LECTURE 3 SUMMARY

Topics Covered: Reconstruction from samples, Aliasing, Discrete systems.

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Review of Lecture 2:

Introduction:

In the last lecture discussion on the idea of sampling was initiated i.e., we are trying find out the conditions under which we can store the values of the signal only at particular time instances i.e., sampling instances without creating any confusion & without losing any information.

Confusion in Reconstruction of Original Signal from Samples:

Let us consider a sinusoidal wave whose frequency is 0.1kHz, phase ϕ_0 and a sampling rate of 1kHz. When we try to reconstruct the original sinusoidal wave then there is confusion raised which is as follows:

Let the original sinusoid be $y(t)=A_0\cos(2\pi(100)t+\phi_0)$. When the original sinusoid is sampled at the above stated sampling frequency i.e., 1kHz the samples obtained at all the sampling instances will be same for all the sinusoids of the form:

$$Z_n(t)= A_0\cos(2\pi(1000n\pm100)t\pm\phi_0). \text{ Where } n=0,1,2,3.....$$

So, there is a confusion in the reconstruction of original sinusoid.

Note: For the above "±'s" consider both + or both -.

Challenge 1: Add the above continuous sinusoids represented by $Z_n(t)$. Prove that the sum is constructive at the points of sampling i.e., at $t=kT$ and destructive at all other t.

In this challenge the constructive sum part is intuitively expected as all the sinusoids represented by $Z_n(t)$ have same samples so they add up constructively!

Hints: Use trigonometric identities (or) use the concept of decomposing a sinusoid into two oppositely rotating complex numbers i.e., phasors.

Idealized Sampling:

Sampling a signal: It is process in which small pulses are created at the instances of sampling whose amplitude is equal to value of signal at that instance and at other instances its value is zero. The sketch of the same can be seen in the lecture video.

- Idealized sampling is nothing but replacing a continuous signal by a train of “sharp pulses.”

Reconstruction of original Signal:

Let the original continuous signal be x(t) and the sampled signal be $x_s(t)$. The reconstruction is the process of getting back x(t) from $x_s(t)$. We have already seen that there are infinitely many possible x(t) which can be constructed from $x_s(t)$. The difficulty lies in identifying the correct x(t)!

A) Spectrum of sampled Signals:

- As we want to look at the spectrum of sampled signal let us analyse what sampling does to frequency axis.

On the frequency axis, sampling creates “Copies of the original signal” at a collection of frequencies. In detail we can state that sampling creates Copies of original signal spectrum after the multiples of sampling frequency and mirror imaged version of original signal spectrum with phase reversal before the multiples of sampling frequencies.

Note:

- The spectrum is the frequency domain representation of decomposed version of original signal into various sinusoids that comprise the original signal. It is essentially a Fourier transform. Its physical meaning is it is the collection of sinewaves specifying the frequencies, amplitudes and phases.
- We can have a continuous band of frequencies (Ex: Aperiodic signals) i.e., the frequency axis is continuously occupied in a small segment unlike the example stated in the lecture video which has discrete set of frequencies occupied!

By looking at the frequency domain we can state that reconstruction of x(t) from $x_s(t)$ means to retain the original signal spectrum and throw away all copies and mirror images of original signal spectrum which are formed around the multiples of sampling frequencies.

The above retention can be done if and only if the copies and mirror images of original signal spectrum do not pollute the original signal spectrum. (i.e., do not overlap with original signal spectrum).

Nyquist Criterion:

From the frequency domain we can see that the first mirror imaged version of original signal spectrum is the troublemaker i.e., it can overlap with original signal spectrum without the overlap of other copies and mirror images of original signal spectrum with original signal spectrum. If we prevent this troublemaker overlap it implies that we have indeed prevented the overlap of all copies and mirror images of original signal spectrum with the original signal spectrum.

Let the maximum frequency component of the original signal spectrum be f_m and the sampling frequency be f_s. From the frequency domain analysis, we can see that the first mirror image occupies a region on the frequency axis from f_s-f_m to f_s. In order to prevent the overlap, we need to have,

$$f_s - f_m > f_m$$

$$f_s > 2f_m$$

The above principle i.e., the lower bound on sampling frequency is known as Nyquist Principle of sampling.

Aliasing:

When the Nyquist Criterion is not satisfied there is an overlap with the original signal spectrum. This phenomenon is known as Aliasing.

Note: Aliasing is not always undesirable. It some situations it is desirable but as far as this course is concerned, we treat aliasing as undesirable as it makes the reconstruction process easier.

- The word ALIAS means false name. Here if the sampling rate does not satisfy Nyquist criterion then there will be an overlap as stated before which means we have some frequency components which are false original signal frequency components (imposters). So, we use the word Aliasing for such phenomenon.

Challenge 2: Let the original signal spectrum be non-zero between two frequencies f_L and f_H and zero everywhere else. From the Nyquist criterion if the sampling frequency $f_s > 2f_H$ then there is no problem for reconstruction. Can one sample at a frequency less than $2f_H$ and still reconstruct? If yes, then state the sampling frequency and reconstruction method with the stated sampling frequency.

Filters:

An ideal filter modifies the amplitudes and phases of the sinewaves given to it in a certain way i.e., it does not depend on the amplitudes and phases of the input sinewave given to it.

Examples:

1) An ideal low pass filter with cut off frequency f_m retains the amplitude and phases of the sinewaves upto frequency f_m and beyond the frequency f_m it makes the amplitude zero irrespective of amplitudes and phases of the input sinewave given to the filter.
2) An ideal high pass filter with cut off frequency f_m retains the amplitudes and phases of the sinewaves beyond the frequency f_m and it makes the amplitude zero for the sinewaves below f_m irrespective of amplitudes and phases of the input sinewave given to the filter.

- The ideal filters cannot be achieved practically, we can design the ideal filters approximately. As we progress in this course this aspect will be discussed in detail.

For designing a system (i.e., a filter) which is impartial. Here impartial means the system's output i.e., whether to retain or to eliminate should not depend on the amplitude and phase of the input sinewave given to it. For getting this impartial behaviour on the frequency axis for both continuous and discrete independent variable we need the following properties:

1) Linearity.
2) Shift Invariance.

<u>Note</u>: The necessity of these properties will be proved in the upcoming lectures.

<u>Discrete System:</u>

This system has a sampled input and it produces a sampled output. Here we are assuming that the Nyquist criterion is satisfied i.e., the sampling frequency is more than twice the maximum frequency component of the input signal.

- As the Nyquist criterion is satisfied, we can reconstruct the output signal by passing the samples through an ideal low pass filter whose cut off frequency is the maximum frequency component of the input signal i.e., we can use the same reconstruction filter which can used for the input samples reconstruction. Here we are assuming that the discrete system does not introduce any new frequency components.
- The analog system equivalence of the discrete system can be stated as follows:
  If the continuous signal is passed through an analog system and the output of analog system is sampled according to Nyquist criterion we get samples which are identical with the output of a discrete system whose input is the sampled (According to Nyquist criterion and at the same instance as that of output) version of the continuous signal which is given to the analog system.
  Here we are assuming that the discrete system does exactly same as what analog system want to do.

A discrete system is a relationship between all the samples $y[n]\ \forall\ n \in \mathbb{Z}$ and all the samples $x[n]\ \forall\ n \in \mathbb{Z}$ in general! It is not point to point relationship.