EE 338 Digital Signal Processing: Tutorial 5

Spring 2015, Instructor: Prof. Vikram M. Gadre

1. Consider the system in Figure 1. $h_1[n] = \delta[n-1]$ and $h_2[n] = a^n u[n]$. Find the impulse response $h[n]$ of overall system. Find the frequency response of overall system. Can you comment on the stability and causality of the overall system? What happens if $h_1[n]$ and $h_2[n]$ are interchanged?

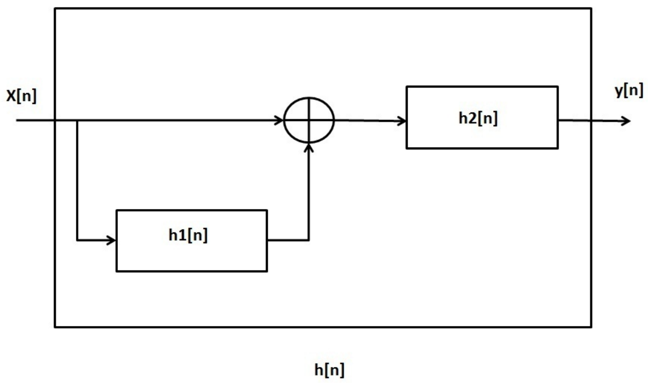

Figure 1: Figure for question 1

2. Consider a continuous time signal $x_c(t) = sin(10\pi t) + cos(30\pi t)$ which is sampled with period T to get the discrete time signal $x[n] = sin(\frac{\pi}{5}n) + cos(\frac{3\pi}{5}n)$. Determine the value of T ? Is it unique? Give one example where T is not unique i.e. can you obtain the same discrete-time signal form continuous-time signal having two sinusoids.

3. Consider a discrete time signal $x[n]$ which has an even symmetry. The DTFT of $x[n]$ is periodic with period π. Find the value of $x[1]$. If there exist sequences $y_1[n]$ and $y_2[n]$ such that $y_1[n] = x[2n]$ and $y_2[n] = x[3n]$, how can one obtain $x[n]$ from $y_1[n]$ and $x[n]$ from $y_2[n]$? **(Reference: Problem 4.9, Discrete Time Signal Processing by Oppenheim, Schafer, Buck, 2nd Edition, Pearson Education)**

4. Consider the system shown in Figure 2. DTFT of discrete time system is given

by

$$H(e^{j\omega}) = \frac{j\omega}{T} \qquad -\pi \leq \omega < \pi \tag{1}$$

$T = \frac{1}{20}$. Find the output $y_c(t)$ for each of the following inputs $x_c(t)$.

(a) $x_c(t) = cos(12\pi t)$

(b) $x_c(t) = cos(28\pi t)$

Note that the system is differentiator. Can you directly obtain outputs $y_c(t)$ from inputs $x_c(t)$ by differentiating? Do the outputs obtained using two different methods match? **(Reference: Problem 4.12, Discrete Time Signal Processing by Oppenheim, Schafer, Buck, 2nd Edition, Pearson Education)**

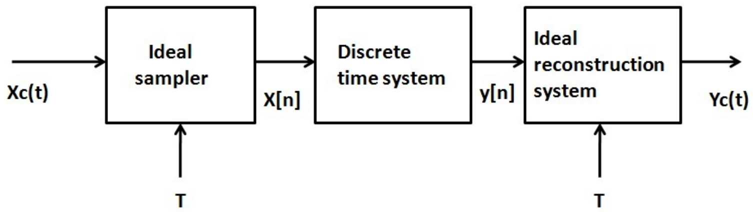

Figure 2: Figure for question 4

5. Consider a continuous time signal $x_c(t)$ such that $x_c(t) = 0$ for $t < 0$ and $t > t_0$. Also, $x_c(t)$ is band limited to $3\pi \times 10^4$ i.e. $X_c(j\Omega) = 0$ for $|\Omega| \geq 3\pi \times 10^4$. We want to estimate E which is the area under the signal $x_c(t)$ for time duration from $t = 0$ to $t = t_0$. However this needs to be done using discrete time system by initially sampling the $x_c(t)$ with sampling period of T and then using an appropriate discrete time system. Assume ideal sampling. Is it possible to obtain the estimate E? If no, explain why. If yes, what are the bounds on sampling period T and which discrete time system will be used? What is the role of t_0 in the realization of discrete time system? **(Reference: Problem 4.45, Discrete Time Signal Processing by Oppenheim, Schafer, Buck, 2nd Edition, Pearson Education)**

6. Consider a causal LTI system described by

$$y[n] + \frac{1}{3}y[n-1] = x[n] \tag{2}$$

Find the response of the system to each of the input given below.

(a) $x[n] = \delta[n] + 3\delta[n-1]$

(b) $x[n] = \frac{1}{3}u[n]$

Find the response of the system to the input with following DTFT.

(a) $X(e^{j\omega}) = \frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}$

(b) $X(e^{j\omega}) = 1 + 2e^{-3j\omega}$

7. Consider the signal

$$x[n] = sin\left(\frac{\pi n}{6}\right) - cos\left(\frac{\pi n}{3}\right) \tag{3}$$

Determine the output for each of the following LTI system.

(a) $h[n] = \frac{sin(\pi n/6)}{\pi n}$

(b) $h[n] = \frac{sin(\pi n/4)}{\pi n} + \frac{sin(\pi n/2)}{\pi n}$

(c) $h[n] = \frac{sin(\pi n/4) sin(\pi n/8)}{\pi^2 n^2}$

(d) $h[n] = \frac{sin(\pi n/4) sin(\pi n/8)}{\pi n}$

8. Consider a signal $x[n] = \alpha e^{j\omega_0 n} + \beta e^{j\omega_1 n} + \gamma e^{j\omega_2 n}$. What is the length of impulse response $h[n]$ of a system (non-trivial) such that $x[n] * h[n] = 0$.