

Exam Solution

Course: AE4870A Rocket Motion

Exam Source: Old source / studocu. Attached.

Exam Date: 2014-10-28

Author:

collaborative effort

October 23, 2019

1 Fundamentals:MC understanding

True or False

1. The so-called "Rocket Equation" (Tsiolkovsky) relates fuel mass to velocity increment. **FALSE**. It relates velocity increment to mass **ratio**.
2. It is impossible to reach space with a single-stage rocket **FALSE** Impossible to get into orbit
3. For launch velocities up to 60% of the circular velocity, a flat-Earth approximation will give about the same shooting range as the spherical-Earth approach **TRUE** From the slide 7.26.
4. To keep a constant acceleration, all stages in a multi-stage rocket need to be equally powerful **FALSE** Latter stages are less powerful, as they carry less mass.
5. The accuracy requirements are easier met for a high trajectory than for a low trajectory **TRUE** Slide 8.29
6. Specific impulse is the thrust to weight ratio of a specific rocket **TRUE** Definition of Specific impulse $I_{sp} = \frac{T}{Mg_0}$
7. The burn time for constant thrust load is longer than the burn time for constant mass flow if we assume identical specific impulse I_{sp} and mass-ratio **TRUE** Slide 2.16
8. The influence of aerodynamic drag on rocket performance can be reduced by decreasing the rocket size (scaling) **TRUE (doubt)**. Aerodynamic drag is proportional to Surface
9. The roll rate of a rocket should have a positive slope when crossing the pitch frequency to avoid so-called lock-in **FALSE (doubt)** Not mentioned in the lecture notes while talking about lock-in. Page 48
10. A gravity turn maneuver is applied to reduce gravity losses **FALSE** Its applied to use the gravity to modify the trajectory.

2 Multi stage rockets:mass. construction mass. V-e.id

In this question we consider the launch of a $M_0 = 2,000\text{kg}$ payload with a multi-stage rocket ($N=3$, number of stages). Consider the case that all three stages are identical, i.e., the effective exhaust velocity per stage is $c_{eff} = 3.2\text{ km/s}$ and the construction-mass ratio per stage is $\epsilon = 0.1$. The required characteristic velocity to reach orbit is $V_{e,id} = 12\text{ km/s}$. Note: always state the used equations (without derivation).

2.1 (a)5 points. What is the total mass of the rocket?

From the slide 5.24 (velocity increment needed for a multi-stage rocket with identical stages):

$$\lambda_{TOT} = \left(\frac{e^{\frac{(-v_e)}{N c_{eff}}} - \epsilon}{1 - \epsilon} \right)^N = \left(\frac{e^{\frac{(-12)}{3 \times 3.2}} - 0.1}{1 - 0.1} \right)^3 = 8.899 \times 10^{-3} \quad (\text{SLIDE 5.24})$$

The payload ratio for each stage:

$$\lambda_i = \lambda_{TOT}^{1/N} = 0.2072 \quad (1)$$

From the definition of total payload ratio:

$$\lambda_{TOT} = \frac{M_u}{M_0} \quad M_0 = \frac{M_u}{\lambda_{TOT}} = 224.743t \quad (2)$$

2.2 (b) 20 points. Calculate for each of the stages, the construction mass M_c and the propellant mass M_p .

From the definition of payload ratio:

$$\lambda_i = \frac{M_u}{M_0} \Big|_i = \frac{M_{0,i+1}}{M_{0,i}} \quad (3)$$

Thus, the payload mass for each stage is:

$$M_{u,1} = M_{0,2} = 0.2072 \times 224.743t = 46.566t \quad (4)$$

$$M_{u,2} = M_{0,3} = 0.2072 \times 46.566t = 9,648.66kg \quad (5)$$

$$M_{u,3} = 0.2072 \times 9,648.66kg = 2,000kg \quad (6)$$

Taking into account that, for every stage, it holds the following relations:

$$M_0 = M_u + M_c + M_p \quad M_c + M_p = M_0 - M_u, \quad (7)$$

we have a way to compute the sum of the construction and fuel masses. If we also consider the construction-mass ratio definition:

$$\epsilon = \frac{M_c}{M_c + M_p} \quad M_c = \epsilon(M_c + M_p), \quad (8)$$

as we now the ratio r , we can compute the construction and fuel mass in every step. The solution is given in the following table:

Stage	M0	Mu	M0-Mu	Mc	Mp
1	224,743.69	46,566.89	178,176.80	17,817.68	160,359.12
2	46,566.89	9,648.66	36,918.23	3,691.82	33,226.41
3	9,648.66	2,000.00	7,648.66	764.87	6,883.79

2.3 (c) 5 points. If we increase the payload mass and add boosters to the first stage, what can you say in general over the contribution to $V_{e,id}$ per stage?

SLIDES 6.29 Same problem, with even the same numbers...

If we add boosters, the ΔV of the first stage will increase, while the ΔV of the other two stages will decrease.

3 Ballistic flight over earth: Ballistic spherical flight range angle ϕ_0

In this question we consider the ballistic flight over a stationary, spherical Earth ($R_e = 6,378 \text{ km}$, $\omega = 398,600 \text{ km}^3/\text{s}^2$).

3.1 Sketch for the geometry

Draw a clear sketch for the geometry of the ballistic flight over a spherical Earth. Indicate the rocket state related to the launch point and a local point in the trajectory, the flight range, true anomaly (for the launch point and local point), perigee and apogee height, and semi-major axis.

3.3 Numerical application

For a flight range $d = 9,020$ km and a launch velocity $V_0 = 7,905.4$ m/s, calculate the corresponding launch angle θ .

First of all, we calculate the adimensional initial velocity parameter S_0 :

$$S_0 = \frac{v_0}{\sqrt{\frac{\mu}{R_e}}} = \frac{7,905.4 \text{ m/s}}{\sqrt{\frac{3.986 \times 10^{14} \text{ m}^3/\text{s}^2}{6.278 \times 10^6 \text{ m}}}} = 1.0000$$

From Equation 15, we obtain that $Psi_0 = 0.7071 \text{ rad} = 40.51^\circ$.

Substituting $S_0 = 1$ into that Equation, we can isolate the flight path angle:

$$\cos(\Psi_0) = \frac{1 - \cos^2(\gamma_0)}{\sqrt{1 - \cos^2(\gamma_0)}} = \sqrt{1 - \cos^2(\gamma_0)} \quad (16)$$

$$\cos(\gamma_0) = \sqrt{1 - \cos^2(\Psi_0)} \quad (17)$$

$$\gamma_0 = 49.4851^\circ \quad (18)$$

4 Launch Trajectories: Wind. constant thrust EOM vert. motion

Consider the flight of a single-stage sounding rocket in a homogeneous gravity field. The rocket starts vertically along a guide rail. The subsequent flight takes place in an atmosphere in which a constant horizontal winds prevails. It is assumed that the rocket is infinitely (statically) stable, meaning that it responds instantly to the impinging airflow. The influence of atmospheric drag may be ignored. Use the following data:

guide rail length	50m
wind speed	10 m/s
rocket initial (total) mass	1000 kg
rocket burnout mass	200 kg
specific impulse	300s
initial thrust load ψ_0	3
gravitational acceleration	9.81 m/s ²

(a) 10 points.

- Assuming that the **thrust is constant** during powered flight, an equation for the instantaneous mass of the rocket as a function of time, specific impulse, and initial thrust load can be found. You are asked to derive this equation
- Compute the total burn time of the rocket

(b) 15 points.

- Set up the equation of motion for the vertical motion of the rocket during its trajectory along the guide rail
- Derive from this equation still for constant thrust relationships for the velocity and altitude of the rocket as a function of the initial thrust load, specific impulse, and instantaneous mass (which indirectly is time!) during this phase of the flight. Use $\int \ln x dx = x \ln x - x$ if needed
- Starting from these equations, compute the instant of time at which the (center of mass of the) rocket leaves the guide rail. Hint: first solve the instantaneous mass by trial and error, an estimate in kg is sufficient.
- Compute the velocity at the instant of the rocket leaving the guide rail

(c) 5 points.

- Indicate angle of attack α and flight path angle γ_0 in a sketch
- Compute the flight path angle, γ_0 , for the instant just after the rocket leaves the guide rail.

4.1 Derivation of the Mass equation

By definition of mass flow (Slides FRM.11):

$$t = \frac{M_0 - M}{m} = \frac{M_0 - M}{T/c_{eff}} = \frac{I_{sp}}{\Psi_0} \left(1 - \frac{M}{M_0}\right) \quad (19)$$

In our case:

$$t = \frac{I_{sp}}{\Psi_0} \left(1 - \frac{M}{M_0}\right) = \frac{300}{3} \left(1 - \frac{200}{1000}\right) = 80s$$

4.2 Derivation of the trajectory equation

The equation of motion while in the rail is:

$$M \frac{dV_z}{dt} = T - Mg_0 = mc_{eff} - Mg_0 \quad (20)$$

Operating and integrating each term,

$$dV_z = \frac{T}{M} - g_0 = -c_{eff} \frac{dM}{M} - g_0 dt \quad (21)$$

$$\frac{dh}{dt} = V_z = c_{eff} \ln\left(\frac{M_0}{M}\right) - g_0 t \quad (22)$$

If we integrate again, we obtain the altitude at each time (detailed explanation at slides FRM.11 - FRM.13):

$$h = \frac{I_{sp}^2}{\Psi_0} g_0 \left(\frac{M}{M_0} \ln\left(\frac{M}{M_0}\right) + 1 - \frac{M}{M_0} \right) - \frac{1}{2} g_0 t^2 \quad (23)$$

Substituting Equation 19 into Equation 23 and inserting the numerical values for the end of the guide rail, i.e., $h = 50m$, and if we call the coefficient $x = \frac{M_0}{M_e}$ we obtain an equation for the mass:

$$h \frac{\Psi_0}{I_{sp}^2 g_0} = 50 \frac{3}{300^2 \times 9.81} = \frac{1}{x} \ln\left(\frac{1}{x}\right) + 1 - \frac{1}{x} - \frac{1}{2\Psi_0} \left(1 - \frac{1}{x}\right)^2 \quad (24)$$

By solving this equation you obtain the value for x (TIP: if you got a CASIO fx-991 or similar, you can plug the equation, insert an estimate value for x , greater than 1 in this case, and press SOLVE. Otherwise, any numerical method or try and guess can work).

$$x = \frac{M_0}{M_e} = 1.022964 \quad M_e = 977.55kg$$

The final time is:

$$t_e = \frac{I_{sp}}{\Psi_0} \left(1 - \frac{1}{x}\right) = \frac{300}{3} \left(1 - \frac{1}{1.022964}\right) = 2.2449s \quad (25)$$

Recalling Equation 22, you obtain the final velocity after the rail:

$$v_e = I_{sp} g_0 \ln(x) - g_0 t_e = 300 \times 9.81 \ln(1.022964) - 9.81 \times 2.2449 = 44.798m/s \quad (26)$$

4.3 Angle of attack and flight path angle

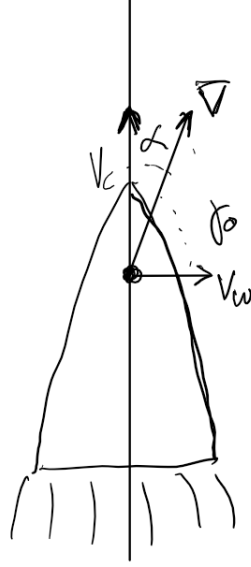


Figure 2:

In the instant after leaving the guide rail, the glith path angle is calculated as

$$\gamma = \text{atan}\left(\frac{v_e}{v_w}\right) = \text{atan}\left(\frac{44.798}{10}\right) = 77,41^\circ \quad (27)$$