Advanced Robot Control and Learning

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Part 1

Advanced Robot Control

Advanced Robot Control - Outline

Differential Geometry in Robotics

Basics of Task Space Modeling and Control

Modern Methods of Robot Control I

- Definition of Passivity
- Passivity Preservation for Interconnections
- Passive Representation of a Robot
- Stability and Passivity
- Passivity-Based Position and Motion Control

Modern Methods of Robot Control II

Linear Parametrization of Robot Dynamics

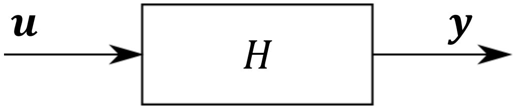
Robot Dynamics Identification

Bio-Inspired Robot Control

Chapter 3: Modern Methods of Robot Control I

Definition of Passivity

Definition of Passivity



- Consider the system H with input $\boldsymbol{u}(t) \in \mathbb{R}^p$ and output $\boldsymbol{y}(t) \in \mathbb{R}^p, t \in \mathbb{R}_+, \mathbb{R} \coloneqq [0, \infty)$
- H is the mapping from input signal space $\mathcal{U}$ to output signal space $\mathcal{V}$

Definition 1

- The System $H\colon \mathcal{U} \mapsto \mathcal{V}$ with $\boldsymbol{u} \in \mathcal{U}$ and $\boldsymbol{y} \in \mathcal{V}$ is called *passive* if there exists a constant $\beta \geq 0$ such that

$$\int_0^\tau \boldsymbol{y}(t)^T \boldsymbol{u}(t) dt \geq -\beta, \qquad \forall \boldsymbol{u} \in \mathcal{U}, \tau \in \mathbb{R}_+$$

Additionally

- System H: *input strictly passive,* if there exists a scalar $\delta_u > 0$ such that

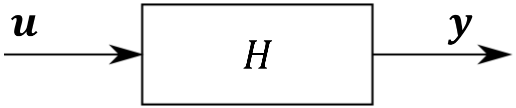
$$\int_0^\tau \boldsymbol{y}(t)^T \boldsymbol{u}(t) dt \geq -\beta + \delta_u \int_0^\tau \|\boldsymbol{u}(t)\|^2 dt, \qquad \forall \boldsymbol{u} \in \mathcal{U}, \tau \in \mathbb{R}_+$$

- System H: *output strictly passive,* if there exists a scalar $\delta_y > 0$ such that

$$\int_0^\tau \boldsymbol{y}(t)^T \boldsymbol{u}(t) dt \geq -\beta + \delta_y \int_0^\tau \|\boldsymbol{y}(t)\|^2 dt, \qquad \forall \boldsymbol{u} \in \mathcal{U}, \tau \in \mathbb{R}_+$$

Note: $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T \boldsymbol{x}}$ (Vector 2-norm)

Definition of Passivity



- Consider now the state space model of the system with input $\boldsymbol{u}(t) \in \mathbb{R}^p$, output $\boldsymbol{y}(t) \in \mathbb{R}^p$ and state vector $\boldsymbol{x}(t) \in \mathbb{R}^n$

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbb{R}^n \qquad (1.1)$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) \qquad (1.2)$$

- Assume that the system has an equilibrum at the origin: $\boldsymbol{f}(0,0) = 0,\ \boldsymbol{h}(0,0) = 0$

Definition 2

- The system (1) from $\boldsymbol{u}(t)$ to $\boldsymbol{y}(t)$ is called *passive* if there exists a positive semidefinite function (*storage function*) $S\colon\ \mathbb{R}^n \mapsto \mathbb{R}_+$ such that

$$S(x(\tau)) - S(x_0) \leq \int_0^\tau \boldsymbol{y}(t)^T\, \boldsymbol{u}(t)dt, \qquad \forall \boldsymbol{u}\colon [0, \tau] \mapsto \mathbb{R}^p, \boldsymbol{x}_0 \in \mathbb{R}^n, \tau \in \mathbb{R}_+$$

Additionally

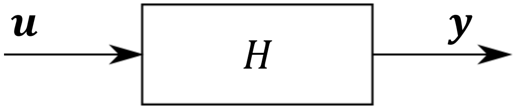
- System (1): *input strictly passive*, if there exists a scalar $\delta_u > 0$ such that

$$S\big(x(\tau)\big) - S(x_0) \leq \int_0^\tau (\boldsymbol{y}(t)^T\, \boldsymbol{u}(t) - \delta_u \|\boldsymbol{u}(t)\|^2)dt, \qquad \forall \boldsymbol{u}\colon [0, \tau] \mapsto \mathbb{R}^p, \boldsymbol{x}_0 \in \mathbb{R}^n, \tau \in \mathbb{R}_+$$

- System (1): *output strictly passive*, if there exists a scalar $\delta_y > 0$ such that

$$S\big(x(\tau)\big) - S(x_0) \leq \int_0^\tau (\boldsymbol{y}(t)^T\, \boldsymbol{u}(t) - \delta_y \|\boldsymbol{y}(t)\|^2)dt, \qquad \forall \boldsymbol{u}\colon [0, \tau] \mapsto \mathbb{R}^p, \boldsymbol{x}_0 \in \mathbb{R}^n, \tau \in \mathbb{R}_+$$

The Storage Function



- If the storage function $S(\boldsymbol{x})$ is continuously differentiable, the time derivative $S(\boldsymbol{x}(t))$ is given by

$$\dot{S} = \frac{d}{dt} S\big(\boldsymbol{x}(t)\big) = \frac{\partial S}{\partial \boldsymbol{x}} \dot{\boldsymbol{x}} = \frac{\partial S}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

- The system is passive if

$$\frac{\partial S}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \leq \boldsymbol{y}^T \boldsymbol{u} = \boldsymbol{h}^T(\boldsymbol{x}, \boldsymbol{u})\boldsymbol{u}, \qquad \forall \boldsymbol{x} \in \mathbb{R}^n, \forall \boldsymbol{u} \in \mathbb{R}^p$$

  Additionally

  - *input strictly passive*, if there exists a scalar $\delta_u > 0$ such that

$$\frac{\partial S}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \leq \boldsymbol{y}^T \boldsymbol{u} - \delta_u \|\boldsymbol{u}\|^2, \qquad \forall \boldsymbol{x} \in \mathbb{R}^n, \forall \boldsymbol{u} \in \mathbb{R}^p$$

  - *output strictly passive*, if there exists a scalar $\delta_y > 0$ such that

$$\frac{\partial S}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \leq \boldsymbol{y}^T \boldsymbol{u} - \delta_y \|\boldsymbol{y}\|^2, \qquad \forall \boldsymbol{x} \in \mathbb{R}^n, \forall \boldsymbol{u} \in \mathbb{R}^p$$

- $S(\boldsymbol{x})$ is the total energy of the system
- $\boldsymbol{y}(t)^T \boldsymbol{u}(t)$ is the power supplied to the system at time t.
- $\int_0^\tau \boldsymbol{y}(t)^T \boldsymbol{u}(t) dt$ is the energy supplied to the system within time interval $[0, \tau]$.

Passivity Preservation for Interconnections

Passivity Preservation for Interconnections – Feedback connection

- When interconnecting passive systems, passivity is preserved!
- Consider two the subsystems

H_1:

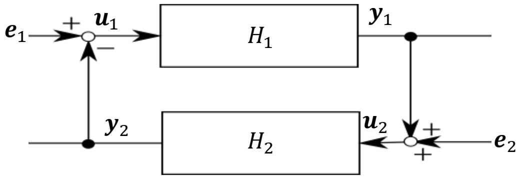
$$\dot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{u}_1), \quad \boldsymbol{x}_1(0) = \boldsymbol{x}_{10} \in \mathbb{R}^{n_1} \qquad (1.a)$$
$$\boldsymbol{y}_1 = \boldsymbol{h}_1(\boldsymbol{x}_1, \boldsymbol{u}_1)$$

H_2:

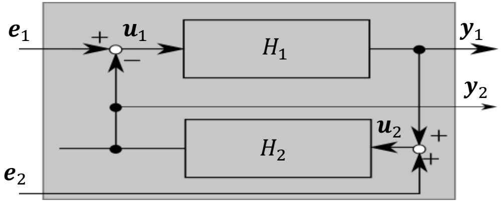
$$\dot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(\boldsymbol{x}_2, \boldsymbol{u}_2), \quad \boldsymbol{x}_2(0) = \boldsymbol{x}_{20} \in \mathbb{R}^{n_2} \qquad (1.b)$$
$$\boldsymbol{y}_2 = \boldsymbol{h}_2(\boldsymbol{x}_2, \boldsymbol{u}_2)$$

With $\boldsymbol{u}_1(t), \boldsymbol{u}_2(t), \boldsymbol{y}_1(t), \boldsymbol{y}_2(t) \in \mathbb{R}^p$

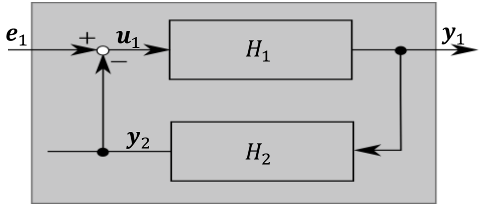
- $\boldsymbol{u}_1 = \boldsymbol{e}_1 - \boldsymbol{y}_2, \boldsymbol{u}_2 = \boldsymbol{e}_2 + \boldsymbol{y}_1$ (2)
- $\boldsymbol{u} := \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix}, \ \boldsymbol{e} := \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \end{bmatrix}, \ \boldsymbol{y} := \begin{bmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{bmatrix}$
- Feedback connection (1) and (2) yield system H_{ey} from from input $\boldsymbol{e}$ to output $\boldsymbol{y}$ by eliminating $\boldsymbol{u}$ through substitution of (2) into (1).
- Suppose H_1 and H_2 are passive, then closed-loop system H_{ey} is also passive from input $\boldsymbol{e}$ to output $\boldsymbol{y}$.
- Additionally, if $\boldsymbol{e}_2 \equiv 0$, the closed-loop system $H_{e_1 y_1}$ with input $\boldsymbol{e}_1$ and output $\boldsymbol{y}_1$ is also passive.



a) Standard feedback connection



b) Block diagram of H_{ey}



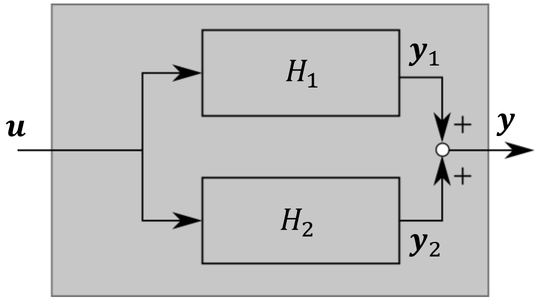
b) Block diagram of $H_{e_1 y_1}$

Proof : Spong, M.W., Hutchinson, S., Vidyasagar, M.: Robot Modeling and Control. Wiley, New York (2005)

Passivity Preservation for Interconnections – Parallel connection

- Consider now the parallel connection of the subsystems H_1 and H_2 with $\boldsymbol{u}_1(t)$, $\boldsymbol{u}_2(t)$, $\boldsymbol{y}_1(t)$, $\boldsymbol{y}_2(t) \in \mathbb{R}^p$

- If both subsystems are passive, then the system from input $\boldsymbol{u} = \boldsymbol{u}_1 = \boldsymbol{u}_2$ to output $\boldsymbol{y} = \boldsymbol{y}_1 + \boldsymbol{y}_2$ is also passive



Parallel connection of the systems H_1 and H_2

Proof: Spong, M.W., Hutchinson, S., Vidyasagar, M.: Robot Modeling and Control. Wiley, New York (2005)

Stability and Passivity

Stability and Passivity –Lyapunov Stability

Given the following system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}), \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbb{R}^n$$

With an equilibrium at $\boldsymbol{x}_e = 0$ such that $\boldsymbol{f}(\boldsymbol{x}_e) = 0$

1. Equilibrium is *Lyapunov stable:* if for each $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that: $\|\boldsymbol{x}_0\| < \delta \rightarrow \|\boldsymbol{x}(t)\| < \epsilon \qquad \forall\, t \in \mathbb{R}_+$
2. Equilibrium is *asymptotically stable* if it is stable and there exists $\delta > 0$ such that: $\|\boldsymbol{x}_0\| < \delta \rightarrow \lim_{t\to\infty} \boldsymbol{x}(t) = 0$
3. Equilibrium is *exponentially stable* if there exists $k > 0, \lambda > 0, \delta > 0$ such that: $\|\boldsymbol{x}(t)\| \leq k\|\boldsymbol{x}_0\| e^{-\lambda t} \qquad \forall \|\boldsymbol{x}_0\| \leq \delta$

Proof: Khalil, H.K.: Nonlinear Systems, 3rd edn. Prentice-Hall, Upper Saddle River (2002)

Stability and Passivity –Lyapunov Stability

Local asymptotic stability:

- Let $\boldsymbol{x} = 0$ be an equilibrium point for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{D} \in \mathbb{R}^n$ a domain containing $\boldsymbol{x} = 0$.
- Let $\boldsymbol{V}: \boldsymbol{D} \to \boldsymbol{R}$ be a continuously differentiable function such that:
  - $\boldsymbol{V}(0) = 0$ and $\boldsymbol{V}(x) > 0$ in $\boldsymbol{D} - \{0\}$
  - $\dot{\boldsymbol{V}}(x) \leq 0$ in D
  - $\dot{\boldsymbol{V}}(x) < 0$ in $D - \{0\}$

Then $x = 0$ is asymptotically stable.

Global asymptotic stability:

- Let $\boldsymbol{x} = 0$ be an equilibrium point for $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ and $\mathrm{V}: \mathbb{R}^n \to \mathbb{R}^n$ continuously differentiable function such that:
  - $\boldsymbol{V}(0) = 0$ and $\boldsymbol{V}(x) > 0$ **,** $\forall \boldsymbol{x} \neq \boldsymbol{0}$
  - $\|\boldsymbol{x}\| \to \infty$ then: $\boldsymbol{V}(\boldsymbol{x}) \to \infty$
  - $\dot{\boldsymbol{V}}(\boldsymbol{x}) < 0$ **,** $\forall \boldsymbol{x} \neq \boldsymbol{0}$

Then $x = 0$ is globally asymptotically stable.

Proof: Khalil, H.K.: Nonlinear Systems, 3rd edn. Prentice-Hall, Upper Saddle River (2002)

Stability and Passivity –Lyapunov Stability

- Consider the following system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbb{R}^n$$
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$$

- Use the storage function $S\big(\boldsymbol{x}(t)\big)$ as Lyapunov function
  - Recall definition of passivity: $\dot{S}\big(\boldsymbol{x}(t)\big) \le \boldsymbol{y}^T\boldsymbol{u}$
  - If the system is passive with respect to S and $\boldsymbol{u} \equiv 0$:

$$\dot{S}\big(\boldsymbol{x}(t)\big) \le 0$$

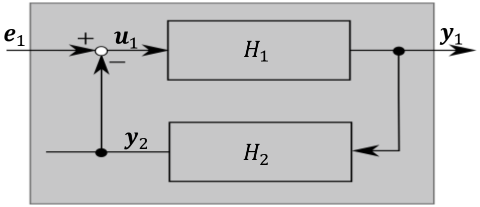
  ➢ Stability of the origin of the system in case S is positive definite!
    - However, definition of passivity requires S to be only positive *semidefinite*!

- System is *zero-state observable* if no solution of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ with $\boldsymbol{u} \equiv 0$ can stay identically in the set of states satisfying $\boldsymbol{h}(\boldsymbol{x}) = 0$, other than $\boldsymbol{x} \equiv 0$
  ➢ If the system is zero-state observable and output strictly passive with respect to $S(\boldsymbol{x})$, then $\boldsymbol{x} = 0$ is an asymptotically stable equlibrium with $\boldsymbol{u} \equiv 0$

Stability and Passivity –Lyapunov Stability

- Consider the feedback connection with
  - subsystem H_1 passive (not necessary to be output strictly passive)
  - subsystem H_2 as static map $\boldsymbol{y}_2 = k\boldsymbol{u}_2, k > 0$ (thus H_2 is input strictly passive)

  Then, using the previous theorems, it can be concluded that:



- The origin ($\boldsymbol{x} = 0$) of a passive zero-state observable system can be asymptotically stabilized with the negative feedback $\boldsymbol{u} = -k\,\boldsymbol{y},\ k > 0$

Passive Representation of a Robot

Passive Representation of a Robot

- Robot dynamics in joint space (revisited)

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}$$

  with:
  - $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}} \in \mathbb{R}^n$: joint positions, velocities, accelerations
  - $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$: robot mass matrix
  - $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$: centrifugal and Coriolis matrix
  - $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^n$: gravitational force
  - $\boldsymbol{\tau} \in \mathbb{R}^n$: command torques
- Note: $\boldsymbol{g}(\boldsymbol{q}) = \left(\frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}}\right)^T$ with robot potential energy $U(\boldsymbol{q})$
- We know that the robot mass matrix $\boldsymbol{M}(\boldsymbol{q})$ is positive definite and that the matrix $\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew-symmetric by defining $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ using the Christoffel symbols[1]
- Define the summation of the kinetic energy and the potential energy as the storage function:

$$S(\boldsymbol{x}) = T(\boldsymbol{q},\dot{\boldsymbol{q}}) + U(\boldsymbol{q}) = \frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q}), \qquad \boldsymbol{x} = \begin{bmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\end{bmatrix}$$

[1]Richard M. Murray, S. Shankar Sastry, and Li Zexiang. 1994. A Mathematical Introduction to Robotic Manipulation (1st ed.). CRC Press, Inc., Boca Raton, FL, USA.

Passive Representation of a Robot

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$S(x) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + U(q), \qquad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

- The time derivative of the storage function is

$$\begin{aligned}\dot{S} &= \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \left(\frac{\partial U(q)}{\partial q}\right)\dot{q} \\ &= \dot{q}^T\big(\tau - C(q,\dot{q})\dot{q} - g(q)\big) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + g(q)^T\dot{q} \\ &= \dot{q}^T\tau - \frac{1}{2}\dot{q}^T(\dot{M}(q) - 2C(q,\dot{q}))\dot{q} \\ &= \dot{q}^T\tau\end{aligned}$$

Passivity of the manipulator dynamics from the commanded torque τ to the joint velocity $\dot{q}$

Passivity-Based Position and Motion Control

PD-Control with Gravity Compensation *(Takegaki & Arimoto[1])* *PD-g(q) Control*

- Objective: *Regulation,* position control ($\boldsymbol{q}_d = \text{const.}$) in free space, displacement error vector $\tilde{\boldsymbol{q}} := (\boldsymbol{q} - \boldsymbol{q}_d)$
- PD Control law with gravity compensation:

$$\boldsymbol{\tau} = \boxed{\boldsymbol{g}(\boldsymbol{q})} - \boldsymbol{K}_\text{P}\tilde{\boldsymbol{q}} + \boldsymbol{u} \qquad (2)$$

where $\boldsymbol{K}_\text{P}, \in \mathbb{R}^{n\times n}$ is a positive definite gain matrix

note: controller makes explicit use of partial knowledge of manipulator model!

- Inserted in dynamics equation:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{K}_\text{P}\tilde{\boldsymbol{q}} = \boldsymbol{u} \qquad (3)$$

- Definition of storage function:

$$\boldsymbol{S}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) := \frac{1}{2}(\dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\text{P}\tilde{\boldsymbol{q}})$$

- Time derivative of storage function:

$$\begin{aligned}\dot{\boldsymbol{S}}(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}}) &= \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\text{P}\dot{\tilde{\boldsymbol{q}}} \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T(-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{K}_\text{P}\tilde{\boldsymbol{q}} + \boldsymbol{u}) + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\text{P}\dot{\boldsymbol{q}} \\ &= \frac{1}{2}\dot{\boldsymbol{q}}^T\left(\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T\boldsymbol{u} = \dot{\boldsymbol{q}}^T\boldsymbol{u}\end{aligned}$$

$\Rightarrow$ System (3) is passive from input $\boldsymbol{u}$ to output $\dot{\boldsymbol{q}}$

[1] Takegaki M., Arimoto S., 1981,*"A new feedback method for dynamic control of manipulators"*, Transactions ASME, Journal of Dynamic Systems, Measurement and Control, Vol. 103, pp. 119–125

PD-Control with Gravity Compensation *(Takegaki & Arimoto)* *PD-g(q) Control*

- Substituting $\dot{\boldsymbol{q}} \equiv 0$ and $\boldsymbol{u} \equiv 0$ into System (3) yields $\tilde{\boldsymbol{q}} \equiv 0$ ⟶ system is **zero-state observable** by taking $\tilde{\boldsymbol{q}} = (\boldsymbol{q} - \boldsymbol{q}_d)$ as a state variable instead of $\boldsymbol{q}$
- To guarantee asymptotic stability of the origin $\tilde{\boldsymbol{q}} = 0, \dot{\boldsymbol{q}} = 0$ the loop is closed by a negative feedback

$$\boldsymbol{u} = -\boldsymbol{K}_{\mathrm{D}}\dot{\boldsymbol{q}} \qquad (4)$$

where $\boldsymbol{K}_{\mathrm{D}}, \in \mathbb{R}^{n \times n}$ is a positive definite gain matrix

Storage function manipulator (open loop)	**Storage function closed-loop control**
$S(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q}),$	$\boldsymbol{S}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = \frac{1}{2}(\dot{\boldsymbol{q}}^T \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{q}} + \underbrace{\tilde{\boldsymbol{q}}^T \boldsymbol{K}_{\mathrm{P}}\tilde{\boldsymbol{q}}})$

potential energy function is shaped by the local feedback loop (2)

- *Passivation:* energy shaping, such that unique minimal value is taken at desired state $\tilde{\boldsymbol{q}} = 0$
- *Damping injection:* closing the loop with (4)

Passivity-based control

PD+ Control[1] (Paden & Panja)

- Objective: *tracking,* motion control $(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)$ in free space (extension of PD-g(q) Control)
- PD+ Control

$$\boldsymbol{\tau} = \boxed{\boldsymbol{M}(\boldsymbol{q})}\ddot{\boldsymbol{q}}_d + \boxed{\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})}\dot{\boldsymbol{q}}_d + \boxed{\boldsymbol{g}(\boldsymbol{q})} - \boldsymbol{K}_\mathrm{P}\,\tilde{\boldsymbol{q}} + \boldsymbol{u}$$

where $\boldsymbol{K}_\mathrm{P}, \boldsymbol{K}_\mathrm{D} \in \mathbb{R}^{n\times n}$ are positive definite matrices and $\tilde{\boldsymbol{q}} := (\boldsymbol{q}_d - \boldsymbol{q})$

- Inserted in dynamics equation:

$$\boldsymbol{M}(\boldsymbol{q})\,\ddot{\tilde{\boldsymbol{q}}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\,\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_\mathrm{P}\tilde{\boldsymbol{q}} = \boldsymbol{u} \qquad (5)$$

- Definition of storage function:

$$S(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}) := \frac{1}{2}(\dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{M}(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\mathrm{P}\tilde{\boldsymbol{q}})$$

- Time derivative of storage function:

$$\begin{aligned}\dot{S}(\tilde{\boldsymbol{q}},\dot{\tilde{\boldsymbol{q}}}) &= \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{M}(\boldsymbol{q})\ddot{\tilde{\boldsymbol{q}}} + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\mathrm{P}\dot{\tilde{\boldsymbol{q}}}\\ &= \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\dot{\boldsymbol{M}}(\boldsymbol{q})\dot{\tilde{\boldsymbol{q}}} + \dot{\tilde{\boldsymbol{q}}}^T\left(-\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_\mathrm{P}\tilde{\boldsymbol{q}} + \boldsymbol{u}\right) + \tilde{\boldsymbol{q}}^T\boldsymbol{K}_\mathrm{P}\dot{\tilde{\boldsymbol{q}}}\\ &= \frac{1}{2}\dot{\tilde{\boldsymbol{q}}}^T\left(\dot{\boldsymbol{M}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\right)\dot{\tilde{\boldsymbol{q}}} + \dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{u} = \dot{\tilde{\boldsymbol{q}}}^T\boldsymbol{u}\end{aligned}$$

System (5) is passive from input $\boldsymbol{u}$ to output $\dot{\tilde{\boldsymbol{q}}}$

[1] Paden, B., & Panja, R. (1988). Globally asymptotically stable 'PD+'controller for robot manipulators. *International Journal of Control*, *47*(6), 1697-1712.

PD+ Control (Paden & Panja)

- Storage function shaped so that minimal value taken at desired state $\tilde{\boldsymbol{q}} = \dot{\tilde{\boldsymbol{q}}} = 0$

- Substituting $\dot{\tilde{\boldsymbol{q}}} \equiv 0$ and $\boldsymbol{u} \equiv 0$ into System (5) yields $\tilde{\boldsymbol{q}} \equiv 0$ $\longrightarrow$ system is zero-state
- Damping injection

$$\boldsymbol{u} = -\boldsymbol{K}_{\mathrm{D}}\dot{\tilde{\boldsymbol{q}}}$$

guarantees asymptotic stability of the origin $\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}} = 0$

Joint Control Summary – Performance vs. Robustness

$\boldsymbol{\tau} = \boldsymbol{K}_{\mathrm{P}}(\boldsymbol{q}_d - \boldsymbol{q}) - \boldsymbol{K}_{\mathrm{D}}(\dot{\boldsymbol{q}}_d - \dot{\boldsymbol{q}}) + \ \ldots$

Performance (Theoretical)

Robustness

$\ldots\, \boldsymbol{g}(\boldsymbol{q})$

PD-g(q) (regulation controller, $\dot{\boldsymbol{q}}_d = 0$)

Global asymptotic stability

$\ldots\, \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_d + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_d + \boldsymbol{g}(\boldsymbol{q})$

PD+ controller (tracking controller)

Global asymptotic stability

$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})[\ddot{\boldsymbol{q}}_d + \boldsymbol{K}_{\mathrm{P}}(\boldsymbol{q}_d - \boldsymbol{q}) - \boldsymbol{K}_{\mathrm{D}}(\dot{\boldsymbol{q}}_d - \dot{\boldsymbol{q}})] + \boldsymbol{c}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{g}(\boldsymbol{q})$

Computed torque controller[1]

Global exponential stability, linear decoupled

Challenge: deal with model errors and distubances

- Scale of joint, inaccurate data about load/robot, external disturbances

Possible solutions:

Model-based friction compensation, Including integrator in controller, Disturbance observer and disturbance controller, Adaptive controller

[1] *See lecture basics of task space modeling and control for the computed torque controller in task space*

References

- The content of this lecture is based on
  - Passivity-based Control and Estimation in Networked Robotics, *T. Hatanaka et al.*, Springer international publishing, 2015
  - Slides of the lecture Sensor-based Robotic Manipulation and Locomotion, *Prof. Dr. Alin Albu-Schäffer*, Technical University of Munich, Wintersemester 2018/19
  - M. W. Spong, S. Hutchinson, M. Vydiasagar: Robot Modelling and Control, John Wiley & Sons, 2006
  - Takegaki M., Arimoto S., 1981,*"A new feedback method for dynamic control of manipulators"*, Transactions ASME, Journal of Dynamic Systems, Measurement and Control, Vol. 103, pp. 119–125
  - Paden, B., & Panja, R. (1988). Globally asymptotically stable 'PD+'controller for robot manipulators. *International Journal of Control*, *47*(6), 1697-1712.
  - Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control* (Vol. 199, No. 1). Englewood Cliffs, NJ: Prentice hall.
  - R. Kelly, V. Davila, A. Loria (2005). *Control of Manipulators in Joint Space*