Apply of Bellman's equation for Q function

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Why this equation is important

$$Q_i(x, u) = R(x_i, u_i) + \mathbb{E}_e\left[\max_{u'} Q_{i+1}\left(f(x_i, u_i, e_i), u'\right)\right]$$

- tool for solving optimal value problems
- the basis of policy iteration in RL
- It allows us to express the value of one state as the value of another state [1]
- the dynamic optimization problem is turned into simple sub-problems
- simplify the reinforcement learning or Markov decision problems. Just make complex problems simple!

Its limitations

Linear equations, theoretically the solution can be solved:

$$\begin{aligned} \mathrm{v} &= \mathcal{R} + \gamma \mathcal{P} \mathrm{v} \\ (\mathrm{I} - \gamma \mathcal{P})\mathrm{v} &= \mathcal{R} \\ \mathrm{v} &= (\mathrm{I} - \gamma \mathcal{P})^{-1} \mathcal{R} \end{aligned}$$

- complexity is $\mathrm{O}\left(\mathrm{n}^3\right)$
- only suitable for small-scale MRPS.
- large-scale MRP usually needs to use other iterative method.

Its origin

- Markov Decision Processes (MDP)
  - Elements
  - MDP problem
  - Optimal Policy

Dynamic Programming (DP)

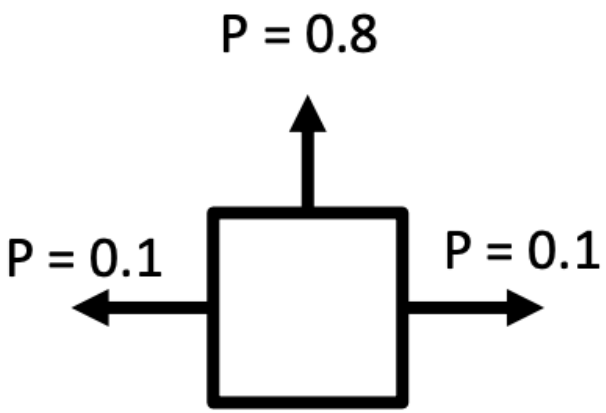
- Solving MDP using DP
- Policy Iteration
- Value Iteration

$V_{k+1}(s) = \max_a \left[r(s,a) + \gamma \sum_{s'} p\left(s' \mid s,a\right) V_k\left(s'\right) \right] v$

- immediate reward $r(s,a)$
- discounted value of successor state $\gamma \sum_{s'} p\left(s' \mid s,a\right) V_k\left(s'\right)$

Example

Selected direction with probability 0.8 and in the perpendicular directions with prob. 0.1 and the rewords discount is 0.9



0	0	0	1
0		0	-100
0	0	0	0

Rewards

$\gamma = 0.9$

Example

$V_k(s)$

k=1			
0	0	0.72	1.81
0		0	-99.9
0	0	0	0

k=5			
0.81	1.60	2.47	3.74
0.27		0.30	-99.5
0	0.03	0.12	0.00

k=10			
2.68	3.53	4.40	5.81
2.02		1.09	-98.8
1.39	0.90	0.74	0.12

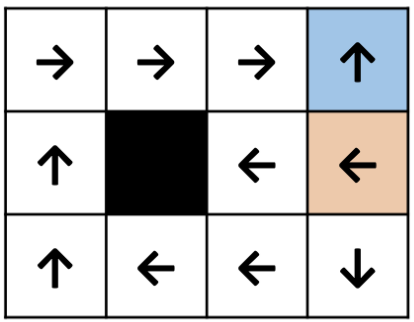
k=1000			
5.47	6.31	7.19	8.67
4.80		3.34	-96.7
4.16	3.65	3.22	1.53

k=1000

5.47	6.31	7.19	8.67
4.80		3.34	-96.7
4.16	3.65	3.22	1.53

$V_k(s)$

$$\pi(s) = \arg\max_a \left[r(s,a) + \gamma \sum_{s'} p(s'|s,a) V(s') \right]$$



Optimal Policy

Improvement

The solution of large-scale MRP usually needs to use iterative method. Except the DP, some other iteration methods are also used:

- Monte-Carlo evaluation [2]
  - agent run multiple
  - sampling average value and update
- Temporal-Difference learning [3]
  - don't need complete environment
  - improvement on existing estimates without entire knowledge.

Thanks for your attention

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