EE 334
Response of LTI Circuits to AC and DC Inputs

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AC and DC for Electrical Supply Systems

1. Why is DC or (single frequency) AC generally preferred?

   (a) Is it easy to generate?

   (b) Is it easy to utilize?

   (c) Is it easy to transmit?
   Important: behaviour of linear time-invariant circuits

2. If single frequency AC is used, why 50 Hz or 60 Hz?

Obtaining the response of LTI circuits to sinusoidal excitation

1. Analytical solution of linear ODEs.

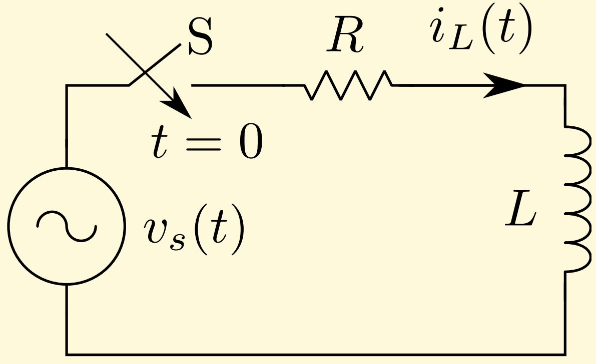
   (a) Direct Time Domain Solution

   (b) Indirect solution:

   - Go to the frequency domain (Laplace Transformation, with ($s = j\omega$)).
   - Sinusoidal steady state can be inferred from phasor analysis (complex numbers).
   - Solution of the “characteristic polynomial” gives you the components of the natural response

2. Numerical Integration (Example: Eulers Method).

Example 1



If $v_s(t) = V_m \sin(\omega t + \alpha)$, then analytical solution of ODE is:

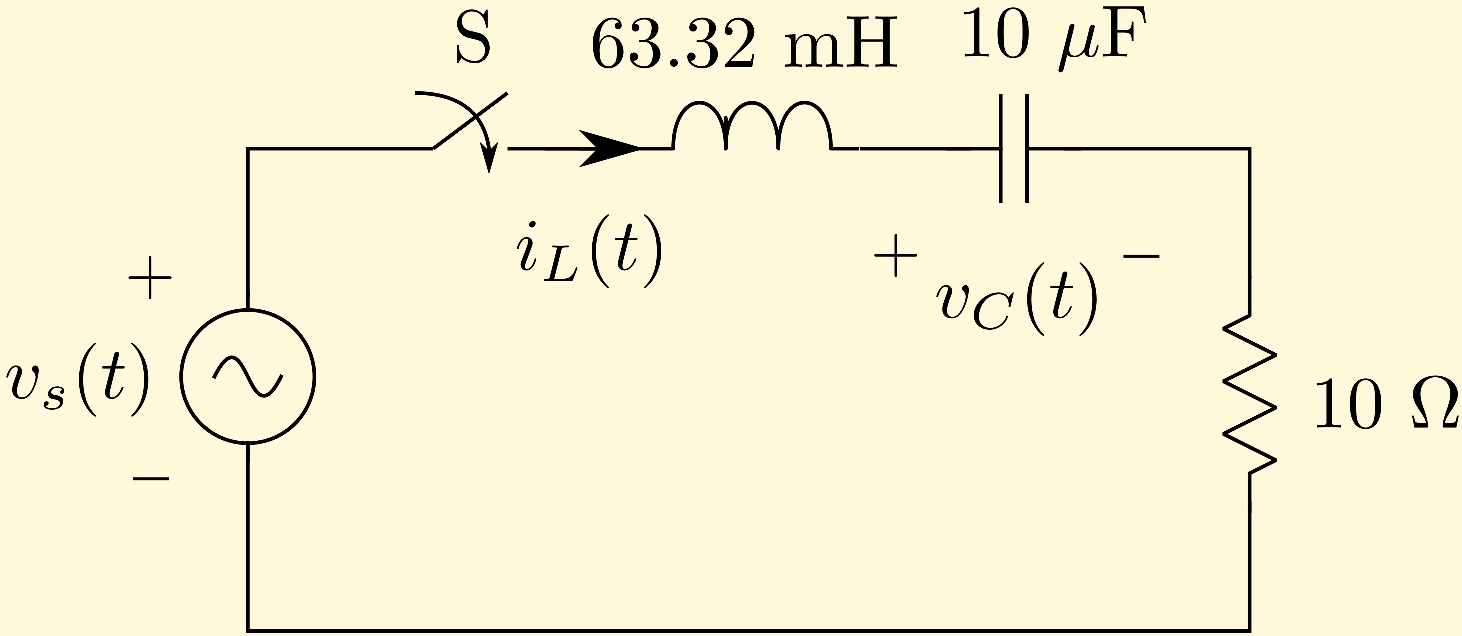
$$i_L(t) = [i_L(0) + \frac{V_m}{|Z|}\sin(\phi - \alpha)]e^{-\frac{R}{L}t} + \frac{V_m}{|Z|}\sin(\omega t + \alpha - \phi)$$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}, \quad \phi = \tan^{-1}(\frac{\omega L}{R})$$

Note: $Z = |Z|\angle\phi = R + j\omega L$ ("impedance")

Natural Transient *adjusts itself* so that $i_L(t = 0_+) = i_L(0)$ (inductor current cannot change instantaneously).

Example 2



Need to specify two initial conditions to obtain natural response.

Roots of characteristic polynomial are complex. Natural response is a damped sinusoid.

Summary: Behaviour of Linear Time Invariant (LTI) Circuits

LTI circuit with (single frequency) ac excitation

1. In steady state (sinusoidal steady state), waveshape is preserved for currents and voltages in the circuit (but gain or phase-shift may be present).

2. In addition, there is a natural transient, which (usually) dies down.

Summary: Behaviour of Linear Time Invariant (LTI) Circuits

LTI with (multiple frequency) ac excitation

1. In steady state (periodic steady state), *overall* waveshape may not be preserved (gain or phase-shift are frequency dependent!).

2. Steady state response to each sinusoidal component in the input can be obtained by superposition.

3. Again, there is a natural transient, which (usually) dies down.

Some side issues

1. LTI Circuits

   (a) Natural transients in "passive" R-L-C circuits usually die down due to R.

   (b) In some circuits (using active elements like amplifiers), natural transients may not die down - e.g., oscillators.

2. **Non-linear** circuits with sinusoidal excitation

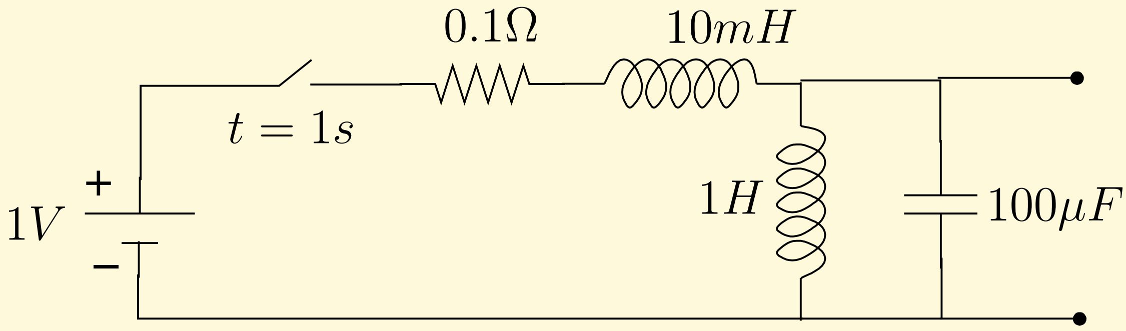
   (a) Periodic steady state may be reached, but response may have frequency components not present in the input.

   (b) "Natural Transient" can be quite complicated.

Now, **before we go on to 50 Hz sinusoidal steady state analysis**, three interesting points:

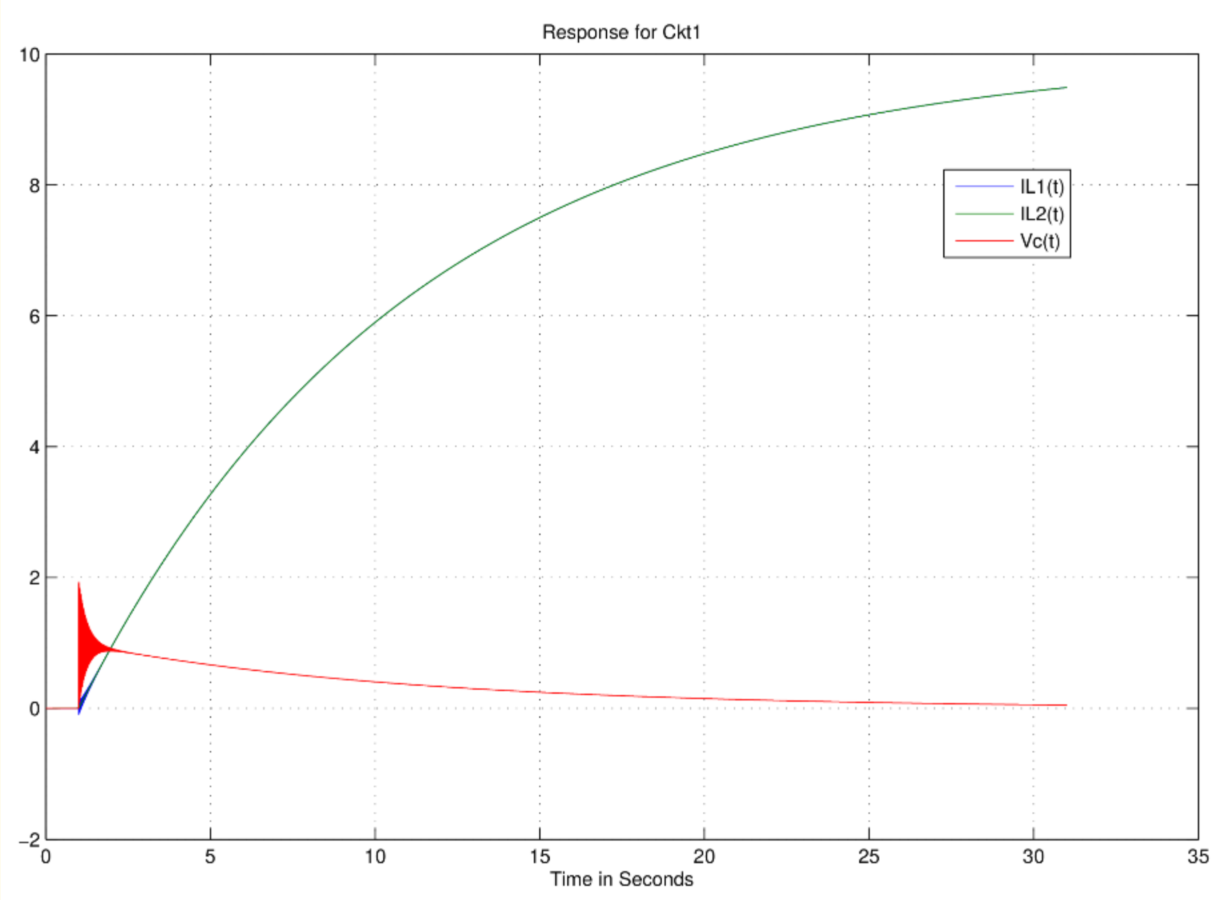
1. Fast and Slow Subsystems: Modeling
2. Real Life Elements
3. Frequency Response through measurement

1. An Interesting Example

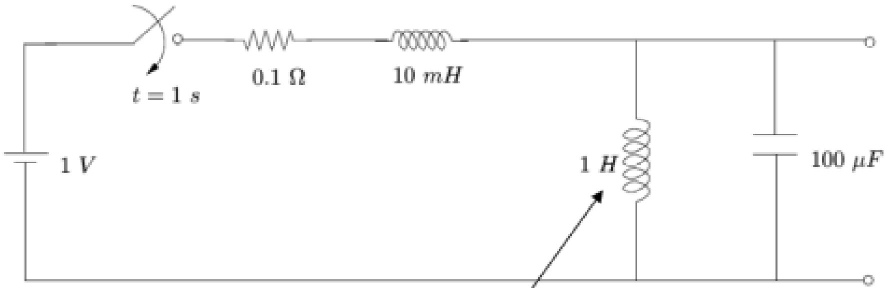


$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v_c \end{bmatrix} = \begin{bmatrix} -10 & 0 & -100 \\ 0 & 0 & 1 \\ 10^4 & -10^4 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v_c \end{bmatrix} + \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvalues $\lambda = -0.099, -4.95 \pm j1005.$

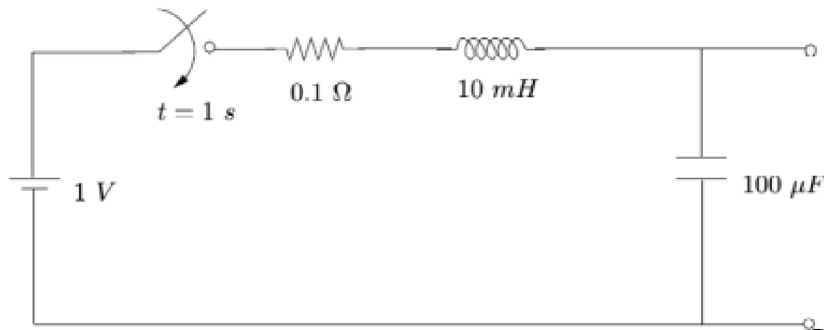

Response for Ckt1
10
8
6
4
2
0
−2
0
5
10
15
20
25
30
35
Time in Seconds
IL1(t)
IL2(t)
Vc(t)

Fast Transients Simplification in Modeling

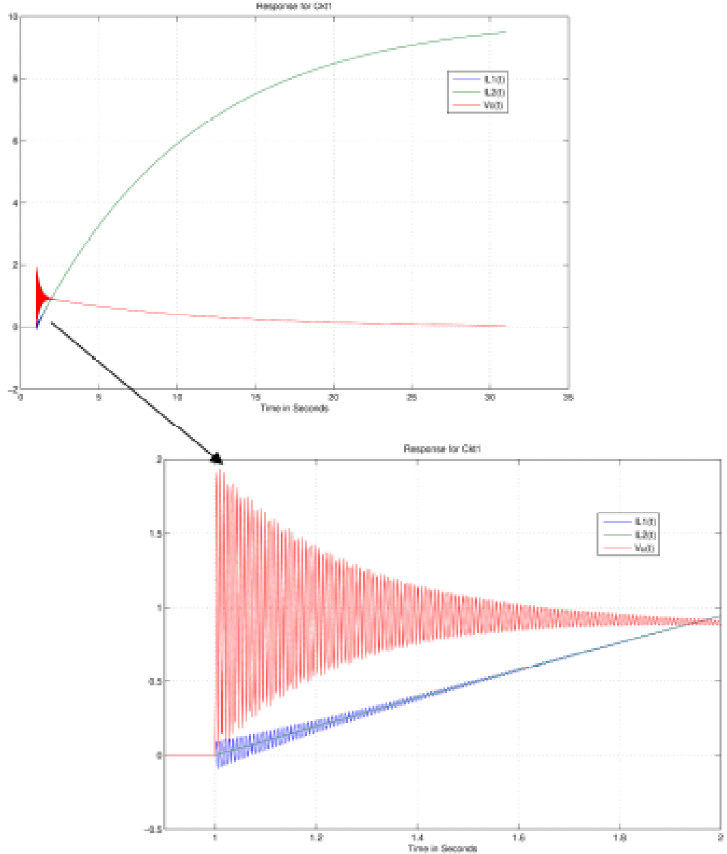


When studying fast transients for a short duration, current through the **large inductor** is virtually unchanged (it remains at its pre disturbance value, **zero** in this case).

Approximate model

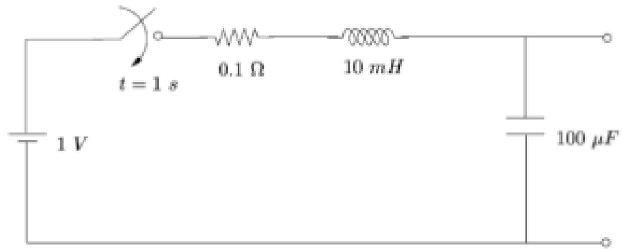


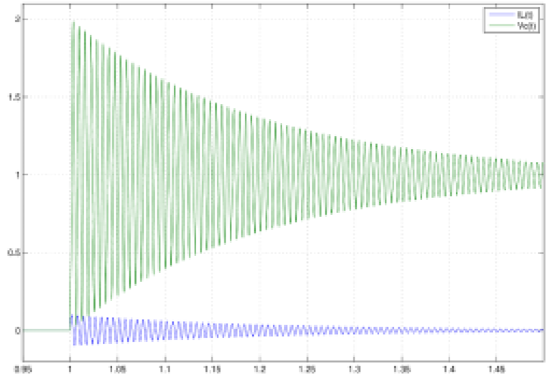
Fast Transients Simplification in Modeling



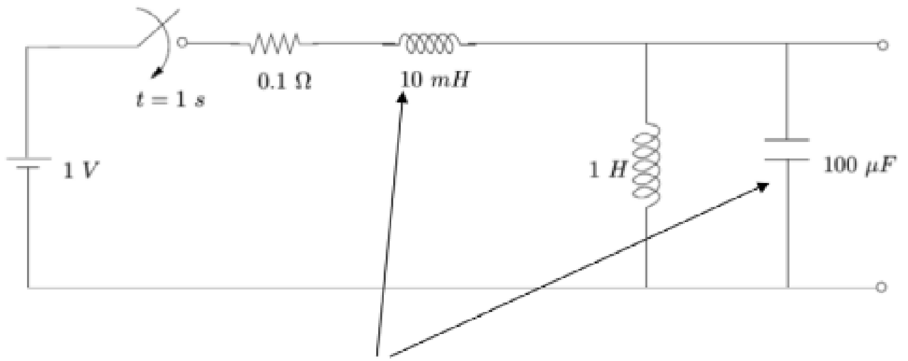
Actual Response

Approximate Model



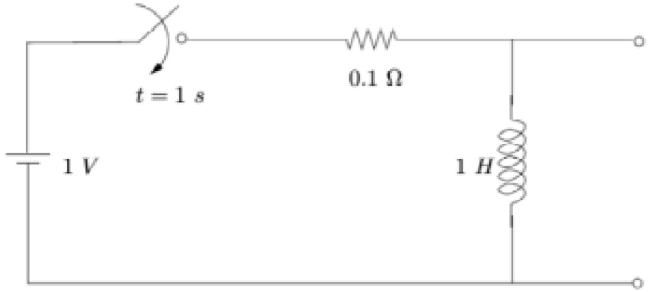


Slow Transients Simplification in Modeling

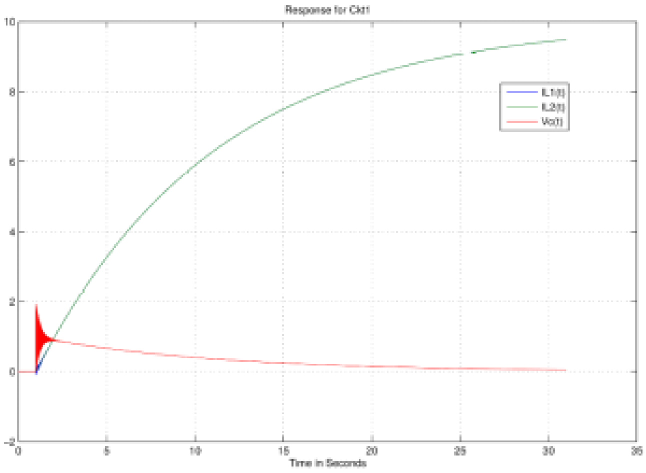


When studying slow transients, rate of change being small, the drop across the smaller inductor and the current through the small capacitor is almost zero.

Approximate Model

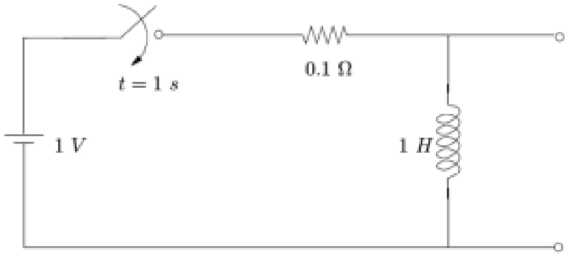


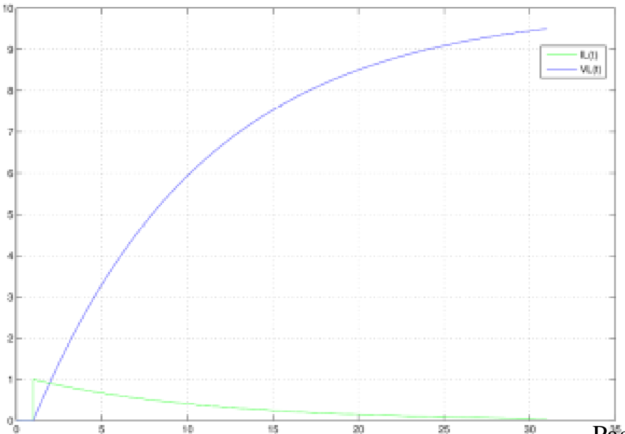
Slow Transients Simplification in Modeling



Actual Response

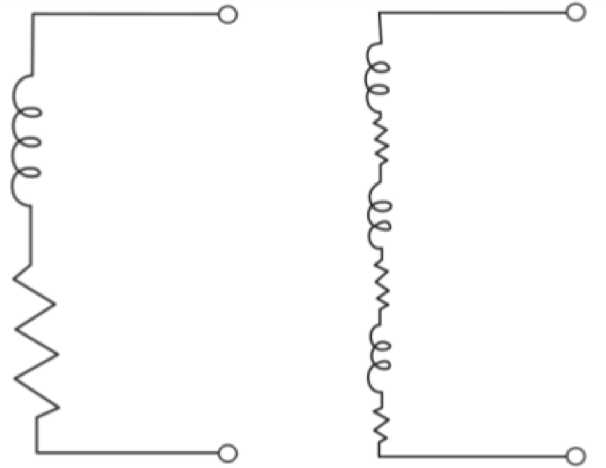
Approximate Model





Non-ideal Inductor:

Equivalent at **low** frequencies.



(a) (b)

Fig. 23-2. The equivalent circuit of a real inductance at low frequencies.

Non-ideal Inductor:

Equivalent at **high** frequencies.