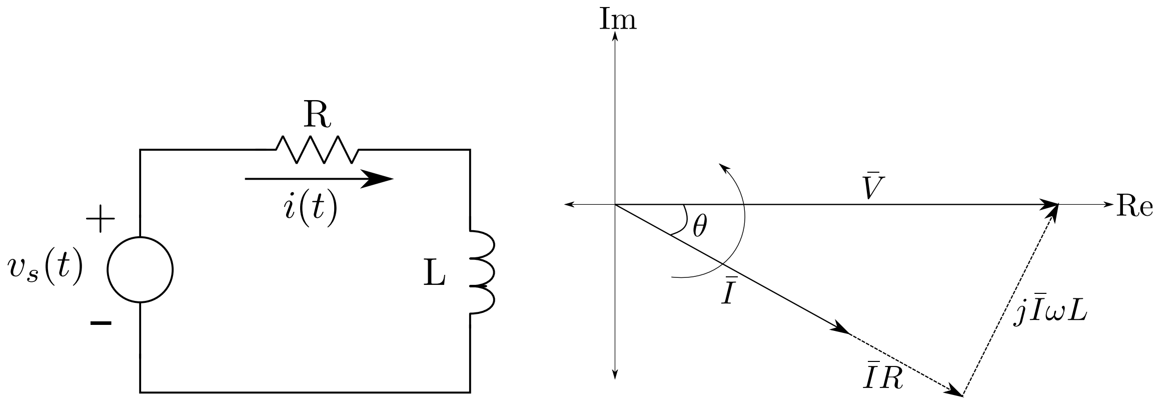
Tutorial 1: A Review of AC Circuit Analysis and Frequency Response Analysis

---

Problem 1 A phasor diagram for a series R-L circuit (shown in Figure 1a) is shown in Figure 1b. The phasor diagram has been drawn by taking $\sin(2\pi 50t)$ as the reference phasor ($1\angle 0°$). If $\sin(2\pi\, 50\,(t - 0.005))$ is taken as the reference phasor, then redraw the phasor diagram with respect to the new reference.



(a) Circuit for Problem 1

(b) Phasor diagram for Problem 1

Figure 1: Circuit and phasor diagram for Problem 1

Note: The angle of a phasor is measured as positive in the anti-clockwise direction, as shown in Figure 1b.

Problem 2 A capacitor C is connected as shown in Fig. 2a. The input voltage source $v_s(t)$ is

$$v_s(t) = 100\sin(2\pi \times 50t)\ \text{V}$$

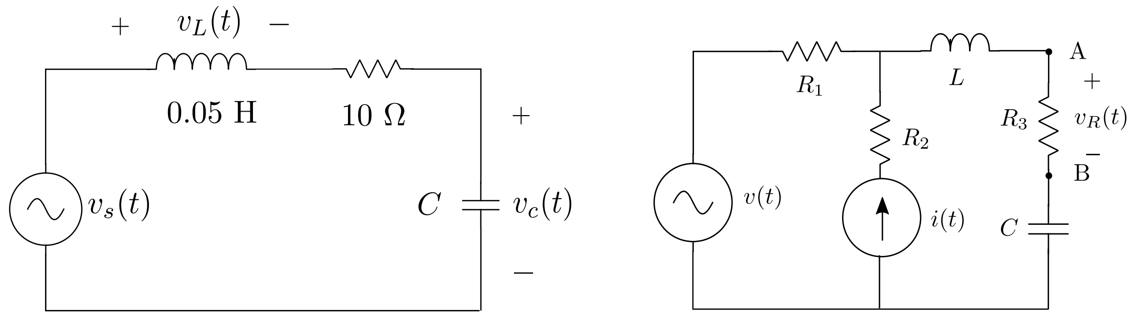
(i) Evaluate the expression of the sinusoidal steady state current $i(t)$ for C = 100 μF.

(ii) Find the value of C that will maximise the amplitude of the steady state current $i(t)$. Write the expressions for $v_L(t)$ and $v_C(t)$ in steady state under this condition. Comment on the amplitudes of $v_c(t)$ and $v_L(t)$ with reference to that of $v_s(t)$.

Problem 3 For the circuit shown in Fig. 2b, the parameters are as follows:

$$v(t) = 50\sin(2\pi \times 50t + 30°)\ \text{V}, \quad i(t) = 10\sin(2\pi \times 50t - 10°)\ \text{A}$$

and R_1 = 5 Ω, R_2 = 3 Ω, L = 10 mH, R_3 = 2 Ω and C = 0.8 mF. Find the steady state expression $v_R(t)$ of the voltage across the resistor R_3.



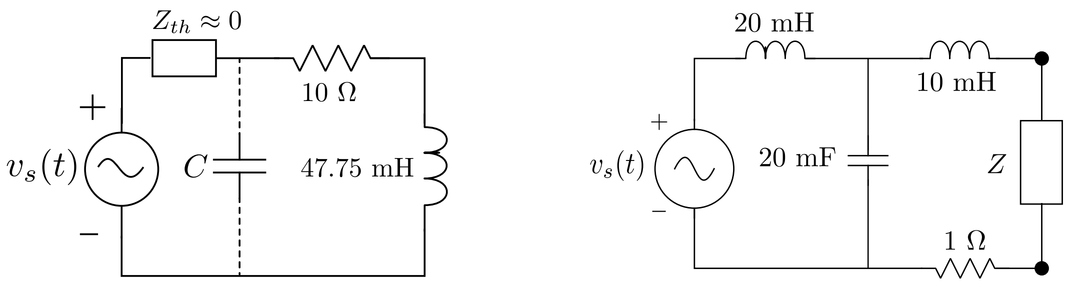
(a) Circuit for Problem 2 (b) Circuit for Problem 3

Figure 2: Circuits for Problem 2 and 3

Problem 4 For the circuit shown in Figure 3a, the voltage source $v_s(t)$ is given as

$$v_s(t) = 100 \sin(2\pi \times 50t) \text{ V}$$

Initially the capacitor C is not connected. For the given values of R and L, find the power factor of the R-L load. Find the value of the capacitor C required to improve the power factor as seen by the source to 0.95 (lag).



(a) Circuit for Problem 4 (b) Circuit for Problem 5

Figure 3: Circuits for Problem 4 and 5

Problem 5 For the circuit shown in Figure 3b, the voltage source $v_s(t)$ is given as

$$v_s(t) = 50 \sin(2\pi \times 50t + 45°) \text{ V}.$$

Find the value of the impedance Z that will extract maximum real power from the source.

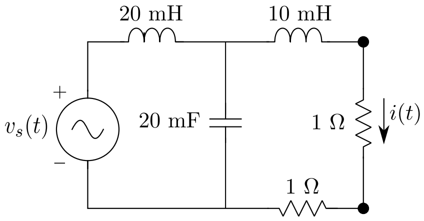
Problem 6 For the circuit shown in Figure 4a, the expression of the voltage source $v_s(t)$ is

$$v_s(t) = 50 \sin(100t - 25°) \text{ V}.$$

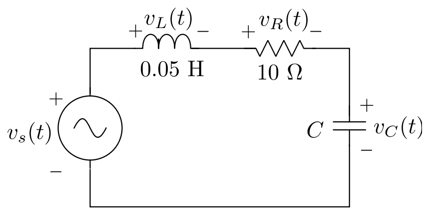
Find the steady state expression of $i(t)$ by using (i) KCL and KVL applied to the two loops, and (ii) Thevenin equivalent circuit.

Problem 7 For the circuit shown in Figure 4b, the expression of the voltage source $v_s(t)$ is

$$v_s(t) = 100 \sin(2\pi \times 50t + 45°) \text{ V}.$$



(a) Circuit for Problem 6



(b) Circuit for Problem 7

Figure 4: Circuits for Problem 6 and 7

Find the steady state expressions of the current $i(t)$, inductor voltage $v_L(t)$, capacitor voltage $v_C(t)$ and the resistor voltage $v_R(t)$ for C = 150 μF, 202.85 μF and 250 μF. Draw the phasor diagrams of the aforementioned quantities for the different values of C.

Problem 8 For the circuit shown in Fig. 5, consider the initial inductor current is zero. The AC voltage source $v_s(t)$ is given as

$$v_s(t) = 100 \sin(2\pi \times 50t + \alpha) \text{ V}$$

- If the switch S is closed at time $t = 0$, find the initial phase of the voltage source α so that there is no natural transient present in the current $i(t)$. Hence, find the expression of the current $i(t)$ in **steady state**.
- For α = 0, find the time instant t ($0 \leq t \leq 10$ ms) at which the switch S should be closed so that there is no natural transient present in the current $i(t)$.

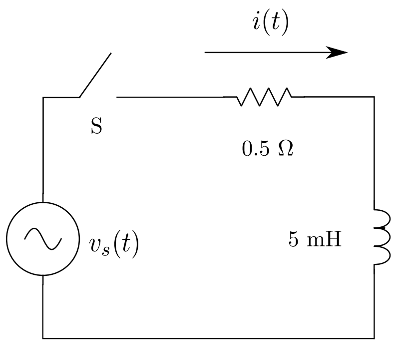
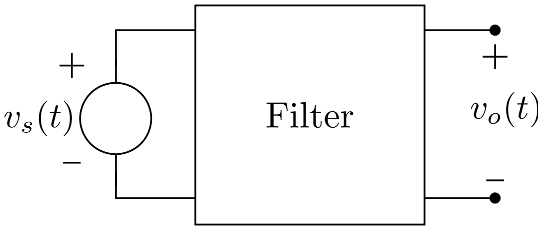


Figure 5: Circuit for Problem 8

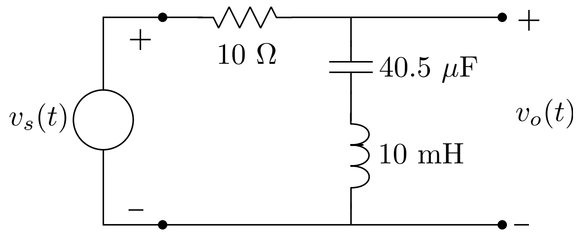
Problem 9 For the circuit shown in Figure 6a, the expression of the voltage source $v_s(t)$ is

$$v_s(t) = 50 \sin(2\pi \times 50t + 30°) + 10 \sin(2\pi \times 500t) \text{ V}$$

(a) A low pass filter is to be designed to eliminate the 10^{th} harmonic component. Design a first-order filter for this purpose. Realize the filter using R-C components and comment on the nature of the frequency response of the filter. Write down the steady state expression of the filtered output voltage $v_o(t)$.
(b) The filtered output is to be given as an input to a resistive load R. What is the overall transfer function when a resistance R of (i) 200 Ω, (ii) 20 kΩ is connected at the filter output? Comment on the nature of the modification of the transfer function in each case.



(a) Circuit for Problem 9



(b) Circuit for Problem 10

Figure 6: Circuits for Problem 9 and 10

Problem 10 For the circuit shown in Figure 6b, the expression of the voltage source $v_s(t)$ is

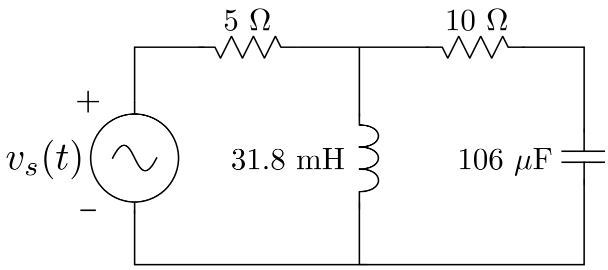
$$v_s(t) = 50\sin(2\pi \times 50t - 3^\circ) + 10\sin(2\pi \times 250t) + 5\sin(2\pi \times 750t + 15^\circ) \text{ V}.$$

Evaluate the transfer function between the input $v_s(t)$ and the output $v_o(t)$. Comment on the nature of the frequency response. Evaluate the expression of the output voltage $v_o(t)$ in sinusoidal steady state.

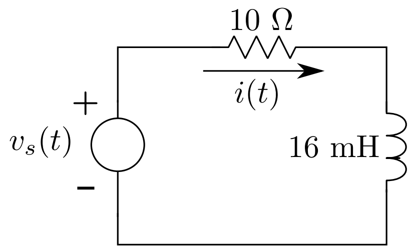
Problem 11 For the circuit shown in Figure 7a, the voltage source $v_s(t)$ is given as

$$v_s(t) = 100\sin(2\pi \times 50t + 30^\circ) \text{ V}$$

Calculate the steady state branch voltages and currents. If the branch voltage and branch current of the i^{th} branch be $\bar{V}_k$ and $\bar{I}_k$, then show that $\Sigma \bar{V}_k \bar{I}_k^* = 0$ over the entire network.



(a) Circuit for Problem 11



(b) Circuit for Problem 12

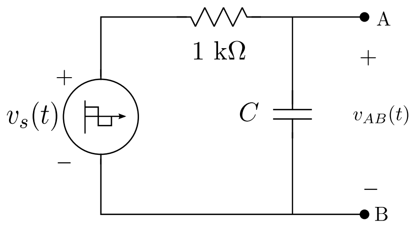
Figure 7: Circuits for Problems 11 and 12

Problem 12 For the circuit shown in Figure 7b, the voltage source $v_s(t)$ is given by

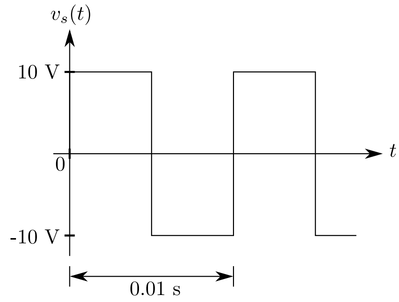
$$v_s(t) = 5 + 10\sin(2\pi \times 50t + 30^\circ) + 3\sin(2\pi \times 150t - 60^\circ) + 0.5\sin(2\pi \times 300t) \text{ V}$$

Find the average power dissipated in the 10 Ω resistance in steady state.

Problem 13 For the circuit shown in Figure 8a, the voltage source $v_s(t)$ is a square wave of frequency 100 Hz, as shown in Figure 8b. For $C = 0.25\ \mu$F and $C = 50\ \mu$F and assuming initial capacitor voltage $v_c(0) = 0$ V, determine the transfer function between $v_{AB}(t)$ and $v_s(t)$. Find the nature of response and the steady state expression of $v_{AB}(t)$ for the different values of C.



(a) Circuit for Problem 13



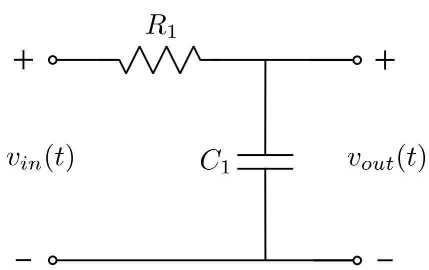
(b) Voltage waveform $v_s(t)$

Figure 8: Circuit and voltage waveform for Problem 13

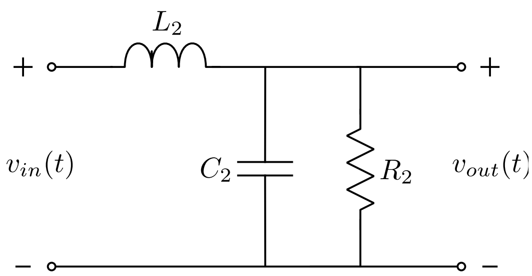
Problem 14 Two low-pass filters are as shown in Figure 9. Compute the following things for each of the filters:

- transfer function between the output voltage and the input voltage,
- low frequency and high frequency gain,
- characterisation of low frequency and high frequency,
- gain sensitivity $\frac{dG}{d\omega}$ at high frequency.

Based on the results obtained, which one will you prefer to use as a low-pass filter?



(a)



(b)

Figure 9: Low pass filters