Hiva Mohammadzadeh 3036919598

## **Question 1: Honor Code**

"I certify that all solutions are entirely in my own words and that I have not looked at another student's solutions. I have given credit to all external sources I consulted."

**Question 2: Logistic Regression with Newton's Method** 

## **Question 3: Wine Classification with Logistic Regression**

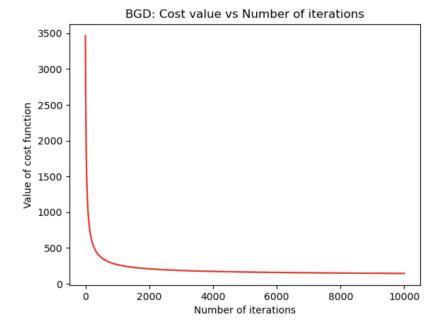
Preprocessing: Normalizing, shuffling and splitting:

Part 1: Batch Gradient Descent Update Rule:

```
W \leftarrow W + (x^{T}(y - \sigma(xw)) - \frac{\lambda}{n} \stackrel{d}{\underset{j=1}{\leq}} W_{j} supdate rule
```

Part 2: Batch Gradient Descent Code:

```
---- Training Batch gradient Descent---
Training accuracy: 0.9928
Validation accuracy: 0.995
```



## Setting hyperparameters:

```
Setting Hyperparameters for Batch Gradient Descent:
Parameters(1e-07, 0.001, 100): Validation accuracy: 0.537
Parameters(1e-07, 0.001, 500): Validation accuracy: 0.568
Parameters(1e-07, 0.001, 1000): Validation accuracy: 0.595
Parameters(1e-07, 0.001, 10000): Validation accuracy: 0.889
Parameters(1e-07, 0.001, 10000): Validation accuracy: 0.889
Parameters(1e-07, 0.001, 10000): Validation accuracy: 0.889
Parameters(1e-07, 0.001, 10000): Validation accuracy:
Parameters(1e-07, 0.001,
                         10000):
                                 Validation accuracy:
Parameters(1e-07, 0.001, 10000): Validation accuracy: 0.889
Parameters(1e-07, 0.001,
                         10000): Validation accuracy: 0.889
Parameters(1e-07,
                         10000):
                                 Validation accuracy:
```

```
Best validation accuracy: 0.992

Best hyperparameters combo:(learning rate, regularization parameter, number of iteration): (1e-05, 0.001, 10000)
```

Part 3: Stochastic Gradient Descent (SGD) Update Rule:

```
3) w = w + \alpha \Gamma(y; -\sigma(x; w)) X; - \frac{\lambda}{n} \stackrel{f}{\underset{j=1}{\stackrel{f}{=}}} w_j \rightarrow update \ rule
```

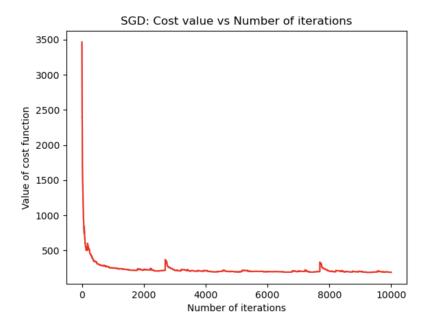
### Part 4: Stochastic Gradient Descent Code:

```
---- Training Stochastic Gradient Descent ---
Training accuracy: 0.993
Validation accuracy: 0.995
```

### Setting hyperparameters:

```
Setting Hyperparameters for Stochastic Gradient Descent (SGD):
Parameters(1e-07, 0.001, 100): Validation accuracy: 0.945
```

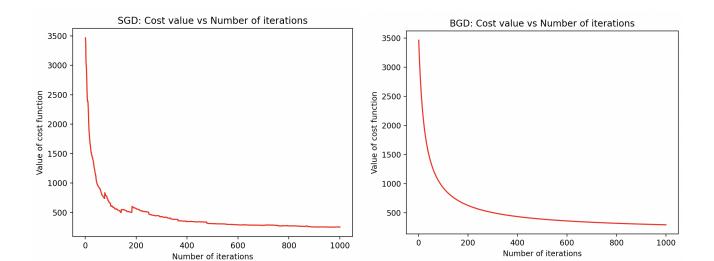
```
Parameters(0.1, 0.001, 10000): Validation accuracy: 0.992
best validation accuracy: 0.992
best hyperparameters combo:(learning rate, regularization parameter, number of iteration): (0.1, 0.001, 10000)
```



replacement. So, we pick a random point and label every iteration.

Compare your plot here with that of question 3.2. Which method converges more quickly?

I trained both BGD and SGD with 1000 training samples to see the difference in their convergence. As we can see Stochastic Gradient Descent converges faster. In SGD, we sample with



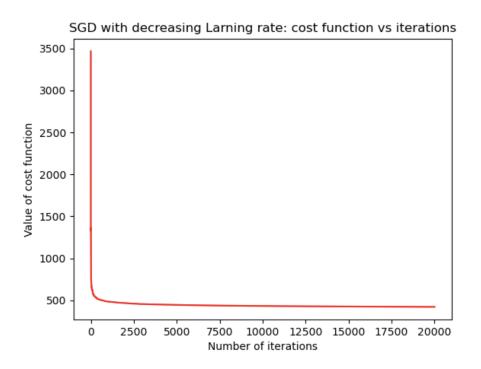
Part 5: SGD with Decreasing Learning Rate

---- Training Stochastic Gradient Descent with decaying learning rate --Training accuracy: 0.9848
Validation accuracy: 0.985

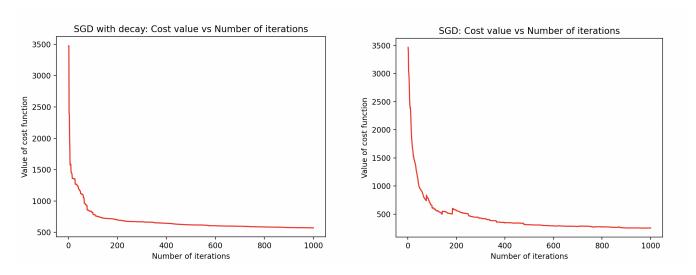
## Setting hyperparameters:

```
Setting Hyperparameters for Stochastic Gradient Descent (SGD) with decaying learning rate:
Parameters(1e-07, 0.001, 100, 0.1): Validation accuracy: 0.927
Parameters(1e-07, 0.001, 500, 0.1): Validation accuracy: 0.936
Parameters(1e-07, 0.001, 1000, 0.1): Validation accuracy: 0.941
Parameters(1e-07, 0.001, 10000, 0.1): Validation accuracy: 0.952
Parameters(1e-07, 0.001, 20000, 0.1): Validation accuracy: 0.954
```

```
Parameters(1e-07, 0.01, 20000, 2): Validation accuracy: 0.985
best validation accuracy: 0.985
best hyperparameters combo:(learning rate, regularization parameter, number of iteration, delta): (1e-07, 0.01, 20000, 2)
----- Training Stochastic Gradient Descent with decaying learning rate ---
Training accuracy: 0.9848
Validation accuracy: 0.985
```



How does this compare to the convergence of your previous SGD code?



I trained both SGD and SGD with decaying learning rate with 1000 training samples to see the difference in their convergence. As we can see SGD with decaying learning rate converges faster. The delta decreases the learning rate allowing it to converge more quickly.

Part 6: Kaggle Submission

---- Training Best Performing Batch gradient Descent for testing--Tested the data and Saved the predictions

Kaggle username: Hiva Mohammadzadeh

Kaggle Scores:

a) WINE: Score: 0.97580 = 97.6%

Briefly describe what your best classifier does to achieve that score.

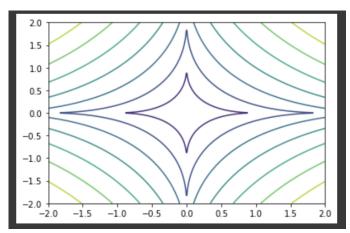
I did hyperparameter tuning of learning rate, regularization parameter and the number of training iterations (I tuned the number of training iterations because I was getting different accuracies when I was using different ones so I just thought I tune it to the best one.) by just having 3 for loops (4 in the case of SGD with decaying learning rate) and took the combination that gave the highest validation accuracy. Then I trained the model that had the highest validation accuracy on the entire dataset and tested it on the test set.

# **Question 4: A Bayesian Interpretation of Lasso**

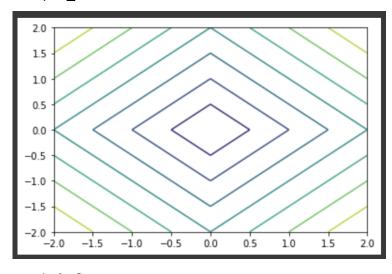
# Question 5: L\_1- regularization, L\_2- regularization, and Sparsity

Part 1:

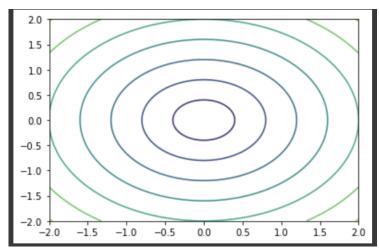
a) L\_0.5



b) L\_1



c) L\_2



## **CODE APPENDIX:**

## **Question 3: Wine Classification with Logistic Regression**

Random Seed = 100

Preprocessing: Normalizing, shuffling and splitting:

```
import numpy as np
import scipy.io as sio
import scipy
import matplotlib.pyplot as plt
from save csv import results to csv
from scipy.special import *
data = sio.loadmat('data.mat')
train data = data['X']
print("Main data: {}" .format(train_data.shape))
train labels = data['y']
print("Main labels: {}" .format(train_labels.shape))
test data = data['X test']
print("Main test: {}" .format(test data.shape))
full_set = np.concatenate((train_data, train_labels), axis=1)
training_data_full = full_set[:5000, :-1]
training labels = full set[:5000, -1:].reshape(-1,)
validation data full = full set[5000:, :-1]
validation labels = full set[5000:, -1:].reshape(-1,)
print("\nShapes after shuffle and split:")
print("Main data: {}" .format(training data full.shape))
print("Main labels: {}" .format(training_labels.shape))
print("Validation data: {}" .format(validation data full.shape))
print("Validation labels:{}" .format(validation_labels.shape))
```

```
# Function to normalize the training and test data with each feature's mean and standard
deviation
mean_f = None
std_f = None

def normalize(data, mean=None, standard_deviation=None):
    samples, features = data.shape
    if not mean:
        mean = data.mean(axis=0)
        standard_deviation = data.std(axis=0)
        data = (data - mean) / standard_deviation
    # Add a fictitious dimension by adding a vector of ls at the end of the training data set.
    data = np.concatenate((data, np.ones((samples, 1))), axis=1)
    return data
# Normalize the training and test data
training_data = normalize(training_data_full)
print("\nAfter normalizing training")
print("Nafter normalizing training_data.shape))
print(training_data)

test_data = normalize(test_data)
print("Nafter normalizing test")
print("Test_data {}" .format(test_data.shape))
print(test_data)
```

#### Part 2: Batch Gradient Descent Code:

```
##### QUESTION 3.2: Batch Gradient Descent Code.

# Batch Gradient Descent Code. Implement your batch gradient descent algorithm for logistic regression and include your code here.

# Choose reasonable values for the regularization parameter and step size (learning rate), specify your chosen values in the write-up, and train your model from question 3.1.

# Shuffle and split your data into training/validation sets and mention the random seed used in the write-up.

# Plot the value of the cost function versus the number of iterations spent in training.

# Random seed = 100

def sigmoid_func(X, w):

# print('hi')

# print(X.shape)

# # print(w.shape)

# print(w.shape)

return scipy.special.expit(np.matmul(X, w.T))

# Loss function of the logistic regression for BGD

def loss_func(data, label, weight, reg):

regularizer = (reg / 2) * weight.T.dot(weight)

# w = np.reshape(w, (w.shape[0], 1))
```

```
loss 1 = np.dot(label.T, np.log(sigmoid func(data, weight)))
  loss 2 = np.dot((1 - label).T, np.log(1 - sigmoid func(data, weight)))
def update rule(data, label, weight, reg, lr):
X train = normalize(training data full)
def train_BGD(num_iterations, X_train, y_train, w_train, regularization_param, lr):
      loss = loss_func(X_train, y_train, w_train, regularization_param)
      next_w = update_rule(X_train, y_train, w_train, regularization_param, lr)
      training_loss.append(int(loss))
print("\nSetting Hyperparameters for Batch Gradient Descent:")
```

```
num_iterations = [100, 500, 1000, 10000]
regularization param = [0.001, 0.01, 0.1, 1.0, 10]
learningRates = [1e-7, 1e-6,1e-5, 1e-3, 0.1, 1, 10]
best validation accuracy so far = 0.0
best hyperparameters so far = None
for lr in learningRates:
           batch best w, batch tl = train BGD(i, X train, training labels, w train, l, lr)
           val data = normalize(validation data full, mean f, std f)
           out_labels = sigmoid_func(val_data, batch_best_w)
           predicted_labels = (out_labels > 0.5).astype(np.int32)
           predicted_labels = predicted_labels.reshape(-1,)
           val accuracy = (validation labels == predicted labels).astype(np.int32).mean()
           if val accuracy > best validation accuracy so far:
               best hyperparameters so far = (lr, l, i)
           print("Parameters{}: Validation accuracy: {}" .format(best_hyperparameters_so_far,
best_validation_accuracy_so_far))
best validation acc = best validation accuracy so far
best hyperparams = best hyperparameters so far
print("\nBest validation accuracy: ",best validation acc)
print("\nBest hyperparameters combo:(learning rate, regularization parameter, number of
iteration): ",best hyperparams)
print("\n---- Training Batch gradient Descent---")
w_train = np.zeros(13)
num iterations = 10000
regularization param = 0.001
lerningRate = 1e-5
batch_GD_best_w, batch_trainingloss = train_BGD(num_iterations, X_train, training_labels,
w_train, regularization param, lerningRate)
olt.figure(1)
```

```
plt.title("BGD: Cost value vs Number of iterations")
plt.xlabel("Number of iterations")
plt.ylabel("Value of cost function")
plt.savefig('Batch Gradient Descent.png')
out labels = sigmoid func(X train, batch GD best w)
predicted labels = (out labels > 0.5).astype(np.int32)
predicted labels = predicted_labels.reshape(-1,)
(training labels == predicted labels).astype(np.int32).mean()
print("Training accuracy: {}" .format((training labels ==
predicted labels).astype(np.int32).mean()))
val data = normalize(validation data full, mean f, std f)
out labels = sigmoid func(val data, batch GD best w)
predicted labels = (out labels > 0.5).astype(np.int32)
predicted labels = predicted labels.reshape(-1,)
(validation labels == predicted labels).astype(np.int32).mean()
print("Validation accuracy: {}" .format((validation labels ==
predicted_labels).astype(np.int32).mean()))
```

#### Part 4: Stochastic Gradient Descent Code

```
#### QUESTION 3.4 : Stochastic Gradient Descent Code.

# Stochastic Gradient Descent Code. Implement your stochastic gradient descent algorithm for logistic regression and include your code here.

# Choose a suitable value for the step size (learning rate), specify your chosen value in the write-up, and run your SCD algorithm from question 3.3.

# Shuffle and split your data into training/validation sets and mention the random seed used in the write-up.

# Plot the value of the cost function versus the number of iterations spent in training.

# Compare your plot here with that of question 3.2. Which method converges more quickly?

Eriefly describe what you observe.

def sigmoid_single(x_i, w):
    return 1 / (1 + np.exp(-np.dot(x_i, w)))

# Update rule for SGD

def sgd_compute_update(x_i, y_i, w, reg, lr):
    sig = sigmoid_single(x_i, (y_i-sig))
    step = reg * w - gradient
    return w - (lr * step)

X_train = normalize(training_data_full)

# Function to train the SGD Logistic Regression
```

```
train_SGD(num_iterations, X_train, y_train, w_train, regularization_param, lr):
  N, d = X train.shape
       loss = loss_func(X_train, y_train, w_train, regularization_param)
      w train = sgd compute_update(training_sample, labels_sample, w_train,
regularization param, lr)
      training loss history.append(float(loss))
print("Setting Hyperparameters for Stochastic Gradient Descent (SGD):")
w train = np.zeros(13)
num iterations = [100, 500, 1000, 10000]
regularization param = [0.001, 0.01, 0.1, 1.0, 10]
learningRates = [1e-7, 1e-6, 1e-5, 1e-3, 0.1, 1, 10]
best validation accuracy so far = 0.0
best_hyperparameters_so_far = None
          batch best w, batch tl = train SGD(i, X train, training labels, w train, l, a)
          pred label = (out labels > 0.5).astype(np.int32)
          pred_label= pred_label.reshape(-1,)
          val_accuracy = (validation_labels == pred_label).astype(np.int32).mean()
           print("Parameters{}: Validation accuracy: {}" .format(best_hyperparameters_so_far,
best_validation_accuracy_so_far))
best valudation acc = best validation accuracy so far
best hyperparams = best hyperparameters so far
print("best validation accuracy: ",best valudation acc)
```

```
iteration): ",best hyperparams)
print("--- Training Stochastic Gradient Descent ---")
w train = np.zeros(13)
regularization_param = 0.001
lr = 0.1
num iterations = 10000
sgd best w, sgd trainingloss = train SGD(num iterations, X train, training labels, w train,
regularization param, lr)
iterations = list(np.arange(1, num iterations + 1, 1))
plt.figure(2)
plt.plot(iterations, sgd_trainingloss, 'r-')
plt.title("SGD: Cost value vs Number of iterations")
plt.xlabel("Number of iterations")
plt.ylabel("Value of cost function")
plt.savefig('Stochastic Gradient Descent.png')
out labels = sigmoid func(X train, sgd best w)
predicted labels = (out labels > 0.5).astype(np.int32)
predicted labels= predicted labels.reshape(-1,)
(training labels == predicted labels).astype(np.int32).mean()
print("Training accuracy: {}" .format((training labels ==
predicted labels).astype(np.int32).mean()))
out labels = normalize(validation data full, mean f, std f)
out labels = sigmoid func(out labels, sgd best w)
predicted_labels = (out_labels > 0.5).astype(np.int32)
predicted labels = predicted labels.reshape(-1,)
(validation labels == predicted labels).astype(np.int32).mean()
print("Validation accuracy: {}" .format((validation labels ==
predicted_labels).astype(np.int32).mean()))
```

## Part 5: Stochastic Gradient Descent with decaying learning rate

```
#### QUESTION 3-5: Stochastic Gradient Descent with decaying learning rate # Instead of using a constant step size (learning rate) in SGD, you could use a step size that slowly shrinks from iteration to iteration. # Run your SGD algorithm from question 3.3 with a step size \epsilon t = \delta/t where t is the iteration number and \delta is a hyperparameter you select
```

```
X train = normalize(training data full)
def train_sgd_decay(num_iterations, X_train, y_train, w_train, lr, regularization_param,
  N, d = X_train.shape
       loss = loss_func(X_train, y_train, w_train, regularization_param)
       w_train = sgd_compute_update(X_sample, y_sample, w_train, regularization_param, lr)
       training_loss_history.append(float(loss))
print("\nSetting Hyperparameters for Stochastic Gradient Descent (SGD) with decaying learning
rate:")
w train = np.zeros(13)
deltas = [0.1, 0.2, 0.5, 0.7, 0.9, 1.0, 2, 5]
num iterations = [100, 500, 1000, 10000, 20000]
regularization param = [ 0.001, 0.01, 0.1, 1.0, 10]
learningRates = [1e-7, 1e-6, 1e-5, 1e-3, 0.1, 1, 10]
best validation accuracy so far = 0.0
best hyperparameters so far = None
for d in deltas:
               batch best w, batch tl = train sgd decay(i, X train, training labels, w train,
               predicted labels = (out labels > 0.5).astype(np.int32)
               predicted labels = predicted labels.reshape(-1,)
               val accuracy = (validation labels == predicted labels).astype(np.int32).mean()
```

```
best hyperparameters so far = (a, 1, i, d)
format(best hyperparameters so far, best validation accuracy so far))
best_val_acc = best_validation_accuracy_so_far
best_hyperparams = best_hyperparameters_so_far
print("best validation accuracy: ",best val acc)
print("best hyperparameters combo:(learning rate, regularization parameter, number of
iteration, delta): ", best hyperparams)
print("---- Training Stochastic Gradient Descent with decaying learning rate ---")
w train = np.zeros(13)
regularization param = 0.01
lr = 1e-07
delta = 2.0
num iterations = 20000
sgd_decay_best_w, sgd_decay_tl = train_sgd_decay(num_iterations, X_train, training_labels,
iterations = list(np.arange(1, num iterations+1, 1))
plt.figure(3)
plt.plot(iterations, sgd_decay_tl, 'r-')
plt.title("SGD with decreasing Learning rate: cost function vs iterations")
plt.xlabel("Number of iterations")
plt.ylabel("Value of cost function")
plt.savefig('SGD with decreasing learning rate.png')
predicted labels = (out labels > 0.5).astype(np.int32)
predicted labels = predicted labels.reshape(-1,)
(training labels == predicted labels).astype(np.int32).mean()
print("Training accuracy: {}" .format((training labels ==
predicted labels).astype(np.int32).mean()))
out labels = normalize(validation data full, mean f, std f)
out_labels = sigmoid_func(out_labels, sgd_decay_best_w)
predicted labels = (out labels > 0.5).astype(np.int32)
```

```
predicted_labels = predicted_labels.reshape(-1,)

(validation_labels == predicted_labels).astype(np.int32).mean()

print("Validation accuracy: {}" .format((validation_labels == predicted_labels).astype(np.int32).mean()))
```

## Part 6: Kaggle

```
### QUESTION 3.6: Kaggle
X train = normalize(training data full)
print("\n--- Training Best Performing Batch gradient Descent for testing---")
w train = np.zeros(13)
num iterations = 10000
regularization param = 0.001
lr = 1e-5
batch best w, batch train loss = train BGD(num iterations, X train, training labels, w train,
regularization param, lr)
test data full = data["X test"]
X_test = normalize(test_data_full, mean_f, std_f)
out_labels = sigmoid_func(X_test, batch_best_w)
predicted labels = (out labels > 0.5).astype(np.int32)
predicted labels = predicted labels.reshape(-1,)
results to csv(predicted labels)
print("Tested the data and Saved the predictions")
```