EE 338 Digital Signal Processing: Tutorial 7

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1. Determine the inverse Z-transform of

$$X(z) = \frac{1 + z^{-1}}{1 - 5z^{-1} + 6z^{-2}}$$

for all possible choices of ROCs of $X(z)$.

2. Determine the right sided signal $x[n]$ if its Z-transform $X(z)$ is given by:

   (a) $X(z) = \frac{1-\frac{1}{4}z^{-1}}{1+\frac{1}{4}z^{-1}}$

   (b) $X(z) = \frac{z^{-6}+z^{-7}}{1-z^{-1}}$

   (c) $X(z) = \frac{1-2z^{-1}}{z^{-1}-2}$

3. Let $h[n] = (\frac{1}{3})^n u[n]$ be the impulse response of an LTI system. Find the Z-transform of $h[n]$. Find the output of the system to input $x[n] = 5^n + (\frac{1}{4})(\frac{1}{2})^n u[n]$ without doing convolution.

4. Obtain the Z-transform of $x[n] = -(\frac{1}{2})^n u[-n-1]$. Does the DTFT of $x[n]$ converge? Find the range of α for which the DTFT of $\alpha^{-n}x[n]$ converges. Comment on the statement:*Existence of the DTFT ensures existence of Z-transform for any signal.*

5. Consider an LTI system with impulse response $h[n]$. The Z-transform of the input $x[n]$ is $X(z)$ and output $y[n]$ is $Y(z)$ as shown below.

$$\begin{aligned} X(z) &= \frac{z-8}{z-3} && |z| > 3 \\ Y(z) &= \frac{z}{(z-3)(z-4)} && |z| > 4 \end{aligned}$$

   (a) Find $x[n]$, $y[n]$.

   (b) Find $h[n]$ for the following conditions.

   - System is causal.
   - System is neither stable nor causal.

Is it possible to obtain $h[n]$ for stable system?

6. A stable and causal system has a rational system function $H(z)$ and an impulse response $h[n]$. The information about the total number of poles and zeros and their locations is not given. But it is specified that one of the poles is located at $z = \frac{1}{3}$ and one of the zeros is located on the unit circle. Comment on the following statements.

   (a) Fourier transform of $(\frac{1}{4})^n h[n]$ converges.

   (b) DTFT of $h[n]$ is zeros at some ω.

   (c) The system has finite duration impulse response.

   (d) $h[n]$ is real.

   (e) The system with impulse response $n(h[n] * h[n])$ is stable.

7. Consider a signal $x[n]$ whose Z-transform is given by

$$X(z) = H(z^2) + z^{-1}G(z^2)$$

   where ROC of $H(z^2)$ is $0.8 < |z| < 2$ and ROC of $G(z^2)$ is $0.5 < |z| < 3$. Find the sequence $x[n]$ in terms of $h[n]$ and $g[n]$. Find the ROC of $X(z)$.

8. Find the Z-transform of the following sequences and mention the ROCs.

   (a) $(-1)^n a^{-n} u[n]$

   (b) $\{a^n cos(\beta n) + b^n sin(\alpha n)\}u[n], \qquad a > 1,\ b > 1$

   (c) $n^2 \alpha^{n-1} u[n-2]$

   (d) $\sum_{k=0}^{k=\infty} a^k b^{n-k}, \qquad a < b$

9. Consider an LTI system with input and output denoted by $x[n]$ and $h[n]$. The impulse response of the system is $h[n]$ where

$$h[n] = \alpha^n u[n] + \left(\frac{1}{4}\right)^n u[n]$$

(a) Find the Z-transform of $h[n]$ assuming $|\alpha| < 1$. Is the system stable? Obtain the pole-zero plot.

(b) If we cascade this system with another system whose impulse response is $h_1[n]$, output of this second system is an original input $x[n]$. Obtain $h_1[n]$ and draw the corresponding pole-zeros plot. Find the values of α for which both $h[n]$ and $h_1[n]$ are stable.

10. For each of the following systems comment on poles and zeros of the system and length of the impulse response of the system. It is given that each LTI system below is causal and has real valued impulse response.

(a) $H_1(z)$ has a pole at $z = 0.2e^{j\pi/4}$. The response of the system to unit step input decays to zero as $n \to \infty$.

(b) $H_2(z)$ has a zero at $z = 0.3e^{j\pi/3}$. Given that $\measuredangle H_2(e^{j\omega}) = -2.5\omega$ and $\sum_n h_2[n] = 0$.

11. A homogeneous solution to difference equation is the solution to the equation with zero input, but with some constraints imposed on the output at given points. The particular integral is a sequence, which satisfies the difference equation and has a form largely similar to the input excitation sequence. Consider the difference equation

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + 2x[n]$$

(a) Obtain the general form of the homogeneous solution to this difference equation. Obtain the homogeneous solution when $y[-1] = 2$, $y[-2] = 4$.

(b) Can we impose two conditions on $y[n]$ at two arbitrarily chosen points $n = n_1$ and $n = n_2$?. Obtain the homogeneous solution with conditions $y[n_1] = y_1$ and $y[n_2] = y_2$ for the most general situation.

(c) Obtain a particular integral for the input $x[n] = (1/2)^n u[n]$.

(d) In what cases would the particular integral resemble the input, and in what cases would it be a modified version of the input? (Hint: repeated poles)

(e) Now obtain the general solution to the difference equation with conditions as chosen in part (a) above. Can one obtain a general solution in part (b)? What would it be?

12. Obtain the approximate magnitude spectrum for following pole-zeros plots shown in Figure 1. Comment on the magnitude spectrum. Assume impulse response corresponding to each pole-zero plot is real.(Hollow circle represents a zero, cross represents a pole.)

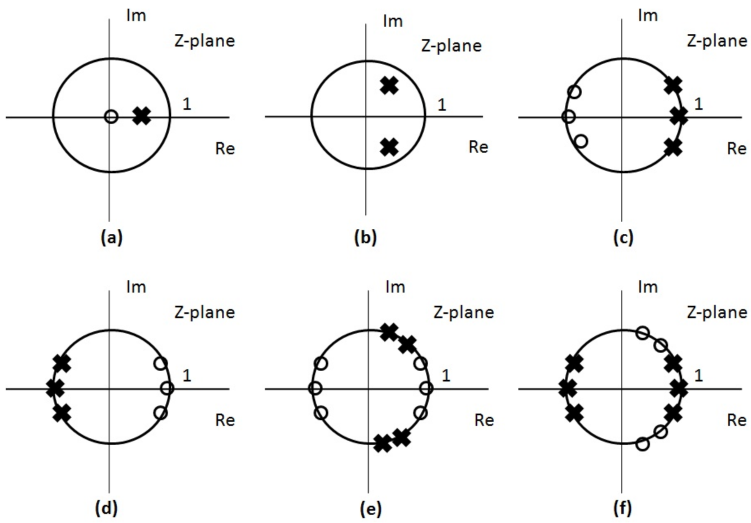

Figure 1: Figure for question 12

13. Draw the pole zero plot of the following signal.

$$\begin{aligned} h[n] &= \alpha^n \qquad & 0 \le n \le L-1, \alpha > 0 \\ &= 0 & \text{otherwise} \end{aligned}$$

If $x[n]$ is the input and $y[n]$ is the output of the system with impulse response given above, write down difference equations such that one is having recursion and the other is without recursion. Here recursion refers to the output $y[n]$ being expressed in term of delayed and/or advanced samples of $y[n]$ itself along with $x[n]$.

14. The aperiodic **autocorrelation sequence** $R_{xx}[n]$ for a real, absolutely summable sequence $x[n]$ is defined as

$$R_{xx}[n] = \sum_k x[k]x[n+k]$$

   (a) Determine its Z-transform $C_{xx}(z)$ in terms of $X(z)$ which has ROC $0 \leqslant r_1 \leqslant z \leqslant r_2)$ and specify the ROC in terms of that of $X(z)$.

   (b) Determine $C_{xx}(z)$ along with ROC if

$$x[n] = a^n u[n], x[n] = b^n u[-n], x[n] = a^n u[n] + b^n u[-n]$$

   Are there any constraints on a and b for the autocorrelation sequence to exist?

   (c) Is the operation of finding the autocorrelation sequence corresponding to a given sequence a linear operation? Explain with an example.

   (d) Show that the sequence corresponding to a given autocorrelation sequence is not unique, by exhibiting three different sequences $x[n]$ that give

$$C_{xx}(z) = \{(1 - az^{-1})(1 - az)\}^{-1}$$

   What are the constraints on a and what would the ROC be?

15. The input to a causal LTI system is given by $x[n]$. The system output has Z-transform denoted by $Y(z)$ as shown below.

$$\begin{aligned} x[n] &= u[-n-1] + 0.5^n u[n] \\ Y(z) &= \frac{-0.5z^{-1}}{(1 - 0.5z^{-1})(1 + z^{-1})} \end{aligned}$$

   (a) Determine the system function $H(z)$ and obtain its ROC.

   (b) Determine $y[n]$ while specifying the ROC of $Y(z)$.

16. A causal and stable system is known as the minimum phase system if its inverse system is also causal and stable. What are the constraints on the pole-zero locations of the minimum phase system? Show that the cascade of two minimum phase system is also a minimum phase system.