SC-635 Advanced Topics in Mobile Robotics

Experiment Module : Filtering

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Systems and Control Engineering

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Overview

1. Filter Algorithm
2. Assignment

Filter Algorithm

Kalman Filter Equations

The Kalman filter maintains the estimates of the state:

$\hat{\mathbf{x}}(k|k)$ – estimate of $\mathbf{x}(k)$ given measurements $z(k), z(k-1), \ldots$

$\hat{\mathbf{x}}(k+1|k)$ – estimate of $\mathbf{x}(k+1)$ given measurements $z(k), z(k-1), \ldots$

and the error covariance matrix of the state estimate

$\mathbf{P}(k|k)$ – covariance of $\mathbf{x}(k)$ given $z(k), z(k-1), \ldots$

$\mathbf{P}(k+1|k)$ – estimate of $\mathbf{x}(k+1)$ given $z(k), z(k-1), \ldots$

We shall partition the Kalman filter recursive processing into several simple stages with a physical interpretation:

[1]

[1] source: http://biorobotics.ri.cmu.edu/papers/sbp_papers/integrated3/kleeman_kalman_basics.pdf

Filter Algorithm (continued)

State Estimation

0. Known are $\hat{\mathbf{x}}(k|k), \mathbf{u}(k), \ \mathbf{P}(k|k)$ and the new measurement z(k+1).

1. State Prediction $\hat{\mathbf{x}}(k+1|k) = \mathbf{F}(k)\hat{\mathbf{x}}(k|k) + \mathbf{G}(k)\mathbf{u}(k)$ *Time update*

2. Measurement Prediction: $\hat{\mathbf{z}}(k+1|k) = \mathbf{H}(k)\hat{\mathbf{x}}(k+1|k)$

3. Measurement Residual: $\mathbf{v}(k+1) = \mathbf{z}(k+1) - \hat{\mathbf{z}}(k+1|k)$ *measurement update*

4. Updated State Estimate: $\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{W}(k+1)\mathbf{v}(k+1)$

   where **W**(k+1) is called the Kalman Gain defined next in the state covariance estimation.

[2]

[2] source: http://biorobotics.ri.cmu.edu/papers/sbp_papers/integrated3/kleeman_kalman_basics.pdf

Filter Algorithm (continued)

State Covariance Estimation

1. State prediction covariance: $\mathbf{P}(k+1|k) = \mathbf{F}(k)\mathbf{P}(k|k)\mathbf{F}(k)' + \mathbf{Q}(k)$

2. Measurement prediction covariance:

$$\mathbf{S}(k+1) = \mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}(k+1)' + \mathbf{R}(k+1)$$

3. Filter Gain $\mathbf{W}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}(k+1)'\ \mathbf{S}(k+1)^{-1}$

4. Updated state covariance

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) - \mathbf{W}(k+1)\mathbf{S}(k+1)\mathbf{W}(k+1)'$$

[3]

[3] source: http://biorobotics.ri.cmu.edu/papers/sbp_papers/integrated3/kleeman_kalman_basics.pdf

Assignment 4

The robot loads with pose (0, 0, 0). Three landmarks are placed at [7,7], [-7,-7], [7,-7], the distance and identity of the landmarks is being published under the topic name **/trilateration_data**.

- The goal of this simulation exercise is to implement EKF to obtain robot pose $x(k+1|k+1) = [x, y, \theta]^T$

  using $x(k|k), y_m, y(k+1|k)$ etc. Following functions are provided:
  - predict_state : To calculate $x(k+1|k)$
  - predict_measurement : To calculate $y(k+1|k)$
  - get_current_H : To calculate $H(k+1)$
- Read carefully the comments in the project template (line 134-157) and handout (cheetsheet)
- A script named vis.py is provided to see the real-time motion of the robot (you may extend the same to display waypoints). This is purely for debugging purpose and doesn't carry any marks.
- Track a circular trajectory with radius 5 meter centered at origin.
- Plot the waypoints and the tracked trajectory. Save figure with labels and title.
- Calculate the mean squared error for one complete traversal of the circular trajectory.
  - Sample 100 points along the tracked trajectory
  - For each robot pose X_r = (x , y, θ) find the closest point X_c on the circle $x^2 + y^2 = 5^2$
  - Calculate the distance to D_{error} and square it
  - Sum all D^2_{error} terms and divide by 100 to obtain you MSE. Report this number in a file named RESULT.txt.

The template project is located at :

http://bit.ly/2tdpTemplateA5

Thank you