$$x[n] = \begin{cases} \alpha^n & 0 \le n \le N_1 \\ 0 & N_1 < n < N_2 \\ \alpha^{n-N_2} & N_2 \le n \le N_1 + N_2 \\ 0 & \text{otherwise} \end{cases}$$

$h[n] = \beta^n u[n]$.

$$x[n] \longrightarrow \boxed{\begin{matrix} LSI \\ h[n] \end{matrix}} \longrightarrow y[n]$$

let signal, $g[n] = \begin{cases} \alpha^n & 0 \le n \le N_1 \\ 0 & \text{otherwise} \end{cases}$

then, we can see that $x[n]$ can be written as summation of $g[n]$ & $g[n-N_2]$

$\Rightarrow \quad x[n] = g[n] + g[n-N_2]$

Now if $\quad g[n] \longrightarrow \boxed{\begin{matrix} LSI \\ h[n] \end{matrix}} \longrightarrow l[n]$ then,

from linearity & shift invariance property,

$$y[n] = l[n] + l[n-N_2].$$

$$l[n] = g[n] * h[n]$$

$$= \cancel{\sum_{k=0} \alpha^k} \quad \sum_{k=0}^{N_1} \alpha^k \beta^{n-k} u[n-k] \quad \text{for } n \ge 0$$

and $l[n] = 0$ for $n < 0$

$\therefore$ for $n < N_1$

$$l[n] = \sum_{k=0}^{n} \alpha^k \beta^{n-k} = \beta^n \sum_{k=0}^{n} \left(\alpha/\beta\right)^k = \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \quad \text{(from a7 Sollution)}$$

& for $n > N_1$

$$l[n] = \sum_{k=0}^{N_1} \alpha^k \beta^{n-k} = \beta^n \frac{\left[1 - (\alpha/\beta)^{N_1+1}\right]}{1 - \alpha/\beta} = \frac{\beta^{n+1}\left[1 - \left(\frac{\alpha}{\beta}\right)^{N_1+1}\right]}{\beta - \alpha}$$

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$$y[n] = l[n] + l[n-N_2]$$

$$l[n] = \begin{cases} \dfrac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} & 0 \leq n \leq N_1 \\[2ex] \dfrac{\beta^{n+1}\left[1 - (\alpha/\beta)^{N_1+1}\right]}{\beta - \alpha} & n \geq N_1 \\[2ex] 0 & \text{otherwise.} \end{cases}$$

$$l[n-N_2] = \begin{cases} \dfrac{\beta^{n-N_2+1} - \alpha^{n-N_2+1}}{\beta - \alpha} & N_2 \leq n \leq N_1 + N_2 \\[2ex] \dfrac{\beta^{n-N_2+1}\left[1 - (\alpha/\beta)^{N_1+1}\right]}{\beta - \alpha} & n \geq N_1 + N_2 \\[2ex] 0 & \text{otherwise} \end{cases}$$

$$y[n] = \begin{cases} \dfrac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} & 0 \leq n \leq N_1 \\[2ex] \dfrac{\beta^{n+1}\left[1 - (\alpha/\beta)^{N_1+1}\right]}{\beta - \alpha} & N_1 \leq n \leq N_2 \\[2ex] \dfrac{\beta^{n+1}\left[1 - (\alpha/\beta)^{N_1+1}\right]}{\beta - \alpha} + \dfrac{\beta^{n-N_2+1} - \alpha^{n-N_2+1}}{\beta - \alpha} & N_2 \leq n \leq N_1 + N_2 \\[2ex] \dfrac{\left[\beta^{n+1} + \beta^{n-N_2+1}\right]\left[1 - (\alpha/\beta)^{N_1+1}\right]}{\beta - \alpha} & n > N_1 + N_2 \\[2ex] 0 & \text{otherwise} \end{cases}$$