Find the Inverse z-transform of

(i) $$\frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})} \quad \text{ROC } |z|>\frac{1}{2}.$$

Sol: splitting into partial fractions :-

$$\frac{A}{(1-\frac{1}{2}z^{-1})} + \frac{B}{(1-\frac{1}{3}z^{-1})} = \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}.$$

$$A(1-\tfrac{1}{3}z^{-1}) + B(1-\tfrac{1}{2}z^{-1}) = 1$$

$$\left.\begin{array}{l} A+B=1 \\ \frac{A}{3}+\frac{B}{2}=0 \end{array}\right\} \Rightarrow \begin{array}{l} A=3 \\ B=-2 \end{array}.$$

$$\frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})} = \frac{3}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

($\because$ ROC $|z|>\frac{1}{2}$) given.

$$\frac{1}{1-\alpha z^{-1}} \longrightarrow \alpha^n u[n] \quad \text{for ROC } |z|>|\alpha|$$

$$\tfrac{1}{2} > \tfrac{1}{3}.$$

So, Inverse Z-Transform is

$$= 3\left(\tfrac{1}{2}\right)^n u[n] - 2\left(\tfrac{1}{3}\right)^n u[n] \quad \text{— (1)}$$

(ii) Convolve $z^{-1}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$ with $z^{-1}\left(\frac{1}{1-\frac{1}{3}z^{-1}}\right)$.

$$\because \; z^{-1}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) = \left(\tfrac{1}{2}\right)^n u[n] \quad \text{for ROC } |z|>\tfrac{1}{2}.$$

$$z^{-1}\left(\frac{1}{1-\frac{1}{3}z^{-1}}\right) = \left(\tfrac{1}{3}\right)^n u[n]$$

Convolution:

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x(n-k)\, y(k)$$

Ans : $\left(\tfrac{1}{2}\right)^n u[n] * \left(\tfrac{1}{3}\right)^n u[n]$.

$$\left(\tfrac{1}{2}\right)^n u[n] * \left(\tfrac{1}{3}\right)^n u[n] = \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{n-k} u[n-k] \cdot \left(\tfrac{1}{3}\right)^k u[k]$$

$$u[n-k] = \begin{cases} 1 & ; \text{ for } k \le n. \\ 0 & ; \text{ otherwise} \end{cases}$$

$$u[k] = \begin{cases} 1 & ; \text{ for } k \ge 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

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from this,

$$u[n-k]\,u[k] = \begin{cases} 1 & ; \text{for } 0 \le k \le n \\ 0 & ; \text{otherwise} \end{cases}$$

So,

$$\left(\tfrac{1}{2}\right)^n u[n] * \left(\tfrac{1}{3}\right)^n u[n] = \sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^{n-k} \left(\tfrac{1}{3}\right)^k .$$

(In G.P Series)

$$= \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{n} \left(\tfrac{2}{3}\right)^k .$$

$$= \left(\tfrac{1}{2}\right)^n \left(\frac{1-(2/3)^{n+1}}{1-2/3}\right)$$

$$= 3\left(\tfrac{1}{2}\right)^n \left(1-\left(\tfrac{2}{3}\right)^{n+1}\right)$$

$$= \left[3\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{3}\right)^n\right] u[n] \quad \left(\because 0 \le k \le n,\ n \ge 0\right) \quad = (2)$$

So, from (1) & (2) we show that they produce same Result.