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 $\cos(\theta)$ 

0

 $-\sin(\theta)$ 

Out[5]:

 $\sin(\phi)\sin(\theta)$ 

 $\cos(\phi)$ 

0

0.0.1 Visualize WebGL Results in Python1 Compare Accuracy Numerical (WebGL) and Analytic

```
In [1]:
         import sympy as sp
         import numpy as np
         np.set_printoptions(threshold=np.inf) #When we want notebook to display everything
         executed in 6.27s, finished 16:47:27 2021-09-17
In [2]:
         import ipyvolume as ipv # Does not come with default anaconda #Used to show 3D graphs
         import ipywidgets as widgets
         executed in 1.17s, finished 16:47:28 2021-09-17
In [3]: p_x, p_y, p_z, th, phi, = sp.symbols(r'p_x p_y p_z \theta \phi')
         executed in 19ms, finished 16:47:31 2021-09-17
In [4]: A = sp.Matrix([[1,0,0,p_x],
                          [0,1,0,p_y],
                         [0,0,1,-p_z],
                         [0,0,0,1]]
         B = sp.Matrix([[sp.cos(th),0,sp.sin(th),0],
                          [0,1,0,0],
                          [-sp.sin(th),0,sp.cos(th),0],
                          [0,0,0,1]]
         C = sp.Matrix([[1,0,0,0],
                        [0,sp.cos(phi),-sp.sin(phi),0],
                        [0,sp.sin(phi),sp.cos(phi),0],
                        [0,0,0,1]])
         D = sp.Matrix([[1,0,0,-p_x],
                          [0,1,0,-p_y],
                          [0,0,1, pz],
                         [0,0,0,1]]
         executed in 71ms, finished 16:47:31 2021-09-17
In [5]: A*B*C*D
         executed in 1.43s, finished 16:47:34 2021-09-17
```

 $\sin(\theta)\cos(\phi)$ 

 $-\sin(\phi)$ 

0

 $\sin(\phi)\cos(\theta) \cos(\phi)\cos(\theta)$ 

 $-p_x \cos(\theta) + p_x - p_y \sin(\phi) \sin(\theta) + p_z \sin(\theta) \cos(\phi)$ 

 $-p_y\cos(\phi) + p_y - p_z\sin(\phi)$ 

1

 $p_x \sin(\theta) - p_y \sin(\phi) \cos(\theta) + p_z \cos(\phi) \cos(\theta) - p_z$ 

```
In [6]: #Calculate numbers needed for 2D surface plotter
        def GetPoints(x,y):
            input
            x: np array
                the x positions of the grid points
            y: np array
                the y positions of the grid points
            returns
            np array
                with the position of the points going from bottom left to bottom right, then the
             1.1.1
            mesh = np.meshgrid(x,y)
            x_{parts} = np.reshape(mesh[0],(1,-1))[0]
            y_{parts} = np.reshape(mesh[1],(1,-1))[0]
            points = np.array([[x_parts[i],y_parts[i]] for i in range(len(x_parts))])
            return np.reshape(points,(1,-1))[0]
        ##Tests
        actual = GetPoints([0,1,2],[0,1])
        expected = np.array([0,0,1,0,2,0,0,1,1,1,2,1])
        assert max(abs(actual-expected))==0
        executed in 20ms, finished 16:47:52 2021-09-17
```

```
In [7]: def GetIndices(x, y):
            input
            x: np array
               the x positions of the grid points
            y: np array
                the y positions of the grid points
            returns
            np array
                with the indices of all the linking points to make 3 vertex shapes
            w = len(x) #width
            def get_pos(x_pos, y_pos):
                return w*y_pos + x_pos #get the index position
            def get_points(x_pos, y_pos):
                return np.array([get_pos(x_pos,y_pos), get_pos(x_pos+1,y_pos), get_pos(x_pos,y_r
                                get_pos(x_pos+1,y_pos), get_pos(x_pos,y_pos+1), get_pos(x_pos+1)
            indices = []
            for y_pos in range(len(y)-1):
                for x_{pos} in range(len(x)-1):
                    indices.append(get_points(x_pos, y_pos))
            return np.reshape(np.array(indices),(1,-1))[0]
        ##Tests
        actual = GetIndices([0,1,2],[0,1])
        expected = np.array([0,1,3,
                            1,3,4,
                            1,2,4,
                            2,4,51)
        assert max(abs(actual-expected)) == 0
        executed in 18ms, finished 16:47:56 2021-09-17
In [8]: x = np.linspace(-2,2,101)#41)
        y = np.linspace(-2,2,101)#41)
        GetPoints(x,y)
        executed in 367ms, finished 16:47:58 2021-09-17
Out[8]: array([-2. , -2. , -1.96, -2. , -1.92, -2. , -1.88, -2. , -1.84,
               -2. , -1.8 , -2. , -1.76, -2. , -1.72, -2. , -1.68, -2. ,
               -1.64, -2. , -1.6 , -2. , -1.56, -2. , -1.52, -2. , -1.48,
               -2. , -1.44, -2. , -1.4 , -2. , -1.36, -2. , -1.32, -2. ,
               -1.28, -2. , -1.24, -2. , -1.2 , -2. , -1.16, -2. , -1.12,
               -2. , -1.08, -2. , -1.04, -2. , -1. , -2. , -0.96, -2. ,
               -0.92, -2. , -0.88, -2. , -0.84, -2. , -0.8 , -2. , -0.76,
               -2. , -0.72, -2. , -0.68, -2. , -0.64, -2. , -0.6 , -2. ,
               -0.56, -2. , -0.52, -2. , -0.48, -2. , -0.44, -2. , -0.4 ,
               -2. , -0.36, -2. , -0.32, -2. , -0.28, -2. , -0.24, -2. ,
               -0.2 , -2. , -0.16, -2. , -0.12, -2. , -0.08, -2. , -0.04,
               -2. , 0. , -2. , 0.04, -2. , 0.08, -2. , 0.12, -2. ,
               0.16, -2. , 0.2 , -2. , 0.24, -2. , 0.28, -2. , 0.32,
               -2. , 0.36, -2. , 0.4 , -2. , 0.44, -2. , 0.48, -2. ,
               0.52, -2. , 0.56, -2. , 0.6 , -2. , 0.64, -2. , 0.68,
               -2. , 0.72, -2. , 0.76, -2. , 0.8 , -2. , 0.84, -2. ,
               0.88, -2. , 0.92, -2. , 0.96, -2. , 1. , -2. , 1.04,
               -2. , 1.08, -2. , 1.12, -2. , 1.16, -2. , 1.2 , -2. ,
                1.24, -2. , 1.28, -2. , 1.32, -2. , 1.36, -2. , 1.4 ,
```

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In [9]:
         GetIndices(x,y)
          executed in 431ms, finished 16:48:02 2021-09-17
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```

## 0.0.1 Visualize WebGL Results in Python ¶

```
In [1]:
    def read_file(file_name):
        f = open(file_name, "r")
        my_string_data = f.read().splitlines()
        my_Z = []
        for row_string_data in my_string_data:
            row_string_list = row_string_data.split(',')
            my_time_frame_row = []
            for element_index in range(len(row_string_list)-1):
                  my_time_frame_row.append(float(row_string_list[element_index]))
            my_Z.append(np.array([my_time_frame_row]))
            return np.array(my_Z)
    executed in 17ms, finished 13:28:40 2021-09-07
```

```
In [2]: #File not available on computing exporting as PDF
        Z_Num = read_file("E:\Downloads\Test (17).txt")
        Num_xy = len(Z_Num[0][0])**0.5
        print(Num xy)
        a = np.linspace(0, 1, num=int(Num_xy), endpoint=True)
        b = np.linspace(0, 1, num=int(Num_xy), endpoint=True)
        U, V = np.meshgrid(a, b)
        X = U
        Y = V
        ipv.figure()
        s = ipv.plot surface(X, Y, Z Num, color="orange")
        #w = ipv.plot_wireframe(X, Y, Z, color="red")
        ipv.animation_control(s, add = True, interval=200)#, sequence_length=2)
        #mylink = widgets.link((s, 'sequence_index'), (w, 'sequence_index'))
        ipv.show()
        executed in 169ms, finished 13:28:41 2021-09-07
```

22.0

## 1 Compare Accuracy Numerical (WebGL) and Analytic

```
In [138]: #Analytic Solution
MAX_N = 100

def Q_n(n_x,n_y):
    return np.pi*(n_x**2+n_y**2)**0.5

def B_n(n_x, n_y):
    return 4*(-1)**(n_x+n_y)/(n_x*n_y*np.pi**2)

def u(x,y,t):
    total = 0
    for n_x in range(1, MAX_N):
        for n_y in range(1, MAX_N):
            total += B_n(n_x,n_y)*np.sin(n_x*np.pi*x)*np.sin(n_y*np.pi*y)*np.cos(Q_n(n_y)*np.total
    executed in 9ms, finished 00:25:38 2021-08-18
```

```
In [101]: #Put time and run commands
X = U
Y = V # The y-axis is the wrong way around
#Z = np.array([[np.real(u(x,y)) for x in a] for y in b])
Z_Ana = np.array([[[u(x,y,t) for x in a] for y in b] for t in [0,1,2,3,4.4,16]])#np.lins
ipv.figure()
s = ipv.plot_surface(X, Y, Z_Ana, color="orange")
#w = ipv.plot_wireframe(X, Y, Z, color="red")
ipv.animation_control(s, add = True, interval=200)
#mylink = widgets.link((s, 'sequence_index'), (w, 'sequence_index'))
ipv.show()
executed in 6.69s, finished 23:43:09 2021-08-17
```

VBox(children=(Figure(animation=200.0, camera=PerspectiveCamera(fov=45.0, position=(0.0, 0.0, 2.0), quaternion...

```
In [ ]: #Compare
In [ ]: #Z_Num[0]-Z_Ana[0].reshape(1,-1)
    print(u(0.90,0.75,0))
    Z_Num[0][0]
```