Exam Solution

Course: AE4870B Re-Entry Systems
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0 Introduction

This contains an exam solution. If you wish to contribute to this exam solution:

1. Create a github account, (you can create an "anonymous" one).
2. git clone ...
3. edit your changes in the document.
4. open cmd, and browse to inside the folder you downloaded and edited
5. git pull (updates your local repository=copy of folder, to the latest version in github cloud)
6. git status shows which files you changed.
7. git add "/some folder with a space/someFileYouChanged.tex"
8. git commit -m "Included solution to question 1c."
9. git push

It can be a bit initimidating at first, so feel free to click on "issue" in the github browser of this repository and ask :) (You can also use that to say "Hi, I'm having a bit of help with this particular equation, can someone help me out?")

If you don't know how to edit a latex file on your own pc iso on overleaf, look at the "How to use" section of `https://github.com/a-t-0/AE4872-Satellite-Orbit-Determination`.

0.1 Consistency

To make everything nice and structured, please use very clear citations:

1. If you copy/use an equation of some slide or document, please add the following data:
   (a) Url (e.g. if simple wiki or some site)
   (b) Name of document
   (c) (Author)
   (d) PAGE/SLIDE number so people can easily find it again
   (e) equation number (so people can easily find it again)
2. If you use an equation from the slides/a book that already has an equation number, then hardcode that equation number in this solution manual so people directly see which equation in the lecture material it is, this facilitates remembering the equations.
3. Here is an example is given in eq. (10.32[?]) (See file references.bib [?]).

$$R_n^E = \sum_{t=1}^{n} l_m(p_t, z_t) - \min_i \sum_{t=1}^{n} {z_t}^i \tag{10.32[?]}$$

1 Question 1

a. False. A ballistic entry is characterised by zero lift (reader p. 218)

b. True. Drag goes up along with the nose radius, so the lift to drag ratio decreases.

c. True. The large angle of attack at the beginning of entry is used to minimize heating rates and deceleration. Therefore it should follow a shallow path during descent (large angle of attack) (reader p. 254)

d. False. The minimum is 1 unit axis rotation. (reader p.195 onwards)

e. False.

f. True. In the exponential atmosphere model the temperature is constant wrt altitude. Since the speed of sound is dependent on the temperature, as altitude increases, the speed of sound remains constant. (reader p.28)

g. (True. at least 4 satellites are needed to determine the position. (reader p.393)

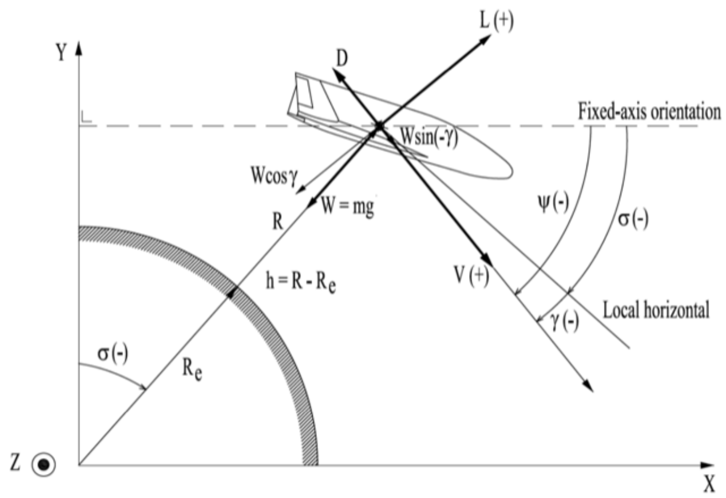
h. False. This only holds for linear systems (reader 330 - 333)

i. True. Reefing slows down the opening of a parachute. (reader p.422)

j. False. For both entry flights the mechanical loads occur after the maximum heat flux. (reader p.323-324 figure 5-70, 5-71) and (reader p239 figure 5.16)

2 Question 2

a. The asked sketch is given below.



b.

$$m\frac{dV}{dt} = -D - mg\sin\gamma \tag{1}$$

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right) \tag{2}$$

$$\frac{dR}{dt} = \frac{dh}{dt} = V\sin\gamma \tag{3}$$

The derivation of the EOM's can be found in (reader p.215-219). For the test no derivations are asked for, the equations should be memorized by heart.

c.

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma\left(1 - \frac{V^2}{V_c^2}\right) \tag{4}$$

By definition of a ballistic flight the lift encountered lift by the vehicle is zero $L = 0$. As a consequence the mass can be divided out. We introduce the altitude h as an independent variable, instead of t, such that:

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dh}\frac{dh}{dt} = V\sin\gamma\frac{d\gamma}{dh} \tag{5}$$

Substituting 5 into 4 gives us:

$$\mathrm{V}^2\sin\gamma\frac{\mathrm{d}\gamma}{\mathrm{dh}} = -\mathrm{g}\cos\gamma\left(1-\frac{\mathrm{V}^2}{\mathrm{V}_\mathrm{c}^2}\right) \tag{6}$$

$$\tan\gamma\frac{\mathrm{d}\gamma}{\mathrm{dh}} = -\mathrm{g}\left(\frac{1}{\mathrm{V}^2}-\frac{1}{\mathrm{V}_\mathrm{c}^2}\right) \approx 0 \tag{7}$$

Assuming $V_c >> V$ and $V^2 >> g$ we obtain the following relationship.

$$\frac{\mathrm{d}\gamma}{\mathrm{dh}} \approx 0 \tag{8}$$

$$\gamma \approx constant = \gamma_\mathrm{E} \tag{9}$$

Since the Flight Path Angle is constant and does not change w.r.t. height, the trajectory of the ballistics craft is said to be a straight line from entry until it reaches the surface.

d. This question follows the derivation given in (reader p.262). For a gliding entry we assume that $\frac{\mathrm{d}\gamma}{\mathrm{dt}} = 0$ and that the flight path angle γ goes to zero.

$$\frac{\mathrm{dV}}{\mathrm{dt}} = -\frac{\mathrm{D}}{\mathrm{m}} = -\frac{\mathrm{D}}{\mathrm{L}}\mathrm{g}\left(\frac{\mathrm{L}}{\mathrm{mg}}\right) = -\frac{\mathrm{D}}{\mathrm{L}}\mathrm{g}\left(1-\frac{\mathrm{V}^2}{\mathrm{V}_\mathrm{c}^2}\right) \tag{10}$$

$$\mathrm{L} = \mathrm{mg}\left(1-\frac{\mathrm{V}^2}{\mathrm{V}_\mathrm{c}^2}\right) \tag{11}$$

Now we substitute 11 back into 10 and obtain the following expression for dt:

$$\mathrm{dt} = -\frac{\frac{\mathrm{L}}{\mathrm{D}}}{\mathrm{g}\left(1-\frac{\mathrm{V}^2}{\mathrm{V}_\mathrm{c}^2}\right)}\mathrm{dV} \tag{12}$$

For simplicity means, we substitute $V/V_c = x$ and integrate the expression we found for dt to obtain an expression for the flight time t_{flight}

$$\mathrm{x} = \frac{\mathrm{V}}{\mathrm{V}_\mathrm{c}}; \mathrm{dx} = \frac{\mathrm{dV}}{\mathrm{V}_\mathrm{c}} \Rightarrow \mathrm{dV} = \mathrm{V}_\mathrm{c}\mathrm{dx} \tag{13}$$

$$\mathrm{dt} = -\frac{\frac{\mathrm{L}}{\mathrm{D}}}{\mathrm{g}\left(1-\mathrm{x}^2\right)}\mathrm{V}_\mathrm{c}\mathrm{dx} = -\frac{\mathrm{L}}{\mathrm{D}}\frac{\mathrm{V}_\mathrm{c}\left(\frac{\mathrm{dx}}{1-\mathrm{x}^2}\right)}{\mathrm{g}} \tag{14}$$

$$\int(\frac{\mathrm{dx}}{1-\mathrm{x}^2}) = \frac{1}{2}\int\frac{\mathrm{dx}}{1+\mathrm{x}} + \frac{1}{2}\int\frac{\mathrm{dx}}{1-\mathrm{x}} = \frac{1}{2}\ln(1+\mathrm{x}) - \frac{1}{2}\ln(1-\mathrm{x}) + \mathrm{c} = \frac{1}{2}\ln\frac{1+\mathrm{x}}{1-\mathrm{x}} + \mathrm{c} \tag{15}$$

$$\mathrm{t_{flight}} = -\frac{\mathrm{V}_\mathrm{c}}{\mathrm{g}}\frac{\mathrm{L}}{\mathrm{D}}\int_{\frac{\mathrm{V_E}}{\mathrm{V}}\mathrm{C}}^{\frac{\mathrm{V_F}}{\mathrm{V_C}}}\frac{1}{1-\frac{V^2}{V_c^2}}\mathrm{d}\left(\frac{\mathrm{V}}{\mathrm{V}_\mathrm{c}}\right) = -\frac{\mathrm{V}_\mathrm{c}}{2\mathrm{g}}\frac{\mathrm{L}}{\mathrm{D}}\left[\ln\left(\frac{1+\mathrm{V_F}/\mathrm{V_c}}{1+\mathrm{V_E}/\mathrm{V_c}}\right) - \ln\left(\frac{1-\mathrm{V_F}/\mathrm{V_c}}{1-\mathrm{V_E}/\mathrm{V_c}}\right)\right] \tag{16}$$

$$\mathrm{t_{flight}} = -\frac{\mathrm{V}_\mathrm{c}}{2\mathrm{g}}\frac{\mathrm{L}}{\mathrm{D}}\ln\left\{\frac{1+\mathrm{V_F}/\mathrm{V_c}}{1-\mathrm{V_F}/\mathrm{V_c}}\cdot\frac{1-\mathrm{V_E}/\mathrm{V_c}}{1+\mathrm{V_E}/\mathrm{V_c}}\right\} \tag{17}$$

Assuming $V_F << V_c$, we obtain the following expression for the flight time.

$$\mathrm{t_{flight}} = \frac{\mathrm{V}_\mathrm{c}}{2\mathrm{g}}\frac{\mathrm{L}}{\mathrm{D}}\ln\left(\frac{1+\mathrm{V_E}/\mathrm{V_c}}{1-\mathrm{V_E}/\mathrm{V_c}}\right) \tag{18}$$

e. Filling in the numbers gives us,

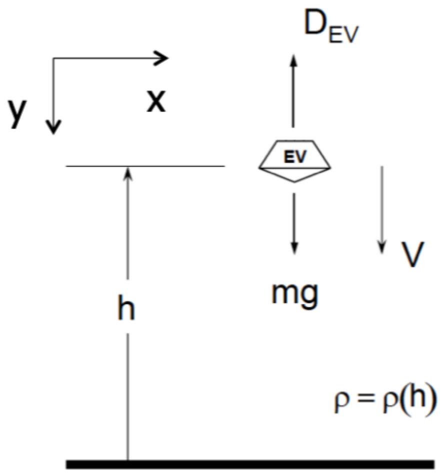
$$\mathrm{t_{flight}} = \frac{7920}{29.81} * (-2) * \ln\left(\frac{1+0.8}{1-0.8}\right) \tag{19}$$

$$\mathrm{t_{flight}} = 1773.9s \tag{20}$$

3 Question 3

a. The figure below shows the free-body diagram of the vehicle after the parachute has been released and it is in free fall trough Mars' atmosphere. The only forces acting on the vehicle at that time is the acceleration of gravity due to Mars and the drag from the atmosphere. Since these are the only two forces acting on the vehicle at free fall thrust is equal to zero (T=0). $h_0 = 1.8km$, $V_0 = -100m/s$

$$\mathrm{m\dot{V} = D + T - mg_M} \tag{21}$$



b. Known Values:
$D = 2500N$
$T = 0N$
$m = 2196kg$
$g_m = 3.96m/s^2$
$t = 5s$

$$\mathrm{m\dot{V} = D + T - mg_M} \tag{22}$$

$$\frac{\mathrm{V_f - V_0}}{\mathrm{t}} = -\frac{D}{m} - g_m \tag{23}$$

$$\frac{\mathrm{V_f - (-100)}}{5} = -\frac{2500}{2196} - 3.96 \tag{24}$$

$V_f = -112.76m/s$

Since the vehicle has a velocity towards the surface of Mars the sign of the component is negative. After the parachute gets released, the vehicle starts accelerating toward the surface of Mars which is why the velocity increases in a negative direction.

c. To find the position after the vehicle free falls for 5 seconds after the parachute is released, integrate the acceleration formula twice to find the position. The constants when integrating are equal to the initial value ($\mathrm{V_0}$ and $\mathrm{h_0}$).

Known Values:
$D = 2500N$
$T = 0N$
$m = 2196kg$
$g_M = 3.96m/s^2$
$t = 5s$
$h_0 = 1800m$
$V_0 = -112.76m/s$

$$\mathrm{\dot{V} = \frac{D}{m} - g_M} \tag{25}$$

$$\mathrm{V} = V0 + \frac{\mathrm{D}}{\mathrm{m}} * t - \mathrm{g_M} * t \tag{26}$$

$$h = h_0 + V_0 * t + \frac{1}{2}\frac{\mathrm{D}}{\mathrm{m}} * t^2 - \frac{1}{2}\mathrm{g_M} * t^2 \tag{27}$$

$h = 1268.01m$

d. For this part, the vehicle has decelerated to a hovering position using the thrusts. The vehicle will stay in an equilibrium position to lower the rover to the surface of Mars. Since the vehicle used fuel to slow down to a hovering position, the mass at the beginning of the hovering phase is 1200 kg. Since the vehicle is no longer moving, the drag component and acceleration during this part is equal to zero newtons and metres per second respectively. The equation of motion for this portion of the flight is stated below.

$$\mathrm{T} - \mathrm{m} * \mathrm{g_M} = 0 \tag{28}$$

$$\mathrm{T} = \mathrm{m} * \mathrm{g_M} = (1200) * (3.69) = 4428N \tag{29}$$

$$Throttle_{setting} = \frac{T}{T_{Tot}} = \frac{4428}{4 * 3000} = 0.1845 = 18.45\% \tag{30}$$

$$\mathrm{I_{sp}} = \frac{T}{\dot{m} * g_0} \tag{31}$$

$$\dot{\mathrm{m}} = \frac{T}{I_{sp} * g_0} = \frac{4428}{200 * 9.81} = 2.26kg/s \tag{32}$$

$$M_p = \dot{m} * t_{hover} = (2.26) * (13) = 29.34kg \tag{33}$$

e. In Part e of this problem, you can find the maximum acceleration error that will be produced having assumed that the mass remains constant by solving for the acceleration and inputting the mass as the total mass minus the mass of propellant used. This is the maximum acceleration error since the thrust used in this calculation will be the thrust needed to keep the initial mass at zero acceleration while using the final mass of the vehicle.

$$\dot{\mathrm{V}} = \frac{\mathrm{T}}{\mathrm{m}} - \mathrm{g_M} = \frac{4428}{1200 - 29.34} - 3.69 = 0.0925m/s^2 \tag{34}$$

f. If the thrusters are not throttled to compensate for this change in mass error, the vehicle will accelerate upward away from the surface of Mars. The acceleration will start to increase from zero as the propellant mass is depleted.

4 Question 4

a. The three components of a PID controller are the Proportional, Integrator and Derivative term. (reader p.444-445 eq[7-86])
Proportional term:

$$u(t) = K_p q_{c,err}(t) \tag{35}$$

Integrator term:

$$u(t) = K_i \int_0^t q_{c,err}(\tau) d\tau \tag{36}$$

Derivative term:

$$u(t) = K_d \frac{dq_{c,err}(t)}{dt} \tag{37}$$

b. The leading derivation for this problem can be found in (reader p.489-491) and in (lecture slides(19/20) GNC, slide. 103)
As stated in the question, for heat-flux tracking the heat flux should remain constant during a major portion of the flight. Hence, $\dot{\mathrm{q}}_\mathrm{c} = 0$. If we now take the time derivative of the Chapman equation ($\mathrm{q_c} = \frac{\mathrm{c_1}}{\sqrt{\mathrm{R_N}}}\sqrt{\rho}\mathrm{V}^3$), we obtain the following expression for $\dot{\mathrm{q}}_\mathrm{c}$. V and ρ are both time dependent variables.

$$\dot{\mathrm{q}}_\mathrm{c} = \frac{\mathrm{c_1}}{\sqrt{\mathrm{R_N}}}\left(\frac{1}{2\sqrt{\rho}}\frac{\mathrm{d}\rho}{\mathrm{dt}}\mathrm{V}^{\frac{2}{8}} + 3\sqrt{\rho}\mathrm{V}^2\frac{\mathrm{dV}}{\mathrm{dt}}\right) = 0 \tag{38}$$

$$\left(\frac{1}{2}\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dt}}\mathrm{V} + 3\frac{\mathrm{dV}}{\mathrm{dt}}\right) = 0 \tag{39}$$

$$\frac{\mathrm{dV}}{\mathrm{dt}} = -\frac{\mathrm{V}}{6\rho}\frac{\mathrm{d}\rho}{\mathrm{dt}} = -\frac{V}{6}\frac{1}{\rho}\frac{d\rho}{dt} \tag{40}$$

We can write $\frac{1}{\rho}\frac{d\rho}{dt}$ as,

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dh}}\frac{\mathrm{dh}}{\mathrm{dt}} \tag{41}$$

An exponential atmosphere is assumed, this means we can make use of

$$\rho = \rho_0 \mathrm{e}^{-\beta \mathrm{h}} \tag{42}$$

$$\frac{d\rho}{dh} = -\beta\rho_0 e^{-\beta h} = -\rho\beta \tag{43}$$

The equations of motion describing the (2D) planar motion of a mass point around a spherical central body can derived from (reader p.215 Figure 5-2) and should be known by heart. The following two EOM's will be used.

$$\frac{dh}{dt} = V\sin\gamma \tag{44}$$

$$\mathrm{m}\frac{\mathrm{dV}}{\mathrm{dt}} = -\mathrm{D} - \mathrm{mg}\sin\gamma \tag{45}$$

Let us know substitute equation 44 and equation 43 in equation 41, which gives us equation 46

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dh}}\frac{\mathrm{dh}}{\mathrm{dt}} = -\beta V\sin\gamma \tag{46}$$

If we substitute equation 46 back into equation 40 we get equation 47

$$\frac{dV}{dt} = \frac{V}{6}\beta L\sin\gamma \tag{47}$$

If we now rearrange equation 45 and substitute our found expression for $\frac{dV}{dt}$, we obtain the following equation for the nominal drag force.

$$\mathrm{D}_{\mathrm{cmd},0} = -\mathrm{mg}\sin\gamma - \frac{\beta}{6}\mathrm{mV}^2\sin\gamma \tag{48}$$

c. We use the proportional and the integrator term described in question 4a. The PI regulator can then be given as (reader p.490 eq.7-205)

$$\Delta\mathrm{D}_{\mathrm{cmd}} = -\mathrm{K}_{\mathrm{p}}\mathrm{q}_{\mathrm{c,err}} - \mathrm{K}_{\mathrm{I}}\int_0^{\mathrm{t}}\mathrm{q}_{\mathrm{c,err}}\mathrm{dt} \tag{49}$$

The complete expression for the commanded drag then becomes,

$$\mathrm{D}_{\mathrm{cmd}} = \mathrm{D}_{\mathrm{cmd},0} + \Delta\mathrm{D}_{\mathrm{cmd}} = -\mathrm{mg}\sin\gamma - \frac{\beta}{6}\mathrm{mV}^2\sin\gamma + \Delta\mathrm{D}_{\mathrm{cmd}} \tag{50}$$

d. By definition of the drag ($D = C_D\frac{1}{2}\rho V^2 S$) and assuming a drag-coefficient, C_D(dependent on Mach number and angle of attack) this can easily be translated into a commanded angle of attack, α_{cmd}. (reader p.491 eq.7-216)

$$\mathrm{D}_{\mathrm{cmd}} = \mathrm{C}_{\mathrm{D,cmd}}\frac{1}{2}\rho\mathrm{V}^2\mathrm{s} \tag{51}$$

$$\mathrm{C}_{\mathrm{D,cmd}} = \mathrm{C}_{\mathrm{D,cmd}}\left(\alpha_{\mathrm{cmd}}, \mathrm{M}\right) = \frac{\mathrm{D}_{\mathrm{cmd}}}{\frac{1}{2}\rho\mathrm{V}^2\mathrm{s}} \tag{52}$$

e. The angle of attack α_{cmd} is retrieved for a given flight condition (h and V) for example. h and V are thus the parameters that should be obtained by the navigation system. Several types of sensors can be used to do so, think of pressure sensors (relate atmospheric pressure to altitude), GPS, radar systems etc. For more examples and context see (reader p.388).

Conclusion