

Tutorial -3

1) a) $\sum_n x[n] = \sum_x$ $\sum_n h[n] = \sum_h$

Both are finite

Suppose $\sum_n y[n]$ exists,

$$\begin{aligned}\sum_n y[n] &= \sum_n \left\{ \sum_k x[k] h[n-k] \right\} \\ &= \sum_k x[k] \left\{ \sum_n h[n-k] \right\} \\ &= \sum_k x[k] \sum_h \\ &= \underline{\underline{\sum_x \sum_h}}\end{aligned}$$

$$\begin{aligned}\sum_n h[n-k] &= \sum_{n=-\infty}^{\infty} h[n-k] \\ n-k &= l \\ &= \sum_{l=-\infty}^{\infty} h[l] \\ &= \sum_h\end{aligned}$$

b) $\sum_n |x[n]| = X_0$ $\sum_n |h[n]| = H_0$

Both are finite

Suppose $\sum_n |y[n]|$ exists

$$\sum_n |y[n]| = \sum_n \left| \sum_k x[k] h[n-k] \right|$$

Using the property $|\sum_i a_i| \leq \sum_i |a_i|$, we get

$$\begin{aligned}\sum_n |y[n]| &\leq \sum_n \sum_k |x[k] h[n-k]| \\ &= \sum_n \sum_k |x[k]| |h[n-k]| \\ &= \sum_k |x[k]| \left\{ \sum_n |h[n-k]| \right\} \\ &= \sum_k |x[k]| H_0 \\ &= X_0 H_0\end{aligned}$$

Therefore, $\sum_n |y[n]| \leq X_0 H_0$. Thus the output if absolutely summable has the upper bound given by $X_0 H_0$.