The aim of these problems is to illustrate the generalization of the Fast Fourier Transform algorithm to arbitrary composite N. We consider two examples to illustrate the idea :

i. N=15
ii. $N=3^p$.

In drawing signal flow graphs as required below, you may simply indicate repetitive structures in a block ONCE, in detail. At other locations, you may simply show the block as a unit. But, even when you do not thus unnecessarily repeat segments, make sure the other details are shown properly, when important. In other words, your SFG should be easy to understand but precise.

Q1- Let the 'time-index' n be decomposed as n=5n1+n2 with N=15; and the 'frequency-index' k as k=3k1+k2 with appropriate limits on n1, n2, k1, k2 for n, k=0,1,2,3,4. Regard the 15-point-DFT as

$$x[3k1+k2]=\sum_{n1}\sum_{n2}x[5n1+n2]W_{15}^{-3k1n2}W_{15}^{-5n1k2}W_{15}^{-n2k2}$$

$$=\sum_{n2}W_5^{-n2k1}\left\{\sum_{n1}x[5n1+n2]W_{15}^{-(5n1+n2)k2}\right\}$$

where, $\tilde{x}[n2,k2]=\sum_{n2}W_5^{-n2k1}\left\{\sum_{n1}x[5n1+n2]W_{15}^{-(5n1+n2)k2}\right\}$

This corresponds to three 5-point-DFTs; one each for k2=0,1,2, Each DFT has input index n2, output index k1.
Construct completely the signal flow graph for this version of the FFT for N=15.

Q2- Now repeat Q1, but decomposing n=3n1+n2 and k=5k1+k2 again with appropriate limits on n1, n2, k1, k2: here we would in effect compute five 3-point-DFTs.

Q3- Now repeat Q1 and Q2 taking $N=3^p$ p>2 (p is an integer). In Q1 take the decomposition n=3n1+n2; $k=3^{p-1}k1+k2$. In Q2 reverse the roles of n and k from Q1 as done earlier.

Q4- What are the TRANSPOSES of the SFG's in each case in Q1, Q2, Q3 ? Explain any pattern that you see and comment.

Q5- Make a PRECISE EVALUATION BOTH of the number of multiplications AND additions required in each of the cases above; AND those required for "naïve" or direct DFT implementation. Make sure to discount multiplications by +/- 1 properly. (Use insight, not brute force!) Compare these computational requirements