Advanced Robot Control and Learning

Prof. Sami Haddadin

Modern Methods of Robot Control II

Advanced Robot Control - Outline

Differential Geometry in Robotics

Basics of Task Space Modeling and Control

Modern Methods of Robot Control I

Modern Methods of Robot Control II

- Impedance and Admittance Control
- Port-Hamiltonian Systems in Robotics
  - Motivation
  - Dirac Structure and Power Ports
  - Port-based Modeling of a Manipulator
  - Passivity Analysis

Linear Parametrization of Robot Dynamics

Robot Dynamics Identification

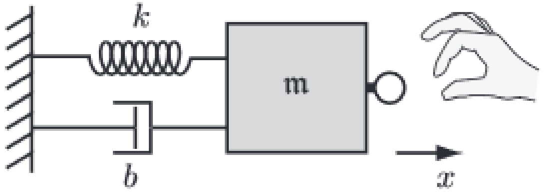
Bio-Inspired Robot Control

References

- The content of this lecture is based on
  - Passivity-based Control and Estimation in Networked Robotics, *T. Hatanaka et al.*, Springer international publishing, 2015
  - Slides of the lecture Sensor-based Robotic Manipulation and Locomotion, *Prof. Dr. Alin Albu-Schäffer*, Technical University of Munich, Wintersemester 2018/19
  - M. W. Spong, S. Hutchinson, M. Vydiasagar: Robot Modelling and Control, John Wiley & Sons, 2006
  - Takegaki M., Arimoto S., 1981,*“A new feedback method for dynamic control of manipulators”*, Transactions ASME, Journal of Dynamic Systems, Measurement and Control, Vol. 103, pp. 119–125
  - Paden, B., & Panja, R. (1988). Globally asymptotically stable ‘PD+’controller for robot manipulators. *International Journal of Control*, *47*(6), 1697-1712.
  - Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control* (Vol. 199, No. 1). Englewood Cliffs, NJ: Prentice hall.
  - Lynch, Kevin M., and Frank C. Park. *Modern Robotics*. Cambridge University Press, 2017.

Impedance/Admittance Control

Impedance & Admittance – Concept



- The robot end-effector is asked to render particular mass, spring, and damper properties.
- The dynamics for a one-dof robot rendering an impedance can be written as:

$$m\ddot{x} + b\dot{x} + kx = f$$

- Taking the Laplace transform yields:

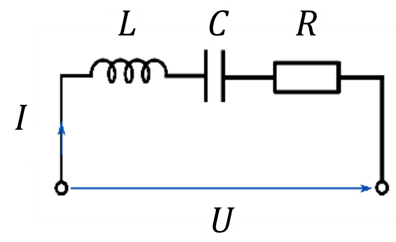
$$(ms^2 + bs + k)X(s) = F(s)$$

- The impedance is defined by the transfer function from position perturbations to forces: $Z(y) = \frac{F(s)}{X(s)}$
- The admittance is the inverse of the impedance: $Y(s) = Z^{-1}(s) = \frac{X(s)}{F(s)}$

Impedance & Admittance – Concept

- From electrical circuits:

$$U = RI + L\dot{I} + \frac{1}{C}\int I \ dt$$



- **Impedance:** $Z(s) = \frac{U(s)}{I(s)}$ e.g. $Z(s) = R + Ls + \frac{1}{Cs}$
- **Admittance:** $\mathrm{Y}(s) = \frac{I(s)}{U(s)}$ e.g. $\mathrm{Y}(s) = \frac{1}{R+Ls+\frac{1}{Cs}}$

- Analogy between mechanical and electrical systems:

$$U = L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q$$
$$F = M\ddot{X} + D\dot{X} + KX$$

$$U \leftrightarrow F, \quad I \leftrightarrow \dot{X}, \quad Q \leftrightarrow X,$$
$$R \leftrightarrow D, \quad L \leftrightarrow M, \quad (\frac{1}{C}) \leftrightarrow K$$

- **Impedance:** $Z(s) = \frac{F(s)}{X(s)}$ Impedance causality: Input: $X(s)$, Output: $F(s)$
- **Admittance:** $\mathrm{Y}(s) = \frac{X(s)}{F(s)}$ Admittance causality: Input: $F(s)$, Output: $X(s)$

Impedance-Control Algorithm

- The goal of impedance control is to implement the task-space behavior: $M\ddot{x} + B\dot{x} + Kx = f_{ext}$
- The robot senses the endpoint motion x(t) and commands joint torques and forces to create $-f_{ext}$, the force to display to the user.
- Theoretically, an impedance-controlled robot should only be coupled to an admittance-type environment
- A good control law might be

$$\tau = J^T(\theta)\Big(\underbrace{\tilde{\Lambda}(\theta)\ddot{x} + \tilde{\eta}(\theta, \dot{x})}_{arm\ dynamics\ compensation} - \underbrace{(M\ddot{x} + B\dot{x} + kx)}_{f_{ext}}\Big)$$

- The task-space dynamics model $\{\tilde{\Lambda}, \tilde{\eta}\}$ is expressed in terms of the coordinates

Admittance -Control Algorithm

- The robot senses f_{ext} using a wrist force–torque sensor and controls its motions in response.
- In an admittance-control algorithm the force f_{ext} applied by the user is sensed by the wrist load cell, and the robot responds with an end-effector acceleration.
- A simple approach is to calculate the desired end-effector acceleration $\ddot{x}_d$ according to:

$$M\ddot{x}_d + B\dot{x} + Kx = f_{ext}$$

- Solving with respect to $\ddot{x}_d$, we get:

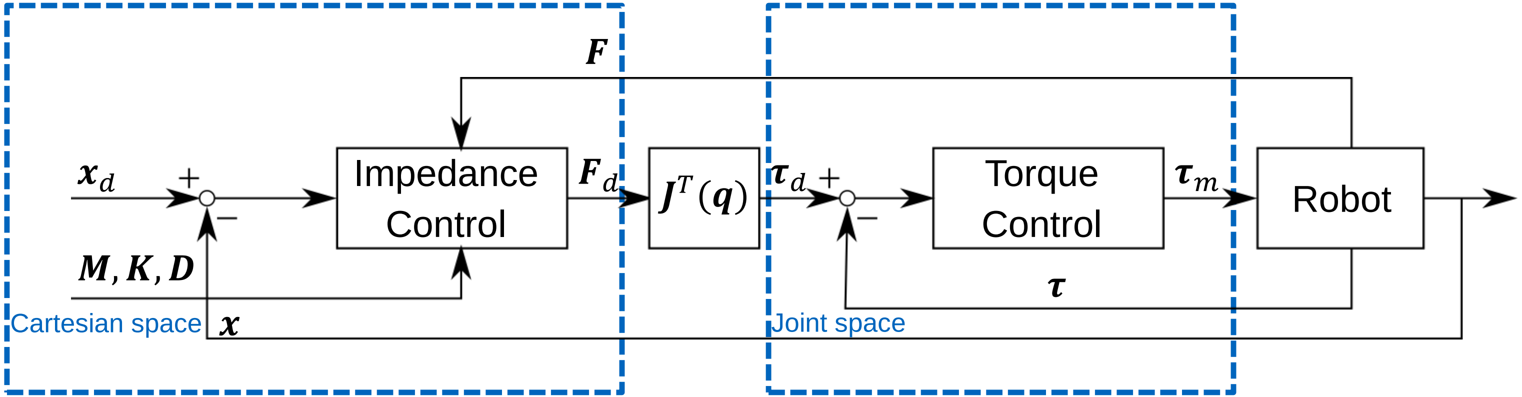
$$\ddot{x}_d = M^{-1}(f_{ext} - B\dot{x} - Kx)$$

- Considering $\dot{x} = J(\theta)\dot{\theta}$, the desired joint accelerations can be solved as:

$$\ddot{\theta}_d = J^{-1}(\theta)(\ddot{x}_d - \dot{J}(\theta)\dot{\theta})$$

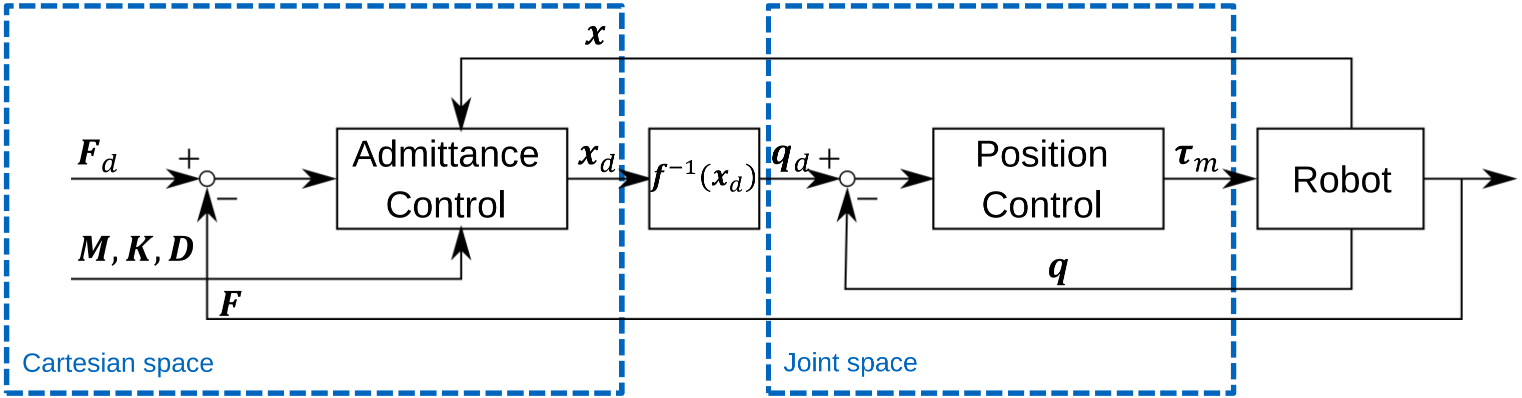
- Then, the inverse dynamics can be used to calculate the commanded joint forces and torques.

Cartesian Controller with Impedance Causality



- Inner loop: compliance
- Outer loop: high stiffness

Cartesian Controller with Admittance Causality



- Inner loop: high stiffness
- Outer loop: compliance

Considerations – Cartesian Space vs. Joint Space

- For the non-redundant, non-singular robot, the same controllers can be applied as in joint space
- Robot model in $SE(3)$ space

$$\boldsymbol{M}_{\mathrm{C}}(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{C}_{\mathrm{C}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{x}} + \boldsymbol{F}_{\mathrm{g}}(\boldsymbol{q}) = \boldsymbol{F}_{\mathrm{in}}$$

- PD controller with gravity compensation

$$\boldsymbol{F}_{\mathrm{in}} = \boldsymbol{K}_{\mathrm{P}}(\boldsymbol{x}_d - \boldsymbol{x}) - \boldsymbol{K}_{\mathrm{D}}\dot{\boldsymbol{x}}$$
$$\boldsymbol{\tau}_{\mathrm{in}} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{F}_{\mathrm{in}} + \boldsymbol{g}(\boldsymbol{q})$$

- Closed loop dynamics

$$\Rightarrow \boldsymbol{M}_{\mathrm{C}}(\boldsymbol{q})\ddot{\boldsymbol{x}} + \boldsymbol{C}_{\mathrm{C}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{x}} + \boldsymbol{K}_{\mathrm{P}}(\boldsymbol{x}_d - \boldsymbol{x}) - \boldsymbol{K}_{\mathrm{D}}\dot{\boldsymbol{x}} = 0$$

- Differences to joint control:
  - All statements are only *local*, not global
  - In case of redundant robot, convergence of coordinate $\boldsymbol{x}$ does not imply stability of the whole system (Null space motion)

Chapter 5: Port-Hamiltonian Systems in Robotics

Motivation – Interconnection by Energy

- **Energy**
  - Transferable property of an object
  - Can be described in an equivalent nature in different domains

- Each system consists of subsystems, with their own network of interconnected atomic elements (Ports) that deal with a specific energy type

- Interconnection through exchange of energy per unit time: **Power**

- Power P as the dual product (i.e. $\langle \varepsilon | \zeta \rangle = \varepsilon^T \zeta$) of *Power-conjugate variable*, namely **effort** and **flow**

Power-conjugate variables

- Belonging to the dual spaces[1] (e.g. $\mathcal{F} = \mathbb{R}^k, \mathcal{E} \coloneqq \mathcal{F}^* = (\mathbb{R}^k)^*$), *flow* ($f \in \mathcal{F}$) and *effort* ($e \in \mathcal{E}$) permit a power description in a unified way
- They can be defined for each subsystem according to its related domain
- Examples

Energy Domain	**Effort**	**Flow**
Kinetic – translational	Force	Linear velocity
Kinetic – rotational	Torque	Angular velocity
Electrical	Voltage	Current
Magnetic	Current	Voltage
Hydraulic	Pressure	Volume flow
Thermal	Temperature	Entropy flow

[1] See first lecture on Differential Geometry in Robotics, tangent and cotangent spaces

Dirac-structure and Power-ports

Dirac Structure

- On the space $\mathcal{F} \times \mathcal{F}^*$ a Dirac structure is a subspace $\mathcal{D}$ such that

  i. $\langle e|f \rangle = 0, \forall (f, e) \in \mathcal{D}$

  ii. $\dim\mathcal{D} = \dim\mathcal{F}$

- First condition $\Longrightarrow$ the power-conservation property of the physical systems
- Second condition $\Longrightarrow$ the impossibility in controlling of both effort and flow[1]
- Belonging to the dual spaces[1] (e.g. $\mathcal{F} = \mathbb{R}^k, \mathcal{E} := \mathcal{F}^* = (\mathbb{R}^k)^*$), *flow* ($f \in \mathcal{F}$) and *effort* ($e \in \mathcal{E}$) permit a power description in a unified way
- They can be defined for each subsystem according to its related domain
- Dirac structure as a power-conserving interconnection of atomic elements (i.e. *ports*)
- The subspace $\mathcal{D}$ is called the space of *port variables*

[1] See fourth lecture on Modern Methods of Robot Control, Impedance control

Internal Ports

Energy Storage Port $(\mathcal{S})$:

- Energy storage is describable by a finite-dimensional state space manifold $\mathcal{H}$ with coordinates x, together with the energy function (namely Hamiltonian) $H\colon \mathcal{H} \to \mathbb{R}$
- Examples:

Energy Domain	State
Kinetic – translational	Linear momentum (P)
Kinetic – rotational	Angular momentum (L)
Potential – translational	Linear displacement (x)
Potential – rotational	Angular displacement (θ)
Electrical	Charge (Q)
Magnetic	Magnetic flux linkage (λ)

- For the related port variables (f_S, e_S), which are interconnected to the energy storage of the system we have

$$P_S = e_S^T f_S = \left(\frac{\partial H}{\partial x}\right)^T (-\dot{x}) = -\dot{H}$$

Energy Dissipation Port $(\mathcal{R})$:

- Corresponds to internal energy dissipation (due to friction, resistance, etc.)
- Port variables are denoted by (f_R, e_R)
- Static resistive relation $R(f_R, e_R) = 0,$ with the property of $\langle \varepsilon | f \rangle \leq 0 \; \forall \; (f_R, e_R)$

- Without the presence of additional external ports, Dirac structure of port-Hamiltonian system satisfies power balance

$$e_S^T f_S + e_R^T f_R = 0$$

- Hence

$$\frac{d}{dt} H = -e_S^T f_S = e_R^T f_R \leq 0$$

External Ports

Interaction Port $(\mathcal{I})$:

- Corresponding with the interaction of the system with the environment
- Assumed to be known and described by the port variables (f_I, e_I)

Control Port $(\mathcal{C})$:

- Corresponding to the control action driving the system
- Assumed to be known and to be described by the port variables (f_C, e_C)
- The presence of sources may be included in this port

All Ports Together

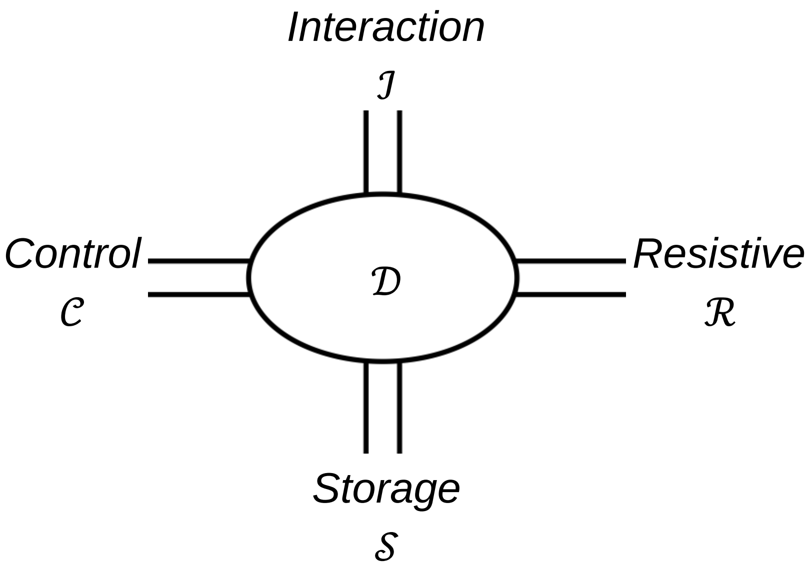
- Dirac structure $\mathcal{D}$ of the port-Hamiltonian syste, satisfies the power balance

$$e_S^T f_S + e_R^T f_R + e_C^T f_C + e_I^T f_I = 0$$

- Hence, for the energy balance

$$\frac{d}{dt} H = e_R^T f_R + e_C^T f_C + e_I^T f_I$$

➢ The variation of internal energy is equal to the sum of the power provided by the external ports (interaction and control) and the power dissipated by the resistive port



Port-Hamiltonian Representation

- Multiple ways to represent Dirac structures resultion
- Kernel Representation:

  Dirac structure $\mathcal{D} \subset \mathcal{F} \times \mathcal{E}$, with $\mathcal{E} = \mathcal{F}^*$

  $$\mathcal{D} = \{(f, e) \in \mathcal{F} \times \mathcal{F}^* | Ff + Ee = 0\}$$

  for linear maps $\mathrm{F}\colon \mathcal{F} \rightarrow \mathcal{V}$ and $\mathrm{E}\colon \mathcal{E} \rightarrow \mathcal{V}$ satifying

  i. $EF^* + FE^* = 0$
  ii. $\mathrm{rank}(F + E) = \mathrm{dim}\mathcal{F}$

  where $\mathcal{V}$ is a linear space with same dimensions as $\mathcal{F}$ with $F^*\colon \mathcal{V}^* \rightarrow \mathcal{E}$ and $E^*\colon \mathcal{V}^* \rightarrow (\mathcal{F}^*)^* = \mathcal{F}$ are adjoint maps of F and E

- And for a system with all four mentioned ports:

  $$\mathcal{D} = \{(f_S, e_S, f_R, e_R, f_C, e_C, f_I, e_I) \in \mathcal{F} \times \mathcal{F}^* | F_S f_S + E_S e_S + F_R f_R + E_R e_R + F_C f_C + E_C e_C + F_I f_I + E_I e_I = 0\}$$

Port-Hamiltonian Representation

- By a proper substitution for the matrices of Kernel representation one will reach to the well-known port-Hamiltonian system representation

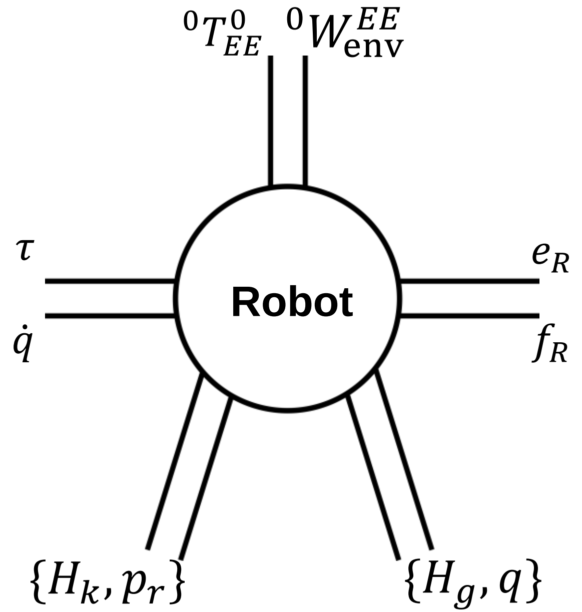
$$\begin{cases} \dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + G(x)u_C + K(x)u_I \\ y_C = G^T(x)\frac{\partial H}{\partial x} \\ y_I = K^T(x)\frac{\partial H}{\partial x} \end{cases}$$

(H, x): Storage port Hamiltonian and states
$J(x)$: Skew-symmetric matrix (assuring the power-conservation)
$R(x)$: Symmetric positive-definite matrix (representing the dissipation port)
u_I, u_C, y_I, y_C: the input and output port-variables of external ports
G, K: Mapping matrices for the external ports

Port-based Modeling of a Manipulator

Modeling of a Robot

- **Storage port – Gravitational** $\{H_g, q\}$
  State: Joint positions: $x_g = q$
  Hamiltonian: Robot‘s gravitational energy: $H_g = U(q)$
- **Storage port – Kinetic** $\{H_k, p_r\}$
  State: Joint generalized momenta: $x_k = p_r = M(q)\dot{q}$ (M: mass matrix)
  Hamiltonian: Robot‘s kinetic energy: $H_k = \frac{1}{2} p_r^T M^{-1}(q) p_r$
- **Dissipative port** (f_R, e_R)
  Flow: Joint velocities: $f_R = \dot{q}$
  Effort: Dissipative joint torques: $e_R = -D(p, q)\dot{q}$ (D: symmetric positive matrix)
- **Control port** $(\dot{q}, \tau)$
  Flow: Joint velocities: $f_C = \dot{q}$
  Effort: Input joint torques: $e_C = \tau$
- **Interaction port** ${}^0T_{EE}^0, {}^0W_{\text{env}}^{EE}$
  Flow: Robot‘s end-effector twist with regard to and described in the base frame: $f_I = {}^0T_{EE}^0$
  Effort: Wrench acting on the robot‘s end-effector described in the base frame: $e_I = {}^0W_{\text{env}}^{EE}$

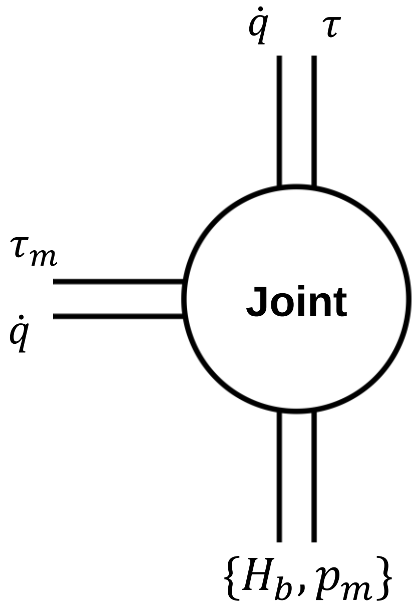


Modeling of a Joint

- **Storage port – Rotational kinetic** $\{H_b, p_m\}$
  State: Motor generalized momenta: $x_b = p_m = b\dot{q}$ (b: motor inertia)
  Hamiltonian: Robot's kinetic energy: $H_b = \frac{1}{2} p_m^T b^{-1} p_m$

- **Control port** $(\dot{q}, \tau_m)$
  Flow: Joint velocities: $f_C = \dot{q}$
  Effort: Input motor torques: $e_C = \tau_m$

- **Interaction port** $(\dot{q}, \tau)$
  Flow: Joint velocities: $f_C = \dot{q}$
  Effort: Input joint torques: $e_C = \tau$

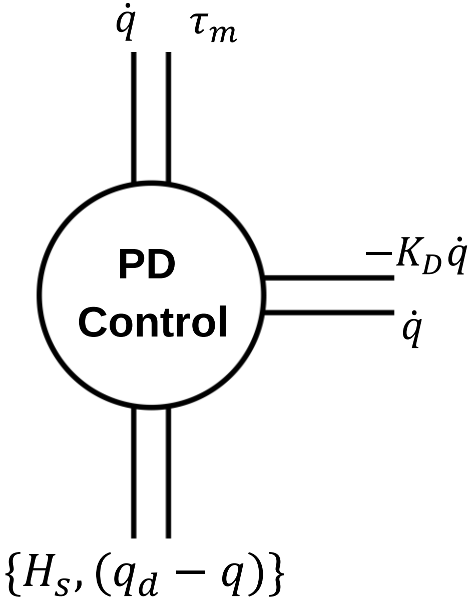


Modeling of Controller

- **Storage port – Proportional term** $\{H_s, (q_d - q)\}$
  State: Position error: $x_s = q_d - q$
  Hamiltonian: Energy stored in the controller due to the position error:
  $H_s = \frac{1}{2} x_s^T K_P x_s$ (K_P: controller proportional gain matrix)

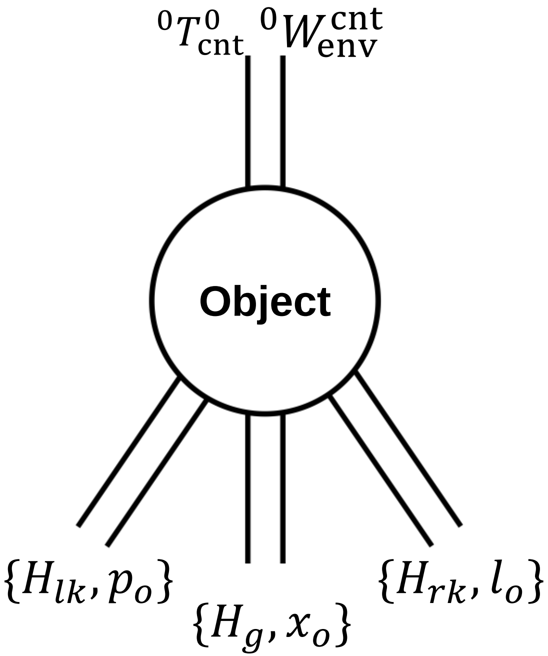
- **Dissipative port** $(\dot{q}, -K_D \dot{q})$
  Flow: Joint velocities: $f_R = \dot{q}$
  Effort: Dissipative torques due to damping effect:
  $e_R = -K_D \dot{q}$ (K_D: controller damping gain matrix)

- **Interaction port** $(\dot{q}, \tau_m)$
  Flow: Joint velocities: $f_C = \dot{q}$
  Effort: Input joint torques: $e_C = \tau_m$



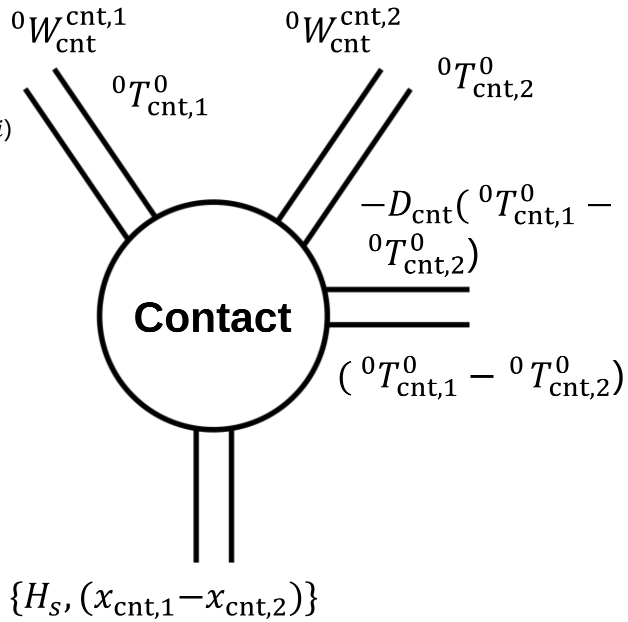
Modeling of Object

- **Storage port – Gravitational** $\{H_g, x_o\}$
  State: Object's center of mass position: $x_g = x_o$
  Hamiltonian: Object's gravitational energy: $H_g = mg^T x_o$ (m: object's mass, g: gravity vector)
- **Storage port – Linear Kinetic** $\{H_{lk}, p_o\}$
  State: Object's linear momentum: $x_{lk} = p_o = m\dot{x}_o$
  Hamiltonian: Object's linear kinetic energy: $H_{lk} = \frac{1}{2m} p_o^T p_o$
- **Storage port – Angular Kinetic** $\{H_{rk}, l_o\}$
  State: Object's rotational momentum: $x_{rk} = l_o = I_o \omega_o$
  Hamiltonian: Object's rotational kinetic energy:
  $H_{rk} = \frac{1}{2} l_o^T I_o^{-1} l_o$ (I_o: inertia tensor, ω_o: angular velocity of object)
- **Interaction port** ${}^0T_{\text{cnt}}^0$, ${}^0W_{\text{env}}^{\text{cnt}}$
  Flow: Object's contact point twist with regard to and described in the base frame: $f_I = {}^0T_{\text{cnt}}^0$
  Effort: Wrench acting on the contact point of the object described in the base frame:
  $e_I = {}^0W_{\text{env}}^{\text{cnt}}$



Modeling of Contact

- **Storage port – Elastic** $\{H_s, x_s\}$
  State: Contact elasticity‘s deflection: $x_s = x_{\text{cnt},1} - x_{\text{cnt},2}$ ($x_{\text{cnt},i}$: the contact point‘s pose from side i)
  Hamiltonian: Stored energy in contact‘s elastic element:
  $H_s = \frac{1}{2} x_s^T K_{\text{cnt}} x_s$ (K_{cnt}: the contact elasicity matrix)
- **Dissipative port** (f_R, e_R)
  Flow: Relative twists on the damping element of the contact: Joint $f_R = {}^0T^0_{\text{cnt},1} - {}^0T^0_{\text{cnt},2}$
  Effort: Wrench due to the contact‘s damping effect:
  $e_R = -D_{\text{cnt}} f_R$ (D_{cnt}: symmetric positive matrix)
- **Interaction port** (to one side) ${}^0T^0_{\text{cnt},1}, {}^0W^{\text{cnt},1}_{\text{cnt}}$
  Flow: Twist on one side of the contact are: $f_I = {}^0T^0_{\text{cnt},1}$
  Effort: Wrench acting on the contact area from one side: $e_I = {}^0W^{\text{cnt},1}_{\text{cnt}}$
- **Interaction port** (to another side) ${}^0T^0_{\text{cnt},2}, {}^0W^{\text{cnt},2}_{\text{cnt}}$
  Flow: Twist on another side of the contact are: $f_I = {}^0T^0_{\text{cnt},2}$
  Effort: Wrench acting on the contact area from another side: $e_I = {}^0W^{\text{cnt},2}_{\text{cnt}}$



Overall System

Now, connecting all the aforementioned sub-systems, we reach to one unified system with the internal ports of:

- The overall Hamiltonian function will be the summation of all Hamiltonians:

$$H_{tot} = H_{k,controller} + H_{k,robot} + H_{g,robot} + H_{s,contact} + H_{lk,object} + H_{rk,object} + H_{g,object}$$

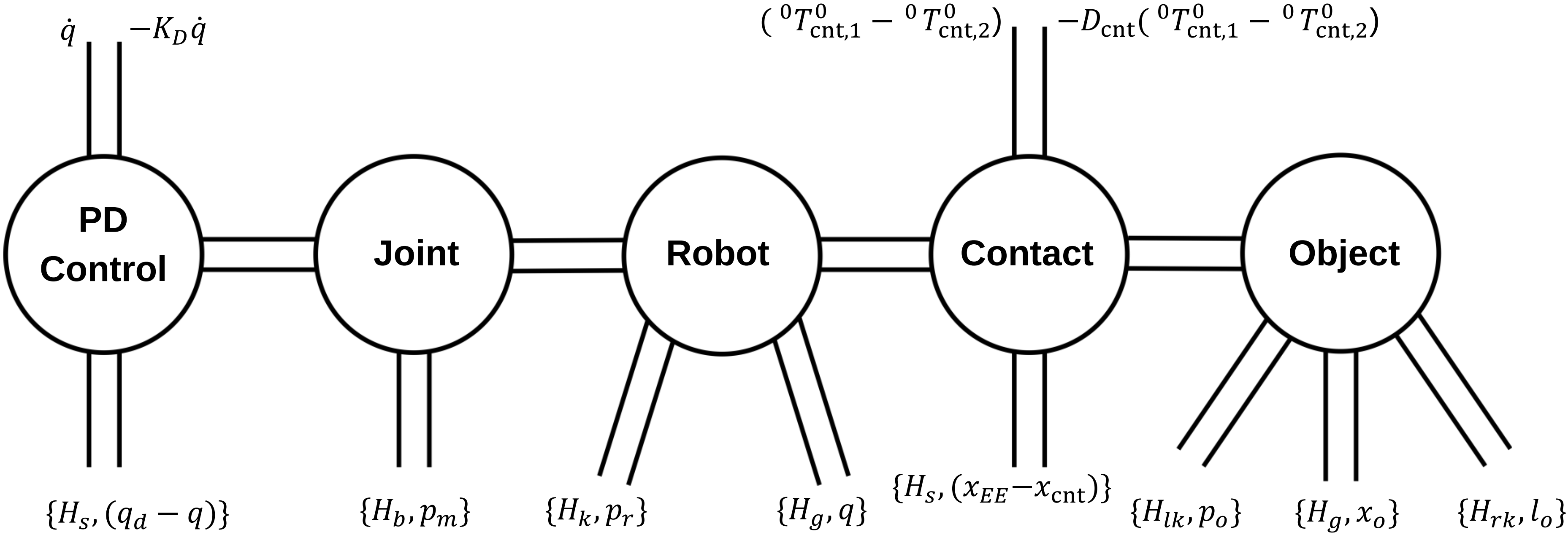
- Assumption: the dissipation port of the robot can be disregarded, and as a result there will be two dissipation ports in the system for the controller and contact

Possible Extensions:

- One can consider an elastic joint, for which there would be an extra storage port for the elasticity, and an extra dissipative port for the damping effect
- One can consider a rigid contact, for which ${}^{0}T^{0}_{\mathrm{cnt},1} = {}^{0}T^{0}_{\mathrm{cnt},2}$ and ${}^{0}W^{\mathrm{cnt},1}_{\mathrm{cnt}} = -\,{}^{0}W^{\mathrm{cnt},2}_{\mathrm{cnt}}$. This can be modelled as a constraint on the twists

After the proper connection, the whole system can be shown as a single port-based model.

Overall System



Passivity Analysis

How to Prove Passivity by Port-based Modeling?

As mentioned before, for a single system with all kinds of ports, the fundamental property in port-based modeling is:

$$\frac{d}{dt}H = e_R^T f_R + e_C^T f_C + e_I^T f_I$$

For our system, this means:

$$\frac{d}{dt}H_{tot} = e_{R,controller}^T \cdot f_{R,controller} + e_{R,contact}^T \cdot f_{R,contact}$$

$$\rightarrow \dot{H}_{tot} = -\dot{q}^T K_D \dot{q} - \left({}^0T_{\mathrm{cnt},1}^0 - {}^0T_{\mathrm{cnt},2}^0 \right)^T D_{\mathrm{cnt}} \left({}^0T_{\mathrm{cnt},1}^0 - {}^0T_{\mathrm{cnt},2}^0 \right) \leq 0$$

Hence, the overall system is proven to be passive.