Nonuniform Quantizer
(Lloyd-Max Quantizer)

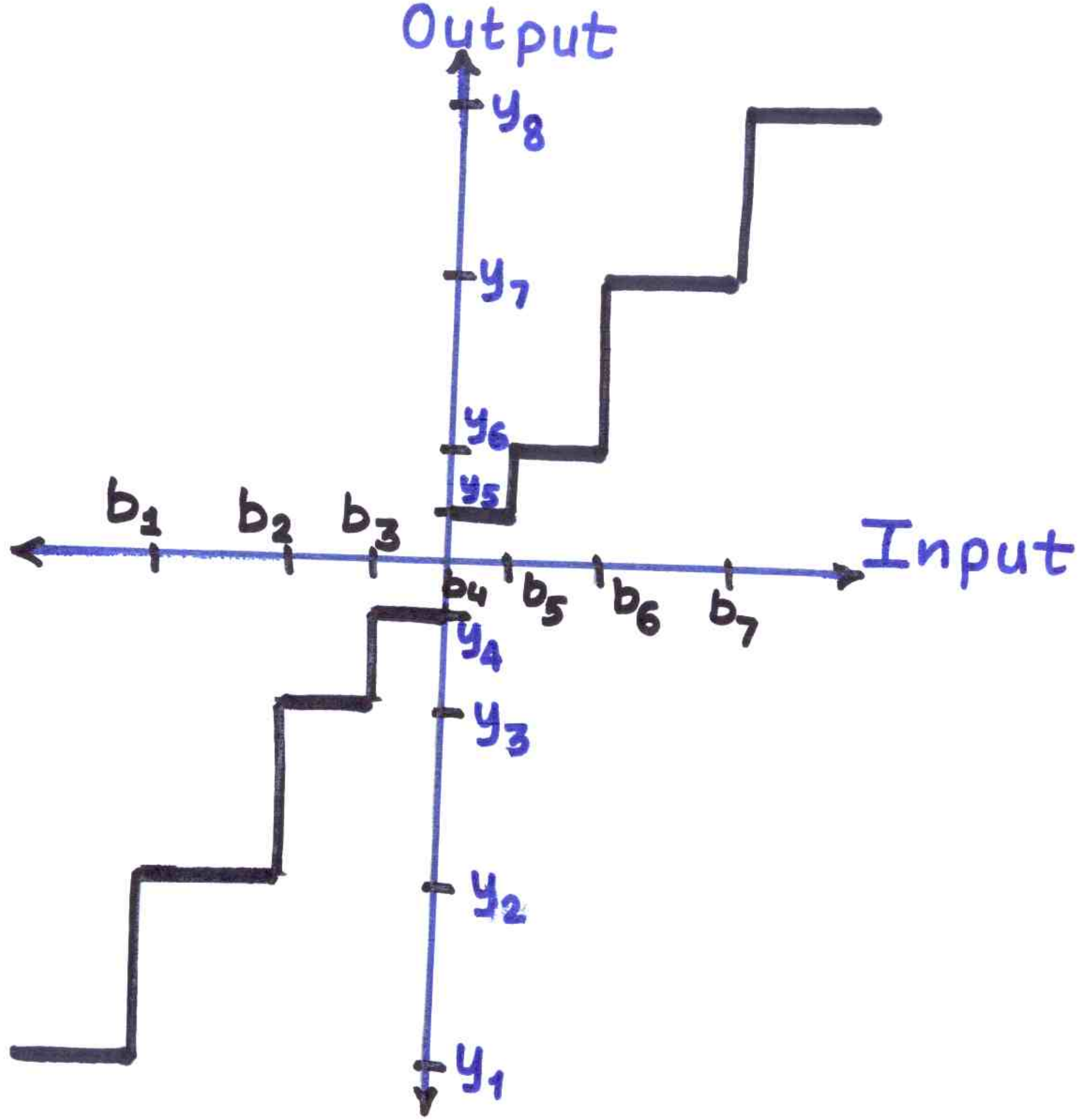
Uniform Quantizer : Δ

DBs: $b_1, b_2, \ldots b_{L-1}$

RLs: $y_1, y_2, \ldots y_L$

minimize $\sigma_q^2(b_1, b_2, \ldots b_{L-1}, y_1, y_2 \ldots y_L)$

A Nonuniform midrise Quantizer



$$\sigma_q^2 = \sum_{j=1}^{L} \int_{b_{j-1}}^{b_j} (x - y_j)^2 f_X(x)\,dx$$

$$\frac{\partial \sigma_q^2}{\partial y_j} = -\int_{b_{j-1}}^{b_j} 2(x - y_j) f_X(x)\,dx = 0$$

$$y_j = \frac{\int_{b_{j-1}}^{b_j} x f_X(x)\,dx}{\int_{b_{j-1}}^{b_j} f_X(x)\,dx}$$

$$\sigma_q^2 = \sum_{j=1}^{L} \int_{b_{j-1}}^{b_j} (x - y_j)^2 f_X(x)\, dx$$

$$= \cdots\cdots + \int_{b_{j-1}}^{b_j} (x - y_j)^2 f_X(x)\, dx + \int_{b_j}^{b_{j+1}} (x - y_{j+1})^2 f_X(x)\, dx$$

$$+ \cdots\cdots$$

Leibnitz's rule states that if $b(x)$ and $a(x)$ are monotonic, then

$$\frac{\partial}{\partial x}\left[\int_{a(x)}^{b(x)} \phi(\alpha, x)\, d\alpha\right] = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x}\phi(\alpha, x)\, d\alpha$$

$$+\phi\left(\alpha = b(x); x\right)\frac{\partial}{\partial x} b(x)$$

$$-\phi\left(\alpha = a(x); x\right)\frac{\partial}{\partial x} a(x)$$

$$\frac{\partial \sigma_q^2}{\partial b_j} = 0 \Rightarrow$$

$$f_x(b_j)(b_j - y_j)^2 - f_x(b_j)(b_j - y_{j+1})^2 = 0$$

$$\Rightarrow (b_j - y_j)^2 = (b_j - y_{j+1})^2$$

$$\Rightarrow (b_j - y_j) = y_{j+1} - b_j \quad \left(\because \; b_j > y_j \;\&\; b_j < y_{j+1} \right).$$

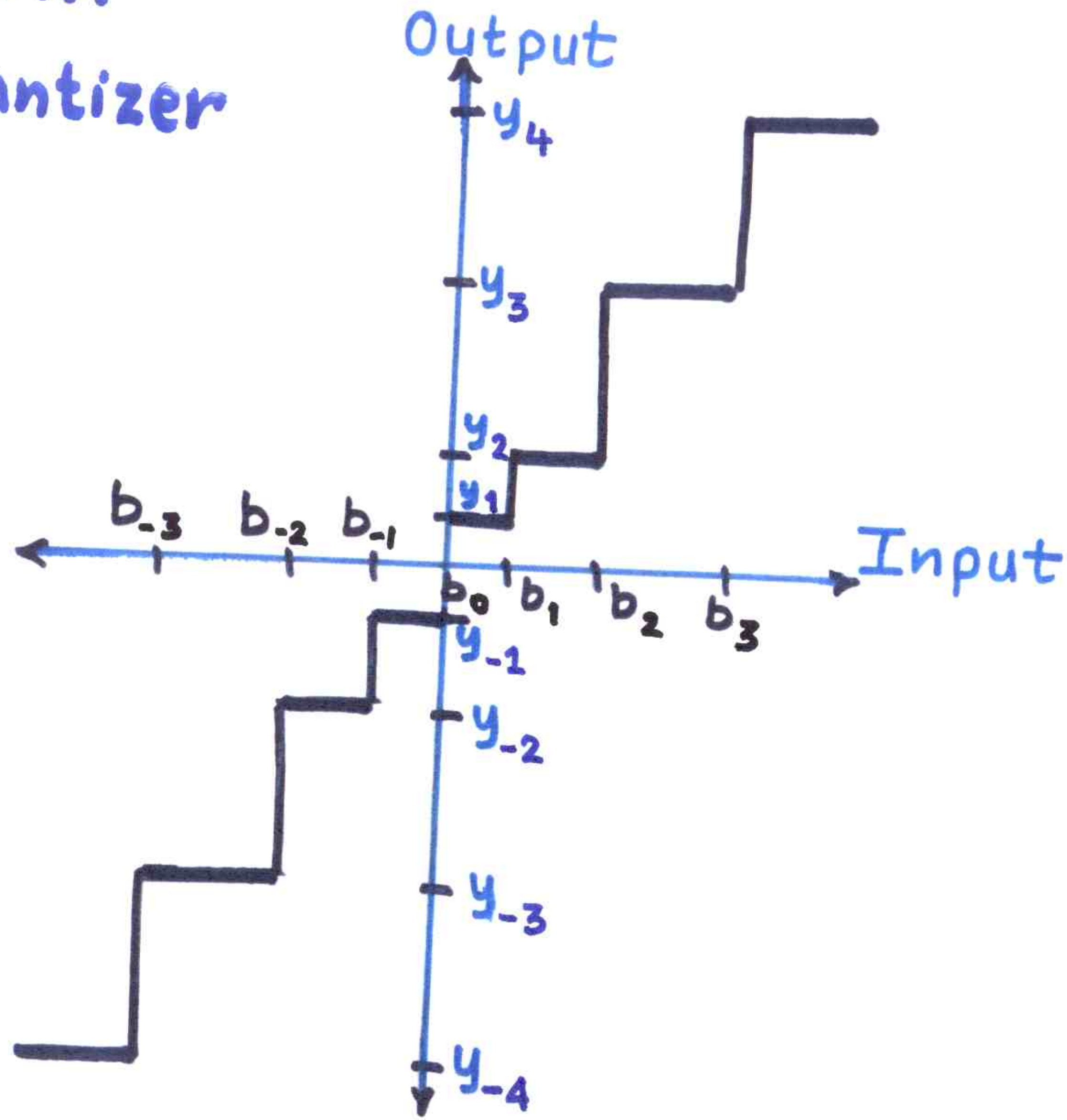
$$\Rightarrow b_j = \frac{(y_{j+1} + y_j)}{2}$$

Lloyd-Max Algorithm

- L-level symmetric midrise quantizer
- Obtain the reconstruction levels (RLs): $\{y_1, y_2, \dots y_{\frac{L}{2}}\}$

DBs: $\{b_1, b_2, \dots b_{\frac{L}{2}-1}\}$

A nonuniform midrise quantizer



- $y_1 = \dfrac{\int_{b_0}^{b_1} x f_X(x)\,dx}{\int_{b_0}^{b_1} f_X(x)\,dx}$

b_0 is ZERO

Guess y_1 and solve for b_1 numerically

$b_1 = \dfrac{y_2 + y_1}{2} \Rightarrow y_2 = 2b_1 - y_1$

$$y_2 = \frac{\int_{b_1}^{b_2} x f_X(x)\,dx}{\int_{b_1}^{b_2} f_X(x)\,dx}$$

Solve for b_2 numerically

$$y_3 = 2b_2 - y_2$$

$\{ y_1, y_2, \cdots y_{\frac{L}{2}} \}$ and

$\{ b_1, b_2, \cdots b_{\frac{L}{2}-1} \}$

$y_{\frac{L}{2}}$ is the centroid of the probability mass of the interval $[b_{\frac{L}{2}-1}, b_{\frac{L}{2}}]$

$$\hat{y}_{\frac{L}{2}} = \frac{\displaystyle\int_{b_{\frac{L}{2}-1}}^{b_{\frac{L}{2}}} x f_X(x)\,dx}{\displaystyle\int_{b_{\frac{L}{2}-1}}^{b_{\frac{L}{2}}} f_X(x)\,dx}$$

TABLE: QUANTIZER BOUNDARY AND RECONSTRUCTION LEVELS FOR NONUNIFORM GAUSSIAN AND LAPLACIAN QUANTIZERS

Levels	Gaussian			Laplacian		
	b_i	y_i	$(SNR)_q$	b_i	y_i	$(SNR)_q$
4	0.0	0.4528		0.0	0.4196	
	0.9816	1.510	9.3 dB	1.1269	1.8340	7.54 dB
6	0.0	0.3177		0.0	0.2998	
	0.6589	1.0		0.7195	1.1393	
	1.447	1.894	12.41 dB	1.8464	2.5535	10.51 dB
8	0.0	0.2451		0.0	0.2334	
	0.7560	0.6812		0.5332	0.8330	
	1.050	1.3440		1.2527	1.6725	
	1.748	2.1520	14.62 dB	2.3796	3.0867	12.64 dB
8-level PDF optimised UQ			14.27 dB			11.39 dB