Tutorial 2

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1. Calculate the overall channel capacity of the cascade of two identical binary symmetric channels.

2. Determine the maximum differential entropy of a continuous random variable that has a uniform distribution between $-M$ and $+M$.

3. The binary erasure channel has two source symbols, 0 and 1, and three destination symbols, 0, 1 and E, where E denotes a detected by uncorrectable error. The forward transition probabilities are:

$$\begin{aligned} P(0|0) &= P(1|1) = 1-\alpha \\ P(E|0) &= P(E|1) = \alpha \\ P(1|0) &= P(0|1) = 0 \end{aligned}$$

   Determine the capacity of the erasure channel.

4. An 8-level PAM signal is defined by

$$s_i(t) = A_i \mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$

   where $A_i = \pm 1, \pm 3, \pm 5, \pm 7$. Formulate and draw the signal constellation of $\{s_i(t)\}_{i=1}^{8}$.

5. Consider the signals

$$\begin{aligned} s_1(t) &= u(t) - u(t - T/3) \\ s_2(t) &= u(t) - u(t - 2T/3) \\ s_3(t) &= u(t - T/3) - u(t - T) \\ s_4(t) &= u(t) - u(t - T) \end{aligned}$$

   Use the Gram-Schmidt orthogonalization procedure to find an orthonormal basis for these signals. Construct the corresponding signal space diagram.

6. Expand the signal $\mathrm{sinc}(3t/2)$ as an orthonormal expansion in the set of signals $\{\mathrm{sinc}(t-n); -\infty < n < \infty\}$.

7. A pair of signals $s_i(t)$ and $s_k(t)$ have a common duration T. Show that the inner product of the pair of signals is given by

$$\int_0^T s_i(t) s_k(t) dt = \mathbf{s}_i^T \mathbf{s}_k$$

   where $\mathbf{s}_i$ and $\mathbf{s}_k$ are the vector representations of $s_i(t)$ and $s_k(t)$ respectively.

8. A set of $2M$ biorthogonal signals is obtained from a set of M orthogonal signals by augmenting it with the negative of each signal in the set.

   a. The extension of orthogonal signals to biorthogonal signals leaves the dimensionality of the signal set unchanged. Explain how.

   b. Construct the constellation for the biorthogonal constellation for the orthogonal signal set $\{u(t) - u(t-1), u(t) - 2u(t-0.5) + u(t-1)\}$.