Comparison of Direct and Indirect Data-Driven Predict Control

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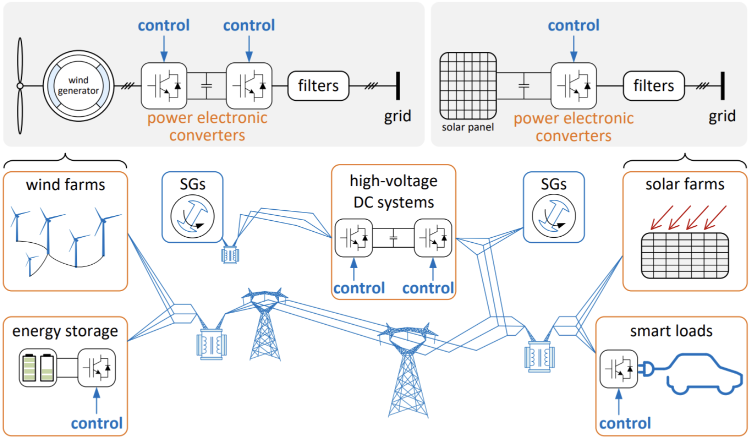
Why We Need Data-Driven Control

- system complex, variable
- proprietary models unknown



An illustration of a power electronics dominated power system

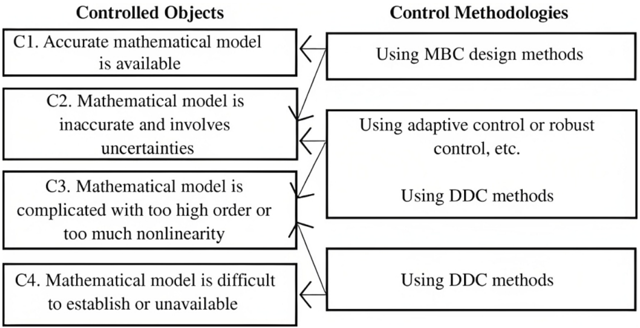
Comparsion Between MBC And DDC

MBC(Model Based Control)

- Very mature
  optimal control adaptive control robust control,etc. ✓
- Complex systems, inaccurate
  A large traffic network ✗

DDC(Data Driven Control)

- independent on system model
  approximate dynamic programming, model-free adaptive control, pseudo control, iterative learning control, etc. [Krishnan+ 2021]



Controlled objects of DDC

Data-Driven Control Design

Considering a (LTI) system: [De Persis+ 2020]

$$x_{t+1} = Ax_t + Bu_t + w_t, \quad y_t = Cx_t$$

where $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{p \times n}$
$x_t \in \mathbb{R}^n, u_t \in \mathbb{R}^m$ and $y_t \in \mathbb{R}^p$ are the system state, control input and output
$w_t \in \mathbb{R}^n$ being the process noise.
in order to study the relationship between input and output when the measurement noise exists. There are two ways:

- **Direct Data-Driven Controls**
- **Indirect Data-Driven Controls**

[Tang+ 2017]

Applications of DDC(Related Work)

DDDC for autonomous vehicle

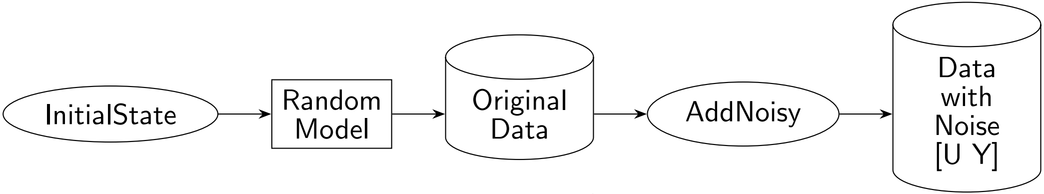
- The control signal generated based on the real-time data from various sensors. ✓
- mathematical model of the vehicle dynamics.✗
- control the vehicle's speed and steer it to follow the predicted path
- handle real-world uncertainties and changing environments
- more flexible and adaptive

IDDC for robotic arm

- data from various sensors $\xrightarrow{ML}$ model of the robotic arm's dynamics$\rightarrow$ design a control algorithm$\rightarrow$achieve the desired control objectives.
- trained neural network model replaces the mathematical model of the arm's dynamics.
- overcomes the limitations of model-based control methods, which rely on precise mathematical models.

An open question is whether the direct data-driven control has advantages over the indirect data-driven control or vice versa.

The Process of the Experiment



Step 1: Original Data Generating

Step 2: Noisy-added Data

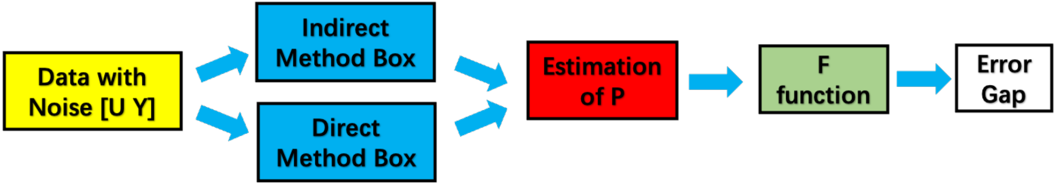


Fig. Different methods to track the original data

According to the error gap, we can determine which method is better.

Mathematics Analysis of DDC Design

LTI system equation

$$x_{t+1} = Ax_t + Bu_t + w_t, \quad y_t = Cx_t$$

can be written as: $\mathbf{y} = \mathcal{G}\mathbf{u} + \mathcal{G}'\mathbf{w}$, where :

$$\mathcal{G} = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{T-1}B & CA^{T-2}B & \dots & CB \end{bmatrix}$$

and the cost function can be used:

$$F(\mathbf{u}, \mathbf{y}) = \mathbf{u}^\top \mathcal{Q}\mathbf{u} + (\mathbf{y} - \mathbf{y}_{\mathrm{ref}})^\top \mathcal{R}\,(\mathbf{y} - \mathbf{y}_{\mathrm{ref}})$$

We want to find: $\mathbf{u}^* = \arg\min_{\mathbf{u}\in\mathbb{R}^{mT}} F(\mathbf{u}, \mathcal{G}\mathbf{u}) = \min_{\mathbf{u}\in\mathbb{R}^{mT}} F(\mathcal{P}\mathbf{u}),$ $\boxed{\mathrm{P}} = \begin{bmatrix} I \\ \mathcal{G} \end{bmatrix}$

Mathematics Analysis of DDC Design

Solve the equation above,

$$\nabla F(\mathbf{u},\mathbf{y}) = 2\begin{bmatrix} \mathcal{Q} & 0 \\ 0 & \mathcal{R} \end{bmatrix}\begin{pmatrix} \mathbf{u} \\ \mathbf{y} \end{pmatrix} - 2\begin{pmatrix} 0 \\ \mathcal{R}\mathbf{y}_{\mathrm{ref}} \end{pmatrix}.$$

Simplifying the above, we get:

$$\mathbf{u}^* = \left(\mathcal{Q} + \mathcal{G}^\top \mathcal{R} \mathcal{G}\right)^{-1} \mathcal{G}^\top \mathcal{R} \mathbf{y}_{\mathsf{ref}}$$

[Nortmann+ 2020] small summary:

$$A, B, C \longrightarrow \mathcal{G}$$
$$\mathcal{G} \longrightarrow \mathbf{u}^*, \mathcal{P}$$

Question: What if we don't have the information of A, B, C, how can we get the information of $\mathbf{u}^*, \mathcal{P}$

Estimation of $\mathcal{P}$

Noisy input-output behavior data:

$$U, Y \longrightarrow \widehat{\mathcal{P}}$$

The data-driven control design problem is formulated by replacing $\widehat{\mathcal{P}}$ for $\mathcal{P}$ in:

$$\min_{\mathbf{u}\in\mathbb{R}^{mT}} F(\widehat{\mathcal{P}}\mathbf{u}), \quad \widehat{\mathcal{P}} = \begin{bmatrix} I \\ \widehat{\mathcal{G}} \end{bmatrix}.$$

Also, let $\widehat{\mathbf{u}} = \arg\min_{\mathbf{u}\in\mathbb{R}^{mT}} F(\widehat{\mathcal{P}}\mathbf{u})$. Following similar steps as in the computation of $\mathbf{u}^*$:

$$\widehat{\mathbf{u}} = \left(\mathcal{Q} + \hat{\mathcal{G}}^\top \mathcal{R}\widehat{\mathcal{G}}\right)^{-1} \hat{\mathcal{G}}^\top \mathcal{R}\mathbf{y}_{\text{ref}}$$

$$\mathrm{Gap}(\widehat{\mathbf{u}}) = F(\mathcal{P}\widehat{\mathbf{u}}) - F\left(\mathcal{P}\mathbf{u}^*\right).$$

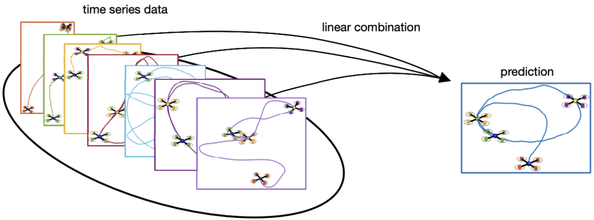
We use the gap to estimate whether the methhod is good or not.

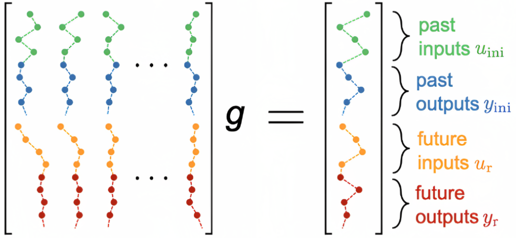
Find the $\mathrm{Gap}(\hat{\mathbf{u}})$ in DDDC and IDDC

The mathematical analysis above is for general DDC methods. However, for DDDC and IDDC, we need to design experiments to process signals with noise using both methods and calculate the $\mathrm{Gap}(\hat{\mathbf{u}})$ between the processed signal and the original signal in more detail.

Fig. Different methods to calculate the noise-polluted data

Direct Data-Driven Control



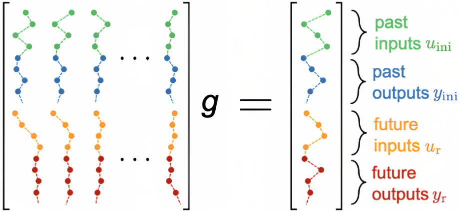


Trajectories from a library are linearly combined to optimally synthesize the future control trajectory [Nortmann+ 2020]

- Hankel matrix $\rightarrow$ time series data
- $g \rightarrow$ linear combination
- future in-outputs $\rightarrow$ prediction

Hankel Matrix

$$
\begin{bmatrix}
u(0) & u(1) & \cdots & u(T-L) \\
\vdots & \vdots & & \vdots \\
u(L-1) & u(L) & \cdots & u(T-1) \\
y(0) & y(1) & \cdots & y(T-L) \\
\vdots & \vdots & & \vdots \\
y(L-1) & y(L) & \cdots & y(T-1)
\end{bmatrix}
$$



Hankel matrices of these inputs and outputs

$$
\begin{bmatrix} \mathcal{H}_L\left(u_{[0,T-1]}\right) \\ \mathcal{H}_L\left(y_{[0,T-1]}\right) \end{bmatrix} g = \begin{bmatrix} \bar{u}_{[0,L-1]} \\ \bar{y}_{[0,L-1]} \end{bmatrix}
$$

The Results Simulated by MatLab

In direct method, I did some job in order to illustrate the equation blow better to be understood.

$$\begin{bmatrix} \bar{u}_{[0,L-1]} \\ \bar{y}_{[0,L-1]} \end{bmatrix} = \begin{bmatrix} \mathcal{H}_L\left(u_{[0,T-1]}\right) \\ \mathcal{H}_L\left(y_{[0,T-1]}\right) \end{bmatrix} g,$$

Given the initial state $y_{[0,0]}, u_{[0,N]}, \bar{u}_{[0,N]}, \bar{y}_{[0,N]}$

- **step1: given** $y_{[0,0]}, u_{[0,T-1]}$ **and use it to generate the** $\mathcal{H}_L(u_{[0,T-1]}, y_{[0,0]})$
- **step2: calculate the** g **via the pseudoinverse of** $\mathcal{H}_L(u_{[0,T-1]}, y_{[0,0]})$
- **step3:update the Hankle Matrix, and time** g **to get** $y_{[1,T]}$
- **step4: put** $y_{[T,T]}$ **as the initial value to replace** $y_{[0,0]}$**, back to the step1, till get all** $y_{[0,N]}$

end

Indirect Data-Driven Control

We want to use least-square algorithm to minimize the error generated by the measurement noise or disturbance

$$x(k+1) = Ax(k) + Bu(k)$$

$$X_{1,T} = \begin{bmatrix} B & A \end{bmatrix} \left[\frac{U_{0,T}}{X_{0,T}} \right]$$

where

$$X_{1,T} = \begin{bmatrix} x_d(1) & x_d(2) & \dots & x_d(T) \end{bmatrix}$$

it follows immediately that

$$\begin{bmatrix} B & A \end{bmatrix} = X_{1,T} \left[\frac{U_{0,T}}{X_{0,T}} \right]^{\dagger}$$

the right-hand side of the above identity is simply the minimizer of the least-square problem. After getting the model we put the initial state into it and compare the result with the experiment data.

The Reconstructed Model Matrix A And B

Here I provide some data generated by MatLab for you to have a better understanding. The model I used to generate the experiment data is:

$$A = \begin{bmatrix} -0.3304 & 0.3083 \\ 0.3083 & -0.0379 \end{bmatrix} B = \begin{bmatrix} -0.7342 & 0.2323 \\ -0.0308 & 0.4264 \end{bmatrix}$$

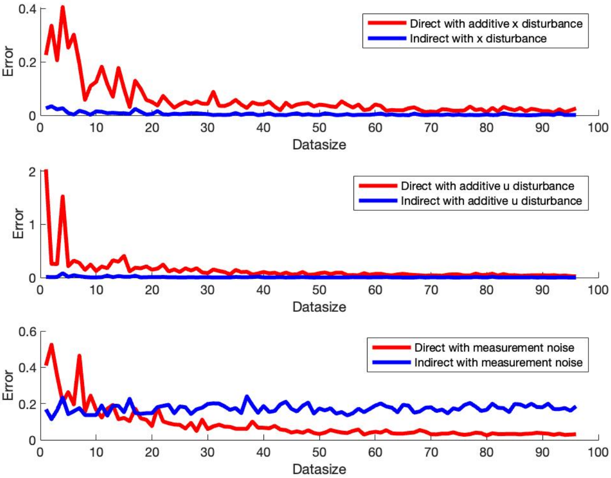
The calculated model is:

$$A_{\text{cal}} = \begin{bmatrix} -0.3301 & 0.3093 \\ 0.3094 & -0.0405 \end{bmatrix} B_{\text{cal}} = \begin{bmatrix} -0.7344 & 0.2319 \\ -0.0300 & 0.4270 \end{bmatrix}$$

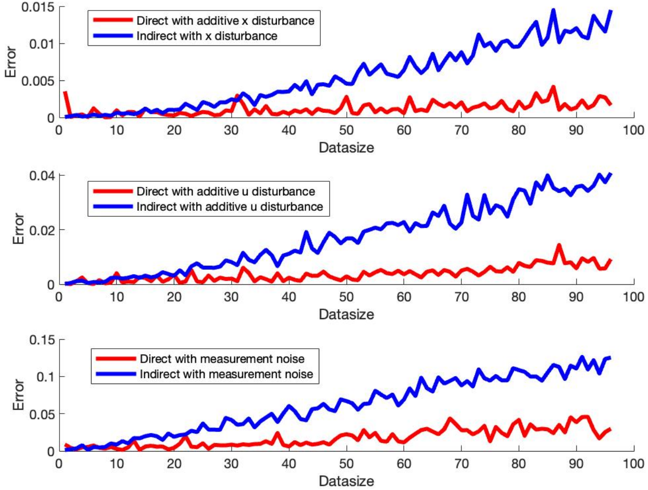
As we can see these two model is very close.

Random Gaussien Noise Influence

- Datasize is small
  $\mathbf{error}_{\text{direct}} > \mathbf{error}_{\text{indirect}}$
- $N \to \infty$, $\mathbf{error}_{\text{direct}} \to 0$,
  $\mathbf{error}_{\text{indirect}} \to \neq 0$
- And when it comes to the measurement noise, direct approach outperformers the indirect one

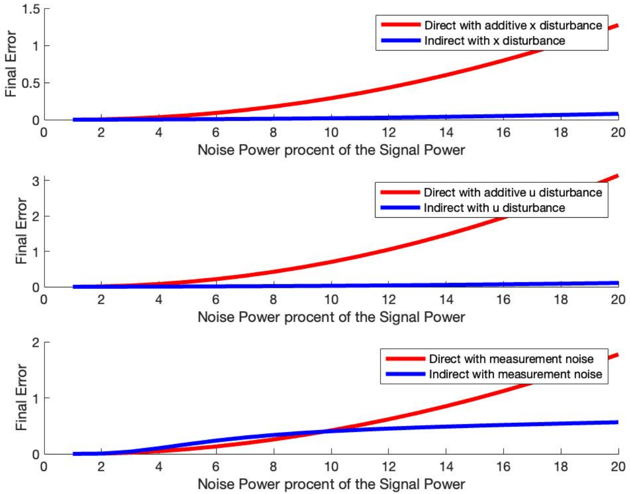


The Influence of constant offset



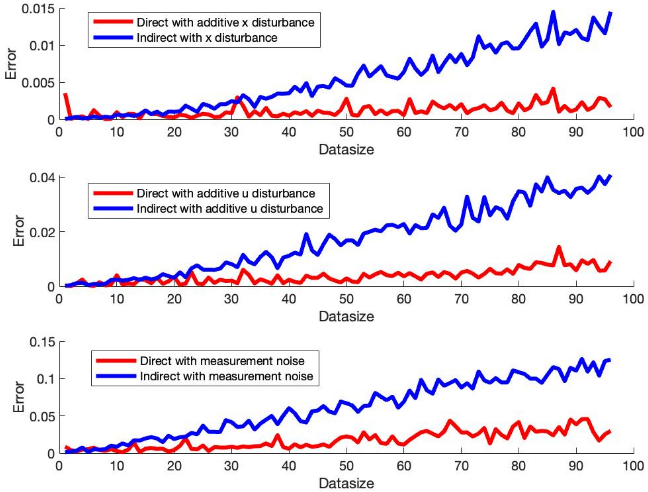
the performance of both approaches is poor, but the indirect approach performs even worse. Additionally, there is a linear relationship between the gap and data size in both approaches

The Influence of noise intensity in Gaussien random noise



When data size is fixed, the performance of the direct approach deteriorates with increasing strength of random noise, while the indirect approach can maintain relatively good performance. However, when the measurement noise exceeds a certain intensity, the indirect approach also exhibits poor performance.

The Influence of offset value



When data size is fixed and the noise is a constant offset, the opposite is observed. The performance of the indirect approach deteriorates with increasing strength of the offset, while the direct approach can maintain relatively good performance. However, when the offset exceeds a certain intensity, the direct approach also exhibits poor performance.

Conclusion

The study precludes the conclusion that either approach is always superior. [De Persis+ 2020]
Both approaches do not perform well when dealing with constant offsets, but the direct approach outperforms the indirect approach when dealing with measurement noise. When the data size is small, the indirect approach performs better.

Prospect: DDC method [Nortmann+ 2020]

- handle complex and nonlinear systems, which difficult to mathematically model.
- less sensitive to modeling errors and uncertainties.
- the availability and quality of data limits to handle high-dimensional systems.
- powerful in various fields, including robotics, manufacturing, transportation, and energy systems.

Thanks for your attention

References

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