EE302 : Control Systems : Tutorial Sheet 4

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1 Question : 1

Draw Root Locus for the following:

- $\frac{1}{(s+1)(s+2)(s+3)}$ for $k < 0$
- $\frac{(2-s)}{(s+3)(s+4)}$ for $k > 0$
- $\frac{(s-z)}{(s+10)(s^2+2)}$ for $k \ \epsilon \ (-\infty, +\infty)$ take $z = -15$ at first.

For the third part,

- Find angle of arrival/departure at all poles/zeros for $z = -15$
- Find a value of $z \ \epsilon \ R$ such that break-away and break-in points coincide

2 Question : 2

Plot the root locus for $\frac{1}{[(s-1)^2+5^2]^2}$ by first plotting for $\frac{1}{[s^2+25]^2}$ and check using root locus plotter(simulation)

3 Question : 3

Consider the polynomial $2s^3+3ps^2-6ps-3p+6s^2+6s+2$. Plot the roots of this polynomial in s, as the parameter p varies from $-\infty$ to $+\infty$, using root locus methods

4 Question : 4

Consider $G(s) = \frac{n(s)}{d(s)}$ with $n(s)$ and $d(s)$ having real, distinct roots

- Obtain formula for break-away/break-in points using maxima/minima property of k on the real axis i.e. derive $\sum_{p_i}(\frac{1}{s-p_i}) = \sum_{z_i}(\frac{1}{s-z_i})$ this relation
- Use repeated roots property of $d(s) + kn(s)$ at that value of k to get the same formula above for break-away/break-in point
- Look up Sylvester Resultant of two polynomials, discriminant of a polynomial and relate it with the discriminant of $ax^2 + bx + c = 0$ where $a \neq 0$

5 Question : 5

Consider the plant given by $G(s) = \frac{1}{s(s^2+\omega^2)}$ with $\omega > 0$ and real. Suppose one wants to design a controller by first cancelling the jR poles. Consider the following controllers:

- $C(s) = s^2 + (s+\epsilon)^2$, $\epsilon > 0$
- $C(s) = s^2 + (s-\epsilon)^2$, $\epsilon > 0$

Use root locus to establish which mismatch (whether in first or second controller) is acceptable and justify your answer

6 Question : 6

- Consider a system $\dot{x} = 3x + u$, in which $x(0) = 7$ and using $u = -5x$, check that $x(t) \to 0$, $u(t) \to 0$ as $t \to +\infty$
- Now find $u(t)$ explicitly as a function of time (using $x(0) = 7$)
- Use laplace transform to get output for the system $u(t) \to (\frac{1}{(s-3)}) \to x(t)$, with $u(t)$ as in (b) and $x(0) = 7$
- Now solve (c) but with system $\dot{x} = (3+\epsilon)x$
- Solve (c) but with initial condition $x(0) = 7 + \epsilon$

7 Question : 7

- Consider $G(s) = \frac{1}{(s+1)(s+2)}$ with 15% OS for closed loop
- Design a PD controller which has the same % OS as (a) but half the 2% settling time
- Design two different lead compensators for the same specifications as in (b). By different, we mean with different pole/zero locations.
- Design a PI controller to achieve steady state error = 0
- Design a lag compensator to get steady state error as one tenth of that of (c) (design two different lag compensators)

8 Question : 8

Sketch (on the same figure) Bode magnitude and phase plots of the following:

- $\frac{s+1}{s+8}$
- $\frac{s-1}{s+8}$
- e^{-20s}
- $\frac{s-1}{s+1}$
- $\frac{(s-1)(s-3)}{(s+3)(s-1)}$

For each of the above, correct by 3dB at the corner frequency

9 Question : 9

- Sketch $G(s) = \frac{5}{s^2+20\delta s+100}$ for $\delta = 0.1$ and $\delta = 0.9$
- Find the range of values of δ for which there is a peak in magnitude plot
- Find the frequency at which peak happens as a function of δ

10 Question : 10

Classify into lag, lead, PD, PID and plot bode plot. Also classify from bode plot and get transfer function from bode plot for the following:

- $\frac{s+8}{s+20}$
- $3s + 9$
- $6 + \frac{s}{3} + 14s$
- $25\frac{(s+3)}{(s+0.5)}$

11 Question : 11

Look up lag, lead, lag-lead compensator realization from Norman Nise :

- by using RC components
- by using op-amps

12 Question : 12

Consider the following set-up given in the figure below consisting of an input source, an LTI system and an oscilloscope. Suppose the input signals generated by the source are of the form $u(t) = Asin(\omega t)$ with adjustable $\omega \epsilon R$

- Show that for a given ω, the oscilloscope will display an ellipse or a straight line
- How can you compute the phase $\Phi(\omega)$ and the magnitude $M(\omega)$ of the LTI system by inspecting the aforementioned ellipse or straight line ?