Tentamen Oktober 2014, vragen

Rocket Motion (Technische Universiteit Delft)

ROCKET MOTION AE4870A

EXAMINATION

October 28, 2014

Delft University of Technology

Faculty of Aerospace Engineering

Space Engineering Division

This exam contains 4 questions.

PLEASE NOTE

Always write down the correct units for each computed parameter value. Be mindful for any required conversion before making any computations. **Always** write down the derivations of your answers, unless otherwise stated.

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Question 1 (10 points)

Determine whether the following statements are "True" or "False". No motivation/explanation has to be given. Each wrong answer (or no answer) deducts 2 points from the total.

(a) The so-called "Rocket Equation" (Tsiolkovsky) relates fuel mass to velocity increment

(b) It is impossible to reach space with a single-stage rocket

(c) For launch velocities up to 60% of the circular velocity, a flat-Earth approximation will give about the same shooting range as the spherical-Earth approach

(d) To keep a constant acceleration, all stages in a multi-stage rocket need to be equally powerful

(e) The accuracy requirements are easier met for a high trajectory than for a low trajectory

(f) Specific impulse is the thrust to weight ratio of a specific rocket

(g) The burn time for constant thrust load is longer than the burn time for constant mass flow if we assume identical specific impulse I_{sp} and mass-ratio Λ

(h) The influence of aerodynamic drag on rocket performance can be reduced by decreasing the rocket size (scaling)

(i) The roll rate of a rocket should have a positive slope when crossing the pitch frequency to avoid so-called lock-in

(j) A gravity turn maneuver is applied to reduce gravity losses

Question 2 (30 points)

In this question we consider the launch of a 2,000 kg payload with a multi-stage rocket (N = 3, number of stages). Consider the case that all three stages are identical, *i.e.,* the effective exhaust velocity per stage is c_{eff} = 3.2 km/s and the construction-mass ratio per stage is ε = 0.1. The required characteristic velocity to reach orbit is $V_{e,id}$ = 12 km/s.

Note: always state the used equations (without derivation).

(a) **5 points.** What is the total mass of the rocket?

(b) **20 points.** Calculate for each of the stages, the construction mass M_c and the propellant mass M_p.

(c) **5 points.** If we increase the payload mass and add boosters to the first stage, what can you say in general over the contribution to $V_{e,id}$ per stage?

Question 3 (30 points)

In this question we consider the ballistic flight over a stationary, spherical Earth (R_e = 6,378 km, μ = 398,600 km^3/s^2).

(a) **5 points.** Draw a clear sketch for the geometry of the ballistic flight over a spherical Earth. Indicate the rocket state related to the launch point and a local point in the trajectory, the flight range, true anomaly (for the launch point and local point), perigee and apogee height, and semi-major axis.

(b) **20 points.** Starting from the trajectory equation for conic sections,

$$r = \frac{p}{1 + e\cos\theta} \tag{1}$$

show that the flight-range angle ψ_0 is given by

$$\psi_0 = \frac{d/2}{R_e} = \arccos\left(\frac{1 - S_0^2\cos^2\gamma_0}{\sqrt{1 - S_0^2\cos^2\gamma_0\left(2 - S_0^2\right)}}\right) \tag{2}$$

Note: use can be made of the definition of semi-latus rectum $p = \frac{H^2}{\mu}$. Realize that the orbital momentum H is constant. The relation between eccentricity, e, semi-major axis, a, and p is given by $e = \sqrt{1 - \frac{p}{a}}$.

(c) **5 points.** For a flight range d = 9,020 km and a launch velocity V_0 = 7,905.4 m/s, calculate the corresponding launch angle γ_0.

Question 4 (30 points)

Consider the flight of a single-stage sounding rocket in a homogeneous gravity field. The rocket starts vertically along a guide rail. The subsequent flight takes place in an atmosphere in which a constant horizontal winds prevails. It is assumed that the rocket is infinitely (statically) stable, meaning that it responds instantly to the impinging airflow. The influence of atmospheric drag may be ignored. Use the following data:

guide rail length	50m
wind speed	10 m/s
rocket initial (total) mass	1000 kg
rocket burnout mass	200 kg
specific impulse	300s
initial thrust load ψ_0	3
gravitational acceleration	9.81 m/s^2

(a) **10 points.**

- Assuming that the **thrust is constant** during powered flight, an equation for the instantaneous mass of the rocket as a function of time, specific impulse, and initial thrust load can be found. You are asked to derive this equation
- Compute the total burn time of the rocket

(b) **15 points.**

- Set up the equation of motion for the vertical motion of the rocket during its trajectory along the guide rail
- Derive from this equation still for constant thrust relationships for the velocity and altitude of the rocket as a function of the initial thrust load, specific impulse, and instantaneous mass (which indirectly is time!) during this phase of the flight. Use $\int \ln x \, \mathrm{d}x = x \ln x - x$ if needed
- Starting from these equations, compute the instant of time at which the (center of mass of the) rocket leaves the guide rail. Hint: first solve the instantaneous mass by trial and error, an estimate in kg is sufficient.
- Compute the velocity at the instant of the rocket leaving the guide rail

(c) **5 points.**

- Indicate angle of attack α and flight path angle γ_0 in a sketch
- Compute the flight path angle, γ_0, for the instant just after the rocket leaves the guide rail.