min}}$$

$-0.18 \leq x_1 \leq 0.82$ $\quad$ $-0.46 \leq x_2 \leq 0.54$

Feature Scaling Part 2

Z-score normalization



$300 \le x_1 \le 2000$ $\quad$ $0 \le x_2 \le 5$

$$x_1 = \frac{x_1 - \mu_1}{\sigma_1} \qquad x_2 = \frac{x_2 - \mu_2}{\sigma_2}$$

$-0.67 \le x_1 \le 3.1$ $\quad$ $-1.6 \le x_2 \le 1.9$

Feature Scaling Part 2

Feature scaling

aim for about $-1 \le x_j \le 1$ for each feature x_j

$$-3 \le x_j \le 3$$
$$-0.3 \le x_j \le 0.3$$

acceptable ranges

$0 \le x_1 \le 3$ okay, no rescaling

$-2 \le x_2 \le 0.5$ okay, no rescaling

$-100 \le x_3 \le 100$ too large $\rightarrow$ rescale

$-0.001 \le x_4 \le 0.001$ too small $\rightarrow$ rescale

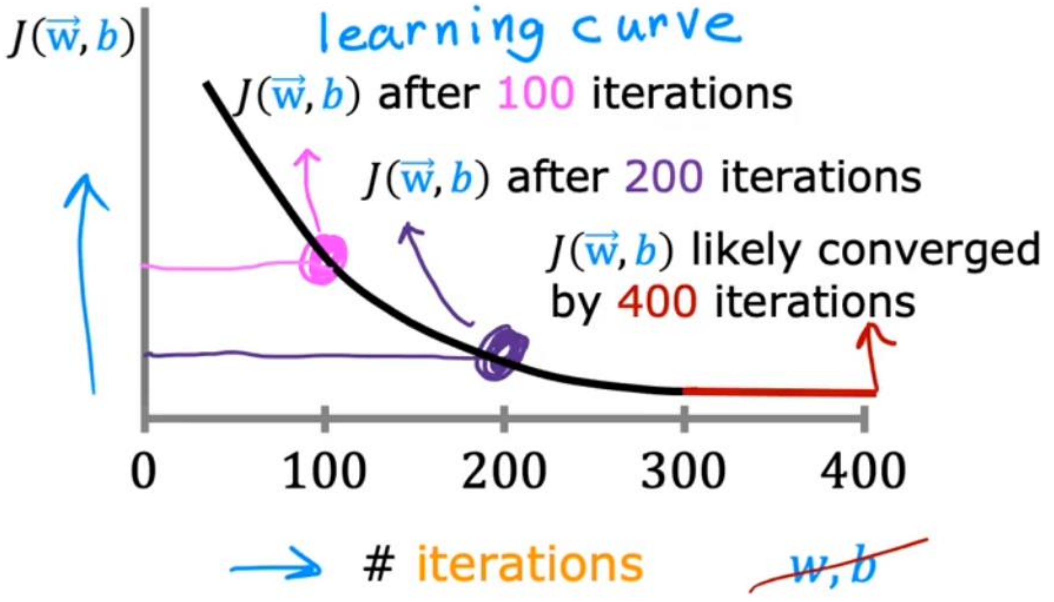
$98.6 \le x_5 \le 105$ too large $\rightarrow$ rescale

Checking Gradient Descent for Convergence

Make sure gradient descent is working correctly

objective: $\min_{\vec{w},b} J(\vec{w},b)$

$J(\vec{w},b)$ should decrease after every iteration



\# iterations needed varies 30 1,000 100,000

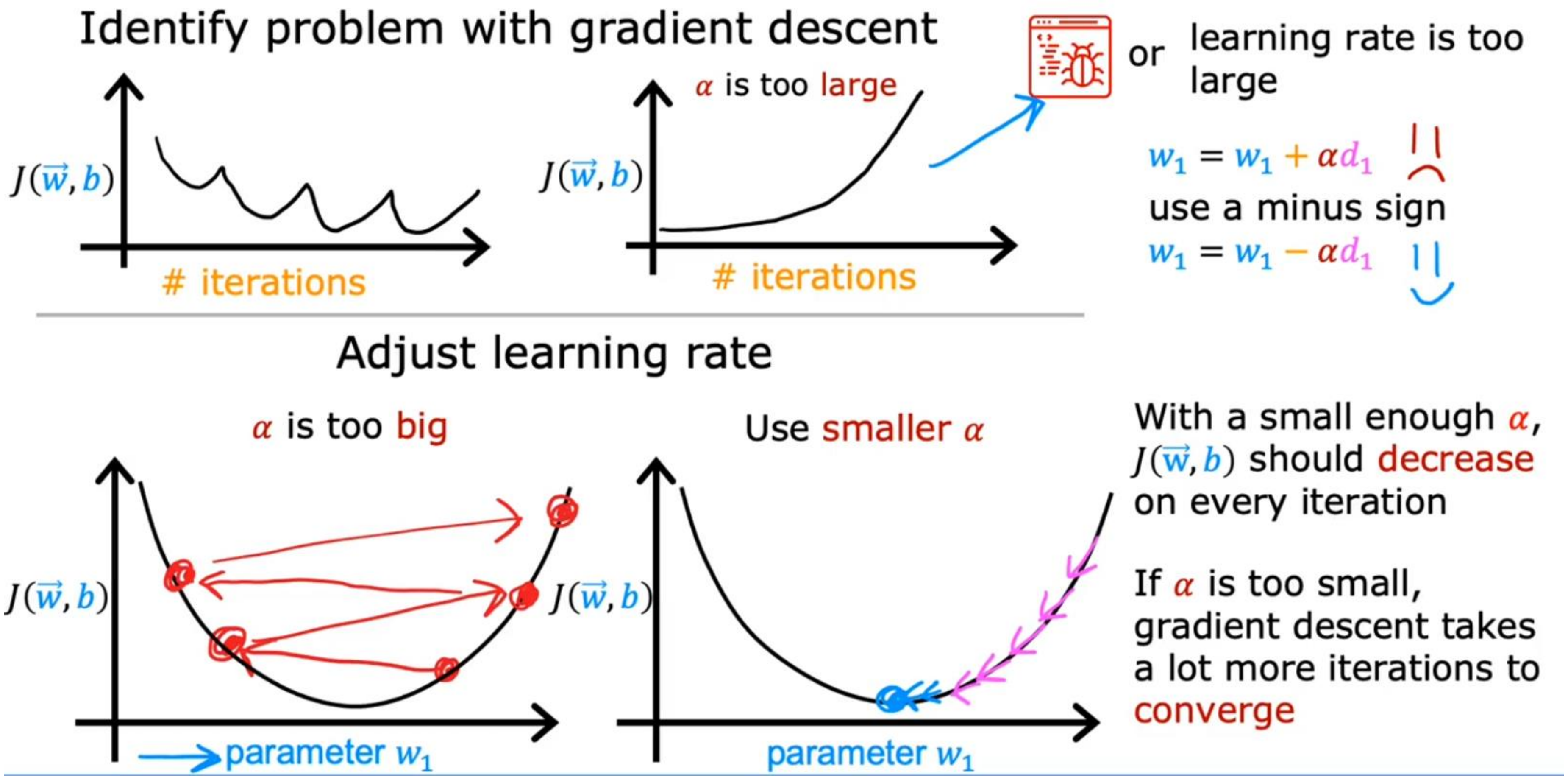
Automatic convergence test

Let ε "epsilon" be 10^{-3}.
0.001

If $J(\vec{w},b)$ decreases by $\leq \varepsilon$ in one iteration, declare convergence.

(found parameters $\vec{w}, b$ to get close to global minimum)

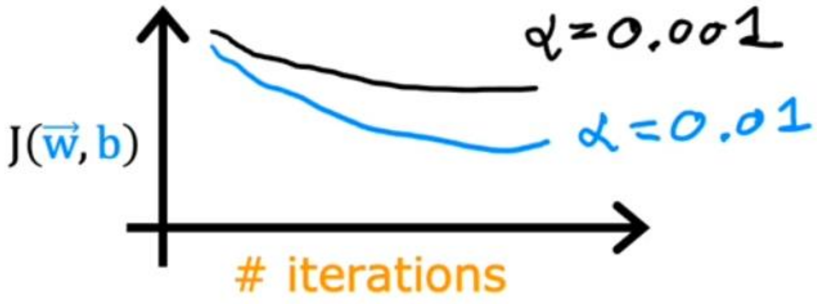
Choosing the Learning Rate

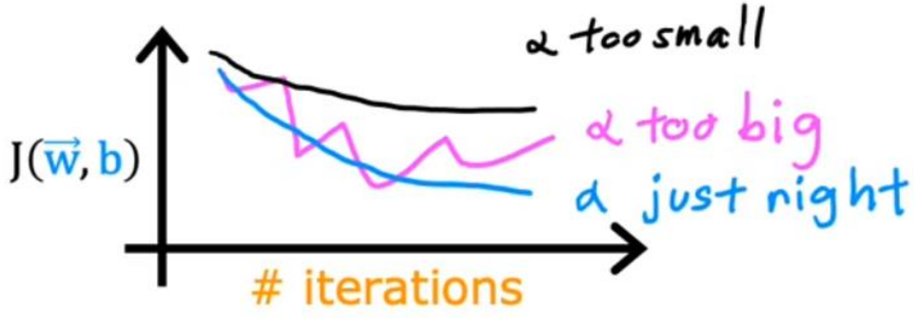


Choosing the Learning Rate

Values of α to try:

$$\ldots\ 0.001 \xrightarrow{3X} 0.003 \xrightarrow{\approx 3X} 0.01 \xrightarrow{3X} 0.03 \xrightarrow{\approx 3X} 0.1 \xrightarrow{3X} 0.3 \xrightarrow{\approx 3X} 1\ \ldots$$





Feature Engineering

Feature engineering

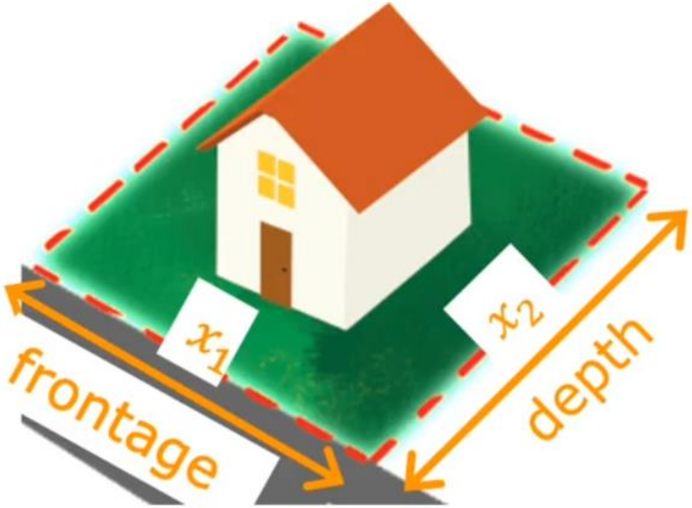
$$f_{\vec{w},b}(\vec{x}) = w_1 \underbrace{x_1}_{\text{frontage}} + w_2 \underbrace{x_2}_{\text{depth}} + b$$

$$area = frontage \times depth$$

$$x_3 = x_1 x_2$$

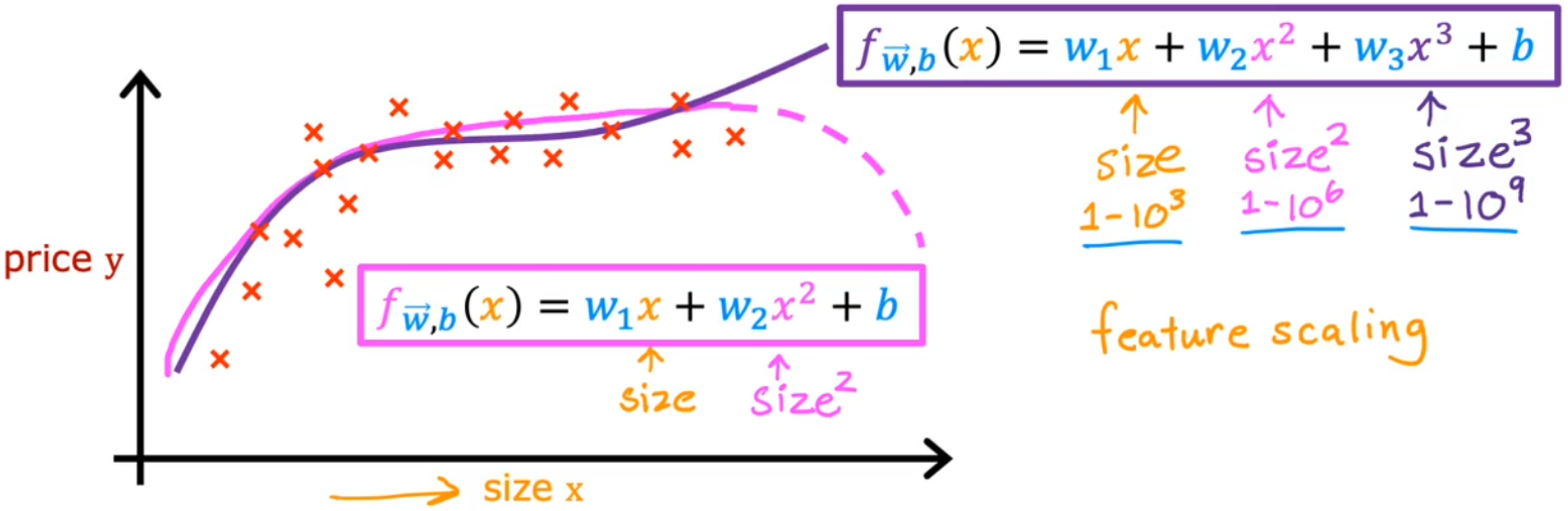
new feature

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$



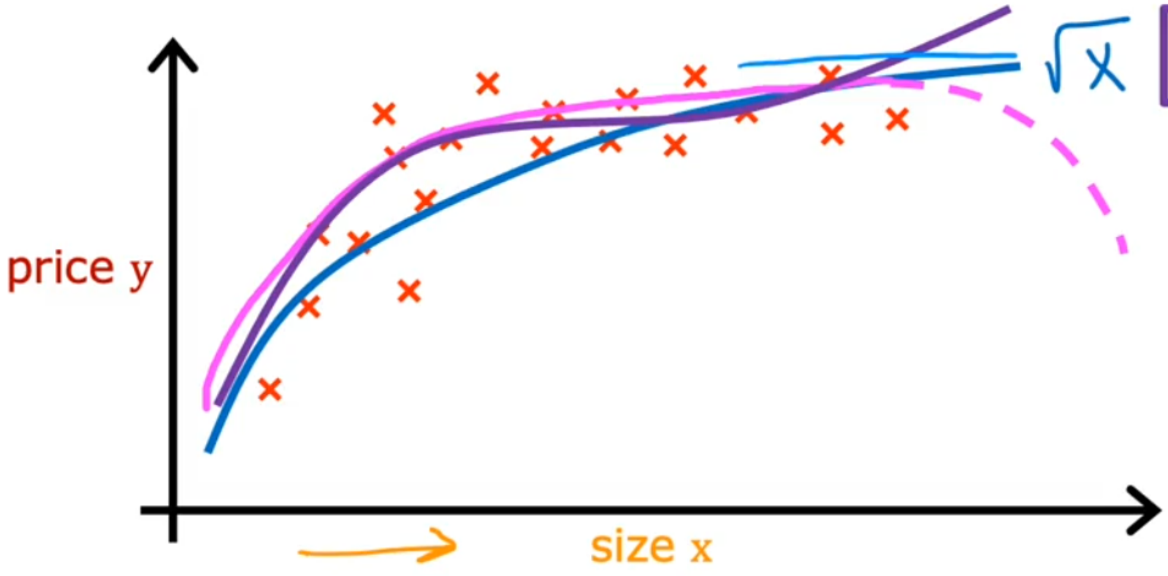
Feature engineering:
Using intuition to design new features, by transforming or combining original features.

Polynomial Regression



Polynomial Regression

Choice of features



$$\boxed{f_{\vec{w},b}(x) = w_1 x + w_2\sqrt{x} + b}$$

$\sqrt{x}$

size

$\sqrt{size}$