# Lab2: Satellite Position

## Satellite Navigation Laboratory, Summer Semester 2022

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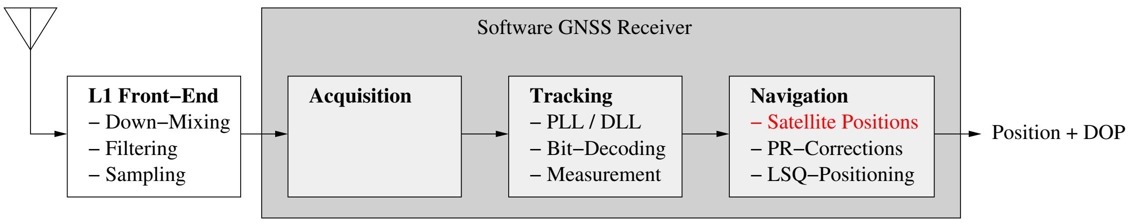
## Contents



## 1 Introduction




To estimate the user position in Lab1, GPS satellite positions are assumed to be known. In this lab session, we will discuss how to compute these satellite positions using ephemeris data that are sent with navigation messages from GPS satellites.

## 2 Keplerian Parameters

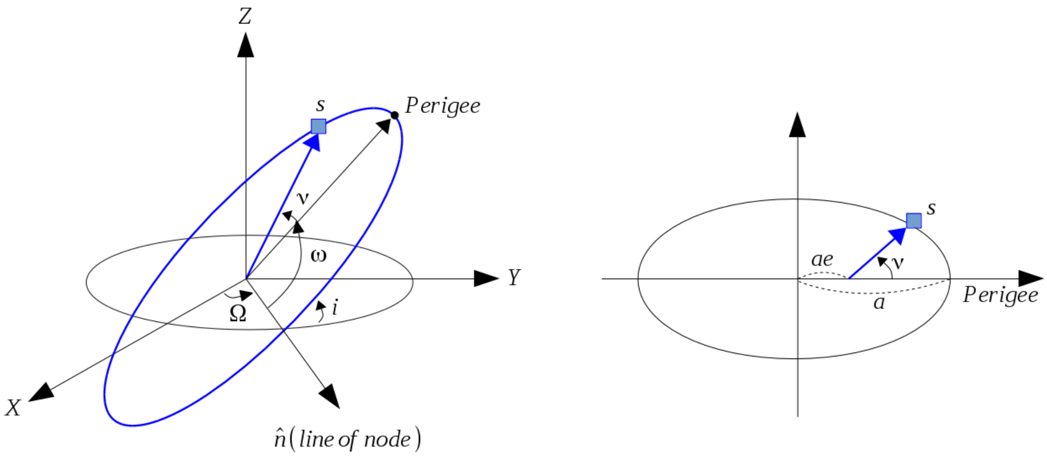



Figure 1: Keplerian parameters of satellite orbits [1]

For an ideal situation, we can describe a satellite orbit as a Keplerian motion with the six Keplerian parameters as illustrated in Fig. 1:

- $a$: Semi-major axis
- $e$: Eccentricity
- $\nu$: True anomaly
- $i$: Orbital inclination
- $\Omega$: Right ascension of ascending node (RAAN)
- $\omega$: Argument of perigee

The first three elements $(a, e, \nu)$ describe the orbit's shape, as well as the satellite position on the orbit plane. The last three parameters $(i, \Omega, \omega)$ define how this orbit plane is posed in the reference frame, earth-center, earth fixed (ECEF).

## 3 Satellite Position Computation with Ephemeris Data

Each satellite transmits ephemeris data with navigation messages. With these data and orbit models, we can estimate the current satellite position, and predict the position in the future.

In the real world, satellites do not exactly follow a Keplerian orbit since some effects induce errors in their orbit, such as inhomogeneity of the Earth's gravitational field, equatorial bulge of the Earth, gravitational effects from the Sun and the Moon, and solar radiation effect. Hence, in addition to the Keplerian elements, additional parameters for correcting the errors are also included in ephemeris data.

The following steps explain the method that is used for GPS [2][3] to compute satellite positions with ephemeris data.

**Compute the satellite positions on the orbit plane** First, mean anomaly $M(t)$ is computed with a linear model of time. The initial value is denoted with $M_0$, and which is the mean anomaly at the time of ephemeris $t_{oe}$. The mean motion (slope) is denoted with $n$:

$$M = M_0 + n \cdot t_k \tag{1}$$
$$t_k = t - t_{\mathrm{oe}} \tag{2}$$
$$n = \sqrt{\frac{\mathrm{GM}}{a^3}} + \delta n. \tag{3}$$

Using this mean anomaly, the eccentric anomaly $E$ is described as a nonlinear equation:

$$E = M + e \sin(E). \tag{4}$$

With the estimated eccentric anomaly $E$ and eccentricity $e$, the true anomaly $\nu$ can be computed:

$$\nu = \arctan\left(\frac{\sqrt{1-e^2} \cdot \sin(E)}{\cos(E) - e}\right). \tag{5}$$

Then, the radial distance between the satellite and the origin of the orbit is computed as

$$r = a(1 - e\cos(E)) + C_{\mathrm{rc}} \cos(2\phi) + C_{\mathrm{rs}} \sin(2\phi) \tag{6}$$

where $\phi = \nu + \omega$, and $C_{\mathrm{rc}}, C_{\mathrm{rs}}$ are the sinusoidal correction terms transmitted in ephemeris data.

The argument of latitude is calculated with

$$u = \nu + \omega + C_{\mathrm{uc}} \cos(2\phi) + C_{\mathrm{us}} \sin(2\phi). \tag{7}$$

Finally, the satellite position on the orbital plane [1] can be computed with

$$\vec{r}^{s}_{\mathrm{orb}} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} r\cos(u) \\ r\sin(u) \\ 0 \end{pmatrix}. \tag{8}$$

**Compute the orbit pose with respect to the reference frame** After estimating the satellite position on the orbital plane (8), we need to compute the position and orientation of the orbit plane with respect to the earth-centered reference frame.

First, the inclination of the orbital plane is computed with

$$i = i_0 + \dot{i} \cdot t_k + C_{\mathrm{ic}} \cos(2\phi) + C_{\mathrm{is}} \sin(2\phi), \tag{9}$$

where $i_0$ is the initial inclination at the reference time $t_{\mathrm{oe}}$ and $\dot{i}$ is the rate of the inclination.

RAAN $\Omega$ can be calculated with

$$\Omega = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_{\mathrm{E}}) \cdot t_k - \dot{\Omega}_{\mathrm{E}} \cdot t_{oe}, \tag{10}$$

where $\dot{\Omega}$ denotes the rate of RAAN, and $\dot{\Omega}_{\mathrm{E}}$ is the Earth's rotation rate.

[1] In this orbital coordinate system the x-axis is aligned with the line of nodes and the y-axis is perpendicular to it.

**Compute the satellite position with respect to the reference frame** Using the previous results, we can finally compute the satellite positions with respect to the earth-centered reference frame, such as the ECEF frame.

$$\vec{r}^{\,s}_{\mathrm{ECEF}} = R_z(-\Omega)R_x(-i)\vec{r}^{\,s}_{\mathrm{orb}} \tag{11}$$

$$= \begin{pmatrix} r_x\cos(\Omega) - r_y\cos(i)\sin(\Omega) \\ r_x\sin(\Omega) + r_y\cos(i)\cos(\Omega) \\ r_y\sin(i) \end{pmatrix}, \tag{12}$$

where $r_x$ and $r_y$ are the satellite coordinates in the orbit plane (8), and $R_x$ and $R_z$ are the rotation matrices:

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \quad R_y = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \quad R_z = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

## References

[1] P. Misra and P. Enge, *Global Positioning System – Signals, Measurements, and Performance.* Ganga-Jamuna Press, 2006.

[2] G. N. JPO, *Navstar GPS Space Segment / Navigation User Interfaces.* ICD-GPS-200c, 2000.

[3] O. Montenbruck and E. Gill, *Satellite Orbits - Models, Methods and Applications.* Springer, 2001.

[4] C. Günther, *Lecture notes of Satellite Navigation*, 2017/2018.

[5] P. Massatt and M. Zeitzew, *The GPS Constellation Design - Current and Projected.* ION GPS-98 Meeting, 1998.

# 4 Homework

1. Show an iterative method to estimate the eccentric anomaly $E$ with Kepler's equation(4) using a given mean anomaly $M$.

2. The radius of the Galileo satellite orbits is approximately 29601 km. Consider a cone that is centered at the satellite, and encloses the complete Earth. Compute the apex angle of the cone.

3. Roughly estimate the minimum number of Medium Earth Orbit (MEO) satellites (e.g. Galileo) to ensure the visibility of at least four satellites at any location on the Earth. How is the minimum number changed for Low Earth Orbit (LEO) satellites (altitude 700km)?

# 5 Lab Tasks

1. **Satellite position computation with ephemeris data**

   Ephemeris data transmitted from GPS satellites include satellite's location, time, and health information. Using them, we can compute the GPS satellites positions, and predict their states in the future. There are some organizations collecting ephemeris data with a wide GNSS receiver network and provide them to the publics. For today's lab, we will use the file provided from the National Geodetic Survey (NGS) network of Continuously Operating Reference stations (CORS).

   (a) Download a data file from the NGC website `http://www.ngs.noaa.gov/CORS/standard1.shtml`. Select the *SITE* and *DATE*, then set the *OPTION-Non Site Specific* as *Global Navigation*. The file will be saved in the receiver independent exchange navigation format (RINEX). `brdc3070.18n` in the `LabFiles` is a sample (SD Rapid City, SDRC, Nov. 3 2018).

   (b) Compute the ephemeris matrix using the downloaded navigation file by `rinexe('filename_ephdatafile', 'filename_output')`. This function reads a RINEX file, parses it to the 21 ephemeris parameters of $K$ satellites, then save a $21 \times$ K matrix in the output file. The stored matrix in the output file can be read with `Eph = get_eph('filename_output')`. Analyze the 21 rows of the matrix with referring Table 1.

   (c) Modify the function `satpos.m` to compute the satellite position using the $k$-th column of the ephemeris matrix `Eph(:,k)` at time $t$. The earth universal gravitational constant GM is $3.986005 \cdot 10^{14} \mathrm{m}^3/\mathrm{s}^2$, and the earth rotation rate $\dot{\Omega}_{\mathrm{E}}$ is $7.2921151467 \cdot 10^{-5}$rad/s [2].

   (d) Extend `GPSConstellationEphemeris.m` to calculate (and plot) all satellite orbits using the latest ephemeris information of the satellites. How many GPS satellites are shown in your figure?
   *Hint: The plot will be similar with Fig. 2.*

| Ephermeris # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | PRN | $a_{\mathrm{f2}}$ | $M_0$ | $\sqrt{a}$ | $\delta n$ | $e$ | $\omega$ | $C_{\mathrm{uc}}$ | $C_{\mathrm{us}}$ | $C_{\mathrm{rc}}$ | $C_{\mathrm{rs}}$ |
| Ephermeris # | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | |
| Value | $i_0$ | $\dot{i}$ | $C_{\mathrm{ic}}$ | $C_{\mathrm{is}}$ | $\Omega_0$ | $\dot{\Omega}$ | $t_{oc}$ | $a_{\mathrm{f0}}$ | $a_{\mathrm{f1}}$ | $t_{\mathrm{oe}}$ | |

Table 1: Ephemeris data format including the Keplerian parameters and sinusoidal and linear correction terms

2. **Accuracy of the orbits computed with ephemeris data**

   The International GNSS Service (IGS) [2] uses the largest network with more than

[2] `http://www.igs.org`

500 stations to determine the satellite positions, so the accuracy of the provided orbit is less than 5cm.

(a) Compute a satellite position at a single epoch with the IGS method by `satposIGS(time, year, day of year, PRN)`.

(b) Extend `ComparisonBroadcastIGS.m` to compare the orbits calculated with broadcast ephemeris data with the IGS orbits.

**3. Sinusoidal and linear correction terms of ephemeris data**

(a) Compute satellite positions without the linear correction term $\delta n$ and the sinusoidal terms in `satpos_nolincorr.m` and `satpos_nosincorr.m`, respectively. In addition, compute satellite positions without any correction terms in `satpos_nocorr.m`.
*Hint: Start with copying the codes in* `satpos.m`, *then remove the parts related with the corrections*

(b) Write a new function `GPSBenefitCorrections.m` to computes the satellite positions without the correction terms, and compare them with the IGS' orbits. The results will be similar with Fig. 3

**4. Update of ephemeris data**

With ephemeris data at 0:00AM (outdated) and 2:00AM (up-to-date), compute the orbits from 2:00AM to 4:00AM, using `GPSBenefitEphemerisUpdate.m`. Then, plot the norm of the differences between the estimates calculated with two PRNs
*Hint: The result should look similar to Fig. 4.*

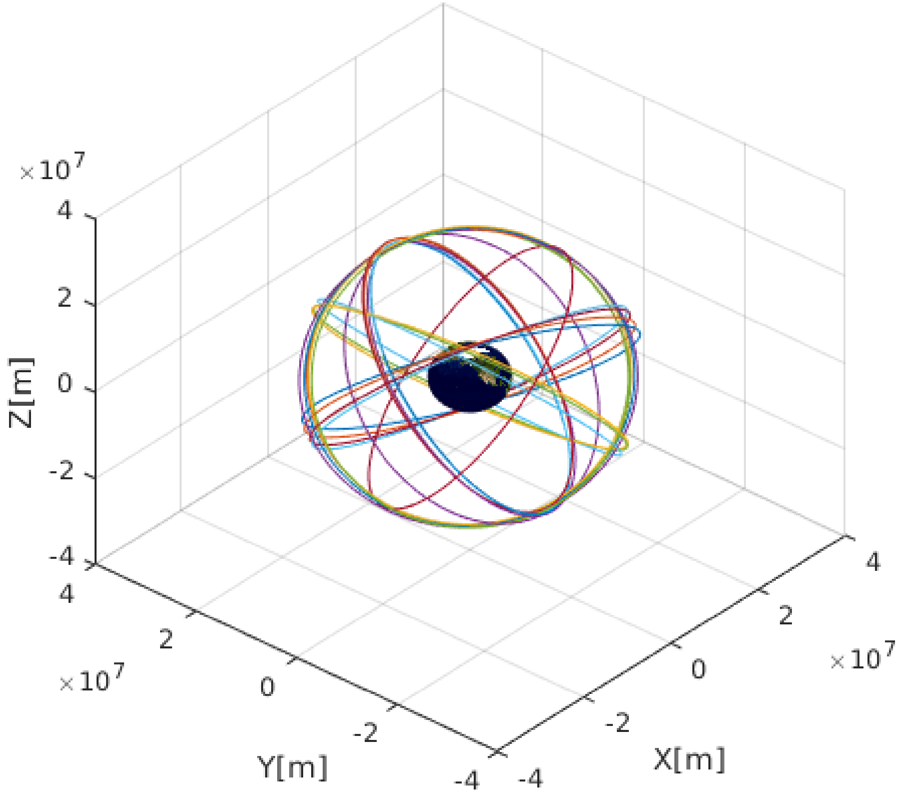



Figure 2: GPS satellite ephemerides on Nov. 3, 2018.

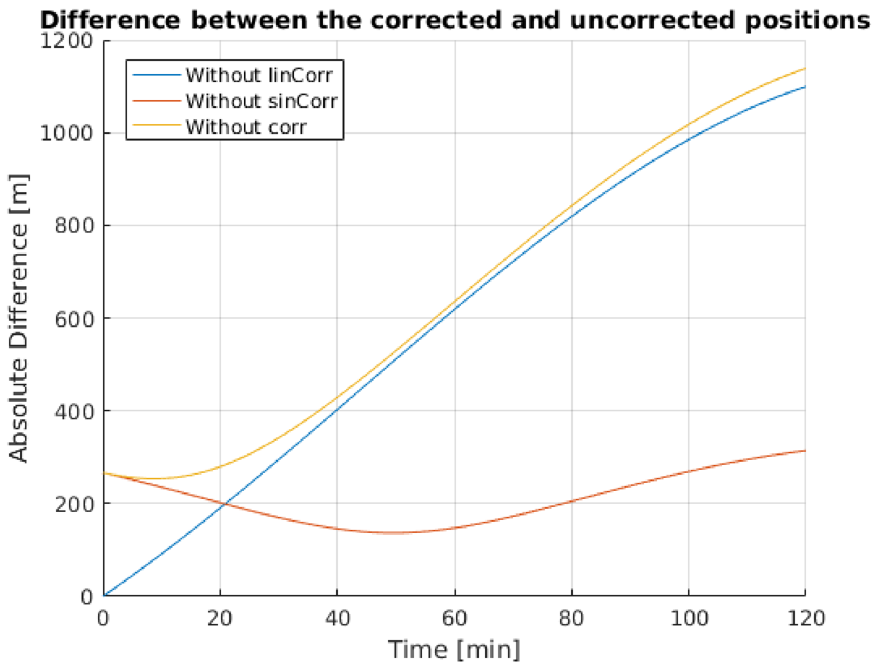



Figure 3: Impact of linear and sinusoidal correction terms on the orbit estimation. Ephemeris data transmitted on Nov. 3, 2018 are used.

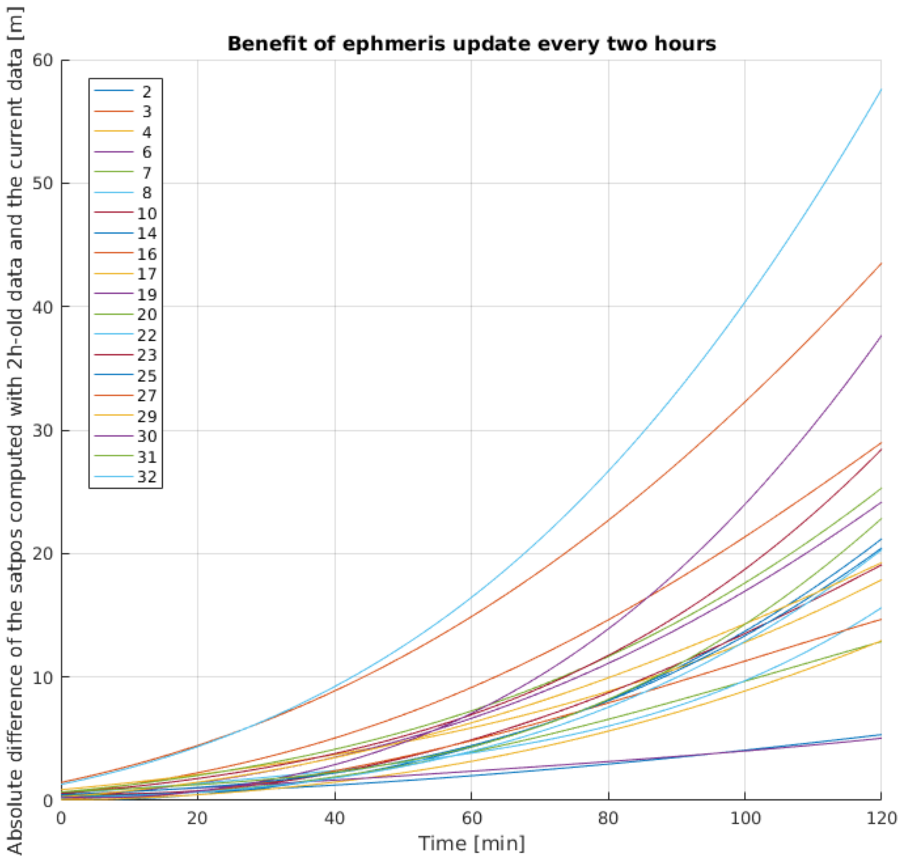



Figure 4: Absolute difference of the satpos computed with 2h-old data and the current data [m]. Ephemeris data transmitted on Aug. 8, 2018 are used.