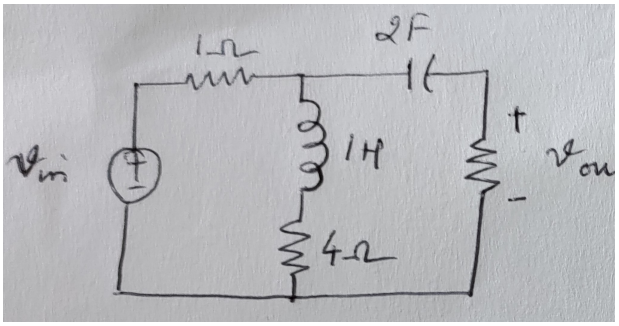
EE 302 : Control Systems

Tutorial 1

27 January, 2020

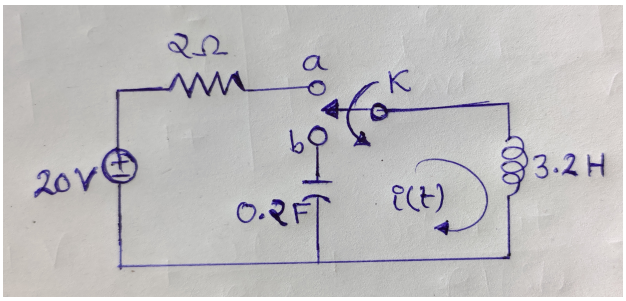
1. Find the Laplace transform of the following functions:
   (a) $2e^{-2t}\sin(2t)u(t)$
   (b) $t^2e^{-2t}u(t)$
   (c) $(1-e^{-10t})u(t)$
   (d) $e^{2t}u(t)+e^{-3t}u(-t)$
   (e) $(t\sin(2t)+e^{-2t})u(t)$
   (f) $(e^{4t}\sin(3t)+2e^{4t}\cos(3t))u(t)$
   (g) $t^2e^{t}\sin(5t)u(t)$
   (h) $t^3u(t)$

2. (a) Find $G(s)=\dfrac{V_{on}(s)}{V_{in}(s)}$ for the following RLC network.

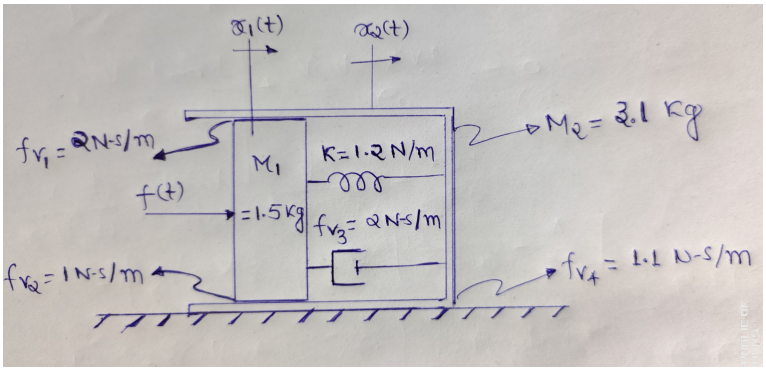


   (b) Use series-parallel based simplification rules for RLC.
   (c) Write KCL/KVL in Laplace transformed variables (instead of time-domain variables)
   (d) Find $\dfrac{I(s)}{V_{in}(s)}$ and $\dfrac{V_{in}(s)}{I(s)}$.

3. In the figure given below, switch K is moved from position 'a' to position 'b' at $t=0$ (a steady state existed at position 'a' prior to $t=0$). Solve for current $i(t)$, using the Laplace transform.



4. For the given system of figure,
   a) Use free-body diagram to obtain the differential equations in terms of $x_1(t)$, $x_2(t)$ and their derivatives.
   b) Hence, obtain the transfer function G(s) $=\dfrac{X_2(s)}{F(s)}$
   c) What are the poles and zeroes of G(s)?



5. Find the final values of $x(t)$, $\dot{x}$(t) as t$\to \infty$ for the following Laplace transforms.

   (a) $\dfrac{2s+1}{s^4+8s^3+16s^2+s}$ (b) $\dfrac{2}{s(s^2-s-2)}$

   (c) $\dfrac{5(s+2)}{s(s^2+4)}$ (d) $\dfrac{4s^2+3s}{s^2+as-4}$, $a \in \mathbb{R}$

   (e) $\dfrac{2s-a}{s(s^2+\epsilon s+4)}$, $a \in \mathbb{R}$ and $\epsilon > 0$ (f) $\dfrac{6s-31}{s^2(4s+3)}$

6. Find the initial values for the following Laplace transform (for $f(0^+)$ and $f'(0^+)$)

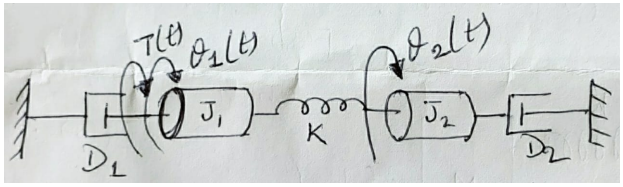
   (a) $F(s) = \dfrac{s^2+4s+7}{s^2+s+5}$ (b) $F(s) = \dfrac{2}{s^2+4s+7}$

   (c) $F(s) = \dfrac{s^2+3s-2}{s(s+a)}$, a$\in$R (d) $F(s) = \dfrac{as+5}{s^2+3s-2}$, $a \in \mathbb{R}$

7. Solve the following initial value problem using Laplace Transform:

$$\ddot{y} - \dot{y} - 6y = 2;\ y(0) = 1,\ \dot{y}(0) = 0$$

8. For the rotational mechanical system given below, draw both "through" and "across" analog circuits and explain using dynamical equations for the system.



9. An input $x(t) = e^{-2t}u(t) + \delta(t-6)$ is applied to an LTI system with impulse response $h(t) = u(t)$. Find the output y(t) using Laplace transform. Verify your answer by computing $y(t)$ in time domain.

10. For the Op-Amp circuit shown in figure below, find transfer function $\dfrac{V_{out}(s)}{V_{in}(s)}$