Transmission lines and cables

Maxwell's equations – dynamic conditions

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_o} \int_V \rho \, dV$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S}$$

$$\frac{1}{\mu_o} \oint_L \vec{B} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S} + \epsilon_o \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{S}$$

Maxwell's equations – dynamic conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{1}{\mu_o}\vec{\nabla} \times \vec{B} = \vec{J} + \epsilon_o \frac{\partial \vec{E}}{\partial t}$$

Maxwell’s equations – static conditions

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_o} \int_V \rho \, dV$$

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

$$\frac{1}{\mu_o} \oint_L \vec{B} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$$

Maxwell's equations – static conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$\frac{1}{\mu_o}\vec{\nabla} \times \vec{B} = \vec{J}$$

Maxwell's equations – static conditions

Potential V is given as:

$$\nabla.(\nabla V) = -\frac{\rho}{\epsilon_o}$$

Maxwell’s equations – static conditions

Potential V is given as:

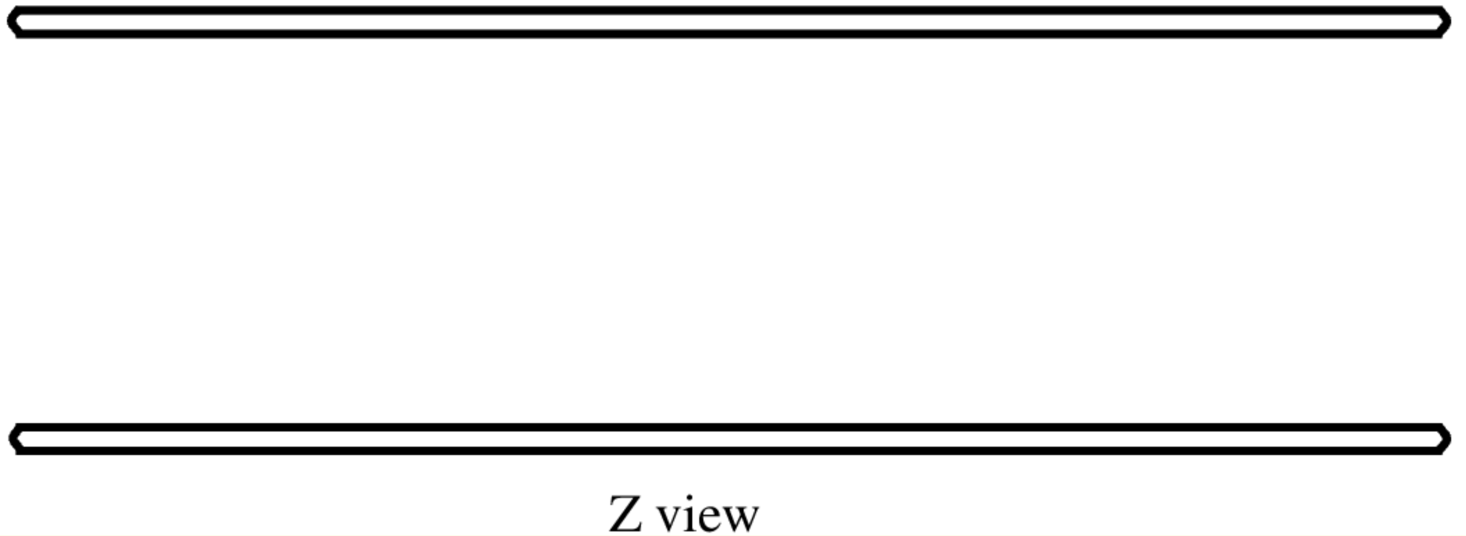
$$\nabla.(\nabla V) = -\frac{\rho}{\epsilon_o}$$

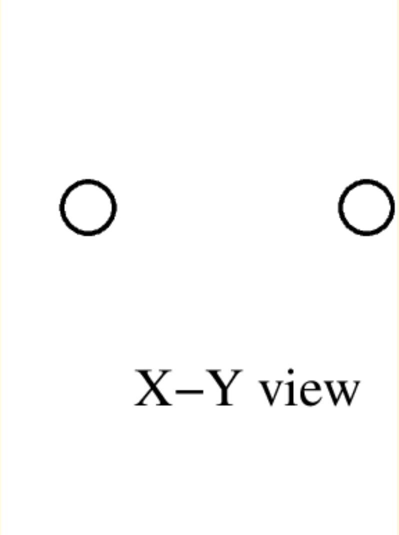
For charge free region:

$$\nabla.(\nabla V) = 0$$

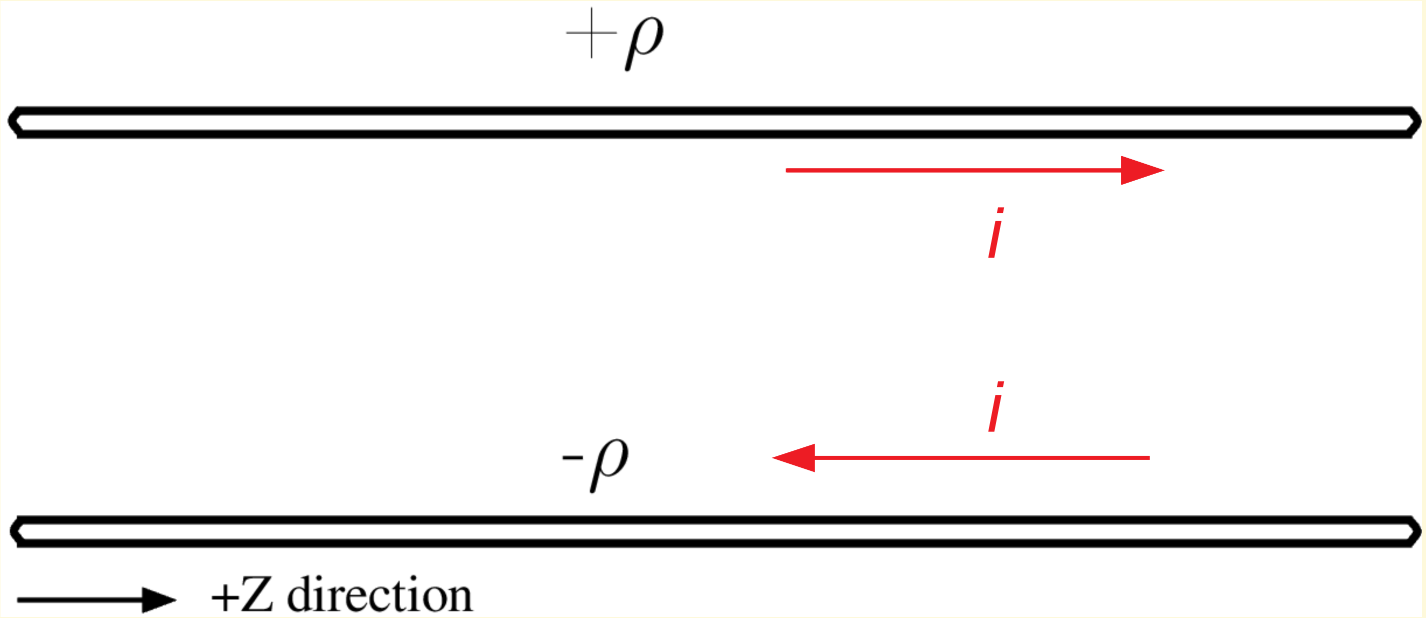
Laplace equation

Two wire line under static (dc) conditions





Static conditions

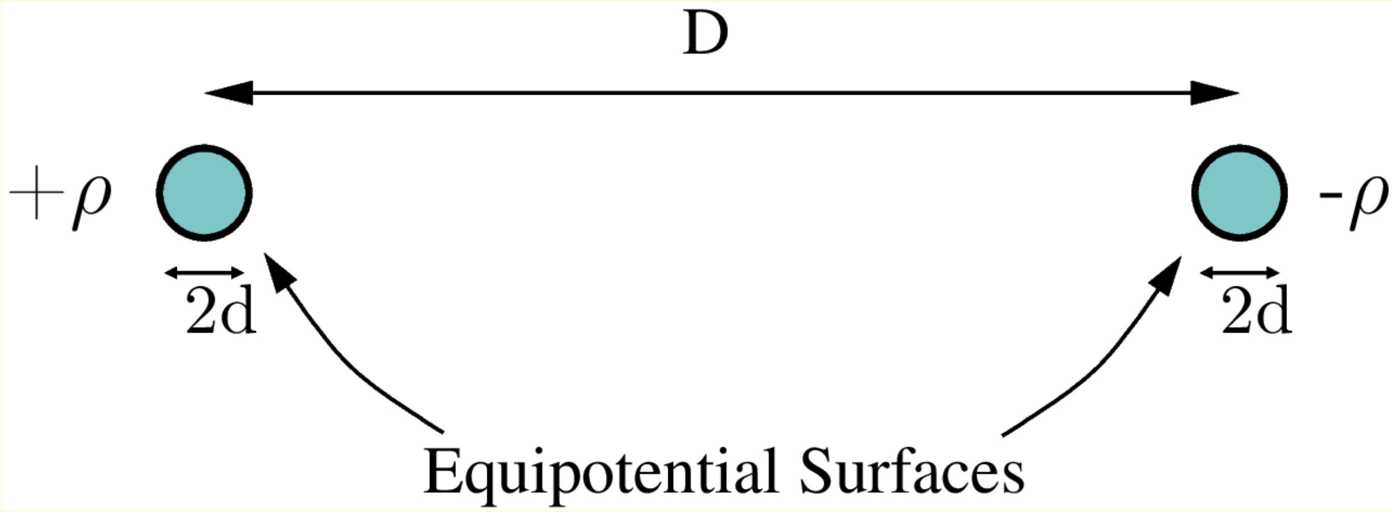


ρ :line charge density

Infinite length lossless transmission line is considered.

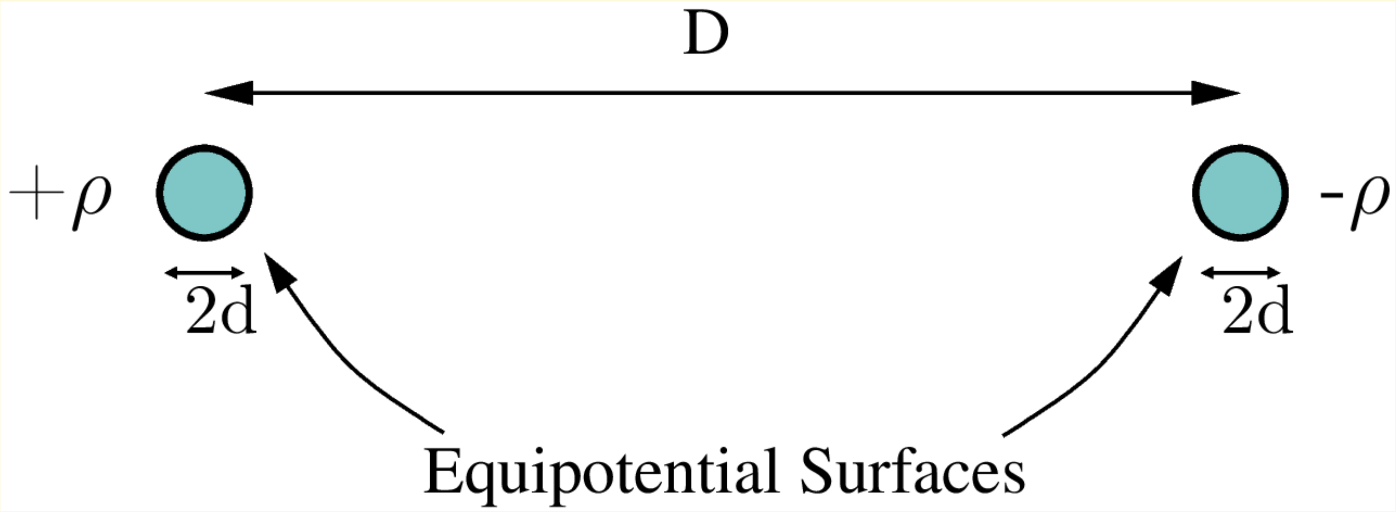
No $\vec{E}$ and $\vec{B}$ in z direction.

Two dimensional Laplace equation



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Two dimensional Laplace equation

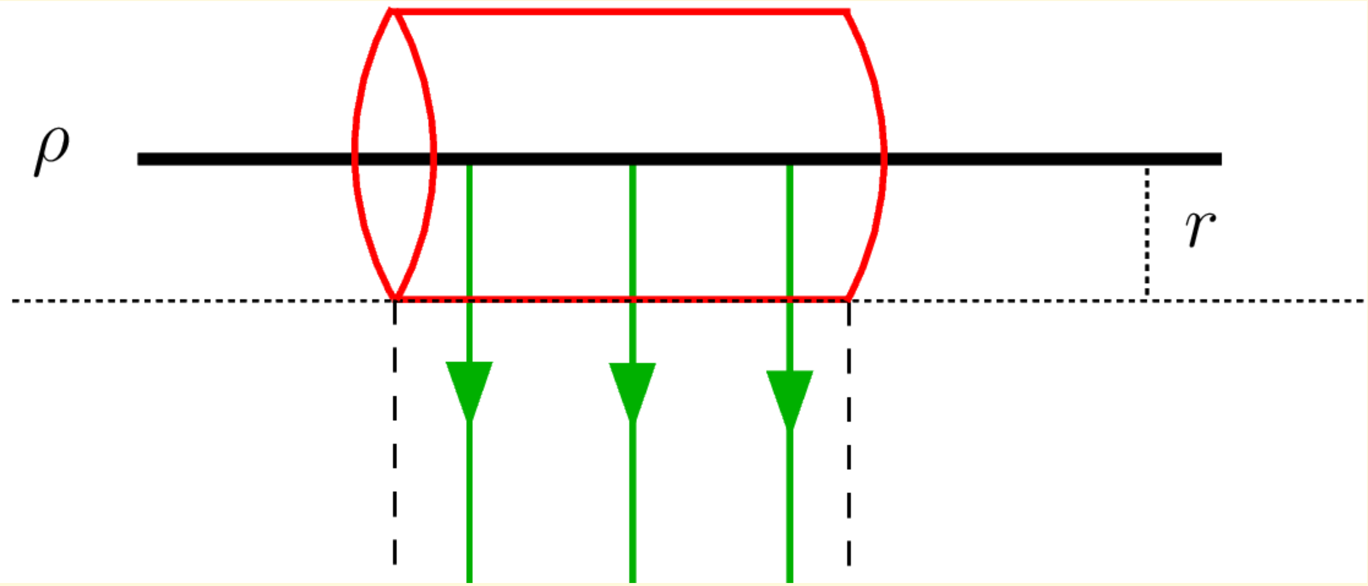


$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

d << D : proximity effects neglected.

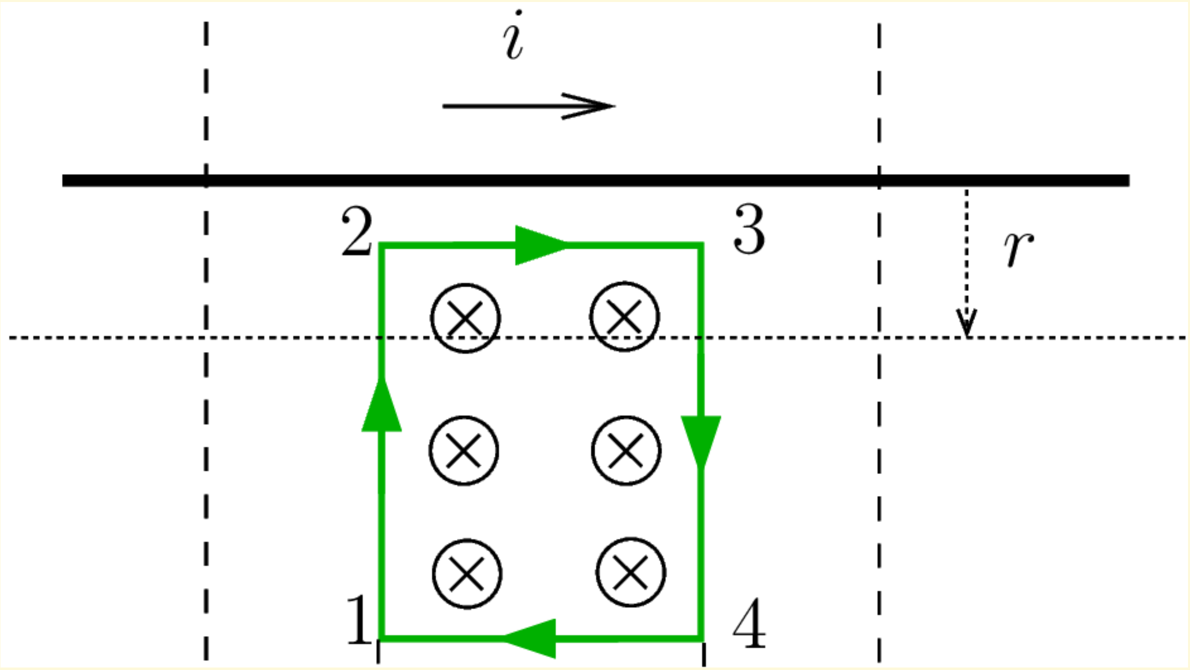
Hence *charge density* is uniform along the periphery of the conductor.

$\vec{E}$ as a function of distance from a conductor



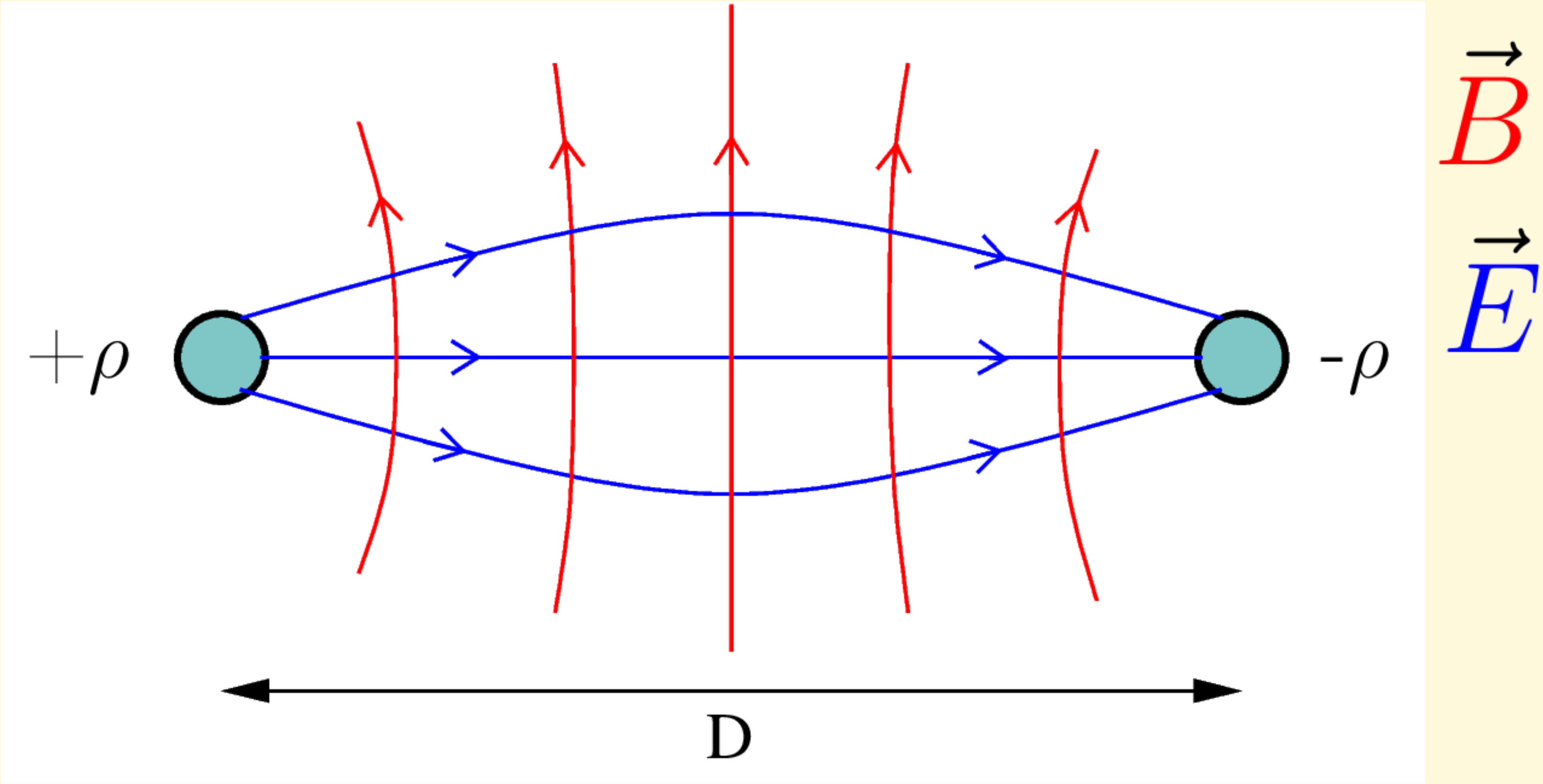
$$|\vec{E}| = \frac{\rho}{2\pi r \epsilon_o}$$

$\vec{B}$ as a function of distance from a conductor



$$|\vec{B}| = \frac{\mu_o\, I}{2\,\pi\, r}$$

$\vec{E}$ on $\vec{B}$ fields



Time varying fields

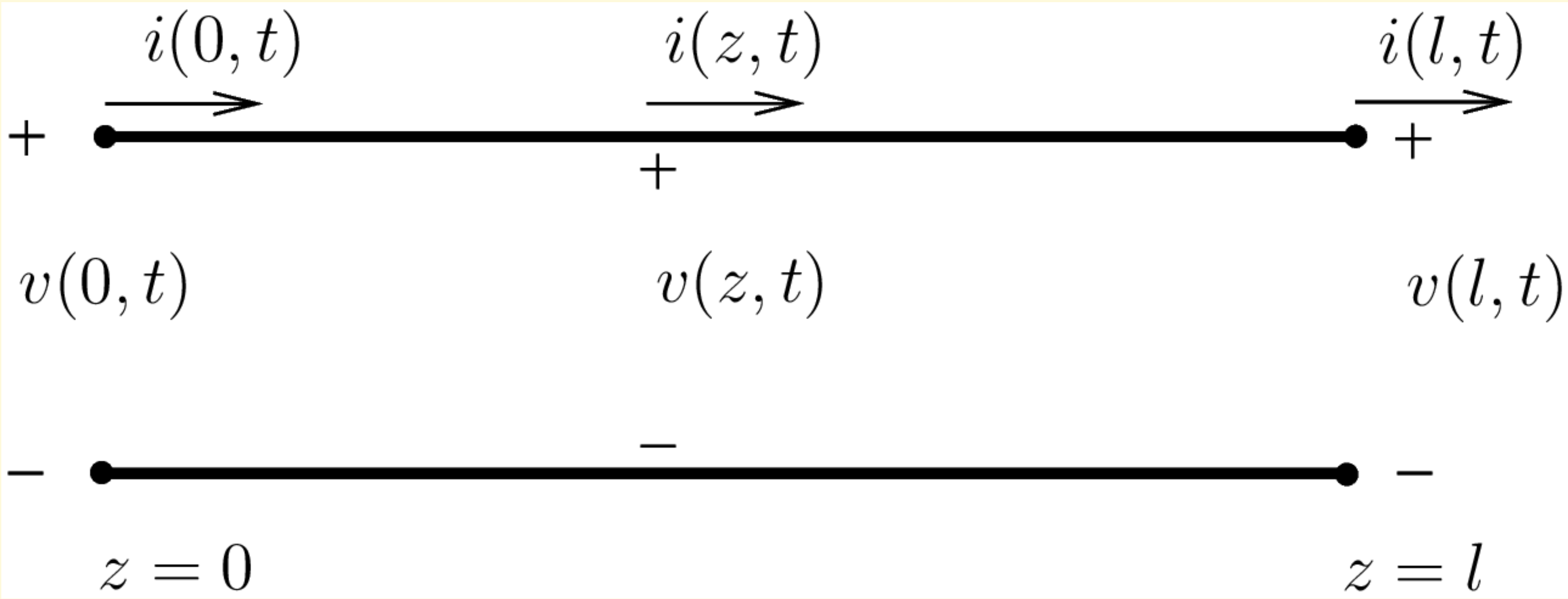
- TEM is only one of the valid patterns.
  Other patterns may exist.

- Frequency *f* tends to 0, then (dimensions of the line) << $\frac{c}{\lambda}$

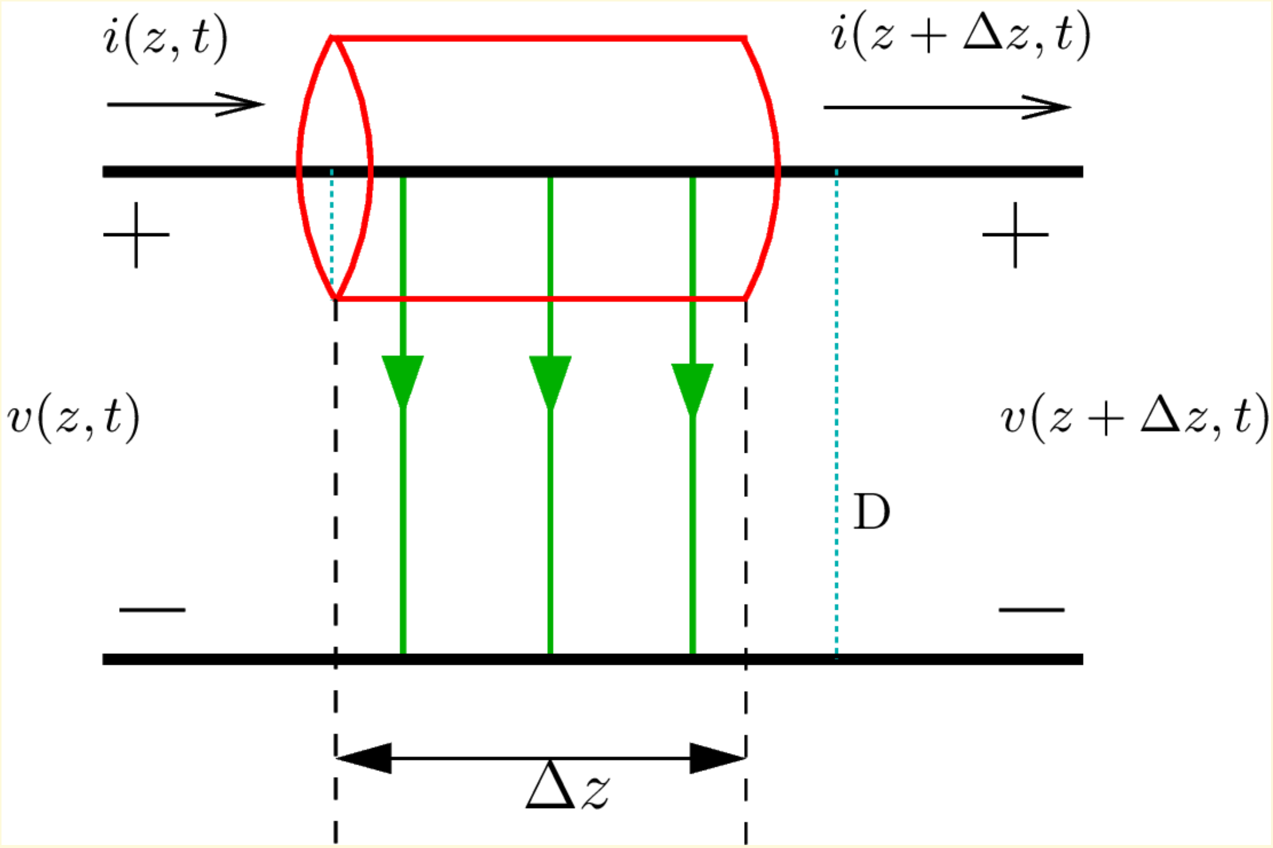
- In such cases, TEM is the “dominant mode”.

* TEM: transverse electromagnetic

Consider a system:

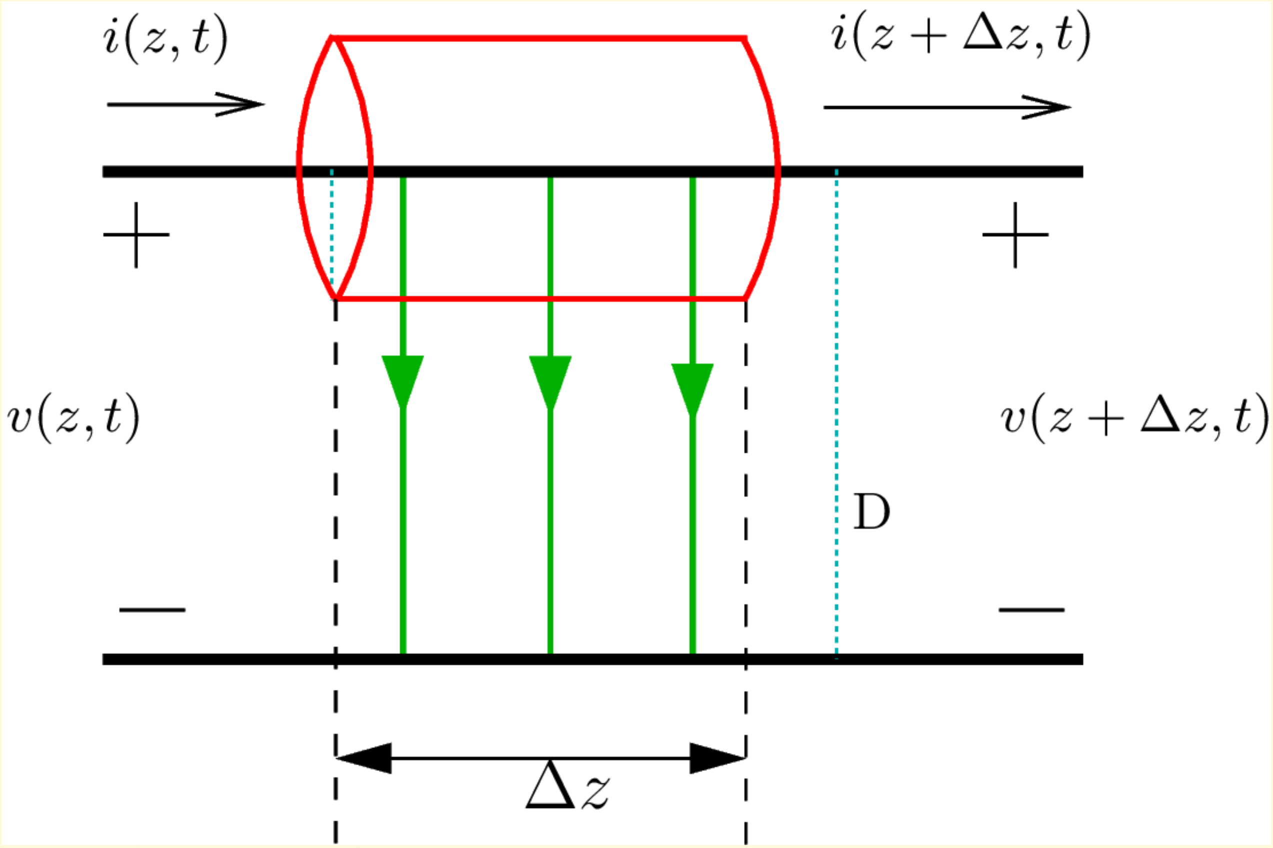


Electric field effects for two wire line system



r : radius of the conductors

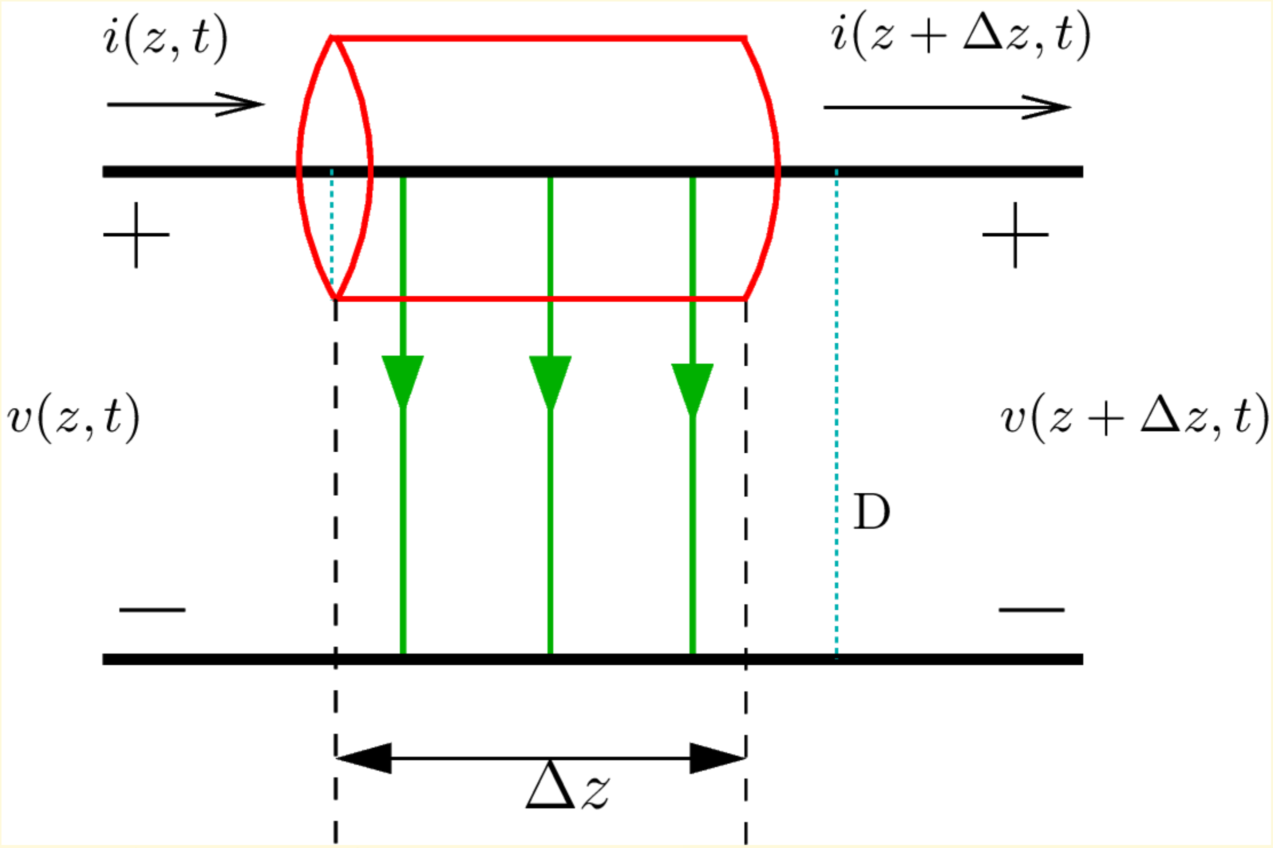
Electric field effects for two wire line system



$$-\frac{\partial i(z,t)}{\partial z} = \left(\frac{\pi\,\epsilon_o}{\log_e \dfrac{D}{r}} \right) \frac{\partial v(z,t)}{\partial t}$$

r : radius of the conductors

Electric field effects for two wire line system



$$-\frac{\partial i(z,t)}{\partial z} = \left(\frac{\pi\,\epsilon_o}{\log_e \dfrac{D}{r}} \right) \frac{\partial v(z,t)}{\partial t} \quad ==> \quad -\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t}$$

$\mathcal{C}'$: capacitance per unit length of the line.

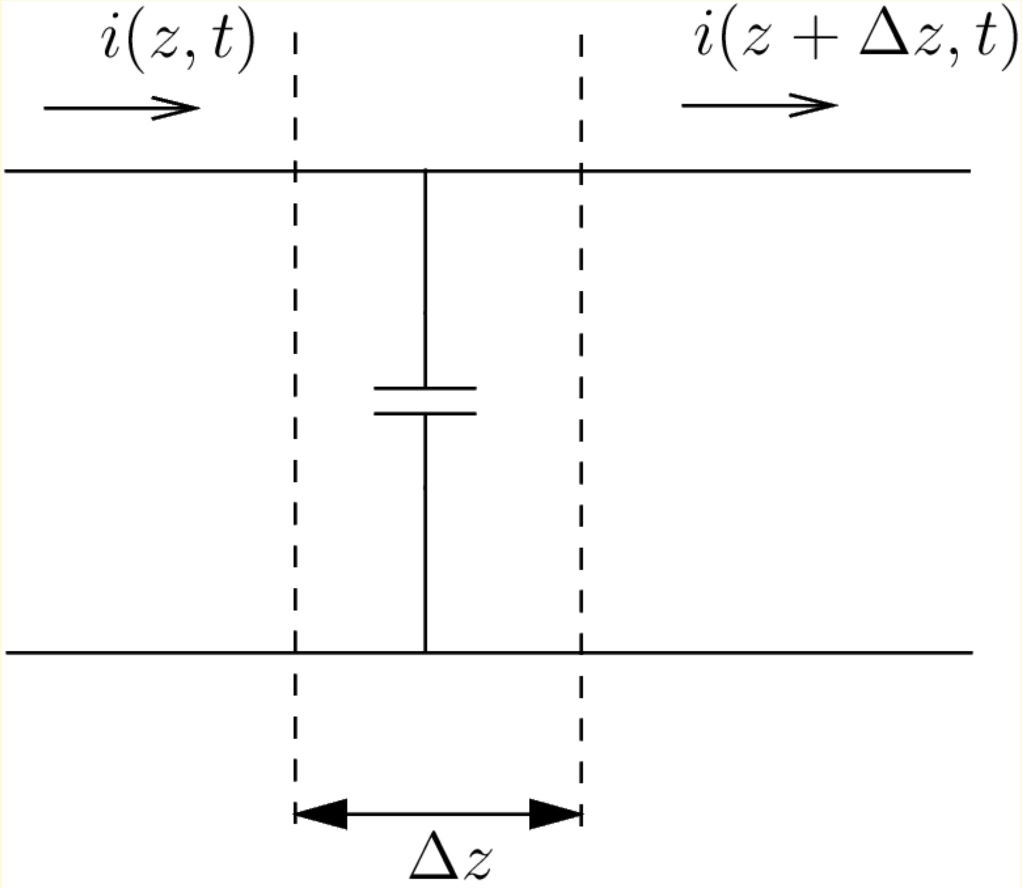
r : radius of the conductors

Electric field effects for two wire line system

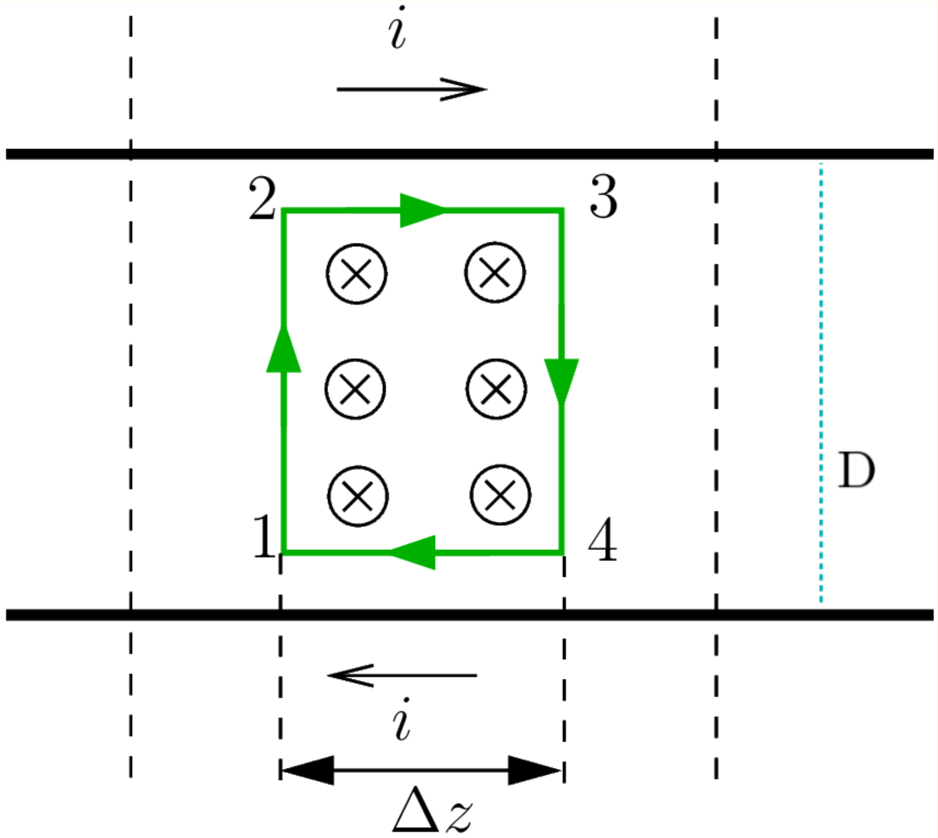
$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t}$$

Electric field effects for two wire line system

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}'\,\frac{\partial v(z,t)}{\partial t}$$

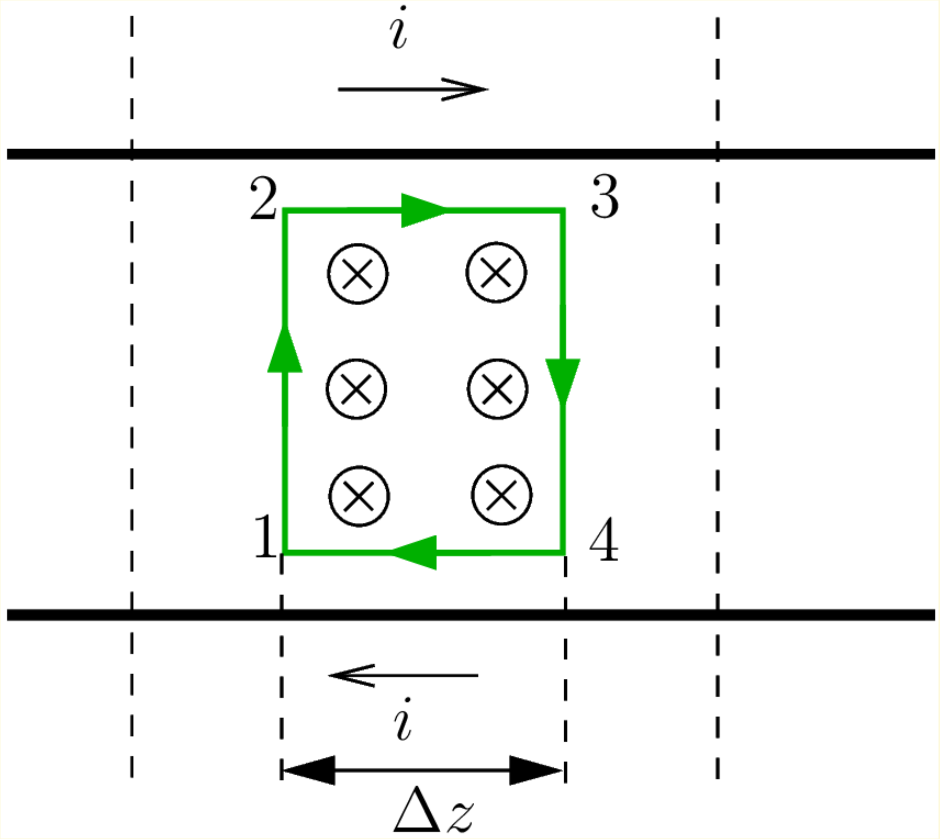


Magnetic field effects for two wire line system



r : radius of the conductors

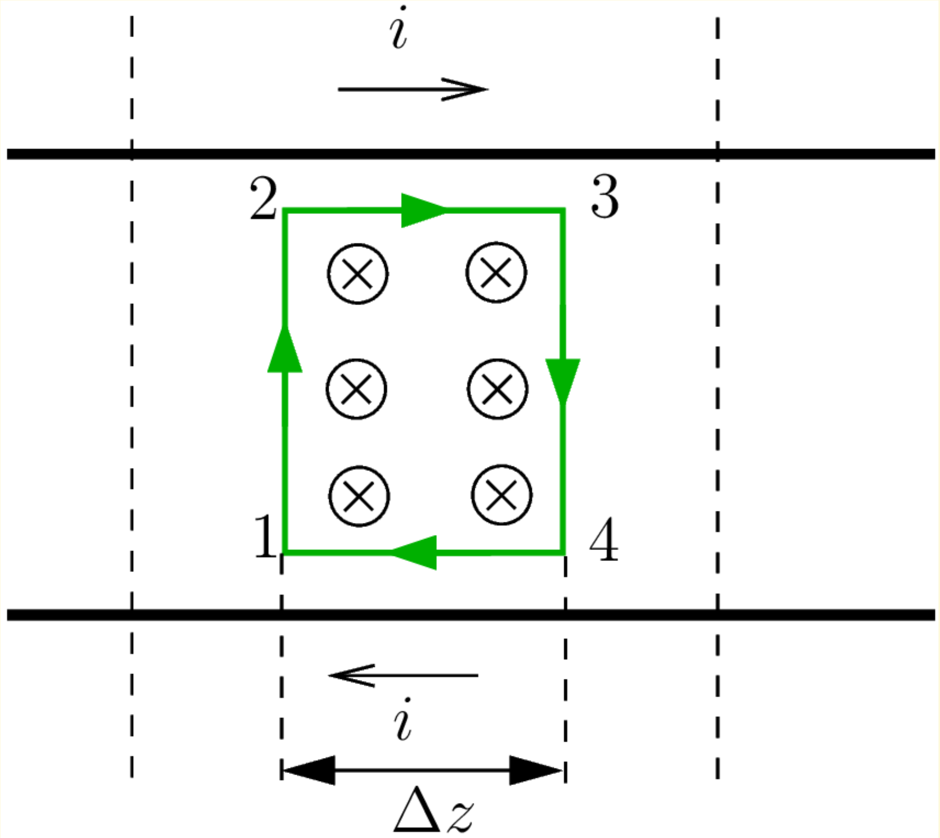
Magnetic field effects for two wire line system



$$-\frac{\partial v(z,t)}{\partial z} = \left(\frac{\mu_o}{\pi}\log_e\frac{D}{r}\right)\frac{\partial i(z,t)}{\partial t}$$

r : radius of the conductors

Magnetic field effects for two wire line system



$$-\frac{\partial v(z,t)}{\partial z} = \left(\frac{\mu_o}{\pi}\log_e\frac{D}{r}\right)\frac{\partial i(z,t)}{\partial t} \quad \text{==>} \quad -\frac{\partial v(z,t)}{\partial z} = \mathcal{L}'\frac{\partial i(z,t)}{\partial t}$$

$\mathcal{L}'$: inductance per unit length of the line.

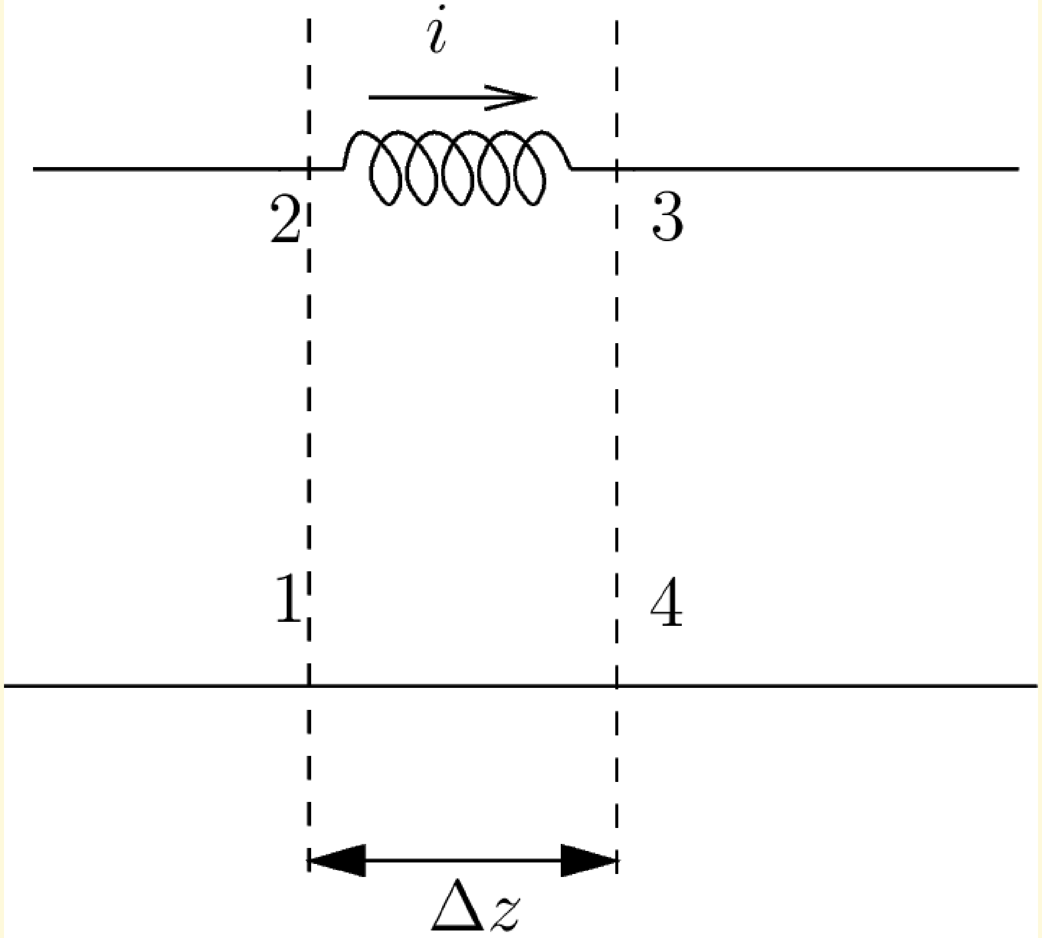
r: radius of the conductors

Magnetic field effects for two wire line system

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \, \frac{\partial i(z,t)}{\partial t}$$

Magnetic field effects for two wire line system

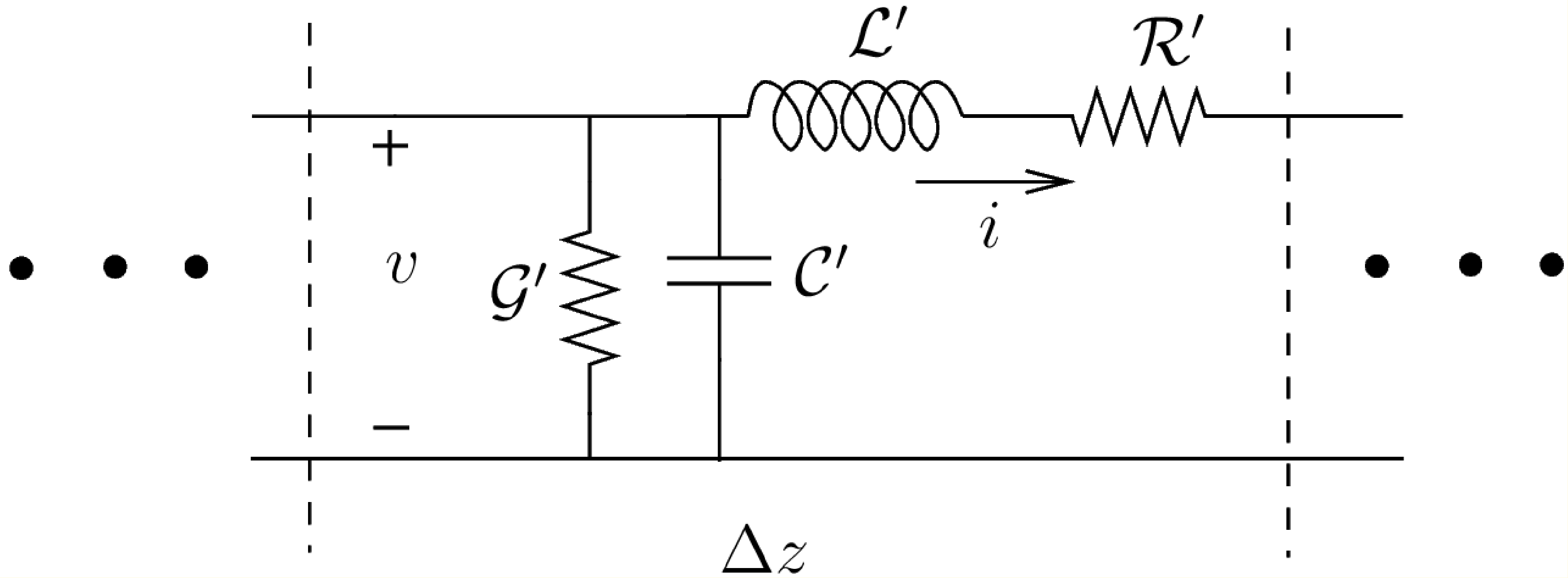
$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t}$$



Similarly, the lossy effects can also be accommodated as

$\mathcal{R}'$: series resistance per unit length of the line.

$\mathcal{G}'$: leakage conductance per unit length of the line.



Telegrapher Equations

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t} + \mathcal{R}' \, i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t} + \mathcal{G}' \, v(z,t)$$

Telegrapher Equations

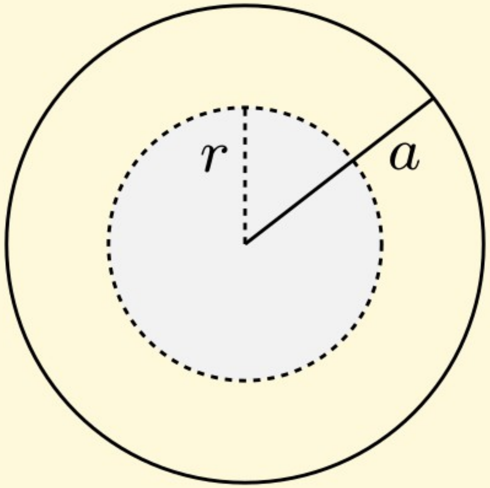
$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t} + \mathcal{R}' \, i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t} + \mathcal{G}' \, v(z,t)$$

Above equations do not have closed form solution, except for special cases.

Fields within the conductor (Skin effect)

Assume $\vec{J} = \sigma\,\vec{E}$ within the conductors.



Z view

$$\vec{J}_z(r) = \frac{k\,\vec{I}}{2\,\pi\,a}\,\frac{J_o(k\,r)}{J_1(k\,a)}$$

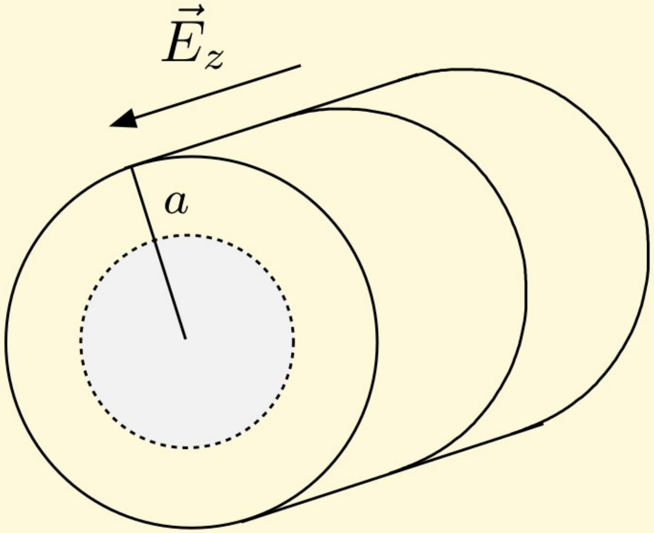
where,

$$k = \sqrt{-j\,\omega\,\mu\,\sigma}$$

J_o and J_1 are Bessel functions of order zero and one respectively.

Fields on the surface of conductor (Skin effect)

$$\vec{E}_z(a) = \frac{\vec{J}_z(a)}{\sigma}$$



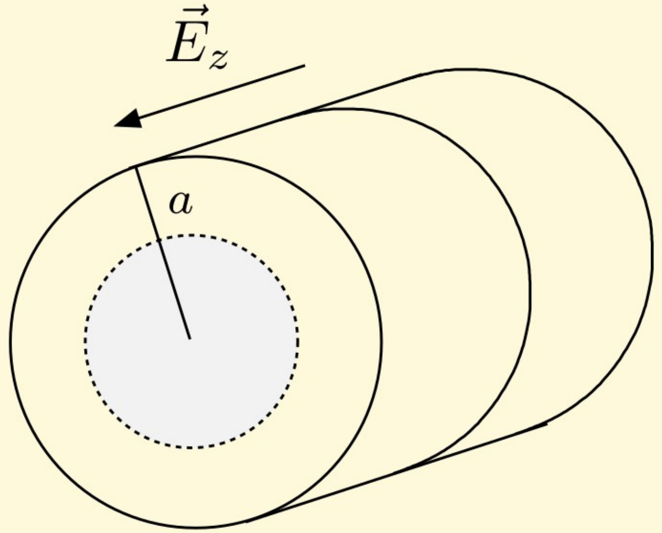
where,

$\vec{E}_z(a)$ (voltage per unit length) on the surface

$\vec{J}_z(a)$ can be written in terms of $\vec{I}$.

Fields on the surface of conductor (Skin effect)

$$\vec{E}_z(a) = \frac{\vec{J}_z(a)}{\sigma}$$



where,

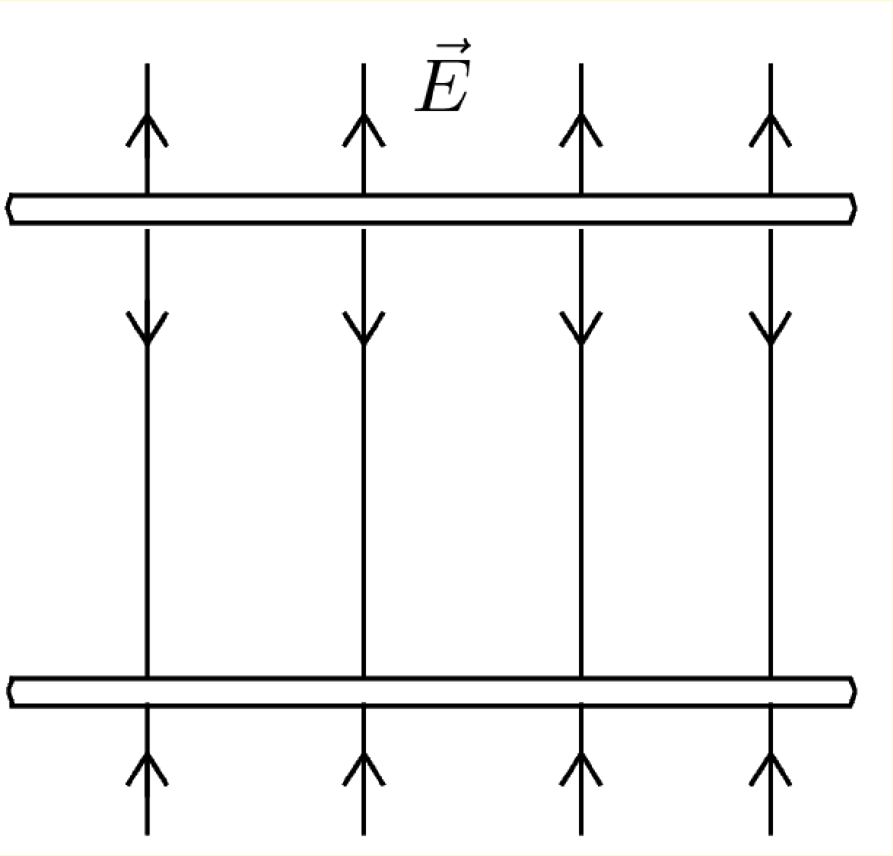
$\vec{E}_z(a)$ (voltage per unit length) on the surface

$\vec{J}_z(a)$ can be written in terms of $\vec{I}$.

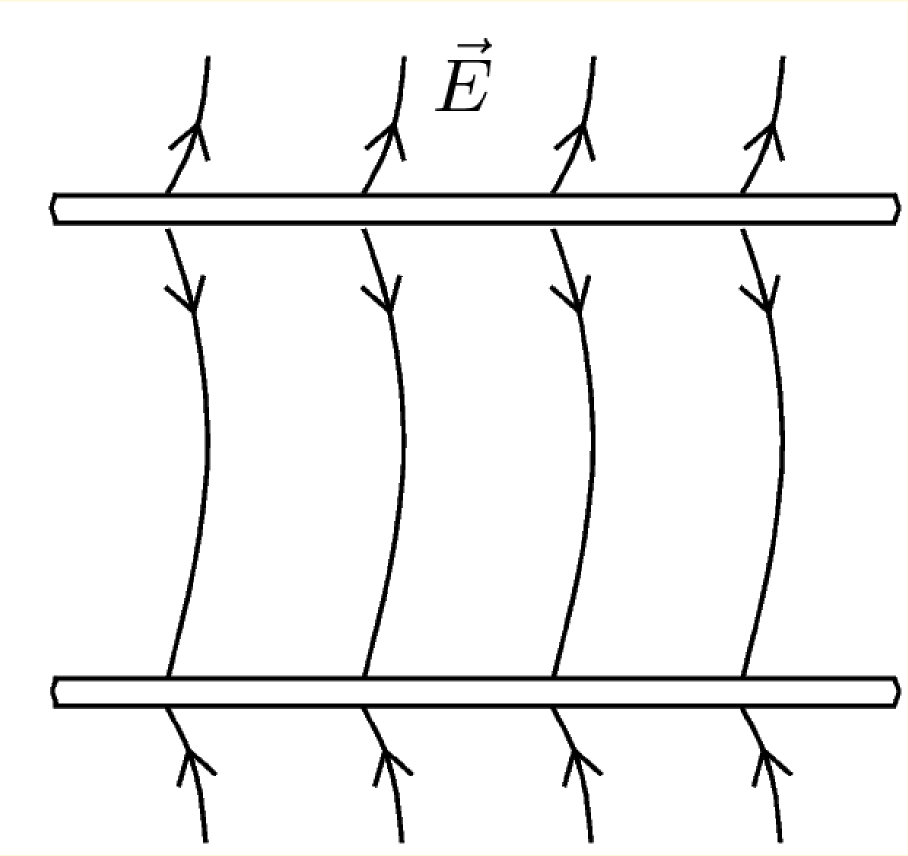
Impedance per unit length $\mathcal{Z}_i$ is called internal impedance.

$$\mathcal{Z}_i = \frac{\vec{J}_z(a)}{\sigma\,\vec{I}}$$

$\vec{E}$ distribution



Lossless line



Lossy line

Fields within the conductor (Skin effect)

The internal impedance for low frequencies

$$\mathcal{Z}_{\text{int}} = \frac{1}{\sigma\,\pi\,a^2} + j\,\omega\,\frac{\mu_{\text{int}}}{8\,\pi}$$

Note: $\mathcal{Z}_{\text{int}}$ is internal impedance per unit length of the line.

Telegrapher Equations

Special case: $\mathcal{R}' = \mathcal{G}' = 0.$

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t}$$

Telegrapher Equations

The general solution is

$$v(z,t) = f_1\left(t - \frac{z}{v_p}\right) + f_2\left(t + \frac{z}{v_p}\right)$$

$$i(z,t) = \frac{1}{Z_c}\left[f_1\left(t - \frac{z}{v_p}\right) - f_2\left(t + \frac{z}{v_p}\right)\right]$$

Telegrapher Equations

The general solution is

$$v(z,t) = f_1\left(t - \frac{z}{v_p}\right) + f_2\left(t + \frac{z}{v_p}\right)$$

$$i(z,t) = \frac{1}{Z_c}\left[f_1\left(t - \frac{z}{v_p}\right) - f_2\left(t + \frac{z}{v_p}\right)\right]$$

where,

$Z_c = \sqrt{\dfrac{\mathcal{L}'}{\mathcal{C}'}}$: characteristic impedance of the line,

$v_p = \dfrac{1}{\sqrt{\mathcal{L}'.\mathcal{C}'}}$: speed of propagation.

Telegrapher Equations

The general solution is

$$v(z,t) = f_1\left(t - \frac{z}{v_p}\right) + f_2\left(t + \frac{z}{v_p}\right)$$

$$i(z,t) = \frac{1}{Z_c}\left[f_1\left(t - \frac{z}{v_p}\right) - f_2\left(t + \frac{z}{v_p}\right)\right]$$

where,

$Z_c = \sqrt{\dfrac{\mathcal{L}'}{\mathcal{C}'}}$: characteristic impedance of the line,

$v_p = \dfrac{1}{\sqrt{\mathcal{L}'.\mathcal{C}'}}$: speed of propagation.

Boundary conditions are applied to eliminate f_1 and f_2.

Low frequency model of transmission lines

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t} + \mathcal{R}' \, i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t} + \mathcal{G}' \, v(z,t)$$

In sinusoidal steady state conditions, $\dfrac{\partial}{\partial t}$ is replaced by $j\,\omega$.

Low frequency model of transmission lines

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \, \frac{\partial i(z,t)}{\partial t} + \mathcal{R}' \, i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \, \frac{\partial v(z,t)}{\partial t} + \mathcal{G}' \, v(z,t)$$

In sinusoidal steady state conditions, $\dfrac{\partial}{\partial t}$ is replaced by $j\,\omega$.

$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}(j\,\omega)\,\bar{I}(j\,\omega) \quad , \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

Low frequency model of transmission lines

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}'\,\frac{\partial i(z,t)}{\partial t} + \mathcal{R}'\,i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}'\,\frac{\partial v(z,t)}{\partial t} + \mathcal{G}'\,v(z,t)$$

In sinusoidal steady state conditions, $\dfrac{\partial}{\partial t}$ is replaced by $j\,\omega$.

$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}'(j\,\omega)\,\bar{I}(j\,\omega) \quad , \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

where,

$\mathcal{Z}(j\,\omega) = \mathcal{R}' + j\,\omega\,\mathcal{L}'$ and $\mathcal{Y}(j\,\omega) = \mathcal{G}' + j\,\omega\,\mathcal{C}'$.

Low frequency model of transmission lines

$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}(j\,\omega)\,\bar{I}(j\,\omega) \quad , \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

Low frequency model of transmission lines

$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}(j\,\omega)\,\bar{I}(j\,\omega) \quad , \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

General solution for voltage and current phasors at a distance ***z*** from **sending end** of the line is given as:

$$\bar{V}_z = A\,\mathrm{e}^{-\gamma\,z} + B\,\mathrm{e}^{\gamma\,z} \qquad \bar{I}_z = \frac{1}{Z_c}\left(A\,\mathrm{e}^{-\gamma\,z} - B\,\mathrm{e}^{\gamma\,z}\right)$$

Low frequency model of transmission lines

$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}(j\,\omega)\,\bar{I}(j\,\omega) \quad , \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

General solution for voltage and current phasors at a distance ***z*** from **sending end** of the line is given as:

$$\bar{V}_z = A\,\mathrm{e}^{-\gamma\,z} + B\,\mathrm{e}^{\gamma\,z} \qquad \bar{I}_z = \frac{1}{Z_c}\left(A\,\mathrm{e}^{-\gamma\,z} - B\,\mathrm{e}^{\gamma\,z}\right)$$

where,

$$\gamma = \sqrt{\mathcal{Z}(j\,\omega).\mathcal{Y}(j\,\omega)} \qquad Z_c = \sqrt{\frac{\mathcal{Z}(j\,\omega)}{\mathcal{Y}(j\,\omega)}}$$

Propagation constant

Characteristic impedance of the line.

Low frequency model of transmission lines

$$\bar{V}_z = A\,\mathrm{e}^{-\gamma\,z} + B\,\mathrm{e}^{\gamma\,z} \qquad \bar{I}_z = \frac{1}{Z_c}\left(A\,\mathrm{e}^{-\gamma\,z} - B\,\mathrm{e}^{\gamma\,z}\right)$$

Applying boundary conditions to eliminate *A* and *B.*

Low frequency model of transmission lines

$$\bar{V}_z = A\,\mathrm{e}^{-\gamma\,z} + B\,\mathrm{e}^{\gamma\,z} \qquad \bar{I}_z = \frac{1}{Z_c}\left(A\,\mathrm{e}^{-\gamma\,z} - B\,\mathrm{e}^{\gamma\,z}\right)$$

Applying boundary conditions to eliminate *A* and *B.*

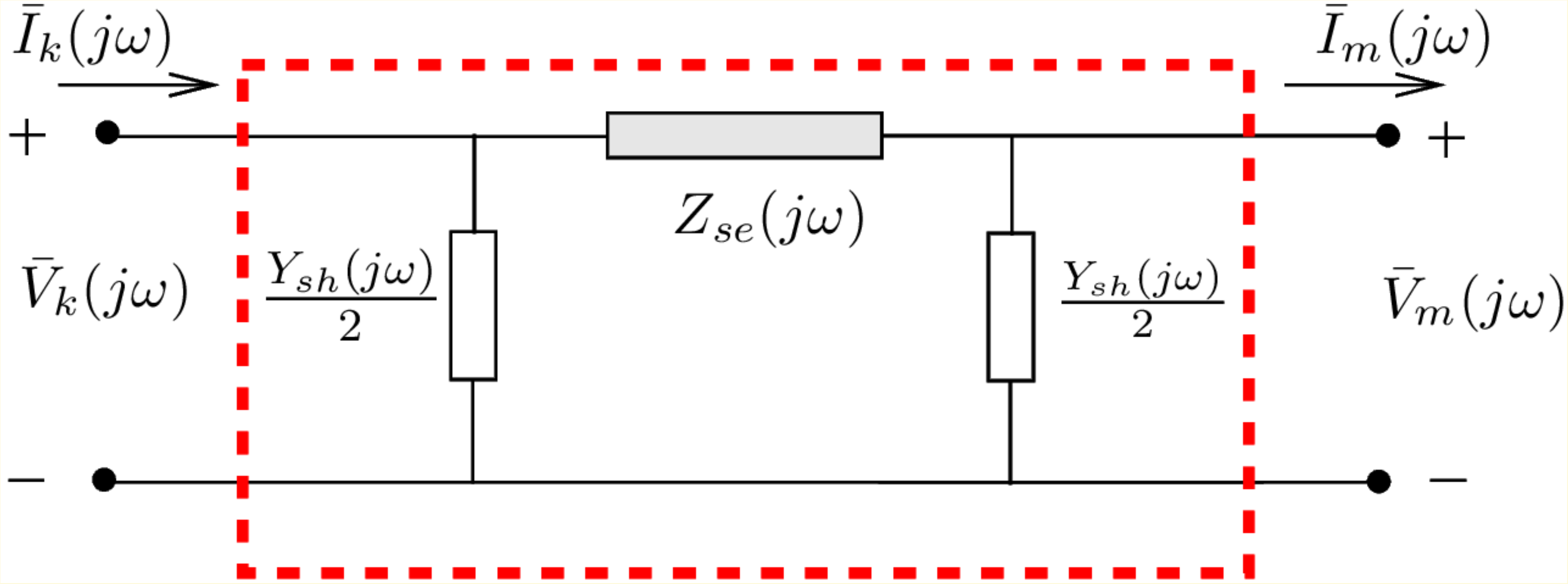
Following relations are obtained:

$$\bar{V}_k = \cosh(\gamma\,l).\bar{V}_m + Z_c\,\sinh(\gamma\,l).\bar{I}_m$$

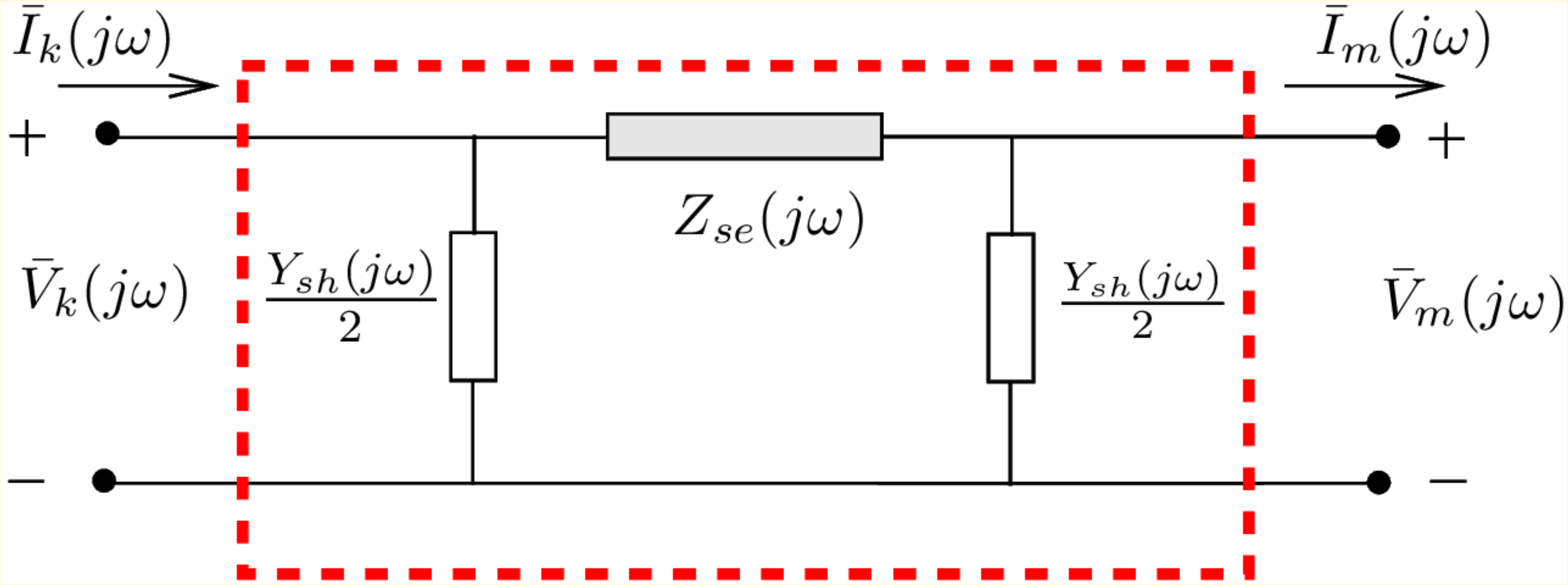
$$\bar{I}_k = \frac{1}{Z_c}\,\sinh(\gamma\,l).\bar{V}_m + \cosh(\gamma\,l).\bar{I}_m$$

‘*k*’ and ‘*m*’ subscripts refer to sending and receiving end variables.

Above equations can be used to synthesize an equivalent PI circuit :



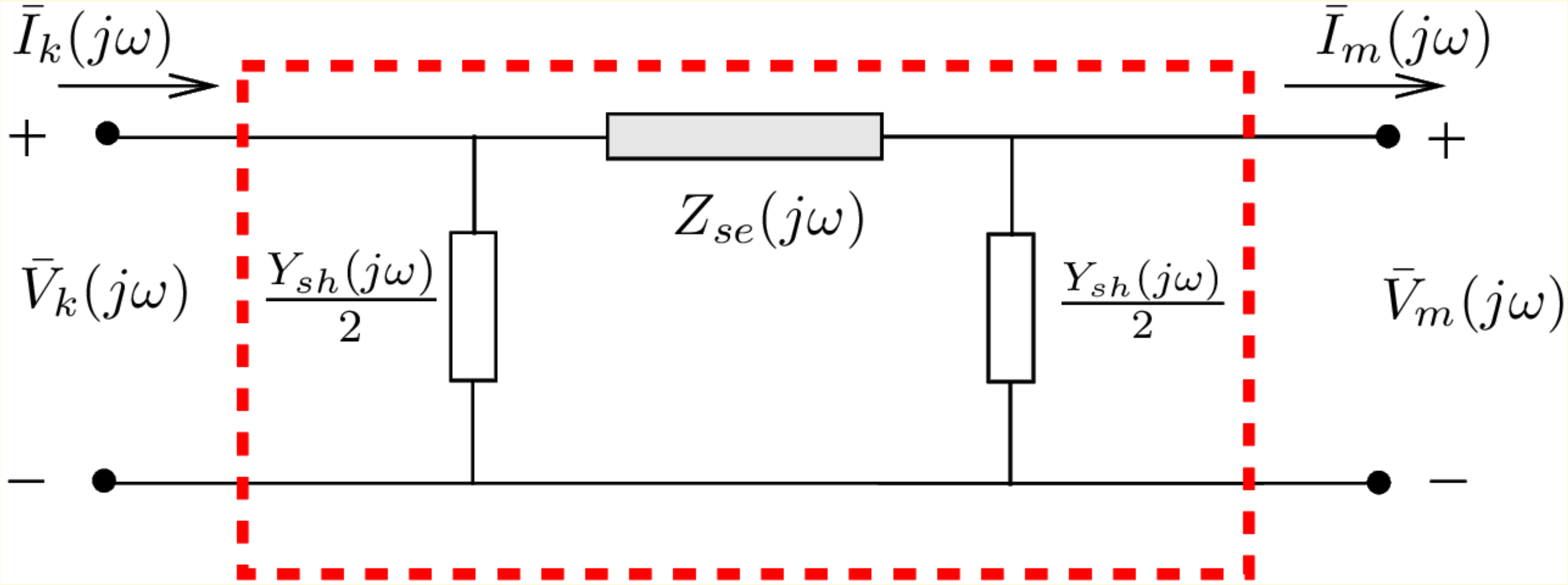
Above equations can be used to synthesize an equivalent PI circuit :



$$Z_{se} = Z_c \sinh(\gamma\, l) = \frac{Z \sinh(\gamma\, l)}{\gamma\, l}$$

$$Z = (\mathcal{R}' + j\,\omega\,\mathcal{L}').l$$

Above equations can be used to synthesize an equivalent PI circuit :



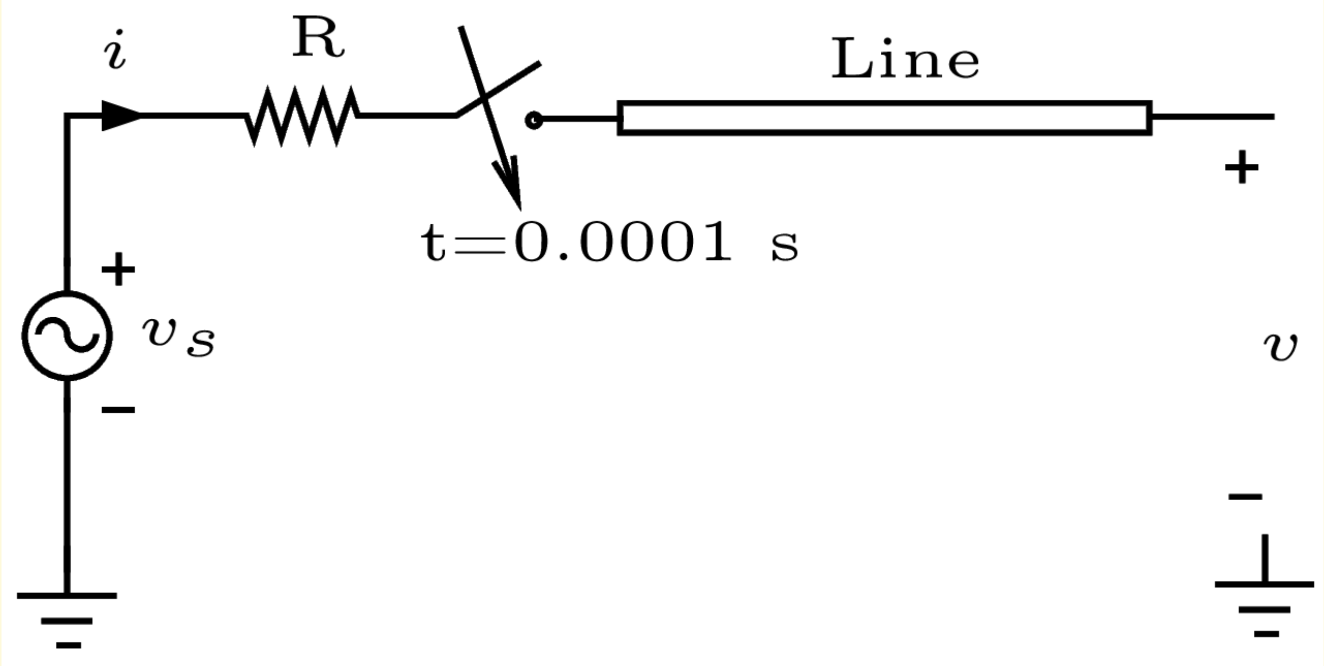
$$Z_{se} = Z_c \sinh(\gamma\, l) = \frac{Z\, \sinh(\gamma\, l)}{\gamma\, l}$$

$$Z = (\mathcal{R}' + j\,\omega\,\mathcal{L}').l$$

$$\frac{Y_{sh}}{2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma\, l}{2}\right) = \frac{Y}{2} \frac{\tanh\left(\frac{\gamma\, l}{2}\right)}{\left(\frac{\gamma\, l}{2}\right)}$$

$$Y = (\mathcal{G}' + j\,\omega\,\mathcal{C}').l$$

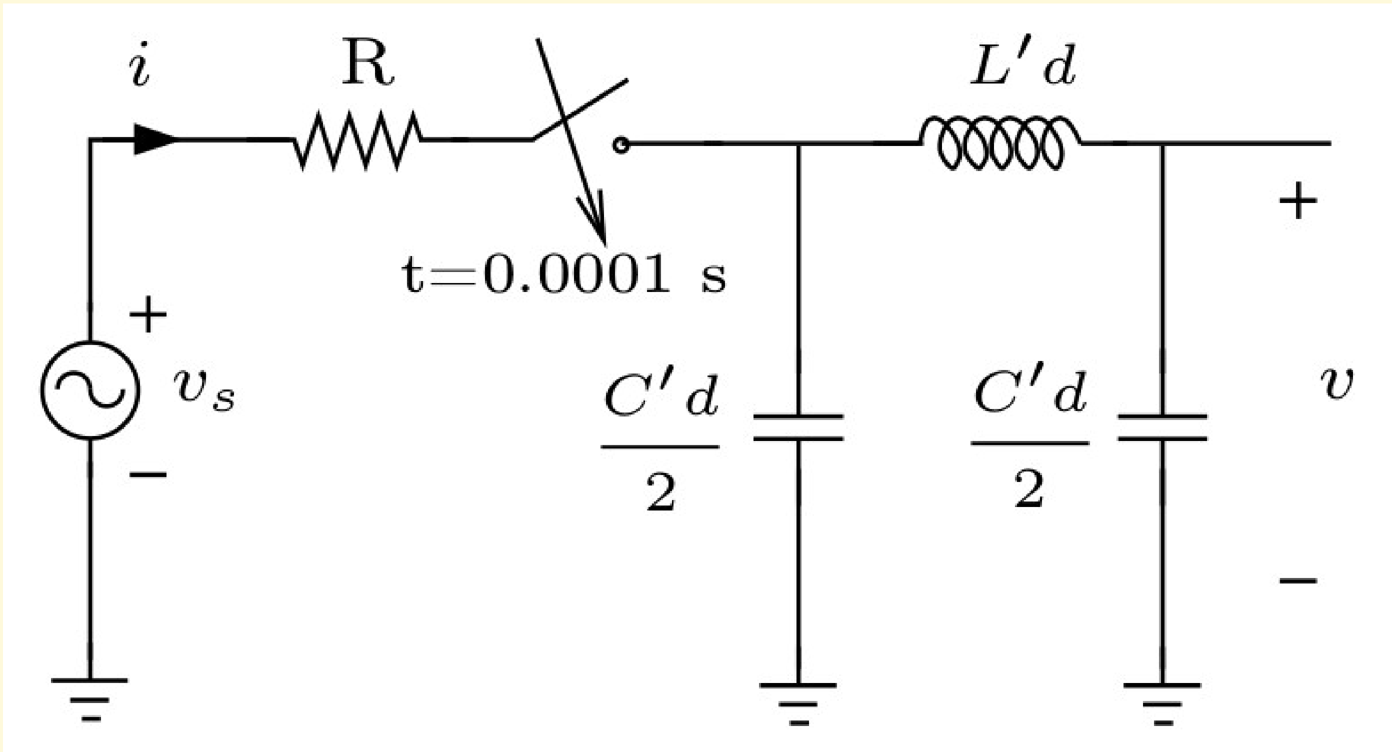
Consider a system



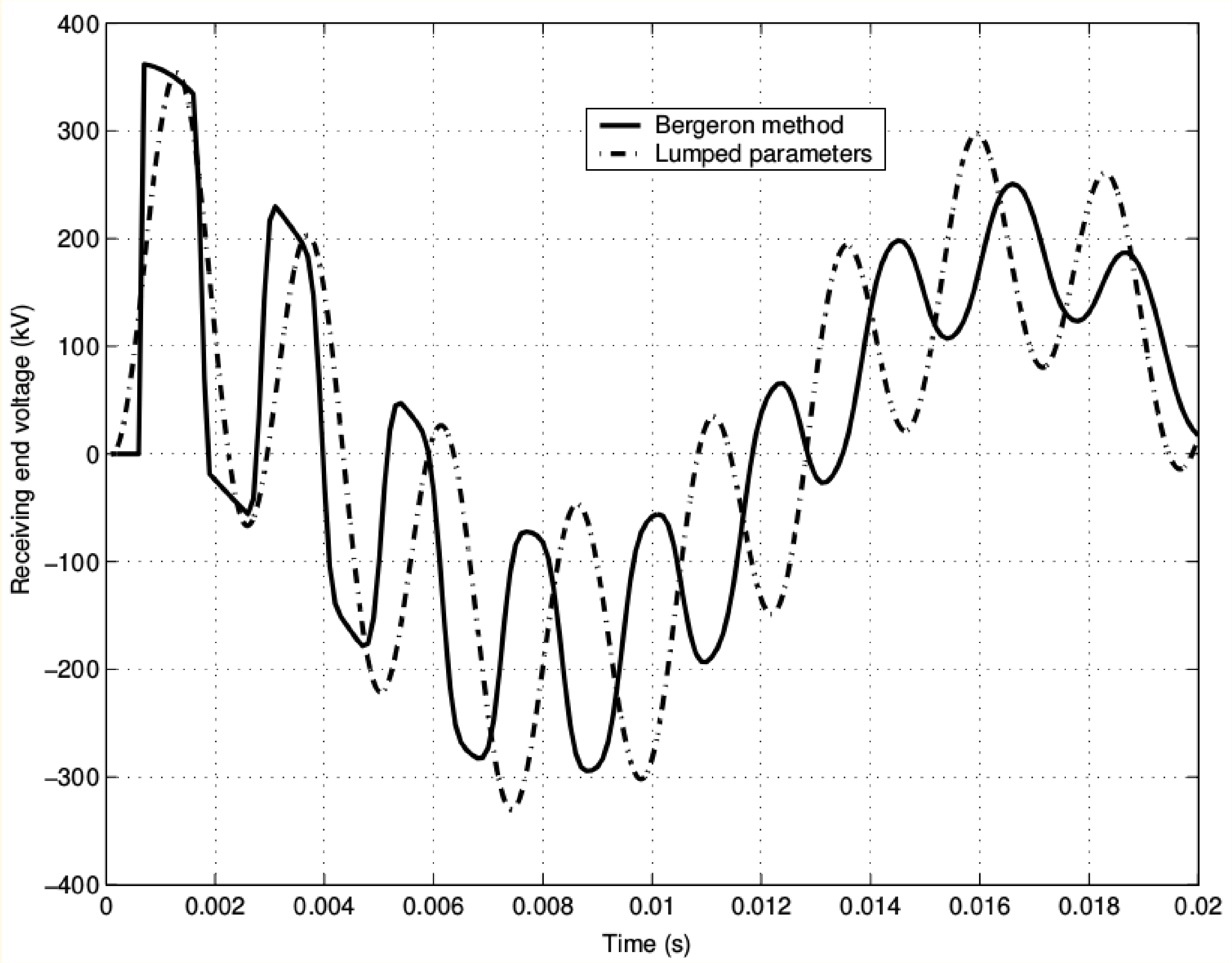
$$v_s = \frac{230000\sqrt{2}}{\sqrt{3}} \cos(2\pi 60 t) \text{ V. } \quad \text{L'} = 1.5 \text{ mH/mi},$$

$$\text{C'} = 0.02\ \mu\text{F/mi}, \quad \text{d} = 100 \text{ mi}, \quad \text{R} = 10\ \Omega.$$

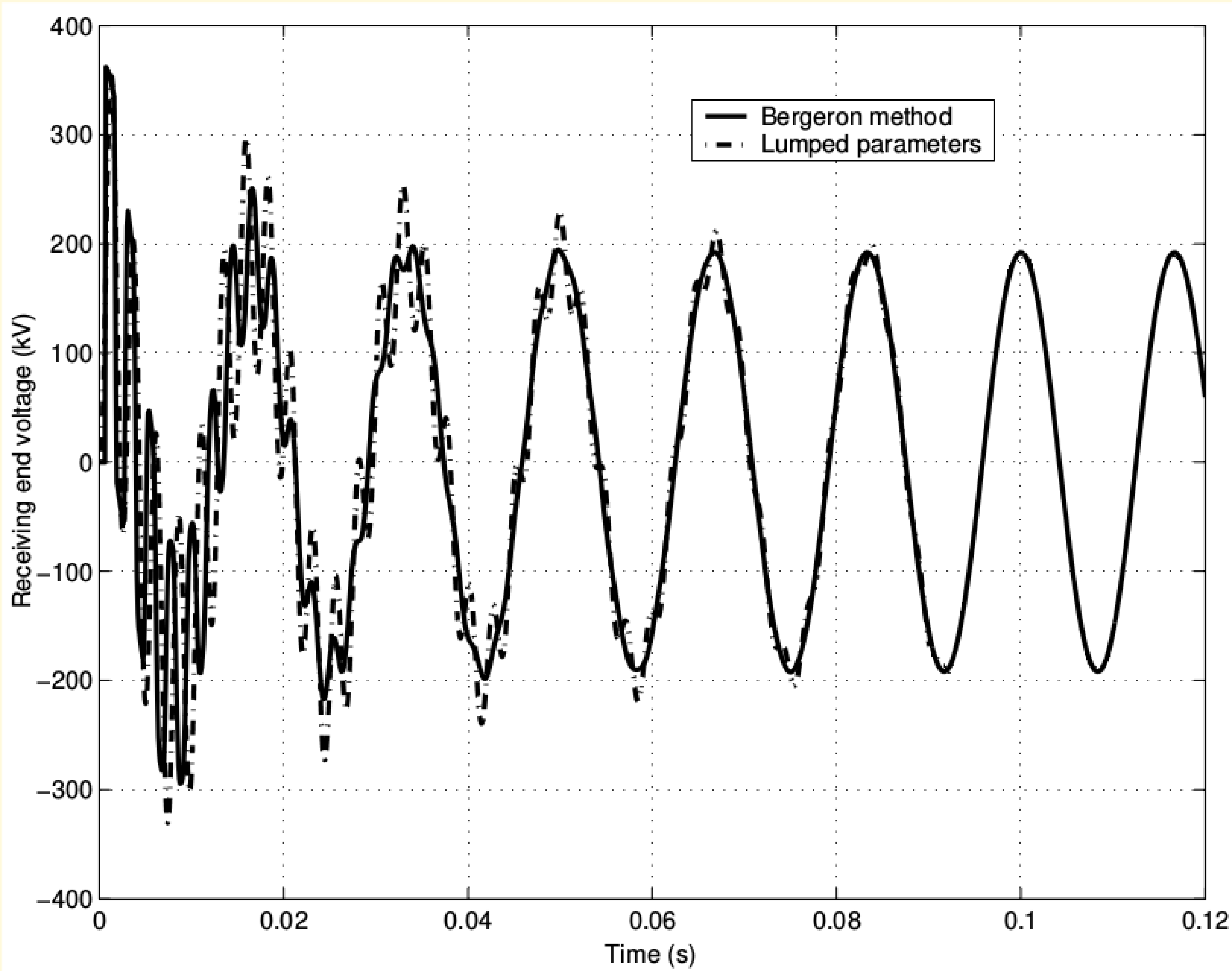
Source: Sauer P. W. And M. A. Pai (1998), *Power System Dynamics ad Stability,* Upper Saddle River, NJ, Prentice Hall.



i
R
L'd
t=0.0001 s
+
v_s
−
C'd/2
C'd/2
+
v
−



Bergeron method
Lumped parameters
Receiving end voltage (kV)
Time (s)
400
300
200
100
0
−100
−200
−300
−400
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02

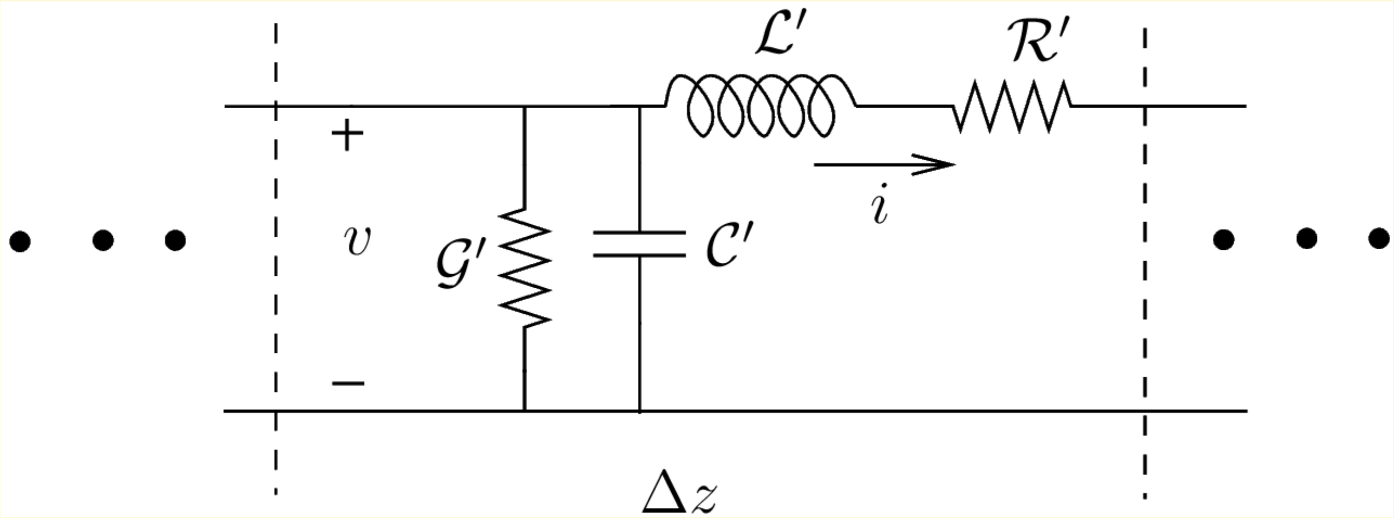

Bergeron method
Lumped parameters
Receiving end voltage (kV)
Time (s)
400
300
200
100
0
−100
−200
−300
−400
0
0.02
0.04
0.06
0.08
0.1
0.12

Summary of transmission lines concept:

Distributed parameter line model (travelling wave model)

$$-\frac{\partial v(z,t)}{\partial z} = \mathcal{L}' \frac{\partial i(z,t)}{\partial t} + \mathcal{R}' \, i(z,t)$$

$$-\frac{\partial i(z,t)}{\partial z} = \mathcal{C}' \frac{\partial v(z,t)}{\partial t} + \mathcal{G}' \, v(z,t)$$



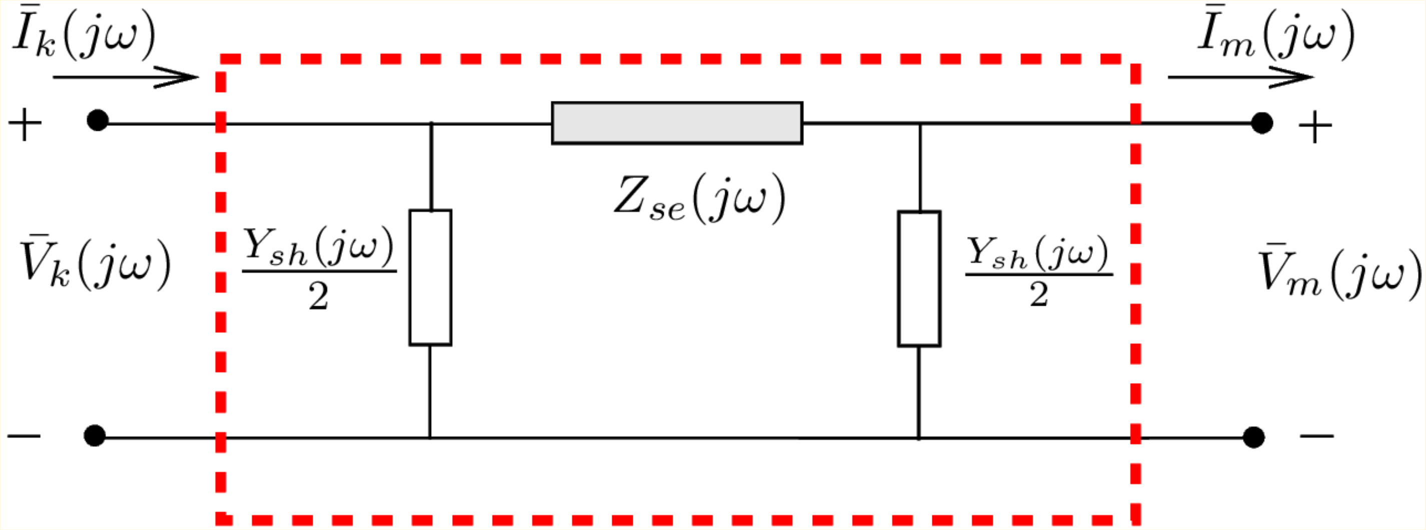
Summary of transmission lines concept:

Lumped parameter line model (sinusoidal steady-state model)

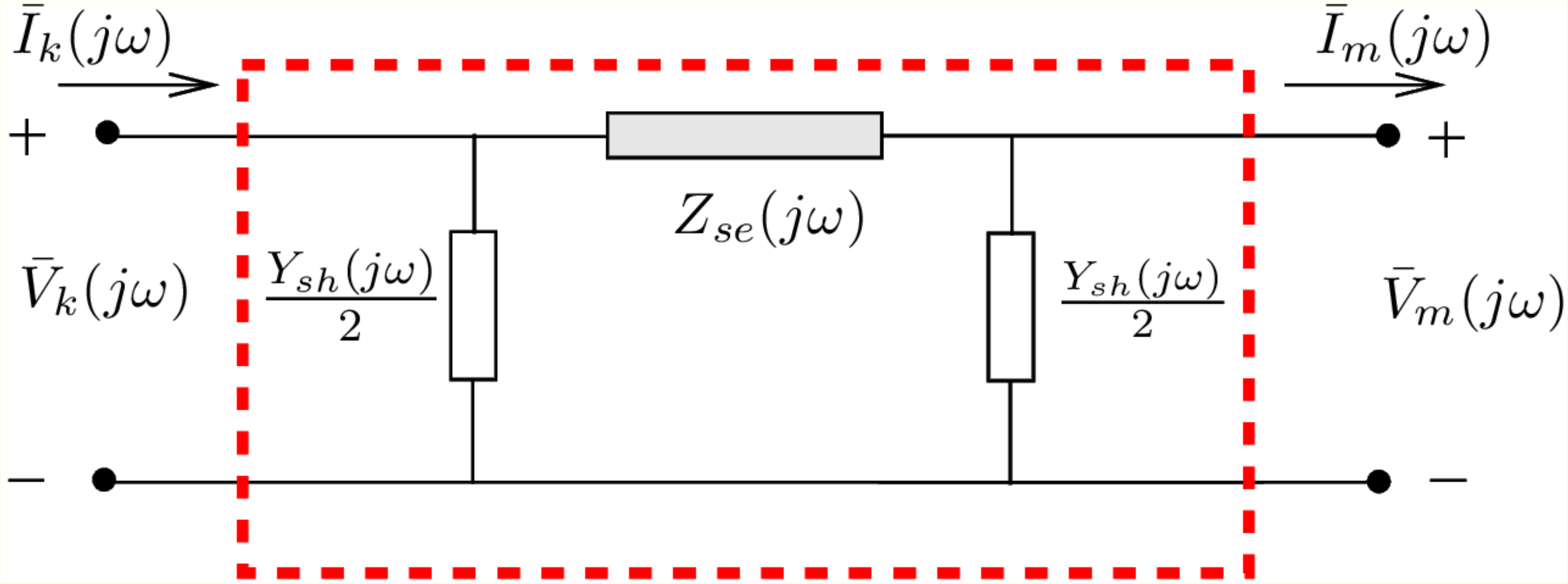
$$-\frac{d\,\bar{V}(j\,\omega)}{dz} = \mathcal{Z}(j\,\omega)\,\bar{I}(j\,\omega)\,, \quad -\frac{d\,\bar{I}(j\,\omega)}{dz} = \mathcal{Y}(j\,\omega)\,\bar{V}(j\,\omega)$$

where,

$$\mathcal{Z}(j\,\omega) = \mathcal{R}' + j\,\omega\,\mathcal{L}' \qquad \mathcal{Y}(j\,\omega) = \mathcal{G}' + j\,\omega\,\mathcal{C}'.$$



Above equations can be used to synthesize an equivalent PI circuit :



$$Z_{se} = Z_c \sinh(\gamma\, l) = \frac{Z \sinh(\gamma\, l)}{\gamma\, l}$$

$$Z = (\mathcal{R}' + j\,\omega\,\mathcal{L}').l$$

$$\frac{Y_{sh}}{2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma\, l}{2}\right) = \frac{Y}{2} \frac{\tanh\left(\frac{\gamma\, l}{2}\right)}{\left(\frac{\gamma\, l}{2}\right)}$$

$$Y = (\mathcal{G}' + j\,\omega\,\mathcal{C}').l$$

Polyphase transmission system

Bundled conductors

Courtesy: Prof. Sanjay Dambhare, College of Engineering Pune, India.

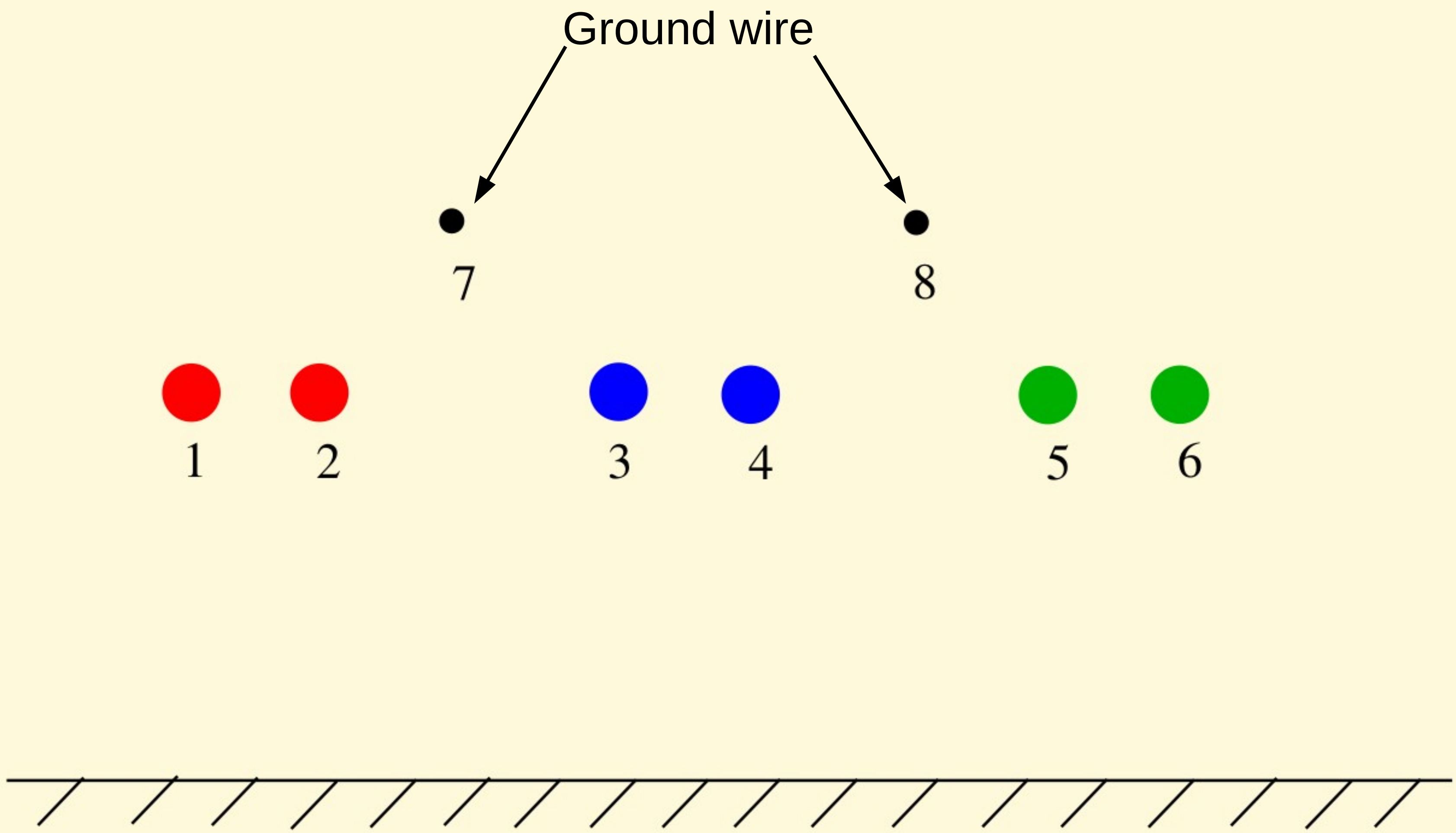
Bundled conductors

Performance of Bundles at 500 kV (Constant Conductor Cross Section)

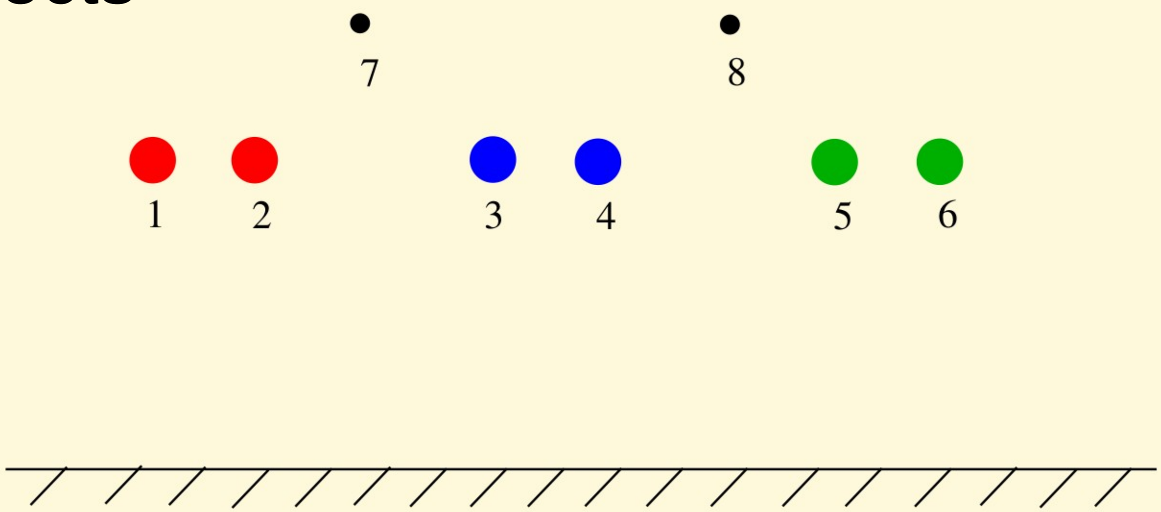
Characteristic	1 conductor 6.35 cm diam.	2 cohductors 4.5 cm diam.	3 conductors 3.66 cm diam.	4 conductors 3.18 cm diam.
Max. Gradient kV(rms)/cm				
Center phase	16.14	15.80	14.87	14.07
Outer phase	15.22	14.68	13.72	12.89
Av Gradient kV(rms)/cm				
Center phase	16.14	14.38	13.06	12.26
Outer phase	15.22	13.37	12.05	11.23
Audible Noise dB over 0.0002 μBar, 30.5 m from outer phase in heavy rain	63.8	57.5	51.7	48.8
Corona loss in heavy rain kW/3ϕ mile	187.5	111.5	76.3	58.3

Source: Walter Weeks, Transmission and Distribution of Electrical Energy, Harper and Row, 1981.

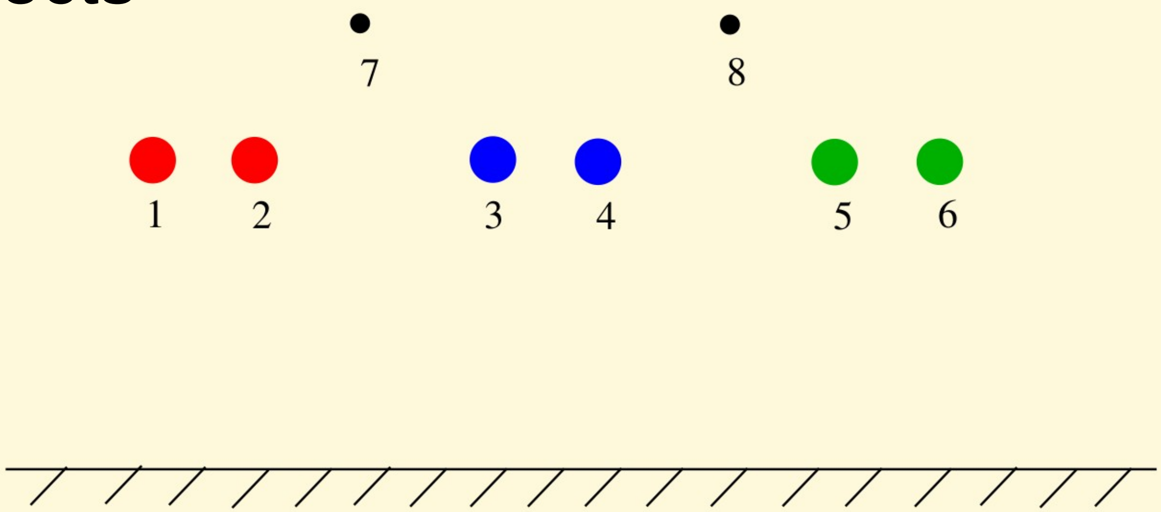
Consider a three-phase transmission system



Series Effects



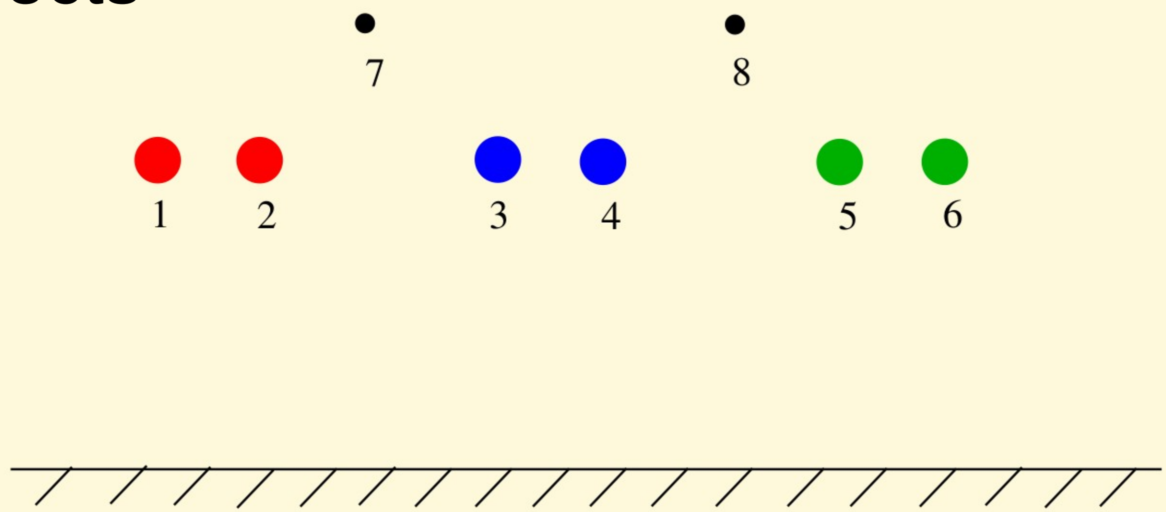
Series Effects



$$-\frac{d}{dz}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} & Z_{18} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} & Z_{28} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} & Z_{38} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} & Z_{47} & Z_{48} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} & Z_{57} & Z_{58} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} & Z_{67} & Z_{68} \\ Z_{71} & Z_{72} & Z_{73} & Z_{74} & Z_{75} & Z_{76} & Z_{77} & Z_{78} \\ Z_{81} & Z_{82} & Z_{73} & Z_{84} & Z_{85} & Z_{86} & Z_{87} & Z_{88} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

Note: variables are functions of distance from sending end as ‘*z*’.

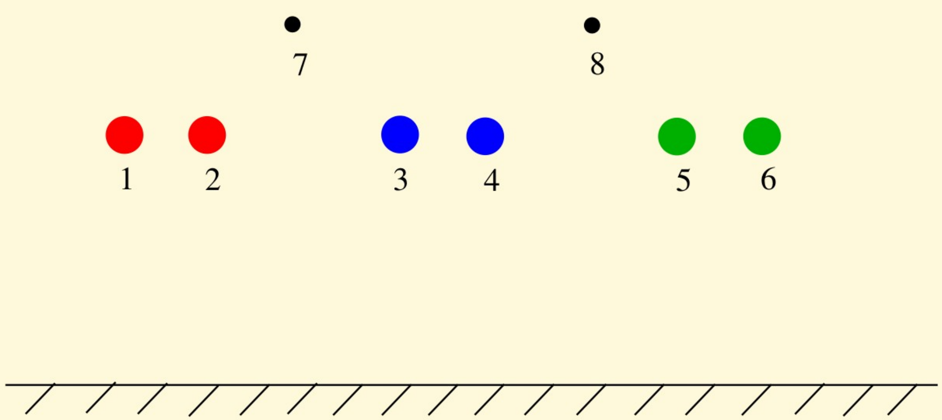
Series Effects



$$-\frac{d}{dz}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} & Z_{18} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} & Z_{28} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} & Z_{38} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} & Z_{47} & Z_{48} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} & Z_{57} & Z_{58} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} & Z_{67} & Z_{68} \\ Z_{71} & Z_{72} & Z_{73} & Z_{74} & Z_{75} & Z_{76} & Z_{77} & Z_{78} \\ Z_{81} & Z_{82} & Z_{73} & Z_{84} & Z_{85} & Z_{86} & Z_{87} & Z_{88} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

Note: variables are functions of distance from sending end as ‘*z*’.

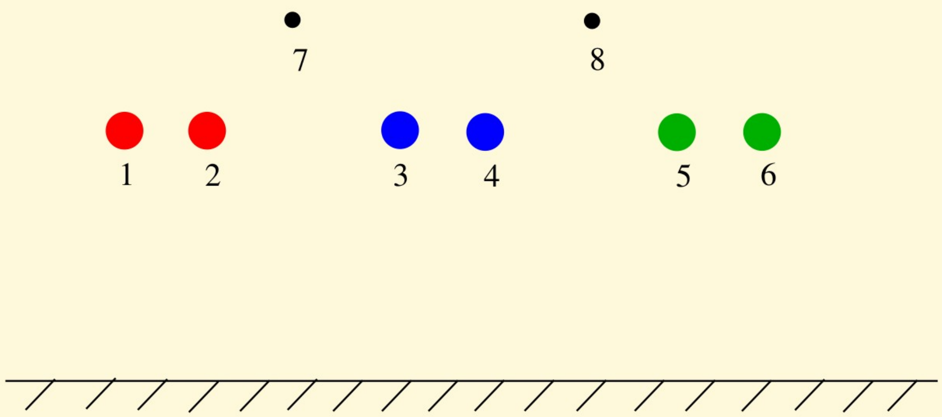
$$-\frac{d}{dz}\begin{pmatrix}\boldsymbol{V}_p \\ \boldsymbol{V}_g\end{pmatrix} = \begin{pmatrix}\boldsymbol{Z}_{pp} & \boldsymbol{Z}_{pg} \\ \boldsymbol{Z}_{gp} & \boldsymbol{Z}_{gg}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}_p \\ \boldsymbol{I}_g\end{pmatrix}$$



$$-\frac{d}{dz}\begin{pmatrix}\boldsymbol{V}_p\\ \boldsymbol{V}_g\end{pmatrix} = \begin{pmatrix}\boldsymbol{Z}_{pp} & \boldsymbol{Z}_{pg}\\ \boldsymbol{Z}_{gp} & \boldsymbol{Z}_{gg}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}_p\\ \boldsymbol{I}_g\end{pmatrix}$$

Conditions imposed on the above system are,

==> $\boldsymbol{V}_g = \begin{bmatrix}0 & 0\end{bmatrix}^T$

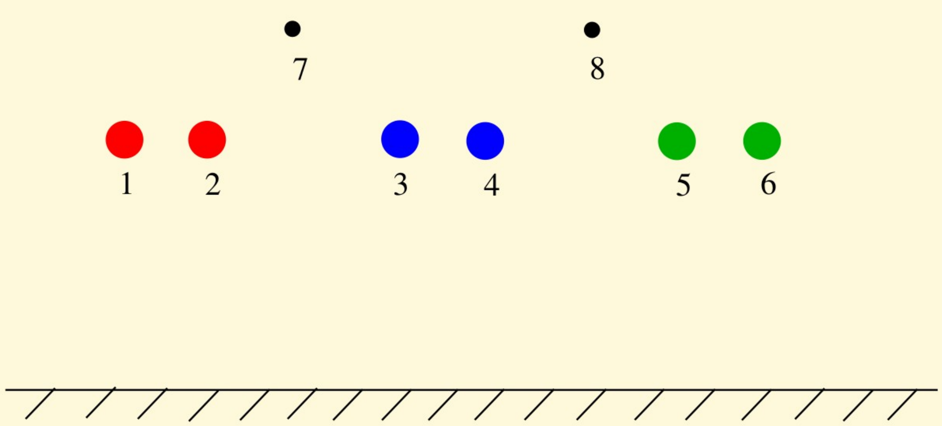


$$-\frac{d}{dz}\begin{pmatrix}\boldsymbol{V}_p \\ \boldsymbol{V}_g\end{pmatrix} = \begin{pmatrix}\boldsymbol{Z}_{pp} & \boldsymbol{Z}_{pg} \\ \boldsymbol{Z}_{gp} & \boldsymbol{Z}_{gg}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}_p \\ \boldsymbol{I}_g\end{pmatrix}$$

Conditions imposed on the above system are,

==> $\boldsymbol{V}_g = [0 \quad 0]^T$

==> $I_a = I_1 + I_2$ $I_b = I_3 + I_4$ $I_c = I_5 + I_6$



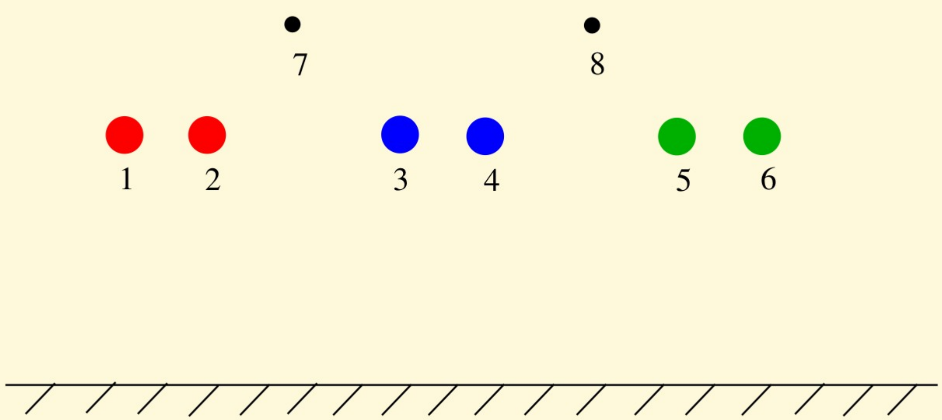
$$-\frac{d}{dz}\begin{pmatrix}\boldsymbol{V}_p\\ \boldsymbol{V}_g\end{pmatrix} = \begin{pmatrix}\boldsymbol{Z}_{pp} & \boldsymbol{Z}_{pg}\\ \boldsymbol{Z}_{gp} & \boldsymbol{Z}_{gg}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}_p\\ \boldsymbol{I}_g\end{pmatrix}$$

Conditions imposed on the above system are,

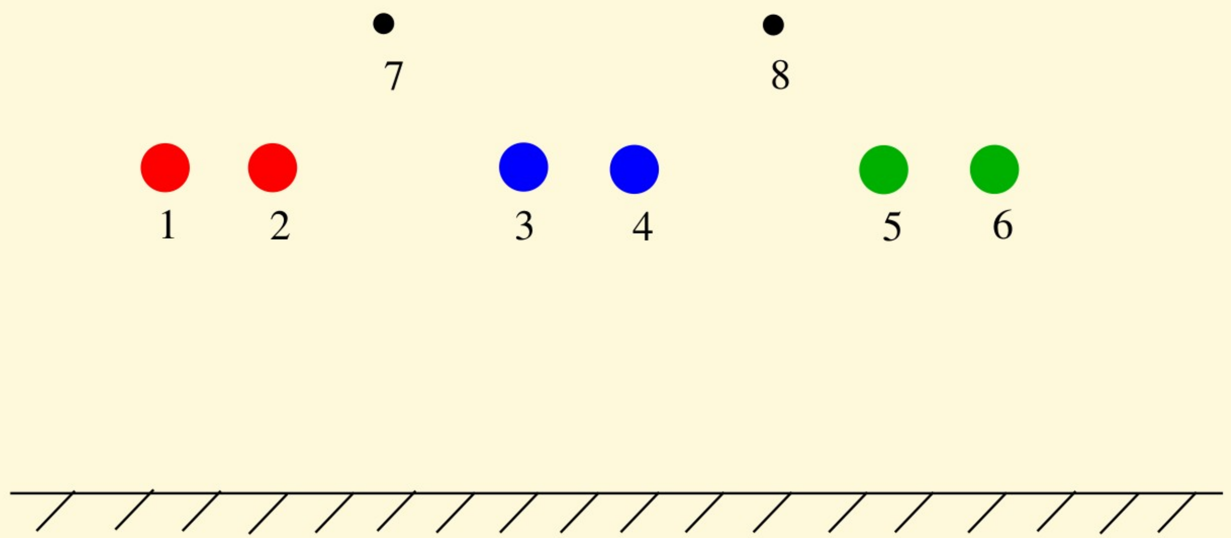
==> $\boldsymbol{V}_g = \begin{bmatrix}0 & 0\end{bmatrix}^T$

==> $I_a = I_1 + I_2$ $\quad I_b = I_3 + I_4$ $\quad I_c = I_5 + I_6$

==> $V_a = V_1 = V_2$ $\quad V_b = V_3 = V_4$ $\quad V_c = V_5 = V_6$



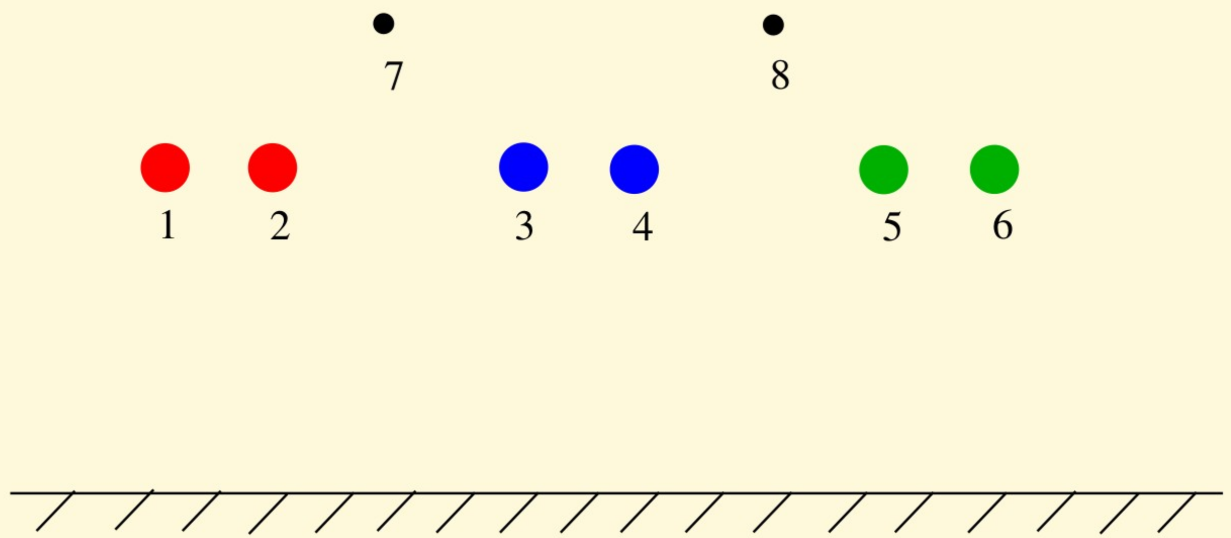
Series Effects



$$-\frac{d}{dz}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z'_{11} & Z'_{12} & Z'_{13} \\ Z'_{21} & Z'_{22} & Z'_{23} \\ Z'_{31} & Z'_{32} & Z'_{33} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Note: variables are functions of distance from sending end as ‘*z*’.

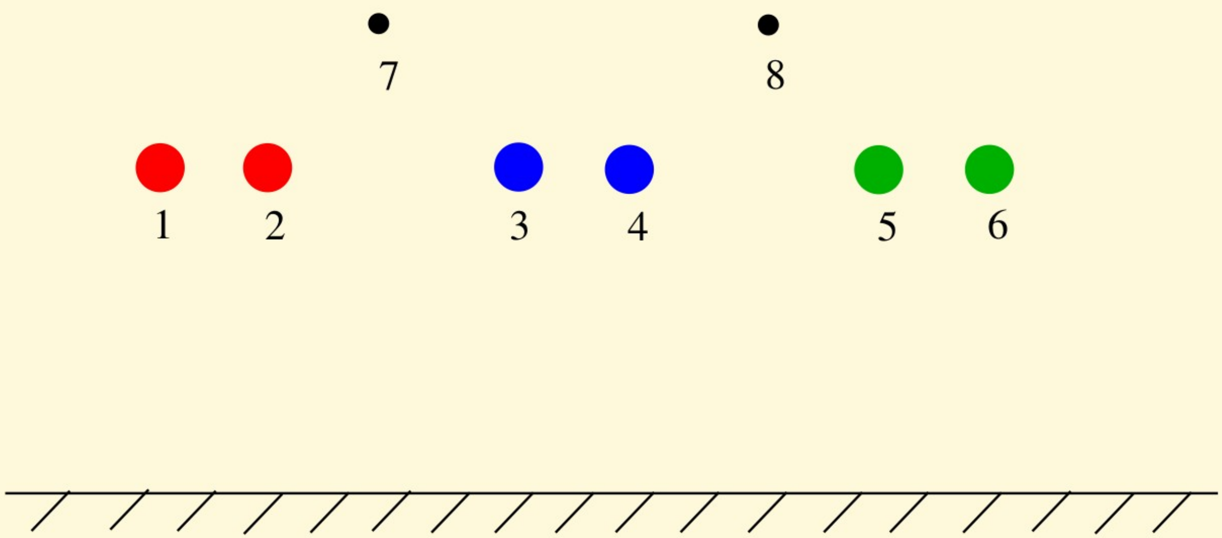
Shunt Effects



$$
-\frac{d}{dz}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} & Y_{17} & Y_{18} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} & Y_{27} & Y_{28} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} & Y_{36} & Y_{37} & Y_{38} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} & Y_{46} & Y_{47} & Y_{48} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} & Y_{56} & Y_{57} & Y_{58} \\ Y_{61} & Y_{62} & Y_{63} & Y_{64} & Y_{65} & Y_{66} & Y_{67} & Y_{68} \\ Y_{71} & Y_{72} & Y_{73} & Y_{74} & Y_{75} & Y_{76} & Y_{77} & Y_{78} \\ Y_{81} & Y_{82} & Y_{73} & Y_{84} & Y_{85} & Y_{86} & Y_{87} & Y_{88} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix}
$$

Note: variables are functions of distance from sending end as ‘*z*’.

Shunt Effects



$$-\frac{d}{dz}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} Y'_{aa} & Y'_{ab} & Y'_{ac} \\ Y'_{ba} & Y'_{bb} & Y'_{bc} \\ Y'_{ca} & Y'_{cb} & Y'_{cc} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Note: variables are functions of distance from sending end as ‘*z*’.

Transposition of lines

Courtesy: Prof. Sanjay Dambhare, College of Engineering Pune, India.

Formulae for lumped parameter line model

$$\gamma = \sqrt{\mathcal{Z}(j\,\omega).\mathcal{Y}(j\,\omega)}$$

$$Z_c = \sqrt{\frac{\mathcal{Z}(j\,\omega)}{\mathcal{Y}(j\,\omega)}}$$

where,

$$\mathcal{Z}(j\,\omega) = \mathcal{R}' + j\,\omega\,\mathcal{L}'$$

$$\mathcal{Y}(j\,\omega) = \mathcal{G}' + j\,\omega\,\mathcal{C}'$$

$$\gamma = \alpha + j\,\beta$$

Table 6.1 Typical overhead transmission line parameters

Nominal Voltage	230 kV	345 kV	500 kV	765 kV	1,100 kV
R (Ω/km) $x_L = \omega L$ (Ω/km) $b_C = \omega C$ (μs/km)	0.050 0.488 3.371	0.037 0.367 4.518	0.028 0.325 5.200	0.012 0.329 4.978	0.005 0.292 5.544
α (nepers/km) β (rad/km)	0.000067 0.00128	0.000066 0.00129	0.000057 0.00130	0.000025 0.00128	0.000012 0.00127
Z_C (Ω) SIL (MW)	380 140	285 420	250 1000	257 2280	230 5260
Charging MVA/km $= V_0^2 b_C$	0.18	0.54	1.30	2.92	6.71

Notes:
1. Rated frequency is assumed to be 60 Hz.
2. Bundled conductors used for all lines listed, except for the 230 kV line.
3. R, x_L, and b_C are per-phase values.
4. SIL and charging MVA are three-phase values.

Source: Prabha Kundur, Power System Stability and Control, TMH, 1982.

Table 6.2 Typical cable parameters

Nominal Voltage	**115 kV**	**115 kV**	**230 kV**	**230 kV**	**500 kV**
Cable Type	PILC	PIPE	PILC	PIPE	PILC
R (Ω/km) $x_L=\omega L$ (Ω/km) $b_C=\omega C$ (μs/km)	0.0590 0.3026 230.4	0.0379 0.1312 160.8	0.0277 0.3388 245.6	0.0434 0.2052 298.8	0.0128 0.2454 96.5
α (nepers/km) β (rad/km)	0.00081 0.00839	0.000656 0.00464	0.000372 0.00913	0.000824 0.00787	0.000127 0.00487
Z_C (Ω) SIL (MW)	36.2 365	28.5 464	37.1 1426	26.2 2019	50.4 4960
Charging MVA/km $= V_0^2 b_C$	3.05	2.13	13.0	15.8	24.1

Performance equations of transmission lines

For lossless lines $\mathcal{R}' = \mathcal{G}' = 0.$

$$\gamma = j\omega\sqrt{\mathcal{L}'\,\mathcal{C}'} = j\,\beta \quad , \qquad Z_c = \sqrt{\frac{\mathcal{L}'}{\mathcal{C}'}}$$

Performance equations of transmission lines

For lossless lines $\mathcal{R}' = \mathcal{G}' = 0.$

$$\gamma = j\omega\sqrt{\mathcal{L}'\,\mathcal{C}'} = j\,\beta \quad , \qquad Z_c = \sqrt{\frac{\mathcal{L}'}{\mathcal{C}'}}$$

$$\bar{V}_k = \cos(\beta\,l).\bar{V}_m + j\,Z_c\,\sin(\beta\,l).\bar{I}_m$$

$$\bar{I}_k = j\,\frac{1}{Z_c}\,\sin(\beta\,l).\bar{V}_m + \cos(\beta\,l).\bar{I}_m$$

Substituting $\theta = \beta\,l$

Performance equations of transmission lines

Substituting $\theta = \beta\, l$ (electrical length of the line.)

$$\bar{V}_k = \cos(\theta).\bar{V}_m + j\, Z_c\, \sin(\theta).\bar{I}_m$$

$$\bar{I}_k = j\,\frac{1}{Z_c}\,\sin(\theta).\bar{V}_m + \cos(\theta).\bar{I}_m$$

Performance equations of transmission lines

1) Purely resistive loading at receiving end

Thevenin equivalent at receiving end:

$$\bar{V}_{th} = \frac{\bar{V}_k}{\cos(\theta)}$$

Open circuit voltage at receiving end

$$Z_{th} = -\frac{\bar{V}_m}{\bar{I}_m}\Big|_{\bar{V}_k=0} = j\, Z_c\, \tan(\theta)$$

Thevenin impedance seen from receiving end

Performance equations of transmission lines

1) Purely resistive loading at receiving end

Thevenin equivalent at receiving end:

$$\bar{V}_{th} = \frac{\bar{V}_k}{\cos(\theta)}$$

Open circuit voltage at receiving end

$$Z_{th} = -\frac{\bar{V}_m}{\bar{I}_m}\Big|_{\bar{V}_k=0} = j\, Z_c\, \tan(\theta)$$

Thevenin impedance seen from receiving end