4 Homework

1. Show an iterative method to estimate the eccentric anomaly E with Kepler's equation(4) using a given mean anomaly M.

Using Newton-Method to do iteration

① $F(E) = E - e\sin(E) - M$

$F'(E) = 1 - e\cos(E)$

② $F(E_n) = E_n - e\sin(E_n) - M$

$F'(E_n) = 1 - e\cos(E_n)$

③ $E_{n+1} = E_n - \frac{f(E_n)}{f'(E_n)}$

$= E_n - \frac{E_n - e\sin(E_n) - M}{1 - e\cos(E_n)}$

$= \frac{-E_n e\cos(E_n) + e\sin(E_n) + M}{1 - e\cos E_n}$

④ check if $\left| \frac{-E_n e\cos(E_n) + e\sin(E_n) + M}{1 - e\cos E_n} \right| < \epsilon$

⑤ check

if ④ is No. back to ③

if ④ is Yes. END.

or

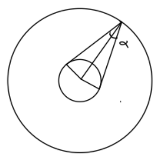
① $E_{n+1} = M - e\sin E_n$

② check if $|E_{n+1} - E_n| < \epsilon$

if NOT back to ①

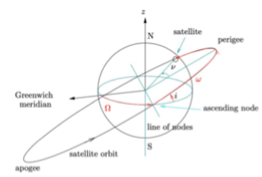
if YES, end.

2. The radius of the Galileo satellite orbits is approximately 29601 km. Consider a cone that is centered at the satellite, and encloses the complete Earth. Compute the apex angle of the cone.



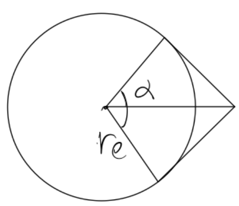
$\tan\frac{\alpha}{2} = \frac{r_e}{r_{Ga}} = \frac{6371}{29601}$

$\Rightarrow \alpha = 12.1464^\circ \times 2 = 24.2928^\circ$



3. Roughly estimate the minimum number of Medium Earth Orbit (MEO) satellites (e.g. Galileo) to ensure the visibility of at least four satellites at any location on the Earth. How is the minimum number changed for Low Earth Orbit (LEO) satellites (altitude 700km)?

GPS requires 24 satellites to be the bare minimum. This number can ensure that at least 4 satellites can be seen at any time in every place in the world, and four variables of three-dimensional coordinates and time can be solved through four equations. The 24 satellites are distributed in 6 orbital planes, each with 4 satellites.



$\cos\frac{\alpha}{2} = \frac{r_e}{r_e + h} = \frac{6371}{6371 + 700}$

$\Rightarrow \alpha = 51.42^\circ \quad n = \left[\frac{360^\circ}{\alpha}\right] = 8$

So we need at least 9 satelliates on one orbital planes. (8 for localization, 1 for delay time).

So we totally need $9 \times 6 = 54$ satelliates together.