# Exam Solution

Course: AE4870A Rocket Motion
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# 0 Introduction

This contains an exam solution. If you wish to contribute to this exam solution:

1. Create a github account, (you can create an "anonymous" one).
2. git clone ...
3. edit your changes in the document.
4. open cmd, and browse to inside the folder you downloaded and edited
5. git pull (updates your local repository=copy of folder, to the latest version in github cloud)
6. git status shows which files you changed.
7. git add "/some folder with a space/someFileYouChanged.tex"
8. git commit -m "Included solution to question 1c."
9. git push

It can be a bit initimidating at first, so feel free to click on "issue" in the github browser of this repository and ask :) (You can also use that to say "Hi, I'm having a bit of help with this particular equation, can someone help me out?")

If you don't know how to edit a latex file on your own pc iso on overleaf, look at the "How to use" section of `https://github.com/a-t-0/AE4872-Satellite-Orbit-Determination`.

## 0.1 Consistency

To make everything nice and structured, please use very clear citations:

1. If you copy/use an equation of some slide or document, please add the following data:
   (a) Url (e.g. if simple wiki or some site)
   (b) Name of document
   (c) (Author)
   (d) PAGE/SLIDE number so people can easily find it again
   (e) equation number (so people can easily find it again)
2. If you use an equation from the slides/a book that already has an equation number, then hardcode that equation number in this solution manual so people directly see which equation in the lecture material it is, this facilitates remembering the equations.
3. Here is an example is given in eq. (10.32[**?**]) (See file references.bib [**?**]).

$$R_n^E = \sum_{t=1}^{n} l_m(p_t, z_t) - \underset{i}{min} \sum_{t=1}^{n} {z_t}^i \tag{10.32[?]}$$

# 1

Starting from

$$M\frac{dV}{dt} = T - Mg_0$$

after simple manipulations it gives

$$\frac{dV}{dt} = c_{eff}\frac{m}{M} - g_0$$

Then, trivial integration yields

$$\Delta V = \int_0^t \frac{c_{eff}dM}{Mdt} - g_0dt = I_{sp}g_0ln\Lambda - g_0t$$

Next, the distance travelled during the burn is computed by integrating the equation

$$\frac{dh}{dt} = I_{sp}g_0ln\Lambda - g_0t$$

which results in

$$\Delta h = \int_0^t (I_{sp}g_0ln\Lambda - g_0t)dt = I_{sp}g_0\int_0^t ln\frac{M_0}{M}dt - 0.5g_0t^2$$

where

$$I_{sp}g_0\int_0^t ln\frac{M_0}{M}dt = -\int_{M_0}^{M_e}\frac{c_{eff}}{m}ln\frac{M_0}{M}dM = \frac{c_{eff}}{m}\int_{M_e}^{M_0}(lnM_0 - lnM)dM =$$
$$\frac{c_{eff}}{m}(M_0 - M_elnM_0 + M_elnM_e - M_e) = \frac{c_{eff}}{m}M_0\left(1 - \frac{1}{\Lambda}(1 + ln\Lambda)\right) = \frac{I_{sp}^2g_0}{\Psi_0}\left(1 - \frac{1}{\Lambda}(1 + ln\Lambda)\right)$$

Finally, the distance travelled during the burn $h$ is

$$\Delta h = \frac{I_{sp}^2g_0}{\Psi_0}\left(1 - \frac{1}{\Lambda}(1 + ln\Lambda)\right) - 0.5g_0t^2$$

# 2

Knowing the relations

$$\epsilon = 0.12$$

$$T = mc_{eff}$$

and departing from the basic equation

$$M_0 = M_u + M_c + M_p$$

where $M_u$ is the mass of upper stages with fuel and payload of the whole rocket, $M_c$ is the collective mass of 3 boosters and $M_p$ is the propellant mass contained in the boosters. Relating propellant to construction mass gives

$$M_p + M_c = \frac{M_p}{1 - \epsilon}$$

From the definition of specific impulse

$$c_{eff} = I_{sp}g_0$$

and thrust

$$m = \frac{T}{c_{eff}}$$

where $T$ signifies the total thrust provided by the three boosters. The propellant mass might be calculated by

$$M_p = mt_b$$

Hence, the total mass is

$$M_0 = M_u + \frac{M_p}{1 - \epsilon}$$

## 3

The indices used in this section relate to the numbers of points in the question.

$$M(1) = M_0$$

$$a(1) = \frac{T(1)}{M(1)} - g_0 = \frac{3T_b}{M(1)} - g_0$$

$$M(2) = M_0 - 3mt_{50}$$

$$a(2) = \frac{T(2)}{M(2)} - g_0 = \frac{3T_b}{M(2)} - g_0$$

$$M(3) = M_0 - 3mt_{50} = M(2)$$

$$a(3) = \frac{T(3)}{M(3)} - g_0 = \frac{2.5T_b}{M(3)} - g_0$$

$$M(4) = M(3) - 2.5mt_{200}$$

$$a(4) = \frac{T(4)}{M(4)} - g_0 = \frac{2.5T_b}{M(4)} - g_0$$

$$M(5) = M(4)$$

$$a(5) = \frac{T(5)}{M(5)} - g_0 = \frac{0.5T_b}{M(5)} - g_0$$

$$M(6) = M(5) - t_{rest}$$

$$a(6) = \frac{T(6)}{M(6)} - g_0 = \frac{0.5T_b}{M(6)} - g_0$$

$$\text{where } t_{50} = 50s$$

$$t_{200} = 200s$$

$$t_{rest} = t_b - t_{50} - \tfrac{1}{2}t_{200} = 100s$$

## 4

Here, the same index notation as in previous section is used.

$$V(2) = c_{eff} ln\frac{M(1)}{M(2)} - g_0 t_{50}$$

$$h(2) = \frac{I_{sp}^2 g_0}{\Psi_0}\left(1 - \frac{\frac{1}{M(1)}}{M(2)}(1 + ln\frac{M(1)}{M(2)})\right) - 0.5g_0 t_{50}^2$$

$$V(4) = V(2) + c_{eff} ln\frac{M(3)}{M(4)} - g_0 t_{200}$$

$$h(4) = h(2) + V(2)t_{200} + \frac{I_{sp}^2 g_0}{\Psi_0}\left(1 - \frac{\frac{1}{M(3)}}{M(4)}(1 + ln\frac{M(3)}{M(4)})\right) - 0.5g_0 t_{200}^2$$

$$V(6) = V(4) + c_{eff} ln\frac{M(5)}{M(6)} - g_0 t_{rest}$$

$$h(6) = h(4) + V(4)t_{rest} + \frac{I_{sp}^2 g_0}{\Psi_0}\left(1 - \frac{\frac{1}{M(5)}}{M(6)}(1 + ln\frac{M(5)}{M(6)})\right) - 0.5g_0 t_{rest}^2$$

## 5

To calculate the culmination height, mechanical energy conservation principle is used.

$$\frac{V^2}{2} = \Delta h g_0$$

$$h_c = 271km + \Delta h = 271km + \frac{V^2}{2g_0}$$

# Conclusion

Attention has to be paid to including all the terms (it cannot be forgotten to count the gravity losses or distance covered due to already built-up velocity). Moreover, even though booster part in the reader is not really necessary to solve this exercise, it might turn out to be quite useful.