

## TUTORIAL 2 : QUESTION 7

$$x[n] = \alpha^n u[n]$$

$$h[n] = \beta^n u[n]$$

$$x[n] \rightarrow \boxed{\begin{matrix} \text{LSI} \\ h[n] \end{matrix}} \rightarrow y[n] \quad y[n] = (x * h)[n]$$

$$\therefore y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$y[n] = \sum_{k=0}^n \alpha^k \beta^{n-k}$$

$$y[n] = 0 \text{ for } n < 0$$

$$y[n] = \beta^n \sum_{k=0}^n \left( \frac{\alpha}{\beta} \right)^k$$

$$y[n] = \beta^n \frac{1 - \left( \frac{\alpha}{\beta} \right)^{n+1}}{1 - \frac{\alpha}{\beta}} \quad \text{using G.P. with common ratio } \frac{\alpha}{\beta}.$$

$$y[n] = \frac{\beta^{n+1} \left[ 1 - \left( \frac{\alpha}{\beta} \right)^{n+1} \right]}{\beta - \alpha}$$

$$\therefore y[n] = \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \text{ for } n \geq 0$$

$$= 0 \text{ for } n < 0$$

$$\therefore y[n] = \left( \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right) u[n]$$