The Complete Manual to Surviving CIS 1600

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> What are the basic mathematical concepts and techniques needed in computer science? This course provides an introduction to proof principles and logics, functions and relations, induction principles, combinatorics and graph theory, as well as a rigorous grounding in writing and reading mathematical proofs.
>
> — *Description of CIS 1600 in course catalog*

Contents

§1 Introduction

Keep in mind that I took this course specifically during Spring 2023 with Val Tannen. Material and conditions might be different depending on the year and if you take it with Rajiv in the fall.

Anyway, this course has a reputation for being one of the hardest at UPenn relative to its level. But it helps to have the preconceptions of why it is so dreaded so that you can tackle the course with the right mindset:

- The thought process behind solving problems in this course is not something that can be easily described with black and white principles.
- The TAs are extremely nitpicky about grading on the homeworks and tests.
- Concepts are not always explained in plain English. The course is not exactly friendly to those who don't have a background in math.
- If you fail to include certain buzzwords they're looking for in the homework, they won't hesitate to take points off. They don't always tell you what buzzwords they're looking for, too.
- The sheer amount of material thrown at you is insane and the pace of the course is unforgivingly fast.
- The midterms and final are closed notes and are fast paced. See more on "Test Strategies."

What this guide is supposed to do:

- Help you understand concepts in Layman's terms
- Show examples of the concepts in action with the important parts highlighted
- Point out when and where a concept is applicable to build a thought process on tackling HW and test problems
- Make you feel more comfortable about what you're learning to give you momentum in the course

However, I do NOT want you thinking:

- You can rely on this guide solely for the material and then skip class
- I will do your homework for you just because I typed all this out

That being said, I would greatly appreciate constructive criticism on people who use this guide. If a section is poorly explained, I have no issue in improving it.

Some boxes to keep in mind as you read this guide:

Example 1.1

Examples boxes will have a simple problem that demonstrates the concept at a basic level.

Caveat 1.2. Caveat boxes will have information on what you may need to say in a write-up of a homework problem. Pay attention to these.

Lemma 1.3

Lemma boxes will have claims we want to prove and take more work or reasoning to show than an example.

§2 General Course Advice

This goes for the homework and how you could utilize the resources you have available.

- They mean it when they say start the homework early and go to office hours early.
- Don't feel discouraged if you don't see the solution immediately on a HW problem. Sometimes it takes some thought or trial and error to arrive at a complete solution.
- **In general, if you're using a rule, principle, or technique in a solution to a HW or test problem, explicitly namedrop it.**
- You never need to prove anything that is proven in the lectures or recitation.
- When in doubt, ask on Piazza if you need to prove something.
- If you're stuck, try looking back on the problems solved during the lecture. The intuition can give hints on what to consider.
- Be comfortable with converting words to math and vice versa. I will point out when this is applicable throughout the guide.
- The first third of the course is usually people getting used to the format and circumstances. During the second third, people generally know what to expect. But the last third is a difficulty spike because graph theory problems are abnormally challenging.

§3 Counting

In simplest terms, counting problems involves finding how many of this satisfies some condition. But the "this" can vary. It could be sandwiches, sequences, arrangements, etc.

Counting will primarily be in the first third of the course, where things being counted and methods for counting will be introduced to you.

§3.1 Addition Rule

The Addition Rule is a way to add up things that we sort into different cases.

Example 3.1

Suppose in my collection of cards, I have 20 red cards, 23 blue cards, and 24 green cards. How many cards do I have total?

Intuitively, I have $20 + 23 + 24 = 67$ cards, total right? That's true, but we need to take a closer look on why we can add these. This introduces the concept of disjoint cases and exhaustive cases.

The cases being "disjoint" means there are no overlap between any of the cases. So we have 20 in the sum, we can think of that 20 as the cards in the case where the card is red. Similarly, the 23 comes from the case when the card is blue, and 24 comes from the case when the card is green. But a card isn't more than one color, so no card belongs to more than one case. Imagine if a card belonged to more than one case. A card would be counted excessive times, disrupting the total. That's what it means for the cases to be disjoint: the thing being counted is never included in more than one case.

Sometimes, you'll find no matter what you do, you cannot create cases that are disjoint. We'll see a tool later on that will help with just that.

Now, onto what "exhaustive" means. Well, what if we just did $20+23 = 43$, accounting for only the red and blue cards? The case where the card is red is disjoint from the case where the card is blue. There's no overlap. But we missed a possibility! We need to cover every possibility for the correct total. That's what it means for the cases to be exhaustive: every possibility of the thing being counted is covered.

The example above is a simple application of the Addition Rule in action. Usually, it will be harder to establish what the cases are and/or count the number in each case. But the basic idea of adding up each possibility exactly once still applies.

A more general statement is, suppose you have some things in k cases that are disjoint and exhaustive (where k is some integer). You have t_1 things in the first case, t_2 things in the second case, and so on, until t_k things in the kth case. Then, the number of things you have total is $t_1 + t_2 + \cdots + t_k$. In the cards example, $k = 3$ with $t_1 = 20$, $t_2 = 23$, and $t_3 = 24$. **You cannot use the Addition Rule if the cases are not disjoint or exhaustive.**

> **Caveat 3.2.** Always cite the Addition Rule (abbreviated as "AR") whenever you use it in the course, ever. Also, always say the cases are disjoint and exhaustive when appropriately using it.

§3.2 Multiplication Rule

The Multiplication Rule helps to count things that require a procedure with steps.

Example 3.3

For the dinner, I have to select 1 entree and then 1 dessert. The menu has 9 different entrees and 4 different desserts. How many ways can I select my dinner?

So to select my dinner, I have two steps: choose my entree and then choose my desert. I have 9 ways to select my entree, and then 4 ways to select my dessert. The important thing to note here is, the number of ways I do the second step does not effect the number of ways I do the first step. Thus, I have $9 \cdot 4 = 36$ ways to choose my dinner.

What allows me to use the Multiplication Rule is that the steps are "independent." This means that what happens in one step never effects the number of ways to perform any of the other steps. If the steps are not independent, you cannot use the Multiplication Rule. An easy way to check if the steps are independent is, for each rule in the procedure, does the number of ways to perform this step change depending on what happened in prior steps?

Now, keep in mind, the choices available may be effected by prior steps. But that doesn't matter as long as the number of ways to do the step doesn't change.

Example 3.4

I have 10 different cookies. I want to give one to my sister and then one to my brother. How many ways can I give them cookies?

Suppose one of the cookies was a chocolate chip one and another was a peanut butter one. The first step would presumably be to give my sister a cookie. Suppose I give her the chocolate chip cookie. Now, I no longer have the chocolate chip cookie to give to my brother, but I can still give him the peanut butter cookie. Now, suppose I give my sister the peanut butter cookie. Now, I no longer have the peanut butter cookie to give to my brother, but I can still give him the chocolate chip cookie. So it is clear that based on what I give my sister, the choices on what I give my brother are different. However, notice that when I give my sister a cookie, regardless of which one it is, I have 9 cookies left. And one of those 9 cookies goes to my brother.

So even if what I had available to do the second step (give my brother a cookie) changes, the number of ways does not. This is enough for the steps to be independent. In this example, the procedure would be to first give my sister a cookie and then my brother. There are 10 ways to do the first step and then 9 ways to do the second step. Since the steps are independent, by the Multiplication Rule, I have $10 \cdot 9 = 90$ ways total.

A more general statement is, suppose your procedure has k independent steps (where k is some integer). You have w_1 ways to do the first step, w_2 ways to do the second step, and so on, until w_k ways to do the last step. Then, the total number of outcomes is $w_1 \cdot w_2 \cdot \ldots \cdot w_k$. In the dinner example, $k = 2$ with $w_1 = 9$ and $w_2 = 4$. **You cannot use the Multiplication Rule if the steps are not independent.**

> **Caveat 3.5.** Like AR, always cite the Multiplication Rule (abbreviated as "MR") whenever you use it in the course, ever. Also, always say the steps are independent when appropriately using it.

You will often see the AR and MR together in action. Sometimes, you may be creating a procedure and then realize the steps aren't independent partway through. Perhaps divide into cases that address when the steps no longer become independent. Use MR for the procedure in each separate case and add up the cases with AR at the end.

§3.3 Subsets, Words, Strings, and Permutations

§3.4 Combinations

§3.5 Stars and Bars

§3.6 Anagrams

§3.7 Pascal's Triangle

§3.8 Combinatorial Proof

§3.9 Injections

§3.10 Principle of Inclusion/Exclusion

§3.11 Pigeonhole Principle

§4 Proofs

This is what this course revolves around. You have to be thorough with proofs and account for every possibility, as well as making sure the steps are valid to get from start to finish. If you don't do this, you may end up "proving" incorrect things that don't make sense.

For the sake of demonstration, consider the general statement "All cars are red." We may find a red car. But just because we find a red car, that doesn't mean all of cars are red. We would in theory need to check every single car's color.

We'll see general statements all the time that we need to prove, where we can't just use one example and call it a day. We need to figure out how to be general in our proofs, too. Not only that, but we also need to learn different strategies for proving something and when these proofs may be applicable.

§4.1 Odd and even

Definition 4.1 (Odd integer)**.** If n is odd, then there is an integer k such that $n = 2k + 1$. This works the other way around: if there is an integer k such that $n = 2k + 1$, then n is odd.

For instance, since we know $n = 5$ can be written as $2 \cdot 2 + 1$, we can say $k = 2$ is an integer such that $5 = 2k + 1$. So by the definition, we can say 5 is odd.

We can use this definition the other way around. Suppose now that $k = 2023$ but n is yet to be computed out. Then, we can say $n = 2 \cdot 2023 + 1$ is odd.

Definition 4.2 (Even integer)**.** If n is even, then there is an integer k such that $n = 2k$. This works the other way around: if there is an integer k such that $n = 2k$, then n is even.

Similar logic applies here. For instance, since we know $n = 12$ can be written as $2 \cdot 6$, we can say $k = 6$ is an integer such that $6 = 2k$. So by the definition, we can say 12 is even.

And we can use this definition the other way around. Suppose now that $k = 2023$. Then, we can say $n = 2 \cdot 2023$ is even.

In this course, you'll want to use these two definitions whenever you are given something is odd/even, or whenever you want to prove something is odd/even.

Example 4.3

Suppose x is an odd integer and y is an even integer. Prove that $5x + 7y$ is an odd integer.

So suppose we use $x = 1$ and $y = 2$. Then, $5x + 7y = 19$, which is odd. But like the "all cars are red" example at the beginning of Section 4, just because it works for one choice of x and y, that doesn't mean we can call it a day. We need to be more general. We don't know anything specific about x and y beyond the fact that x is odd and y is even.

Instead, we want to use our definitions of odd and even integer because those definitions are general (they hold for any odd integer and any even integer). So we can say $x = 2a + 1$ for some integer a because x is odd. Also, $y = 2b$ for some integer b because y is even. Now, we want to consider the expression we want to prove as odd. Plugging our representations of x and y in, as well as some algebra implies

$$5x + 7y = 5(2a + 1) + 7(2b) = 10a + 5 + 14b = 10a + 14b + 4 + 1 = 2(5a + 7b + 2) + 1.$$

Something that is helpful to remember is that the sum, difference, or product of any two integers is always an integer. So we can use this to reason out that $5a + 7b + 2$ is some integer. As we can see from the algebra, $5x + 7y = 2(\text{some integer}) + 1$. Using our definition, this means $5x + 7y$ is odd.

Essentially, to solve these types of problems, you want to convert the words to math. You want to use the definitions of odd and even with the given information because they are general. Then, you want to use algebra to get the end goal expression using the new variables. Last, you conclude using the definitions of odd and even.

> **Caveat 4.4.** You don't have to prove that the sum, difference, or product of any two integers is always an integer, but you should mention it in homeworks when you're using it.

§4.2 Divisibility and Primes

Definition 4.5 (Divisibility)**.** For integers n and d, we say n is divisible by d if there is an integer k such that $n = d \cdot k$.

For example, suppose we wanted to check if 15 is divisible by 5. Then, we would plug in $n = 15$ and $d = 5$ above so that we have $15 = 5 \cdot k$. Then, $k = 3$, which is an integer. So there is an integer k, meaning yes, 15 is divisible by 5.

Furthermore, suppose n is divisible by d. Then, we can say d is a divisor of n. We can also say n is a multiple of d.

Often times, when given a condition like "this is divisible by this," you will want to convert the condition to math, similar to the odd and even section. For example, if you are told n is divisible by 3, then you want to write $n = 3k$ for some integer k.

Definition 4.6 (Primes)**.** An integer p is prime if it has exactly 2 positive divisors: itself and 1. Note that 1 is not a prime number.

A property we can use for all primes p (without needing to prove) is $p \geq 2$.

Example 4.7

Suppose p is a prime number and r and s are positive integers such that $p = r \cdot s$. What can we say about r and s?

Based on our definition of divisibility, p is divisible by r, since r and s are integers. But p's only divisors (by the definition of a prime) are itself and 1. So either $r = p$ or $r = 1$. This yields the only possibilities of $r = p$ and $s = 1$, or the other way around. Our conclusion is that whenever $p = r \cdot s$, r and s can only be 1 and p in some order. This example on primes is helpful when you have a prime equals the product of two integers because it forces the integers to be specific values.

Lemma 4.8

2 is the only even prime.

This could be a useful fact on the HW, but you would need to prove it. So let p be an even prime and let r and s be integers such that $p = r \cdot s$. Remember from the example above that whenever $p = r \cdot s$, r and

s can only be 1 and p in some order. By the definition of even, we know 2 is a divisor of p. So we could choose $r = 2$ and that would mean s comes out to an integer. Since r isn't 1, s would have to be the variable equal to 1. So $s = 1$. This forces $p = 2$.

Also, for proving something is not prime, you generally want to find a divisor of that number that is not equal to 1 or the number itself. Alternatively, show that the number is the product of two integers that are both greater than 1.

§4.3 Logic Behind Proofs

§4.4 Predicates and Quantifiers

§4.5 Negation, Converse, and Contrapositive

§4.6 Proof by Contradiction

§4.7 Induction

§4.8 Strong Induction

§5 Terminology and Notation

§5.1 Sets

This section will be definition heavy. You will have to get used to notation. Try to understand it in plain English first, then, use whatever method you used for memorizing vocabulary to get accustomed to the notation.

§5.1.1 Introductory

Definition 5.1 (Set)**.** A set is simply a collection of things. The things could be anything. It could be integers, letters, animals, whatever. It could contain a mix of different things too. A set can have finite size or infinite size.

A set is denoted with square brackets and different elements in the set are separated with commas. For example, $\{1, a, L\}$ is a set containing only 1, a, and L.

Definition 5.2 ($\in$)**.** Let e be a thing and S be a set. We say $e \in S$ (reads e is in S) if e is a member of S. Likewise, we say $e \notin S$ (reads e is NOT in S) if e is NOT a member of S.

For example, let $S = \{1, 2, 3\}$, $e = 2$, and $f = 4$. We can see $e \in S$ and $f \notin S$.

Definition 5.3 (Subset)**.** A subset of a set is a set containing some (maybe all, maybe none) of the elements of the original set. The formal way to think of a subset is, every element of the subset is also an element of the original set. But the intuitive way to think of it is, take the original set. Delete some of its elements (maybe all of them, maybe none of them). Then, boom, you have a subset.

For example, let $S = \{1, 2, 3\}$. We can clearly see $\{1, 2\}$ is a subset of S. But $\{1, 4\}$ wouldn't because 4 isn't in S.

If A is a subset of B, then we use the notation $A \subseteq B$.

Definition 5.4 (Proper Subset)**.** A subset of a set that isn't just the original set.

For example, if $S = \{1, 2, 3\}$, $\{1, 2, 3\}$ is a subset of S but not a proper subset. $\{1, 2\}$ would still be a proper subset of S.

If A is a proper subset of B, then we use the notation $A \subsetneq B$.

Definition 5.5 (Empty Set)**.** The set with no elements. It is noted as $\emptyset$. It is a subset of any set.

Definition 5.6 ($\mathbb{Z}$)**.** The set of all integers. This includes the positive integers: $1, 2, 3, \cdots$. It also includes 0. It also includes the negative integers: $-1, -2, -3, \cdots$.

Definition 5.7 ($\mathbb{Z}^+$)**.** The set of all positive integers. 0 isn't considered positive, so it's not included.

Definition 5.8 ($\mathbb{N}$)**.** The set of all natural numbers. These are the positive integers including 0. (Other definitions of natural numbers don't include 0, but for this course, 0 is a natural number)

§5.1.2 Intermediate

§5.1.3 Set Builder Notation

In plain English, set builder notation goes like: $S = \{\text{thing} \mid \text{some property}\}$. This defines S as the set consisting of all the "things" out there that satisfy the "some property."

For example, $S = \{x \mid x \text{ is an even integer}\}$ defines S as the set of all x such that x is an even integer.

§5.2 Logical Structure

§5.3 Truth Tables

§5.4 Functions

§5.5 Bijections

§6 Probability

§6.1 Probability Space and Events

§6.2 Uniformity

§6.3 Bernoulli Trials

§6.4 Probability Properties

§6.5 Principle of Inclusion/Exclusion for Probability

§6.6 Independence

§6.7 The Monty Hall Problem

§6.8 Conditional Probability

§6.9 Chain Rule

§6.10 Random Variables

§6.11 Expectation

§6.12 Linearity of Exepectation and Indicators

§6.13 Independent Random Variables

§6.14 Variance

§6.15 Correlation

§6.16 Binomial Distribution

§7 Graphs

A graph in this course is a mathematical structure used to model objects and any connections or relations between the objects.

Graphs are fundamental to networks. On social media like Facebook, you may have friendships and your friends may have friendships of their own. That can be modeled with a graph. Look at a map on your phone and see the intersections and roads. That can also be modeled with a graph. Interactions between objects is such a widespread occurrence that graphs can appear in places you wouldn't expect at first.

§7.1 Basics and Definitions

§7.1.1 Introductory

A graph has two key components: vertices and edges.

Think of a vertex as a dot. Then, edges connect pairs of dots.

For now, we will work with simple graphs. This means there is never more than one edge between any pair of vertices. This also means the endpoints of each edge cannot be the same; we cannot have an edge that starts and ends at the same vertex.

The graphs will also be undirected, meaning the edges are all two-way. Later, we will work with graphs that have one-way edges.

So to construct a graph, first, we establish how many vertices we have. Then, we take every possible pair of two vertices and decide whether or not we want to draw an edge between them. After all that is done, boom, we have a graph. It is possible for a graph to have no edges at all.

Definition 7.1 (Neighbors)**.** Vertex a is neighbors with vertex b if a and b are connected via an edge.

Definition 7.2 (Degree)**.** The degree of a vertex a is the number of neighbors that a has. For a simple undirected graph that we are working with now, the degree of a vertex is equal to the number of edges emanating from it. One way of denoting the degree of a vertex a is $\deg(a)$.

Here's some advice to make your life easier: **if you are stuck on a graph problem, try proof by contradiction or proof by contrapositive.** What makes graph theory problems challenging is how unrestricted graphs can be. Think about it: with just 7 vertices, you can have all sorts of configurations of edges. And the number of configurations only increase when there are more vertices. By considering proof by contradiction or contrapositive, you often create a heavy restriction on the graph that you can work with.

§7.1.2 Intermediate

§7.2 Special Graphs

Definition 7.3 (Complete graph)**.** The complete graph with n vertices (denoted as K_n) is the graph where an edge is drawn between every single possible pair of vertices.

Definition 7.4 (Path graph)**.** The path graph with n vertices (denoted as P_n) has the vertices arranged in a line with edges connecting the vertices along the line.

Definition 7.5 (Cycle graph)**.** The cycle graph with n vertices (denotes as C_n) has the vertices arranged in a circle with edges connected the vertices around the circle.

Definition 7.6 (Grid graph)**.** The $m \times n$ grid graph has the vertices arranged like a grid with m rows and n columns with edges creating gridlines.

§7.3 Handshaking Lemma

Lemma 7.7 (Handshaking Lemma)

For any graph with V as its set of vertices and E as its set of edges,

$$\sum_{a \in V} \deg(a) = 2|E|.$$

In plain English, the left hand side is saying that we take every a in the set V and sum the degrees up. So this lemma is saying the sum of the degrees of all the vertices in the graph is twice the number of edges in the graph.

To prove this, consider how we construct the graph, where we first decide the vertices but not drawing any edges yet. At this point, the total sum degree is 0 because no vertex has edges out of it yet. So the equation in the lemma makes sense at this point with both sides being 0. If we add an edge, which involves two endpoints, those endpoints are having their degree increased by 1. So the degree sum increases by 2, increasing the left hand side by 2. The number of edges increases by 1, increasing the right hand side by 2. So the equation still holds if we add an edge. If we add any more edges, it is easy to see, repeating this logic, that the equation still holds. So this proves the lemma because it will hold no matter how many edges or which edges we add.

Whenever you have a problem where you're given a specific condition about the degree of each vertex (like every vertex is of degree 1 or 3), that is generally a sign to use Handshaking Lemma. It also works well with problems involving the parity of the degree of each vertex (like every vertex is of odd degree).

§7.4 Walks and Paths

Definition 7.8 (Walk)**.** A walk is simply a sequence of vertices, where vertices that are adjacent in the sequence have an edge connecting them.

Definition 7.9 (Path)**.** A path is a walk where a vertex never shows up more than once in the sequence. All paths are walks, but not all walks are paths.

Lemma 7.10

A walk in any graph can always be converted into a path.

If the walk contains no repeated vertices, then it's a path by definition.

But what if there are repeated vertices? Here's the intuitive proof that isn't formal, but is easier to understand than how they teach you. Suppose the walk contains the vertex a more than once. So the walk will start somewhere, visit a at some point, do some stuff in between, and then revisit a, and finally continue to finish. Do we really need to do that stuff in between? Why not just continue to the finish right when we get to a instead of goofing off with the stuff in between? We can just delete that "stuff in between" and remove the repeated use of a. If there are still repeated vertices, we can reuse this logic to remove unnecessary parts to the walk until we no longer have repeated vertices. Then, it becomes a path.

Lemma 7.11 (Maximal Path Technique)

There is always a path of maximal length in any graph.

This relies on something called the Well-Ordering Principle, which says in any finite set of real numbers, there is a largest element. You don't have to prove this, just take their word for it.

Imagine if we take a graph, find all the possible paths, and made a set with the lengths of each possible path. This is a finite set. Why? Intuitively, a path can never use the same edge twice (that would easily lead to repeated vertices), and there are only a finite number of edges in the graph. So there is only a finite number of sets of edges and a finite number of orders those edges can be traversed in. Not all sets of edges and orders lead to an actual path, but that means the number of paths in the graph is less than or equal to a finite number.

So the set with the lengths of each possible path is a finite set. Using the Well-Ordering Principle, this set has a largest element. So there is a path of maximal length in the graph. This path may not be unique; there could be more than one such path. The point though is, there is a path of maximal length.

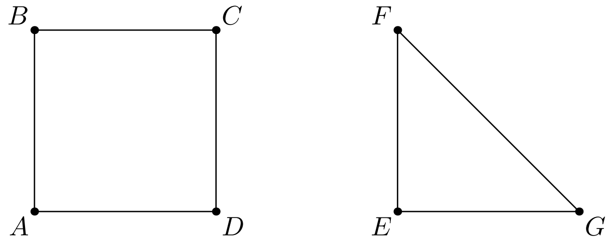
Don't forget this technique. It can be very useful in the homework problems. We'll see later on how it can be powerful when we get to trees and cycles.

§7.5 Connected Components

Definition 7.12 (Connected vertices)**.** Two vertices a and b are connected in a graph if there exists a walk (and by extension, a path, since a walk can be converted to a path) from a to b.

Definition 7.13 (Connected component)**.** A set of vertices such that any two vertices in the set are connected, and we cannot add another vertex to that set so that any two vertices in the set are connected.

It is easier to understand this definition intuitively and with an example. Consider the graph below with vertices A, B, C, D, E, F, and G.



Consider the set of vertices $\{A, B, C\}$. It is obvious any two vertices in the set are connected. However, it is not a connected component as it violates "we cannot add another vertex to that set so that any two vertices in the set are connected." If we add D to the set, then any two vertices in the set are connected. But the set of vertices $\{A, B, C, D\}$ cannot be expanded any further to have any two vertices in the set being connected. So we know $\{A, B, C, D\}$ is a connected component. So we can think of connected components as these isolated "islands" in the graph that include all the vertices in the island. It is also easy to tell by the logic that $\{E, F, G\}$ forms a connected component but something like $\{E, F\}$ would not.

Note that a lone vertex without any edges coming out of it is a connected component on its own.

Definition 7.14 (Connected graph)**.** A connected graph is a graph where any two vertices are connected. The graph is one connected component.

Definition 7.15 (Disconnected graph)**.** The opposite of a connected graph: the graph has more than one connected component. Some two vertices in the graph are not connected.

§7.6 Cycles

§7.7 Forests, Trees, and Leaves

Lemma 7.16

A tree always has at least two leaves.

§7.8 Spanning Trees

§7.9 Coloring

§7.10 Cliques and Independent Sets

§7.11 Directed Graphs

§7.12 Reachability and Strong Connectivity

§7.13 Directed Acyclic Graphs (DAGs)

§7.14 Topological Sorts

§7.15 Binary Tree

§8 Test Strategies

§8.1 General

Both midterms were 60 minutes long, while the final was 2 hours long. There's no time to waste on these tests, and the time is a lot less than you think. Keep in mind, you not only have to solve the problem, but write it up by hand, while making sure to explain your steps. Making errors or not being sure what to do all cost time. And that costed time adds up. Not to mention being mentally fatigued from solving problems back to back, and your arm physically getting tired from writing.

Tips:

- The problems are designed so you can write the solution as you're working through the details.
- You generally need to know what to do immediately or at least know what to try out because of the time pressure. If you have absolutely no clue what to do at first glance, give it a minute to think and if you're stuck, skip it and come back to it.
- Review solutions from homework and in-class. The intuition can help in similar problems that show up on the test.
- Similarly, check the homework rubrics so you know what the TAs want to see in general on solutions. This way, you don't lose points for silly errors.
- Don't cram the day of. Problem solving intuition is something that is built over a period of time and is seen through solving and reflecting on problems.
- Try sitting down and do the practice problems they give you in a timed format and back to back, with write ups being handwritten. That way, you're not as shocked with having to do back to back problems.
- To be more prepared on what will appear, it often helps to think as if you're a TA constructing the test. Look at the possible topics and think "If I were the TA, what would I want to test the students on?"

§8.2 Midterm 1

When I took the course, the semester begun on January 11, while Midterm 1 was February 16. So it takes place a little over a month into the course.

Test statistics (out of 120 points):

- Mean: 74.75
- Standard Deviation: 20.62
- Median: 76
- Max: 113

Tips:

- Expect a combinatorial proof problem.
- The AR and MR rule will show up

§8.3 Midterm 2

Midterm 2 was March 30. So it takes place two and a half months into the course.

Test statistics (out of 120 points):

- Mean: 90.71
- Standard Deviation: 18.81
- Median: 95
- Max: 120

Note that Val said he made this midterm easier after midterm 1. So this might not be the case when you take midterm 2. But it shouldn't be a shocker that scores are higher in this midterm. People now know what to expect on the midterm and aren't easily losing points because they forgot to include a buzzword. They also are no longer surprised by the fast pace of the midterm and the time pressure.

§8.4 Final