Exam Solution

Course: AE4870B Re-Entry Systems
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collaborative effort

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0 Introduction

This contains an exam solution. If you wish to contribute to this exam solution:

1. Create a github account, (you can create an "anonymous" one).
2. git clone ...
3. edit your changes in the document.
4. open cmd, and browse to inside the folder you downloaded and edited
5. git pull (updates your local repository=copy of folder, to the latest version in github cloud)
6. git status shows which files you changed.
7. git add "/some folder with a space/someFileYouChanged.tex"
8. git commit -m "Included solution to question 1c."
9. git push

It can be a bit initimidating at first, so feel free to click on "issue" in the github browser of this repository and ask :) (You can also use that to say "Hi, I'm having a bit of help with this particular equation, can someone help me out?")

If you don't know how to edit a latex file on your own pc iso on overleaf, look at the "How to use" section of `https://github.com/a-t-0/AE4872-Satellite-Orbit-Determination`.

0.1 Consistency

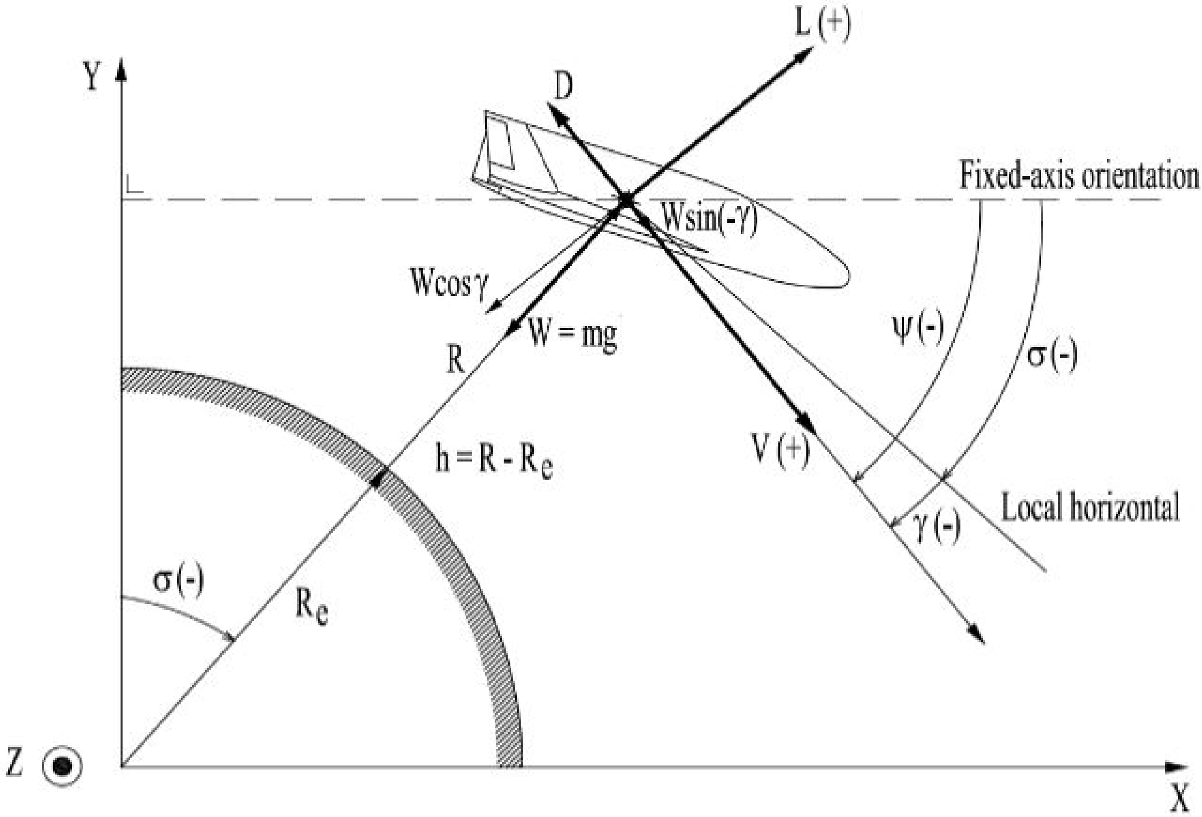
To make everything nice and structured, please use very clear citations:

1. If you copy/use an equation of some slide or document, please add the following data:
   (a) Url (e.g. if simple wiki or some site)
   (b) Name of document
   (c) (Author)
   (d) PAGE/SLIDE number so people can easily find it again
   (e) equation number (so people can easily find it again)
2. If you use an equation from the slides/a book that already has an equation number, then hardcode that equation number in this solution manual so people directly see which equation in the lecture material it is, this facilitates remembering the equations.
3. Here is an example is given in eq. (10.32[1]) (See file references.bib [1]).

$$R_n^E = \sum_{t=1}^{n} l_m(p_t, z_t) - \underset{i}{min} \sum_{t=1}^{n} {z_t}^i \tag{10.32[1]}$$

1 Gliding and ballistic entry

a)



b)

1.

$$m\frac{dV}{dt} = -D - mg\sin\gamma$$

2.

$$mV\frac{d\Psi}{dt} = L - mg\cos\gamma$$

$$\text{where } \Psi = \sigma + \gamma, \text{ hence } \frac{d\Phi}{dt} = \frac{d\gamma}{dt} + \frac{d\sigma}{dt}$$

$$\text{and } R\frac{d\sigma}{dt} = -V\cos\gamma$$

what after some substitutions into the initial equation gives

$$mV\frac{d\gamma}{dt} = L - mg\cos\gamma(1 - \tfrac{V^2}{V_c^2})$$

3.

$$\frac{dh}{dt} = \frac{dR}{dt} - \frac{dR_e}{dt} = \frac{dR}{dt} = V\sin\gamma$$

c)

By taking

$$\frac{a}{g} = \frac{\frac{dV}{dt}}{g} = \frac{D}{mg} - \sin\gamma$$

and expressing deceleration in positive terms as

$$\frac{\bar{a}}{g} = -\frac{\frac{dV}{dt}}{g} = \frac{D}{mg} + \sin\gamma$$

Then, by substituting $\bar{\gamma} = \sin\bar{\gamma} = -\frac{1}{\beta R_e}\frac{2}{L/D}\frac{V_c^2}{V^2}$
and $L = mg(1 - V^2/V_c^2)$

it might be found that

$$\frac{\bar{a}}{g} = -\frac{\frac{dV}{dt}}{g} = \frac{D}{L}\left(1 - \frac{V^2}{V_c^2} - \frac{1}{\beta R_e}\frac{2}{L/D}\frac{V_c^2}{V^2}\right)$$

From that, the maximum deceleration is found by comparing the derivative of the above equation and substituting the result in the expression, as presented below:

$$\frac{d\bar{a}}{d\frac{V}{V_c}} = 0$$

$$\frac{V}{V_c} = \left(\frac{2}{\beta R_e}\right)^{0.25}$$

$$\text{hence, } \bar{a}_{max} = \frac{D}{L}\left(1 - 2\sqrt{\frac{2}{\beta R_e}}\right)$$

d)

Once again, starting from $m\frac{dV}{dt} = -D - mg\sin\gamma$ and expressing the deceleration in positive terms as

$$\bar{a} = -\frac{dV}{dt} = \frac{Dg}{W} - g\sin\gamma$$

The last term is dropped due to $D >> W$

$$\bar{a} = -\frac{dV}{dt} = \frac{1/2C_D S\rho V^2 g}{W} = \frac{\rho V^2 g}{2K}$$

$$\text{where } K = \frac{W}{C_D S}$$

Departing from $ln\frac{V}{V_E} = \frac{g\rho}{2K\beta\sin\gamma_E}$ the density is found to be

$$\rho = \frac{1}{g}2K\beta\sin\gamma_E ln\frac{V}{V_E}$$

then, the deceleration is given by

$$\bar{a} = -\frac{dV}{dt} = \beta\sin\gamma_E ln\frac{V}{V_E}(\frac{V}{V_E})^2 V_E^2$$

and again, by comparing the derivative to zero it is found that for maximum deceleration the relation holds

$$ln\frac{V}{V_E} = -0.5$$

$$\frac{V}{V_E} = \frac{1}{\sqrt{e}}$$

which, after substitution to the expression for deceleration, gives

$$\bar{a}_{max} = -\frac{\beta\sin\gamma_E V_E^2}{2e}$$

e)

Ballistic parameter K does not affect the magnitude of maximum deceleration, however it determines the altitude at which this deceleration occurs.

For gliding entry, increasing gliding ratio L/D lowers the maximum deceleration value. Contribution of mass reduces the maximum deceleration.

f)

For ballistic entry, the trajectory does depend on the initial velocity and follows the path along the initial path angle. The same holds w.r.t. maximum deceleration - it depends on the path angle and entry velocity. The greater the values of entry velocity and path angle, the greater the deceleration max.

The initial conditions of flight do not affect the maximum deceleration if the vehicle follows constant L/D path.

g)
Gliding entry is performed:

$$\bar{a}_{max} = g_0 \tfrac{D}{L} \left(1 - 2\sqrt{\frac{2}{\beta R_e}}\right) = 0.38 g_0$$

h)
Here, ballistic entry is performed:

$$\bar{a}_{max} = -\tfrac{\beta \sin\gamma_E V_E^2}{2e} = 118.7 m/s^2 = 12.1 g_0$$

Conclusion

It seems to be a pretty standard set of questions. It is usually necessary to know the derivation and be able to play with it under new assumptions.

References

[1] Some author. *Advanced tree dynamics*, volume lecture 5 of *AE2344 Some course*, page 15. Accessed: 2019-04-27.