EE 338 Digital Signal Processing, Spring 2015-2016

Tutorial 6

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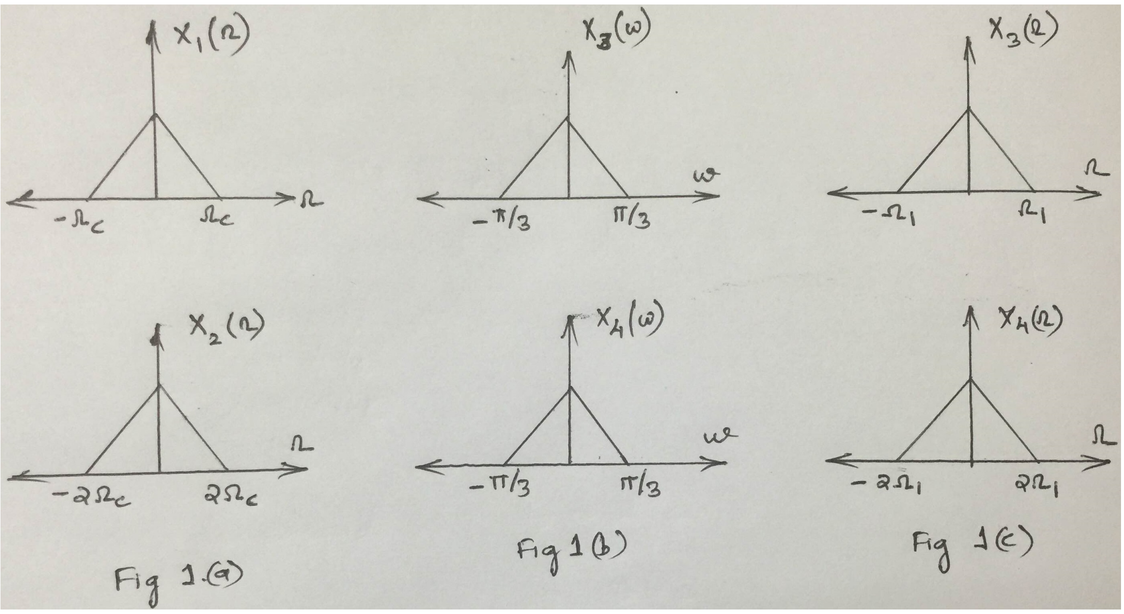
1. Consider two continuous time signals $x_1(t)$ and $x_2(t)$, with CTFT $X_1(\Omega)$ and $X_2(\Omega)$ respectively, as shown in Fig.1(a). They are sampled at various angular frequencies Ω_S, as listed below, to generate discrete signals $x_1[n]$ and $x_2[n]$.
   (i) $\Omega_S = 2\Omega_C$, (ii) $\Omega_s = 4\Omega_C$ (iii) $\Omega_s = 8\Omega_C$
   (a) Plot the DTFT $X_1(\omega)$ and $X_2(\omega)$ for each of above three cases.
   (b) Consider two another sampled signals $x_3[n]$ and $x_4[n]$ having DTFTs $X_3(\omega)$ and $X_4(\omega)$ respectively as shown in Fig.1(b). The CTFT of these two signals are shown in Fig.1(c). Calculate the corresponding sampling frequencies used for each of the signals $x_3[n]$ and $x_4[n]$.



2. (a) For the system described by difference equation,

$$y[n] + 0.5y[n-1] = x[n]$$

   i. Find the maximum value of output y[n] for input $x[n] = (-1)^n$
   ii. Find impulse response h[n] of this system. Is it BIBO stable? Is it causal?. (Assume $y[-\infty] = 0$)

(b) Consider another system described by difference equation

$$y[n] + y[n-1] = x[n]$$

i. Find impulse response h[n] of this system. Is it BIBO stable? Is it causal?. (Assume $y[-\infty] = 0$)

(c) Now consider the modified system to improve stability.

$$y[n] + \alpha y[n-1[= x[n]$$

What values of α will you choose to make the system BIBO stable.

3. A discrete-time system has input $x[n]$ and output $y[n]$. The Fourier transforms of these signals are related by the following equation:

$$Y(w) = 2X(w) + e^{-jw}X(w) - \frac{dX(w)}{dw}$$

(a) Is the system linear? Why?

(b) Is the system time-invariant? Why?

(c) What is $y[n]$ if $x[n] = \delta[n]$?

4. Show that the output of an **LTI** system can be expressed in terms of its unit step response $s[n]$ as follows.

$$y[n] = \sum_{k=-\infty}^{\infty} (\mathrm{s[k] - s[k-1])x[n-k]}$$

$$= \sum_{k=-\infty}^{\infty} (\mathrm{x[k] - x[k-1)])s[n-k]}$$

5. Three systems A,B and C have the inputs and outputs indicated in Table below. Determine whether each system could be **LTI**. If your answer is yes, specify whether there could be more than one **LTI** system with the given input-output pair. Explain your answer.

System	Input	Output
System A	$\left(\frac{1}{2}\right)^n$	$\left(\frac{1}{4}\right)^n$
System B	$e^{jn/7}u[n]$	$3e^{jn/7}u[n]$
System C	$e^{jn/7}$	$2e^{jn/7}$

6. Consider a stable discrete time signal $x[n]$ whose DTFT $X(e^{j\omega})$ satisfies the equation

$$X(e^{j\omega}) = X(e^{j(\omega-\pi)})$$

and has even symmetry, i.e., $x[n] = x[-n]$.

(a) Show that $X(e^{j\omega})$ is periodic with a period π.

(b) Find the value of x[1].

(c) Let $y[n]$ be the decimated version of $x[n]$, i.e, $y[n] = x[2n]$. Can you reconstruct $x[n]$ from $y[n]$ for all n. If yes, how? If no, justify your answer.

7. Consider the frequency response $H(e^{j\omega})$ of a discrete-time LTI system. Let $h[n]$ be the corresponding impulse response. Assume that $h[n]$ satisfies the following five properties:

   (a) The system is causal.

   (b) $H(e^{j\omega}) = H^*(e^{-j\omega})$

   (c) The DTFT of the sequence $h[n+1]$ is real.

   (d) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \mid H(e^{j\omega}) \mid^2 d\omega = 2$

   (e) $H(e^{j\pi}) = 0$

Answer the following questions:

(i) Show that properties (a)-(c) imply that h[n] is non-zero for only a finite duration.

(ii) Find all possible discrete signals h[n] that satisfy properties (a)-(e).

8. Let g[n] be a finite length sequence defined for $N_1 \leq n \leq N_2$, with $N_2 > N_1$. Likewise h[n] be a finite length sequence defined for $M_1 \leq n \leq M_2$, with $M_2 > M_1$. Let $y_1[n] = g[n] * h[n]$ and $y_2[n] = g[n] \circledast h[n]$

a) What is length of $y_1[n]$ and $y_2[n]$?

b) What is the range of indices for which $y_1[n]$ is defined?

c) What should be the minimum value of window length(N), so that the circular convolution is same as the linear convolution?

(For circular shift - we perform [n mod N] where N is window length)