Exam Solution

Course: AE4870A Rocket Motion
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Author:
collaborative effort

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1 Fundamentals:MC understanding

True or False

1. *The so-called "Rocket Equation" (Tsiolkovsky) relates fuel mass to velocity increment.* **FALSE.** It relates velocity increment to mass **ratio.**
2. *It is impossible to reach space with a single-stage rocket* **FALSE** Impossible to get into orbit
3. *For launch velocities up to 60% of the circular velocity, a flat-Earth approximation will give about the same shooting range as the spherical-Earth approach* **TRUE** From the slide 7.26.
4. *To keep a constant acceleration, all stages in a multi-stage rocket need to be equally powerful* **FALSE** Latter stages are less powerful, as they carry less mass.
5. *The accuracy requirements are easier met for a high trajectory than for a low trajectory* **TRUE** Slide 8.29
6. *Specific impulse is the thrust to weight ratio of a specific rocket* **TRUE** Definition of Specific impulse $I_{sp} = \frac{T}{Mg_0}$
7. *The burn time for constant thrust load is longer than the burn time for constant mass flow if we assume identical specific impulse Isp and mass-ratio* **TRUE** Slide 2.16
8. *The influence of aerodynamic drag on rocket performance can be reduced by decreasing the rocket size (scaling)* **TRUE (doubt)**. Aerodinamic drag is proportional to Surface
9. *The roll rate of a rocket should have a positive slope when crossing the pitch frequency to avoid so-called lock-in* **FALSE (doubt)** Not menctioned in the lecture notes while talking about lock-in. Page 48
10. *A gravity turn maneuver is applied to reduce gravity losses* **FALSE** Its applied to use the gravity to modify the trajectory.

2 Multi stage rockets:mass. construction mass. V-e.id

In this question we consider the launch of a $M_0 = 2,000kg$ payload with a multi-stage rocket (N =3, number of stages). Consider the case that all three stages are identical, i.e., the effective exhaust velocity per stage is ceff = 3.2 km/s and the construction-mass ratio per stage is "ϵ= 0.1. The required characteristic velocity to reach orbit is Ve,id = 12 km/s. Note: always state the used equations (without derivation).

2.1 (a)5 points. What is the total mass of the rocket?

From the slide 5.24 (velocity increment needed for a multi-stage rocket with identical stages):

$$\lambda_{TOT} = \left(\frac{e^{\frac{(-v_e)}{Nc_{eff}}} - \epsilon}{1-\epsilon} \right)^N = \left(\frac{e^{\frac{(-12)}{3\times 3.2}} - 0.1}{1-0.1} \right)^N = 8.899 \times 10^{-3} \quad \text{(SLIDE 5.24)}$$

The payload ratio for each stage:

$$\lambda_i = \lambda_{TOT}^{1/N} = 0.2072 \quad (1)$$

From the definition of total payload ratio:

$$\lambda_{TOT} = \frac{M_u}{M_0} \quad M_0 = \frac{M_u}{\lambda_{TOT}} = 224.743t \quad (2)$$

2.2 (b) 20 points. Calculate for each of the stages, the construction massMc and the propellant mass Mp.

From the definition of payload ratio:

$$\lambda_i = \frac{M_u}{M_0}\bigg\|_i = \frac{M_{0,i+1}}{M_{0,i}} \tag{3}$$

Thus, the payload mass for each stage is:

$$M_{u,1} = M_{0,2} = 0.2072 \times 224.743t = 46.566t \tag{4}$$
$$M_{u,2} = M_{0,3} = 0.2072 \times 46.566t = 9,648.66kg \tag{5}$$
$$M_{u,3} = 0.2072 \times 9,648.66kg = 2,000kg \tag{6}$$

Taking into acount that, for every stage, it holds the following relations:

$$M_0 = M_u + M_c + M_p \quad M_c + M_p = M_o - M_u, \tag{7}$$

we have a way to compute the sum of the construction and fuel masses. If we also consider the construction-mass ratio definition:

$$\epsilon = \frac{M_c}{M_c + M_p} \quad M_c = \epsilon(M_c + M_p), \tag{8}$$

as we now the ratio r, we can compute the construction and fuel mass in every step. The solution is given in the following table:

Stage	M0	Mu	M0-Mu	Mc	Mp
1	224,743.69	46,566.89	178,176.80	17,817.68	160,359.12
2	46,566.89	9,648.66	36,918.23	3,691.82	33,226.41
3	9,648.66	2,000.00	7,648.66	764.87	6,883.79

2.3 (c) 5 points. If we increase the payload mass and add boosters to the first stage, what can you say in general over the contribution to Ve,id per stage?

SLIDES 6.29 Same problem, with even the same numbers...

If we add boosters, the deltaV of the first stage will increase, while the deltaV of the other two stages will decrease.

3 Balistic flight over earth:Balistic spherical flight range angle phi-0

In this question we consider the ballistic flight over a stationary, spherical Earth (Re = 6,378 km, = 398,600 km3/s2).

3.1 Sketch for the geometry

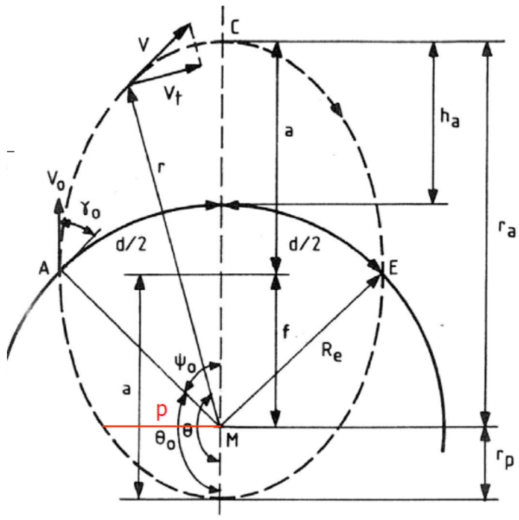

Figure 1:

3.2 Derive the flight-range angle equation

Derivation in slides 7.8 - 7.16.

Assuming a keplerian trajectory between the shooting point and the final range, and dismissing atmospheric effects, you can apply the vis-viva ecuation in the point A:

$$\frac{v^2}{2} - \frac{\mu}{r} = \frac{v_0^2}{2} - \frac{\mu}{R_e} = -\frac{\mu}{2a} \tag{9}$$

After some manipulation (see slides for details):

$$\frac{R_e v_0^2}{\mu} - 2 = -\frac{R_e}{a} \tag{10}$$

Introducing the adimensional velocity (w.r.t.) the velocity of the rotation of the earth:

$$S_0 = \frac{v_0}{\sqrt{\frac{\mu}{R_e}}}$$

The semi-axis is:

$$\frac{a}{Re} = \frac{1}{2 - S_0^2} \tag{11}$$

For the semilatus rectum:

$$\frac{p}{Re} = \frac{H^2}{\mu R_e} = \frac{R_e v_0^2 cos^2(\gamma_0)}{\mu} = S_0^2 cos^2(\gamma_0) \tag{12}$$

Thus, for the eccentricity, we have:

$$e = \sqrt{1 - \frac{p}{a}} = \sqrt{1 - S_0^2 cos^2(\gamma_0)(2 - S_0^2)} \tag{13}$$

Substituting this into the trajectory equation, we have:

$$\frac{r}{R_e} = \frac{p/R_e}{1 + ecos(\theta)} = \frac{S_0^2 cos^2(\gamma_0)}{1 + \sqrt{1 - S_0^2 cos^2(\gamma_0)(2 - S_0^2)} cos(\theta)} \tag{14}$$

As in the initial point $r = R_e$ and $\theta = \Psi_0 = \frac{d/2}{R_e}$, we can isolate from the previous expression $cos(\Psi_0)$:

$$cos(\Psi_0) = \frac{1 - S_0^2 cos^2(\gamma_0)}{\sqrt{1 - S_0^2 cos^2(\gamma_0)(2 - S_0^2)}} \tag{15}$$

3.3 Numerical application

For a flight range d = 9,020 km and a launch velocity V0 = 7,905.4 m/s, calculate the corresponding launch angle 0.

First of all, we calculate the adimensional initial velocity parameter S_0:

$$S_0 = \frac{v_0}{\sqrt{\frac{\mu}{R_e}}} = \frac{7,905.4m/s}{\sqrt{\frac{3.986 \times 10^{14} m^3/s}{6.278 \times 10^6 m}}} = 1.0000$$

From Equation 15, we obtain that $Psi_0 = 0.7071rad = 40.51°$.

Substituting $S_0 = 1$ into that Equation, we can isolate the flight path angle:

$$cos(\Psi_0) = \frac{1 - cos^2(\gamma_0)}{\sqrt{1 - cos^2(\gamma_0)}} = \sqrt{1 - cos^2(\gamma_0)} \tag{16}$$

$$cos(\gamma_0) = \sqrt{1 - cos^2(\Psi_0)} \tag{17}$$

$$\gamma_0 = 49.4851° \tag{18}$$

4 Launch Trajectories:Wind. constant thrust EOM vert. motion

Consider the flight of a single-stage sounding rocket in a homogeneous gravity field. The rocket starts vertically along a guide rail. The subsequent flight takes place in an atmosphere in which a constant horizontal winds prevails. It is assumed that the rocket is infinitely (statically) stable, meaning that it responds instantly to the impinging airflow. The influence of atmospheric drag may be ignored. Use the following data:
guide rail length 50m
wind speed 10 m/s
rocket initial (total) mass 1000 kg
rocket burnout mass 200 kg
specific impulse 300s
initial thrust load 0 3
gravitational acceleration 9.81 m/s2

4.1 Derivation of the Mass equation

By definition of mass flow (Slides FRM.11):

$$t = \frac{M_0 - M}{m} = \frac{M_0 - M}{T/c_{eff}} = \frac{I_{sp}}{\Psi_0}(1 - \frac{M}{M_0}) \tag{19}$$

In our case:

$$t = \frac{I_{sp}}{\Psi_0}(1 - \frac{M}{M_0}) = \frac{300}{3}(1 - \frac{200}{1000}) = 80s$$

4.2 Derivation of the trajectory equation

The equation of motion while in the rail is:

$$M\frac{dV_z}{dt} = T - Mg_0 = mc_{eff} - Mg_0 \tag{20}$$

Operating and integrating each term,

$$dV_z = \frac{T}{M} - g_0 = -c_{eff}\frac{dM}{M} - g_0 dt \tag{21}$$

$$\frac{dh}{dt} = V_z = c_{eff} ln(\frac{M_0}{M}) - g_0 t \tag{22}$$

If we integrate again, we obtain the altitude at each time (detailed explanation at slides FRM.11 - FRM.13):

$$h = \frac{I_s p^2}{\Psi_0} g_0 \left(\frac{M}{M_0} ln(\frac{M}{M_0}) + 1 - \frac{M}{M_0} \right) - \frac{1}{2} g_0 t^2 \tag{23}$$

Substituting Equation 19 into Equation 23 and inserting the numerical values for the end of the guide rail, i.e., $h = 50m$, and if we call the coefficient $x = \frac{M_0}{M_e}$ we obtain an ecuation for the mass:

$$h\frac{\Psi_0}{I_{sp}^2 g_0} = 50\frac{3}{300^2 \times 9.81} = \frac{1}{x}ln(\frac{1}{x}) + 1 - \frac{1}{x} - \frac{1}{2\Psi_0}\left(1 - \frac{1}{x}\right)^2 \tag{24}$$

By solving this ecuation you obtain the value for x (TIP: if you got a CASIO fx-991 or similar, you can plug the equation, insert an estimate value for X, greater than 1 in this case, and press SOLVE. Otherwise, any numerical method or try and guess can work).

$$x = \frac{M_0}{M_e} = 1.022964 \quad M_e = 977.55kg$$

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The final time is:

$$t_e = \frac{I_{sp}}{\Psi_0}\left(1 - \frac{1}{x}\right) = \frac{300}{3}\left(1 - \frac{1}{1.022964}\right) = 2.2449s \tag{25}$$

Recalling Equation 22, you obtain the final velocity after the rail:

$$v_e = I_{sp}g_0 ln(x) - g_0 t_e = 300 \times 9.81 ln(1.022964) - 9.81 \times 2.2449 = 44.798m/s \tag{26}$$

4.3 Angle of attack and flight path angle

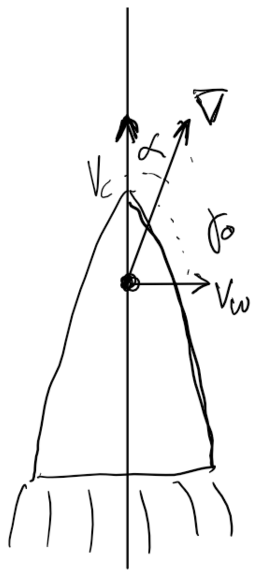

Figure 2:

In the instant after leaving the guide rail, the glith path angle is calculated as

$$\gamma = atan(\frac{v_e}{v_w}) = atan(\frac{44.798}{10}) = 77,41° \tag{27}$$