SC635

Assignment 4 - cheat sheet

February 2020

Kalman Filter for linearized systems. Linearization of a nonlinear dyanmical systems involves taylor decomposition of the state and measurement equations at an *equilibrium point*. Given a nonlinear system dynamics and measurement function:

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = h(x(t))$$

one can compute the equilibrium point(s) by solving for x_* and u_* such that $f(x_*, u_*) = 0$.

The taylor expansion of the system dynamics and the measurement function gives:

$$\frac{d}{dt}(x_* + \tilde{x}) = f(x_*, u_*) + \frac{\partial f}{\partial x}\bigg|_{x_*,u_*}\tilde{x} + \frac{\partial f}{\partial x}\bigg|_{x_*,u_*}\tilde{u} + \textit{higher order terms}$$

$$y_* + \tilde{y} = h(x_*) + \frac{\partial h}{\partial x}\bigg|_{x_*}\tilde{x} + \textit{higher order terms}$$

where $\frac{d}{dt}x_* = 0$ and $f(x_*, u_*) = 0$. On neglecting the higher order terms, the resulting expression becomes:

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

$$\tilde{y} = H\tilde{x}$$

where,

$$A = \frac{\partial f}{\partial x}\bigg|_{x_*,u_*} \quad B = \frac{\partial f}{\partial x}\bigg|_{x_*,u_*} \quad H = \frac{\partial h}{\partial x}\bigg|_{x_*}$$

Kalamn filter for linearized state-space system (assuming constant matrices) is written as:

Prediction:

$$\tilde{x}_{k+1|k} = D\tilde{x}_{k|k} + G\tilde{u}_{k|k}$$
$$P_{k+1|k} = DP_{k|k}D^\top + Q$$
$$\tilde{y}_{k+1|k} = H\tilde{x}_{k+1|k}$$

Update:

$$S_{k+1} = HP_{k+1|k}H^\top + R$$
$$W_{k+1} = P_{k+1|k}H^\top S_{k+1}^{-1}$$
$$\tilde{x}_{k+1|k+1} = \tilde{x}_{k|k} + W_{k+1}(\tilde{y}_m - \tilde{y}_{k+1|k})$$
$$P_{k+1|k+1} = P_{k+1|k} + W_{k+1}S_{k+1}W_{k+1}^\top$$

where, D and G are discretized version of the state and input matrix. Also note that $\tilde{y}_m$ is a perturbed measurement and not the actual measurement.

The key idea of Kalman filter design is to predict the states $\tilde{x}_{k+1|k}$, the state-covariance matrix $P_{k+1|k}$, and the measurement $\tilde{y}_{k+1|k}$. When measured data of the $(k+1)^{th}$ time instant arrives, correction is made to the predicted state using kalman gain W_{k+1}. The result is a filtered state $x_{k+1|k+1}$ which is mathematically the best combination of prediction and measuremnt data.

Extended Kalman Filter. The state and output matrices were assumed constant in case of linearized system, however it needs to be re-evaluated at each sample time.

The final form of this algorithm is:

Prediction:

$$x_{k+1|k} = f(x_{k|k}, u_{k|k})$$
$$P_{k+1|k} = D_kP_{k|k}D_k^\top + Q_k$$
$$y_{k+1|k} = h(x_{k+1|k})$$

Update:

$$S_{k+1} = H_{k+1}P_{k+1|k}H_{k+1}^\top + R_{k+1}$$
$$W_{k+1} = P_{k+1|k}H_{k+1}^\top S_{k+1}^{-1}$$
$$x_{k+1|k+1} = x_{k|k} + W_{k+1}(y_m - y_{k+1|k})$$
$$P_{k+1|k+1} = P_{k+1|k} + W_{k+1}S_{k+1}W_{k+1}^\top$$

Summary.

- Identify system dynamics
- Identify the equilibrium points
- Obtain the matrices: A, B, and H
- Obtain discrete versions of the matrices: D, and G
- Apply the EKF algorithm