Advanced Robot Control and Learning

Munich Institute of Robotics and Machine Intelligence

Peng,Xie

Technische Universität München Arcisstraße 21, 80333 München

February, 2023

Abstract

Advanced Robot Control and Learning is a field that focuses on developing advanced control systems and learning algorithms for robots to operate more efficiently and effectively in complex and dynamic environments. This field leverages cutting-edge technologies such as artificial intelligence, machine learning, computer vision, and sensor fusion to improve the performance of robots. The article is split into two sections. The first section provides an overview of the theoretical knowledge of mechanical machinery grounded in Newtonian mechanics, which includes the topics of space conversion and the force analysis of a robot arm. The latter section focuses primarily on machine learning.

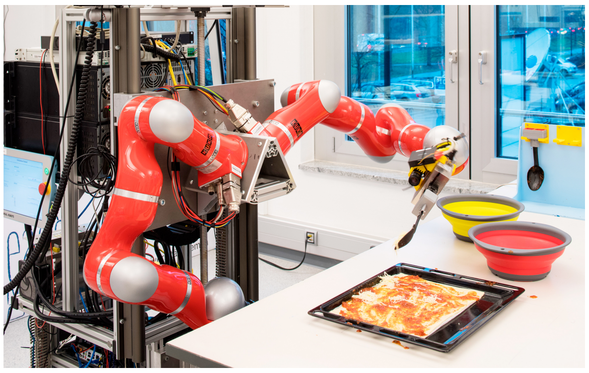


Figure 1: Artificial intelligence: degree courses in Germany

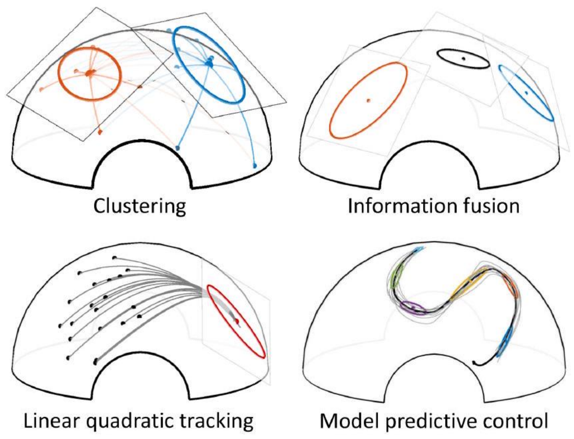


Figure 2: Gaussian-based representation on Riemannian manifolds

1. Differential Geometry

Differential geometry is an important tool in robotics, as it allows robots to navigate complex surfaces with ease. It can be used to model the shape of objects, determine the motion of the robot, and even provide information about the environment.

1.1 Riemannian manifolds

Two basic notions of Riemannian geometry are crucial for robot learning and adaptive control applications

Geodesics: Geodesics refer to the shortest possible paths between two points on a Riemannian manifold. Just like straight lines in Euclidean space, the second derivative of a geodesic is constant and equal to zero along its entire length.The exponential map $\mathrm{Exp}_{\boldsymbol{x}_0} : \mathcal{T}_{\boldsymbol{x}_0}\mathcal{M} \rightarrow \mathcal{M}$ maps a point $\boldsymbol{u}$ in the tangent space of $\boldsymbol{x}_0$ to a point $\boldsymbol{x}$ on the manifold, so that $\boldsymbol{x}$ lies on the geodesic starting at $\boldsymbol{x}_0$ in the direction $\boldsymbol{u}$. The norm of $\boldsymbol{u}$ is equal to the geodesic distance between $\boldsymbol{x}_0$ and $\boldsymbol{x}$. The inverse map is called the logarithmic map $\log_{\boldsymbol{x}_0} : \mathcal{M} \rightarrow \mathcal{T}_{\boldsymbol{x}_0}\mathcal{M}$.

Parallel transport:$\Gamma_{g\rightarrow h} : \mathcal{T}_g\mathcal{M} \rightarrow \mathcal{T}_{\boldsymbol{h}}\mathcal{M}$The process of moving vectors between tangent spaces while maintaining their inner product is facilitated by the use of a connection. A connection defines the relationship between vectors in tangent spaces that are infinitesimally close to each other. This connection enables a smooth transport of a vector from one tangent space to another by sliding it along a curve with tiny, incremental moves.

1.2 Manifolds in robot applications

The most common manifolds in robotics Two Lie

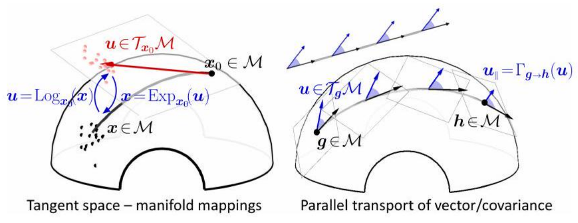

$u \in \mathcal{T}_{x_0}\mathcal{M}$
$x_0 \in \mathcal{M}$
$u = \mathrm{Log}_{x_0}(x)$
$x = \mathrm{Exp}_{x_0}(u)$
$x \in \mathcal{M}$
$u \in \mathcal{T}_g\mathcal{M}$
$u_\parallel = \Gamma_{g\to h}(u)$
$h \in \mathcal{M}$
$g \in \mathcal{M}$
Tangent space – manifold mappings
Parallel transport of vector/covariance

Figure 3: Applications in robotics using Riemannian manifolds

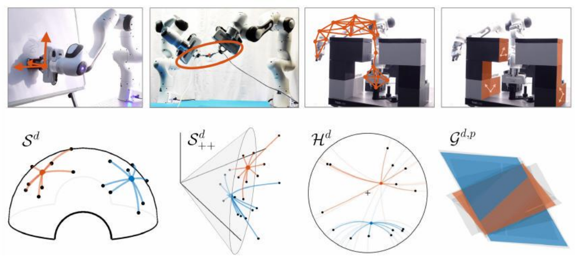


Figure 4: Structured manifolds in robotics

groups: the special orthogonal group SO(3) and the special Euclidean group SE(3)

- **The sphere manifold** $\mathcal{S}^d$ is a key concept in robotics for encoding directions and orientations. This is achieved by using unit quaternions $\mathcal{S}^3$ to depict the orientation of the end effector (tool tip). Surfaces' perpendicular unit directional vector, required for contact planning, can be represented through $\mathcal{S}^2$. Articulatory joints in robotics can be symbolized on the torus $\mathcal{S}^1 \times \mathcal{S}^1 \times \ldots \times \mathcal{S}^1$. The Kendall shape space, utilized to encode 3D skeletal motion capture data, is also based on unit spheres.
- **The special orthogonal group** SO(d) represents the group of rotations around the origin in a d-dimensional space. The groups SO(2) and SO(3) are widely used in robotics.
- **The special Euclidean group** SE(3) is the group of rigid body transformations, which is comprised of rotations and translations. The geometry of SE(3) can be used to describe the kinematics and the Jacobian of robots, making it a popular choice for describing robot motion and pose estimation. $\mathbb{R}^d$.
- **The manifold of symmetric positive definite (SPD) matrices** $\mathcal{S}^d_{++}$ is utilized in a variety of ways within robotics.
- **Hyperbolic manifolds** $\mathcal{H}^d$ are the analogues of spheres with constant negative curvature instead of constant positive curvature.
- **Grassmannian** $\mathcal{G}^{d,p}$ is the manifold of all p-dimensional subspaces of $\mathbb{R}^d$. [2]

2. Basics of Task Space Modeling and Control

2.1 Inverse Kinematics

The inverse kinematics of the system translates position, velocity, and acceleration from Cartesian space backward to configuration space and string lengths. Therefore admissible control inputs for the system are obtained via the inverse kinematics of desired trajectories in Cartesian space. First, we define a vector for each space, collecting all variables in a specific order.

$$\underline{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} \quad \underline{c} = \begin{bmatrix} r \\ \theta \\ \varphi \end{bmatrix} \quad \underline{e} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The function mapping positions in Cartesian space to configuration space is called $\underline{c}(\underline{e})$ and is defined as

$$\underline{c}(\underline{e}) = \begin{bmatrix} r(\underline{e}) \\ \theta(\underline{e}) \\ \varphi(\underline{e}) \end{bmatrix} = \begin{bmatrix} \frac{x^2+y^2+z^2}{2\sqrt{x^2+y^2}} \\ \cos^{-1}\left(\frac{-x^2-y^2+z^2}{x^2+y^2+z^2}\right) \\ \operatorname{atan} 2(y,x) \end{bmatrix}$$

2.2 Control Theory

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \tau$$

where $\mathbf{q} \in \mathrm{Re}^n$ is the joint angle vector, $\mathbf{M}(\mathbf{q})$ is the inertia for complex humanoids. $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the Coriolis/centrifugal vector. $\mathbf{g}(\mathbf{q})$ is the ultimate gravity vector and τ is the joint torque vector.

$$g(\mathbf{q}) = \frac{1}{2}\left(\mathbf{q} - \mathbf{q}_{\text{rest}}\right)^T \mathbf{K}_w \left(\mathbf{q} - \mathbf{q}_{\text{rest}}\right)$$

where $q \in \mathrm{Re}^n$ is the joint angle vector, $\mathbf{M}(\mathbf{q})$ is the inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the Coriolis/centrifugal vector, $\mathbf{g}(\mathbf{q})$ is the gravity vector, and τ is the joint torque vector. The forward kinematics and differential relationship between the joint coordinates and the task coordinates are given as

$$\begin{aligned} \mathbf{x} &= \mathbf{f}(\mathbf{q}) \\ \dot{\mathbf{x}} &= \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \\ \ddot{\mathbf{x}} &= \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} \end{aligned}$$

where $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix and $\mathbf{x} \in \mathrm{Re}^m$, where $m < n$, is the task coordinate vector. The

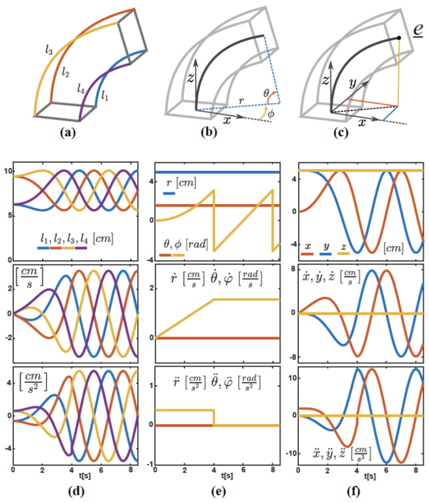


Figure 5: (a) actuator space. (b) configuration space. (c) Cartesian space. (d) kinematics in actuator space. (e) kinematics in configuration space. (f) kinematics in Cartesian space. rigid body dynamics equations

basic idea in redundancy resolution in the velocity and acceleration levels is to compute the desired joint velocities and accelerations respectively as

$$\dot{\mathbf{q}}_d = \mathbf{J}^\dagger \dot{\mathbf{x}}_d + \left(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}\right) \boldsymbol{\xi}_1$$
$$\ddot{\mathbf{q}}_d = \mathbf{J}^\dagger \left(\ddot{\mathbf{x}}_d - \dot{\mathbf{J}}\dot{\mathbf{q}}\right) + \left(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}\right) \boldsymbol{\xi}_2$$

where $\mathbf{J}^\dagger$ is the pseudoinverse defined by $\mathbf{J}^\dagger = \mathbf{J}^T \left(\mathbf{J}\mathbf{J}^T\right)^{-1}$, ξ_1 and ξ_2 are arbitrary vectors, and $\dot{\mathbf{x}}_d$ and $\ddot{\mathbf{x}}_d$ are the given desired task space velocities and accelerations, respectively. $\left(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}\right)$ projects arbitrary vectors ξ_1 and ξ_2 onto the null space of the Jacobian J. Note that by equating and the analytical differentiation, sufficient conditions for these two resolution schemes to be consistent are derived, respectively, as

$$\boldsymbol{\xi}_2 = \dot{\mathbf{J}}^\dagger \mathbf{J} \left(\dot{\mathbf{q}} - \boldsymbol{\xi}_1\right) + \dot{\xi}_1$$

and

$$\xi_2 = \dot{\mathbf{J}}^T \left(\mathbf{J}^\dagger\right)^T \left(\dot{\mathbf{q}} - \boldsymbol{\xi}_1\right) + \dot{\xi}_1$$

where $\mathbf{J}^\dagger = \frac{d}{dt}\left(\mathbf{J}^\dagger\right)$ for notational convenience. In the torque based redundancy resolution, the inertia weighted pseudoinverse $\overline{\mathbf{J}} = \mathbf{M}^{-1}\mathbf{J}^T \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1}$ is used to compute the desired joint torque command exploiting the dynamical decoupling property of the joint torques that create task space forces, and the joint torques that act only in the null space of the task.[8]

3. Modern Methods of Robot Control

3.1 Definition of Passivity

The system S with $m = q$ (equivalent number of input and output variables) is called passive if it is dissipative with respect to the special supply rate

$$s(\underline{u}, \underline{y}) = \underline{y}^T \underline{u}.$$

In other words, there exists a positive semidefinite function $V(\underline{x})$, such that the integral passivity inequality

$$\int_0^t \underline{y}^T \underline{u} d\tau + V(\underline{x}(0)) \geq V(\underline{x}(t)), \quad \forall t \geq 0$$

holds for all initial values $\underline{x}_0$ and all input variables $\underline{u}(t)$.

3.2 Stability and Passivity – Lyapunov Stability

An equilibrium point $\underline{x}^* = \underline{0}$ of the system is - stable in the sense of Lyapunov (i.S.o.L) if for every $\varepsilon > 0$, there exists a $\delta\left(\varepsilon, t_0\right) > 0$ such that

$$\left\|\underline{x}\left(t_0\right)\right\| < \delta \Rightarrow \|\underline{x}(t)\| < \varepsilon, \forall t \geq t_0.$$

- - asymptotically stable in the sense of Lyapunov (i.S.o.L) if $\underline{x}^*$ is stable and $\delta\left(\varepsilon, t_0\right) > 0$ is chosen in a way such that

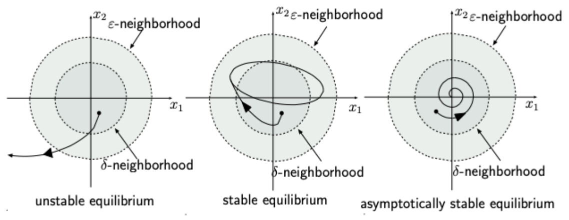
$$\left\|\underline{x}\left(t_0\right)\right\| < \delta \Rightarrow \lim_{t \to \infty} \|\underline{x}(t)\| = 0.$$

- - uniformly stable in the sense of Lyapunov (i.S.o.L) if for every $\varepsilon > 0$ there exists a $\delta(\varepsilon) > 0$ independent of t_0 such that

$$\left\|\underline{x}\left(t_0\right)\right\| < \delta \Rightarrow \|\underline{x}(t)\| < \varepsilon, \forall t \geq t_0.$$

- - uniformly asymptotically stable in the sense of Lyapunov (i.S.o.L), if $\underline{x}^*$ is uniformly stable and $\delta(\varepsilon) > 0$ is independent of t_0, such that $\left\|\underline{x}\left(t_0\right)\right\| < \delta \Rightarrow \lim_{t \to \infty} \|\underline{x}(t)\| = 0$ and $\underline{x}(t)$ converges uniformly in t_0. Uniform convergence of a trajectory $\underline{x}(t)$ in t_0 means that there exists a time interval $T\left(R_1, R_2\right)$ for every pair R_1, R_2 with $0 < R_2 < R_1$ independent of t_0, such that from $\left\|\underline{x}\left(t_0\right)\right\| < R_1$ for all time $t \geq t_0 + T\left(R_1, R_2\right)$ it follows that $\|\underline{x}(t)\| < R_2$ holds.
- - unstable if $\underline{x}^*$ is not stable.

[6]



x_2 ε-neighborhood
x_1
δ-neighborhood
unstable equilibrium
stable equilibrium
asymptotically stable equilibrium

Figure 6: Lyapunov-stability

3.3 Passive Representation of a Robot

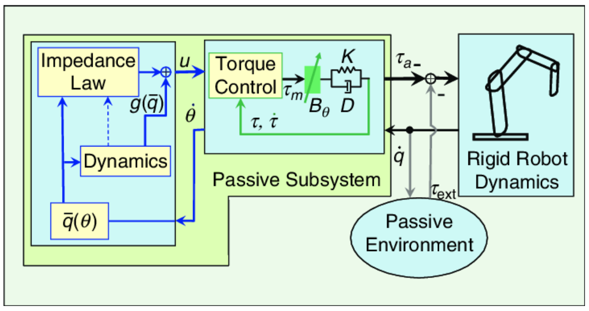


Figure 7: Papline of Passive Representation

3.4 Control Algorithm

3.4.1 Velocity based Control

Given the velocities and positions of the target in task space, we define the task space velocity command $\dot{\mathbf{x}}_r$ from the addion of a position error term with a target velocity feedforward term as

$$\dot{\mathbf{x}}_r = \dot{\mathbf{x}}_d + \mathbf{K}_p (\mathbf{x}_d - \mathbf{x})$$

where $\mathbf{K}_p$ is a positive definite gain matrix. Note that this scheme does not use the target acceleration information $\ddot{\mathbf{x}}_d$. The reference joint velocities are calculated based on a pseudoinverse solution with minimization of the cost function in the null space

$$\dot{\mathbf{q}}_r = \mathbf{J}^\dagger \dot{\mathbf{x}}_r - \left(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}\right) \alpha \nabla g.$$

The null-space vector is chosen to be the negative gradient of the cost function as $\xi_1 = -\alpha \nabla g = -\alpha \left(\frac{\partial q}{\partial q}\right)^T$ where α is a positive constant. The reference joint accelerations and positions are obtained by numerical differentiation and integration of the reference joint velocities (11), respectively as

$$\ddot{\mathbf{q}}_r = \frac{d}{dt}\dot{\mathbf{q}}_r \simeq \frac{\dot{\mathbf{q}}_r(t) - \dot{\mathbf{q}}_r(t - \Delta t)}{\Delta t}$$
$$\mathbf{q}_r = \int_{t_0}^{t} \dot{\mathbf{q}}_r dt' \simeq \mathbf{q}_r(t - \Delta t) + \dot{\mathbf{q}}_r(t)\Delta t$$

where Δt is the sampling period. The final control signal is calculated using the computed torque control method with a PD controller as

$$\begin{aligned} \boldsymbol{\tau} =& \mathbf{M}(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}_r, \dot{\mathbf{q}}_r) + \mathbf{g}(\mathbf{q}_r) \\ &+ \mathbf{K}_{q,d}(\dot{\mathbf{q}}_r - \dot{\mathbf{q}}) + \mathbf{K}_{q,p}(\mathbf{q}_r - \mathbf{q}) \end{aligned}$$

where $\mathbf{K}_{q,d}$ and $\mathbf{K}_{q,p}$ are positive definite gain matrices.

3.4.2 Velocity-based Control without Joint Velocity Integration

Task space velocity and the reference joint velocities obtained from the Liegeois' null-space optimization,

$$\begin{aligned} \dot{\mathbf{x}}_r &= \dot{\mathbf{x}}_d + \mathbf{K}_p (\mathbf{x}_d - \mathbf{x}) \\ \dot{\mathbf{q}}r &= \mathbf{J}^\dagger \dot{\mathbf{x}}_r - \alpha \left(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}\right) \nabla g. \end{aligned}$$

An inverse dynamics control law can be formulated with the addition of a joint velocity feedback term as

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{K}_{q,d}(\dot{\mathbf{q}}_r - \dot{\mathbf{q}})$$

$$\begin{aligned} \boldsymbol{\tau} =& \mathbf{M}\ddot{\mathbf{q}}_r + \mathbf{C} + \mathbf{g} + \mathbf{K}_{q,d}(\dot{\mathbf{q}}_r - \dot{\mathbf{q}}) \\ =& \mathbf{M}\left(\ddot{\mathbf{q}}_d + \mathbf{J}^\dagger \mathbf{K}_p \dot{\mathbf{e}}\right) + \mathbf{C} + \mathbf{g} + \mathbf{K}_{q,d}(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) \\ &+ \mathbf{K}_{q,d}\mathbf{J}^\dagger \mathbf{K}_p \mathbf{e} \end{aligned}$$

with $\xi_1 = -\alpha \nabla g$, where $\mathbf{e} = \mathbf{x}_d - \mathbf{x}$.

3.4.3 Acceleration based Control

In this method, the control law is given by where

$$\ddot{\mathbf{x}}_r = \ddot{\mathbf{x}}_d + \mathbf{K}_d \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e}$$

3.4.4 Torque based Control

The dynamics which describe the motion of the task space coordinates are given by

$$\overline{\mathbf{M}}(\mathbf{x})\ddot{\mathbf{x}} + \overline{\mathbf{C}}(\mathbf{x}, \dot{\mathbf{x}}) + \overline{\mathbf{g}}(\mathbf{x}) = \mathbf{F}$$

where

$$\begin{aligned} \overline{\mathbf{M}} &= \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1} \\ \overline{\mathbf{C}} &= \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1} \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{C} - \dot{\mathbf{J}}\dot{\mathbf{q}}\right) \\ \overline{\mathbf{g}} &= \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1} \mathbf{J}\mathbf{M}^{-1}\mathbf{g}. \end{aligned}$$

From these equations, a control law for the task space dynamics is designed to achieve asymptotic tracking of the desired end-effector trajectory $\mathbf{x}_d$ as

$$\mathbf{F} = \overline{\mathbf{M}}\ddot{\mathbf{x}}_r + \overline{\mathbf{C}} + \overline{\mathbf{g}}$$

where

$$\ddot{\mathbf{x}}_r = \ddot{\mathbf{x}}_d + \mathbf{K}_p(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}).$$

$$\ddot{\mathbf{e}} + \mathbf{K}_p \dot{\mathbf{e}} + \mathbf{K}_p \mathbf{e} = 0$$

3.5 Controller Example

3.5.1 Open-Loop Controller

In the beginning, we implemented an open-loop version of the computed torque controller and tested the impact of friction on the system. The function of the controller is given as while the friction is a constant term acting against the motion. Therefore the system equation is

$$\ddot{\underline{l}} = M(\underline{l})^{-1}\left(\tau_l - C(\underline{l},\dot{\underline{l}})\dot{\underline{l}} - G(\underline{l})\right) - K\,\mathrm{sign}(\dot{\underline{l}}),$$

where K is a constant, chosen to be 0.4 N.

3.5.2 Closed-Loop Controller

To counteract deviations in motion created by unaccounted forces and dampen the system, we implemented a closed-loop version of the computed torque controller. By adding a PD controller to the computed torque controller. The controller function is extended to

$$\tau_l = M(\underline{l})\ddot{\underline{l}}_d + C(\underline{l},\dot{\underline{l}})\dot{\underline{l}} + G(\underline{l}) + K_P\left(\underline{l}_d - \underline{l}\right) + K_D\left(\dot{\underline{l}}_d - \dot{\underline{j}}\right),$$

Once we add friction to the system, the open-loop system outputs are not able to follow the system inputs anymore. This can be seen in Fig 3(c). On the other side, the closed loop controller can balance out the friction and the system outputs follow the inputs, which can be seen in (d). Therefore the closed-loop system is stable and can reject small forces generated by friction.[7]

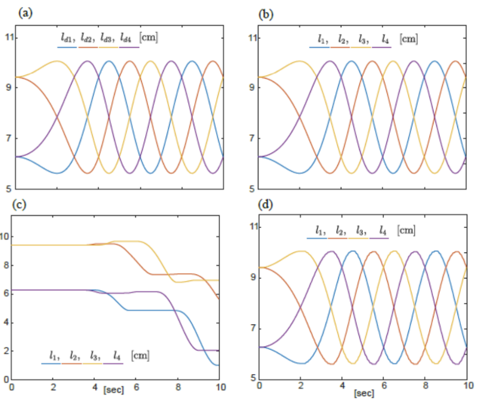


Figure 8: (a) Desired Trajectory, (b) System output of open loop controller with no friction, (c) System output of open loop controller with friction, (d) System output of closed-loop controller with friction.

4. Port-Hamiltonian Systems in Robotics

The port-Hamiltonian framework is a methodology for modeling physical systems based on a known energy function, known as the Hamiltonian ($\mathcal{H}$). This framework provides an energy-consistent description of a system and is particularly useful for modeling complex, interconnected systems. Each individual element in the system is defined by its own Hamiltonian energy function and interacts with other elements by exchanging energy through a "port". These ports are described by a pair of variables, referred to as efforts (e.f) and flows, that represent the input and output of the system respectively. The product of these two variables is referred to as power. A port-Hamiltonian system is described in an input-state-output form, and its behavior is defined by the Hamiltonian function is:

$$\begin{aligned}\dot{\boldsymbol{x}} &= [J(\boldsymbol{x}) - D(\boldsymbol{x})]\frac{\partial \mathcal{H}}{\partial \boldsymbol{x}}(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u}\\ \boldsymbol{y} &= G^T(\boldsymbol{x})\frac{\partial \mathcal{H}}{\partial \boldsymbol{x}}(\boldsymbol{x}),\end{aligned}$$

where $J(\boldsymbol{x})$ is a skew-symmetric structure matrix and $D(\boldsymbol{x}) \succeq 0$ is a symmetric dissipation matrix. Inputs to the system are flows, $\boldsymbol{u}$, and outputs are efforts $\boldsymbol{y}$. The matrix $G(\boldsymbol{x})$ is a mapping matrix and $\boldsymbol{x}$ is the state vector of the system. The Hamiltonian of the system is a positive, semidefinite function $\mathcal{H} \geq 0$, which represents the total energy stored in the system. The rate of the energy change of is:

$$\dot{\mathcal{H}} = \boldsymbol{y}^T\boldsymbol{u} - \frac{\partial^T \mathcal{H}}{\partial \boldsymbol{x}} D(\boldsymbol{x})\frac{\partial \mathcal{H}}{\partial \boldsymbol{x}}$$

Every port-Hamiltonian system is passive.

5. Linear Parametrization of Robot Dynamics

both the kinetic energy and the gravity potential energy can be rewritten so that a new set of dynamic parameters appears only in a linear way

5.1 fundamental kinematic relation

$v_{ci} = v_i + \omega_i \times r_{ci} = v_i + S(\omega_i)\, r_{ci} = v_i - S(r_{ci})\,\omega_i$

kinetic energy of link i

$$\begin{aligned}T_i &= \frac{1}{2}m_i v_{ci}^T v_{ci} + \frac{1}{2}\omega_i^T I_{ci}\omega_i\\ &= \frac{1}{2}m_i\left(v_i - S(r_{ci})\,\omega_i\right)^T\left(v_i - S(r_{ci})\,\omega_i\right) + \frac{1}{2}\omega_i^T I_{ci}\omega_i\\ &= \frac{1}{2}m_i v_i^T v_i + \frac{1}{2}\omega_i^T\left(I_{ci} + m_i S^T(r_{ci})\,S(r_{ci})\right)\omega_i - v_i^T S(m_i r_{ci})\,\omega_i\end{aligned}$$

$$\text{Steiner theorem} \longrightarrow I_i = \begin{pmatrix} I_{i,xx} & I_{i,xy} & I_{i,xz}\\ & I_{i,yy} & I_{i,yz}\\ \text{symm} & & I_{i,zz}\end{pmatrix}$$

5.2 gravity potential energy

the gravitational potential energy of link i

$$U_i = -m_i g_0^T r_i - g_0^T \left(m_i r_{ci}\right)$$

5.3 Overall Energies and Robot Regressor Formulation

using a $N \times 10N$ regression matrix Y_π that depends only on kinematic quantities, the robot dynamic equations can always be rewritten linearly in the standard dynamic parameters as

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = Y_\pi(q,\dot{q},\ddot{q})\pi = u$$
$$\pi^T = \begin{pmatrix} \pi_1^T & \pi_2^T & \cdots & \pi_N^T \end{pmatrix}$$

the open kinematic chain structure of the manipulator implies that the i-th dynamic equation can depend only on the dynamic parameters of links from i to N $\Rightarrow$ Y_π has a block upper triangular structure

$$\boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) = \begin{bmatrix} \boldsymbol{Y}_{1,1} & \boldsymbol{Y}_{1,2} & \boldsymbol{Y}_{1,3} & \cdots & \boldsymbol{Y}_{1,n-1} & \boldsymbol{Y}_{1,n} \\ \mathbf{0} & \boldsymbol{Y}_{2,2} & \boldsymbol{Y}_{2,3} & \cdots & \boldsymbol{Y}_{2,n-1} & \boldsymbol{Y}_{2,n} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{Y}_{3,3} & \cdots & \boldsymbol{Y}_{3,n-1} & \boldsymbol{Y}_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{Y}_{n-1,n-1} & \boldsymbol{Y}_{n-1,n} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{Y}_{n,n} \end{bmatrix}$$

Property: element m_{ij} of $M(q)$ depends at most on $(q_{k+1}, \cdots, q_N)$, with $k = \min\{i,j\}$, and on the dynamic parameters of at most links h to N, with $h = \max\{i,j\}$

6. Bio-Inspired Robot Control

One way to enhance the adaptability of robots is through the implementation of bio-inspired learning algorithms that emulate the methods used by living organisms to adapt to new situations. This can be achieved through the use of machine learning algorithms that allow robots to continuously improve their performance based on previous experiences. Furthermore, by incorporating sensory feedback, robots can adjust their behavior in real-time and respond to changes in their surroundings, leading to optimal performance.

The capability for adaptation to new dynamics is a crucial area of study in robotics, as it holds the potential to transform the field by enabling robots to carry out tasks that are currently not possible. Researchers aim to create machines that are capable of adapting to changing environments and conditions, leading to more efficient and effective performance in real-world settings.

6.1 Adaption to Novel Dynamics

Adaptation to novel dynamics: from humans to robot.

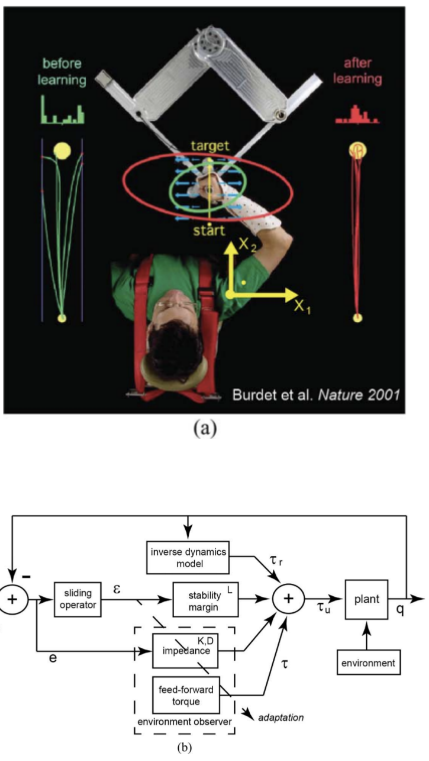


Figure 9: (a) Investigation of point-to-point arm movements with lateral instability. (b) Control diagram of the biomimetic controller that is presented in this paper and was derived from the results of this investigation.

[9]

The control of robots using biological inspiration involves designing robotic systems that emulate the behaviors and movements of living organisms. The aim is to create more efficient and adaptive machines by replicating the control mechanisms found in nature, such as the nervous system, in robots. This is done by using advanced sensors, algorithms, and control systems that allow robots to respond to environmental stimuli in a similar way to living organisms. As a result, robots with bio-inspired control are capable of performing challenging tasks, such as adjusting to new environments, navigating complex terrains, and executing delicate manipulations.[3] Adaptation to novel dynamics refers to the ability of a system to adjust and modify its behavior in response to changes in its environment or operating conditions. In the context of bio-inspired robot control, this means designing robots that can adapt to new or unexpected dynamics in real-time. This is a key feature for robots that are intended to operate in complex and changing environments, where they must be able to respond to different scenarios and conditions.[4]

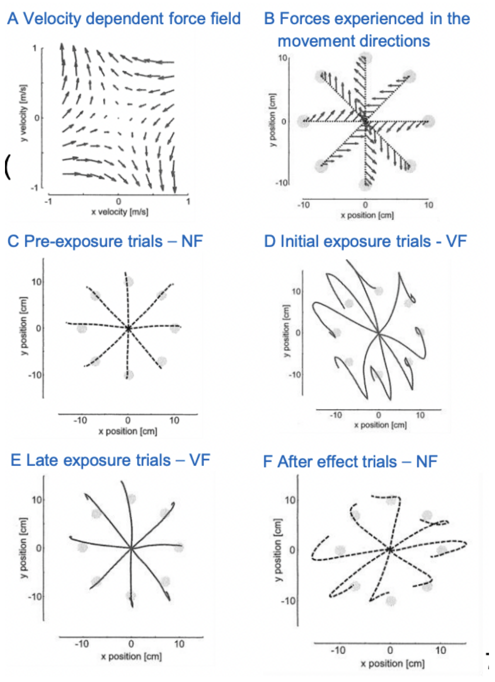


Figure 10: Adaption to Novel Dynamics

7. ML in Robotics

7.1 Neural Networks

Neural networks are a type of artificial intelligence system modeled after the structure and function of the human brain. They are widely used in various applications, including image recognition, natural language processing, and control systems for robots. Neural networks are particularly effective for tasks that require pattern recognition and classification, as they can learn and identify complex relationships between inputs and outputs. In the field of robotics, neural networks are utilized to develop advanced control systems that allow robots to respond to environmental stimuli and operate more efficiently.

This is achieved by using neural networks to process sensory information and generate control signals that drive the robot's actuators. The network is trained using a large dataset of examples, and it continuously improves its performance over time.

One key advantage of using neural networks in robotics is their ability to learn from experience and adapt to changing conditions. This is crucial in real-world applications where robots must operate in unpredictable and dynamic environments. For example, robots equipped with neural networks can adapt to different terrains, obstacles, and other factors that may affect their movements, allowing them to perform tasks that are difficult or impossible for traditional robots.

Another application of neural networks in robotics is in the development of bio-inspired robots, which aim to replicate the behaviors and movements of living organisms. This is achieved by incorporating biologically-inspired features, such as sensory feedback, into the robot's control system. The robot's neural network then processes this information and generates control signals that drive its actuators, enabling it to respond to environmental stimuli in a manner similar to living organisms.

In conclusion, the integration of neural networks in robotics has the potential to revolutionize the field by enabling robots to perform tasks that are beyond their current capabilities. By incorporating advanced AI algorithms, robots can adapt to changing environments, navigate complex terrains, and perform delicate manipulations, leading to new possibilities and applications for these systems in various industries.

7.2 3 Categories

Machine Learning can be divided into three main categories of problems

7.2.1 Supervised Learning

Supervised learning is a type of machine learning that trains a model to make predictions based on labeled data. In robotics, this approach is used for tasks such as object recognition, scene understanding, and control.

Supervised learning is beneficial for robots operating in dynamic environments as they can learn from examples and improve their performance over time.

For example, a robot can learn to perform a task by observing a human demonstration and continuously improving its performance through training. Another application of supervised learning in robotics is in the development of autonomous robots, allowing them to perform tasks such as navigation, grasping, and manipulation more efficiently. The use of supervised learning in robotics has the potential to revolutionize the field and lead to new possibilities for these systems.

7.2.2 Unsupervised Learning

Unsupervised learning is a type of machine learning where the model is not provided with labeled data, but instead must find patterns and relationships in the data on its own. In robotics, unsupervised learning can be used for tasks such as dimensionality reduction, clustering, and anomaly

detection.
One application of unsupervised learning in robotics is in the development of self-organizing maps, which can be used for visual recognition and navigation. In these systems, the robot uses unsupervised learning to map its environment and recognize objects and obstacles.
Another application of unsupervised learning in robotics is in the development of adaptive controllers, which can be used to improve the performance of robots in dynamic environments. By using unsupervised learning, robots can learn and adapt to new dynamics, allowing them to operate more effectively and efficiently in real-world settings.
In conclusion, unsupervised learning is an important tool in robotics, as it allows robots to learn from data without the need for labeled examples. This approach can be used for tasks such as visual recognition, navigation, and adaptive control, and has the potential to revolutionize the field and lead to new possibilities for these systems.

7.2.3 Reinforcement Learning

Reinforcement learning is a type of machine learning that trains a model to make decisions by learning from experiences. In this approach, the model receives rewards for certain actions and learns to optimize its behavior over time to maximize the reward. In robotics, reinforcement learning is used for tasks such as autonomous navigation, grasping, and manipulation.
One application of reinforcement learning in robotics is in the development of autonomous robots, which must be able to make decisions and take actions based on the state of their environment. By using reinforcement learning, robots can learn to perform tasks such as navigation and grasping by receiving rewards for successful actions and adjusting their behavior accordingly.
Another application of reinforcement learning in robotics is in the development of control systems, where the model can learn to control the robot's movements to achieve a desired goal. For example, a robot equipped with a reinforcement learning system can learn to balance on a moving platform by continuously adjusting its behavior based on the rewards it receives.
In conclusion, reinforcement learning is an important tool in robotics, allowing robots to learn and make decisions based on experiences. By incorporating reinforcement learning into their control systems, robots can adapt to changing conditions and perform tasks that are beyond their current capabilities. The use of reinforcement learning in robotics has the potential to revolutionize the field and lead to new possibilities for these systems. [5]

7.3 Optimal Control- Bellman's Equation

$Q_i(x, u) = R(x_i, u_i) + \mathbb{E}_e \left[\max_{u'} Q_{i+1}\left(f(x_i, u_i, e_i), u'\right)\right]$

- tool for solving optimal value problems
- the basis of policy iteration in RL
- It allows us to express the value of one state as the value of another state[6]
- the dynamic optimization problem is turned into simple sub-problems
- simplify the reinforcement learning or Markov decision problems. Just make complex problems simple!

Linear equations, theoretically the solution can be solved:

$$\begin{aligned} v &= \mathcal{R} + \gamma \mathcal{P} v \\ (I - \gamma \mathcal{P}) v &= \mathcal{R} \\ v &= (I - \gamma \mathcal{P})^{-1} \mathcal{R} \end{aligned}$$

- complexity is $O\left(n^3\right)$
- only suitable for small-scale MRPS.
- large-scale MRP usually needs to use other iterative methods.

Markov Decision Processes (MDP)

- Elements
- MDP problem
- Optimal Policy

Dynamic Programming (DP)

- Solving MDP using DP
- Policy Iteration
- Value Iteration

$V_{k+1}(s) = \max_a \left[r(s, a) + \gamma \sum_{s'} p\left(s' \mid s, a\right) V_k\left(s'\right)\right]$v

- immediate reward $r(s, a)$
- discounted value of successor state $\gamma \sum_{s'} p\left(s' \mid s, a\right) V_k\left(s'\right)$

Selected direction with probability 0.8 and in the perpendicular directions with prob. 0.1 and the rewards discount is 0.9[1]

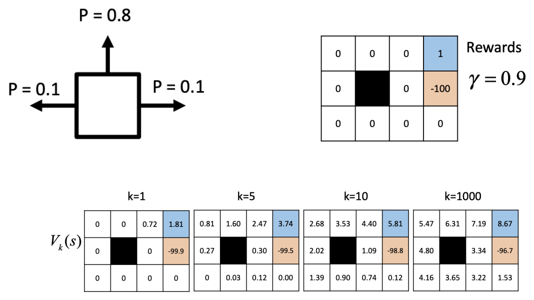


Figure 11: Example

7.4 Examples

7.4.1 Supervised Learning in Robotics (Computer Vision)

Figure 13: Autonomous Driving (Tesla)

Figure 14: space exploration (NASA)

Figure 15: Humanoid Locomotion (Boston Dynamics)

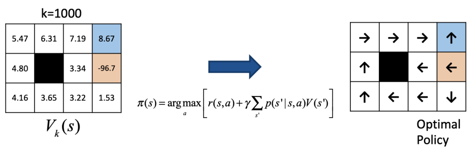


Figure 12: Final Result

7.5 Reinforcement Learning in Robotics

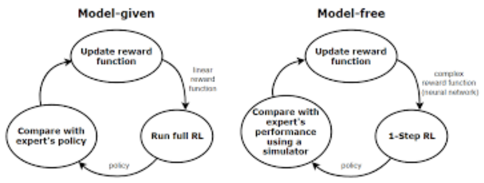

Model-given
Update reward function
linear reward function
Run full RL
policy
Compare with expert's policy
Model-free
Update reward function
complex reward function (neural network)
1-Step RL
policy
Compare with expert's performance using a simulator

Figure 16: Imitation Learning

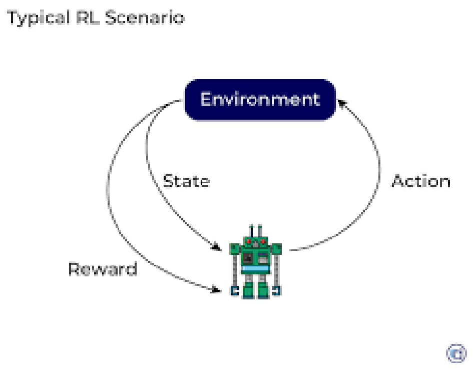


Figure 17: Reinforcement learning

8. Overview of Robot Perception

Robot perception refers to the process by which a robot collects, processes, and interprets sensory data from its environment in order to make informed decisions about its surroundings. This is a crucial aspect of robotics, as it enables robots to interact with their environment and perform tasks more effectively. There are several key components to robot perception, including:

- Sensors: Robots use various sensors, such as cameras, depth sensors, LIDAR, and microphones, to gather information about their environment. The type of sensor used will depend on the task the robot is performing and the information it needs to gather.

- Signal Processing: Once the data is collected by the sensors, it must be processed in order to extract useful information. This involves filtering the raw data, removing noise, and transforming the data into a format that can be used by the robot's control system.

- Feature Extraction: The processed data is then analyzed to extract features that are relevant to the task the robot is performing. These features may include the position of objects, their shape, size, and color, and any other relevant information.

- Object Recognition: The extracted features are used to identify objects in the environment and categorize them. This may involve comparing the features to a database of known objects, or using machine learning algorithms to identify patterns in the data.

- Scene Understanding: The final step in robot perception is to build a comprehensive understanding of the scene, including the relationships between objects and their interactions with each other. This information is then used by the robot's control system to make informed decisions about how to interact with the environment.

In conclusion, robot perception is a complex process that involves several components, including sensors, signal processing, feature extraction, object recognition, and scene understanding. These components work together to provide the robot with a detailed understanding of its environment, allowing it to make informed decisions and interact more effectively with its surroundings.

Conclusion

Advanced robot control and learning is a rapidly evolving field that holds great promise for revolutionizing the way robots operate and interact with their environment. The integration of cutting-edge technologies, such as artificial intelligence, machine learning, computer vision, and sensor fusion, allows robots to perform more efficiently and effectively in complex and dynamic environments. The continued development of this field will likely result in robots that are better equipped to handle a wide range of tasks and environments, and that can operate with greater autonomy and adaptability. As the technology continues to advance, it will likely play an increasingly important role in many areas of society, including manufacturing, healthcare, and transportation, to name just a few. Overall, the future of advanced robot control and learning is exciting and holds immense potential for improving the quality of life for people all around the world.

References

[1]R. Caccavale, M. Saveriano, A. Finzi, and D. Lee, «Kinesthetic teaching and attentional supervision of structured tasks in human–robot interaction», Autonomous Robots **43**, 10.1007/s10514-018-9706-9 (2019) 10.1007/s10514-018-9706-9.

[2]S. Calinon, «Gaussians on riemannian manifolds: applications for robot learning and adaptive control», IEEE Robotics Automation Magazine **27**, 33–45 (2020) 10.1109/MRA.2020.2980548.

[3]S. Haddadin and E. Croft, «Erratum to: physical human–robot interaction», in *Springer handbook of robotics*, edited by B. Siciliano and O. Khatib (Springer International Publishing, Cham, 2016), E1–E1, ISBN: 978-3-319-32552-1, 10.1007/978-3-319-32552-1_81.

[4]S. Haddadin, A. De Luca, and A. Albu-Schäffer, «Robot collisions: a survey on detection, isolation, and identification», IEEE Transactions on Robotics **33**, 1292–1312 (2017) 10.1109/TRO.2017.2723903.

[5]S. Haddadin, S. Haddadin, A. Khoury, T. Rokahr, S. Parusel, R. Burgkart, A. Bicchi, and A. Albu-Schäffer, «On making robots understand safety: embedding injury knowledge into control», The International Journal of Robotics Research **31**, 1578–1602 (2012) 10.1177/0278364912462256.

[6]T. Liu, F. Liu, and M. Buss, «Indirect adaptive control of piecewise affine systems without common lyapunov functions», IEEE Control Systems Letters **6**, 1808–1813 (2022) 10.1109/LCSYS.2021.3133800.

[7]J. Nassour, P. Hénaff, F. B. Ouezdou, and G. Cheng, «Experience-based learning mechanism for neural controller adaptation: application to walking biped robots», in 2009 ieee/rsj international conference on intelligent robots and systems (2009), pp. 2616–2621, 10.1109/IROS.2009.5354797.

[8]J. Nassour, V. Hugel, F. Ouezdou, and G. Cheng, «Qualitative adaptive reward learning with success failure maps: applied to humanoid robot walking», Neural Networks and Learning Systems, IEEE Transactions on **24**, 10.1109/TNNLS.2012.2224370 (2012) 10.1109/TNNLS.2012.2224370.

[9]C. Yang, G. Ganesh, S. Haddadin, S. Parusel, A. Albu-Schäeffer, and E. Burdet, «Experimental safety study on soft-tissue injury in robotics», IEEE Transactions on Robotics **27**, 918–930 (2011).