Solutions Exams Rocket MOTION

Question type 1 - flight of a single stage sounding rocket

$h_{rail} = 50\,m$ $\quad I_{sp} = 300\ s$

$V_W = 10\,m/s$ $\quad \psi = 3 = \text{constant}$

$M_0 = 1000\,kg$ $\quad g = 9{,}81\ m/s^2$

$M_e = 200\,kg$

In homogeneous gravity field without drag. The rocket is infinitely (statically) stable

ⓐ Derive $M(t)$ starting from definition of ψ and determine t_b.

$$\psi = \frac{T}{W} = \frac{C_{eff}\,\dot m}{M g_0} = -\frac{C_{eff}}{g_0}\frac{1}{M}\frac{dM}{dt} \rightarrow dt = \frac{-C_{eff}}{g_0\psi}\frac{1}{M}dM$$

Integration gives:

$$t\Big|_{t_0}^{t_1} = \frac{-C_{eff}}{g_0\psi}\ln M\Big|_{M_0}^{M_1} = \frac{C_{eff}}{g_0\psi}\ln M\Big|_{M_1}^{M_0}$$

If we take $t_0 = 0$ and $t_1 = t$ we get $M_0 = M_0$ and $M_1 = M$

$$t = \frac{C_{eff}}{g_0\psi}\ln\frac{M_0}{M} \rightarrow \frac{t g_0\psi}{C_{eff}} = \ln\frac{M_0}{M} \rightarrow \boxed{M = M_0\exp\left(\frac{-t g_0\psi}{C_{eff}}\right)}$$

at $t = t_b$ $\quad M = M_e$. From this we can obtain t_b:

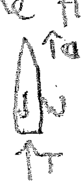
$$t_b = \frac{C_{eff}}{g_0\psi}\ln\Lambda$$

The I_{sp} is defined as $I_{sp} = \frac{T}{\dot m g_0} = \frac{\dot m C_{eff}}{\dot m g_0} = \frac{C_{eff}}{g_0}$. the burn time becomes:

$$t_b = \frac{I_{sp}}{\psi}\ln\Lambda = \frac{300}{3}\ln\left(\frac{1000}{200}\right) = 100\ln 5 = 160{,}9\ sec$$

$$\boxed{t_b = 160{,}9\ sec}$$

ⓑ Set-up the EOM, determine $V(\psi, t)$ and $Z(\psi, t)$ for the vertical part of the flight and determine $t_{leave\ rail}$ and $V_{leave\ rail}$



EOM $\quad Ma = T - W \rightarrow a = \frac{T}{M} - g_0 = g_0(\psi - 1) = \frac{dV}{dt}$

$$V\Big|_{V_0}^{V} = g_0 t(\psi - 1)\Big|_{t_0}^{t}$$

At $t = 0$, $V = 0$:

$$V = g_0 t(\psi - 1) = \frac{dz}{dt}$$

$Z\Big|_{z_0}^{z} = \frac{1}{2} g_0 t^2 (\psi - 1)\Big|_{t_0}^{t}$

for $t=0$, $Z=0$ we get

$Z = \frac{1}{2} g_0 t^2 (\psi - 1)$

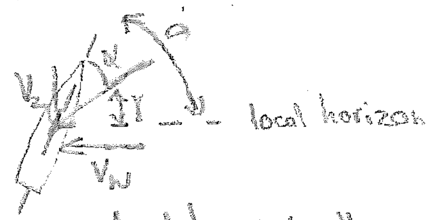
at $Z=50m$, we can solve for t:

$$t_r = \sqrt{\frac{2Z}{g_0(\psi-1)}} = \sqrt{\frac{2\cdot 50}{9{,}81(3-1)}} = 2{,}258 \text{ sec} \qquad \boxed{t_R = 2{,}26 \text{ sec}}$$

at $t = t_r$ we can find V_r:

$$V_r = g_0 t_r (\psi - 1) = 9{,}81 \cdot 2{,}258 \cdot (3-1) = 44{,}34 \text{ m/s} \qquad \boxed{V_r = 44{,}34 \text{ m/s}}$$

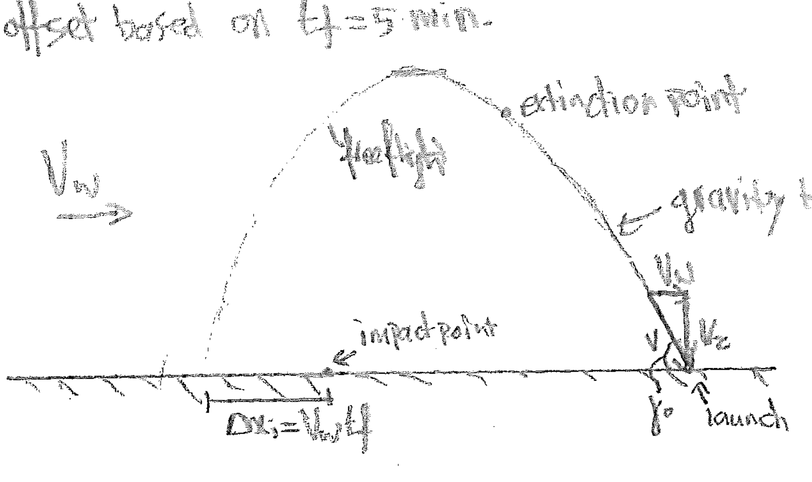
(c) Compute γ_0 for when the rocket has just left the guide rail.



For an infinitely statically stable rocket, α will always be equal to zero. γ, the angle between the air velocity and the local horizon is equal to θ and is given by:

$$\gamma_0 = \arctan\left(\frac{V_z}{V_W}\right) = \arctan\left(\frac{44{,}34}{10}\right) = 77{,}3^\circ \qquad \boxed{\gamma_0 = 77{,}3^\circ}$$

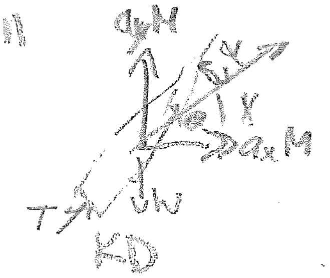
(d) Sketch the trajectory of the rocket after leaving the launch rail until time of impact. Indicate V_W and explain wind offset. Estimate the wind offset based on $t_f = 5$ min.



The rocket describes a gravity turn wrt the reference frame attached to the wind. If $V_W(h) = V_W = \text{const}$, this frame has a uniform motion wrt the inertial reference frame, so the EOM remain unchanged.

The point of impact follows from the motion of the wit in relati to the ground, which is given by the wind offset.
The wind offset is given by:
$\Delta X_i = V_w t_f = 10 \cdot 5 \cdot 60 = 3$ Km $\quad \boxed{\Delta X_i = 3\,\text{Km}}$

ⓔ Derive EOM for grav turn



In gravity turn $\alpha = 0 \Rightarrow \theta = \gamma \Rightarrow \cos\theta = \frac{V_x}{V}$, $\sin\theta = \frac{V_z}{V}$

$a_x M = T\cos\theta = T\,V_x/V = dV_x/dt\, M$

$a_z M = T\sin\theta - W = T\,V_z/V - W = dV_z/dt\, M$

Multiply first by V_x and second by V_z and add:

$$V_x \frac{dV_x}{dt} + \frac{dV_z}{dt} V_z = \frac{1}{2}\frac{dV^2}{dt} = \frac{T}{M}\left(\frac{V_x^2}{V} + \frac{V_z^2}{V}\right) - g_0 V_z = \frac{T}{M} V - g_0 \sin\gamma V = V\frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{T}{M} - g_0 \sin\gamma$$

Multiply first by V_z and second by V_x and substract:

$$M\left[V_z \frac{dV_x}{dt} - V_x \frac{dV_z}{dt}\right] = M g_0 V_x$$

divide by V_x^2:

$$\frac{V_z}{V_x^2}\frac{dV_x}{dt} - \frac{1}{V_x}\frac{dV_z}{dt} = \frac{g_0}{V_x} \Rightarrow \frac{1}{V_x}\frac{dV_z}{dt} - \frac{V_z}{V_x^2}\frac{dV_x}{dt} = -\frac{g_0}{V_x} = \frac{d(V_z/V_x)}{dt}$$

$V_z/V_x = \tan\gamma$. We get

$$\frac{d(\tan\gamma)}{dt} = -\frac{g_0}{V_x} = -\frac{g_0}{V\cos\gamma} = \frac{1}{\cos^2\gamma}\frac{d\gamma}{dt} \rightarrow V\frac{d\gamma}{dt} = -g_0\cos\gamma$$

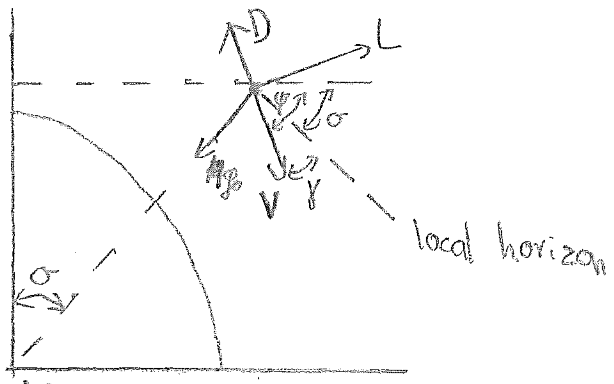
Question type 2 - reentry using repeating skipping trajectory

$\gamma_E = 15°$

$V_E = 8\,km/s$

$L/D = 1 = constant$

ⓐ Set up EOM in direction parallel with and perpendicular to velocity vector. Next, derive $V(\gamma)$ and $\gamma(\rho)$ under assumption of exponential atmosphere. Indicate assumptions



$M\frac{dV}{dt} = Mg_0 \sin\gamma - D$ > minus

$-MV\frac{d\psi}{dt} = L - Mg_0\cos\gamma$

We know $\psi = \sigma + \gamma \Rightarrow \frac{d\psi}{dt} = \frac{d\sigma}{dt} + \frac{d\gamma}{dt}$ $\quad \frac{d\sigma}{dt} = \frac{V}{r}\cos\gamma$

$-MV\frac{d\gamma}{dt} = L - Mg_0\cos\gamma\left(1 - \frac{V^2}{rg_0}\right) = L - Mg_0\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right)$

For skipping flight $\frac{L}{W} \gg 1$ and $\frac{D}{W} \gg 1$. We get

$M\frac{dV}{dt} = -D$

$-MV\frac{d\gamma}{dt} = L$

When we devide these two, we find

$\frac{1}{V}\frac{dV}{d\gamma} = \frac{D}{L} \Rightarrow \ln V\Big|_{V_E}^{V} = \frac{D}{L}\gamma\Big|_{\gamma_E}^{\gamma} \Rightarrow V = V_E \exp\left(\frac{\gamma - \gamma_E}{L/D}\right)$

We can see $d\gamma/dt < 0 \Rightarrow$ vehicle decelerates.

$-MV\frac{d\gamma}{d\rho}\frac{d\rho}{dh}\frac{dh}{dt} = L = \frac{1}{2}\rho V^2 C_L S$ Assuming an isothermal atmosphere

$d\rho/dh = -\rho g_0 \cdot \frac{dh}{dt} = -V\sin\gamma$

We get:

$$-MV \rho g_0 V \sin\gamma \frac{d\gamma}{d\rho} = \frac{1}{2}\rho V^2 C_L S = -W V^2 \rho \sin\gamma \frac{d\gamma}{d\rho}$$

or after integration and assuming $\rho_E = 0$

$$\frac{1}{2W/C_LS} \rho\Big|_{\rho_E}^{\rho} = \cos\gamma\Big|_{\gamma_E}^{\gamma} \Rightarrow \cos\gamma - \cos\gamma_E = \frac{1}{2\frac{W/S}{C_L}}(P - P_E) = \frac{P}{2\frac{W/S}{C_L}}$$

For an assumed exponential atmosphere, it follows $\frac{P}{\rho} = RT = gH$

$$\cos\gamma - \cos\gamma_E = gH \frac{1}{2\frac{W/S}{C_L}} (\rho - \rho_E)$$

ⓑ Draw a sketch of variation of V as function of γ from entry to 2nd exit

Since $d\gamma/dt < 0$ lowest point is when $\gamma = 0$. Since at end of the skip $\rho = \rho_E = 0$ $\cos\gamma_F = \cos\gamma_E$ and since $d\gamma/dt < 0 \Rightarrow \gamma_E = -\gamma_F$.

lowest point first skip: $\gamma = 0$; $V = V_E \exp\left(\frac{\gamma - \gamma_E}{L/D}\right) = 6{,}15$ km/s

exit atmosphere : $\gamma = -\gamma_E$; $V = 4{,}739$ km/s ;

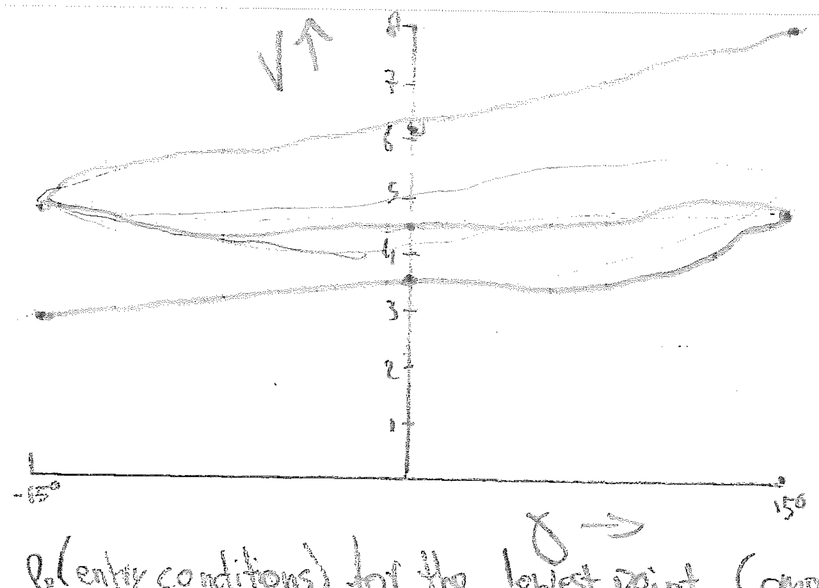
highest point : In this point, there is only a horizontal velocity:

$\gamma = 0$; $V = V\cos\gamma = 4{,}578$ km/s

atmospheric entry: From characteristics of the ellipse shaped trajectory we find $\gamma = \gamma_E$; $V = 4{,}739$ km/s

lowest point : $\gamma = 0$; $V = 3{,}64$ km/s

exit atmosphere : $\gamma = -\gamma_E$; $V = 2{,}8$ km/s



ⓒ Derive ρ_p (entry conditions) for the lowest point. Compute h_p given $\frac{W}{SC_L} = 0$, $H = 7000$.

At the lowes altitude, $\gamma = 0$. At h_E $\rho = \rho_E = 0$. We get

$$1-\cos\gamma_E = gH\frac{1}{2W/C_LS}\rho_P = 2\sin^2\gamma_E/2$$

We get:

$$\rho_P = 4\sin^2\frac{\gamma_E}{2}\frac{W}{C_LS}\frac{1}{gH}$$

$$\rho_P = \frac{4}{9{,}81\cdot 7000}\cdot 2000\sin^2 7{,}5 = 0{,}0019885 \quad /160$$

$$\frac{\rho_E}{\rho_0} = \exp\left(\frac{-h}{H}\right) \rightarrow -H\ln\frac{\rho_E}{\rho_0} = h = 44{,}976\,km = 45\,km$$

Question type 3 – Rocket with booster engines

$(t_b)_1 = 200\,s$	$(T)_1 = 3000\cdot 10^3\,N$
$(t_b)_s = 60\,s$	$(I_{sp})_1 = 300\,s$
$N_s = 3$	$(M_e)_1 = 40000\,kg$
$T_s = 1000\cdot 10^3\,N$	$g = 9{,}81\,m/s^2$
$(M_e)_s = 36000\,kg$	
$(I_{sp})_s = 200\,s$	Three boosters

ⓐ Set up EOM for vertical flight of single stage rocket and derive V_b and Z_b as function of c, Λ and ψ_0. $\int \ln x\,dx = x\ln x - x$

$$a_z M = T - Mg = \frac{dV_z}{dt}M = \dot{m}c - Mg$$

$$\frac{dV_z}{dt} = \frac{c\frac{dM}{dt}}{M} - g \rightarrow V_z = c\ln\Lambda - gt \quad \text{since } V_0 = 0,\ t_0 = 0$$

$$V_z = \frac{dZ}{dt} = c\ln\frac{M_0}{M} - gt \rightarrow dZ = c[\ln M_0 - \ln M]\frac{dt}{dM}dM - gtdt$$

$$dZ = -\frac{c}{\dot{m}}[\ln M_0 - \ln M]dM - g_0 t dt$$

$$Z\Big|_{Z_0}^{Z} = -\frac{c}{\dot{m}}\Big[M\ln M_0 - M\ln M + M\Big]\Big|_{M_0}^{M} - \frac{1}{2}g_0t^2 = Z$$

$$Z = -\frac{c}{\dot{m}}\Big[M\ln M_0 - M\ln M + M - M_0\ln M_0 + M_0\ln M_0 - M_0\Big] - \frac{1}{2}g_0t^2$$

$$= -\frac{c}{\dot{m}}\Big[M\ln M_0 - M\ln M + M - M_0\Big] - \frac{1}{2}g_0t^2$$

since $\dot{m} = M_0 g_0 \psi_0 / c$ we get

$$Z = \frac{c^2}{g_0\psi_0}\left[1 - \cancel{\Lambda} - \frac{1}{\Lambda}\ln\Lambda\right] - \frac{1}{2}g_0t^2$$

(b) Compute the Total starting mass of the rocket

$(M_0)_{tot} = (M_0)_1 + 3(M_0)_s$

$(M_0)_1 = (M_p)_1 + (M_e)_1$

$(M_p) = (\dot{m})_1 t_b$

$I_{sp} = \frac{T}{\dot{m} g_0} \rightarrow \dot{m} = \frac{T}{I_{sp} g_0} \rightarrow (\dot{m})_1 = \frac{3000.10^3}{300.9,81} = 1019\ kg/s$

$(M_p)_1 = 1019.200 = 204.10^3\ kg \rightarrow (M_0)_1 = 244.10^3\ kg$

$(M_0)_{tot} = 244.10^3 + 3.36.10^3 = 352.10^3\ kg$

(c) compute dV/dt at instant of lift off, after jettisoning the boosters and before bur

$\frac{a}{g} = \frac{T}{Mg} - 1$

At liftoff: $T = 6.10^6\ N$, $M = (M_0)_{tot} \Rightarrow a = 0,7375\,g = 7,235\ m/s^2$

After jettisonig $T = 3.10^6\ N$, $M = M_j$

$M_j = (M_0)_1 - (M_p)_1 \frac{t_j}{t_b} = 182{,}8.10^3\ kg \rightarrow a = 0,6729\,g = 6,601\ m/s^2$

before burnout: $T = 3.10^6\ N$, $M = (M_e)_1 \rightarrow a = 6,645\,g = 65,19\ m/s^2$

(d) Compute V and Z after burnout of the boosters

$$Z = \frac{C_{eff}^2}{g_0 \psi_0}\left[1 - \frac{1}{\Lambda} - \frac{1}{\Lambda}\ln\Lambda\right] - \frac{1}{2} g_0 t_{b_s}^2$$

$\Lambda = \frac{(M_0)_{tot}}{(M_e)_s}$ where $(M_e)_s = M_j + 3\left[(M_0)_s - (M_p)_s\right]$

$(M_p)_s = (\dot{m})_s (t_b)_s \qquad (\dot{m})_s = \frac{T}{I_{sp} g_0} = 509,68\ kg/s$

$(M_p) = 30,6.10^3\ kg \rightarrow (M_e)_s = 199.10^3\ kg$

$\Lambda = 1,7688$

$C_{eff} = \frac{(T_1 + 3T)_s}{(\dot{m}_1 + 3\dot{m})_s} = 2356\ m/s$

$V = C \ln\Lambda - g_0 t = 755\ m/s$

$Z = 18,887\ Km$

$\psi_0 = \frac{T}{M_0 g_0} = \frac{6.10^6}{352.10^3.9,81} = 1,7375$

(f) Compute V and Z at the instant of $M(t_b)_1$

$\Lambda_2 = \frac{M_1}{(M_e)_1} = 4{,}57 \qquad c_{eff} = \frac{I}{\dot{m}} = 2944{,}06$ m/s

$(t_b)_1' = 200 - 60 = 140$ sec

$(V_e)_2 = (V_e)_1 + c \ln \Lambda - g_0 (t_b)_1' = 3855$ m/s

$(Z_e)_2 = (Z_e)_1 + (V_e)(t_b)_1' + \frac{c^2}{g_0 \psi_0}\left[1 - \frac{1}{\Lambda} - \frac{1}{\Lambda}\ln\Lambda\right] - \frac{1}{2} g_0 (t_b)_1'^2$

$\psi_0 = 1{,}6729$

$Z = 30{,}813$ km $\rightarrow$ not correct : ≈ 160 [km]

(g) Compute Z_c

find when $V = 0 = 3855 - g_0 (t_c) \qquad \Rightarrow t_c = \frac{3855}{g_0}$

$Z_c = Z + 3855 t_c - \frac{1}{2} g_0 t_c^2 = Z + \frac{3855^2}{g_0} - \frac{1}{2}\frac{3855^2}{g_0} = Z + \frac{3855^2}{2g_0} = 788{,}255$ km

(h) For only the single stage: compute dV/dt at start and end V_b, Z_b and Z_c

start:

$\frac{dV}{dt g} = \frac{I}{W} - 1 = 0{,}2533g = 2{,}485$ m/s² $\qquad M_0 = 244 \cdot 10^3$ kg

end: $a = 6{,}6452g = 65{,}19$ m/s²

$\Lambda = 6{,}1$

$V = c \ln \Lambda - g_0 t_b = 3361{,}7$

$Z_b = 184{,}218$ km

$Z_c = Z_b + \frac{3361{,}7^2}{2g_0} = 760{,}213$ km

Question type 4 - vertical flight of a two stage rocket

In homogeneous gravity field and in a vacuum

(a) Derive $Z_c(t_c, Z_{c0})$ where t_c is coasting time between stages

$\Delta V_1 = c_1 \ln \Lambda_1 - g_0(t_b)_1$, $\Delta V_2 = c_2 \ln \Lambda_2 - g_0(t_b)_2$

for $t_c = 0$ we find

$$h_c = \Delta h_1 + \Delta h_2 + \frac{(\Delta V_2 + \Delta V_1)^2}{2g_0}$$

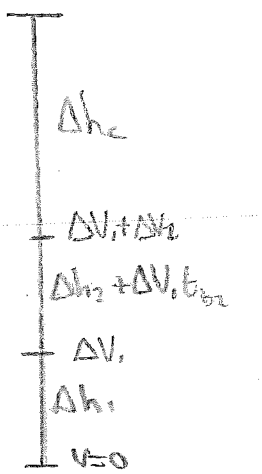
$$\Delta h_{c0} = \Delta V_1 t_c - \tfrac{1}{2} g_0 t_{c0}^2 \rightarrow (V_2)_0 = \Delta V_1 - g_0 t_c \rightarrow (V_2)_e = \Delta V_1 - g_0 t_c + \Delta V_2$$

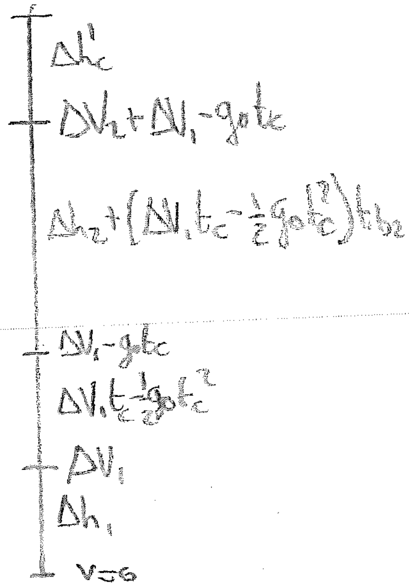
The culmination attitude with coast time becomes:

$$\Delta h_c' = \Delta h_1 + \Delta h_2 + \Delta V_1 t_c - \tfrac{1}{2} g_0 t_c^2 + \frac{(\Delta V_1 + \Delta V_2 - g_0 t_c)^2}{2g_0} + (\Delta V_1 - g_0 t_c) t_{b_2}$$

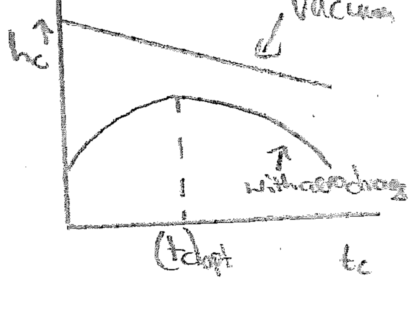
$$= \Delta h_1 + \Delta h_2 + \Delta V_1 t_c - \tfrac{1}{2} g_0 t_c^2 + \frac{(\Delta V_1 + \Delta V_2)^2}{2g_0} - \frac{2\Delta V_1 g_0 t_c}{2g_0} - \frac{2\Delta V_2 g_0 t_c}{2g_0} + \frac{g_0^2 t_c^2}{2g_0} + (\Delta V_1 - g_0 t_c) t_{b_2}$$

$$= h_c - (\Delta V_2 + g_0 (t_b)_2) t_c$$





(b) Present this relation as well for in case of drag. Explain the difference.



It can be advantageous to coast to higher altitudes before firing up the next stage, since higher velocities will occur when the density is lower → less drag. If the t_c becomes to large, velocity reducting negative effect dominates → optimum. It is lower than in vacuum due to

ⓒ Derive $t_{culm}(\ldots)$ and show it is independent of t_c and burn times

$t_c = (t_b)_1 + (t)_c + (t_b)_2 + \Delta t_c$

Δt_c is time until $V=0$: $V=0=(\Delta V_1 + \Delta V_2 - g_0 t_c) - g_0 \Delta t_c$

$t_c = (t_b)_1 + (t_b)_2 + \dfrac{\Delta V_1 + \Delta V_2}{g_0} =$

$\Delta V_1 = c_1 \ln \Lambda_1 - g_0 (t_b)_1$; $\Delta V_2 = c_2 \ln \Lambda_2 - g_0 (t_b)_2$

we get

$t_c = \dfrac{c_1 \ln \Lambda_1 + c_2 \ln \Lambda_2}{g_0}$

Question type 5 - balistic flight

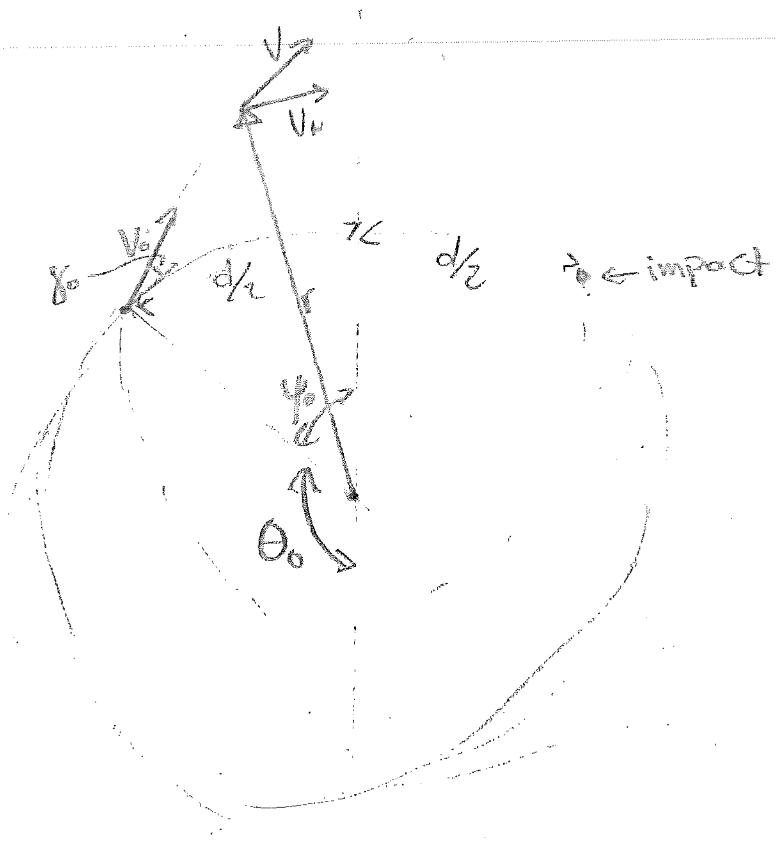
$\Lambda_0 = 5°E$ $\quad\quad$ $\beta_0 = 120°$

$\lambda_0 = 52° N$ $\quad\quad$ $R_e = 6370 \cdot 10^3$ m

$V_0 = 5500$ km/s $\quad\quad$ $\mu = 398600{,}4$ km^3/s^2

$\gamma_0 = 55°$

ⓐ Make a sketch in 2-D plane and explain



ⓑ List the useful relations: ellipse equation, $P(H)$, Energy law

Ellipse equation: $r = \frac{P}{1+e\cos\theta}$

$P(H) = \frac{H^2}{\mu}$

Energy law: $\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = \text{constant}$

ⓒ Derive h_{max} (launch parameters) and compute it.

$h_{max} = r_a - R_e$

$\frac{V_0^2}{2} - \frac{\mu}{R_e} = -\frac{\mu}{2a} \Rightarrow \frac{a}{R_e}\left(\frac{V_0^2}{2}R_e - \mu\right) = -\frac{\mu}{2} \Rightarrow \frac{a}{R_e} = -\mu \frac{1}{2\left[\frac{V_0^2 R_e}{2} - \mu\right]} = \frac{1}{2 - \frac{V_0^2 R_e}{\mu}} = \frac{1}{2 - S_0^2}$

$S_0 = \sqrt{V_0^2 R_e/\mu}$

$r_a = a(1+e)$

$H = V_t r = V_0 \cos\gamma_0 R_e = \text{const} \rightarrow P = \frac{V_0^2 \cos^2\gamma_0 R_e^2}{\mu} \rightarrow \frac{P}{R_e} = S_0^2 \cos^2\gamma_0$

$r_a + r_p = 2a = \frac{P}{1+e} + \frac{P}{1-e} = \frac{2P}{1-e^2} \rightarrow P = a(1-e^2) \rightarrow e = \sqrt{1 - \frac{P}{a}}$

$e = \sqrt{1 - S_a^2 \cos^2\gamma_0 (2 - S_0^2)}$

$\frac{r_a}{R_e} = \frac{a}{R_e}(1+e) = \frac{1}{2-S_0^2}\left(1 + \sqrt{1 - S_0^2 \cos^2\gamma_0 (2 - S_0^2)}\right)$

$\frac{h_a}{R_e} = \frac{r_a}{R_e} - 1 = \frac{S_0^2 + \sqrt{1 - S_0^2\cos^2\gamma_0(2-S_0^2)} - 1}{2 - S_0^2}$ $\qquad S_0^2 = \frac{V_0^2 R_e}{\mu} = 0,483$

$h_a = 4428,01$ KM

ⓓ Derive an expression for the range of rocket and compute it

$\psi_0 = \frac{d/2}{R_e} = \pi - \theta_0 \rightarrow \cos\theta_0 = -\cos\psi_0$

For $r = R_e$:

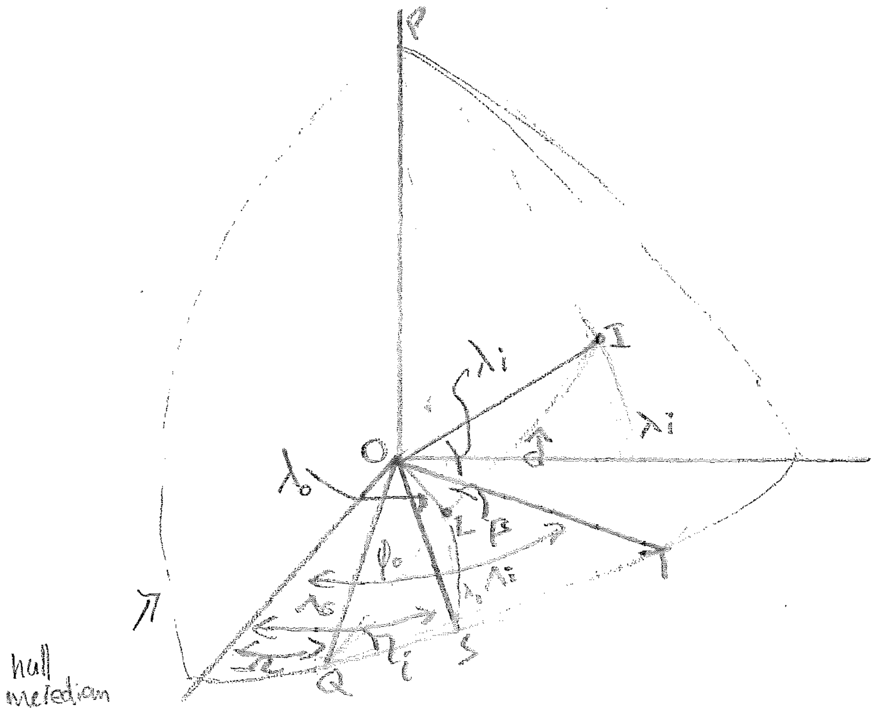
$1 = \frac{r}{R_e} = \frac{P/R_e}{1+e\cos\theta_0} = \frac{S_0^2\cos^2\gamma_0}{1 + \sqrt{1 - S_0^2\cos^2\gamma_0(2-S_0^2)}\cos\theta_0} = \frac{S_0^2 \cos\gamma_0}{1 - \sqrt{1 - S_0^2\cos^2\gamma_0(2-S_0^2)}\cos\psi_0}$

$1 - \frac{P}{R_e} = e\cos\theta_0 = e\cos\psi_0$

$$\cos\varphi_0 = \frac{1 - S_0^2\cos^2\gamma}{\sqrt{1 - S_0^2\cos^2\gamma\,(2 - S_0^2)}}$$

$$\varphi_0 = 35^0 = 0{,}6138 = \frac{d/2}{R_e} \Rightarrow d = 7782{,}4\ \text{km}$$

(e) Sketch all important parameters for 3-D



(f) Derive relation for inclination

$$\cos i = \sin\beta\cos\lambda_0$$

(g) Derive equation for Ω

$$\tan\lambda_0 = \tan\varphi_0\cos\beta_0 \rightarrow \tan\varphi_0 = \frac{\tan\lambda_0}{\cos\beta_0}$$

$$\tan(\Lambda_0 - \Omega) = \tan\varphi_0\cos i = \frac{\tan\lambda_0}{\cos\beta_0}\sin\beta_0\cos\lambda_0 = \sin\lambda_0\tan\beta_0$$

(h) Derive expressions for λ_i an Λ_i and compute them

$$\tan(\Lambda_i - \Omega) = \tan(\varphi_0 + \hat{d})\cos i$$
$$\tan(\lambda_i) = \tan(\varphi_0 + \hat{d})\sin i\cos(\Lambda_i - \Omega)$$

$$i = \arccos\left(\sin\beta \cos\lambda_0\right) = 57{,}7^\circ$$

$$\Lambda_0 - \Omega = \arctan\left(\sin\lambda_0 \tan\beta_0\right) = -53{,}77 \rightarrow \Omega = 58{,}77^\circ \rightarrow \Omega = 238{,}77$$

since $\beta > 90$

$$\phi_0 = \arctan\left(\frac{\tan\lambda_0}{\cos\beta_0}\right) = -68{,}66^\circ \rightarrow \phi_0 = 111{,}33$$

since $\beta > 90$

$$\hat{d} = \frac{d}{R_e} = 30{,}23^\circ$$

$$\Lambda_i - \Omega = \arctan\left(\tan(\phi_0 + \hat{d})\cos i\right) \rightarrow \Lambda_i = 35{,}88^\circ$$

$$\lambda_i = \arctan\left(\tan(\phi_0 + \hat{d}) \sin i \cos(\Lambda_i - \Omega)\right) \rightarrow \lambda_i = 31{,}71$$