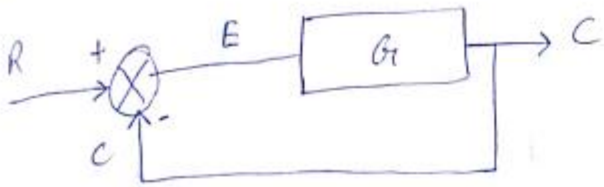
TUTORIAL SOLUTIONS

Q.1)

$$e_{ss} = \lim_{t \to \infty} e(t)$$

steady state error



$$C = GE$$

$$C = G\left[R - C\right]$$

$$\boxed{\frac{C}{R} = \frac{G}{1+G}} \Leftarrow \text{closed loop transfer function}$$

$$\frac{C}{R} = \frac{GE}{R} = \frac{G}{1+G}$$

$$\boxed{\frac{E}{R} = \frac{1}{1+G}} \Rightarrow \text{Error transfer function}$$

$$\boxed{E = \frac{R}{1+G}}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s E(s)$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)}}$$

for type = 0 :-

Let $G(s) = \dfrac{K(s+z_1)\cdots}{(s+P_1)\cdots}$

for step input :-

$$\boxed{R_s = A/s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s}{1 + \frac{K(s+z_1)\cdots}{(s+P_1)\cdots}}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}} \qquad \boxed{K_p = \lim_{s \to 0} G(s)}$$

for Ramp i/p :-

$$R(s) = A/s^2$$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s^2}{1 + \frac{K(s+z_1)\cdots}{(s+P_1)\cdots}} = \infty$$

$$\boxed{e_{ss} = \infty}$$

for parabolic i/p

$$R(s) = A/s^3$$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s^3}{1 + \frac{K(s+z_1)\cdots}{(s+P_1)\cdots}} = \infty \qquad \boxed{e_{ss} = \infty}$$

for type = 1 :

let $G(s) = \dfrac{K(s+z_1)\cdots}{s(s+P_1)\cdots}$

for step input

$R_s = A/s$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s}{1 + \frac{K(s+z_1)\cdots}{s(s+P_1)\cdots}} = 0 \qquad \boxed{e_{ss} = 0}$$

for Ramp input :

$\boxed{R_s = A/s^2}$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s^2}{1 + G(s)} = \lim_{s \to 0} \frac{A}{s + sG(s)} = \frac{A}{K_v}$$

$$\boxed{e_{ss} = A/K_v} \qquad \boxed{K_v = \lim_{s \to 0} sG(s)}$$

for parabolic input :

$R_s = A/s^3$

$$e_{ss} = \lim_{s \to 0} \frac{s \times A/s^3}{1 + \frac{K(s+z_1)\cdots}{s(s+P_1)\cdots}} = \infty$$

$$\boxed{e_{ss} = \infty}$$

for type = 2 :-

let $G(s) = \dfrac{K(s+z_1)\text{-----}}{s^2(s+P_1)\text{-----}}$

for step input :

$R_s = A/s$

$$e_{ss} = \lim_{s\to 0} \frac{s \times A/s}{1 + \frac{K(s+z_1)\text{---}}{s^2(s+P_1)\text{---}}} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

for Ramp input :

$R_s = A/s^2$

$$e_{ss} = \lim_{s\to 0} \frac{s \times A/s^2}{1 + \frac{K(s+z_1)\text{---}}{s^2(s+P_1)\text{---}}} = 0 \Rightarrow \boxed{e_{ss} = 0}$$

for parabolic input :

$R_s = A/s^3$

$$e_{ss} = \lim_{s\to 0} \frac{s \times A/s^3}{1 + G(s)} = \lim_{s\to 0} \frac{A}{s^2 + s^2 G(s)} = \lim_{s\to 0} \frac{A}{s^2 G(s)}$$

$$\boxed{e_{ss} = \frac{A}{K_a}} \qquad \boxed{K_a = \lim_{s\to 0} s^2 G(s)}$$

e_{ss}	type 0	type 1	type 2
step $\boxed{R_s = A/s}$	$\frac{A}{1+K_p}$	0	0
ramp $\boxed{R_s = A/s^2}$	∞	A/K_v	0
parabolic $\boxed{R_s = A/s^3}$	∞	∞	A/K_a

Q.2)

(2) $G(s) = \dfrac{K}{s^3 + 4s^2 + 2s + 9}$; Find range of K for closed loop stability using RH table.

so $T(s) = \dfrac{G(s)}{1 + G(s)H(s)} = \dfrac{K}{s^3 + 4s^2 + 2s + 9 + k}$

Routh-Hurwitz table:

s^3	1	2
s^2	4	$(9+k)$
s^1	$\dfrac{8-(9+k)}{4}$	0
s^0	$9+k$	0

→ for stability,

$9 + k > 0$

$\Rightarrow \boxed{k > -9}$

and $\dfrac{8}{4} - \dfrac{9+k}{4} > 0$

$\Rightarrow 2 > \dfrac{9+k}{4} \Rightarrow 8 > 9 + k$

$\Rightarrow \boxed{k < -1}$

∴ For closed loop stability of the given system, the range of K: $\boxed{-9 < k < -1}$

Q.3) Part 1

Date: / /

Q.3. $G(s) = \frac{K}{(s^2+s+1)(s+5)}$

(a) Closed loop T.F. $\Rightarrow \frac{G(s)}{1+G(s)}$

C.E. is $(s^2+s+1)(s+5) + K = 0$.

$\Rightarrow s^3+5s^2+s^2+5s+s+5+K = 0$.

$\Rightarrow s^3+6s^2+6s+(5+K) = 0$

s^3	1	6
s^2	6	$5+K$
s^1	$\frac{31-K}{6}$	
s^0	$5+K$	

$\therefore$ For stability $\Rightarrow 31-K>0$

$\Rightarrow K<31$.

& $5+K>0 \Rightarrow K>-5$.

$\boxed{\therefore \text{ Range is } -5<K<31}$

(b) Given: K_p for $G(s) = K_p$ for $\tilde{G}(s)$

We know, $K_p = \lim_{s\to 0} G(s)$.

Let $\tilde{G}(s) = \frac{K_1}{s^2+s+1}$.

$\Rightarrow \frac{K}{5} = K1. \quad \Rightarrow \tilde{G}(s) = \frac{K/5}{s^2+s+1}.$

$\therefore$ CLTF $\Rightarrow$ $s^2+s+\left(1+\frac{K}{5}\right) = 0$ 4 C.E.

$\Rightarrow$

$$\begin{array}{c|cc} s^2 & 1 & 1+\frac{K}{5} \\ s^1 & 1 & \\ s^0 & 1+\frac{K}{5} & \end{array}$$

$\therefore 1+\frac{K}{5} > 0$

$\Rightarrow \boxed{K > -5}$

for stability.

Q.3) 2nd part

a) $3s^7+9s^6+6s^5+4s^4+7s^3+8s^2+2s+6$

s^7	3	6	7	2
s^6	9	4	8	6
s^5	$14/3$	$13/3$	0	
s^4	$-\frac{61}{14}$	8	6	
s^3	$\frac{905}{183}$	$\frac{392}{61}=6.426$		
s^2	13.66	6		
s^1	2.45			
s^0	6			

since two sign changes in 1^{st} col^m, there are 2 poles/roots in RHS & rest 5 are in LHS.

b) In $s^7-6s^6+3s^5+3s-10$ which implies not all there is negative sign ($-6s^6$ & -10) → not-Hurwitz roots are in left hand side → not-Hurwitz

Also, there are missing co-efficient (of s^4, s^3, s^2) → not-Hurwitz

we can also see these with routh-Hurwitz table

s^7	1	3	0	3
s^6	-6	0	0	-10
s^5	3	0	$4/3$	
s^4	$\cancel{0}\to\epsilon$	$8/3$	-10	
s^3	$-8/\epsilon$	$\frac{\frac{4}{3}\epsilon+30}{\epsilon}$		
s^2	$\frac{\frac{-64}{3\epsilon}-\left(\frac{4}{3}\epsilon+30\right)}{-8/\epsilon}$	-10		
s^1	α			
s^0	-10			

where,

$$\alpha = \frac{\left\{\left[\frac{\frac{-64}{3\epsilon}-\left(\frac{4}{3}\epsilon+30\right)}{-8/\epsilon}\right]\left[\frac{\frac{4}{3}\epsilon+30}{\epsilon}\right] + \frac{80}{\epsilon}\right\}}{\frac{-\frac{64}{3\epsilon}-\left(\frac{4}{3}\epsilon+30\right)}{-8/\epsilon}}$$

since in 1^{st} col^m, there are sign changes → some poles / roots are in RHS.

c) Given $s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 6 & 3 \\
s^3 & \cancel{0}\ \varepsilon & 7/2 & \\
s^2 & \frac{6\varepsilon - 7}{\varepsilon} & 3 & \\
s^1 & \frac{\frac{7(6\varepsilon-7)}{2\varepsilon} - 3\varepsilon}{\frac{6\varepsilon-7}{\varepsilon}} & & \\
s^0 & 3 & &
\end{array}$$

$$\frac{\frac{7(6\varepsilon-7)}{2\varepsilon} - 3\varepsilon}{\frac{6\varepsilon-7}{\varepsilon}} = \frac{7}{2} - \frac{3\varepsilon^2}{6\varepsilon - 7}$$

as $\varepsilon \to 0$: $\frac{6\varepsilon - 7}{\varepsilon} = 6 - 7/\varepsilon = 6 - \infty = \underline{-\infty}$

& $\frac{7}{2} - \frac{3\varepsilon^2}{6\varepsilon - 7} = \frac{7}{2}$

so, there are two sign changes → 2 poles on RHS
& 3 are in LHS
($\because$ not row of zeroes means no poles on img axis)

By reversing co-efficients.
$3s^5 + 5s^4 + 6s^3 + 3s^2 + 2s + 1$

$$\begin{array}{c|ccc}
s^5 & 3 & 6 & 2 \\
s^4 & 5 & 3 & 1 \\
s^3 & \frac{21}{5} & 7/5 & \\
s^2 & 4/3 & 1 & \\
s^1 & -7/3 & & \\
s^0 & 1 & &
\end{array}$$

two sign changes → 2 poles in RHS
& 3 are in LHS
(no poles on img axis)

d) $p(s) = s^5+7s^4+6s^3+42s^2+8s+56$

s^5	1	6	8
s^4	7	42	56
s^3	28 $\cancel{0}$	84 $\cancel{0}$	0
s^2	21	56	
s^1	$\frac{28}{3}$	0	
s^0	0		

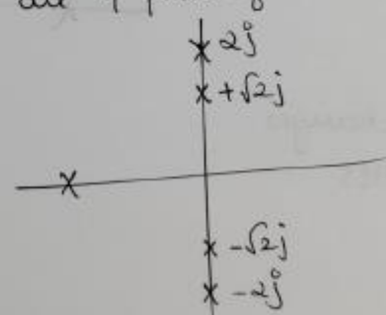
$56 \rightarrow AE = 7s^4+42s^2+56$

$\frac{dAE}{ds} \rightarrow 28s^3+84s+0$

~~no sign change~~ → ~~all p~~

∴ 1 row of zero → some poles ~~are~~ are on img axis [specifically, order of AE is 4 ⇒ 4 poles are symmetrically lies w.r.t. origin]

since no sign change → all 4 poles of AE are on img axis



for $sp(s) = s^6+7s^5+6s^4+42s^3+8s^2+56s$

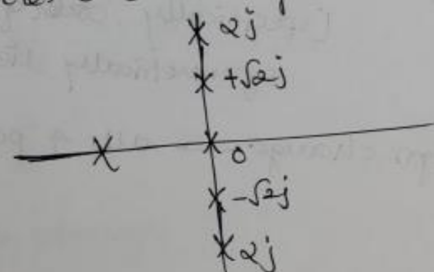
s^6	1	6	8	0
s^5	7	42	56	0
s^4	$\cancel{0}$ 35	$\cancel{0}$ 126	$\cancel{0}$ 56	
s^3	$\frac{84}{5}$	$\frac{224}{5}$	0	
s^2	$\frac{98}{3}$	56	0	
s^1	16	0		
s^0	[illegible]			

$\rightarrow AE = 7s^5 + 42s^3 + 56s$

$\frac{dAE}{ds} = 35s^4 + 126s^2 + 56$

1 row of zero $\longrightarrow$ some poles are on img axis

so.



since, no sign changes
no pole on RHS

Q.4)

(4) $P(s) = s^6 + s^5 - 6s^4 - s^2 - s + 6$

$P(s) = \underbrace{P_e(s)} + \underbrace{P_o(s)}$; $\cdot\, P_e(s) = s^6 - 6s^4 - s^2 + 6 = P_h$

or $P_h + P_l$ $\qquad \cdot\, P_o(s) = s^5 - s = P_l$

* successive division of P_{higher} by P_{lower} :

$\boxed{P_h / P_l} = s$

$$
\begin{array}{r|l}
 & s \\
\hline
[s^5 - s] \; (P_2) & [s^6 - 6s^4 - s^2 + 6] \leftarrow (P_1) \\
 & s^6 \qquad -s^2 \\
\hline
 & [-6s^4 + 6] \leftarrow (P_3)
\end{array}
$$

$$
\begin{array}{r|l}
 & -1/6\, s \\
\hline
-6s^4 + 6 & s^5 - s \\
 & s^5 - s \\
\hline
 & [0] \leftarrow (P_4)
\end{array}
$$

now using $A(s)$ and $A(s)'$,

$$
\begin{array}{r|l}
 & 1/4\, s \\
\hline
[-24s^3] \; (P_4') & -6s^4 + 6 \\
 & -6s^4 \\
\hline
 & [6] \leftarrow (P_5)
\end{array}
$$

using ϵ-method,

$[P_5' = \epsilon s^2 + 6]$

$$
\begin{array}{r|l}
 & -24/\epsilon\, s \\
\hline
[\epsilon s^2 + 6] \; (P_5') & -24s^3 \\
 & -24s^3 - 144/\epsilon\, s \\
\hline
 & \left[\frac{144}{\epsilon} s\right] \leftarrow (P_6)
\end{array}
$$

Finally,

$$
\begin{array}{r|l}
 & (\epsilon^2/144)\, s \\
\hline
\frac{144}{\epsilon} s & \epsilon s^2 + 6 \\
 & \epsilon s^2 \\
\hline
 & [6] \leftarrow (P_7)
\end{array}
$$

R-H table

s^6	1	-6	-1	6	$\leftarrow P_1$
s^5	1	0	-1	0	$\leftarrow P_2$
s^4	-6	0	6	0	$\leftarrow P_3$
s^3	0	0	0		$\leftarrow P_4$

$A(s) = -6s^4 + 6$

$\frac{dA(s)}{ds} = -24s^3$

s^3	-24	0	0	$\leftarrow P_4'$
s^2	$0\ (\epsilon)$	6	0	$\leftarrow P_5 = 6$, $P_5' = \epsilon s^2 + 6$
s^1	$\frac{144}{\epsilon}$	0	0	$\leftarrow P_6$
s^0	6	0		$\leftarrow P_7$

$\therefore$ It's verified that by successive division of P_{higher} by P_{lower} we got the R-H table.

Q.5)

TUTORIAL (3) - Question (5)

Polynomial: $s^3 + \underbrace{(q_1+q_2+2)}_{a_2} s^2 + \underbrace{(q_1+q_2+2)}_{a_1} s + \underbrace{(2.04+6q_1+6q_2+2q_1q_2)}_{a_0}$

By routh-hurwitz criterion :

1) $a_2, a_0 > 0$

$\Rightarrow a_2 > 0$

$\boxed{q_1+q_2 > -2}$

$\Rightarrow a_0 > 0$

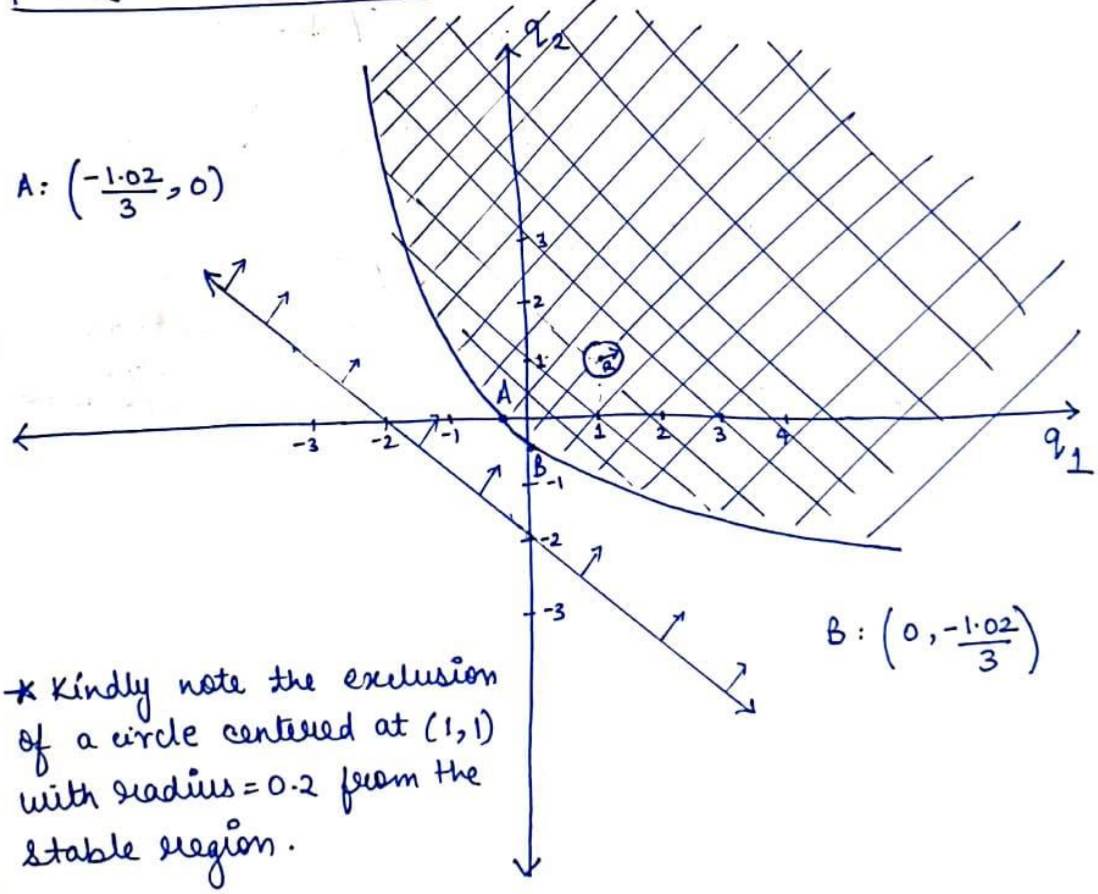
$\boxed{2q_1q_2+6q_1+6q_2+2.04>0}$

2) $a_2 a_1 > a_0$

$\Rightarrow (q_1+q_2+2)^2 > 2q_1q_2+6q_1+6q_2+2.04$

$\Rightarrow q_1^2+q_2^2-2q_1-2q_2+2 > 0.04$

$\boxed{(q_1-1)^2+(q_2-1)^2 > (0.2)^2}$



* Kindly note the exclusion of a circle centered at (1,1) with radius = 0.2 from the stable region.

Q.6)

⑥ Given Equation

$1+s+se^{-s}=0 \Rightarrow 1+s(1+e^{-s})=0$

$\Rightarrow 1+e^{-s}=\frac{-1}{s} \Rightarrow \cancel{e^{-s}=-(1+\frac{1}{s})}$

$e^{-s}=-1-\frac{1}{s}=-\left(\frac{s+1}{s}\right)$

Now let us say $s_1=\sigma \pm j\omega$ be the any root in R.H.P

Then $e^{\cancel{s_1}} = \cancel{\sigma + j\omega}$ $\quad \cancel{s_2 = \sigma - j\omega}$

$$e^{-[\sigma+j\omega]} = -\left[\frac{\sigma+j\omega+1}{\sigma+j\omega}\right] =$$

$$= \cancel{e^{-\sigma} \cdot e^{-j\omega}} \quad e^{-\sigma} \cdot e^{-j\omega} = -\Big[$$

consider R.H.S

$$= \frac{(\sigma+j\omega+1)(\sigma-j\omega)}{\sigma^2+\omega^2} \angle -180$$

$$= \frac{\sigma^2+\omega^2+\sigma-j\omega}{\sigma^2+\omega^2} \angle -180$$

Magnitude $$\frac{(\sigma^2+\omega^2+\sigma)^2+\omega^2}{\sigma^2+\omega^2} > 1 \text{ always}$$

Where as L.H.S Magnitude $e^{-\sigma}$ with $\sigma > 0$

$e^{-\sigma} < 1 \Rightarrow$ There will be no root in open R.H.P

with $\sigma = 0$ $$e^{-j\omega} = -\left[\frac{j\omega+1}{j\omega}\right]$$

$\cancel{j\omega}$ $\cancel{\omega^2+1}$

5

$\tan^{-1}(3/$

$$-\left[\frac{j\omega+1}{j\omega}\right]\cdot\frac{j}{j} = \frac{\cancel{\omega+1}}{\cancel{\omega}} \quad \cancel{\omega}+\frac{1}{\omega}$$

$\cancel{j\omega}$

$$\angle\omega = \tan^{-1}\left(\frac{1/\omega}{1}\right)$$

$$\angle\omega = \tan^{-1}(1/\omega)$$

$$\boxed{\tan(\omega) = 1/\omega}$$

$$\boxed{\omega\tan\omega = 1}$$

$$\boxed{\omega = 0.86j}$$

$$\tan x = 1/x$$

$$x\,\cancel{\tan x} = $$

$$x\frac{\sin x}{\cos x} = 1$$

$$x\sin x - \cos x =$$

Q.7)

First Fig

$$Y(s) = \frac{\left[\frac{K(s)}{1+G_m(s)(1-e^{-sT})K}\right] G(s).e^{-sT}}{1 + \frac{K(s)G(s)e^{-sT}}{1+G_m(s)(1-e^{-sT})K(s)}} . R(s)$$

$$+ \frac{G(s).e^{-sT}}{1 + \frac{k(s).e^{-sT}G(s)}{1+G_m(s)(1-e^{-sT})k(s)}} . D(s)$$

For $G_m = G$,

$$\Rightarrow Y(s) = \frac{k(s)G(s)e^{-sT}}{1+G_m(s).K(s)} R(s)$$

$$+ \frac{G(s).[1+G_m(s)k(s)(1-e^{-sT})]}{1+G_m(s)k(s)} D(s)$$

Taking one input at a time, it follows that for $R(s) = \frac{1}{s}$

$$\lim_{s\to 0} sY_R(s) = \lim_{s\to 0} s.\frac{1}{s} . \frac{k(s)G(s).e^{-sT}}{1+k(s).G_m(s)}$$

with, $G(s)=G_m(s)$ being type-0 and $k(s) = k_p + \frac{k_I}{s}$,

we have

$$\lim_{t\to\infty} y_R(t) = \lim_{s\to 0} \frac{(k_I+k_ps)G(s).e^{-sT}}{s + (k_ps+k_I)G_m(s)}$$

$$= \frac{k_I G(0)}{k_I G_m(0)} = 1.$$

Similarly for $D(s) = \frac{1}{s}$,

$$\lim_{s\to 0} sY_D(s) = \lim_{s\to 0} s.\frac{1}{s} \frac{G(s).[1+G_m(s)K(s)(1-e^{-sT})]}{1+G_m(s).K(s)}$$

$$= \lim_{s\to 0} \frac{G(s)[s+G_m(s)(k_ps+k_I)(1-e^{-sT})]}{s+G_m(s).(k_ps+k_I)}$$

$$= \frac{G(0)k_I G_m(0)(1-1)}{G_m(0).k_I} = 0$$

Thus, disturbance is rejected.

Now suppose $G(s)$ is of type 1.

It ~~and~~ readily follows that for $R(s)=\frac{1}{s}$,

$\lim_{t\to\infty} y_R(t) = 1$, once again.

However, for $D(s)=\frac{1}{s}$,

let $G(s)=G_m(s) = \frac{\tilde{G}(s)}{s}$, where $\tilde{G}(s)$ is of type 0.

Then, $\lim_{s\to 0} s.\frac{1}{s} Y_D(s) =$

$$= \lim_{s\to 0} \frac{s.\tilde{G}(s)\left[s^2+\tilde{G}(s)(k_p s+k_I)(1-e^{-sT})\right]}{s^2\left[s^2+\tilde{G}(s)(k_p s+k_I)\right]}$$

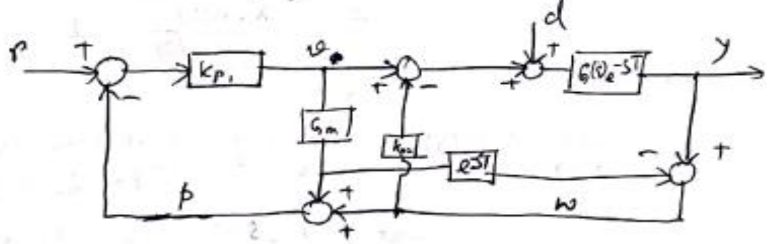
$$= \lim_{s\to 0} \frac{1}{s} \frac{\tilde{G}(s)\left[s^2+\tilde{G}(s)(k_p s+k_I)(1-e^{-sT})\right]}{s^2+\tilde{G}(s)(k_p s+k_I)}$$

$$= \frac{Tk_I\tilde{G}(0)^2}{k_I\tilde{G}(0)} \cdot \lim_{s\to 0}\frac{1-e^{-sT}}{sT} = \tilde{G}(0).T \neq 0$$

Thus, disturbance is not rejected!

* Check my calculations!

For second fig.



$k_{p_1}(R-P) = v$ (i)

$G(s).e^{-sT}(v-k_{p_2}w+d) = Y$ (ii)

$p = G_m(s).v+w$ (iii)

(i) & (iii) give,

$k_{p_1}(R-G_m v-w) = v$ (iv)

$\Rightarrow \boxed{k_{p_1}R = (k_{p_1}G_m+1)v+k_{p_1}w}$ (v)

~~$Y - k_{p_1}(R$~~

$$k_{p_1}\left[R - (G_m V + W)\right] = V \quad (i)$$

And,

~~$W = -G_m e^{-sT} V + Y$~~

~~$\Rightarrow W = Y - k_{p_1} G_m e^{-sT}\left[R - G_m V - W\right]$~~

$\Rightarrow$ ~~W~~

$$\Rightarrow \quad V(1 + k_{p_1} G_m) = k_{p_1}(R - W) \Rightarrow V = \frac{k_{p_1}(R-W)}{1 + k_{p_1} G_m}$$

Again

$$W = Y - G_m e^{-sT} V$$

$$= Y - \frac{G_m e^{-sT} k_{p_1}(R-W)}{1 + k_{p_1} G_m}$$

$$\Rightarrow W\left[1 - \frac{k_{p_1} G_m e^{-sT}}{1 + k_{p_1} G_m}\right] = Y - \frac{k_{p_1} G_m e^{-sT} R}{1 + k_{p_1} G_m}$$

$$\Rightarrow \boxed{W = \frac{Y(1 + k_{p_1} G_m)}{1 + k_{p_1} G_m(1 - e^{-sT})} - \frac{k_{p_1} G_m e^{-sT} R}{1 + k_{p_1} G_m(1 - e^{-sT})}}$$

Finally,
$$G e^{-sT}\left\{\frac{k_{p_1}(R-W)}{1 + k_{p_1} G_m} - k_{p_2}\left[\frac{1 + k_{p_1} G_m}{1 + k_{p_1} G_m(1 - e^{-sT})} Y - \frac{k_{p_1} G_m e^{-sT}}{1 + k_{p_1}(1 - e^{-sT}) G_m} R\right] + D\right\} = Y$$

Substitute W from boxed eqn in above expression, to obtain $(G_m = G)$

$$\frac{Y(s)}{R(s)} = \frac{k_{p_1} G(s) e^{-sT}}{1 + k_{p_1} G_m(s)}$$

&

$$\frac{Y(s)}{D(s)} = \frac{\left[1 + k_{p_1}(G_m - G_m e^{-sT})\right] G(s)}{(1 + k_{p_1} G_m(s))(1 + k_{p_2} G(s) e^{-sT})}$$

Let $R(s) = D(s) = \frac{1}{s}$, and Let $G(s) = G_m(s) = \frac{\tilde{G}(s)}{s}$.

$$\therefore \lim_{t\to\infty} y_R(t) = \frac{k_{p_1}\tilde{G}(0)}{k_{p_1}\tilde{G}(0)} = 1.$$

and. $$\lim_{t\to\infty} y_D(t) = \lim_{s\to 0} \frac{\left[s + k_{p_1}\tilde{G}(s)(1-e^{-sT})\right]\tilde{G}(s)}{\left[s + k_{p_1}\tilde{G}(s)\right]\left[s + k_{p_2}\tilde{G}(s)e^{-sT}\right]}$$

$$= \frac{k_{p_1}\tilde{G}(0)^2}{k_{p_1}k_{p_2}\tilde{G}(0)^2} \cdot \lim_{s\to 0}(1-e^{-sT})$$

$$= \frac{1}{k_{p_2}} \cdot \lim_{s\to 0}(1-e^{-sT}) = 0.$$

Hence disturbance is rejected.

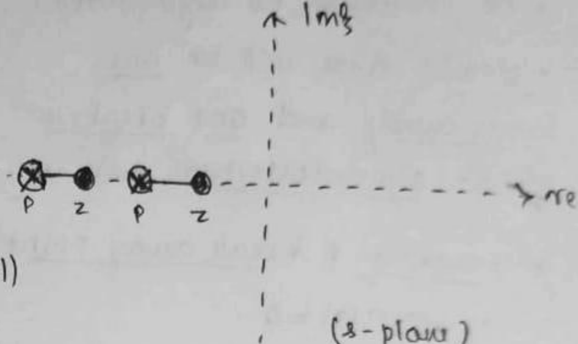
Q.8)

$\boxed{Q-8}$ Root locus:

- break away/in point
- K value
- range of K
- ω & k at img crossing
- real axis segments
- asymptotic angles, intersect

(a) $\dfrac{(s+1)(s+3)}{(s+2)(s+4)}$

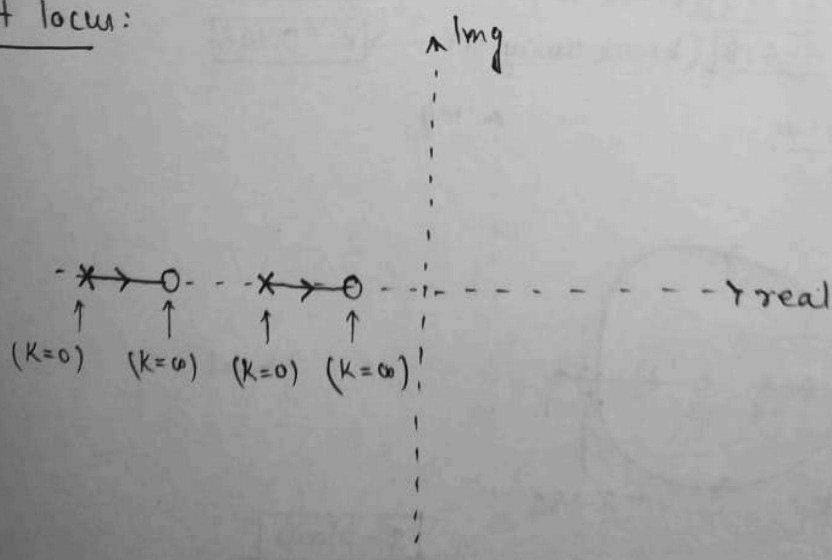
- open loop poles : $-2, -4$
  " zeros : $-1, -3$
- tot. no. of poles $= |P| = 2$
  " zeros $= |z| = 2$
- no. of RL branches $= \max(|P|, |z|)$
  $= 2$
- centroid $(\sigma) = \dfrac{(-2-4)-(-1-3)}{|P|-|z|}$



(s-plane)
(real axis segments)

∴ clearly there will be $\underline{\underline{no}}$ centroid, asymptotes, break-in/away points ; no intersection with imaginary axis.

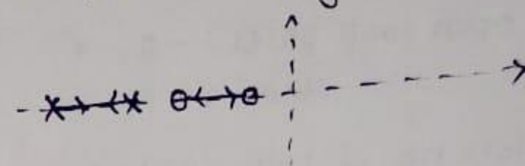
Root locus:



(b) $\dfrac{(s+1)(s+3)}{(s+4)(s+6)}$

- open loop poles : $-4, -6$
- open loop zeros : $-1, -3$
- no. of RL branches $= 2$

- no centroids, no asymptotes
- clearly there will be one break away and one break-in point. No intersection with img axis.
- real axis segments :



* break-in & break away point :

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K(s+1)(s+3)}{(s+4)(s+6)} = 0$$

$$\Rightarrow K(s) = -\frac{(s+4)(s+6)}{(s+1)(s+3)}$$

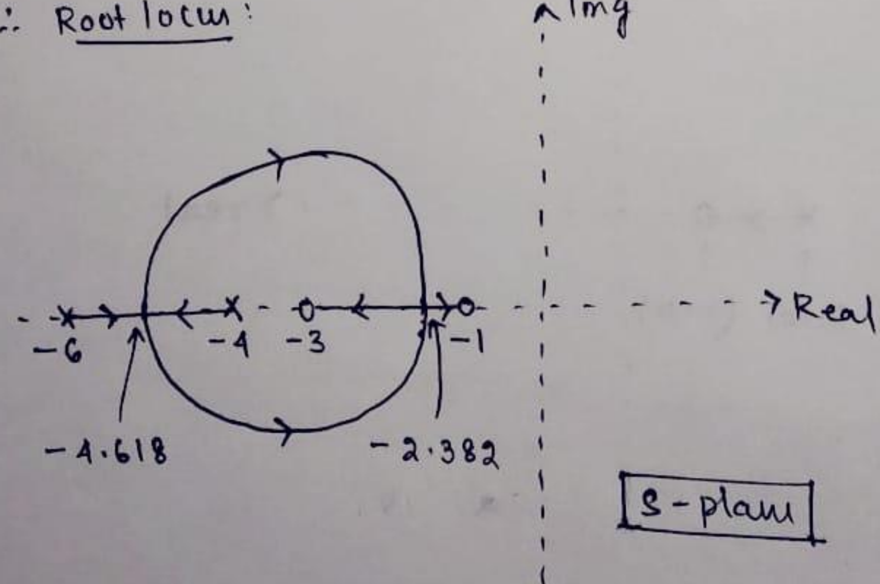
$$\frac{dK(s)}{ds} = 0$$

$$\Rightarrow s^2 + 7s + 11 = 0$$

$$\Rightarrow s = \boxed{-2.382} \text{ (break in point)} \rightsquigarrow \boxed{K = 6.85}$$

$$\boxed{-4.618} \text{ (break away pt)} \rightsquigarrow \boxed{K = 0.146}$$

$\therefore$ Root locus :



(c) $\dfrac{\omega_n s}{s^2+\omega_n^2}$

- no. of RL branches = 2
- centroid : at Origin (0)
- <u>Asymptote</u>:

    asymptotic angle, $\boxed{\theta = 180^\circ}$

- one RL branch : pole to zero

    other " : pole to infinity in the direction of asymptote

$\therefore$ one break-in point exist.

Char. eq^n: $1+\dfrac{K\omega_n s}{s^2+\omega_n^2}=0$

$\Rightarrow s^2+\omega_n^2+K\omega_n s=0$

$\Rightarrow K = \dfrac{-s^2-\omega_n^2}{\omega_n s}$

$\dfrac{dK(s)}{ds}=0 \Rightarrow s^2-\omega_n^2=0$

$\Rightarrow s^2=\omega_n^2$

$\Rightarrow s=\pm\omega_n$

but $+\omega_n$ will not be valid here

$\therefore \boxed{s=-\omega_n} \Rightarrow \boxed{K=2}$

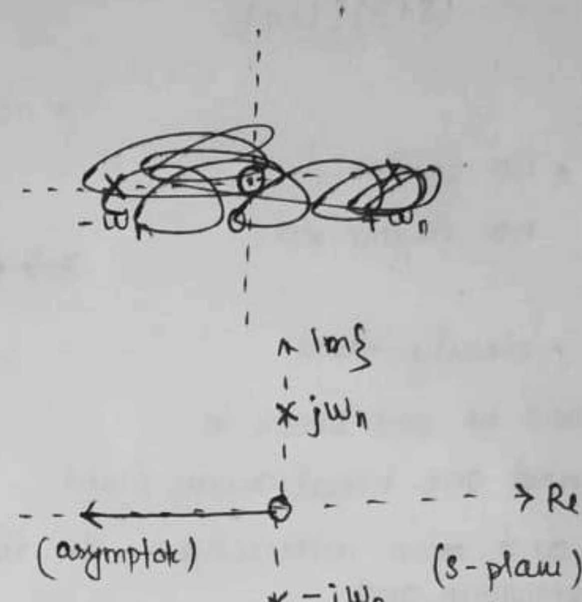
(break in point)

$\rightarrow$ clearly, intersect^n with im$\{$ axis at $\boxed{\pm j\omega_n}$

$(K=0)$

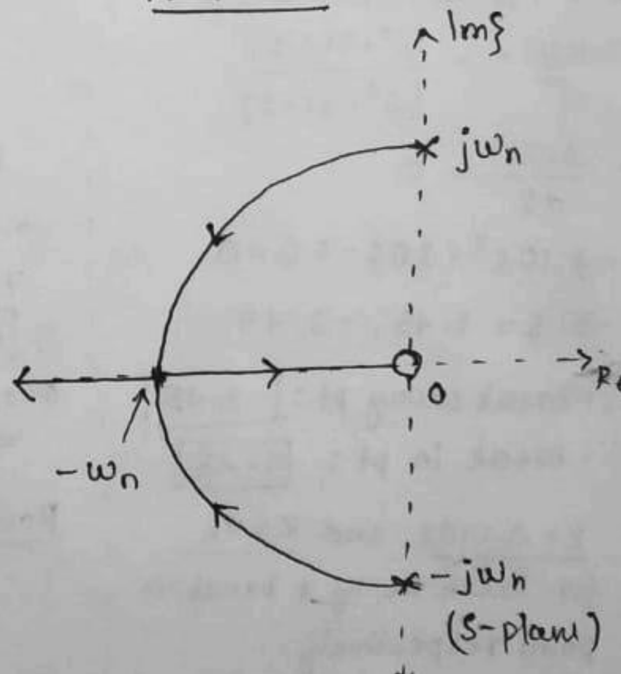
- open loop pole at $\pm j\omega_n$

    zero at 0



(p-z plot & asymptote)

* <u>Root locus</u>



(d) $\dfrac{(s-1)(s-2)}{(s+3)(s+4)}$

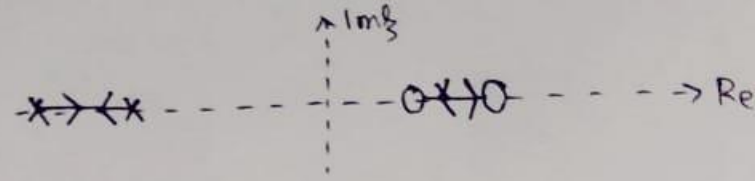
• open loop poles : $-3, -4$

" zeros : $1, 2$

• no. of RL branches : $\boxed{2}$

• no centroid

no asymptotes.



(real axis segments)

• clearly there will be one break in and one break away point and also intersection with img-axis

* break-in and break away point :

$1 + G(s)H(s) = 0$

$\Rightarrow 1 + \dfrac{K(s-1)(s-2)}{(s+3)(s+4)} = 0$

$\Rightarrow s^2 + 7s + 12 + K(s^2 - 3s + 2) = 0$

$\Rightarrow K(s) = -\dfrac{(s^2+7s+12)}{(s^2-3s+2)}$

• $\dfrac{dK(s)}{ds} = 0$

$\Rightarrow 10s^2 + 20s - 50 = 0$

$\Rightarrow s = 1.45, -3.45$

∴ • break away pt : $\boxed{-3.45}$

• break in pt : $\boxed{1.45}$

$\underline{K = 0.0102}$ and $\underline{K = 98}$ for break away & break-in point respectively.

* Intersection point of RL branches with img-axis :

• characteristic eq^n :

$(s+3)(s+4) + K(s-1)(s-2) = 0$

hence, characteristic polynomial :

$s^2 + 7s + 12 + K(s^2 - 3s + 2)$

$= s^2(1+k) + s(7-3k) + (12+2k)$

Routh table :

s^2	$(1+k)$	$(12+2k)$
s^1	$(7-3k)$	0
s^0	$12+2k$	

→ for stability,

$7 - 3k > 0$

$\Rightarrow \boxed{k < 7/3}$

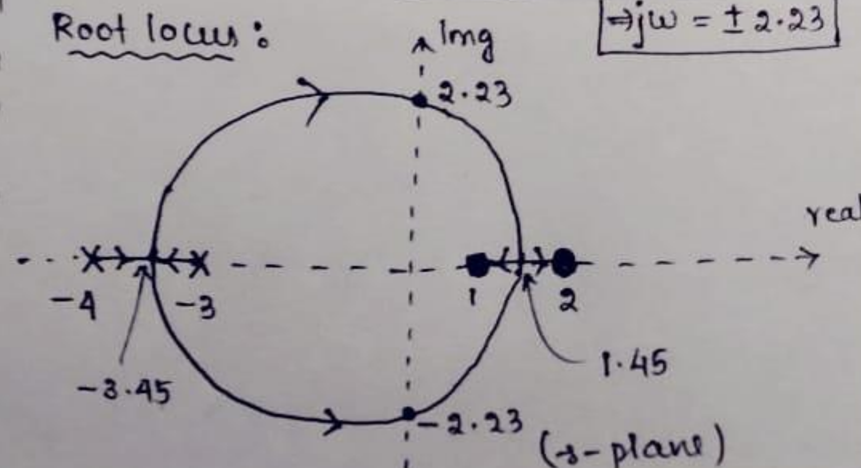
also, $12 + 2k > 0$

$\Rightarrow \boxed{k < 6}$

∴ $\boxed{K_{marginal} = 7/3 = 2.33}$

$\boxed{\Rightarrow j\omega = \pm 2.23}$

Root locus :



Q.9)

*

(a)

~~15~~ Percentage over shoot = 15%

$\Rightarrow \xi = 0.5169$

$\frac{1}{s(s+2)}$ $\Rightarrow$ ~~ch. Eq~~ closed loop ch Eq = $s(s+2) + K = 0$

$s^2 + 2s + K = 0$

$\omega_n = \sqrt{K}$ $\quad$ $\cancel{2}\xi\omega_n = \cancel{2}$

$\omega_n = \frac{1}{\xi} = \frac{1}{0.5169}$

$\Rightarrow \quad K = \left(\frac{1}{0.5169}\right)^2 = \underline{\underline{3.74}}$

(b)

$s^2 + 2.01s + 0.02 + K = 0$

$2\xi\omega_n = 2.01 \Rightarrow \omega_n = \frac{2.01}{2 \times 0.5169}$

$\omega_n = 2.031$

$\omega_n^2 = 0.02 + K \Rightarrow K = 4.106$

(c)

$$\frac{s+1}{(s+4)(s+2)}$$

$$\lim_{s \to 0} G(s) = \frac{1}{(4)(2)} = 1/8$$

$$e(s) = \frac{1}{1 + 1/8} = \frac{8}{9}$$

(c) ch—Eqn

$$(S+4)\left(\frac{S+1}{(S+4)(S+2)} \right.$$

If we observe root locus of given system

∴ No imaginary root will

exist

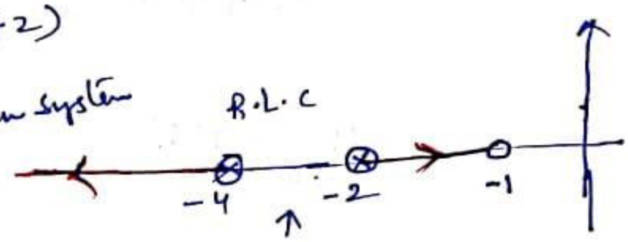
∴ No damping

∀ K > 0 The system is going

to have real roots.

∴ No oscillations will appear in the o/p.

R.L.C



no root locus Branch

(10) ess

Steady state error

1. $\frac{1}{S(S+2)}$ → Type '1' system ∴ $e_{ss} = \infty$

2. $e(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)} = \frac{1}{1+\frac{1}{0}} = \underline{\underline{0}}$

2.

$$G(s) = \frac{1}{(s+0.01)(s+2)}$$

$$\lim_{s\to 0} G(s) = \frac{1}{0.02} = 50$$

$$e(\infty) = \frac{1}{1+50} = \frac{1}{51}$$

Q.10)

• We need settling time to be 10 sec.

$\therefore 4\tau = 10 \text{ sec} \longrightarrow \tau = \frac{10}{4} = \frac{1}{\zeta\omega_n}$

$\therefore$ real part of pole $= -\zeta\omega_n = -\frac{4}{10} = -0.4$

i) $GH = \frac{K}{s(s+2)}$

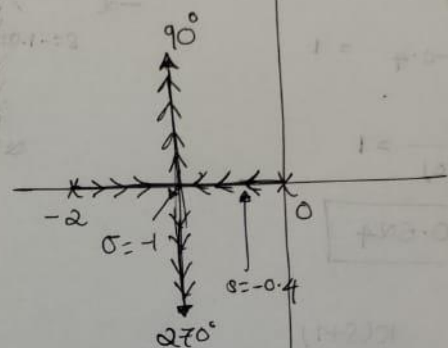
$N = P - Z = 2$

$\theta = \frac{(2q+1)180^\circ}{2}$

$= (2q+1)90^\circ$ for $q = 0, 1$

$= 90^\circ, 270^\circ$

$\sigma = \frac{0-2}{+2} = -1$



Root locus lies on ~~0.4~~ $s = -0.4$

To find value of K for $s = -0.4$. use magnitude condn

$|GH|_{s=-0.4} = 1 \longrightarrow \frac{|K|}{|-0.4|\cdot|-0.4+2|} = 1$

$\frac{K}{0.4(1.6)} = 1$

$\boxed{\therefore K = 0.64}$

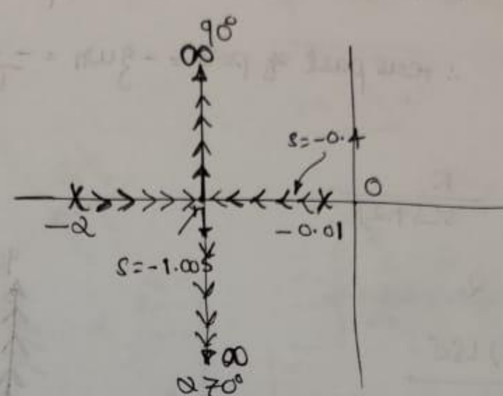
c)

ii) $GH = \dfrac{K}{(s+0.01)(s+2)}$

$N = P - Z = 2$

$\theta = 90^\circ, 270^\circ$

$\sigma = \dfrac{-0.01-2}{2} = 1.005$



Using magnitude condⁿ,

$|GH|_{s=-0.4} = 1$

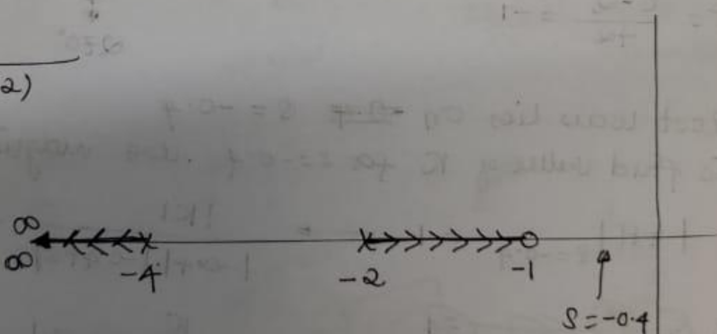
$\dfrac{K}{(0.39)(1.6)} = 1$

$\therefore \boxed{K = 0.624}$

iii) $GH = \dfrac{K(s+1)}{(s+4)(s+2)}$

$N = P - Z = 1$

$\theta = 180^\circ$



Root locus does not lies on $s=-0.4$.
Therefore for this system.. it is not possible to achieve 10 sec settling time.