Signal

front-end: Amplification / Filtering / Down-converting

目地：
① 找 PRN. => to differeniate the signals.
② Arrived time τ, pseudorange measurement, user position and Clock offset
③ Doppler shift f_D => user velocity and Clock frequency.

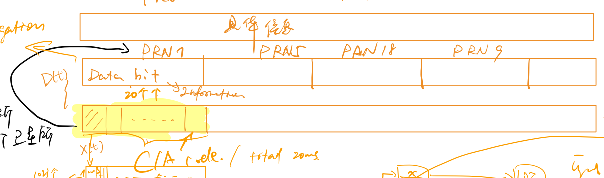
Acqusition

$S(t) = \sqrt{2P_{tmt}}\, D(t)\, x(t) \cos(2\pi f_c t + \theta_{tmt})$

$r(t) = \sqrt{2P_{rcv}}\, D(t-\tau)\, x(t-\tau) \cos(2\pi (f_c + f_D) t + \theta_{rcv}) + n(t)$

滤波后 → $r(t) = \sqrt{2P_{rcv}}\, D(t-\tau)\, x(t-\tau) \cos(2\pi (f_{IF} + f_D) t + \delta\theta) + \eta(t)$

$\delta\theta$: $\theta_{tmt} - \theta_{rcv}$



它创造 回来判断 是哪一个卫星所发送信息

实际发送的 Navigation code + C/A code (D)

如果 D 是 0 则 C/A code 不变. 如果 D 为 1 则翻转

Carrier wipe-off

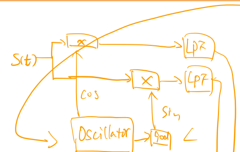
为了去掉其中不必要的信息 如 f_{IF}

利用一组信号不断和 $r(t)$ 信息相乘，改变 $\hat{f}_D$ 和 $\hat{\theta}$ 滤波

$I: \sqrt{2}\cos(2\pi(f_{IF} + \hat{f}_D)t + \hat{\theta})$

$Q: \sqrt{2}\sin(2\pi(f_{IF} + \hat{f}_D)t + \hat{\theta})$

每格



可得到：$I: \sqrt{P_{rcv}}\, D(t-\tau)\, x(t-\tau) \cos(2\pi \Delta f_D t + \Delta\theta)$

$x(t-\tau)$: Code related; Δf_D, $\Delta\theta$: Carrier signal related

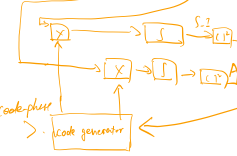
Q: ... sin

去掉了 f_{IF}.

Code wipe-off

$S_I = \frac{\sqrt{P_{rcv}}}{T_{co}} \int_0^{T_{co}(10ms)} x(t-\tau)\, x(t-\hat{\tau}) \cos(2\pi \Delta f_D t + \Delta\theta)\, dt$

$S_Q = \ldots \sin$



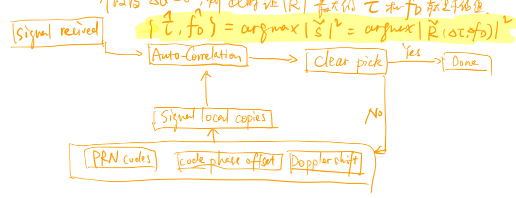
Ambiguity Function:

$\tilde{R}(\Delta\tau, \Delta f_D) = \int_0^{T_{co}} \frac{1}{T_{co}} x(t-\tau)\, x(t-\hat{\tau}) \exp(j 2\pi \Delta f_D t)\, dt$

$\tilde{S} = S_I + j S_Q = \sqrt{P_{rcv}}\, D \exp(j\Delta\theta)\, \tilde{R}(\Delta\tau, \Delta f_D)$

假设 $\Delta\theta = 0$，则 此时让 $|\tilde{R}|^2$ 最大的 τ 和 f_D 就是我们想要值

$\{\hat{\tau}, \hat{f}_D\} = \arg\max |\tilde{S}|^2 = \arg\max |\tilde{R}(\Delta\tau, \Delta f_D)|^2$



Non-Coherent acquisition:

① - serial search. Check all the grids.

② - Parallel frequency space search. Check the Doppler shifts in parallel for each code-phase

针对某一特定的 Doppler offset - candidate

③ - Parallel code space search. check the code-phase offset in parallel for each Doppler-shift candidate

针对某一特定的 code-phase offset

Complexity: ① $O(mnN)$ ② $O(mN\log N)$ ③ $(nN\log N)$

N: 积分区间中的采样个数.

m: 循环中的位移的个数

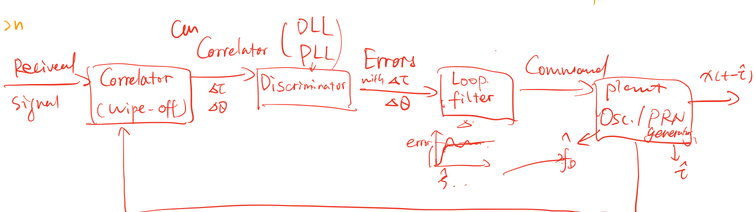
n: 循环中 Δf_D 位移个数 $+1$

Normally $m > n$

找到 acquisition 后. global research 太麻烦，所以用 tracking 来节约时间).

即假设 $\Delta\tau$ 和 $\Delta\theta$ 变化很小. 这里用到 控制论 跟踪的方法.

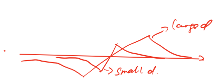
Tracking



① 对于 Discriminator 中一般有

DLL. PLL. FLL. 3选2. (DLL和 PLL) (DLL和 FLL)

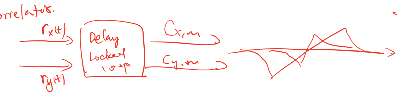
DLL
- coherent ($\Delta\theta = 0$, D known) $I_{E-L} = \frac{1}{2} \frac{I_E - I_L}{I_P}$ → track $\Delta\tau$ → code generator.
- non-coherent ($\Delta\theta \neq 0$) { e-l power $\frac{1}{2}: [(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)]$; product power ; envelope ;



PLL. ($\Delta\tau = 0$, D unknown) track $\Delta\theta$ → Osc. $L_{atan} = \operatorname{atan}(Q_P / I_P)$

↳ insensitive to D.

② 对于 correlator.



③ Navigation Bit demodulation

ephemeric data
pesudorange measurement
Carrier phase measurement.

satellite position

$\rho_c^k = \|\vec{r}_u^k - \vec{r}_c\| + c\delta r + \eta^k$

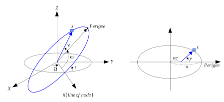


Figure 1: Keplerian parameters of satellite orbits [1]

a ; Semi-major axis
e : eccentricity
ν : true anomaly
2D

i : orbital inclination
Ω : Right acension of ascending node
ω : Argument of prigee
3D

Using ephemeris Data to compute

① 计算 2D 中的 mean. anomaly and eccentric anomaly

$M \rightarrow E$

$M = M_0 + n \cdot t_k$ $t_k = t - t_{oe}$ $n = \sqrt{\frac{GM}{a^3}} + \delta n.$

$\vec{r}_{orb}^s = \begin{pmatrix} r\cos u \\ r\sin u \\ 0 \end{pmatrix}$

$E = M + e\sin(E)$ ⓐ E, 计算 V, 和 r. ⓑ V 计算 u. ⓒ u 计算 $\vec{r}_{orb}$

$V = \arctan\left(\frac{\sqrt{1-e^2}\sin E}{\cos E - e}\right)$

$r = a(1 - e\cos E) + C_{rc}\cos 2\phi + C_{rs}\sin 2\phi$

$u = V + w + C_{uc}\cos 2\phi + C_{us}\sin 2\phi$

② 计算 3D 中的 $\vec{r}_{orb}$

inclination : $i = i_0 + \dot{i} \cdot t_k + C_{ic}\cos(2\phi) + C_{is}\sin(2\phi)$

RAAN: $\Omega = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) \cdot t_k - \dot{\Omega}_E \cdot t_{oe}$

将得到的 i 和 Ω. 转从 orb 到 ECEF.

$\vec{r}_{ECEF}^s = \begin{pmatrix} r_x\cos\Omega - r_y\cos(i)\sin\Omega \\ r_x\sin\Omega - r_y\cos i\cos\Omega \\ r_y\sin i \end{pmatrix},$

pseudorange Corrections

① GPS satellite positions correction

根据 Satellite position 中的 6 要素, 对其中 2 个

做了更改. 1. Mean motion ⇒ instead of true anomaly.

2. Longititude of acending node ⇒ instead of RAAN.

$\rho_{r,f}^s = c(t_r'' - t^{s'}) = c(t_r'' - t_r + t_r - t^s + t^s - t^{s'})$

$= c[(t_r'' - t_r) - (t^{s'} - t^s)] + c(t_r - t^s)$

$= c \cdot \Delta t_r - c \cdot \Delta t^s + \|\vec{r}_r - \vec{r}^s\| = c\Delta t_r - c\Delta t^s + \vec{e}_r^{s\,T}(\vec{r}_r - \vec{r}^s) + I + T + \eta$

$\rho_{f,c} = \rho_{r,f}^s + \underbrace{c \cdot \hat{\Delta t}^s + \vec{e}_r^{s\,T}\hat{\vec{r}}^s - \hat{I} - \hat{T}}_{\text{根据上面可以求出 satellite orbit.}} = c\Delta t_r + \vec{e}_r^{s\,T}\vec{r}_r + \eta$

Satellite-position → Correeted pseudorange measurement.

Ionospheric
$\rho_{f_1} = (\vec{e}_r^s)^T(\vec{r}_r - \vec{r}^s) + c(\Delta t_r - \Delta t^s) + I_{f_1}^s + T_r^s + \eta_{r,f_1}^s$

$\rho_{f_2} = (\vec{e}_r^s)^T(\vec{r}_r - \vec{r}^s) + c(\Delta t_r - \Delta t^s) + \frac{f_1^2}{f_2^2} I_{f_1}^s + T_r^s + \eta_{r,f_2}^s$

$\rho = \alpha_1 f_1 + \alpha_2 f_2,$

$\Rightarrow \underbrace{\begin{pmatrix} \rho_{f_1}^1 \\ \vdots \\ \rho_{f_1}^k \\ \rho_{f_2}^1 \\ \vdots \\ \rho_{f_2}^k \end{pmatrix}}_{2k\times 1} = \underbrace{\left[\begin{array}{c|c} H & I_{k\times k} \\ \hline H & \frac{f_1^2}{f_2^2} I_{k\times k} \end{array}\right]}_{2k\times(4+k)} \underbrace{\begin{pmatrix} \vec{r}_u \\ c\delta_u \\ \hline I_{f_1}^1 \\ \vdots \\ I_{f_1}^k \end{pmatrix}}_{(4+k)} + \vec{\eta}$

Tropospheric 假设

$\begin{pmatrix} \rho_1 \\ \vdots \\ \rho_k \end{pmatrix} = \underbrace{\left[\begin{array}{c|c} H & \begin{matrix} m(E^1) \\ \vdots \\ m(E^k) \end{matrix} \end{array}\right]}_{k\times 5} \underbrace{\begin{pmatrix} \vec{r}_u \\ c\delta_u \\ \hline T_v \end{pmatrix}}_{4+1} + \vec{\eta}$

你还需要计算的:

① $\hat{\Delta t}^s$, Computed with ephemeris data $\Delta t_{sv} = a_{f_0} + a_{f_1}(t - t_{oc}) + a_{f_2}(t - t_{oc})^2 + \Delta t_{rel}$
from Gps satellite.

take value from IGS.

② Atomospheric delays. → Ionospheric delay ($\phi_{pp}, \lambda_{pp}, z_{pp}$)

→ Model with total Electron Content (TEC)

$I_{\rho f} = 40.3\frac{TEC}{f^2}$ $I_{\phi f} = -40.3\frac{TEC}{f^2}$

→ Mapping function $I_{slant} = M_{SLM}(E)\, I_{vertical}$

将研究 I_{slant} 转化为研究 $I_{vertical}$

Vertical ionospheric models.

① klobucher mode ② Global ionospheric model

→ Dual frequency observations.

multiple frequency → a linear combined pseudorange model

$\rho_{LC} = \sum_{i=1}^{N} \alpha_i \rho_{f_i}$

Tropospheric delay. ← ② Atomospheric delays

↓ 一样先将 $T_{slant} \rightarrow T_{vertical}$

Mapping function $m(E, a, b, c)$

elevation angle coefficients

Black and Eisner. Niel Vienna Global

只考虑 evelation angle 海拔角

weather high accuraney

Vienna mapping function 的 average.

↓ 再研究 $T_{vertical}$

① Collins.

② post-processed data from IGS

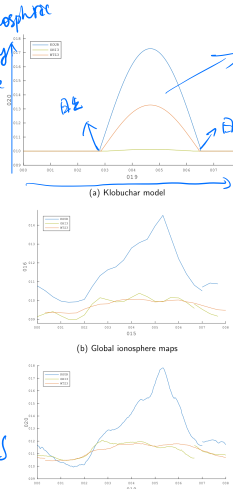
③ Direct estimation with an extended model.

Ionospheric delay time

日出 日落

有太阳的时候 延迟高

一天的时间



(a) Klobuchar model

(b) Global ionosphere maps

(c) Dual-frequency observations

Receiver positioning

$\downarrow$

user-position

Dop.

$$\rho_c^k(\vec{r}_u, c\delta) = \|\vec{r}_u - \vec{r}_c^k\| + c\delta + \eta^k$$

$$\nabla \rho_c^k(\vec{r}_u, c\delta) = [(\vec{e}^k)^T \ \ 1]$$

$$\Delta \rho_c^k(\vec{r}_u, c\delta) = ((\vec{e}^k)^T, 1)\begin{pmatrix}\Delta\vec{r}_u \\ c\Delta\delta_i\end{pmatrix} + \eta^k$$

$$= ((\vec{e}^k)^T, 1)\Delta\vec{\xi}_i + \eta^k$$

$$\widehat{\Delta\xi_i} = \underset{\Delta\vec{\xi}_i}{\operatorname{argmin}} \|\Delta\vec{\rho}_{c,i} - H_i\Delta\vec{\xi}_i\|^2$$

$$\Rightarrow \Delta\hat{\xi}_i = (H_i^T H_i)^{-1} H_i^T \Delta\vec{\rho}_i$$

$$= (H_i^T \Sigma^{-1} H_i)^{-1} H_i^T \Sigma^{-1} \Delta\vec{\rho}_i$$

$$\Rightarrow \hat{\xi}_{i+1} = \hat{\xi}_i + \Delta\hat{\xi}_i$$

Differential positioning

可以通过上述流程找到最后 pseudorange measurement，也可以直接通过 differential pointing 的方法直接得到

Pseudorange measurements:

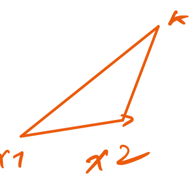
error of satellite orbit.

$$\rho_{u,m}^k(t) = (\vec{e}^k)^T(\vec{r}_u(t) - r^{(k)}(t)) - \underline{\delta r^k(t)} + c(\delta t_u(t) - \delta t^k(t-\tau)) + I_{u,m}^k(t) + T_u^k(t) + \varepsilon_{u,m}^k(t)$$

① Single difference measurement.

$$\rho_{1,m}^k(t) - \rho_{2,m}^k(t) = (\vec{e}_1^k)^T \vec{r}_{12}(t) + c \cdot \delta t_{12}(t) + I_{12,m}^k(t) + T_{12,m}^k(t) + \varepsilon_{12,m}^k(t)$$

unknown.



1. $\vec{r}_{12}(t) \rightarrow$ baseline vector
2. Satellite orbit error and satellite clock bias are removed
3. short baseline $\rightarrow$ atmosphere delay are reduced.

② Double difference measurement.

$$\rho_{12}^{kl}(t) = (\vec{e}^k - \vec{e}^l)^T \vec{r}_{12}(t) + I_{12,m}^{kl} + T_{12,m}^{kl} + \varepsilon_{12,m}^{kl}(t)$$

$\Downarrow$ corrected.

$$\rho_{12,c}^{kl} = \rho_{12}^{kl}(t) - I_{12,m}^{kl} - T_{12,m}^{kl} = (\vec{e}^k - \vec{e}^l)^T \vec{r}_{12}(t) + \varepsilon_{12,m}^{kl}(t)$$

$$\underbrace{\begin{pmatrix} \rho^{11}_{12,c} \\ \rho^{21}_{12,c} \\ \vdots \\ \rho^{k1}_{12,c} \end{pmatrix}}_{\vec{\rho}_{12,c}} = \underbrace{\begin{pmatrix} \vec{e}^{1} - \vec{e}^{l} \\ \vec{e}^{2} - \vec{e}^{l} \\ \vdots \\ \vec{e}^{k} - \vec{e}^{l} \end{pmatrix}}_{H_{dd}} \vec{r}_{12}(t) + \varepsilon_{12,m}^{kl}$$

$$\hat{r}_{12} = (H_{dd}^{T} C^{-1} H_{dd})^{-1} C^{-1} \vec{\rho}_{12,c}$$