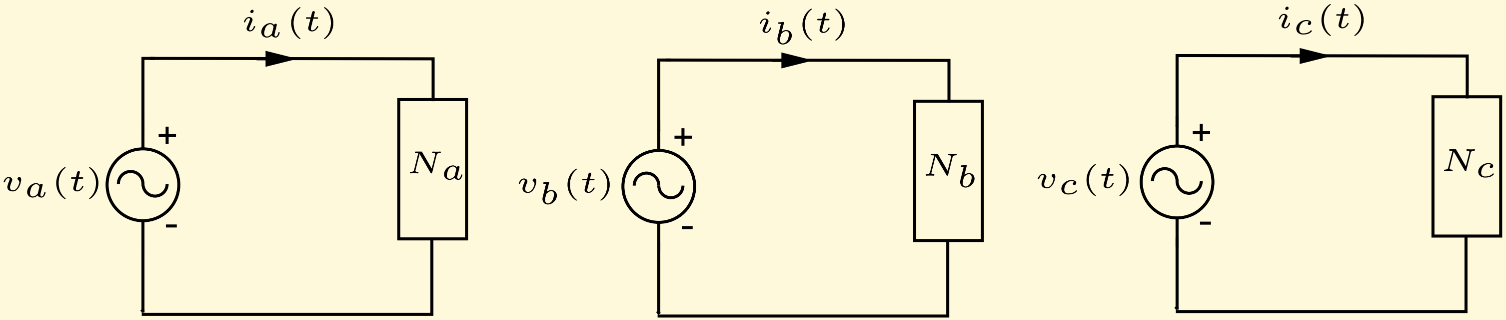
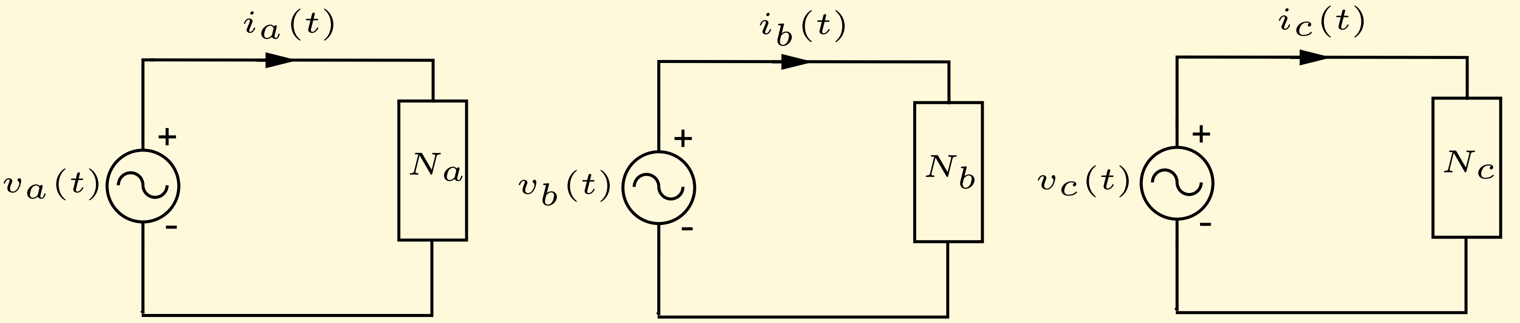
Three Phase Systems

System in time-domain



System in time-domain



$v_a(t) = V_m \sin(\omega t + \alpha)$ $\quad v_b(t) = V_m \sin(\omega t + \alpha - 120°)$ $\quad v_c(t) = V_m \sin(\omega t + \alpha - 240°)$ V

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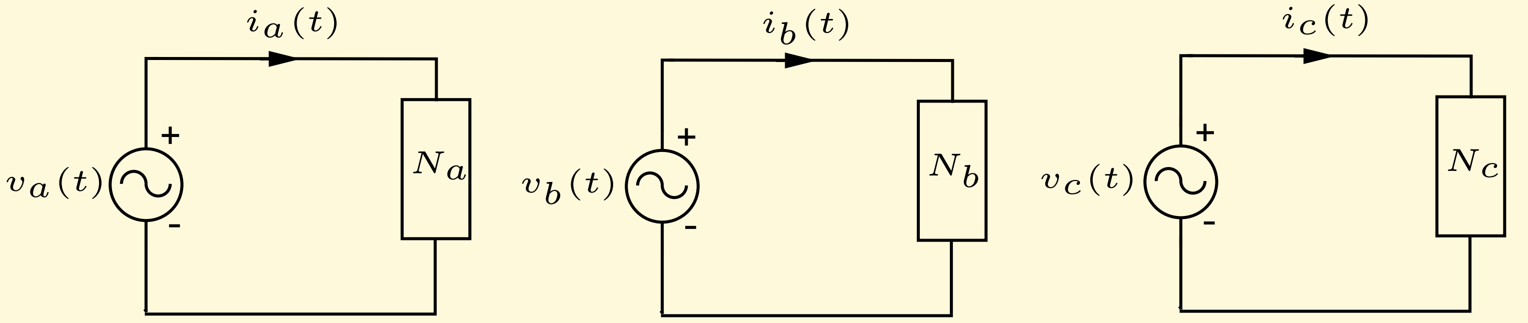

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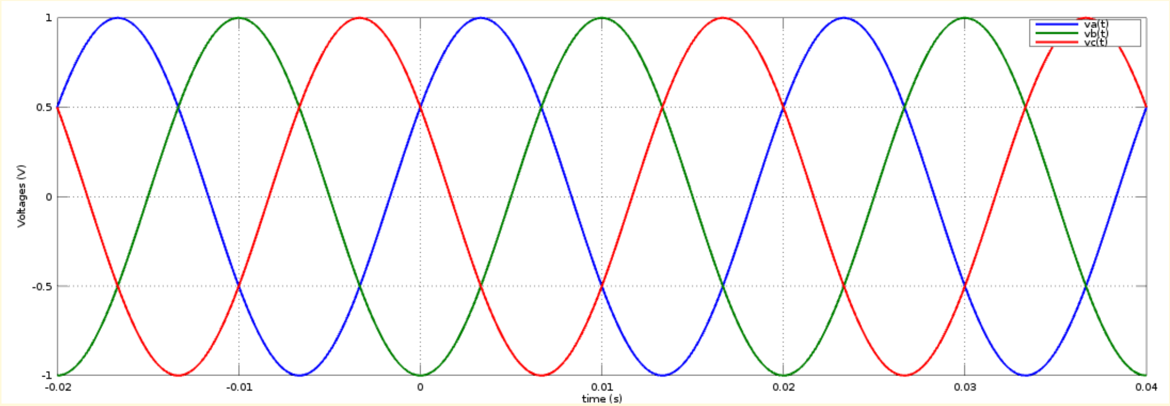
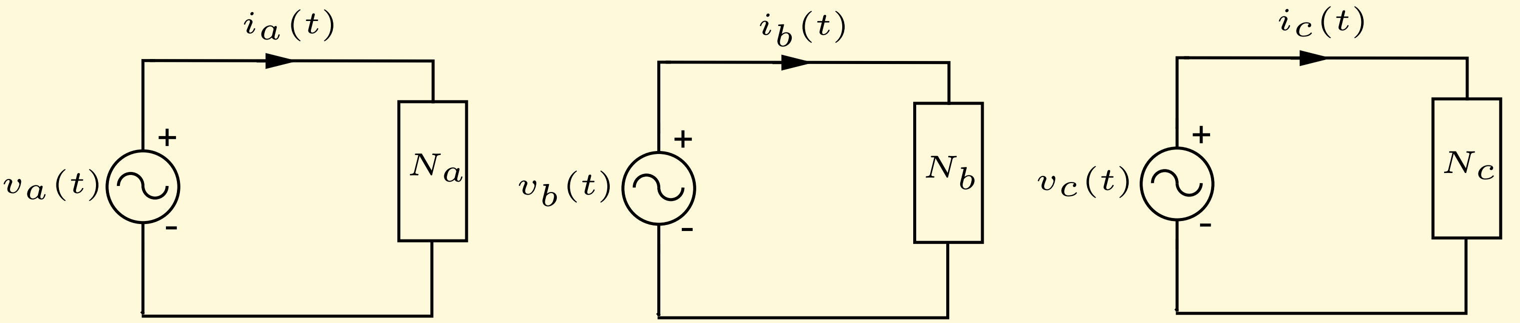


Balanced sets of three phase voltages.

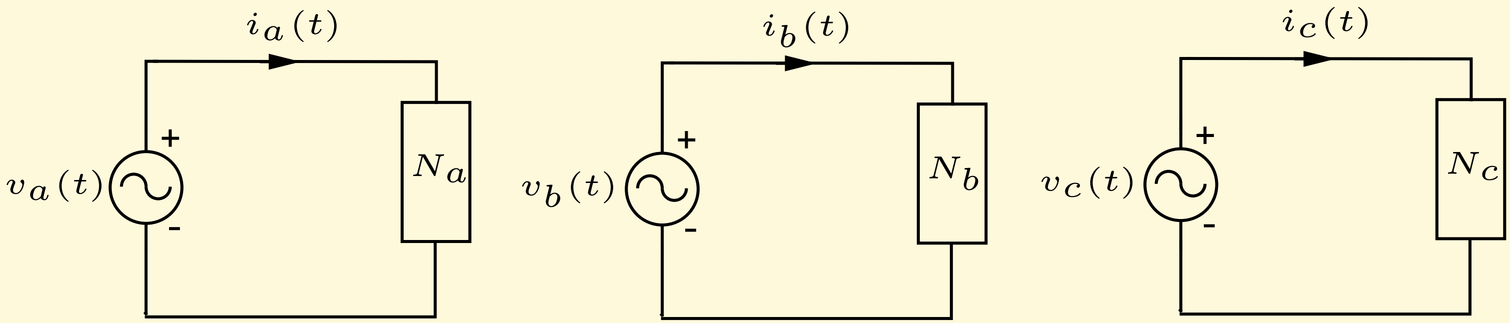
System in time-domain



$v_a(t) = V_m \sin(\omega t + \alpha)$ $\quad v_b(t) = V_m \sin(\omega t + \alpha - 120°)$ $\quad v_c(t) = V_m \sin(\omega t + \alpha - 240°)$V

N_a, N_b and N_c are three linear element network.

System in time-domain



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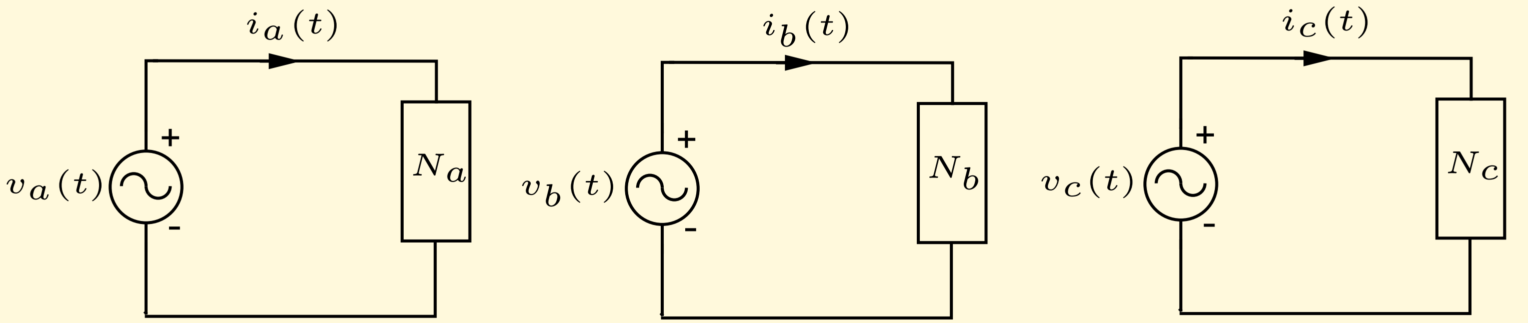
If they are identical, then load is **balanced** and currents are given as:

System in time-domain



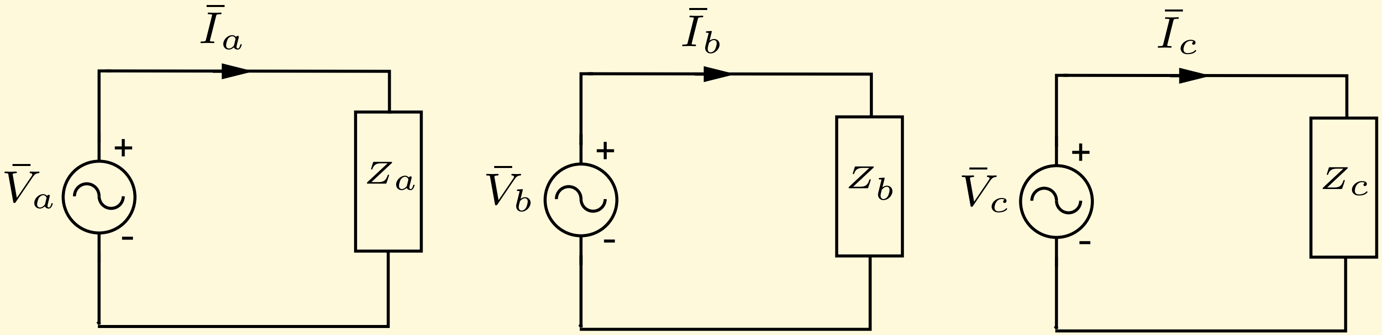
$v_a(t) = V_m \sin(\omega t + \alpha)$ $v_b(t) = V_m \sin(\omega t + \alpha - 120°)$ $v_c(t) = V_m \sin(\omega t + \alpha - 240°)$V

N_a, N_b and N_c are three linear element network.

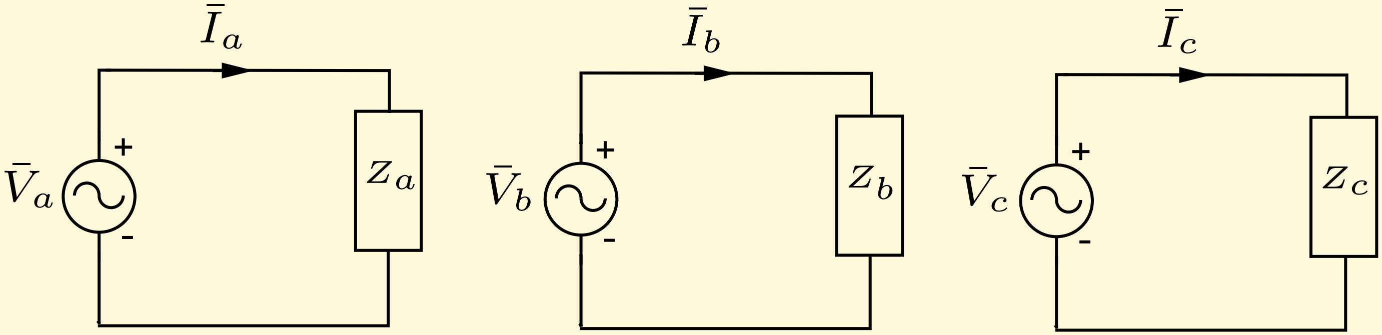
If they are identical, then load is **balanced** and currents are given as:

$i_a(t) = I_m \sin(\omega t + \alpha - \phi)$ $i_b(t) = I_m \sin(\omega t + \alpha - \phi - 120°)$ $i_c(t) = I_m \sin(\omega t + \alpha - \phi - 240°)$A

Balanced system in phasor domain

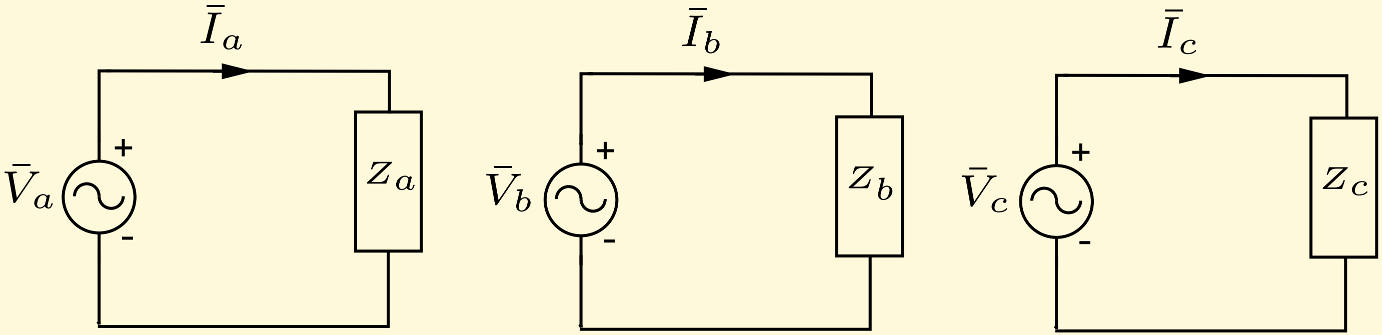


Balanced system in phasor domain



$$\bar{V}_a = \frac{V_m}{\sqrt{2}}\angle\alpha^\circ \qquad \bar{V}_b = \frac{V_m}{\sqrt{2}}\angle(\alpha - 120)^\circ \qquad \bar{V}_c = \frac{V_m}{\sqrt{2}}\angle(\alpha - 240)^\circ \text{ V}$$

Balanced system in phasor domain



$$\bar{V}_a = \frac{V_m}{\sqrt{2}}\angle\alpha^\circ \qquad \bar{V}_b = \frac{V_m}{\sqrt{2}}\angle(\alpha - 120)^\circ \qquad \bar{V}_c = \frac{V_m}{\sqrt{2}}\angle(\alpha - 240)^\circ \text{ V}$$

If $Z_a = Z_b = Z_c$ and $\phi = \angle Z_a = \angle Z_b = \angle Z_c$, the currents are given as.

$$\bar{I}_a = \frac{\bar{V}_a}{Z_a} = \frac{I_m}{\sqrt{2}}\angle(\alpha - \phi)^\circ \quad \bar{I}_b = \frac{\bar{V}_b}{Z_b} = \frac{I_m}{\sqrt{2}}\angle(\alpha - 120 - \phi)^\circ \quad \bar{I}_c = \frac{\bar{V}_c}{Z_c} = \frac{I_m}{\sqrt{2}}\angle(\alpha - 240 - \phi)$$

A

System in phasor domain

Conditions for three-phase system to be **balanced**:

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1. The **input voltages** should be balanced.
   - $|\bar{V}_a| = |\bar{V}_b| = |\bar{V}_c|$
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   i.e. $(Z_a = Z_b = Z_c)$.

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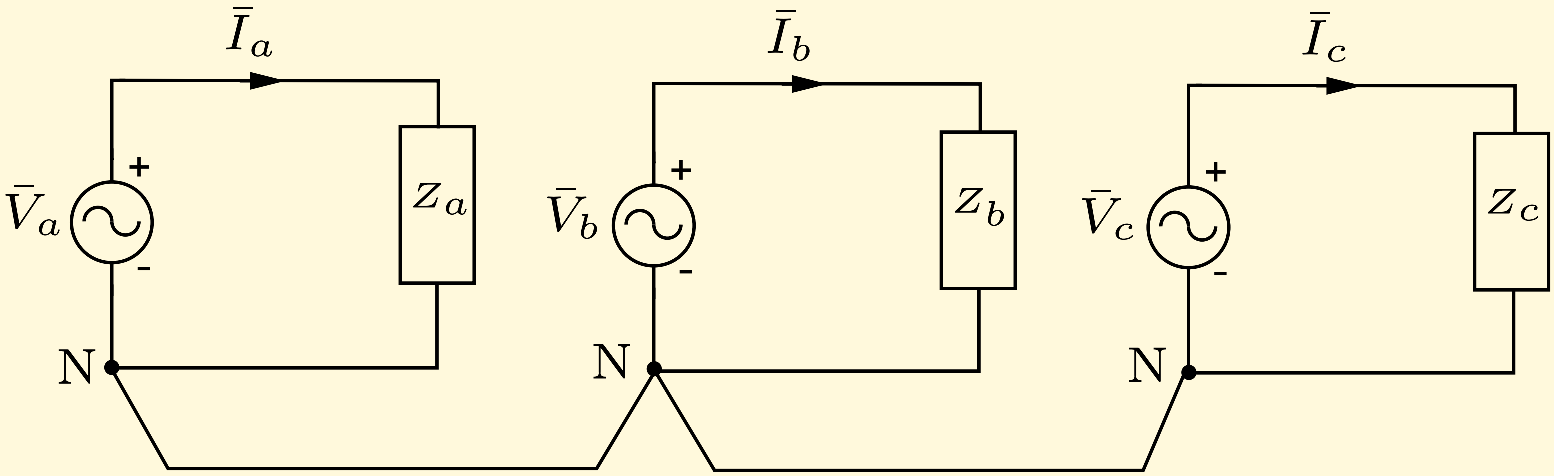
   i.e. ($Z_a = Z_b = Z_c$).

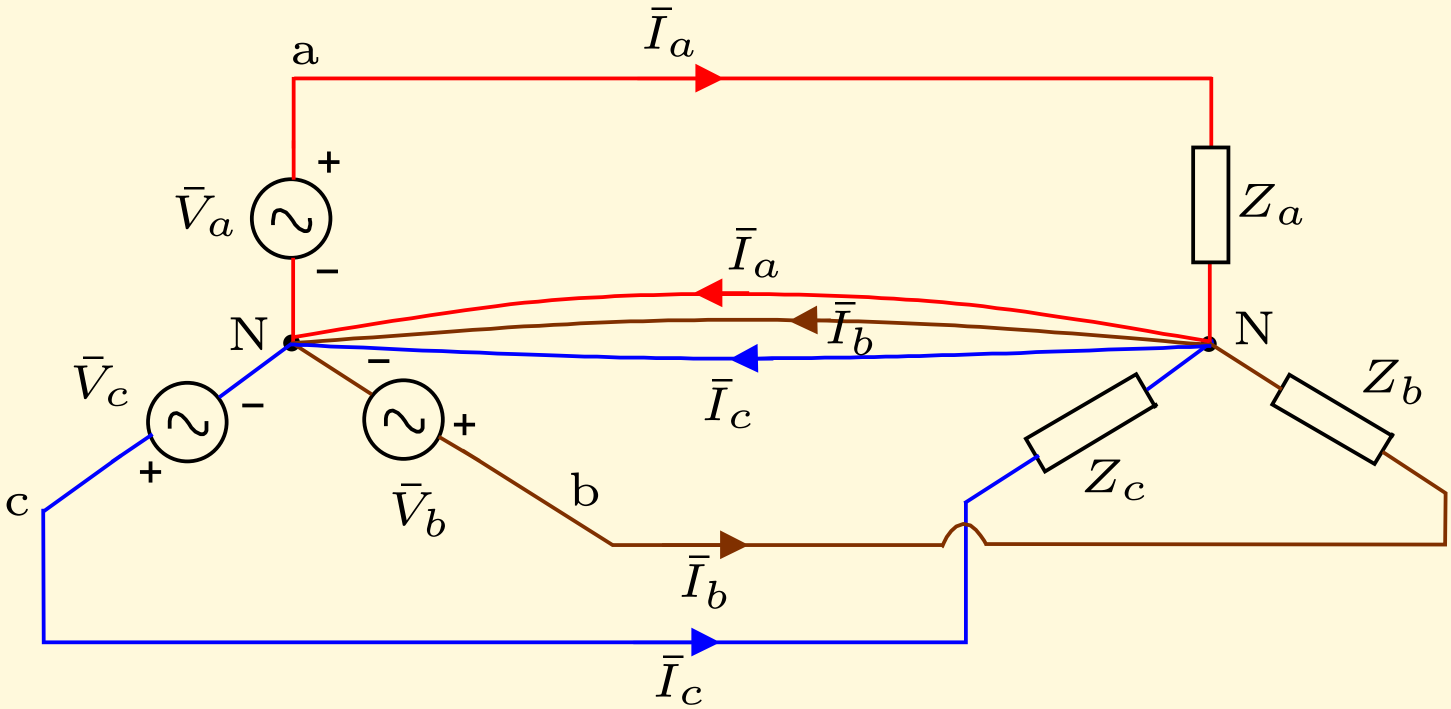
Note:
The convention used for voltage phasor: $\bar{V}_b$ lags $\bar{V}_a$ and $\bar{V}_c$ lags $\bar{V}_b$.

System in phasor domain

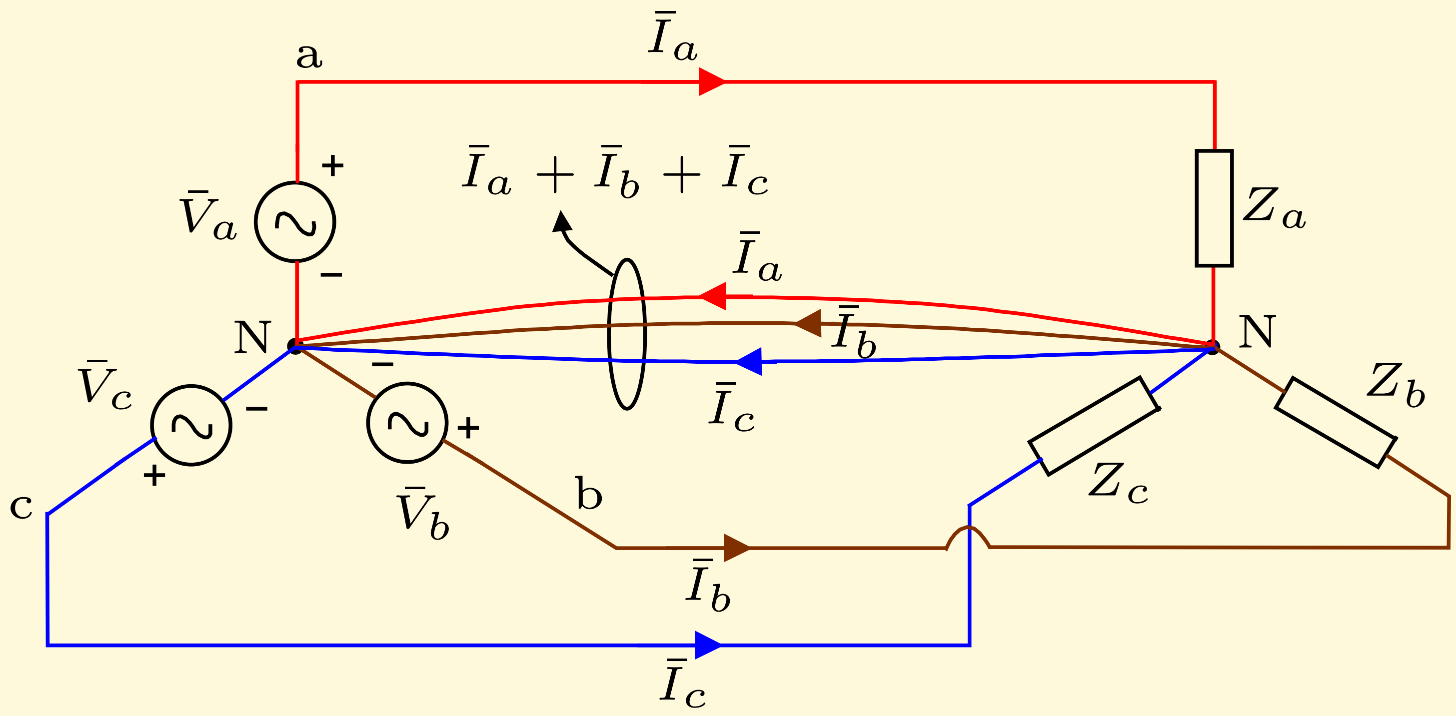
Consider a three single-phase balanced system with a common neutral (N).



System in phasor domain



System in phasor domain



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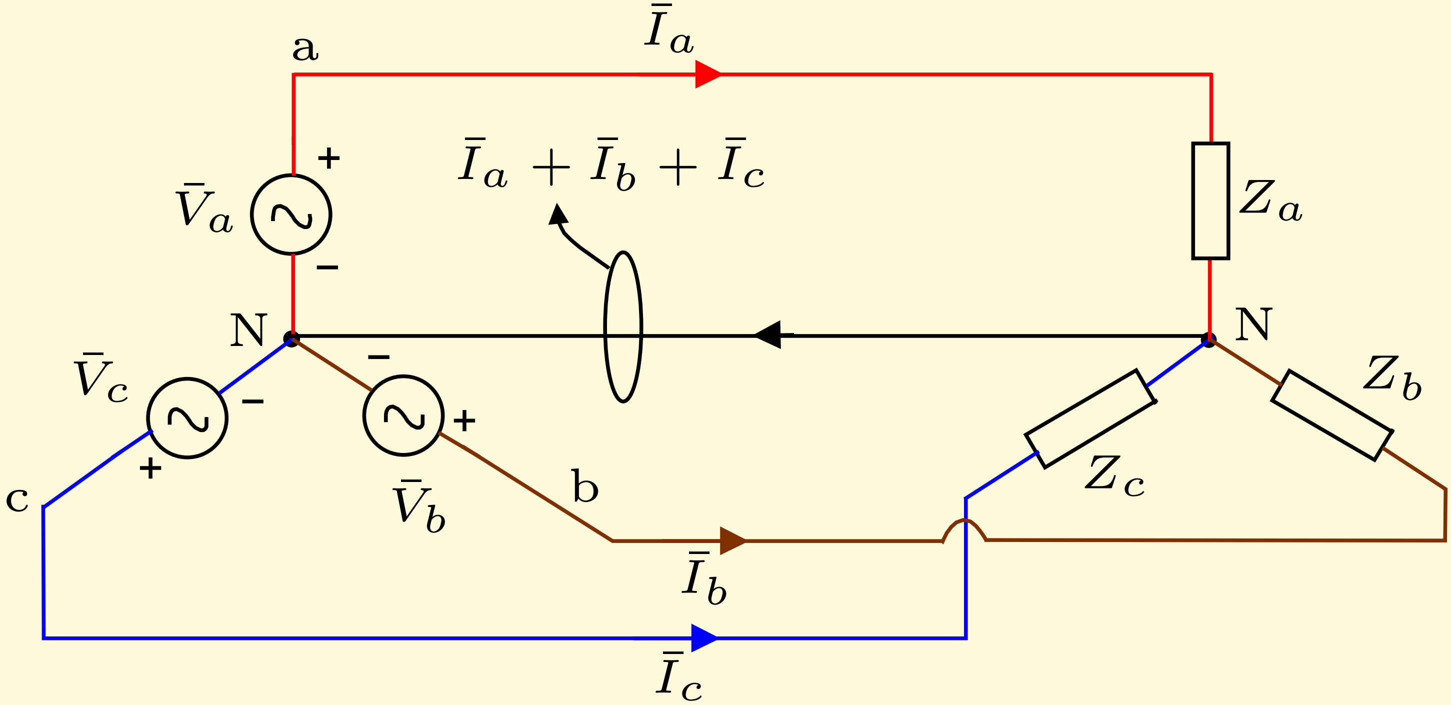
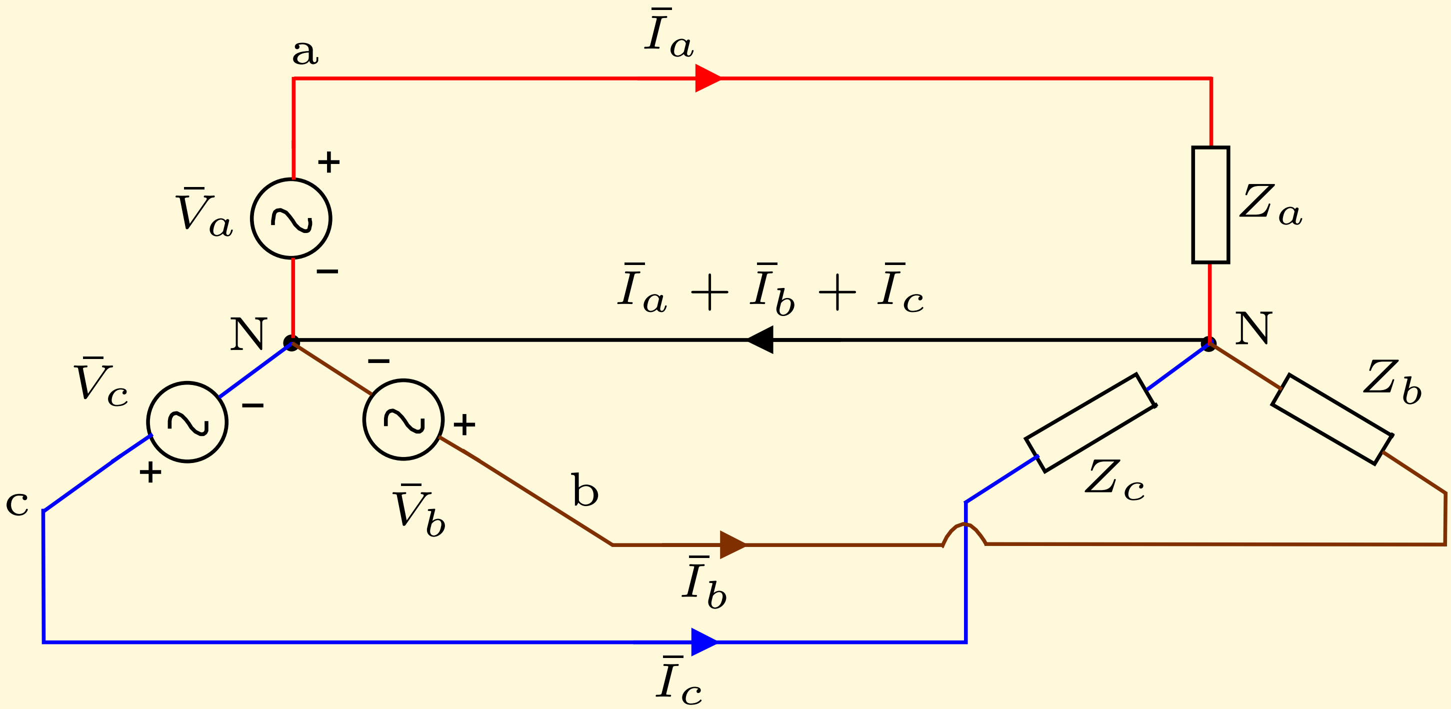
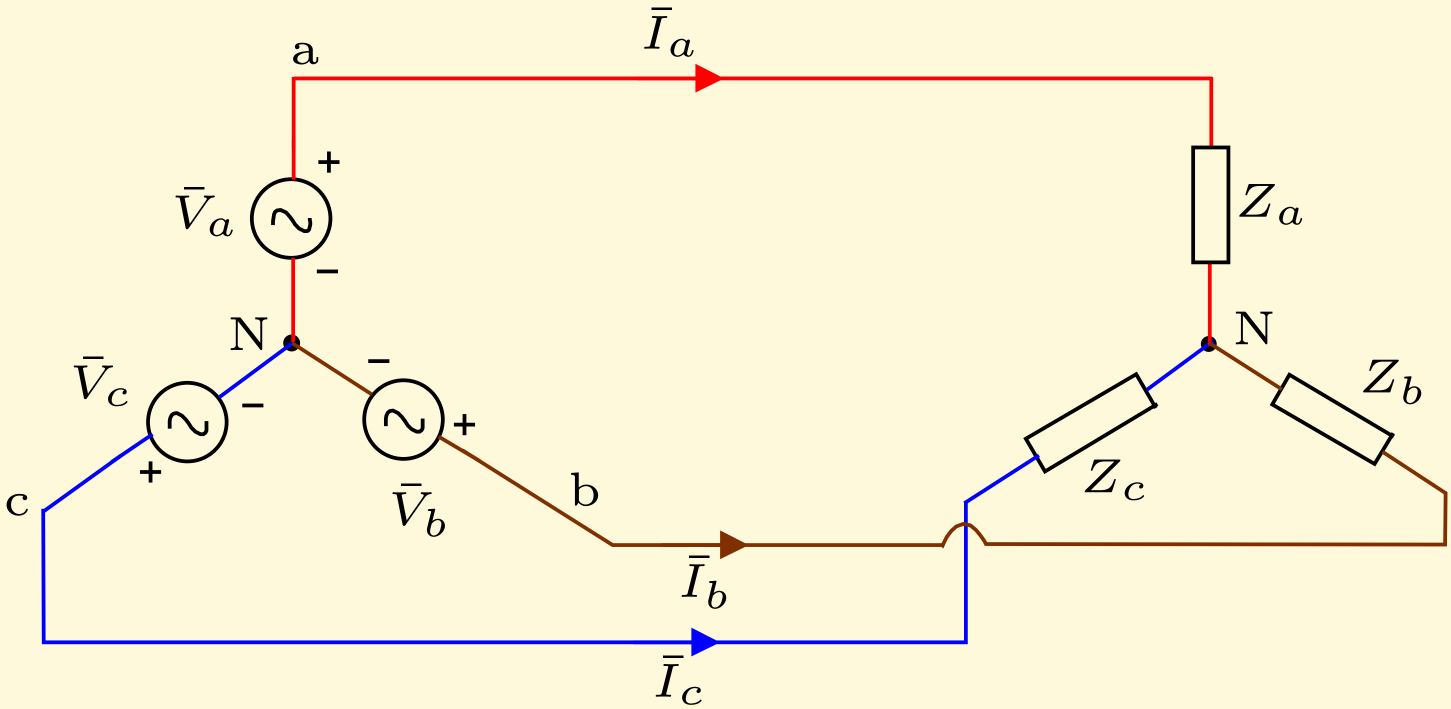


System in phasor domain



System in phasor domain (balanced) case:



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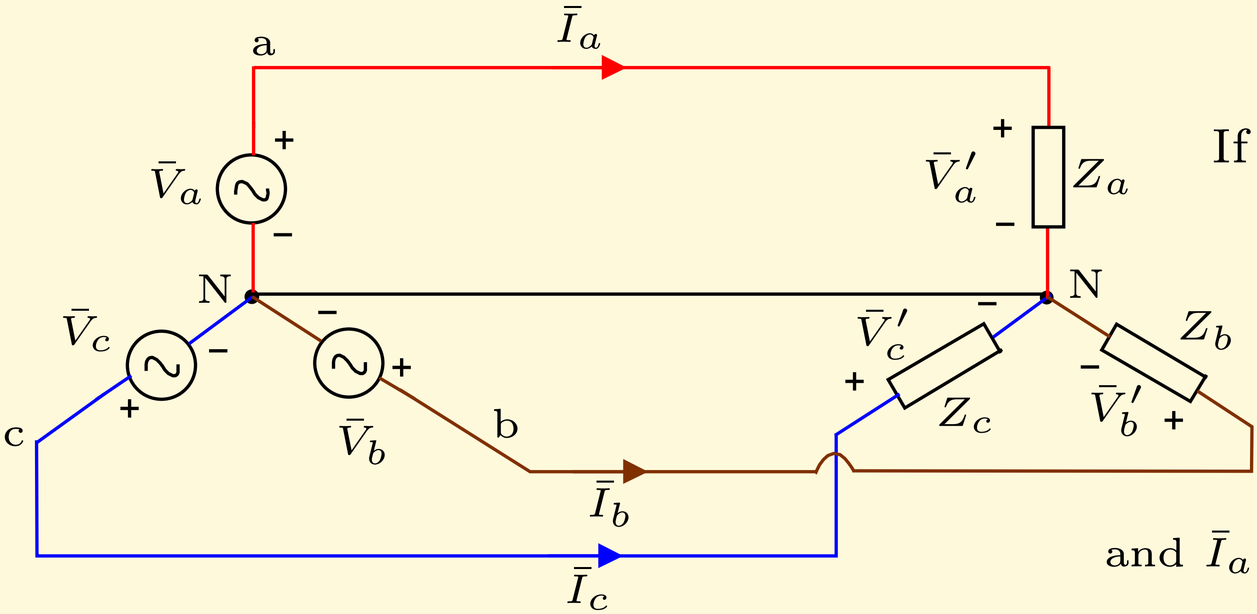


If balanced, then

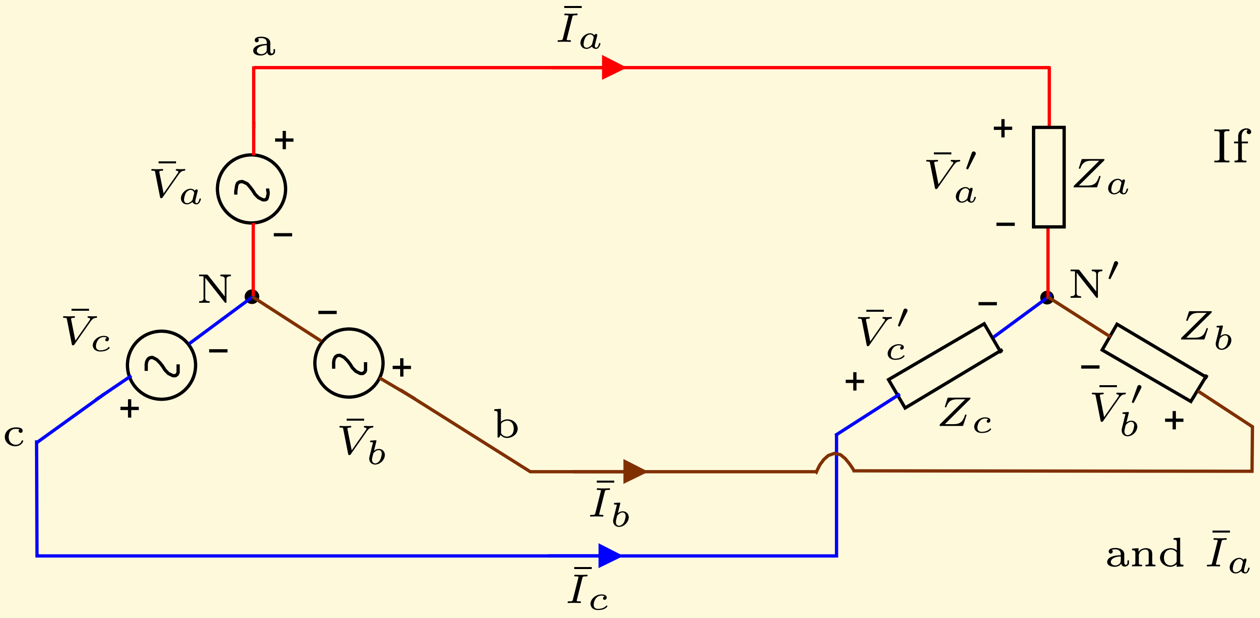
$$\bar{V}_a = \bar{V}_a'$$

$$\bar{V}_b = \bar{V}_b'$$

$$\bar{V}_c = \bar{V}_c'$$

and $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$

System in phasor domain (balanced) case:



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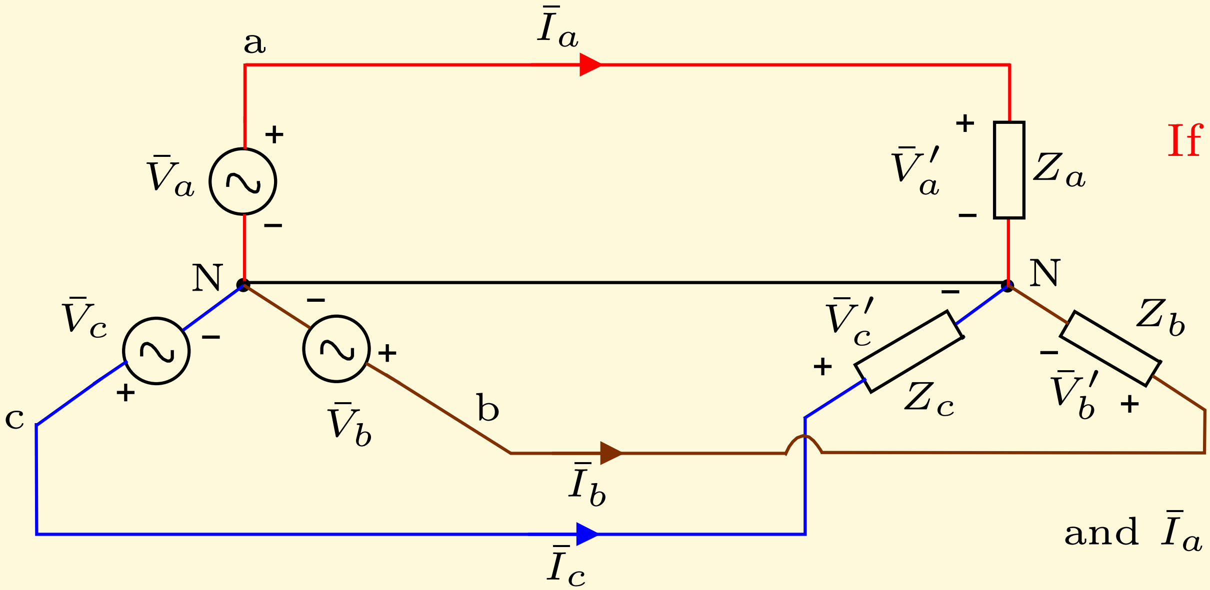
System in phasor domain (unbalanced) case:



If unbalanced, then

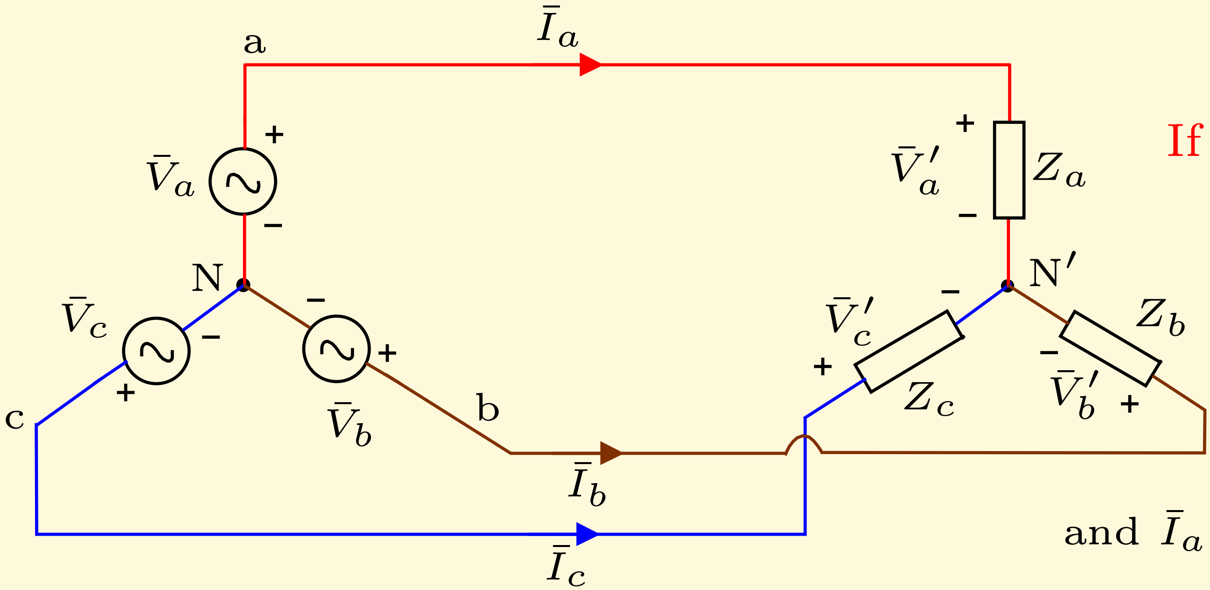
$$\bar{V}_a = \bar{V}_a'$$

$$\bar{V}_b = \bar{V}_b'$$

$$\bar{V}_c = \bar{V}_c'$$

and $\bar{I}_a + \bar{I}_b + \bar{I}_c \neq 0$

System in phasor domain (unbalanced) case:



If unbalanced, then

$$\bar{V}_a \neq \bar{V}_a'$$

$$\bar{V}_b \neq \bar{V}_b'$$

$$\bar{V}_c \neq \bar{V}_c'$$

and $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$

Power calculations:

In time-domain:

$$p(t) \;=\; v_a(t) \times i_a(t) + v_b(t) \times i_b(t) + v_c(t) \times i_c(t)$$

Power calculations:

In time-domain:

$$
\begin{aligned}
p(t) &= v_a(t) \times i_a(t) + v_b(t) \times i_b(t) + v_c(t) \times i_c(t) \\
&= V_m \sin(\omega t + \alpha) \times I_m \sin(\omega t + \alpha - \phi) + \\
&\quad V_m \sin(\omega t + \alpha - 120^\circ) \times I_m \sin(\omega t + \alpha - 120^\circ - \phi) + \\
&\quad V_m \sin(\omega t + \alpha - 240^\circ) \times I_m \sin(\omega t + \alpha - 240^\circ - \phi)
\end{aligned}
$$

Power calculations:

In time-domain:

$$
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&= V_m \sin(\omega t + \alpha) \times I_m \sin(\omega t + \alpha - \phi) + \\
&\quad V_m \sin(\omega t + \alpha - 120^\circ) \times I_m \sin(\omega t + \alpha - 120^\circ - \phi) + \\
&\quad V_m \sin(\omega t + \alpha - 240^\circ) \times I_m \sin(\omega t + \alpha - 240^\circ - \phi) \\
&= 3 \times \frac{V_m I_m}{2} \cos\phi \\
&= 3 \times V_{rms} I_{rms} \cos\phi
\end{aligned}
$$

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Note: V_{rms}:“ phase-to neutral rms voltage”.

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Note: V_{rms}:" phase-to neutral rms voltage".

In phasor domain: $P = 3 * Real\{\bar{V}_a \times \bar{I}_a^*\}$ (in W)

Balanced n-phase systems

Phase voltages:

$$v_k(t) = V_m \sin\left(\omega t + \alpha - \frac{2\pi}{n}(k-1)\right)$$

Phase currents:

$$i_k(t) = I_m \sin\left(\omega t + \alpha - \phi - \frac{2\pi}{n}(k-1)\right) \text{ where, } k = 1 \cdots n$$

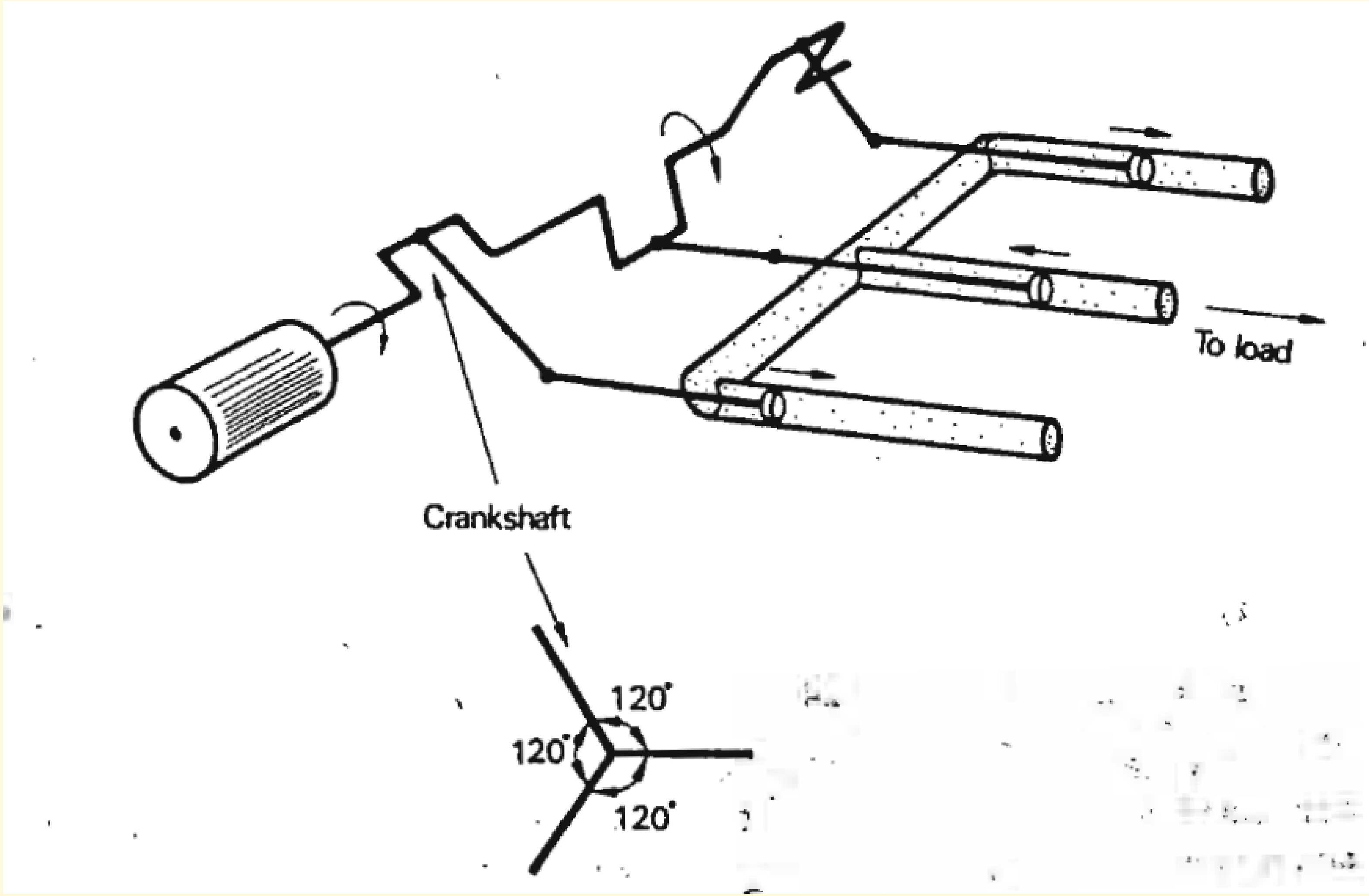
Sum of phase voltages: $\sum_{k=1}^{n} v_k(t) = 0$ for $k \geq 2$.

Sum of phase currents: $\sum_{k=1}^{n} i_k(t) = 0$ for $k \geq 2$.

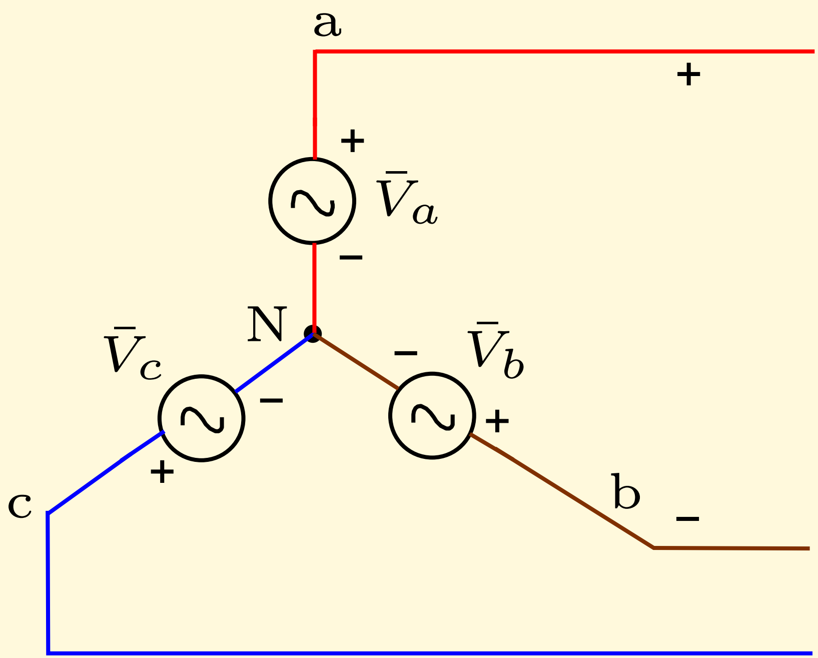
$p(t) = \sum_{k=1}^{n} v_k(t) i_k(t)$ = constant for $k \geq 3$.

Thus, $p(t)$ is time-independent *except* for a two-phase (180^o phase shift) system.

Mechanical equivalent for three phase system



Terminologies in three phase AC systems:



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$\bar{V}_a$, $\bar{V}_a$, $\bar{V}_a$: phase voltages.

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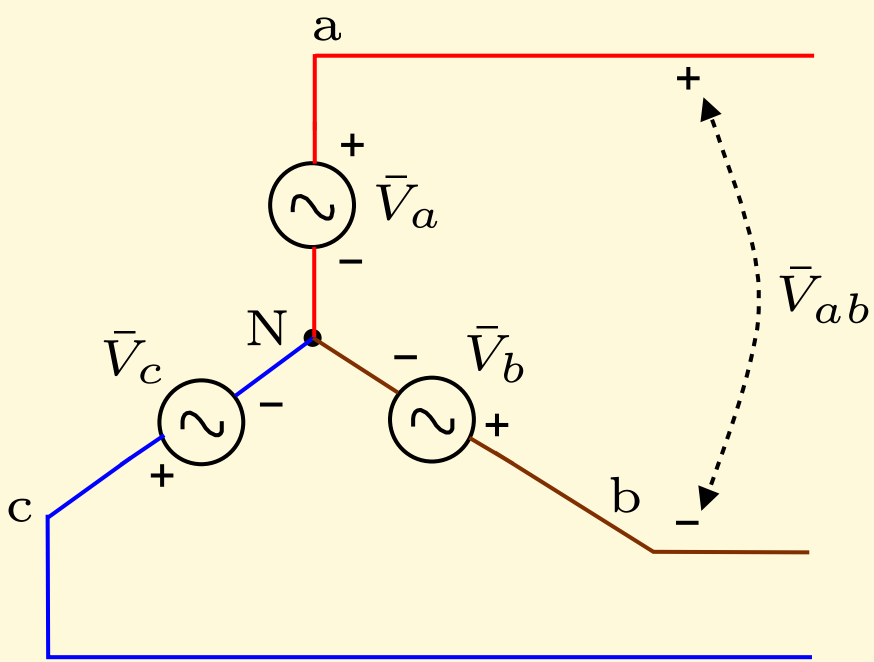

$\bar{V}_a$, $\bar{V}_a$, $\bar{V}_a$: phase voltages.

$|\bar{V}_{ab}|$: **line to line voltage**.

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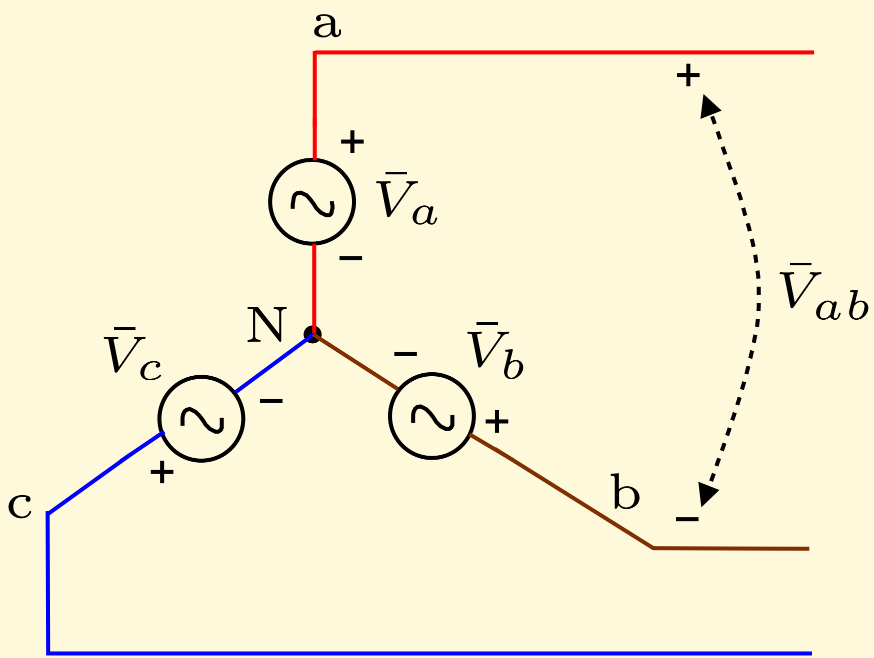

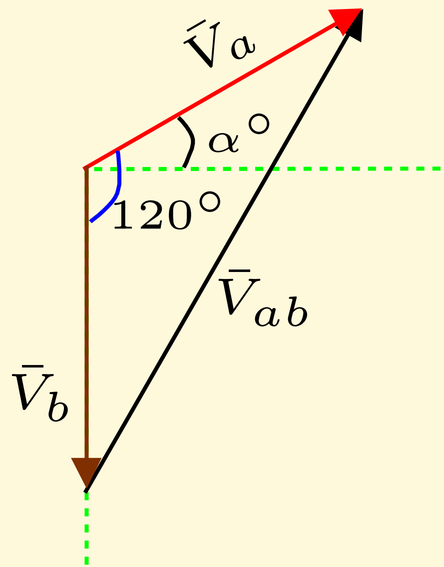

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$|\bar{V}_{ab}|$: **line to line voltage**.

$\bar{V}_{ab} = \bar{V}_a - \bar{V}_b = V_{rms}\angle\alpha - V_{rms}\angle(\alpha - 120^\circ)$

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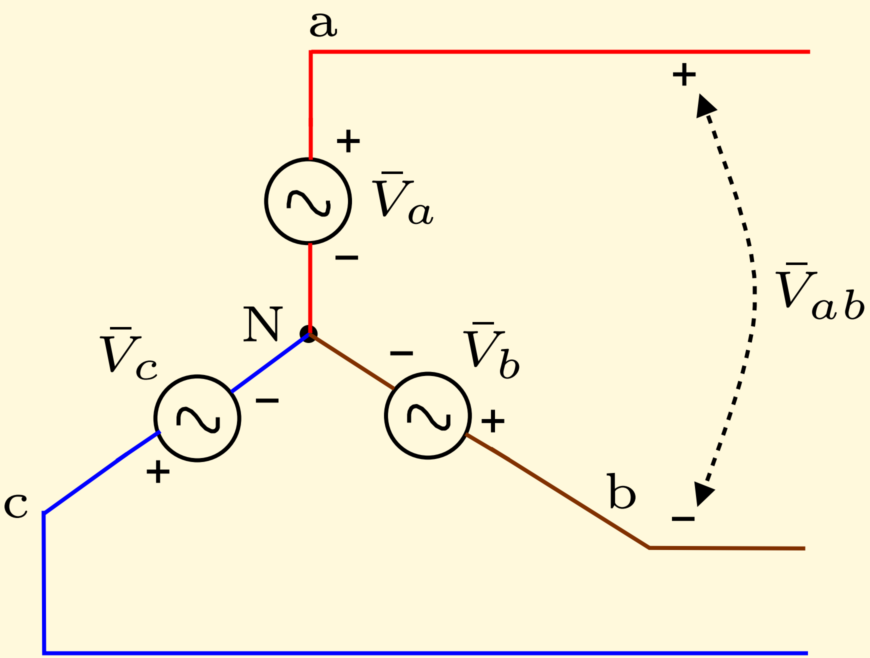

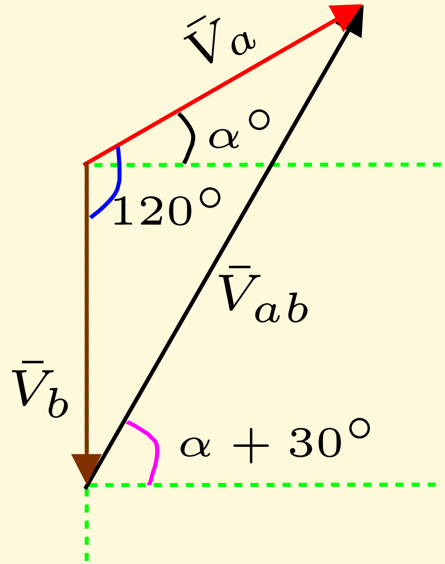

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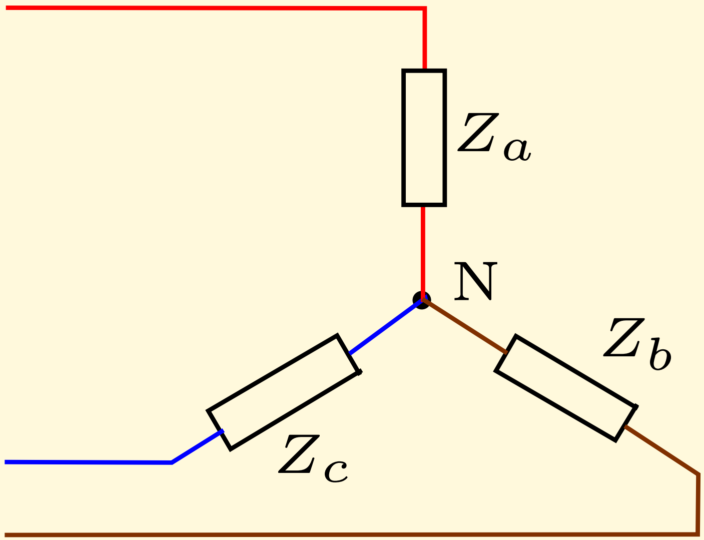
$|\bar{V}_{ab}|$: **line to line voltage**.

$\bar{V}_{ab} = \bar{V}_a - \bar{V}_b = V_{rms}\angle\alpha - V_{rms}\angle(\alpha - 120^\circ)$

$\bar{V}_{ab} = \sqrt{3} \times V_{rms}\angle(\alpha + 30)^\circ$ V.

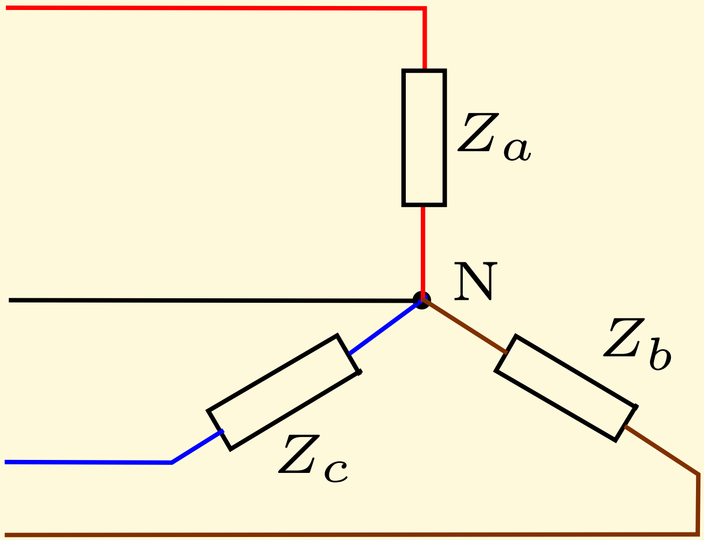
Load configurations ...

Star connected three wire system



Load configurations ...

Star connected four wire system



Load configurations ...

Delta connected three wire system