Notes et Discussions

Meaning as Grammar *plus* Consequences

Ruy J. G. B. de QUEIROZ*

In his lecture notes on the semantics of his *Intuitionistic Type Theory,* which is meant to be an attempt to clarify the syntax and semantics of intuitionistic mathematics, Martin-Löf seems to provide an account of meaning for his 'language' that lacks the force of a convincing new perspective on the clarification of the term *'constructive'*. Staying within Gentzen's principles, which says that the meaning of a proposition is determined by its *conditions for assertion,* he says:

> (...) The introduction rules say what are the canonical elements (and equal canonical elements) of the set, thus giving its meaning (...)
>
> (Martin-Löf (1984), p. 24.)

Now, what aspect does such an 'introduction' rule of (Gentzen's) proof theory cover with respect to the meaning of a logical proposition? I should like to say that it covers *one* and *only one* of its fundamental aspects, which corresponds roughly to the rule of sentence/proposition formation according to Wittgenstein's later account of propositions.

It is often the case that one automatically accepts an explanation of the *meaning* of an assertion by listening to an exposition of its possible *consequences.* And, yet, one hardly appreciates the important rôle of the *consequences* in determining the meaning of the assertion made. Questions of the sort 'What do you mean?' or 'How do you mean?' are invariably answered via an explanation of what conclusions one can draw from the assertion whose meaning is being questioned.

Analytical studies on the nature and the foundations of language, especially in this second half of the XXth century, have had a very strong bias towards grammatical analysis. A number of important issues related to the (syntactic) structure of language have been clarified. Very little, however, has been the contribution towards the understanding of the rôle of language and the nature of semantics. Fundamental aspects which are naturally beyond grammar have been often neglected. Those are the aspects I

* Imperial College of Science, Technology and Medicine, Department of Computing, London SW7 2BZ.

would like to call the *consequences* of assertions. I take a view which appears to be at least partially congenial with that of Ludwig Wittgenstein's, that grammar constitutes *one* of the aspects of language, by saying:

> Grammar does not tell us how language must be constructed in order to fulfil its purpose, in order to have such-and-such an effect on human beings. It only describes and in no way explains the use of signs.
>
> (*Investigations*, Part I, § 496, p. 138[e].)

In a number of other remarks Wittgenstein attempts to make the point on the insufficiency of grammatical rules in explaining the *use*, the *meaning*, of assertions. In the notes of a series of lectures on religious belief, taken by a small group of students, and published as *Lectures and Conversations on Aesthetics, Psychology and Religious Belief*, we can find a remark of such nature.

> When I say he's using a picture I'm merely making a *grammatical* remark: **[What I say] can only be verified by the consequences he does or does not draw.**
>
> (Lectures and Conversations on APRB, p. 72. The **emphasis** is mine.)

Now, I would like to draw attention to the particular case of logic and mathematics where questions of meaning are of great concern. For the 'language' of mathematics, Wittgenstein seemed to have observed such an important rôle of the *consequences* of an assertion with respect to the meaning of its (major) logical sign. Already in his early days of philosophical work he had a particularly bright intuition on matters regarding logic and the meaning of logical signs, which later on becomes one of his great contributions to the understanding of the nature of semantics. And that was the connection of meaning with logical *consequence*. In a letter to Russell, dated 1.7.12, he writes:

> Will you think that I have gone mad if I make the following suggestion?: The sign $(x).\varphi x$ is not a complete symbol but has meaning only in an inference of the kind: from $\vdash \varphi x \supset_x \psi x.\varphi(a)$ **follows** $\psi(a)$. Or more generally: from $\vdash (x).\varphi x.\epsilon_0(a)$ **follows** $\varphi(a)$. I am — of course — most uncertain about the matter but something of the sort might really be true.
>
> (*Letters to RK&M, R.3*, p. 12. The **emphasis** is mine.)

It seems reasonable to take such remark as an attempt to demonstrate how close are the links between the concepts *meaning, use* — where the proposition '$(x).\varphi(x)$' only has *meaning* in a context where it is *used* —, and *consequences* —where the explanation saying that one is allowed to draw the *consequences* '$\varphi(a)$' from the proposition '$(x).\varphi(x)$' gives it *meaning*—.

Uncertain as he seemed to be, apart from a single remark on how the presence of *generality* in the sign '$(x).\varphi x$' was made clear via its logical *consequence*, which was written in 1914 and published in his *Notebooks 1914-16*, he does not come back to the point on the rôle of *consequences* with respect to meaning, until *circa* 1937-38 when he writes remarks on the foundations of mathematics. In the meantime, under the influence of Russell and Frege he takes a position typical of the phase characterised by what is called 'the early Wittgenstein', where *truth-conditions* were said to constitute meaning. Later on, acknowledging drawbacks in that early account of meaning expressed in the *Tractatus*, he seems to go back to what appeared to be his original intuition, and attempts to recover the place of the *consequences* of an assertion as constituents of the meaning of the signs used:

> (...) And what a proposition is is in one sense determined by the rules of sentence formation (in English, for example) and in another sense by the use of the sign in the language-game. (...)
>
> (*Investigations*, Part I, § 136, p. 53[e].)

Based on my own interpretation of such an account of meaning for the mathematical language (de Queiroz 198?), I claim that (Gentzen's) proof-theoretic semantics considers only the 'grammatical' aspect of logic ('*introduction*' rules) as the one which determines the meaning of logical signs. This 'weakness' of the proof-theoretic account of meaning for the language of mathematics makes proof-theory a less intuitively appealing alternative to classical (realist) semantics. Taking the 'codings' of *normalisation* ('*reduction*') rules as explanations of the *consequences*, in the sense that they show the effect of particular *elimination* rules on the result of *introduction* inferences, the *operations*-based account of meaning, which proof-theory itself was meant to be, becomes more convincing. The Lambda-calculus, e.g., is much more convincing than any natural deduction calculus regarding the account of meaning by providing the definition of 'λ' as constituted by a *grammar* aspect — the '*abstraction*' rule —and a *consequences* aspect — the '*β-reduction*' rule —.

This way one can *understand* it, one can see the *consequences* of it.

It would perhaps be convenient to recall a significant remark on the rôle of the *consequences* from Wittgenstein's late writings:

> For the question is not 'What am I doing when ...?' (for this could only be a psychological question) — but rather, 'What meaning does the statement have, what can be deduced from it, *what consequences does it have?*'
>
> (*RPP II*, § 38, p. 8[e]. The *emphasis* is the Translators'.)

Like this one, many remarks of his later phase make explicit reference to the connections between understanding the *meaning* of an assertion and seeing its *consequences. E.g.:*

> What does anyone tell me by saying "Now I see it as..."? What consequences has this information? What can I do with it?
>
> (*Last Writings I*, § 630, p. 80[e].)

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