Institute for
Communications
and Navigation

Satellite Navigation Laboratory SoSe 2022
Lab6. Tracking

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Recap:

- HW2 of Lab5
  - Q: Why N, not 2N?

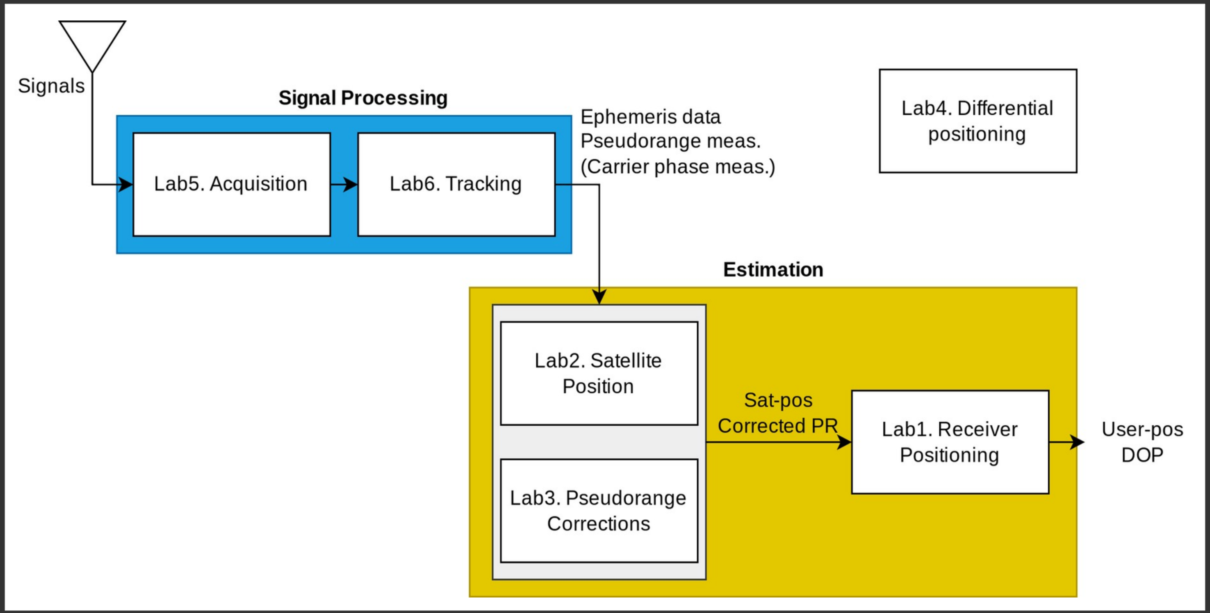
Parameter setting for acquisition:

- The number of samples per integration interval $N = T_{CO} \cdot f_s$ [samples/interval]
- The number of bins for the code-phase offset search $m = 1023 \cdot T_c/\Delta\tau$
- The number of bins for the Doppler shift search $n = 2\omega_{D,max}/\Delta\omega_D + 1$, where $\omega_D = 2\pi f_D$

  - A: Nyquist frequency
    - Sampling frequency should be at least 2f
    - = We can sample the signals with max. fs/2 when using fs
    - Thus,
      - N is correct no matter what fs is.
      - You need to check if our signal's frequency smaller than fs/2? (if the sampling frequency is properly set)

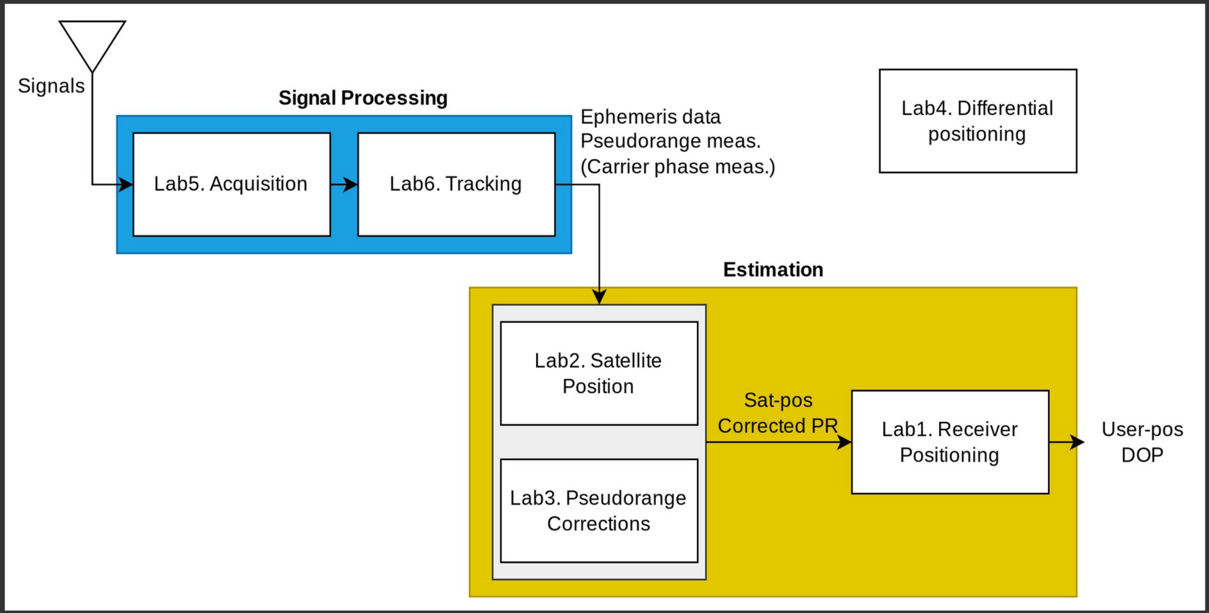
Overview

- Acquisition
  - PRN known / Code phase offset & Doppler shift estimated



Overview

- Acquisition
  - PRN known / Code phase offset & Doppler shift estimated
- Instead of repeating acquisition,
  - Track the code phase & Doppler shift of the signals
  - Keep the errors ~ 0
  - Estimate the phase offset for Precise Point Positioning (PPP)



Tracking loop

- Similar to the feedback control loop

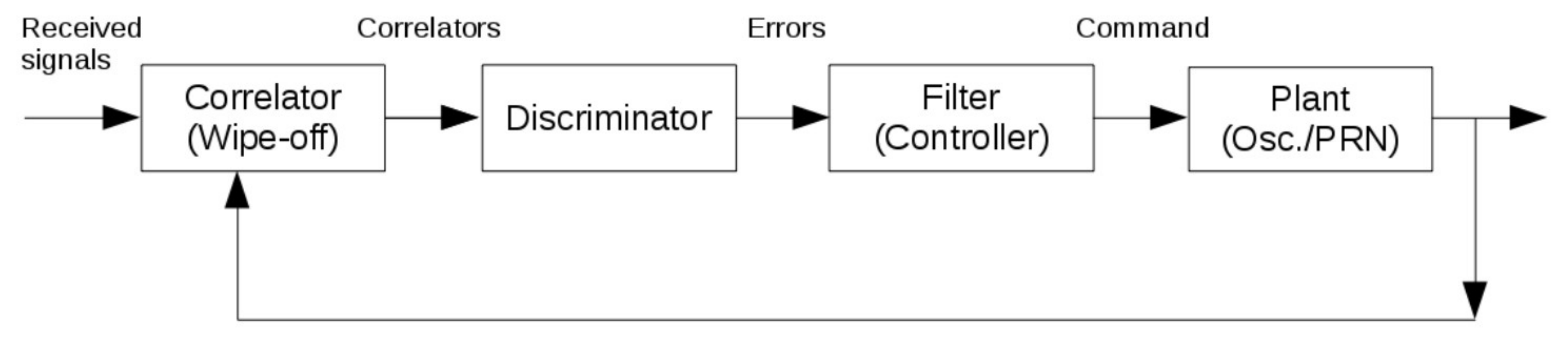


Figure 1: An overview of a tracking loop

Tracking loop

- Similar to the feedback control loop
  - Discriminator: Compute the errors using the correlations
  - Filter: Compute the control input for the plants
  - Plant: Generate the outputs (code replicas, carrier reference signals) using the parameters changed by the control input

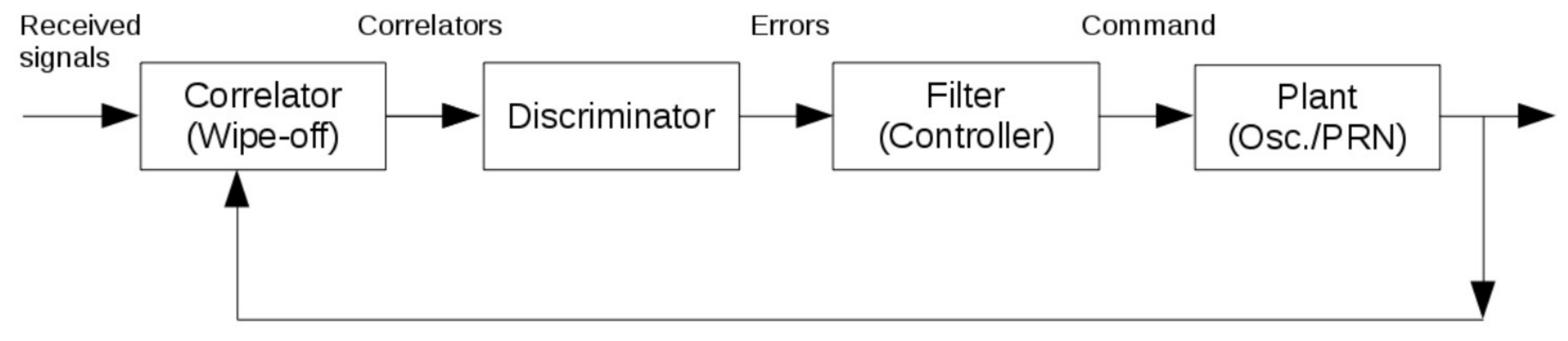


Figure 1: An overview of a tracking loop

Correlator

- Assuming acquisition was successful (del-tau, del-fD ~ 0)

$$\tilde{Z} = \tilde{S} + \tilde{\eta}.$$

- The prompt correlator: $\tilde{Z}_P = \tilde{S}_P + \tilde{\eta}_P \cong \sqrt{P_{rcv}} D \exp(j\Delta\theta) R(\Delta\tau) + \tilde{\eta}_P$
- The Early correlator: $\tilde{Z}_E = \tilde{S}_E + \tilde{\eta}_E \cong \sqrt{P_{rcv}} D \exp(j\Delta\theta) R(\Delta\tau - dT_c/2) + \tilde{\eta}_E$
- The Late correlator: $\tilde{Z}_L = \tilde{S}_L + \tilde{\eta}_L \cong \sqrt{P_{rcv}} D \exp(j\Delta\theta) R(\Delta\tau + dT_c/2) + \tilde{\eta}_L$

$$R(\Delta\tau) = \frac{1}{T_{CO}} \int_0^{T_{CO}} x(t-\tau) x(t-\hat{\tau}) dt.$$

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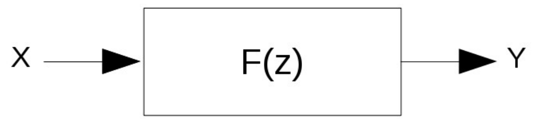
Discriminator:
- Correlators (measurements) → Errors
- Different Disc. for DLL & PLL will be discussed

Loop filter

- Errors => Control inputs of the plants (Osc./Code gen.)
- e.g. 2nd order filter



$$
\begin{aligned}
&F(z) = 2\zeta\omega_n + \frac{\omega_n^2 T_{CO} z^{-1}}{1 - z^{-1}} = \frac{Y}{X} \\
&\Leftrightarrow Y(1 - z^{-1}) = [2\zeta\omega_n(1 - z^{-1}) + \omega_n^2 T_{CO} z^{-1}]X \\
&\Leftrightarrow Y - z^{-1}Y = 2\zeta\omega_n X + (\omega_n^2 T_{CO} - 2\zeta\omega_n) z^{-1} X \\
&\Leftrightarrow Y_k - Y_{k-1} = 2\zeta\omega_n X_k + (\omega_n^2 T_{CO} - 2\zeta\omega_n) X_{k-1} \\
&\Leftrightarrow Y_k = 2\zeta\omega_n X_k + (\omega_n^2 T_{CO} - 2\zeta\omega_n) X_{k-1} + Y_{k-1}
\end{aligned}
$$

Loop filter

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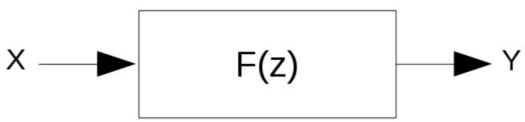

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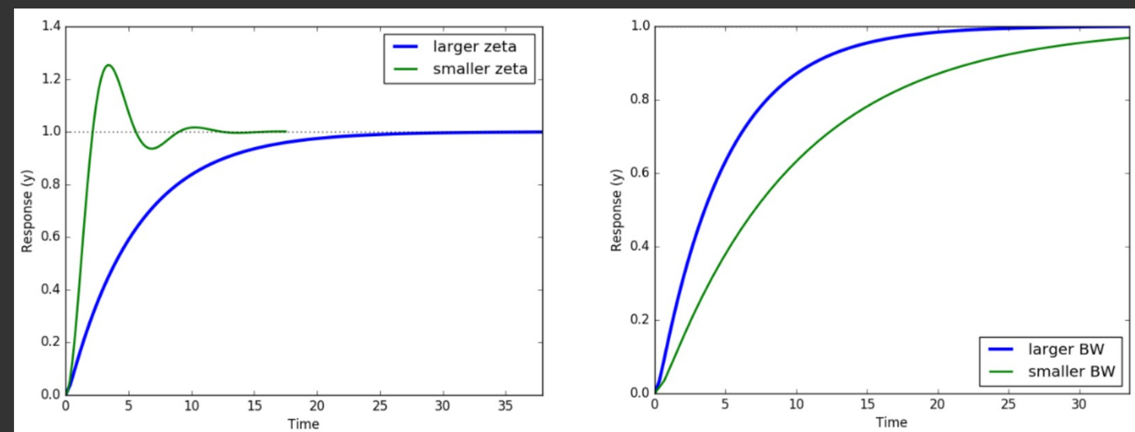

- Bigger ζ (blue) $\rightarrow$ Smaller overshoot, but slower response (longer settling time)
- Wider filter bandwidth (blue) $\rightarrow$ Shorter settling time, but lower robustness to noise

Delay Lock Loop (DLL)

- Coherent DLL
  - The carrier phase is locked (del-theta~0)
  - Navigation bits D are known

    => Track the changes of the code-phase offset (Control the code gen.)
  - e.g. Coherent early-minus-late discriminator

$$\begin{aligned} L_\tau(\Delta\tau, d) &= \frac{1}{2}\left(I_E - I_L\right) + \varepsilon_\tau \\ &= \frac{1}{2}\left[R\left(\Delta\tau - \frac{dT_c}{2}\right) - R\left(\Delta\tau + \frac{dT_c}{2}\right)\right] + \varepsilon_\tau . \end{aligned}$$

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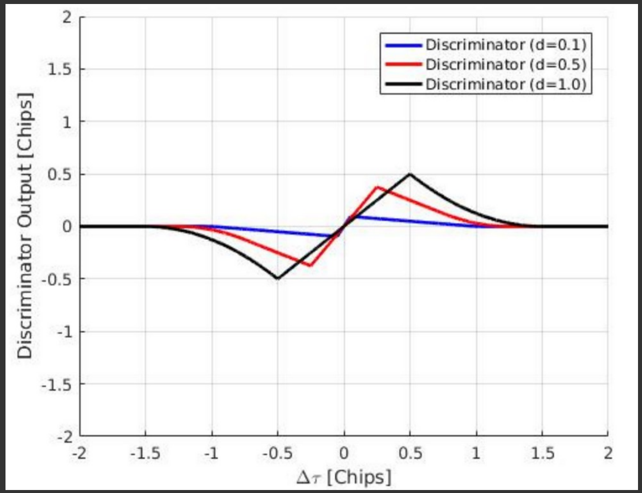
- HW1: Small & large d?
  - Large d (black):
    - Wide valid range
    - Can deal with more dynamics signals
  - Small d (blue):
    - More robust to the discriminator errors



Delay Lock Loop (DLL)

- HW2: How to normalize the E-L discriminator?

The normalized coherent early-late discriminator:

$$l_{\text{E-L}} = \frac{1}{2}\frac{I_{\text{E}} - I_{\text{L}}}{I_{\text{P}}}$$

Proof: Assuming $\Delta\tau \sim 0$, the early-late discriminator can be linearized as

$$L_{\text{E-L}} = \sqrt{P_{rcv}}\frac{\Delta\tau}{T_c} + \varepsilon_\tau = I_{\text{P}}\frac{\Delta\tau}{T_c} + \varepsilon_\tau.$$

Therefore, the normalized coherent early-late discriminator is

$$l_{\text{E-L}} = \frac{L_{\text{E-L}}}{I_{\text{P}}} = \frac{1}{2}\frac{I_{\text{E}} - I_{\text{L}}}{I_{\text{P}}}$$

Delay Lock Loop (DLL)

- Non-coherent DLL
  - The carrier phase is not locked (del-theta is not ~ 0)
  - e.g. DLL discminators and their normalized versions

- Non-coherent early-minus-late power: $\frac{1}{2}\left[(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)\right]$
- Quasi-coherent dot product power: $\frac{1}{2}\left[(I_E - I_L)I_P + (Q_E - Q_L)Q_P\right]$
- Non-coherent early-minus-late envelope: $\frac{1}{2}\left[\sqrt{I_E^2 + Q_E^2} - \sqrt{I_L^2 + Q_L^2}\right]$

$$l_{\text{E-L power}} = \frac{1}{2}\frac{(I_\text{E}^2 + Q_\text{E}^2) - (I_\text{L}^2 + Q_\text{L}^2)}{(I_\text{E}^2 + Q_\text{E}^2) + (I_\text{L}^2 + Q_\text{L}^2)} \cdot (2-d)$$

$$l_{\text{Dot product}} = \frac{1}{4}\left[\frac{I_\text{E} - I_\text{L}}{I_\text{P}} + \frac{Q_\text{E} - Q_\text{L}}{Q_\text{P}}\right]$$

$$l_{\text{E-L envelope}} = \frac{1}{2}\frac{\sqrt{I_\text{E}^2 + Q_\text{E}^2} - \sqrt{I_\text{L}^2 + Q_\text{L}^2}}{\sqrt{I_\text{E}^2 + Q_\text{E}^2} + \sqrt{I_\text{L}^2 + Q_\text{L}^2}} \cdot (2-d)$$

Phase Lock Loop (PLL)

- Assuming
  - The code phase delay is locked (del-tau~0)
  - Navigation bits D are unknown

  => Track the changes of the carrier phase (Control the oscillator)

Phase Lock Loop (PLL)

- HW3: The properties of the PLL disc. ?
  - Data bits are unknown

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    => Disc. should be insensitive to the data bits

    => $L_\theta(\Delta\theta) = L_\theta(\Delta\theta + n\pi),\ n \in \mathbb{Z}.$

Phase Lock Loop (PLL)

- HW3: The properties of the PLL disc. ?
  - Data bits are unknown

    => Disc. should be insensitive to the data bits

    => $L_\theta(\Delta\theta) = L_\theta(\Delta\theta + n\pi),\ n \in \mathbb{Z}.$
  - e.g. Tangent discriminator

$$L_{\mathrm{atan}} = \mathrm{atan}\left(Q_{\mathrm{P}}/I_{\mathrm{P}}\right)$$

$$I_{\mathrm{P}}(\Delta\tau, \Delta\phi) = \mathcal{A} \cdot R(\Delta\tau)\cos(\Delta\theta)$$
$$Q_{\mathrm{P}}(\Delta\tau, \Delta\phi) = \mathcal{A} \cdot R(\Delta\tau)\sin(\Delta\theta)$$

Note that we need to use **atan** instead of atan2 here
- atan(y/x): [-pi/2, +pi/2]
- atan2(y,x): close to -pi, and pi

e.g. x=1, y=0.01
- atan(y/x)=-0.57deg
- atan(y,x) =179.43deg

Phase Lock Loop (PLL)

- HW4: How to normalize the Costas disc. ?

$$L_{\mathrm{IQ}} = \frac{I_{\mathrm{P}} \cdot Q_{\mathrm{P}}}{I_{\mathrm{P}}^2 + Q_{\mathrm{P}}^2}.$$

Navigation Bit Demodulation

- HW5: Estimate the navigation bit D using MAP
  - If DLL and PLL are locked => del-tau~0 & del-theta~0

    => $I_{\mathrm{P}} \approx \sqrt{P_{rcv}} D + \eta_{\mathrm{I}}$
  - If the integration interval of correlation is 1ms

    => 20 samples = 20ms = 1 bit duration

    => Navigation bit can be estimated using MAP with 20 inphase samples

$$D^* = \underset{D}{\operatorname{argmax}}\, P(D = \pm 1 | I_{\mathrm{P}}).$$

Navigation Bit Demodulation

- HW5: Estimate the navigation bit D using MAP

$$D^* = \underset{D}{\operatorname{argmax}}\, P(D = \pm 1 | I_\mathrm{P}).$$

  - Bayesian theorem

$$\underbrace{P(D = \pm 1 | I_\mathrm{P})}_{\text{a posteriori}} = \underbrace{P(I_\mathrm{P} | D = \pm 1)}_{\text{likelihood}} \times \underbrace{P(D = \pm 1)}_{\text{a priori}}$$

  - Assuming a priori is constant, MAP → ML

$$\begin{aligned} D^* &= \underset{D}{\operatorname{argmax}}\, P(D = \pm 1 | I_\mathrm{P}) \\ &= \underset{D}{\operatorname{argmax}}\, P(I_\mathrm{P} | D = \pm 1) \end{aligned}$$

Navigation Bit Demodulation

- HW5: Estimate the navigation bit D using MAP
  - Assuming that the noise of the inphase samples ~ Gaussian

$$P(I_{\mathrm{P},k}|D=\pm 1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(I_{\mathrm{P},k}-\sqrt{P_{rcv}}D)^2}{2\sigma^2}\right).$$

Navigation Bit Demodulation

- HW5: Estimate the navigation bit D using MAP
  - Assuming that the noise of the inphase samples ~ Gaussian

$$P(I_{\mathrm{P},k}|D=\pm 1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(I_{\mathrm{P},k}-\sqrt{P_{rcv}}D)^2}{20^2}\right).$$

$$\sum_{k=1}^{20} -\frac{(I_{\mathrm{P},k}-\sqrt{P_{rcv}}1)^2}{20^2} \underset{D=-1}{\overset{D=+1}{\gtrless}} \sum_{k=1}^{20} -\frac{(I_{\mathrm{P},k}-\sqrt{P_{rcv}}(-1))^2}{20^2}$$

$$\sum_{k=1}^{20} I_{\mathrm{P},k}^2 - 2I_{\mathrm{P},k} + P_{rcv} \underset{D=-1}{\overset{D=+1}{\lessgtr}} \sum_{k=1}^{20} I_{\mathrm{P},k}^2 + 2I_{\mathrm{P},k} + P_{rcv}$$

$$\sum_{k=1}^{20} I_{\mathrm{P},k} \underset{D=-1}{\overset{D=+1}{\gtrless}} 0$$

Lab Schedule

	Tag	Datum	von	bis	Ort	Ereignis	Termintyp	Info	Lerneinheit
Standardgruppe Hinweis: Auf Datum klicken, um Einzeltermin zu verschieben. In der Spalte Serie auf S klicken, um Terminserie zu verschieben.									
☐	Di	03.05.2022	10:00	11:00	N2409, Seminarraum (0104.02.409)	Abhaltung	fix		Kick-off meeting
☐	Di	10.05.2022	10:00	14:00	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab1. Receiver Positioning
☐	Di	24.05.2022	11:30	15:30	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab2. Satellite Position
☐	Di	31.05.2022	10:00	14:00	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab3. Pseudorange Corrections
☐	Di	14.06.2022	10:00	14:00	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab4. Differential Positioning
☐	Di	28.06.2022	10:00	14:00	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab5. Acquisition
☐	Di	12.07.2022	10:00	14:00	N2407B, Praktikum (0104.02.407B)	Abhaltung	fix		Lab6. Tracking
☐	~~Di~~	~~19.07.2022~~	~~10:00~~	~~14:00~~	N2407B, Praktikum (0104.02.407B)	~~Abhaltung~~	~~fix~~		~~Left over~~

Final exam – Time slots

Aug. 2, Tuesday, 2022

Time	Student
09:00-09:30	Alberto Arana Ragel
09:30-10:00	Peng Xie
10:00-10:30	Break
10:30-11:00	Alvaro Pérez-Lozao Alonso
11:00-11:30	Yiming Wei
11:30-12:00	Jonathan Klesse
12:00-12:30	Break
12:30-13:00	Break
13:00-13:30	Akhil Nahar
13:30-14:00	Fahri Mert Ünsal