EE 302 — Tutorial - 2 (solutions)

Q.1 Linearity & Time Invariance

- Linearity:
  - superposition $\rightarrow f(x+y) = f(x) + f(y)$
  - homogeneity $\rightarrow f(\alpha x) = \alpha f(x)$
- time invariance: $f(x(t), t) = f(x(t))$

(a) non-linear, time invariant

(b) non-linear, time variant

(c) linear, time variant

(d) linear, time variant

(e) non-linear, time invariant

Q.2 (a) undamped; poles at $\pm 2j$; $T_p = n\pi$, (oscillatory system)

(b) overdamped; poles at $-1, -2$; dominant pole: -1; $T_s = 4s$.

Q.3 $\omega_n = 4$, $\xi = 1$; critically damped: no peak time & % OS

$T_s = 1s$

Q.4 $T_s = 1s$; $\xi = 0.6$; $\omega_n = 20/3$

$$TF = \frac{9}{9s^2 + 72s + 400}$$

Q.5

$$TF = \frac{32}{s^2 + 16s + 32}$$

Q.7

$$\frac{V_c}{V_i} = \frac{1/cs}{R + Ls + 1/cs} = \frac{1}{RCs + LCs^2 + 1} = \frac{100}{s^2 + 10Rs + 100}$$

$\omega_n = 10$, $2\xi = R$ $\therefore$ R (in Ω) $\rightarrow$
- < 2, under damped
- $= 2$, critically damped
- > 2, over damped

Q.10

T_s : $A > B > C > D$

ξ : $C < B < A$ and D (A, D has repeated poles), so $A = D$ (critically damped)

O.S : $C > B$, no overshoot for A & D.

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Q.8

- $T_{stall} = 100$ N/m
- $\omega_{nL} = 150$ rad/s
- $E_a = 50$ V

} (from T-ω characteristics)

- $\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} \Rightarrow \frac{K_t}{R_a} = \underline{\underline{2}}$
- $\omega_{nL} = E_a/K_b \Rightarrow K_b = \underline{\underline{1/3}}$
- $J_m = J_a + J_L (N_1/N_2)^2 = \underline{\underline{7}}$ kg m^2
- $D_m = D_a + D_L (N_1/N_2)^2 = \underline{\underline{12}}$ Nms/rad

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/R_a J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]} \; ; \quad \theta_L(s) = \frac{1}{3}\theta_m(s)$$

$$\therefore \boxed{\frac{\theta_L(s)}{E_a(s)} = \frac{2/21}{s(s + 38/21)}} \qquad \left(K = 2/21,\ \alpha = 38/21\right)$$

Q.9

$$\frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{T_1}{T_2}$$

Hence, $\left(\left[J_2 + J_1\left(\frac{N_2}{N_1}\right)^2\right]s^2 + \left[D_2 + D_1\left(\frac{N_2}{N_1}\right)^2\right]s + K\left(\frac{N_2}{N_1}\right)^2\right)\theta_2(s) = T(s)\,\frac{N_2}{N_1}$

$$\Rightarrow (19s^2 + 11s + 108)\,\theta_2(s) = 3 \cdot T(s)$$

$$\therefore \boxed{G(s) = \frac{\theta_2(s)}{T(s)} = \frac{3}{19s^2 + 11s + 108}}$$

Q.6 $\underline{\xi < 1}$; roots are complex conj. (underdamped system)

$\underline{O/P}$: $Y(s) = \frac{1}{s}\left[\frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}\right] = \frac{\alpha}{s} + \frac{\sigma^2 + \omega^2}{(s+\sigma)^2 + \omega^2}$

$$L^{-1}[Y(s)] = y(t) = 1 - \frac{1}{\beta} e^{-\xi\omega_n t} \sin(\omega_n \beta t + \theta)$$

where, $\beta = \sqrt{1-\xi^2}$, $\theta = \cos^{-1}\xi$

- $\underline{\text{peak time}}$, $T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ (derive them)
- $\underline{\text{Peak overshoot}} = 100\, e^{-\xi\pi/\sqrt{1-\xi^2}}$