1. $\hat{\xi}_{WLS} = \arg\min_{\xi} \|\Delta\rho_{c,i} - H_i\Delta\xi\|^2_W$

$$\Rightarrow 0 \stackrel{!}{=} \frac{\partial}{\partial\Delta\xi}\left[(\Delta\rho_{c,i} - H_i\Delta\xi)^T W^{-1}(\Delta\rho_{c,i} - H_i\Delta\xi)\right]$$

$$0 \stackrel{!}{=} \frac{\partial}{\partial\Delta\xi}\left[\Delta\rho_{c,i}^T W^{-1}\Delta\rho_{c,i} - 2\rho^T W^{-1} H_i\Delta\xi + \Delta\xi^T H_i^T W^{-1} H_i \Delta\xi\right]$$

$$\Rightarrow 0 \stackrel{!}{=} 2(\Delta\rho_{c,i}^T W^{-1} H^T) + 2 H_i^T W^{-1} H_i \Delta\xi$$

a convariance matrix is symmetric and so is its inverse, i.e

$$(W^{-1})^T = W^{-1}$$

$$0 \stackrel{!}{=} -H_i^T W^{-1}\Delta\rho_{c,i} + H_i^T W^{-1} H_i\Delta\xi$$

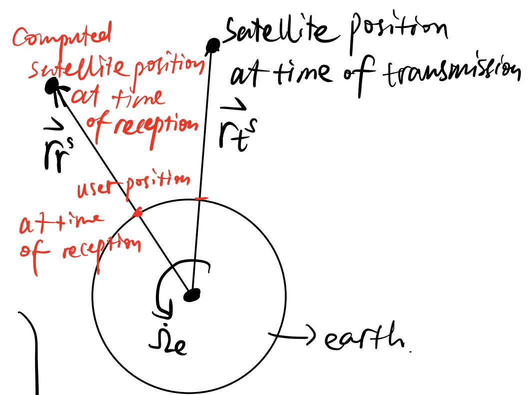
$$\Rightarrow \Delta\xi \stackrel{!}{=} (H_i^T W^{-1} H_i)^T H_i^T W^{-1}\Delta\rho_{c,i}$$

2). Setting a small scalar γ.

after i interations we have

$$\|\xi_{i+1} - \xi_i\| < \gamma$$

and stops.



3). $\vec{r}_r^{\,s} = R_z(\dot{\Omega}_e(t_r - t_t)) \cdot \vec{r}_t^{\,s}$

and $R_z(\dot{\Omega}_e(t_r - t_t))$

$$= \begin{pmatrix} \cos(\dot{\Omega}_e(t_r - t_t)) & \sin(\dot{\Omega}_e(t_r - t_t)) & 0 \\ -\sin(\dot{\Omega}_e(t_r - t_t)) & \cos(\dot{\Omega}_e(t_r - t_t)) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Conculsion: we need to use the transform matrix to compensate the satellite position error

4). from P3 – 2.2 conversion to a Local Coordinates.

we have,

$$\vec{r}_{u,ENU} = R_x(90^\circ - \phi)\, R_z(\lambda + 90^\circ)\, \vec{r}_u$$

with

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \quad R_z = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and we have $\vec{r}_{u,ENU} = (\hat{e}\ \hat{n}\ \hat{u}) = \begin{bmatrix} -\sin\lambda & -\cos\lambda\sin\phi & \cos\lambda\cos\phi \\ \cos\lambda & -\sin\lambda\sin\phi & \sin\lambda\cos\phi \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \vec{r}_u$

and we need to calculate A. s.t.

$$A \cdot \vec{r}_{u,ENU} = \vec{r}_u \Rightarrow A = \begin{bmatrix} -\sin\lambda & -\cos\lambda\sin\phi & \cos\lambda\cos\phi \\ \cos\lambda & -\sin\lambda\sin\phi & \sin\lambda\cos\phi \\ 0 & \cos\phi & \sin\phi \end{bmatrix}^{-1}$$

$$\Rightarrow A = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\phi & -\sin\lambda\sin\phi & \cos\phi \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \end{bmatrix}$$

So we have

$$\hat{x} = (-\sin\lambda \quad -\cos\lambda\sin\phi \quad \cos\lambda\cos\phi)$$
$$\hat{y} = (\cos\lambda \quad -\sin\lambda\sin\phi \quad \sin\lambda\cos\phi)$$
$$\hat{z} = (0 \quad \cos\phi \quad \sin\phi)$$

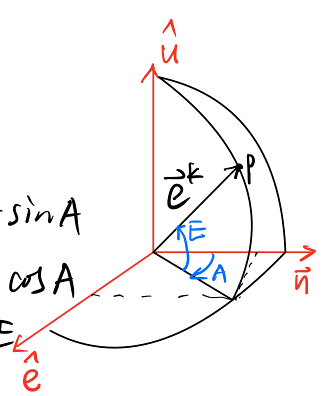
Elevation and azimuth.

Given the line of sight unit vector

$$\vec{e}^{\,k} = \frac{\vec{r}_u - \vec{r}_c^{\,k}}{\|\vec{r}_u - \vec{r}_c^{\,k}\|}$$

we have,

$$\begin{cases} \vec{e}^{\,k} \cdot \hat{e} = \cos E \sin A \\ \vec{e}^{\,k} \cdot \hat{n} = \cos E \cos A \\ \vec{e}^{\,k} \cdot \hat{u} = \sin E \end{cases}$$



and accroding to the equations, we have

$$E = \arcsin(\vec{e}^k \cdot \hat{\vec{u}})$$

$$A = \arctan\left(\frac{\vec{e}^k \cdot \hat{\vec{e}}}{\vec{e}^k \cdot \hat{\vec{n}}}\right)$$