EE 338 DIGITAL SIGNAL PROCESSING
TUTORIAL PROBLEMS - SET 8

1. Consider discrete time signal x[n] defined as below,

$$x[n] = \begin{cases} 0.5^n, & 0 \leq n < N-1 \\ 0, & otherwise \end{cases}$$

   (a) Determine the DTFT of x[n].

   (b) Determine the discrete Fourier series $X_p[k]$ of $x_p[n]$, where $x_p[n]$ is a periodic sequence constructed from x[n] as follows:

$$x_p[n] = \sum_{l=-\infty}^{\infty} x[n + lL]$$

   Where $L = N$.

   (c) How $X_p[k]$ is related to DFT of x[n]?

   (d) Is it possible to recover x[n] from $X_p[k]$? If possible, what are the constraints on L to recover x[n] from $X_p[k]$?

2. Determine the 6-point circular convolution of the $x[n] = \{\underset{\uparrow}{6}, 5,4,3,2,1\}$ and $h[n] = \{\underset{\uparrow}{1}, 0, 1, 0, 0, 0\}$. Determine the values of n for which the linear convolution of x[n] and h[n] is same as the above 6-point circular convolution?

3. Consider the finite duration sequence x[n] such that

   $x[n] = \delta[n]+2\ \delta[n-1]+3\ \delta[n-2]$.

   (a) Obtain the expression for the 5-point DFT.

   (b) Obtain the linear convolution output $y[n] = x[n]*x[n]$ using DFT and IDFT.

4. Consider the sequence $x[n] = (0.3)^n\, u[n]$, with Fourier transform $X(e^{jw})$. Consider another sequence g[n] of length 8 such that its 8 point DFT G[k] corresponds 8 equally spaced samples of $X(e^{jw})$.

   (c) Obtain the expression relating g[n] and x[n] (with reasoning)?

   (d) Determine g[n] without computing DFT or IDFT.

5. Draw a neat labeled Signal Flow graph for a 3 point DFT of 3 input points $y[0]$, $y[1]$, $y[2]$ resulting in the DFT points Y[0], Y[1], Y[2].

6. Similar to decimation in frequency FFT algorithm for radix 2 i.e., $N = 2^v$ we can design algorithms for general case where $N = n^v$, where n is an integer. Such algorithms are

called radix-n FFT algorithms. To design radix-3 algorithm for $N = 9$ case, assuming $x[n]$ is nonzero over n=0 to 8, obtain

(a) sequence $x_1[n]$ in terms of $x[n]$ such that $X_1[k] = X[3k]$ for k = 0, 1, 2.
(b) sequence $x_2[n]$ in terms of $x[n]$ such that $X_2[k] = X[3k+1]$ for k = 0, 1, 2
(c) sequence $x_3[n]$ in terms of $x[n]$ such that $X_3[k] = X[3k+2]$ for k = 0, 1, 2.

7. For a real valued sequence $x[n]$ of length N, the DFT X[k] has following property,
If X[k] can be expressed as sum of real and imaginary parts, i.e.,

$$X[k] = X_R[k]+jX_I[k]$$

Then $X_R[k] = X_R[N\text{-}k]$ and $X_I[k] = -X_I[N\text{-}k]$ for $k=1,2, \ldots\ldots, N\text{-}1$

Using this symmetry property, the computation of the DFT of two real sequences can be effectively reduced or computation can be reduced for real sequence of length $2N$.

(a) Consider two sequences $x_1[n]$ and $x_2[n]$ and form a complex sequence as $g[n] = x_1[n]+j\,x_2[n]$, with corresponding DFT $G[k]=G_R[k]+jG_I[k]$. Show that the DFT $X_1[k]$ and $X_2[k]$ can be obtained from $G_R[k]$ and $G_I[k]$.

(b) Compare the number of multiplications and additions required for the method in (a) to computation of 2 radix-2 FFT of $x_1[n]$ and $x_2[n]$.