EE 334
Response of LTI Circuits to AC and DC Inputs

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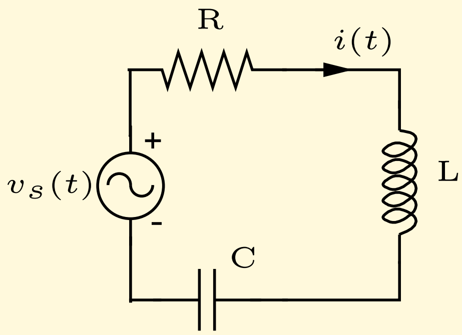
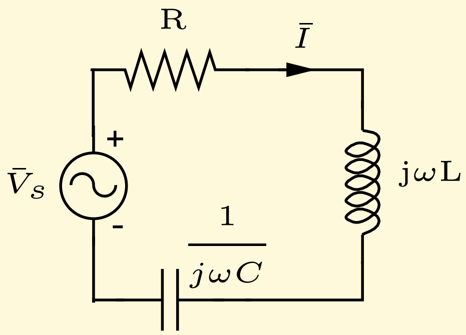
1. Phasors with sine as reference

Considering peak value ...

Time-domain circuit



Phasor equivalent circuit



$$\mathbf{v_s(t) = 100}\ \sin(\mathbf{2\pi 50t})\ \mathrm{V} \quad \rightarrow \quad \mathbf{\bar{V}_s = 100\angle 0^\circ}\ \mathrm{V}$$

R = 10 Ω, L = 0.05 H, C = 100 μF

Applying KVL:

$$\bar{I} = \frac{\bar{V}_s}{\left(R + j\omega\, L + \frac{1}{j\omega\, C}\right)}$$

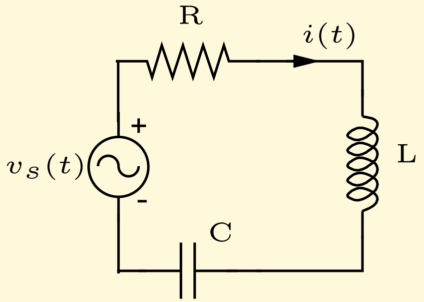
$$\mathbf{i(t) = 5.27}\,\sin(\mathbf{2\pi 50t + 58.2^\circ})\ \mathrm{A} \quad \leftarrow \quad \mathbf{\bar{I} = 5.27\angle 58.2^\circ}\ \mathrm{A}$$

This is sinusoidal steady-state solution.

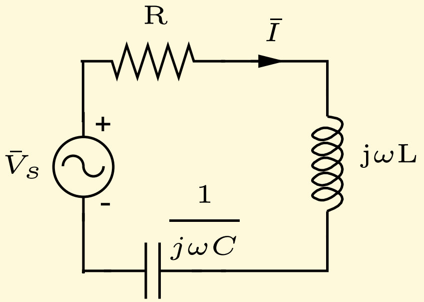
2. Phasors with cosine as reference

Considering peak value ...

Time-domain circuit



Phasor equivalent circuit



$\mathbf{v_s(t) = 100}\ \sin(\mathbf{2\pi 50 t})$ V $\quad \rightarrow \quad \bar{\mathbf{V}}_\mathbf{s} = \mathbf{100}\angle\mathbf{-90^\circ}$ V

Or $\mathbf{v_s(t) = 100}\ \cos(\mathbf{2\pi 50 t - 90^\circ})$ V

R = 10 Ω, L = 0.05 H, C = 100 μF

Applying KVL:

$$\bar{I} = \frac{\bar{V}_s}{\left(R + j\omega L + \frac{1}{j\omega C}\right)}$$

$\mathbf{i(t)} = \mathbf{5.27}\cos(\mathbf{2\pi 50 t - 31.8^\circ})$ A $\quad \leftarrow \quad \bar{\mathbf{I}} = \mathbf{5.27}\angle\mathbf{-31.8^\circ}$ A

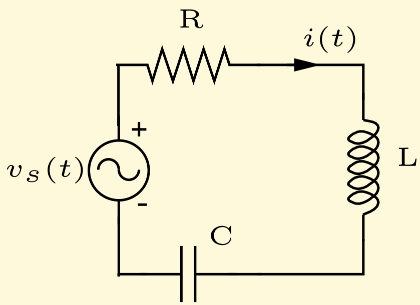
Or $\mathbf{i(t)} = \mathbf{5.27}\sin(\mathbf{2\pi 50 t + 58.2^\circ})$ A

This is sinusoidal steady-state solution.

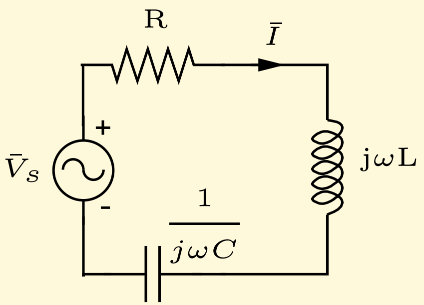
3. Phasors with cosine as reference

Considering RMS value ...

Time-domain circuit



Phasor equivalent circuit



$\mathbf{v_s(t)} = \mathbf{100}\ \sin(\mathbf{2\pi 50t})$ V $\quad\rightarrow\quad$ $\mathbf{\bar{V}_s} = \mathbf{70.71}\angle\mathbf{-90^\circ}$ V

Or $\mathbf{v_s(t)} = \mathbf{100}\ \cos(\mathbf{2\pi 50t - 90^\circ})$ V

R = 10 Ω, L = 0.05 H, C = 100 μF

Applying KVL:

$$\bar{I} = \frac{\bar{V}_s}{\left(R + j\omega\, L + \frac{1}{j\omega C}\right)}$$

$\mathbf{i(t)} = \mathbf{5.27}\cos(\mathbf{2\pi 50t - 31.8^\circ})$ A $\quad\leftarrow\quad$ $\mathbf{\bar{I}} = \mathbf{3.727}\angle\mathbf{-31.8^\circ}$ A

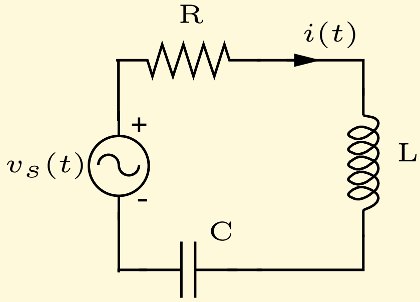
Or $\mathbf{i(t)} = \mathbf{5.27}\sin(\mathbf{2\pi 50t + 58.2^\circ})$ A

This is sinusoidal steady-state solution.

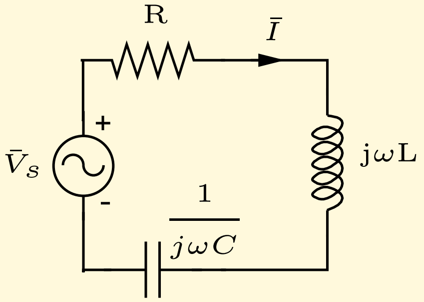
4. Phasors with sine as reference

Considering RMS value ...

Time-domain circuit



Phasor equivalent circuit



$\mathbf{v_s(t) = 100}\ \sin(\mathbf{2\pi 50 t})$ V $\quad\rightarrow\quad$ $\mathbf{\bar{V}_s} = \mathbf{70.71}\angle\mathbf{0}^\circ$ V

R = 10 Ω, L = 0.05 H, C = 100 μF

Applying KVL:

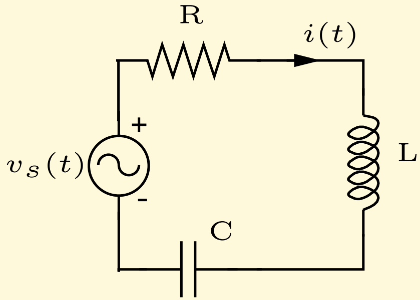
$$\bar{I} = \frac{\bar{V}_s}{\left(R + j\omega L + \frac{1}{j\omega C}\right)}$$

$\mathbf{i(t)} = \mathbf{5.27}\sin(\mathbf{2\pi 50 t} + \mathbf{58.2}^\circ)$ A $\quad\leftarrow\quad$ $\mathbf{\bar{I}} = \mathbf{3.727}\angle\mathbf{58.2}^\circ$ A

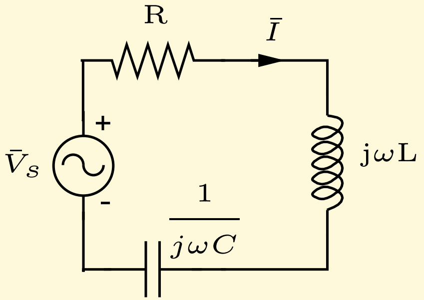
This is sinusoidal steady-state solution.

Phasors with sine as reference

Time-domain circuit

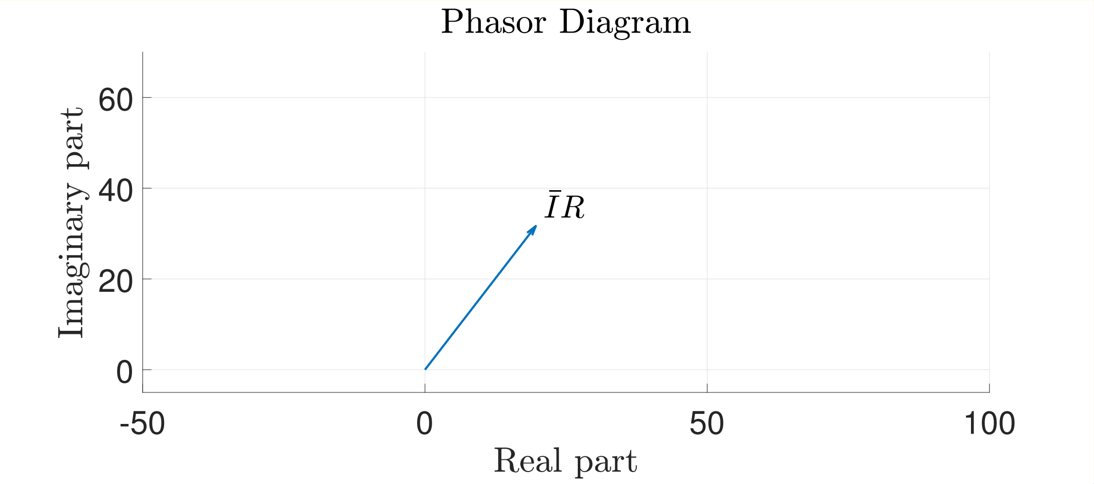


Phasor equivalent circuit



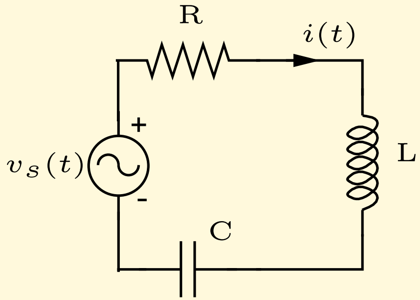
$$\mathbf{v_s(t)} = \mathbf{100}\,\sin(\mathbf{2\pi 50t})\ \mathrm{V} \quad \rightarrow \quad \bar{\mathbf{V}}_{\mathbf{s}} = \mathbf{70.71\angle 0^\circ}\ \mathrm{V}$$

$$\mathbf{i(t)} = \mathbf{5.27}\,\sin(\mathbf{2\pi 50t} + \mathbf{58.2^\circ})\ \mathrm{A} \quad \rightarrow \quad \bar{\mathbf{I}} = \mathbf{3.727\angle 58.2^\circ}\ \mathrm{A}$$

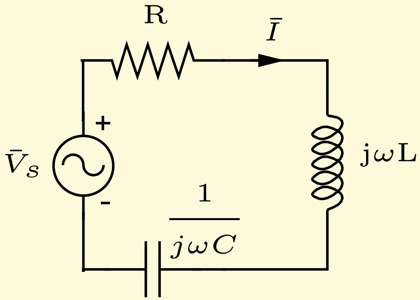


Phasors with sine as reference

Time-domain circuit



Phasor equivalent circuit



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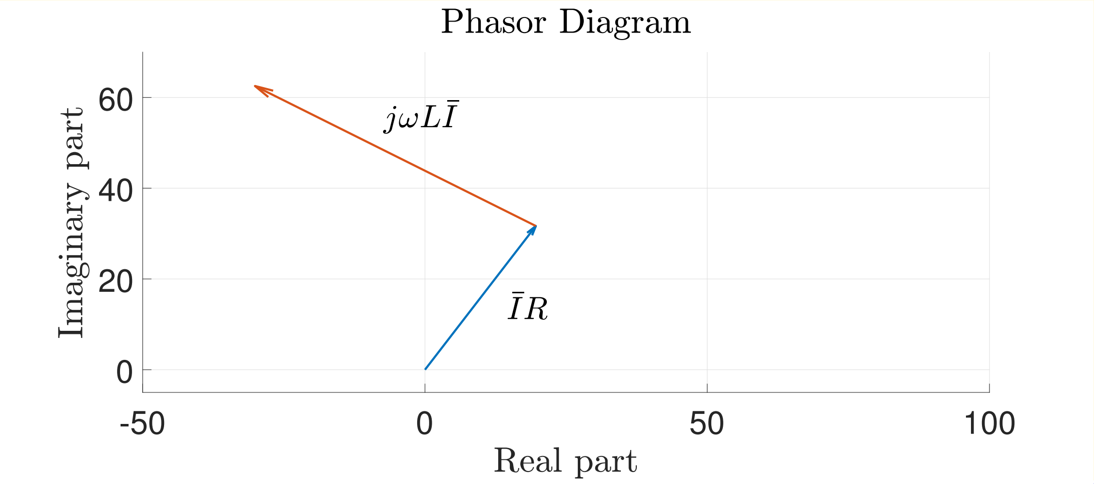
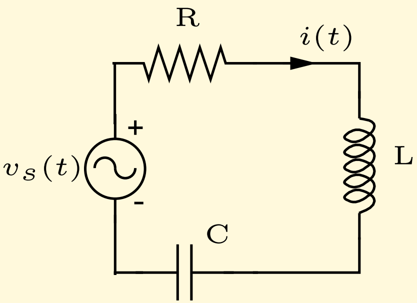
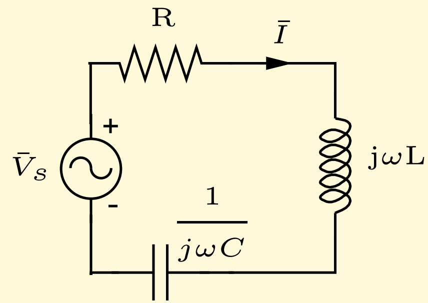


Phasors with sine as reference

Time-domain circuit



Phasor equivalent circuit



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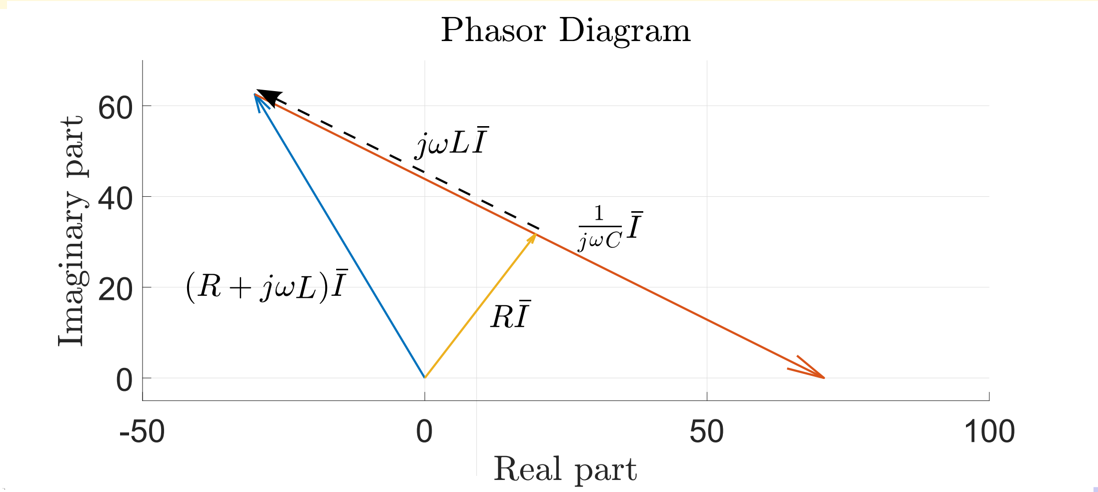
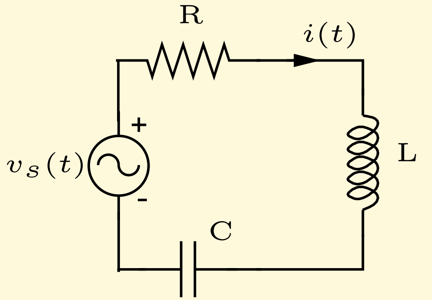
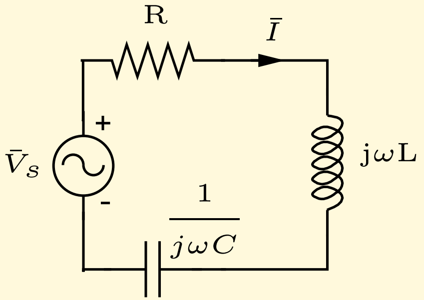


Phasors with sine as reference

Time-domain circuit



Phasor equivalent circuit



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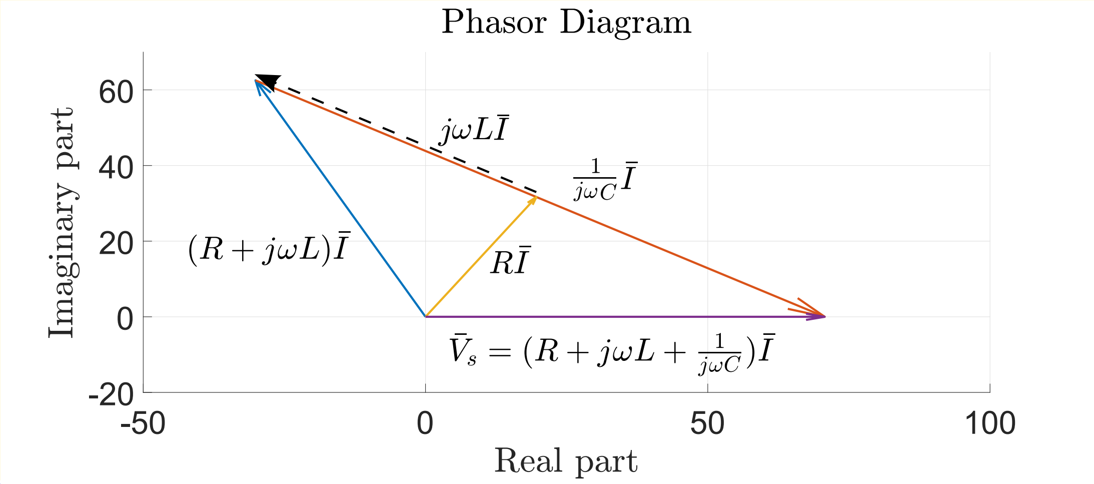


5. What is Reactive Power?

Consider a single phase system with

$v_s(t) = V_m \sin(\omega t)$ and $i(t) = I_m \sin(\omega t - \phi)$:

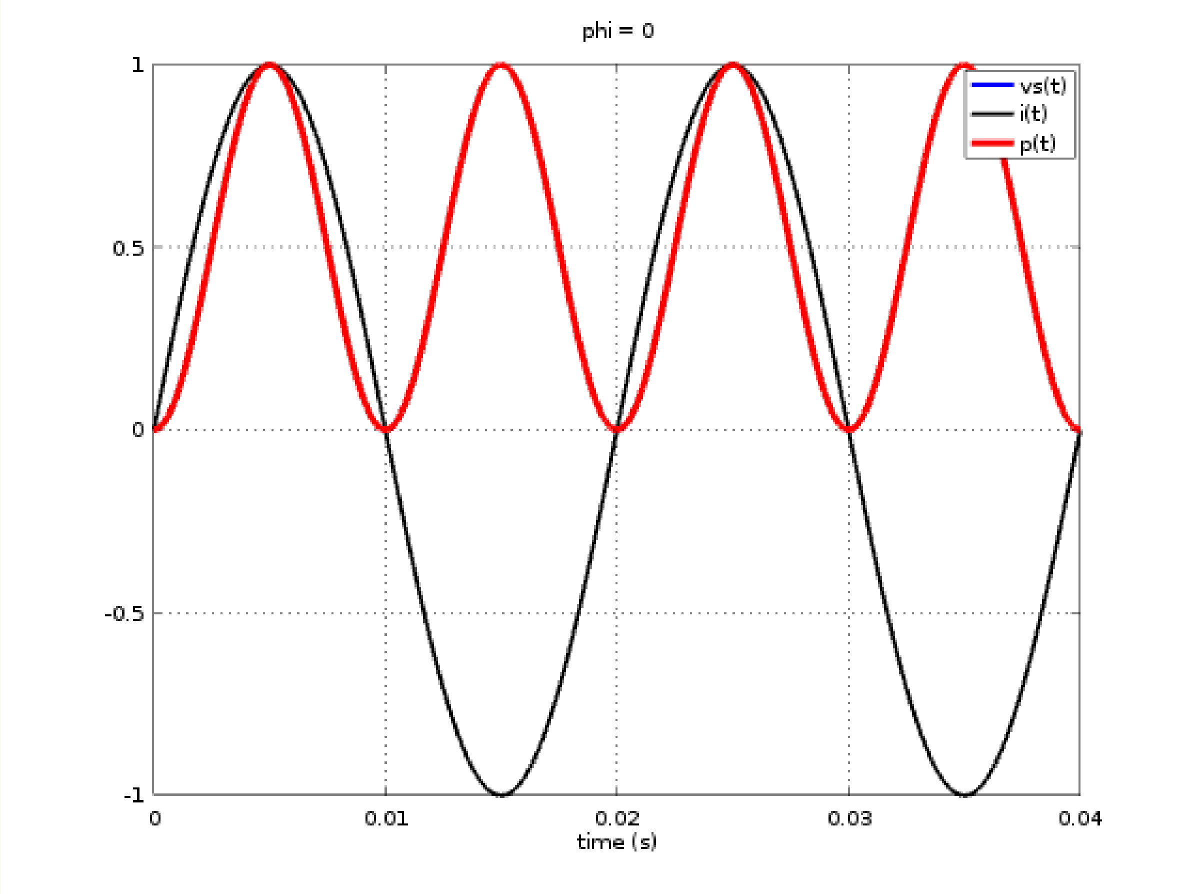
Then the **instantaneous power** is given as:

$$
\begin{aligned}
p(t) &= v_s(t) \times i(t) \\
p(t) &= V_m\, I_m\, \sin(\omega t)\, \sin(\omega t - \phi) \\
&= \frac{1}{2} V_m I_m [\cos(\phi)(1 - \cos(2\omega\, t)) - \sin(\phi) \sin(2\omega\, t)]
\end{aligned}
$$

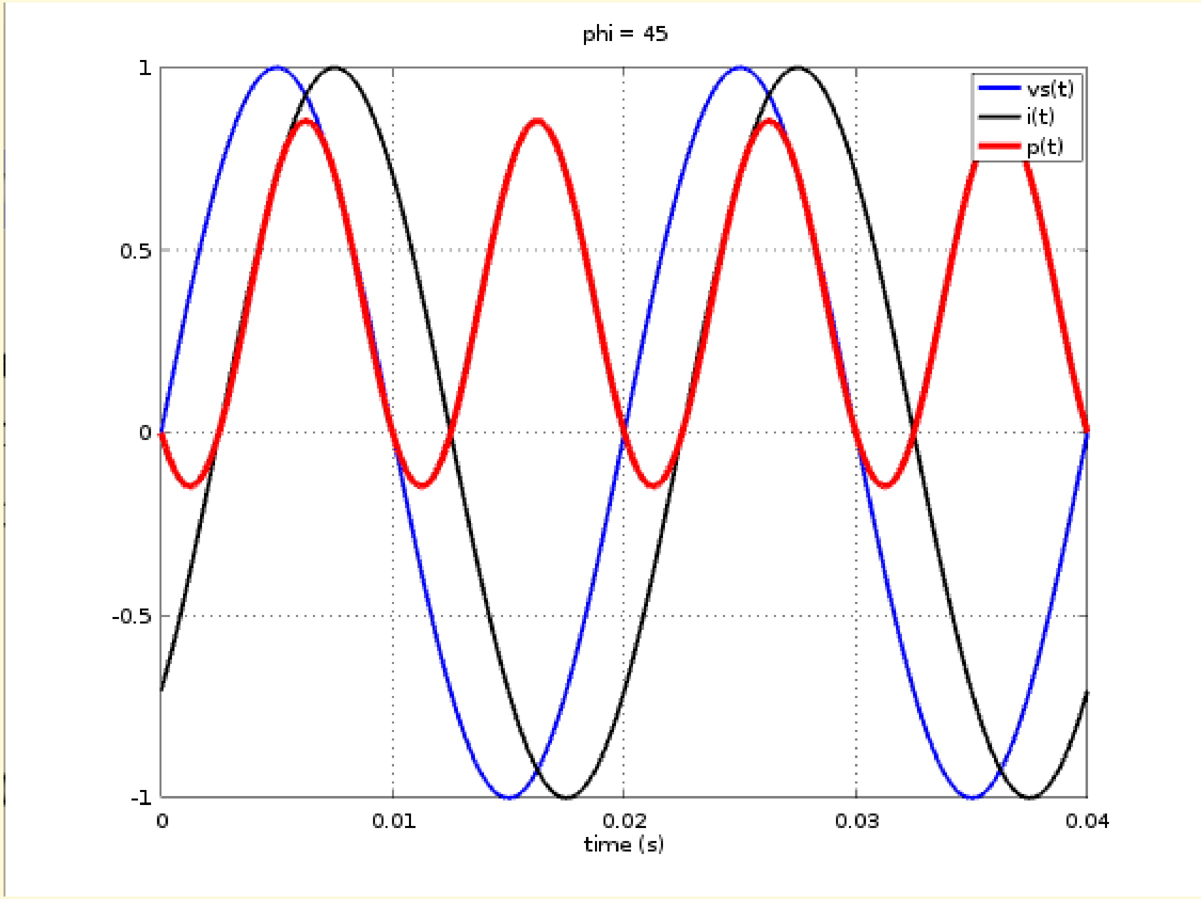
Real Power (or Average P) = $\frac{1}{2} V_m I_m \cos(\phi)$

Reactive Power (Q) = $\frac{1}{2} V_m I_m \sin(\phi)$

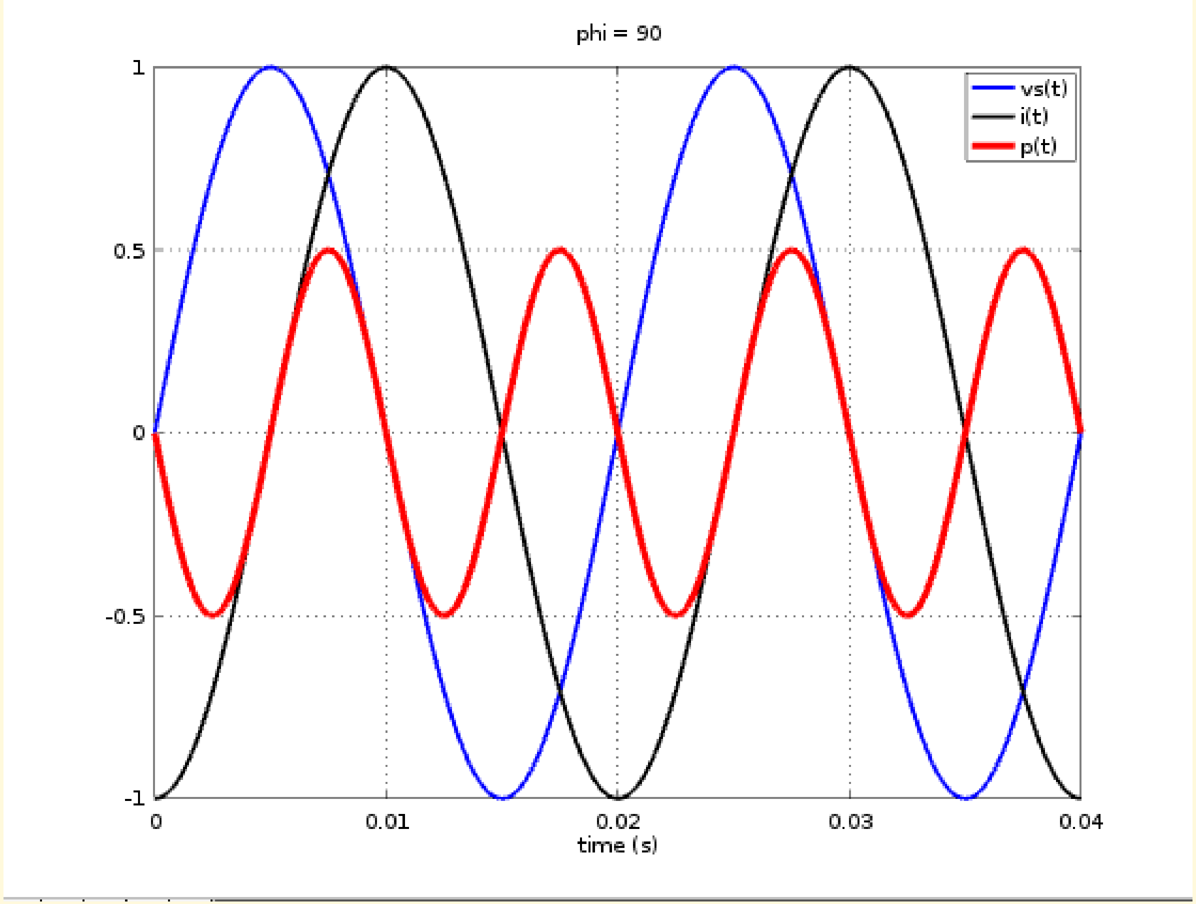
Instantaneous power for $\phi = 0^\circ$:



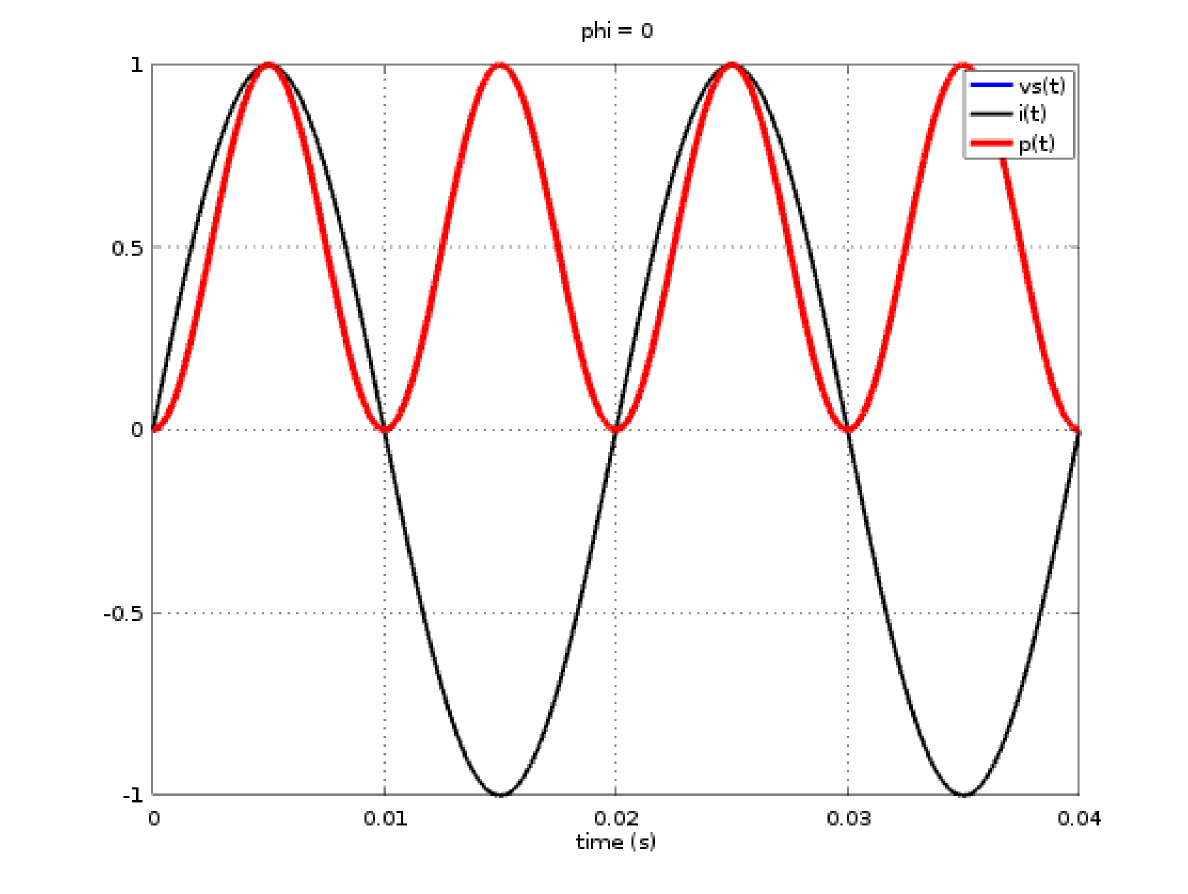
Instantaneous power for $\phi = 45^\circ$:



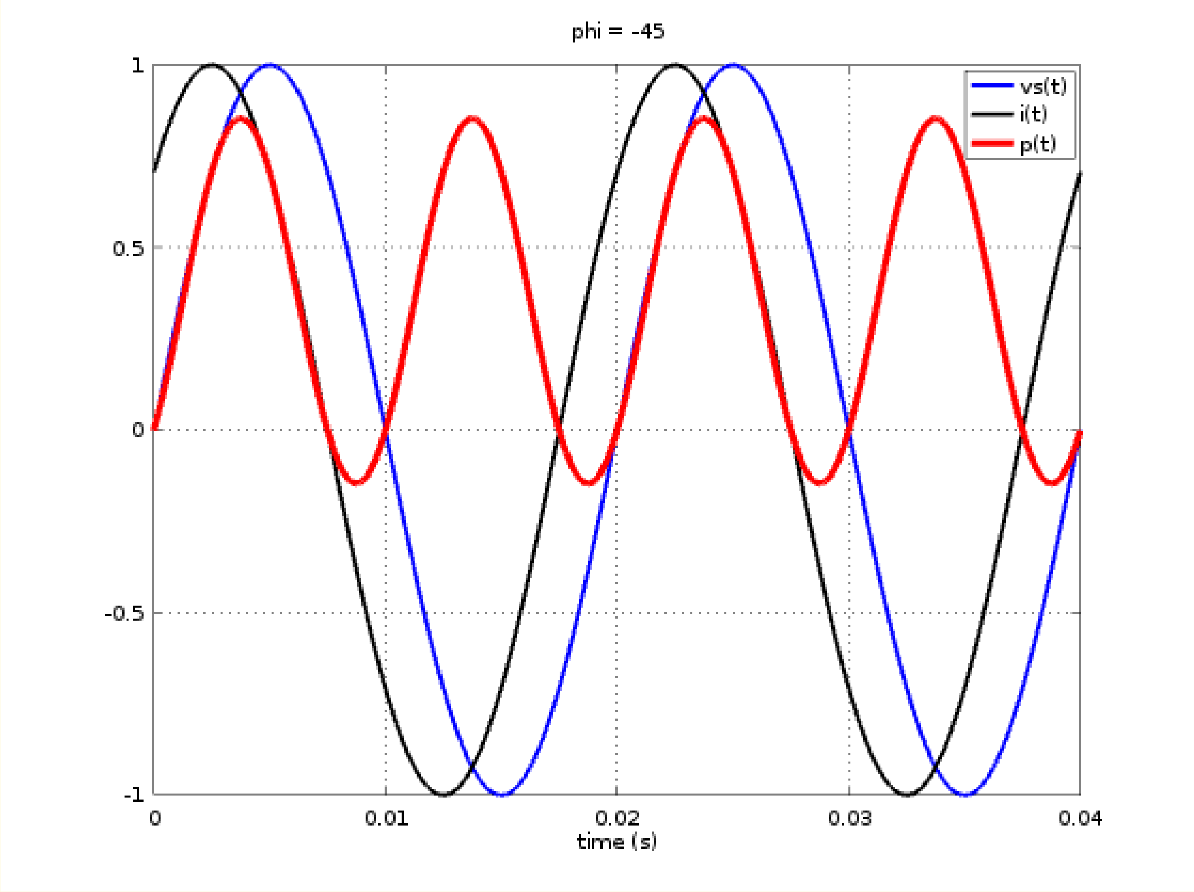
Instantaneous power for $\phi = 90^\circ$:



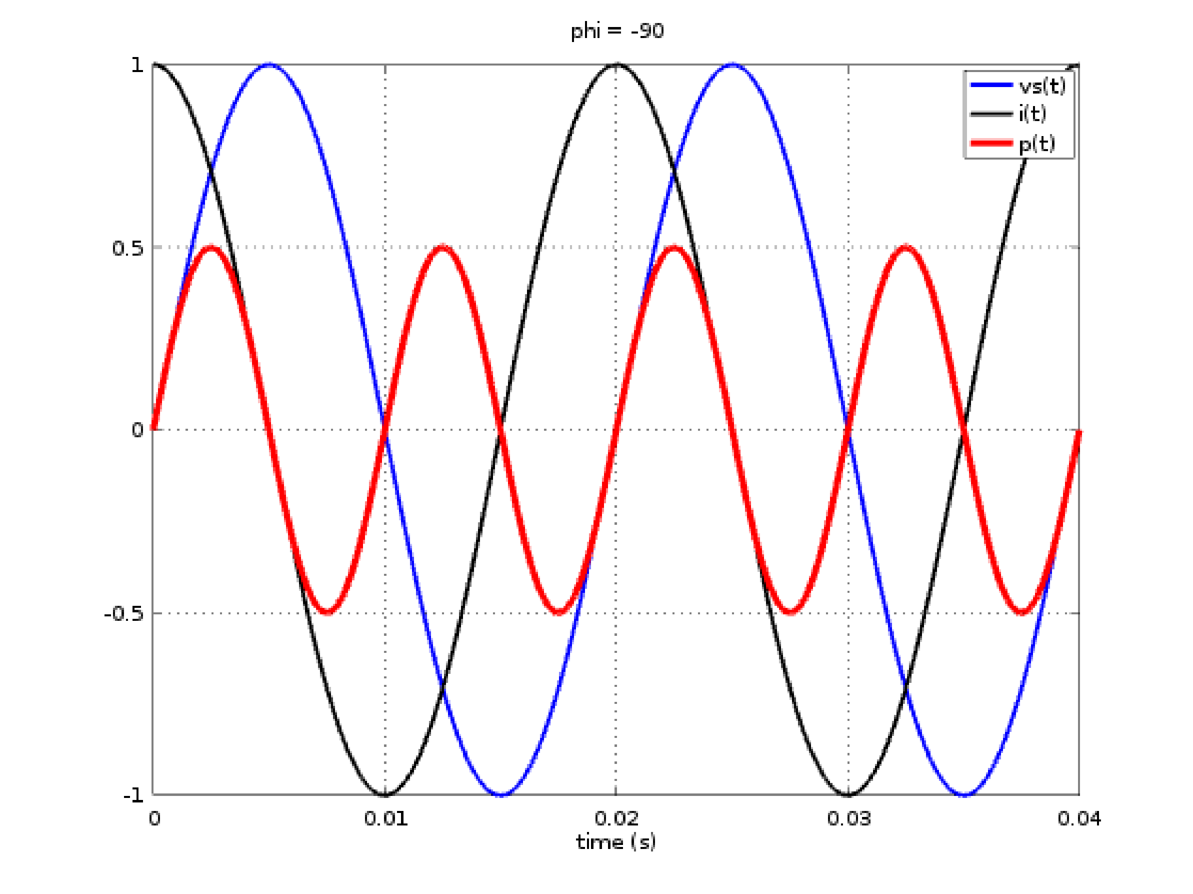
Instantaneous power for $\phi = 0^\circ$:



Instantaneous power for $\phi = -45^\circ$:



Instantaneous power for $\phi = -90^\circ$:



6. Significance of Reactive Power

- Single phase instantaneous power is always **pulsating** at twice the supply frequency.

- If $\phi = 0°$, then instantaneous power is always **positive** (resistor) and **Q = 0**.

- If $\phi = \pm 90°$, then only a **zero average pulsating** component is present (capacitor, inductor) and **P = 0**.

- Also note that:

$$\tan(\phi) = \frac{Q}{P}, \qquad \frac{1}{2} V_m I_m = S = \sqrt{P^2 + Q^2}$$

Significance of Reactive Power

- Phasor analysis can be used: Sum of S absorbed by all branches connected to a node=0

  . $$\mathbf{S} = P + jQ = \bar{V}_s \times \bar{I}^*.$$

- $S > P$ when 'reactive elements are present. Implies that more current is drawn for same amount of P. Results in **more losses, more voltage drops and bigger equipment** for same P rating.

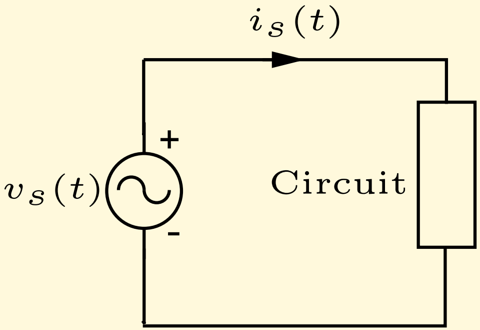
- An inductor "**absorbs**" **positive reactive power**;
  a capacitor **absorbs** **negative reactive power** and vice - versa for supply.

7. Production & Absorption of Reactive Power

- **Synchronous Generators** : Can **generate** or **absorb** reactive power.
- **Overhead Lines** : **Absorb** or **supply** reactive power.
- **Cables** : **Generate** reactive power.
- **Transformers** : Always **absorb** reactive power.
- **Loads** : Normally **absorb** reactive power.
- **Compensating Devices** : **absorb** or **supply** reactive power.

Quick revision:

Time-domain

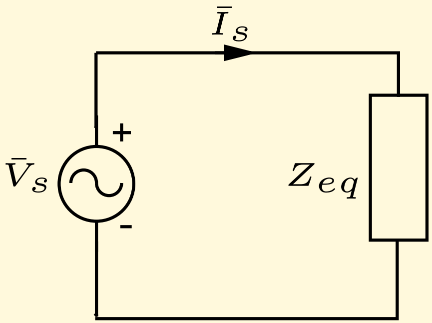


$$v_s(t) = V_m \sin(\omega t + \alpha) \text{ V}$$

$$i_s(t) = \frac{V_m}{|Z_{eq}|} \sin(\omega t + \alpha - \angle Z_{eq})$$

$$= I_m \sin(\omega t + \alpha - \phi) \text{ A.}$$

Phasor domain



$$\bar{V}_s = \frac{V_m}{\sqrt{2}} \angle\alpha^\circ = V_{rms} \angle\alpha^\circ \text{ V}$$

$$\bar{I}_s = \frac{V_m}{\sqrt{2}} \angle\alpha . \frac{1}{Z_{eq}} = \frac{V_{rms}\angle\alpha^\circ}{Z_{eq}}$$

$$= \frac{I_m}{\sqrt{2}} \angle(\alpha - \phi)^\circ = I_{rms} \angle(\alpha - \phi)^\circ \text{ A.}$$

Z_{eq} is the equivalent impedance seen by the source.

Quick revision:

Time-domain

Instantaneous Power

$$p(t) = v_s(t) \times i_s(t)$$

$$p(t) = \frac{1}{2} V_m I_m \left[\cos\phi\,(1 - \cos(2\omega\, t)) - \sin\phi\,\sin(2\omega\, t)\right]$$

Average Power:

$$P = \frac{1}{T}\int_0^T p(\tau)\, d\tau = \frac{V_m I_m}{2}\cos\phi$$

$$P = V_{rms}\, I_{rms} \cos\phi$$

Note: $T = \dfrac{2\pi}{\omega}$

Phasor domain

Average Power is also found to be:

$$\begin{aligned} P &= Real\{\bar{V}_s\, \bar{I}_s^*\} \\ &= Real\{V_{rms}\angle\alpha\,(I_{rms}\angle(\alpha - \phi))^*\} \\ &= Real\{V_{rms}\angle\alpha\,(I_{rms}\angle(\phi - \alpha))\} \\ &= V_{rms}\, I_{rms} \cos\phi \end{aligned}$$

Note: P is Real or active power.

Q is defined as:

$Q = Imag\{\bar{V}_s\, \bar{I}_s^*\} = V_{rms}\, I_{rms} \sin\phi$.

Define:

$$\bar{S} = P + j\,Q = \bar{V}_s \times \bar{I}_s^*$$

$$S = \sqrt{P^2 + Q^2} = V_{rms}\,I_{rms}$$

$$\tan\phi = \frac{Q}{P}$$

where,
S: apparent power in **VA** (volt ampere)

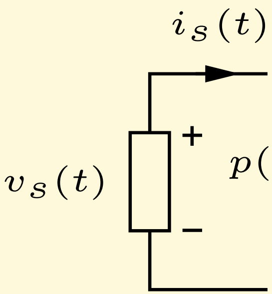
P: real / active / average power in **W** (watt)

Q: reactive power in **VAr** (volt ampere reactive)

$\cos\phi = \dfrac{P}{S}$ = power factor.

Convention for power calculations

Time-domain



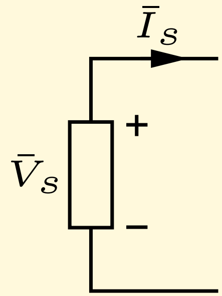
$$p(t) = v_s(t) \times i_s(t)$$

If at any time instant:

$p(t) > 0$: instantaneous power is delivered.

$p(t) < 0$: instantaneous power is absorbed.

Phasor domain



$$P = Real\{\bar{V}_s \ \bar{I}_s^*\}$$

$$Q = Imag\{\bar{V}_s \ \bar{I}_s^*\}$$

In steady state:

$\mathbf{P} > \mathbf{0}$: average power is delivered.

$\mathbf{P} < \mathbf{0}$: average power is absorbed.

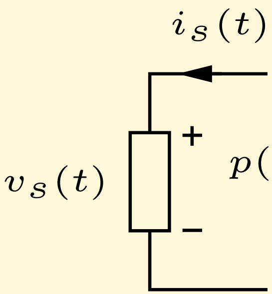
$\mathbf{Q} > \mathbf{0}$: reactive power is delivered.

$\mathbf{Q} < \mathbf{0}$:reactive power is absorbed.

Note: the voltage polarity and the sense of current flow.

Convention for power calculations

Time-domain



$$p(t) = v_s(t) \times i_s(t)$$

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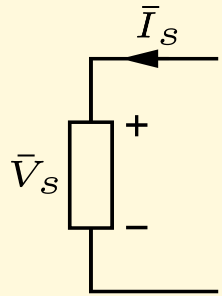
Phasor domain



$$P = Real\{\bar{V}_s\ \bar{I}_s^*\}$$

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$\mathbf{Q} > \mathbf{0}$: reactive power is absorbed.

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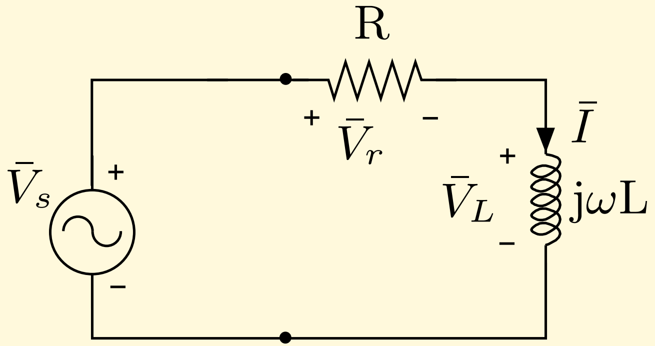
Note: the voltage polarity and the sense of current flow.

Power balancing in AC circuits



$R = 10\ \Omega$, $L = 30$ mH

$\bar{V}_s = 100\angle 30°$ V (rms value)

$\omega = 2\pi 50 = 314.16$ rad/s

Circuit-1:

Solution:

$$\bar{I} = \frac{\bar{V}_s}{R + j\omega L} = 7.277\angle{-13.3°} \text{ A.}$$

$\bar{V}_r = R \times \bar{I} = 72.77\angle{-13.3°}$ V and $\bar{V}_L = j\omega L \times \bar{I} = 68.584\angle 76.7°$ V

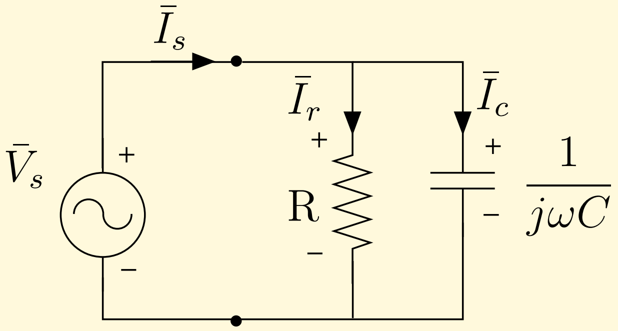
Apparent power **supplied** by source: $S_s = \bar{V}_s\bar{I}^* = (529.55 + j499.08)$ VA

Apparent power **absorbed** by the resistor: $S_R = \bar{V}_r\bar{I}^* = (529.55 + j0)$ VA

Apparent power **absorbed** by the inductor: $S_L = \bar{V}_L\bar{I}^* = (0 + j499.085)$ VA

Supplied apparent power = Absorbed apparent power in the circuit.

Circuit-2:



$R = 10\ \Omega, \quad C = 0.5$ mF

$\bar{V}_s = 100\angle 30^\circ$ V (rms value)

$\omega = 2\pi 50 = 314.16$ rad/s

Solution:

$$\bar{I}_r = \frac{\bar{V}_s}{R} = 10\angle 30^\circ \text{ A} \quad \text{and} \quad \bar{I}_c = \frac{\bar{V}_s}{\left(\frac{1}{j\omega C}\right)} = 15.708\angle 120^\circ \text{ A}$$

$$\bar{I}_s = \bar{I}_r + \bar{I}_c = 18.621\angle 87.52^\circ \text{ A.}$$

Apparent power **supplied** by source: $S_s = \bar{V}_s \times \bar{I}_s^* = (999.96 - j1570.83)$ VA

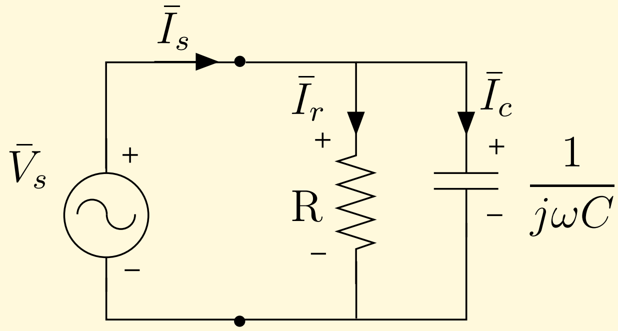
Apparent power **absorbed** by the resistor: $S_R = \bar{V}_s \times \bar{I}_r^* = (1000 + j0)$ VA

Apparent power **absorbed** by the capacitor: $S_c = \bar{V}_s \times \bar{I}_c^* = (0 - j1570.8)$ VA

Supplied apparent power = Absorbed apparent power in the circuit.

(Note: the numerical precision of calculations.)

Circuit-2:



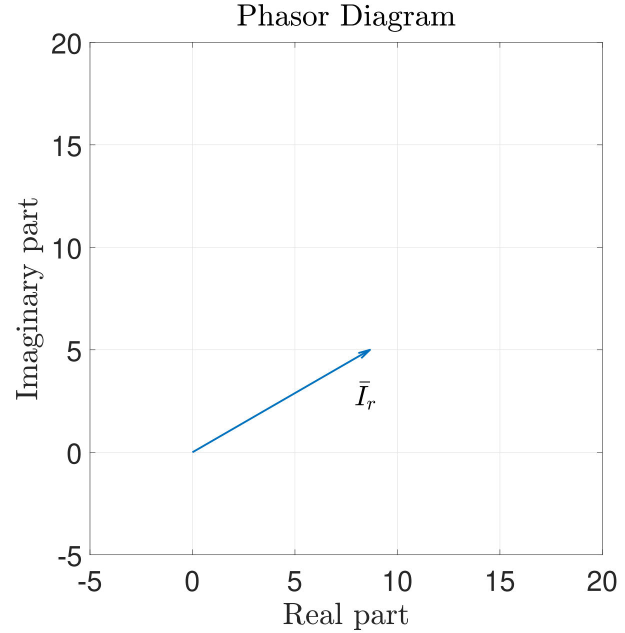
$R = 10\ \Omega, \quad C = 0.5$ mF

$\bar{V}_s = 100\angle 30^\circ$ V (rms value)

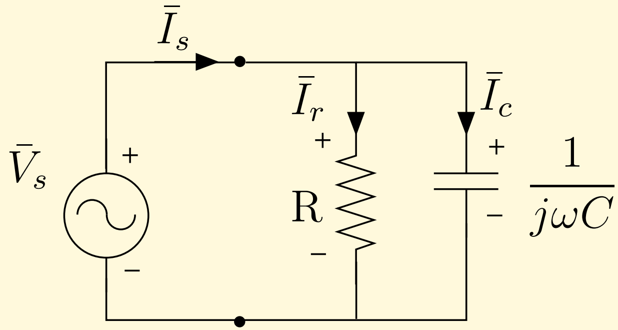
$\omega = 2\pi 50 = 314.16$ rad/s

$\bar{I}_r = 10\angle 30^\circ$ A, $\bar{I}_c = 15.708\angle 120^\circ$ A

$\bar{I}_s = 18.621\angle 87.52^\circ$ A.



Circuit-2:



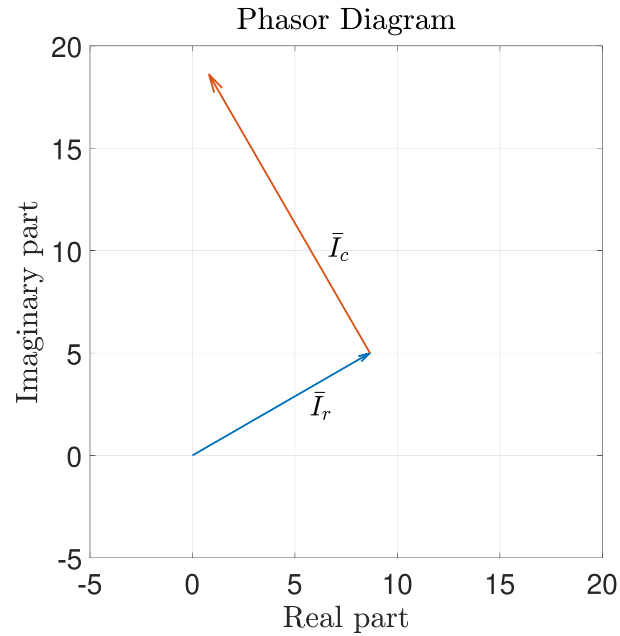
$R = 10\ \Omega, \quad C = 0.5$ mF

$\bar{V}_s = 100\angle 30^\circ$ V (rms value)

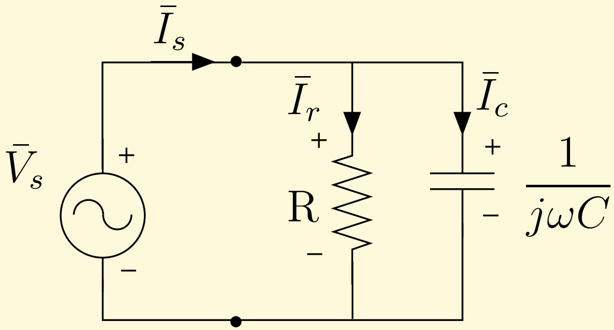
$\omega = 2\pi 50 = 314.16$ rad/s

$\bar{I}_r = 10\angle 30^\circ$ A, $\bar{I}_c = 15.708\angle 120^\circ$ A

$\bar{I}_s = 18.621\angle 87.52^\circ$ A.



Circuit-2:



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