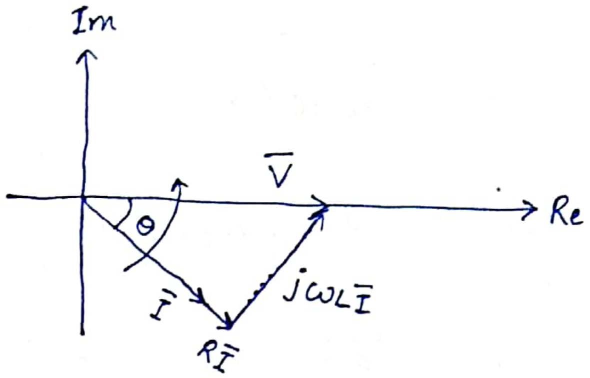
Tutorial- 3

Problem ①:



$Ref = \sin(2\pi 50t)$

$\therefore \bar{V} = V\angle 0°$

$\therefore v_s(t) = V\sin(2\pi 50t)$

$\bar{I} = I\angle -\theta$

$i(t) = I\sin(2\pi 50t - \theta)$

$R\bar{I} = \bar{V}_R = RI\angle -\theta$

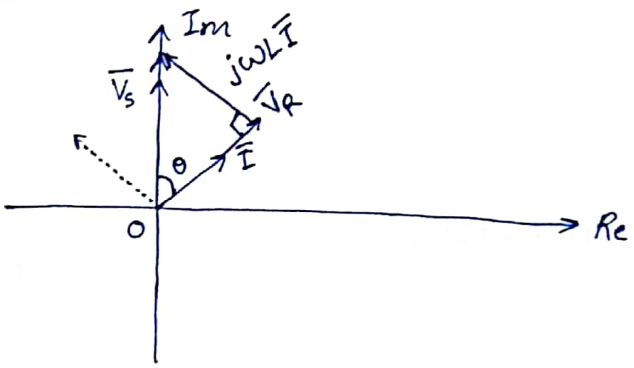
$v_R(t) = RI\sin(2\pi 50t - \theta)$

$\bar{V}_L = j\omega L\bar{I} = \omega LI\angle 90° - \theta$

$v_L(t) = \omega LI\sin(2\pi 50t + 90° - \theta)$

New reference : $\sin(2\pi 50(t - 0.005)) = \sin(2\pi 50t - \pi/2)$

With new reference,



$\bar{V} = V\angle \pi/2$

$\bar{I} = I\angle \pi/2 - \theta$

$R\bar{I} = RI\angle \pi/2 - \theta$

$\bar{V}_L = \omega LI\angle \pi - \theta$

Problem ②:

(i) $v(t) = 100\sin(2\pi 50t)$ volts $= 100\angle 0°$ volts

$\omega = 2\pi \times 50$ rad/s

$Z_L = j\omega L = j(2\pi 50)(0.05) = j15.71\ \Omega$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(2\pi\times 50)(100\times 10^{-6})} = -j31.83\ \Omega$$

$\therefore Z = R + Z_L + Z_C = 10 + j15.71 - j31.83 = 10 - j16.12$

$$Z = 10 - j\,16.12 = 18.96\angle{-58.18^\circ}\ \Omega$$

$$\therefore\ I = \frac{V}{Z} = \frac{100\angle 0^\circ}{18.96\angle{-58.18^\circ}} = 5.274\angle 58.18^\circ$$

$$\therefore\ I(t) = 5.274\sin(2\pi 50t + 58.18^\circ)\ \text{Amps}$$

(ii) $$I = \frac{v\angle\theta}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{v}{Z}$$

$$|I| = \frac{v}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

For maximum current, i.e, for maximum $|I|$

$\frac{d|I|}{dC} = 0 \Rightarrow$ solving this, we get, $\omega L - \frac{1}{\omega C} = 0$

$$\therefore\ C = \frac{1}{\omega^2 L}$$

Alternatively, $|I| = \frac{|V|}{|Z|}$. Now, $|I|$ will be maximum when $|Z|$ is minimum.

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

We can see that $|Z|$ will be minimum only when $\omega L - \frac{1}{\omega C} = 0$

i.e $\omega L = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega^2 L}$

$$\therefore\ C = \frac{1}{(2\pi 50)^2 (0.05)} = 202.64\,\mu F.$$

Under this condition, $Z = 10 + j0 = 10$

$$\therefore \quad I = \frac{V}{Z} = \frac{100\angle 0^\circ}{10}$$

$$\therefore \quad I = 10\angle 0^\circ$$

$I(t) = 10 \sin(2\pi 50t)$ Amps.

$$V_L(t) = X_L\, I(t) = (j\omega L)(10 \sin(2\pi 50t))$$

$$= j(2\pi \times 50)(0.05)(10) \sin(2\pi 50t)$$

$$= (j157.08) \sin(2\pi 50t)$$

$V_L(t) = 157.08 \sin(2\pi 50t + 90^\circ)$ volts.

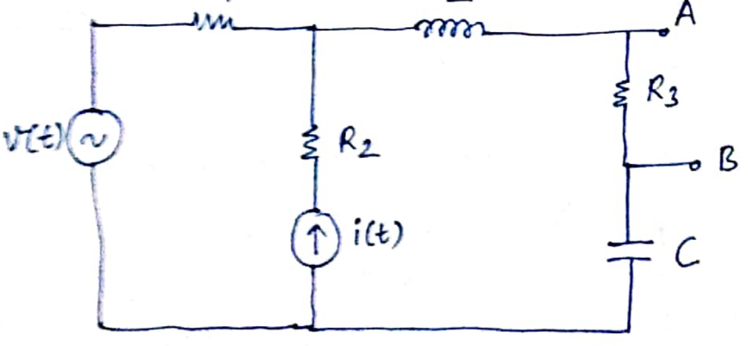
Similarly, $V_C(t) = X_C\, I(t) = \dfrac{-j}{(2\pi 50)(202.64 \times 10^{-6})} \times 10 \sin(2\pi 50t)$

$$= (-j157.08) \sin(2\pi 50t)$$

$V_C(t) = 157.08 \sin(2\pi 50t - 90^\circ)$ volts.

Amplitudes of $V_L(t)$ and $V_C(t)$ are equal (i.e, 157.08 volts), and, amplitudes of $V_L(t)$ and $V_C(t)$ are higher than amplitude of $V_S(t)$ (i.e, 100 v).

Problem ③ :



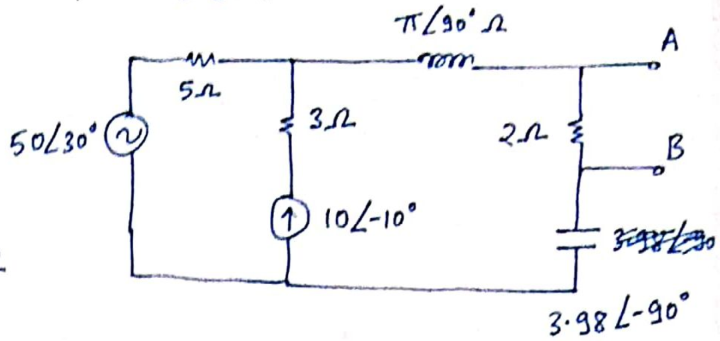
$v(t) = 50 \sin(2\pi 50t + 30^\circ)\, v = 50\angle 30^\circ \; v$

$i(t) = 10 \sin(2\pi 50t - 10^\circ)\, A = 10\angle -10^\circ \; A$

$R_1 = 5\Omega$, $R_2 = 3\Omega$, $L = 10mH$, $R_3 = 2\Omega$, $C = 0.8mF$

To find : $V_R(t)$ expression across R_3

Converting to phasor-form ,

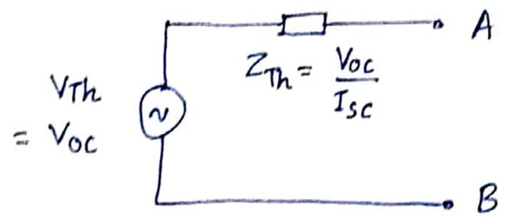


$X_L = j\omega L = j(2\pi \times 50)(10^{-2}) = \pi \angle 90^\circ\ \Omega$

$X_C = \dfrac{-j}{\omega C} = \dfrac{-j}{(2\pi 50)(0.8 \times 10^{-3})} = 3.98 \angle -90^\circ\ \Omega$

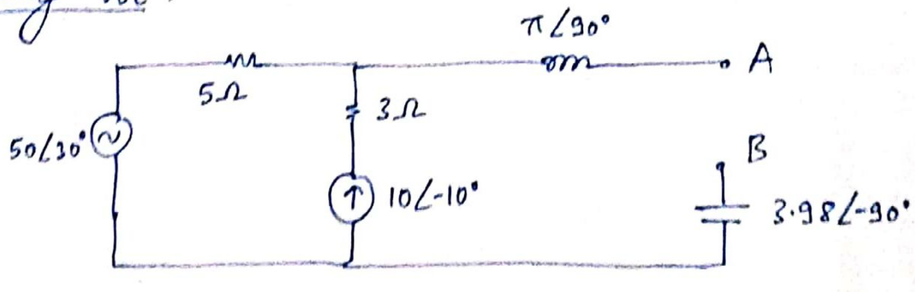
Applying thevenin's equivalent across A-B :

NOTE : For Thevenin's theorem, first we calculate V_{oc} (open circuit voltage) followed by calculation of I_{sc} (short circuit current) and then drawing the equivalent circuit across A and B like :

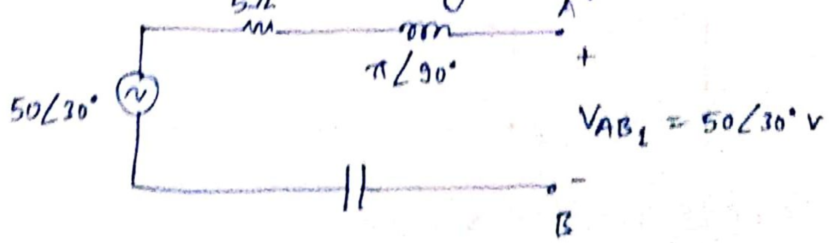


But as given circuit contains no dependent sources, we can compute Z_{Th} by replacing all independent sources with their equivalent impedances and finding total impedance across A-B .

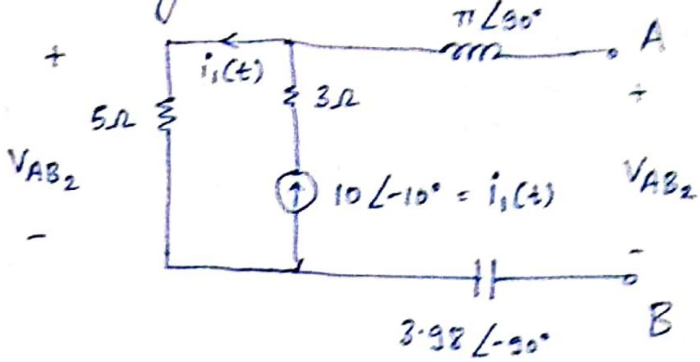
(a) Calculating V_{oc} :



Apply superposition : Considering voltage source :



Considering current source :



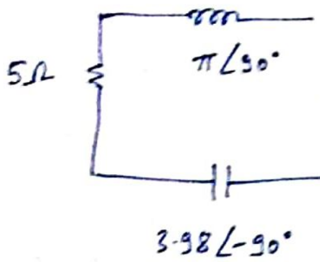
All current will pass through 5Ω branch as open circuit will not allow current to pass.

So, V_{AB_2} = Voltage drop across 5Ω $= 5 \times 10\angle -10^\circ$

$= 50\angle -10^\circ$ V.

So total $V_{AB} = V_{AB_1} + V_{AB_2} = 50\angle 30^\circ + 50\angle -10^\circ = 93.97\angle 10^\circ$ —— (i)

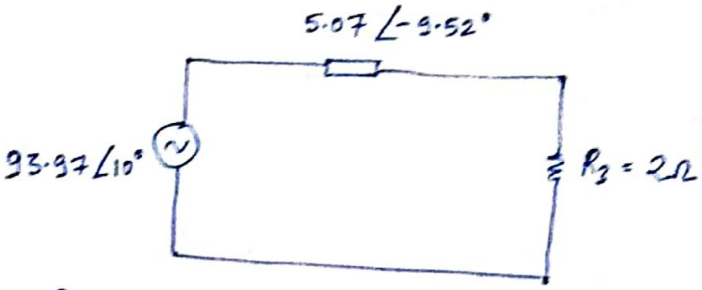
Calculating Z_{Th} : Replace all sources with their equivalent impedances and calculate Z_{AB} .



$\leftarrow$ $Z_{Th} = 5 + \pi\angle 90^\circ + 3.98\angle -90^\circ$

$= 5.07\angle -9.52^\circ\ \Omega$

∴ Thevenin's equivalent :

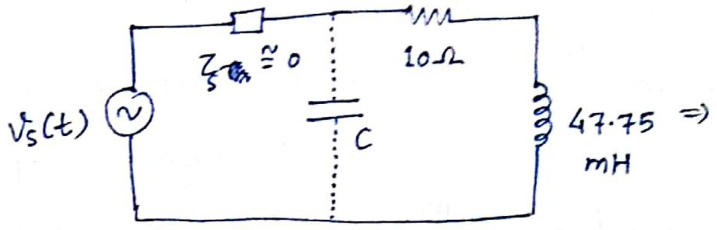


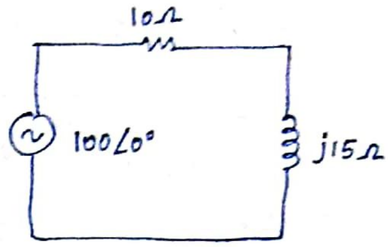
So $V_R(t) = \dfrac{R_3 \times V_{oc}}{R_3 + Z_{Th}} = \dfrac{2}{(2 + 5.07\angle -9.52^\circ)} \times (93.97\angle 10^\circ)$

$V_R(t) = 26.65\angle 16.83^\circ$

$V_R(t) = 26.65 \sin(2\pi 50 t + 16.83^\circ)$

Problem (4) :





$$V_s(t) = 100 \sin(2\pi 50 t) = 100\angle 0^\circ$$

$\therefore f = 50\text{Hz} \Rightarrow \omega = 2\pi \times 50$ rad/sec

$$X_L = 2\pi f L = 2\pi \times 50 \times 47.75 \times 10^{-3} = 15\Omega$$

Now, $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{100\angle 0^\circ}{R + jX_L} = \dfrac{100\angle 0^\circ}{10 + j15} = \dfrac{100\angle 0^\circ}{18.03\angle 56.31^\circ}$

$\bar{I} = 5.546\angle -56.31^\circ$ Amps.

Phase difference between $\bar{V}$ & $\bar{I}$: $\phi = 56.31^\circ$.

$\therefore$ PF $= \cos\phi = \cos(56.31^\circ) = 0.5547$ (lag) $\left[\text{or } \cos\phi = \dfrac{R}{\sqrt{R^2 + X_L^2}} = \dfrac{10}{\sqrt{10^2+15^2}} = 0.5547 \text{ (lag)}\right]$

$$P = \text{Real}\left(\bar{V}_{rms} \cdot \bar{I}_{rms}^{*}\right)$$

$$= \text{Real}\left(\frac{\bar{V}}{\sqrt{2}} \cdot \frac{\bar{I}^{*}}{\sqrt{2}}\right)$$

$$= \frac{1}{2} \text{ Real part of } \left(\bar{V}\bar{I}^{*}\right)$$

$$= \frac{1}{2} \text{Real}\left[(100\angle 0^\circ)(5.546\angle +56.31^\circ)\right]$$

$$= \frac{1}{2} \text{Real}\left[554.6\angle 56.31^\circ\right]$$

$P = 153.85$ Watts.

$$Q = \text{Imaginary}\left(\bar{V}_{rms}\, \bar{I}_{rms}^{*}\right) = \text{Imag}\left(\frac{\bar{V}}{\sqrt{2}} \cdot \frac{\bar{I}^{*}}{\sqrt{2}}\right) = \frac{1}{2}\text{Imag}\left(\bar{V} \cdot \bar{I}^{*}\right)$$

$$Q = \frac{1}{2}\, Imag\left[(100\angle 0^\circ)\cdot(5.546\angle +56.31^\circ)\right]$$

$Q = 230.77$ VAR $\longrightarrow$ (This is 'Q' supplied by V_s)

New Power factor (after adding 'C') $= 0.95$ lag

$$\cos\phi_n = \frac{P}{\sqrt{P^2+Q_n^2}} \Rightarrow Q_n = \left[\left(\frac{P}{\cos\phi_n}\right)^2 - P^2\right]^{1/2}$$

$$Q_n = \left[\left(\frac{153.85}{0.95}\right)^2 - (153.85)^2\right]^{1/2} = 50.568 \text{ VAR}$$

'Q_n' is the new Q supplied by V_s. So the difference of previous Q supplied by V_s and new Q supplied by V_s has to come from Capacitor.

$$\therefore Q_C = 230.77 - 50.568 = 180.2 \text{ VAR}$$

Now, $\bar{I}_c = \dfrac{\bar{V}_c}{\left(\frac{-j}{\omega C}\right)} = (j\omega C)\bar{V}$ ($\bar{V}_c = \bar{V}$. see from figure)

$$\therefore Q_C = \text{Imaginary part of } \left\{\bar{V}_{rms}\cdot \bar{I}^*_{C_{rms}}\right\}$$

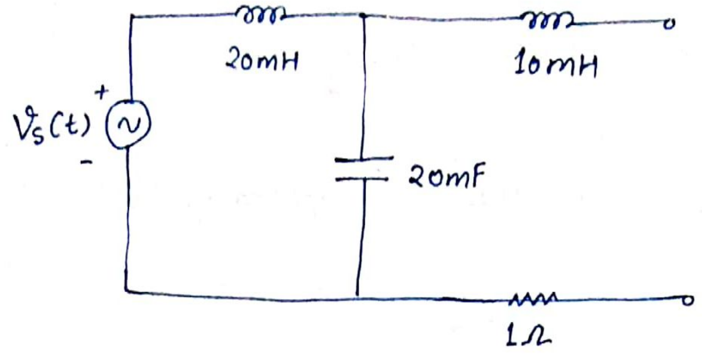
$$= Imag\left[\frac{\bar{V}}{\sqrt{2}}\cdot\frac{\bar{I}_c^*}{\sqrt{2}}\right] = \frac{1}{2}\, Imag\left[\bar{V}\,\bar{I}_c^*\right]$$

$$= \frac{1}{2}\, Imag\left[\bar{V}(-j\omega C\bar{V})\right]$$

$$Q_C = \frac{1}{2}\omega C V^2 \Rightarrow C = \frac{2Q_C}{\omega V^2} = \frac{2\times 180.2}{(2\pi\times 50)\times(100)^2}$$

$$C = 114.72\ \mu F$$

Problem ⑤ :



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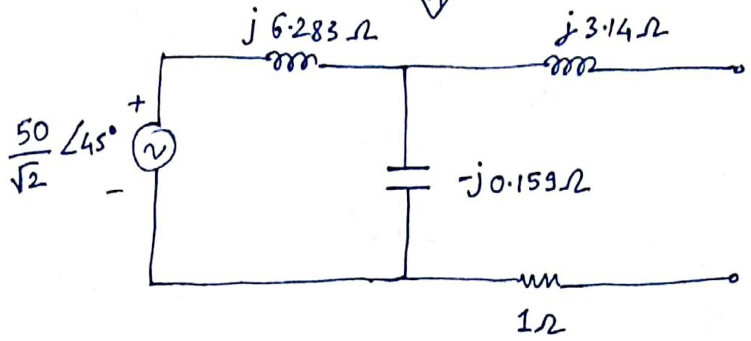


Here, $V_S(t) = 50 \sin(2\pi 50 t + 45^\circ)$ volts

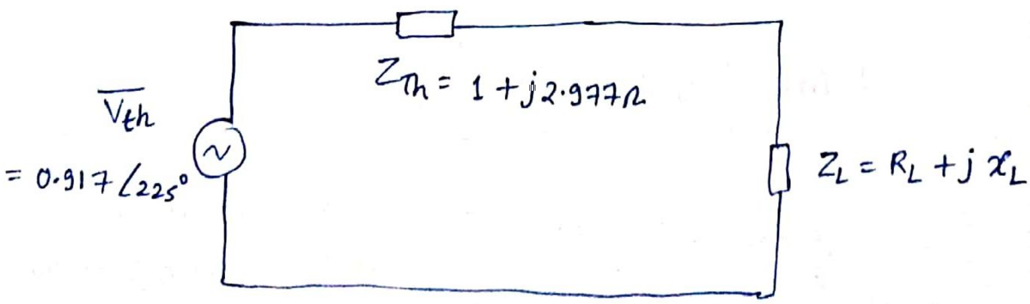
$$\overline{V_S} = \cancel{\frac{50}{\sqrt{2}}\angle 0^\circ} \text{ ~~volts~~} = \frac{50}{\sqrt{2}}\angle 45^\circ \text{ volts}$$

$$\overline{Z_{th}} = j3.14 + (j6.283 \parallel -j0.159) + 1 = 1 + j2.977\ \Omega$$

and $$\overline{V_{th}} = \frac{\overline{V_S} \times (-j0.159)}{j6.283 - j0.159} = \overline{V_S} \times (-0.0259)$$

$$\overline{V_{th}} = \frac{50}{\sqrt{2}} \times 0.0259 \angle 45^\circ + 180^\circ$$

$$\overline{V_{th}} = 0.917\angle 225^\circ$$



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Now, $P_L = \text{Realpart of } (\overline{V_L} \cdot \overline{I}^*)$

$$\overline{V_L} = \overline{V_{th}} \cdot \frac{Z_L}{Z_L + Z_{Th}} = \frac{\overline{V_{th}}\,(R_L + jX_L)}{R_L + jX_L + 1 + j2.977}$$

$$\overline{V_L} = \frac{\overline{V_{th}} \cdot (R_L + jX_L)}{(1+R_L) + j(X_L + 2.977)}$$

$$\overline{I} = \frac{\overline{V_{th}}}{R_L + jX_L + 1 + j2.977} = \frac{\overline{V_{th}}}{(R_L+1) + j(X_L + 2.977)}$$

$$\overline{I}^* = \frac{\overline{V_{th}}^*}{(R_L+1) - j(X_L + 2.977)}$$

$$\therefore P_L = \text{Realpart of } \left[\overline{V_L} \cdot \overline{I}^* \right] = \text{Real} \left\{ \left(\frac{\overline{V_{th}}\,(R_L + jX_L)}{(R_L+1) + j(X_L+2.977)} \right) \left(\frac{\overline{V_{th}}^*}{(R_L+1) - j(X_L+2.977)} \right) \right\}$$

$$P_L = \text{Realpart} \left\{ \frac{|V_{th}|^2 (R_L + jX_L)}{(R_L+1)^2 + (X_L + 2.977)^2} \right\}$$

$$P_L = \frac{|V_{th}|^2 R_L}{(R_L+1)^2 + (X_L + 2.977)^2}$$

Now, $$\frac{\partial P_L}{\partial Z_L} = \frac{\partial P_L}{\partial R_L} \cdot \frac{\partial R_L}{\partial Z_L} + \frac{\partial P_L}{\partial X_L} \cdot \frac{\partial X_L}{\partial Z_L}$$

Now, for max. power, $\dfrac{\partial P_L}{\partial Z_L} = 0$

$$Z_L^2 = R_L^2 + X_L^2 \Rightarrow \frac{\partial Z_L}{\partial R_L} = \frac{R_L}{Z_L} \Rightarrow \frac{\partial R_L}{\partial Z_L} = \frac{Z_L}{R_L}$$

$$\therefore \frac{\partial R_L}{\partial Z_L} = \frac{\sqrt{R_L^2 + X_L^2}}{R_L}$$

Similarly, $\dfrac{\partial Z_L}{\partial x_L} = \dfrac{x_L}{Z_L} \Rightarrow \dfrac{\partial x_L}{\partial Z_L} = \dfrac{Z_L}{x_L} = \dfrac{\sqrt{R_L^2 + x_L^2}}{x_L}$

For $\dfrac{dP_L}{dZ_L} = 0 \Rightarrow \dfrac{\partial P_L}{\partial x_L} = 0$ and $\dfrac{\partial P_L}{\partial R_L} = 0$

$$\frac{\partial P_L}{\partial x_L} = \frac{-2|V_{th}|^2 R_L (x_L + x_{th})}{[(R_L+R_{th})^2 + (x_L+x_{th})^2]^2} = 0$$

This gives us $\underline{x_L = -x_{th}}$

Similarly, $$\frac{\partial P_L}{\partial R_L} = \frac{[(R_L+R_{th})^2 + (x_L+x_{th})^2]\cdot|V_{th}|^2 - (2R_L(R_L+R_{th})|V_{th}|^2)}{[(R_L+R_{th})^2 + (x_L+x_{th})^2]^2}$$

$$= \frac{(|V_{th}|^2)\cdot[(x_{th}+x_L)^2 + (R_{th}-R_L)(R_{th}+R_L)]}{[(R_L+R_{th})^2 + (x_L+x_{th})^2]^2}$$

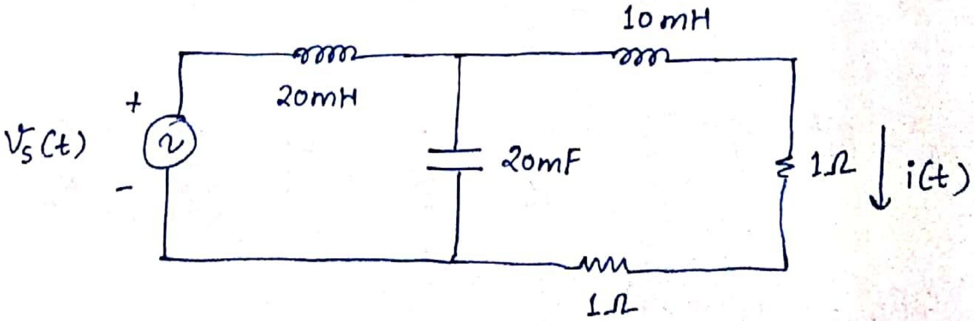
$\dfrac{\partial P_L}{\partial R_L} = 0 \Rightarrow \underline{R_{th} = R_L}$ ($x_{th} + x_L$ also $= 0$ from previous part)

$\therefore Z_L = R_L + jx_L = R_{th} - jx_{th}$

$\therefore \underline{Z_L = Z_{th}^*}$

$\therefore Z_L = 1 - j2.977\ \Omega$

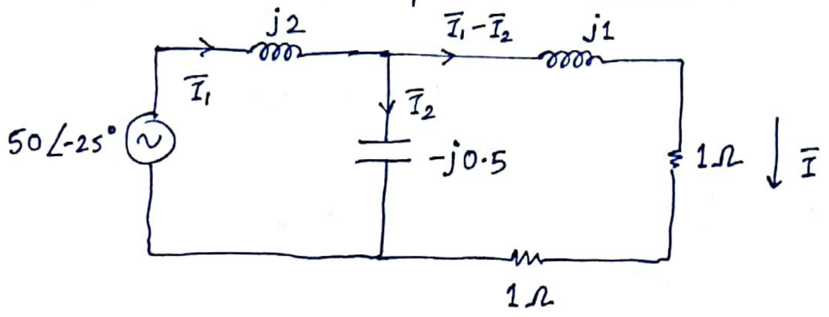
Problem (6) :



$V_s(t) = 50 \sin(100t - 25°)$ V

Here, $\omega = 100$ rad/s. Reference phasor: $\sin(100t)$

$\therefore$ The equivalent circuit in phasor domain is :



Applying KVL, we get,

$$50\angle -25° = \bar{I}_1 (j2) + \bar{I}_2 (-j0.5) \quad \text{--- ①}$$

$$\bar{I}_2 (-j0.5) - (\bar{I}_1 - \bar{I}_2)(j1) - 2(\bar{I}_1 - \bar{I}_2) = 0$$

or, $(2+j1)\bar{I}_1 = (2+j0.5)\bar{I}_2$

$$\bar{I}_1 = \frac{(2+j0.5)}{2+j1}\bar{I}_2 \quad \text{--- ②}$$

Substitute ② in ①,

$$50\angle -25° = \left(\frac{-1+j4}{2+j1} - j0.5\right)\bar{I}_2$$

$$= \frac{(-1+j4 - j1 + 0.5)}{2+j1}\bar{I}_2 = \left(\frac{-0.5+j3}{2+j1}\right)\bar{I}_2$$

$$(50\angle -25°) = (1.35\angle 72.89°)\,\bar{I}_2 \Rightarrow \bar{I}_2 = 37.03\angle -97.89° \text{ Amp.}$$

$$\bar{I}_1 = \left(\frac{2+j0.5}{2+j1}\right)\bar{I}_2$$

$$\bar{I} = \bar{I}_1 - \bar{I}_2 = 8.21\angle 145.53° \text{ Amp.}$$

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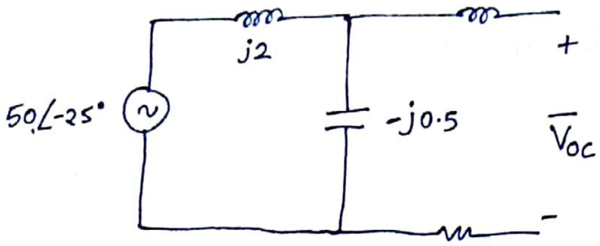
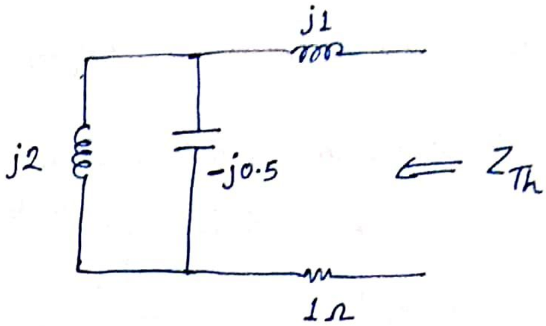
Thevenin's theorem :



$\Rightarrow$

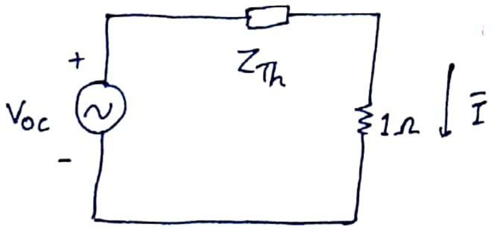


$$\overline{V}_{oc} = (50\angle -25°) \times \frac{-j0.5}{j2 - j0.5} = -16.66\angle -25° \text{ volts}$$

$$\overline{V}_{oc} = 16.66\angle 155° \text{ volts}$$

$$\overline{Z}_{Th} = (j2 \parallel -j0.5) + j1 + 1 = \frac{j2 \times (-j0.5)}{j2 - j0.5} + 1 + j1$$

$$\overline{Z}_{Th} = -j0.66 + 1 + j1 = 1 + j0.33$$

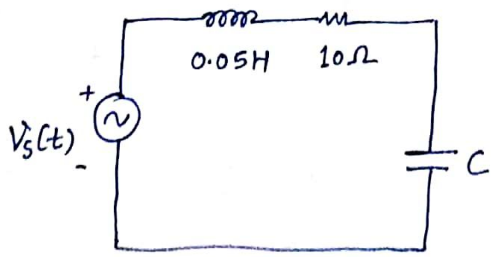


$$\Rightarrow \quad \overline{I} = \frac{\overline{V}_{oc}}{\overline{Z}_{Th} + R} = \frac{16.66\angle 155°}{2 + j0.33} \text{ Amps}$$

$$\overline{I} = \frac{16.66\angle 155°}{2.03\angle 9.37°} = 8.20\angle 145.63° \text{ Amps}.$$

$$\therefore \quad i(t) = 8.20 \sin(100t + 145.63°) \text{ Amps}.$$

Problem 7 :



$V_s(t) = 100 \sin(2\pi 50t + 45°)$ v

Reference : $\sin(2\pi 50t + 45°)$

$\therefore \overline{V}_s = 100\angle 0°$ v

$$\overline{Z} = R + j\omega L - j\left(\frac{1}{\omega C}\right) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

For $C = 150\mu F$, $\bar{Z} = 10 + j\left(2\pi \times 50 \times 0.05 - \frac{1}{(2\pi \times 50 \times 150 \times 10^{-6})}\right)$

$\bar{Z} = 10 + j(15.7 - 21.22) = 10 - j5.51\ \Omega = 11.42\angle -28.85^\circ\ \Omega$

$\bar{I} = \frac{\bar{V_s}}{\bar{Z}} = \frac{100\angle 0^\circ}{11.42\angle -28.85^\circ} = 8.756\angle 28.85^\circ$ Amp

$\therefore \bar{V_R} = \bar{I}\cdot R = (10)(8.756)\angle 28.85 = 87.56\angle 28.85$ volts.

$\bar{V_L} = \bar{I}\cdot j\omega L = (j15.7)\times 8.756\angle 28.85^\circ = 137.56\angle 118.85^\circ$ volts.

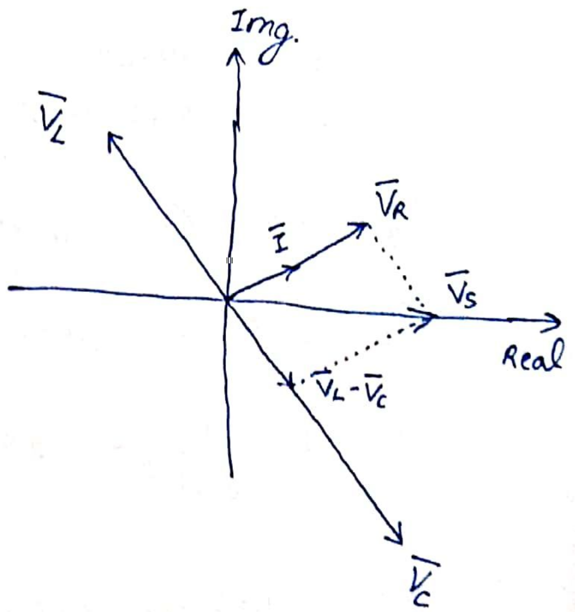
$\bar{V_C} = \bar{I}\cdot\left(\frac{1}{j\omega C}\right) = \frac{8.756\angle 28.85^\circ}{(j\,0.047123)} = 185.81\angle -61.15^\circ$ volts.

$i(t) = 8.756\sin(2\pi 50t + 28.85^\circ)$ A

$v_R(t) = 87.56\sin(2\pi 50t + 28.85^\circ)$ V

$v_L(t) = 137.56\sin(2\pi 50t + 118.85^\circ)$ V

$v_C(t) = 185.81\sin(2\pi 50t - 61.15^\circ)$ V



For $C = 202.85\mu F$,

$\bar{Z} = 10 + j\left(\omega L - \frac{1}{\omega C}\right) = 10 + j\left(2\pi \times 50 \times 0.05 - \frac{1}{(2\pi \times 50 \times 202.85 \times 10^{-6})}\right)$

$\bar{Z} = 10 + j(15.7 - 15.7) = 10$

$\therefore \bar{I} = \frac{\bar{V_s}}{\bar{Z}} = \frac{100\angle 0^\circ}{10} = 10\angle 0^\circ$ Amp.

$\bar{V_R} = \bar{I}\cdot R = 100\angle 0^\circ$

$\bar{V_L} = \bar{I}\cdot(j\omega L) = (10\angle 0^\circ)\cdot(j15.7) = 157\angle 90^\circ$ Volt

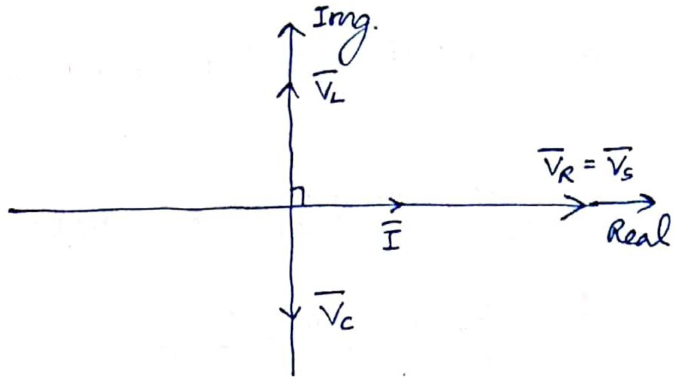
$\bar{V_C} = \bar{I}\left(\frac{1}{j\omega C}\right) = (10\angle 0^\circ)(-j15.7) = 157\angle -90^\circ$ volts.

$\therefore\ i(t) = 10\sin(2\pi 50t + 45^\circ)$ A

$v_R(t) = 100\sin(2\pi 50t + 45^\circ)$ V

$v_L(t) = 157\sin(2\pi 50t + 135^\circ)$ V

$v_C(t) = 157\sin(2\pi 50t - 45^\circ)$ V



For $C = 250\mu F$,

$$\bar{Z} = 10 + j\left(2\pi\times 50\times 0.05 - \frac{1}{(2\pi\times 50\times 250\times 10^{-6})}\right) = 10 + j(15.7 - 12.73)$$

$$\bar{Z} = 10 + j2.97\ \Omega = 10.43\angle 16.59^\circ\ \Omega$$

$$\bar{I} = \frac{\bar{V}_S}{\bar{Z}} = \frac{100\angle 0^\circ}{10.43\angle 16.59^\circ} = 9.58\angle -16.59^\circ$$

$$\bar{V}_R = \bar{I}R = (9.58\angle -16.59^\circ)\times 10 = 95.8\angle -16.59^\circ\ \text{V}$$

$$\bar{V}_L = \bar{I}(j\omega L) = (9.58\angle -16.59^\circ)(j15.7) = 150.623\angle 73.41^\circ\ \text{V}$$

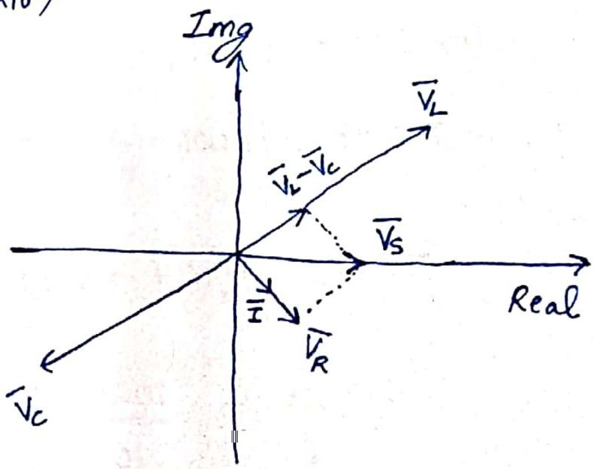
$$\bar{V}_C = \bar{I}\cdot\left(\frac{1}{j\omega C}\right) = (9.58\angle -16.59^\circ)\cdot\left(\frac{1}{j2\pi\times 50\times 250\times 10^{-6}}\right) = 122.05\angle -106.59^\circ\ \text{V}$$

$i(t) = 9.58\sin(2\pi 50t + 28.41^\circ)$ A

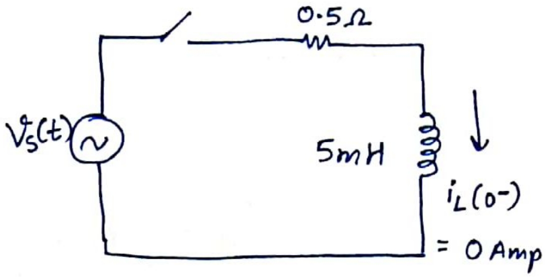
$v_R(t) = 95.8\sin(2\pi 50t + 28.41^\circ)$ V

$v_L(t) = 150.623\sin(2\pi 50t + 118.41^\circ)$ V

$v_C(t) = 122.05\sin(2\pi 50t - 61.59^\circ)$ V



Problem ⑧ : $V_s(t) = 100 \sin(2\pi \times 50 t + \alpha)$ V



(i) Switch is closed at $t=0$,

$V_s(t) = 100 \sin(2\pi 50 t + \alpha)$ can be written as,

$\bar{V_s} = 100 \angle \alpha$

$i(t) =$ Transient response + steady state response

$$= Ae^{-Rt/L} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\omega t + \alpha - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Inductor doesn't allow sudden change in current, so $i_L(0^-) = i_L(0^+) = 0$

At $t=0$, $i(0) = 0$.

Substituting in above equation,

$$i(0) = A + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\alpha - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$0 = A + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\alpha - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\therefore \quad A = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\alpha - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

For transient free response, A must be zero.

$$\therefore \quad \sin\left(\alpha - \tan^{-1}\left(\frac{\omega L}{R}\right)\right) = 0$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

For $\omega = 100\pi$ rad/s, $L = 5$mH, $R = 0.5\Omega$, we get $\alpha = 1.26$ rad $= 72.34°$

$\therefore$ At steady state, $i(t) = \dfrac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin(\omega t)$

$$i(t) = i_{ss} = \frac{100}{\sqrt{(0.5)^2 + (2\pi \times 50 \times 5\times10^{-3})^2}} \sin(\omega t)$$

$$\therefore i_{ss} = 200 \sin(2\pi 50 t) \text{ A} = 200\angle 0^\circ \text{ A}$$

ii) Let switching happens at $t = t_0$ so as to avoid transients,

$\therefore$ $i(t_0^-) = i(t_0^+) = i(t_0) = 0$ and $\alpha = 0$ (given)

$\therefore$ Equation of $i(t)$ can be written as,

$$i(t) = \underbrace{A \cdot e^{-R(t-t_0)/L}} + B \sin(\omega t - \phi)$$

$(t-t_0)$: because 't' is measured after switching | where $B = \dfrac{V_m}{\sqrt{R^2+(\omega L)^2}}$, $\phi = \tan^{-1}\left(\dfrac{\omega L}{R}\right)$

If $t = t_0$ and for transient-free response,

$$i(t_0) = \underbrace{0}_{\text{Transient factor}} + B \sin(\omega t_0 - \phi)$$

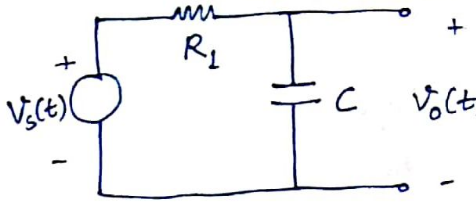
$$0 = B \sin(\omega t_0 - \phi)$$

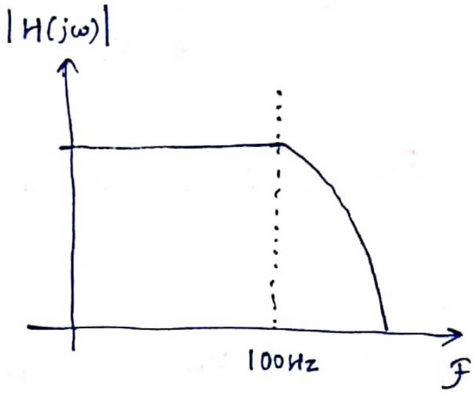
$$\therefore \omega t_0 = \phi \Rightarrow t_0 = \frac{\tan^{-1}\left(\frac{\omega L}{R}\right)}{\omega} = \frac{1.26}{2\pi \times 50}$$

$$\underline{t_0 = 4.02 \text{ msec}}$$

Problem (9):

(a)





$$V_o = V_S \times \frac{\left(\frac{1}{j\omega C}\right)}{R_1 + \left(\frac{1}{j\omega C}\right)} = \frac{1}{1 + jR_1\omega C}$$

$$\therefore \frac{V_o(j\omega)}{V_S(j\omega)} = \frac{1}{(1 + j\omega R_1 C)}$$

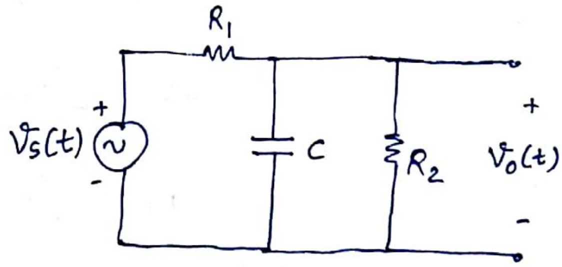
We need a filter with a cut-off frequency of 100Hz as shown above.

If we take $R_1 = 1K\Omega$, we get $R_1 C = \frac{1}{2\pi \times 100}$

$$\therefore C = \frac{1}{2\pi \times 100 \times R_1} = \frac{1}{2\pi \times 100 \times 10^3} = 1.59\,\mu F$$

$\therefore$ Filter transfer function $\Rightarrow$ $H_0(j\omega) = \frac{1}{1 + j(10^3 \times 1.59 \times 10^{-6})\omega} = \frac{1}{(1 + j\,0.00159\,\omega)}$

(b) If a resistor is connected at output,



$$\frac{V_o}{V_S} = \frac{\left[R_2 \cdot \left(\frac{1}{j\omega C}\right)\right] / \left[R_2 + \frac{1}{j\omega C}\right]}{R_1 + \frac{\left[R_2 \cdot \frac{1}{j\omega C}\right]}{R_2 + \left(\frac{1}{j\omega C}\right)}}$$

$$= \frac{\frac{R_2}{1 + j\omega C R_2}}{R_1 + \frac{R_2}{1 + j\omega C R_2}} = \frac{R_2}{(R_1 + R_2 + j\omega_C R_2 R_1)}$$

$$\therefore H(j\omega) = \frac{V_o(j\omega)}{V_S(j\omega)} = \frac{1}{1 + j\omega C R_1 + \left(\frac{R_1}{R_2}\right)}$$

For $R_2 = 200\Omega \Rightarrow H(j\omega) = \frac{1}{(6 + j0.00159\omega)} = \frac{0.166}{1 + j0.00026\omega}$

For $R_2 = 20k\Omega \Rightarrow H(j\omega) = \frac{1}{1.05 + j0.00159\omega} = \frac{0.953}{1 + j0.00159\omega}$

We can see that for $R_2 = 20k\Omega$, $H(j\omega) \cong H_0(j\omega)$.

Steady state output $V_o(t)$ in part ⓐ :

$$V_S(t) = 50\sin(2\pi 50t + 30°) + 10\sin(2\pi 500t) \text{ V}$$

$$= V_{S_1}(t) + V_{S_5}(t)$$

Using superposition theorem, for $V_{S_1}(t)$ source, we get,

$$V_{o_1}(t) = \frac{1}{(1 + j0.00159 \times 2\pi \times 50)} \cdot V_{S_1}(t)$$

→ $\sin(2\pi 50t)$ as reference,

$\overline{V_{S_1}} = 50\angle 30°$ V

$$\therefore \overline{V_{o_1}} = (0.8946\angle -26.54°) \cdot 50\angle 30°$$

$$= 44.73\angle 30° - 26.54°$$

$$= 44.73\angle 3.46°$$

$$\therefore V_{o_1}(t) = 44.73\sin(2\pi 50t + 3.46°) \text{ V}$$

Similarly, for $V_{S_5}(t)$ source, we get,

$$V_{o_5}(t) = \frac{1}{(1 + j0.00159\omega)} \cdot V_{S_5}(t)$$

Reference : $\sin(2\pi 500t)$

$\therefore \overline{V_{S_5}} = 10\angle 0°$ V

$$V_{o_5}(t) = \frac{1}{1 + j(0.00159 \times 2\pi \times 500)} \cdot 10\angle 0^\circ$$

$$= (0.1963\angle -78.67^\circ) \times 10\angle 0^\circ$$

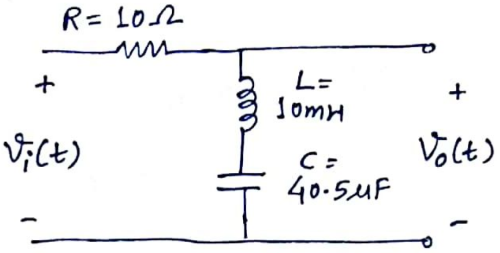
$$= 1.963\angle -78.67^\circ \text{ V}$$

$$= 1.963 \sin(2\pi 500t - 78.67^\circ) \text{ V}$$

$$\therefore \; V_o(t) = V_{o_1}(t) + V_{o_5}(t)$$

$$V_o(t) = 44.73 \sin(2\pi 50t + 3.46^\circ) + 1.963 \sin(2\pi 500t - 78.67^\circ) \text{ volts.}$$

Problem (10) :



$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{\left(\frac{1}{j\omega C} + j\omega L\right)}{R + j\omega L + \frac{1}{j\omega C}}$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = H(j\omega)$$

$$\angle H(j\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega^2 LC - 1}{\omega CR}\right]$$

For $\omega_0 = \frac{1}{\sqrt{LC}} = 1571.3 \text{ rad/s} \Rightarrow |H(j\omega_0)| = 0$

For frequencies around 'ω_0' $\Rightarrow$ For $\omega < \omega_0 \rightarrow$ we get $\frac{1}{\omega C} > \omega L$

$\therefore \angle H(j\omega) \cong -90^\circ$

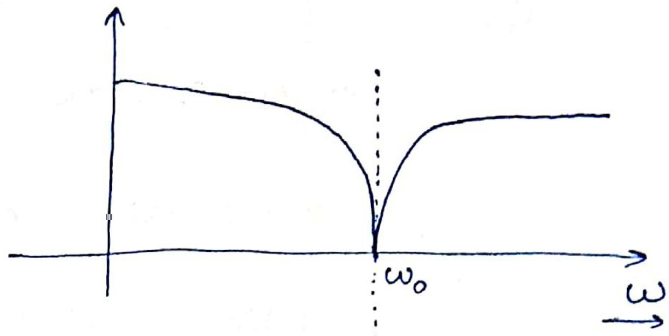
For $\omega > \omega_0 \rightarrow$ we get, $\frac{1}{\omega C} < \omega L$

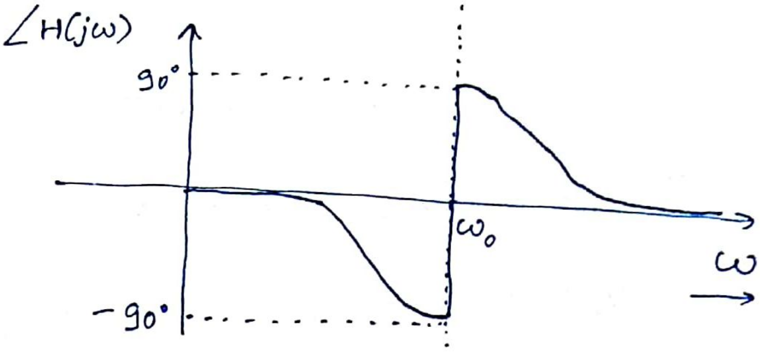
$\angle H(j\omega) \cong 90^\circ$

For 'ω' tends to 0 $\Rightarrow \lim_{\omega \to 0} \angle H(j\omega) = 0^\circ$

Similarly, $\angle H(j\omega) = 0°$
$\lim: \omega \to \infty$

$\therefore$ Frequency response $\Rightarrow$





Output voltage in steady state :

$$v_i(t) = 50\sin(2\pi 50t - 3°) + 10\sin(2\pi\times 250t) + 5\sin(2\pi\times 750t + 15°)\ V$$

$$= v_{i_1}(t) + v_{i_5}(t) + v_{i_{15}}(t)$$

Using superposition theorem, for $v_{i_1}(t)$ source,

$$v_{o_1}(t) = \frac{j\left(2\pi\times 50\times 10\times 10^{-3} - \dfrac{1}{(2\pi\times 50\times 40.5\times 10^{-6})}\right)}{10 + j\left(2\pi\times 50\times 10\times 10^{-3} - \dfrac{1}{2\pi\times 50\times 40.5\times 10^{-6}}\right)} \times v_s(t)$$

$$\overline{V_{o_1}} = (0.9913\angle -7.55°) \times 50\angle -3°$$

$$\overline{V_{o_1}} = 49.566\angle -10.55°\ V$$

$$v_{o_1}(t) = 49.566\sin(2\pi 50t - 10.55°)\ V.$$

Reference :

$\sin(2\pi 50t)$

$\therefore$ ~~$\overline{V_s} = 50\angle -3°$~~

$\overline{V_{i_1}} = 50\angle -3°$

For $V_{i_5}(t)$ source,

$$V_{o_5}(t) = \frac{j\left(2\pi\times250\times10\times10^{-3} - \frac{1}{2\pi\times250\times40.5\times10^{-6}}\right)}{10 + j\left(2\pi\times250\times10\times10^{-3} - \frac{1}{(2\pi\times250\times40.5\times10^{-6})}\right)} \times V_{i_5}(t)$$

$$\simeq \left(\frac{1}{10}\right) \times 10\angle0^\circ$$

Reference :

$\sin(2\pi\times250t)$

$\therefore \overline{V_{i_5}} = 10\angle0^\circ$

$\therefore \overline{V_{o_5}} = 1\angle0^\circ$ V

$\therefore V_{o_5}(t) = \sin(2\pi\times250t)$

For $V_{i_{15}}(t)$ source,

$$V_{o_{15}}(t) = \frac{j\left[2\pi\times750\times10\times10^{-3} - \frac{1}{(2\pi\times750\times40.5\times10^{-6})}\right]}{10 + j\left[2\pi\times750\times10\times10^{-3} - \frac{1}{2\pi\times750\times40.5\times10^{-6}}\right]} \times V_{i_{15}}(t)$$

Reference :

$\sin(2\pi\times750t)$

$\therefore \overline{V_{i_{15}}} = 5\angle15^\circ$

$\therefore \overline{V_{o_{15}}} = (0.9726\angle13.423^\circ) \times 5\angle15^\circ$ V

$= 4.8632\angle28.43^\circ$

$\therefore V_{o_{15}}(t) = 4.8632\sin(2\pi\,750t + 28.43^\circ)$ V.

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∴ Expression for output voltage,

$$V_0(t) = 49.566 \sin(2\pi 50t - 10.55°) + \sin(2\pi \times 250t) + 4.8632 \sin(2\pi \times 750t + 28.43°)$$

Problem (11)

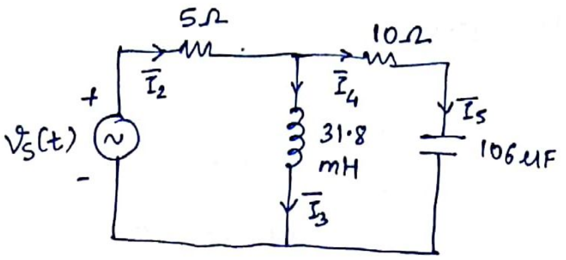


Fig. ①

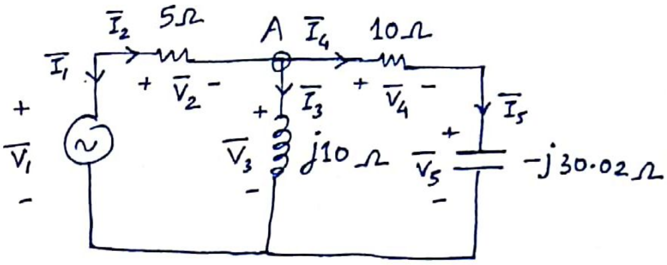


Fig ②

From fig ②,

$\overline{V_1} = \overline{V_S}$

$\overline{I_4} = \overline{I_5}$

$\overline{I_1} = -\overline{I_2}$

$V_S(t) = 100 \sin(2\pi 50t + 30°)$

Taking reference as $\sin(2\pi 50t)$,

$\overline{V_S} = 100\angle 30°$ V. $= \overline{V_1}$

Applying KCL at Node A,

$$\frac{\overline{V_3} - \overline{V_1}}{5} + \frac{\overline{V_3}}{(j10)} + \frac{\overline{V_3}}{(10 - j30.02)} = 0$$

$$\frac{\overline{V_3} - 100\angle 30°}{5} + \frac{\overline{V_3}}{(j10)} + \frac{\overline{V_3}}{(10 - j30.02)} = 0$$

Solving this gives us, $\overline{V_3} = 90.35\angle 48.44°$ v.

$\overline{I_4} = \overline{I_5} = \dfrac{\overline{V_3}}{(10 - j30.02)} = 2.855\angle 120.01°$ A, $\overline{I_3} = \dfrac{\overline{V_3}}{(j10)} = 9.035\angle -41.56°$ A

$\overline{V_4} = (10)\,\overline{I_4} = 28.55\angle 120.01°$ v, $\overline{V_2} = \overline{V_1} - \overline{V_3} = 31.95\angle -33.43°$ v

$\overline{V_5} = (-j30.02)\,\overline{I_4} = (30.02\angle -90°)(2.855\angle 120.01°) = 85.7\angle 30.01°$ v.

$\overline{I_2} = \dfrac{\overline{V_1} - \overline{V_3}}{5} = \cancel{28.558\angle -120.0933° A}$ $6.39\angle -33.43°$ A, $\overline{I_1} = -\overline{I_2} = \cancel{26.958\angle 167.906° A}$ $6.39\angle 146.57°$ A

$$\sum_{K=1}^{5} V_K I_K^*$$

$= V_1 I_1^* + V_2 I_2^* + V_3 I_3^* + V_4 I_4^* + V_5 I_5^*$

$= \cancel{(100\angle 30°)(26.956\angle -167.9°)}$

$= (100\angle 30°)(6.39\angle -146.57°) + (31.95\angle -33.43°)(6.39\angle 33.45°) + (90.35\angle 48.44°)(9.035\angle 41.56°) + (28.55\angle 120.01°)(2.855\angle -120.01°) + (85.7\angle 30.01°)(2.855\angle -120.01°)$

$= 0$

$$\therefore \sum_{K=1}^{5} V_K I_K^* = 0$$

Problem (12) : $V_S(t) = 5 + 10\sin(2\pi \times 50t + 30°) + 3\sin(2\pi \times 150t - 60°) + 0.5\sin(2\pi \times 300t)$ volts.

$= V_{dc} + V_1(t) + V_3(t) + V_6(t)$

Using superposition theorem,

For V_{dc} source, $i_{dc} = \frac{5}{10} = 0.5$ Amp.

For $V_1(t)$ source, $i_1(t) = \frac{10}{\sqrt{(10)^2 + (2\pi \times 50 \times 16 \times 10^{-3})^2}} \cdot \sin\left[2\pi 50t + 30° - \tan^{-1}\left(\frac{2\pi 50 \times 16 \times 10^{-3}}{10}\right)\right]$

$\therefore i_1(t) = 0.893 \sin(2\pi 50t + 30° - 26.686°)$

$i_1(t) = 0.893 \sin(2\pi 50t + 3.31°)$ A.

For $V_3(t)$ source,

$$i_3(t) = \frac{3}{\sqrt{(10)^2+(2\pi\times150\times16\times10^{-3})^2}} \sin\left[2\pi\times150t - 60 - \tan^{-1}\left(\frac{2\pi\times150\times16\times10^{-3}}{10}\right)\right]$$

$$i_3(t) = 0.166 \sin(2\pi150t - 116.45°) \text{ Amp.}$$

For $V_6(t)$ source,

$$i_6(t) = \frac{0.5}{\sqrt{(10)^2+(2\pi\times300\times16\times10^{-3})^2}} \sin\left[2\pi\times300t - \tan^{-1}\left(\frac{2\pi\times300\times16\times10^{-3}}{10}\right)\right]$$

$$i_6(t) = 0.0157 \sin\left(2\pi\times300t - 71.655°\right) \text{ Amp.}$$

Power dissipated across resistor $= I_{rms}^2 \cdot R$

For each Component of Current, we can find power dissipated as,

$$P_{dc} = I_{dc}^2 \cdot R = (0.5)^2\times10 = 2.5\text{ W}$$

Due to $i_1(t) \Rightarrow P_1 = I_{1rms}^2 R = \left(\frac{0.893}{\sqrt{2}}\right)^2\times10 = 3.98\text{ W}$

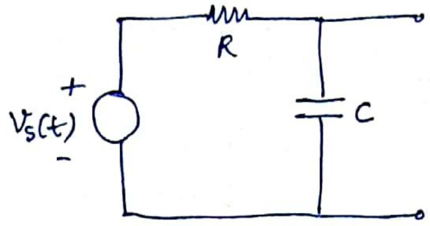
Due to $i_3(t) \Rightarrow P_3 = \left(\frac{0.166}{\sqrt{2}}\right)^2\times10 = 0.13778\text{ W}$

Due to $i_6(t) \Rightarrow P_6 = \left(\frac{0.0157}{\sqrt{2}}\right)^2\times10 = 0.00123\text{ W}$

$\therefore$ Total power dissipated across resistor $= 2.5 + 3.98 + 0.13778 + 0.00123$

$= 6.62$ Watts.

Problem (13) :



$$\frac{V_o}{V_s} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\frac{V_o}{V_s} = \frac{1}{(1+j\omega RC)} \Rightarrow \therefore H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1}{\sqrt{1+(\omega RC)^2}} \angle -\tan^{-1}(\omega RC)$$

From Fourier Series of $V_s(t)$, we can write,

$$V_s(t) = a_0 + \sum_{n \in \mathbb{Z}^+} a_n \sin(n\omega t) + \sum_{n \in \mathbb{Z}^+} b_n \cos(n\omega t)$$

By looking at waveform of $V_s(t)$, we can say that it has zero average value.

$\therefore a_0 = 0$

$V_s(t)$ is also an 'odd' function, $\therefore b_n = 0$ for all values of 'n'.

$V_s(t)$ satisfies half wave symmetry, i.e $V_s(t+T/2) = -V_s(t-T/2)$

$\therefore a_n = 0$ for $n = 2, 4, 6, 8 \ldots\ldots$

$$\therefore V_s(t) = \sum_{n=1,3,5\ldots} a_n \sin(n\omega t) \qquad \text{where } \omega = 2\pi \times 100 \text{ rad/sec.}$$

$$a_n = \frac{1}{T/2} \int_0^{T/2} V_s(t) \sin(n\omega t)\, dt = \frac{4V}{n\pi} \qquad \text{where } V = 10$$

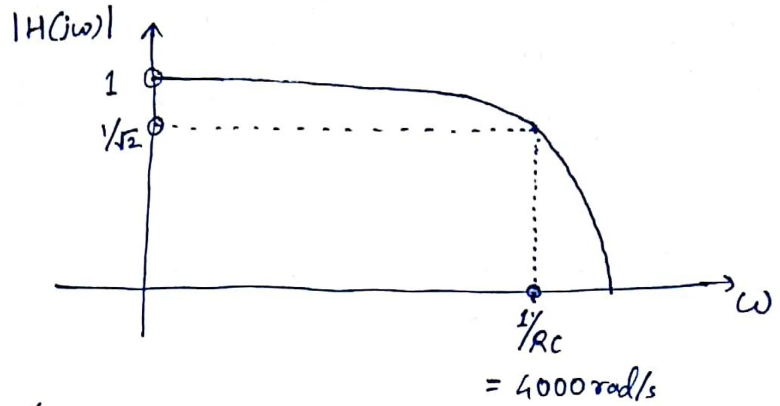
$$\therefore a_n = \frac{40}{n\pi}$$

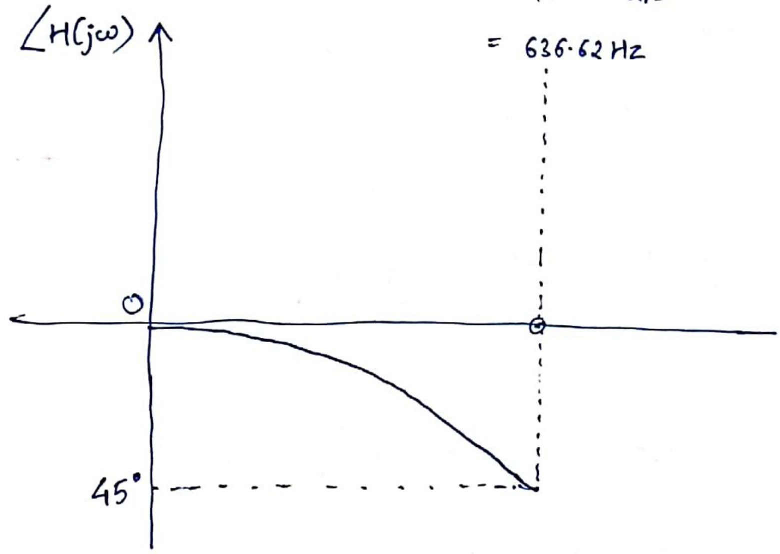
$$\therefore V_s(t) = \sum_{n=1,3,5\ldots} \left(\frac{40}{n\pi}\right) \sin(n\omega t)$$

$$\frac{V_o}{V_s} = \frac{1}{1+j\omega RC} \Rightarrow \cancel{V_o(t) = \left(\frac{40}{n\pi}\right) \times \frac{1}{}}$$

$$\therefore\ V_0(t) = \sum_{n=1,3,5\ldots} \left(\frac{40}{n\pi}\right)\frac{1}{\sqrt{n^2\omega^2R^2C^2+1}} \cdot \sin\left[n\omega t - \tan^{-1}(n\omega RC)\right]$$

Frequency response of $\frac{V_0}{V_S} = \frac{1}{1+j\omega RC}$ $\Rightarrow$





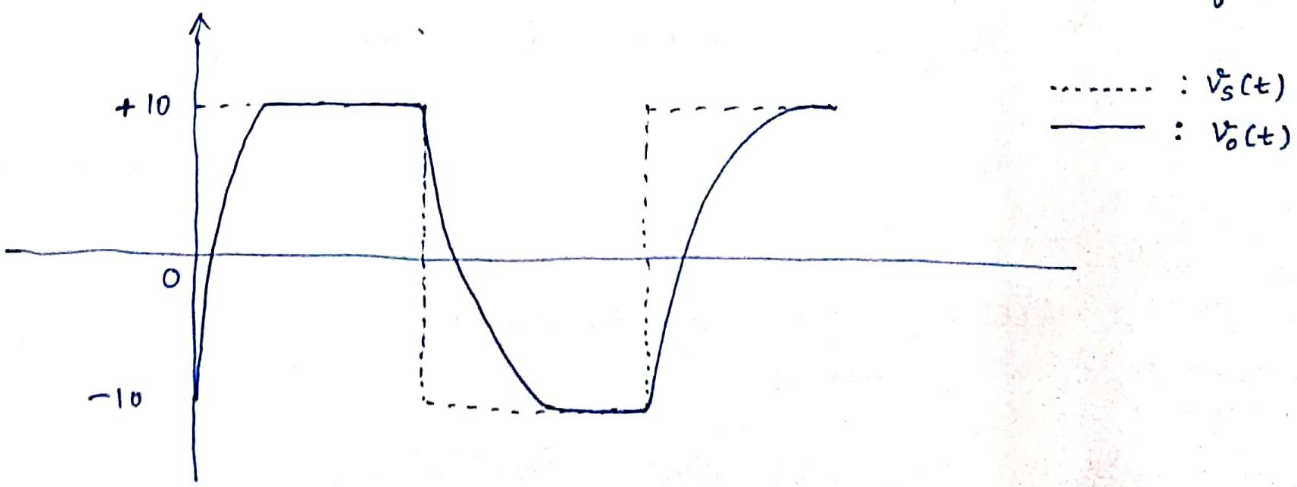
For C = 0.25 μF :

From frequency response of H(jω), we can see that it acts as a low pass filter with cut off frequency of 636.62 Hz .

$\therefore$ $V_0(t)$ will dominantly consists of frequency components upto 636 Hz . (since filter is not ideal, frequency components above 636 Hz will also be present, but they will not be as dominant) .

$\therefore$ $V_0(t)$ will dominant consist of 1^{st}, 3^{rd} and 5^{th} harmonic (i.e, 100 Hz, 300 Hz and 500 Hz) .

So the waveform of $V_0(t)$ will be closer to sinusoidal as compared to that of $V_S(t)$.



For $C = 50\mu F$,

$$\frac{V_o}{V_s} = \frac{1}{1 + j0.05\omega}$$

$$\omega RC = (\omega \times 10^3 \times 50 \times 10^{-6})$$

$$= 0.05\omega$$

For 100Hz, $\omega RC = 31.41$

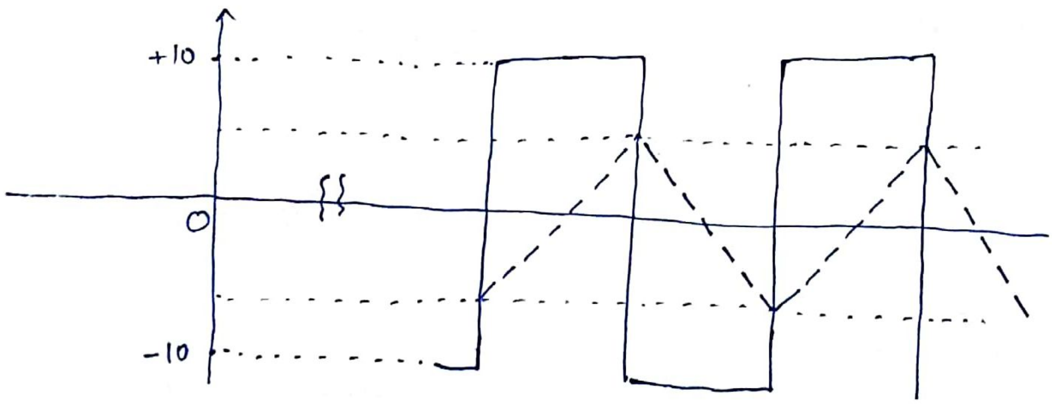
$\therefore$ For significant frequencies, we can write $\frac{V_o}{V_s} \simeq \frac{1}{j(\omega\, 0.05)}$

$$\therefore V_o = \frac{V_s}{j\omega \times 0.05}$$

$$\therefore V_o(t) \simeq \frac{1}{RC} \int V_s(\tau)\, d\tau$$

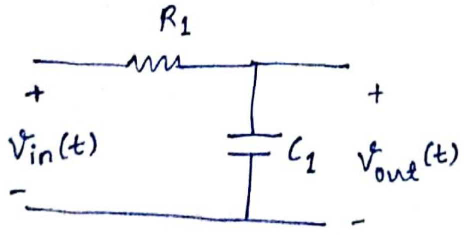
——— : $V_s(t)$

- - - - : $V_o(t)$



Problem (14):

(a)



Transfer Function : $\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{SC_1}}{R_1 + \frac{1}{SC_1}}$

$$\therefore \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + SR_1C_1}$$

$$\text{Gain} = \left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \left|\frac{1}{1+j\omega R_1 C_1}\right| = \frac{1}{\sqrt{\omega^2 R_1^2 C_1^2 + 1}}$$

At low frequencies ($\omega \to 0$) $\Rightarrow$ we have $G \simeq 1$

At high frequencies ($\omega \to \infty$) $\Rightarrow$ $G \simeq 0$

$\Rightarrow$ If $\omega >> R_1C_1$, then it is called High frequencies.

If $\omega << R_1C_1 \Rightarrow$ they are called as Low frequencies.

$$\Rightarrow \underset{(G)}{\text{Gain}} = \frac{1}{\sqrt{1+\omega^2 R_1^2 C_1^2}}$$

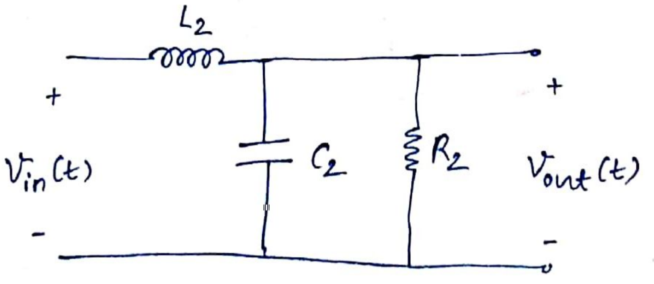
$$\frac{dG}{d\omega} = \frac{\omega R_1^2 C_1^2}{(1+\omega^2 R_1^2 C_1^2)^{3/2}}$$

$\Rightarrow$ At high frequencies (i.e $\omega \to \infty$) $\Rightarrow$ $\dfrac{dG}{d\omega} = \dfrac{\omega R_1^2 C_1^2}{\omega^3 R_1^3 C_1^3}$

$(\omega^2 R_1^2 C_1^2 >> 1)$

$$\therefore \frac{dG}{d\omega} \propto \frac{1}{\omega^2} \text{ at high frequencies.}$$

(b)



$$\text{Transfer Function} = \frac{V_{out}(s)}{V_{in}(s)}$$

$$= \frac{\left(\frac{1}{sC_2}\right) \| (R_2)}{sL_2 + \left[\left(\frac{1}{sC_2}\right) \| R_2\right]}$$

$$= \frac{R_2}{s^2 R_2 L_2 C_2 + sR_2C_2 + R_2}$$

$$\therefore \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s^2 L_2 C_2 + sC_2 + 1}$$

$$\Rightarrow \quad \frac{V_{out}(s)}{V_{in}(s)} = \frac{(1/L_2C_2)}{s^2 + \frac{s}{L_2} + \frac{1}{L_2C_2}}$$

$$\text{Gain } (G_2) = \left| \frac{V_{out}(j\omega)}{V_{in}(j\omega)} \right| = \left| \frac{1/L_2C_2}{(-\omega^2) + \left(\frac{\omega}{L_2}\right)j + \left(\frac{1}{L_2C_2}\right)} \right|$$

$$\therefore \; G_2 = \frac{1}{\sqrt{(1-\omega^2L_2C_2)^2 + \omega^2L_2^2}}$$

At low frequencies $\Rightarrow \omega \to 0 \Rightarrow G \approx 1$

At high frequencies $\Rightarrow \omega \to \infty \Rightarrow G \approx 0$

$\therefore$ If $\omega >> \frac{1}{\sqrt{L_2C_2}}$ $\Rightarrow$ then it is called high frequencies

If $\omega << \frac{1}{\sqrt{L_2C_2}}$ $\Rightarrow$ then it is called as Low frequencies.

$$\frac{dG_2}{d\omega} = \frac{d}{d\omega} \left| \frac{1}{(\omega^4L_2^2C_2^2 + \omega^2(C_2^2 - 2L_2C_2) + 1)^{1/2}} \right|$$

Take $a = L_2^2C_2^2$ & $b = C_2^2 - 2L_2C_2$

$$\therefore \; \frac{dG_2}{d\omega} = \frac{d}{d\omega} \left| \frac{1}{(a\omega^4 + b\omega^2 + 1)^{1/2}} \right|$$

$$\therefore \; \frac{dG_2}{d\omega} = \frac{2a\omega^3 + b\omega}{(a\omega^4 + b\omega^2 + 1)^{3/2}}$$

$\therefore$ At high frequencies, $\frac{dG_2}{d\omega} \propto \frac{1}{\omega^3}$

$\Rightarrow$ So, $\frac{dG_1}{d\omega} \propto \frac{1}{\omega^2}$ and $\frac{dG_2}{d\omega} \propto \frac{1}{\omega^3}$

$\therefore$ Filter in part (b) has lower gain at high frequencies as compared to filter in part (a). $\therefore$ ~~Fil~~ Filter in part (b) is better than that in part (a).

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