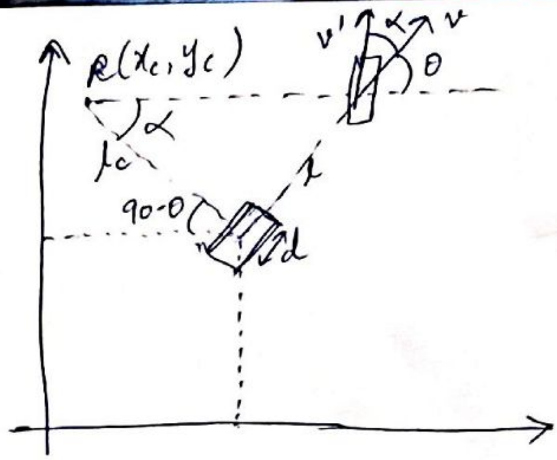
① SC635 - Quiz 1 solutions ①



$$\tan\alpha = \frac{l\dot{\theta}}{v}$$

$$\omega = \frac{v\tan\alpha}{l}$$

The location of bicycle at time t is given by (x_t, y_t)

We assume that control i/ps during time intervals $[t-1\ t]$ are constant, Therefore v & α do not change, therefore the robot moves in a circular motion, with radius 'r' and center (x_c, y_c)

$$\therefore\ u_t = \begin{bmatrix} v \\ \alpha \end{bmatrix}$$

Pose of the robot = $\begin{bmatrix} x \\ y \\ \alpha \\ \theta \end{bmatrix} = x_t$

(a) State variable x_t at time 't' = $\begin{bmatrix} x \\ y \\ \alpha \\ \theta \end{bmatrix}$

(b) Action state variables ~~ut~~ $u_t = \begin{bmatrix} v \\ \dot{\alpha} \end{bmatrix}$

(c) Since u_t is given & we know the poses x_t & x_{t-1}, posterior probability of ~~...~~ state x_t could be found and.
Initially assuming no randomness in motion (x_c, y_c) can be defined as.

$$x_c = x - \frac{v}{\omega}\sin\theta$$

$$y_c = y + \frac{v}{\omega}\cos\theta$$

Now robot moves from x_{t-1} to x_t in Δt. x_t can be defined in terms of (x_c, y_c)

$$x_t = x_c + \frac{v}{\omega}\sin(\theta + \omega\Delta t) \quad \text{--- ①}$$

$$y_t = y_c - \frac{v}{\omega}\cos(\theta + \omega\Delta t) \quad \text{--- ②}$$

$$\theta_t = \theta_{t-1} + \omega\Delta t + \alpha_{t-1} \quad \text{--- ③}$$

$$\alpha_t = \theta_t - \omega\Delta t$$

ε_v = Error in v = $\hat{v} - v$

Error in α = $\hat{\alpha} - \alpha$.

∴ Probability $(X_t | u_{t-1}, X_{t-1})$. i.e. probality for that robot reaches X_t from X_{t-1} in time Δt, on application of u_t.

$$P(X_t | u_t, X_{t-1}) = P(\hat{v} - v, \alpha_1 |v| + \alpha_2 |\alpha|) \cdot P(\hat{\alpha} - \alpha, \alpha_1 |v| + \alpha_2 |\alpha|)$$

where $P(a, b) \Rightarrow$ Probability of 'a' is distribution having zero mean and b variance.

(d) Probality of sensor reading Z_t for a state X_t :- $P(Z_t | X_t)$

$Z_t = \{1, 2 \cdots 360\}$

$\boxed{K = 360}$

The handle can take six different angular positions

$\left\{ -\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3} \right\}_{M=6}$ ∴ $P\left(\frac{\alpha}{X_t}\right)$

And the range sensors can take 360 readings.

At a particular angular position,

$$P(Z_t / X_t) = \prod_{K=1}^{360} P(Z_t^K / X_t).$$

∴ Taking all angular positions in consideration,

$$\boxed{P\left(\frac{Z_t}{X_t}\right) = \sum_{M=1}^{6} \sum_{K=1}^{360} P(Z_t^K / X_t) \cdot P(\alpha^M / X_t)}$$

where $P\left(\frac{\alpha^m}{X_t}\right)$: Prob. of M^{th} angular position at a given location X_t.

(e) (i) Suppose, now, a known map m describing occupancy of a state X_t is given.

∴ Now posterior probability of state 'X_t' becomes
$P(X_t | u_t, X_{t-1}, m)$.

Using Baye's rule, we can write

$$\therefore P(X_t | u_t, X_{t-1}, m) = \eta_1 P(m_r | X_t, u_i) \cdot P(X_t | u_t, X_{t-1}) \quad \text{— (1)}$$

Now, assuming the real motion, the output motion will have certain certainties due to randomness along with the ~~ep~~ inputs. (2)

Therefore actual inputs will become ~~[...]~~ $[\hat{v}, \hat{\alpha}]$

$$\therefore \hat{v} = v + P(\varepsilon_1 |v_0| + \varepsilon_2 |\alpha|)$$

$$\hat{\dot{\alpha}} = \dot{\alpha} + P(\varepsilon_3 |v| + \varepsilon_4 |\alpha|) \quad \{\text{Assuming } \dot{\alpha}_0 = 0\}$$

where $P(\varepsilon_1 |v| + \varepsilon_2 |\alpha|)$ evaluates the probability of $\dot{\alpha}$ with distribution of variable with variance $(\varepsilon_1 |v| + \varepsilon_2 |\alpha|)$.

$\therefore$ ~~Now~~ New state of robot will be as per the following,

where $\hat{\omega} = \frac{\hat{v} \tan \hat{\alpha}}{l}$

$$x' = x^*_{t-1} - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{v}{\omega} \sin(\theta + \hat{\omega}\Delta t)$$

$$y' = y_{t-1} + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}} \cos(\theta + \hat{\omega}\Delta t)$$

$$\theta' = \theta_{t-1} + \hat{\omega}\Delta t + \alpha_{t-1}$$

$$\alpha' = \theta_t - \hat{\omega}\Delta t$$

Now the centre of circular motion of robot becomes.

$$x^* = \frac{x + x'}{2} + u(y - y')$$

$$y^* = \frac{y + y'}{2} + u(x - x')$$

Since the centre lies ~~on~~ midway b/w the two states.

$$\text{where } u = \frac{1}{2}\left[\frac{(x - x')\sin\theta + (y' - y)\cos\theta}{(x - x')\sin\theta - (y' - y)\cos\theta}\right]$$

$\therefore r^*$ = radius of circular motion.

$$= \sqrt{(x - x^*)^2 + (y - y^*)^2} = \sqrt{(x' - x^*)^2 + (y' - y^*)^2}$$

$$\Delta\theta = \tan^{-1}\left(\frac{y' - y^*}{x' - x^*}\right) - \tan^{-1}\left(\frac{y - y^*}{x - x^*}\right)$$

Δd = distance travelled along circular arc.

$$\hat{v} = \frac{\Delta d}{\Delta t} \Rightarrow \quad \Delta d = r^* \Delta\theta$$

$$\hat{v} = r^* \frac{\Delta\theta}{\Delta t}$$

& $\hat{\alpha} = P(\varepsilon_3 |v| + \varepsilon_4 |\alpha|) = \left[\tan^{-1} \frac{l}{r^*}\right]$

Using the same Baye's rule, we can write $P(m_t | x_t, u_t)$
$= \eta_2 . P(x_t | m_t)$ — (2)

from (1) & (2),

$$\boxed{P(x_t | u_1, x_{t-1}, m) = \eta \, P(x_t | m_t) . P(x_t | u_t, x_{t-1})}.$$

ii) For a given known map, probability of sensor reading becomes,

$$\cancel{P(} P(z_t | x_t, m) = \sum_{m=1}^{6} \prod_{k=1}^{360} P(z_t^k | x_t, m) \cdot P(\alpha^m | x_t)$$

Now, the error in sensor reading can be due to four events.

a) Measurement error = P_{hit}.
b) Unexpected obstacles = P_{short}.
c) Missed obstacles = P_{max}.
d) Random Noise = P_{rand}..

Now the sensor ~~ea~~ readings can be divided in 4 subsets, namely $\{Z_{hit}, Z_{short}, Z_{max}, Z_{rand}\}$.

We assume that, we know the type of sensor readings.

$z = 1$ Range.

1) $\underset{z_t \in hit}{P_{hit}} (z / x_t, m) = \eta_1 \, N(x_t^k, \sigma)$ $\quad 0 \le z \le 360$

2) $\underset{z_t \in short}{P_{short}} (z / x_t, m) = \eta_2 \cdot \lambda_{short} \cdot e^{-\lambda_{short} \cdot z}$ $\quad 0 \le z \le 360$

3) $\underset{z_t \in max}{P_{max}} (z / x_t, m) = 1$, $\quad z = z_{max}$

4) $\underset{z_t \in rand}{P_{rand}} (z / x_t, m) = \frac{1}{|z|}$, $\quad 0 \le z \le 360$

Sensor measurement probability will be the weighted sum of (1), (2), (3), (4)

$Z = [Z_{hit}, Z_{short}, Z_{max}, Z_{rand}]$.

$$\sum_{M=1}^{6} \sum_{z' \in Z} \prod_{k=1}^{360} P(z'^k / x_t, m) . P(\alpha^m / x_t) = P(z / x_t, m)$$

(2) For all the features, $\{r_j, \theta_j, s_j\}$ for j^{th} landmark.

(a) Considering the generalized case of any mobile robot, its pose is defined by 3 states :- (x_i, y_i, θ_i).

→ To know the state of such robots, all three states need to be identified,

→ Considering, only single feature with completeness of info is available.

→ so, only ~~the~~ the range & bearing information of single feature w.r.t robot can accurately give its coordinate as :-

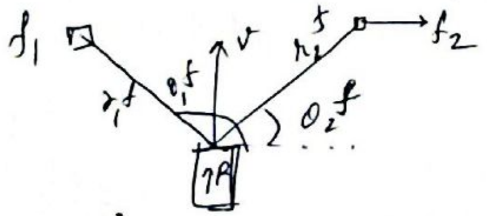
$$\boxed{\begin{aligned} x_i &= r\cos\theta \\ y_i &= r\sin\theta \end{aligned}}$$

∴ where r & θ are the feature's info, but to know robot's orientation, we need another feature.

⇒ for a mobile robot, minimum 2 features must be visible, to uniquely identify the states of robot.

(b) As explained above, at any given time t,

$$Z_t = \{r_i^t, \theta_i^t\}$$



So, $z_t^i = \{r_{it}^f, \theta_{it}^f, f_{it}\}$ $i = 1, 2.$

(c) Here, let the measurements, r & θ are getting corrupted by random noise {particle filter}

So,
$$\boxed{\begin{aligned} \hat{r}_i &= r_i + \text{rand}(0, \sigma_r^2) \\ \hat{\theta}_i &= \theta_i + \text{rand}(0, \sigma_\theta^2) \end{aligned}}$$

so, here, true reading can be calculated as;

$$\boxed{\begin{aligned} \theta' &= \text{atan2}\frac{(\bar{y}' - \bar{y})}{(\bar{x}' - \bar{x})} + \hat{\theta}_i \\ r' &= \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2} + \hat{r}_i \end{aligned}}$$

& $f' = \bar{f} + \hat{f}$

$$\boxed{P(x_t / z_t, u_t) = P(r_i / u_t) \cdot P(\theta_i / u_t) \cdot P(f_i / u_t)}$$