[SOLUTIONS] for Tutorial : 1

Q.1 Laplace Transform :

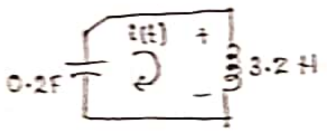
(a) $\frac{4}{(s+2)^2+4}$ (b) $\frac{2}{(s+2)^3}$ (c) $\frac{1}{s} - \frac{1}{s+10} = \frac{10}{s(s+10)}$

(d) $\frac{1}{s-2} + \frac{1}{s+3}$ (e) $\frac{4s}{(s^2+4)^2} + \frac{1}{s+2}$ (f) $\frac{2s-5}{(s-4)^2+9}$

(g) $\mathcal{L}[e^{t}\sin(5t)\,u(t)] = \underbrace{\frac{5}{(s-1)^2+25}}_{= G(s) \text{ let}}$ (h) $6/s^4$

$F(s) = \frac{d^2}{ds^2}[G(s)]$
(solve it)

Q.3 $i(0^-) = 20/2 = 10\text{ A}$ $L\frac{di(t)}{dt} + \frac{1}{C}\int_0^\infty i(t)\,dt = 0$



$\Rightarrow Ls\cdot I(s) - LI(0^-) + \frac{I(s)}{sC} + V_c(0^+)/s = 0$

$\therefore \boxed{I(s) = \frac{32s}{3.2s^2+5}}$

Q.5 Final values ($x(t), \dot{x}(t)$ as $t \to \infty$)

(a) 1, 0   (b) NA (poles on RHP)   (c) oscillating and hence F.V. does not exist.   (d) NA (unstable system) for any a.

(e) $-a/4$, 0   (f) NA (unstable system)

~~Q.6 Initial values (for f(0+), f'(0+))~~

~~(a) does exist (BUT NA)~~ ~~(b) (4, 1)~~ ~~(c) NA~~ ~~(d) (2, NA (for f'(0+)))~~

Q.7 IVP using Laplace :

$(s^2Y(s) - s) - (sY(s) - 1) - 6Y(s) = 2/s \Rightarrow Y(s) = \frac{s^2-s+2}{s^2-s-6}$

$= \frac{8/15}{s-3} + \frac{8/10}{s+2} - \frac{1/3}{s}$

$\therefore \boxed{y(t) = \frac{8}{15}e^{3t} + \frac{8}{10}e^{-2t} - 1/3}$

Q.9 $x(t) = e^{-2t}u(t) + \delta(t-6)$, $h(t) = u(t)$; $y(t) = x(t) * h(t)$

so $Y(s) = X(s)\cdot H(s) = \left[\frac{1}{s+2} + e^{-6s}\right]1/s \Rightarrow Y(s) = \frac{0.5}{s} - \frac{0.5}{s+2} + \frac{e^{-6s}}{s}$

$\therefore \boxed{y(t) = 0.5\left(1-e^{-2t}\right)u(t) + u(t-6)}$

Q.10

- $\frac{-V_d - V_{in}}{1/sC} - \frac{V_d}{R_{in}} - \frac{V_d - V_{out}}{R \parallel sL} = 0$
- $\frac{V_{out} + V_d}{R \parallel sL} + \frac{V_{out} - AV_d}{R_o} = 0$

$\Rightarrow V_{out}\left[\frac{1}{R \parallel sL} + \frac{1}{R_o}\right] = V_d\left[\frac{A}{R_o} - \frac{1}{R \parallel sL}\right]$

Hence, $V_d = V_{out}\underbrace{\left[\frac{R_o(R+sL) + RsL}{ARsL - R_o(R+sL)}\right]}_{R_T}$

or $V_d = V_{out} R_T$

- $\dfrac{V_{out}}{R \| sL} + \left[sC + \dfrac{1}{R_{in}} + \dfrac{1}{R\| sL} \right] V_d = -V_{in}(sC)$; put $V_d = V_{out} \cdot R_T$

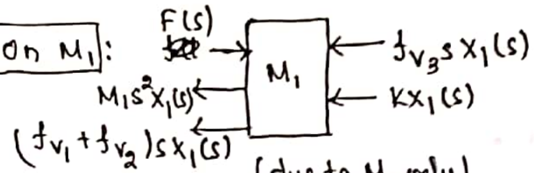
$\Rightarrow V_{out}\left[R_T\left(sC + 1/R_{in} + \dfrac{1}{R\|sL}\right) + \dfrac{1}{R\|sL} \right] = -V_{in} \cdot sC$

$\therefore \dfrac{V_{out}(s)}{V_{in}(s)} = -\dfrac{sC}{R_T\left[sC + \dfrac{1}{R_{in}} + \dfrac{1}{R\|sL}\right] + \dfrac{1}{R\|sL}}$; ideal case: $A \to \infty$, $R_o \to 0$, $R_{in} \to \infty$ so $R_T \to 0$,

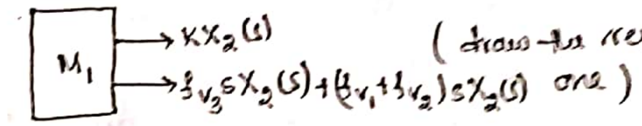
$$\left[TF = \frac{-s^2 LC}{1 + sL/R} \right]$$

$\boxed{Q.1}$ (a) Free body diagram & differential eqⁿ

- $\boxed{\text{On } M_1}$:



(due to M_1 only)

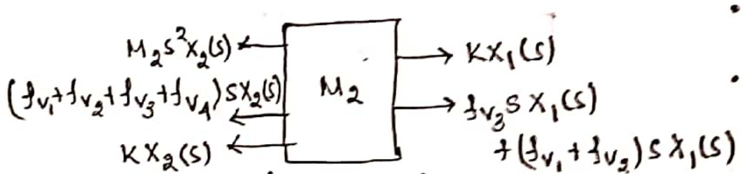


(draw the resultant one)

(due to M_2 only)

- $M_1 = 1.5$, $M_2 = 3.1$ (in Kg)
- $f_{v_1}, f_{v_2}, f_{v_3}, f_{v_4}$: 2, 1, 2, 1.1 (in Ns/m)
- $K = 1.2$ N/m.

- $\boxed{\text{On } M_2}$



(resultant)

dE:
- $[1.5\ddot{x}_1(t) + 5\dot{x}_1(t) + 1.2x_1(t)] - [5\dot{x}_2(t) + 1.2x_2(t)] = f(t)$ (at M_1)
- $[3.1\ddot{x}_2(t) + 6.1\dot{x}_2(t) + 1.2\,x_2(t)] - [5\dot{x}_1(t) + 1.2x_1(t)] = 0$ (at M_2)

(b) $G(s) = \dfrac{X_2(s)}{F(s)} = \dfrac{5s + 1.2}{s(4.65s^3 + 24.65s^2 + 11.02s + 1.32)}$

(c) Poles at: $0, -4.879, -0.239 \pm 0.038j$ (complex conj.) ; zero at: -0.24
(4 poles) (1 zero)

$\boxed{Q.2}$
- $\dfrac{V_{in}(s)}{I(s)} = \dfrac{3s^2 + 14.5s + 2.5}{s^2 + 6s + 0.5}$

- $G(s) = \dfrac{V_{out}(s)}{V_{in}(s)} = \dfrac{2s(s+4)}{s^2 + 6s + 0.5}$

$\boxed{Q.6}$ Initial values (consider $\delta(0^+) = 0$)

(a) $f(0^+) = 3$, $f'(0^+) = -1$

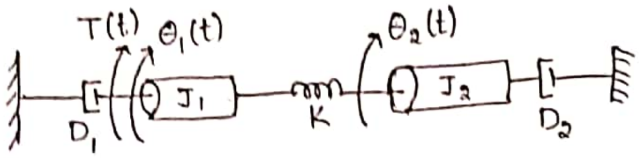
(b) $f(0^+) = 0$, $f'(0^+) = 2$

(c) $f(0^+) = 3 - a$, $f'(0^+) = a^2 - 3a - 2$

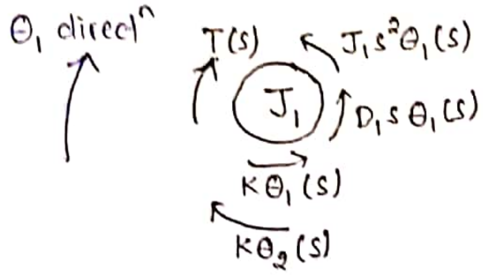
(d) $f(0^+) = a$, $f'(0^+) = 5 - 3a$

[Contd. (Q.6)]

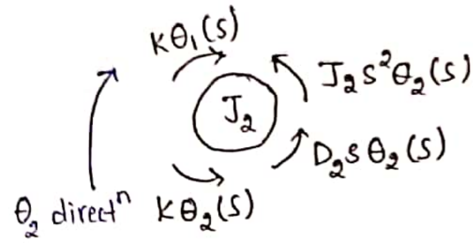
Q.8



- free body diagram: (resultant)



(for J_1)

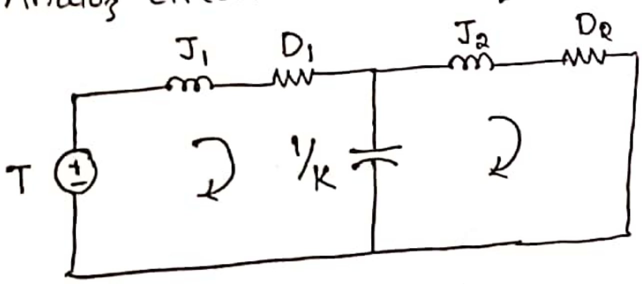


(for J_2)

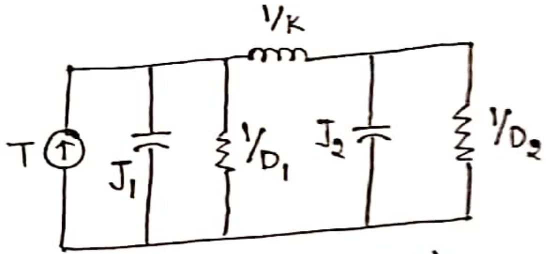
- differential equation:
  - $(J_1 s^2 + D_1 s + K)\,\theta_1(s) - K\theta_2(s) = T(s)$
  - $-K\theta_1(s) + (J_2 s^2 + D_2 s + K)\,\theta_2(s) = 0$

- Transfer function: $\dfrac{\theta_2(s)}{T(s)} = \dfrac{K}{\Delta}$, where $\Delta = \begin{bmatrix} J_1 s^2 + D_1 s + K & -K \\ -K & J_2 s^2 + D_2 s + K \end{bmatrix}$

- Analog circuits: (use torque-voltage and torque-current analogy)



(series analog)



(parallel analog)