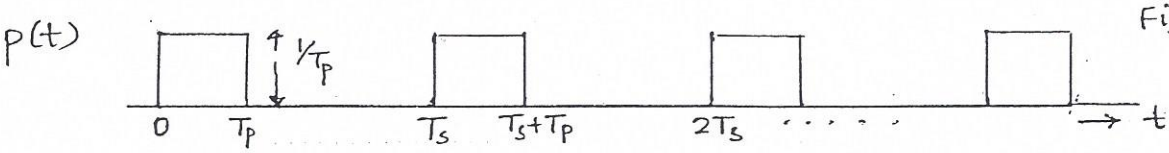
EE338 **DIGITAL SIGNAL PROCESSING**
TUTORIAL PROBLEMS – SET ONE

The aim of these problems is to understand the sampling theorem a little better. One would also like to understand the practical limitations in sampling. These are not mandatory problems for the course, but intend to revise some analog sampling concepts.

Q1. Obtain the Fourier series coefficients of the periodic train of pulses p(t) shown in Fig.1. It is given that $0 < T_p < T_s$. Hence obtain the Fourier Transform of this waveform. (The Fourier Transform would be a train of pulses located at all multiples of the fundamental frequency with strengths proportional to the spectral coefficient amplitudes).

Fig. 1



Q2. Now obtain the Fourier Transform of the product of a signal x(t) with spectrum $X(\Omega)$ as shown below, in Fig. 2; and the train p(t) in Fig. 1. Consider two different cases:

(i) $2\pi / T_s > 2B$.
(ii) $B < 2\pi / T_s < 2B$.

Sketch the resultant spectrum in each case.

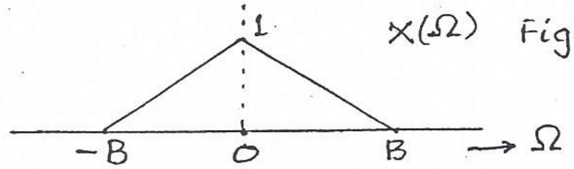


Fig. 2

Q3. What happens to the Fourier Transform in Q1 above, as T_p tends towards zero? Comment on the corresponding changes in the result of Q2.
On the other hand, what happens when T_p tends towards T_s? Similarly comment on the corresponding changes in the result of Q2 and explain.

Q4. (slightly difficult problem – for further thought): Now, consider the sample-and-hold circuits of Figs. 3 and 5. Assume that the pulse train is applied to each of the gates of the Field Effect Transistors in the manner indicated.

(i) Assume T_p is much less than T_s. Show that the system of Fig. 3 is well approximated by the cascade of an Ideal Sampler, and another linear shift-invariant system with the impulse response shown in Fig. 4.
(ii) Suppose the system of Fig. 3 is replaced by the one of Fig. 5. What degradation/ modification would result in the analysis of part (i) above?
(iii) Now assume that T_p is less than T_s but **not** much less. What changes would then result in the analysis?

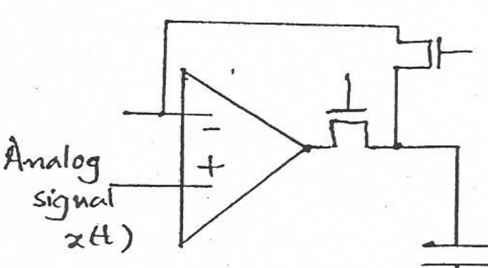


Fig. 3

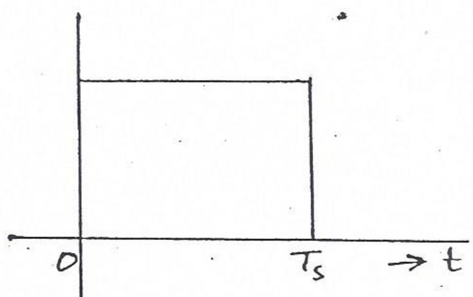


Fig. 4

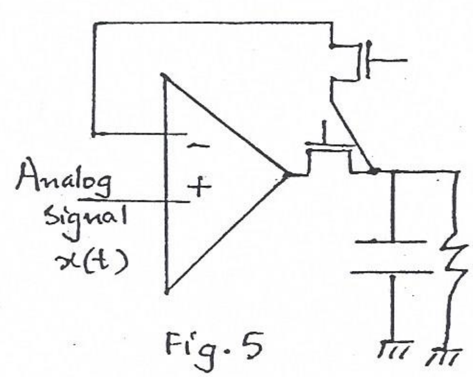


Fig. 5

Assume ideal operational amplifiers

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TUTORIAL PROBLEMS – SET TWO

Q1. Show, through examples, that the following properties of discrete-time systems are completely independent of one another, i.e. a discrete time system may possess ANY subset of them without possessing the others:
(a) Additivity (b) Homogeneity or scaling (c) Causality
(d) Memory (e) Stability (f) Shift-invariance or time-invariance

Q2. Two sequences x[n] and h[n] are nonzero only at the points specified as under. They are zero at all other points. Obtain the sequence y[n] = x[n] convolved with h[n].

n	-2	-1	0	1	2	3
x[n]	1	2	-1	4	3	5

n	2	3	4	5	6
h[n]	1	-1	2	-2	3

Q3. It is given that the input to a system is an all zero sequence, i.e. $x[n] = 0 \ \forall\ n$; and the system obeys *atleast one of the properties of* additivity or homogeneity. Show that the output of the system is also the all-zero sequence $y[n] = 0 \ \forall\ n$.

Q4. Prove that, if the input to a discrete time LSI system is periodic with a period N_0, the output is also periodic with period N_0.

Q5. The nonzero samples of a sequence x[n] lie in the range $N_1 \geq n \geq N_0$. The nonzero samples of another sequence h[n] lie in the range $N_3 \geq n \geq N_2$. Here N_0, N_1, N_2, N_3, are all integers – they could be positive or negative or zero. Show that the nonzero samples of the sequence obtained by convolving x[n] with h[n] lie in the range $(N_1 + N_3) \geq n \geq (N_0 + N_2)$. Illustrate with an example.

Q6. Think of a bank as a discrete-time system where the input is a sequence of deposits made into a given account in the nth month, or withdrawals, by the account holder. Assume that a certain percentage of the balance in the previous month, say p_1 percent, and another percentage of the balance in the month previous to that, say p_2 percent are credited to the balance in the current month as interest. Describe this system mathematically relating x[n] and y[n]: the input and output sequences respectively. Under what circumstances will it be shift-invariant; and under what circumstances not so?

Q7. Obtain the output sequence in an LSI system with input sequence $x[n] = \alpha^n u[n]$; and impulse response $h[n] = \beta^n u[n]$. Assume both $|\alpha|$ and $|\beta|$ to be less than 1.

Q8. In Q7, let the input sequence be replaced by

$x[n] = \{\ 0 \ \forall\ n < 0;\ \alpha^n \ \forall\ N_1 \geq n \geq 0;\ |\alpha| < 1;\ 0 \ \forall\ N_2 > n > N_1;\ \alpha^{n-N2} \ \forall\ N_2 + N_1 \geq n \geq N_2;$ and 0 for all n greater than $N_2 + N_1$. $\}$

Find the output sequence using the properties of linearity and shift-invariance.

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TUTORIAL PROBLEMS – SET THREE

Q1. Let the input sequence x[n] and impulse response sequence h[n] of a discrete time LSI system be:

(a) summable, i.e. Σ_n x[n] is finite, Σ_n h[n] is finite, with sums Σ_x and Σ_h respectively. Show that the output, if summable, has the sum $\Sigma_x\Sigma_h$.

(b) absolutely summable, i.e. Σ_n |x[n]| is finite, Σ_n |h[n]| is finite, with absolute sums X_0 and H_0 respectively. Show that the output, if absolutely summable, has an absolute sum upper bounded by X_0H_0 .

Q2. Consider the following two discrete time LSI systems:

(a) y[n] = { x[n] + x[n-1] } /2; (b) y[n] = { x[n] - x[n-1] } /2;

(i) Obtain their impulse responses; $h_a[n]$ and $h_b[n]$ respectively.

(ii) Obtain their frequency responses $H_a(\omega)$ and $H_b(\omega)$ respectively.

(iii) Let the input sequence $x[n] = \cos \omega_0 n$ be applied to each of these systems. Here ω_0 is between 0 and π. Obtain the output sequences $y_a[n]$ and $y_b[n]$ respectively *without* using the impulse response or frequency response. You may use trigonometric identities. e.g. cos A + cos B = 2 cos….cos…. etc.

(iv) Now correlate your result of part (iii) with that of part (ii).

(v) Find the inverse Discrete Time Fourier Transforms (inverse DTFTs) of $H_a(\omega)$ and $H_b(\omega)$: verify that they are indeed $h_a[n]$ and $h_b[n]$ respectively.

(vi) Obtain and sketch the magnitude and phase of $H_a(\omega)$ and $H_b(\omega)$ as a function of ω for the region ω: 0 to π. Approximately speaking, what kind of filters can we call them?

Q3. The idealized frequency responses of the four standard kinds of digital filters are shown in Fig. 4-1 to Fig. 4-4. Obtain the idealized impulse responses in each case by evaluating the inverse DTFT. *Hint:* Use the result of Fig. 4-1 to evaluate the others. Assume that the impulse responses are real, and that the phase response is zero for all frequencies.