Exam Solution

Course: AE4872 Satellite Orbit Determination
Exam Source:Studeersnel
Exam Date: 2015-01-30

Author:
collaborative effort

06-09-2019

0 Introduction

This contains an exam solution. If you wish to contribute to this exam solution:

1. Create a github account, (you can create an "anonymous" one).
2. git clone ...
3. edit your changes in the document.
4. open cmd, and browse to inside the folder you downloaded and edited
5. git pull (updates your local repository=copy of folder, to the latest version in github cloud)
6. git status shows which files you changed.
7. git add "/some folder with a space/someFileYouChanged.tex"
8. git commit -m "Included solution to question 1c."
9. git push

It can be a bit initimidating at first, so feel free to click on "issue" in the github browser of this repository and ask :) (You can also use that to say "Hi, I'm having a bit of help with this particular equation, can someone help me out?")

If you don't know how to edit a latex file on your own pc iso on overleaf, look at the "How to use" section of `https://github.com/a-t-0/AE4872-Satellite-Orbit-Determination`.

0.1 Consistency

To make everything nice and structured, please use very clear citations:

1. If you copy/use an equation of some slide or document, please add the following data:
   (a) Url (e.g. if simple wiki or some site)
   (b) Name of document
   (c) (Author)
   (d) PAGE/SLIDE number so people can easily find it again
   (e) equation number (so people can easily find it again)
2. If you use an equation from the slides/a book that already has an equation number, then hardcode that equation number in this solution manual so people directly see which equation in the lecture material it is, this facilitates remembering the equations.
3. Here is an example is given in eq. (10.32[?]) (See file references.bib [?]).

$$R_n^E = \sum_{t=1}^{n} l_m(p_t, z_t) - \min_i \sum_{t=1}^{n} {z_t}^i \qquad (10.32[?])$$

1 Part: Statistics and parameter estimation

1.1

In order to check for linearity of the least squares method, that needs to be applied,

$$\bar{y} = A(\bar{x}) + \bar{\epsilon}$$

design matrix A has to be derived. The curve that is being fitted is given by

$r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ Therefore, the LSM equation is

$$\begin{bmatrix} \vdots \\ \Delta x_i \\ \Delta y_i \\ \vdots \end{bmatrix}_k = \begin{bmatrix} & \vdots & \\ 1 & 0 & (x_i - x_{c,k})/r_{c,k} \\ 0 & 1 & (y_i - y_{c,k})/r_{c,k} \\ & \vdots & \end{bmatrix} \begin{bmatrix} \Delta x_{c,k} \\ \Delta y_{c,k} \\ \Delta r_{c,k} \end{bmatrix} + \bar{\epsilon}$$

As it can be observed, the design matrix is dependent on estimated parameters $\bar{x}$. This means it is a non-linear problem, which needs to be solved iteratively.

1.2

Given $Q_{yy} = P_{yy} = \lambda I$, the estimated parameters and their covariances might be found by applying

$$\hat{x} = \left(A^t A\right)^{-1} A^t \overline{y}$$

$$P_{xx} = \lambda \left(A^t A\right)^{-1}$$

which proves that $\hat{x}$ is independent of λ, whereas its covariance matrix P_{xx} indeed is a function of λ.

1.3

The problem statement was presented in class. Below, relevant equations are given.

$$\hat{x} = \left(A^t A\right)^{-1} A^t \overline{y}$$

$$P_{xx} = \lambda \left(A^t A\right)^{-1}$$

This set of equations describes 3-dimensional rotation (by three rotation matrices R) and translation (by vector b) of the point clouds in order to fit them together. Linearization is done by taking partial derivatives of the observation equation w.r.t. three angles, which are parameters to be estimated in this case.

1.4

i) Finding eigenvalues and eigenvectors of the problem enables statement, whether there is a solution of manifolds or a unique solution. Manifold of solutions occur for rank-deficient systems (for which one or more eigenvalues are null).
ii) Yes, the model based on two sets of observations is rank-deficient and will therefore result in a manifold of solutions. This property is manifested by multiple different interpretations of the comet geometry.

2 Part: Satellite Tracking

2.1

Dry tropospheric error depends on pressure at MSL. This means that the height above MSL, at which the measurement is being taken, needs to be accounted for. Moreover, path length proportional to $1/\cos Z$, where Z is zenith angle, has to be taken into consideration.

Wet tropospheric error is dependent on humidity (do not confuse with water!) contained in the atmosphere. It is latitude- and (highly) time-dependent. On average, the greatest WVP (water vapor path) occur in the daytime on low latitudes. The remedy for this problem is to install WVR (water vapor radiometer) onboard, which would, upon measuring WVP, include precise corrections in the data sent to the receiver.

Ionospheric effects (refraction and dispersion) might be taken care of by means of double-frequency measurements, presented below. In this manner, the range and the TEC (total electron content parameter - related to α) might be obtained.

$$\begin{aligned} r_1 &= r_0 + \frac{\alpha}{f_1^2} \\ r_2 &= r_0 + \frac{\alpha}{f_2^2} \end{aligned}$$

2.2

i) Doppler effect, atmospheric errors (twice - upwards and downwards)
ii)
1. Correct for downward atmospheric errors
2. Correct for Doppler effect
3. Mitigate upward atmospheric errors

2.3

i) The primary acceleration that is included in the model is obviously the gravitational acceleration

$$\ddot{\bar{x}} = -\mu/r^3\,\bar{x}$$

apart from which, also perturbations (gravitational pull from other bodies, drag, solar radiation) might be included in the model.
ii) Gravitational, solar wind, gravitational perturbations (distant bodies, other satellites etc.), drag, solar wind
iii) Gravity field (U), position, velocity, clock corrections, atmospheric parameters (e.g. TEC)

2.4

The rotation of the Earth is given by $\bar{\omega} = [0, 0, \omega]^T$. Therefore, translation between the frames is

$$\bar{x}_F = \bar{T} + R_3(\omega\Delta t)\bar{x}_I$$

where F signifies the fixed frame values, I - inertial ones, T is the linear translation (null in this case, because the frames have the same origin - center of Earth) and R_3 is the rotation matrix around 3rd axis (z).

$$R_3(\omega\Delta t)\begin{bmatrix} \cos(\omega\Delta t) & \sin(\omega\Delta t) & 0 \\ -\sin(\omega\Delta t) & \cos(\omega\Delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.5

Gravity field is different for all mentioned cases, which translates to different time experienced by atomic clocks. According to the relativity theory, time experienced by the observer (a clock in this case) depends on the gravity field (or velocity), at which the observer is located.

3 Part: GPS

3.1 I

give more than required 6 for the sake of the benefit of us all (but in the exam it is important to put the exact number asked).

1. Integer ambiguity
2. Atmospheric errors
3. Clock errors
4. Doppler effects
5. Ephemeris error of GNSS satellite

6. Relativistic error
7. Multipath
8. Receiver noise

3.2

Applying double differencing mitigates the following errors:

1. Atmospheric errors
2. Clock errors
3. Ephemeris error of GNSS satellite

3.3

There are 6 possible double differences. For this procedure 2 ground stations are needed (we have two, therefore no possible variations of the configurations) and 2 satellites - we have 4 at our disposal, which gives us 6 different possible pairs of satellites.

3.4

Dilution of precision is a parameter describing quantitatively, how large would be the error caused by geometrical distribution of the satellites on the sky. In other words, the greater the PDOP, the less reliable the position estimation. It can be illustrated by the figure below. Satellites located close to each other in the sky would give large possible area of location, whereas ranges from distanced satellites (in terms of the sky seen by the observer) would give more precise location due to bigger angle between the two radii led from the satellites to the observer.

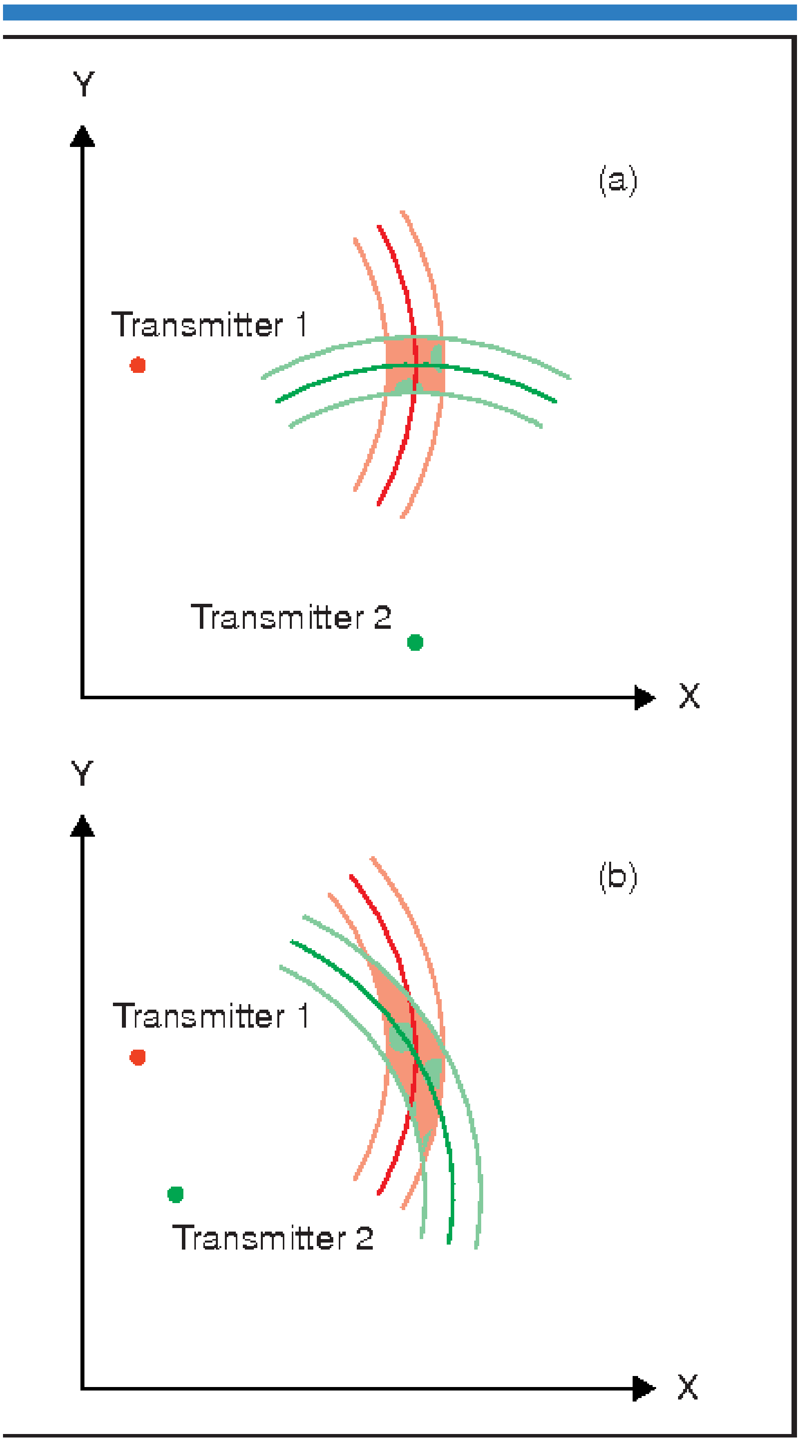

Figure 1: *
Source:Semantic Scholar

3.5

Elevation mask is a technique of ignoring signal from satellite that are low above the horizon (mask might be applied e.g. for elevation of 10 degrees, meaning that all satellites that at the moment of signal emission are below this value of elevation in the local horizon reference frame of the observer would be ignored). This technique serves the purpose of avoiding high atmospheric errors (larger travel paths through the atmosphere result in higher errors). However, selecting only satellites higher up in the sky results in greater value of PDOP (undesirable). Balancing between those two is a tradeoff the designers of navigation systems need to face.

4 Part: Kalman Filter

4.1

The general procedure of Kalman Filter is composed of the following steps:

1. Setting up nitial condition
2. Propagation of state vector
3. Propagation of state covariance matrix
4. Calculating Kalman gain
5. Update of model (estimated state vector according to actual model)
6. Updating error covariance matrix

4.2

By definition:

$$A = \frac{\partial(\frac{\partial \bar{u})}{\partial t}}{\partial \bar{u}}$$

and

$$\Phi = \frac{\partial \bar{u}}{\partial t} \text{ , } \dot{\Phi} = \frac{\partial \Phi}{\partial t}$$

Hence

$$A\Phi = \frac{\partial(\frac{\partial \bar{u})}{\partial t}}{\partial \bar{u}} \frac{\partial \bar{u}}{\partial t} = \frac{\partial^2 \bar{u}}{\partial t^2} = \frac{\partial \Phi}{\partial t} = \dot{\Phi}$$

4.3

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-\mu}{r^3} + \frac{3\mu}{r^5}x^2 & \frac{3\mu xy}{r^5} & \frac{3\mu xz}{r^5} & 0 & 0 & 0 \\ \frac{3\mu xy}{r^5} & \frac{-\mu}{r^3} + \frac{3\mu}{r^5}y^2 & \frac{3\mu xz}{r^5} & 0 & 0 & 0 \\ \frac{3\mu zx}{r^5} & \frac{3\mu zy}{r^5} & \frac{-\mu}{r^3} + \frac{3\mu}{r^5}z^2 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, answering to the question:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ non-zero & non-zero & non-zero & 0 & 0 & 0 \\ non-zero & non-zero & non-zero & 0 & 0 & 0 \\ non-zero & non-zero & non-zero & 0 & 0 & 0 \end{bmatrix}$$

4.4

In order to represent said noise increase in the KF, proper changes need to be made to the noise matrix, namely:

$$R_{yy}(t = 2nd\, day) = 2^2 R_{yy}(t = 2nd\, day)_{predicted}$$

due to the fact that

$$(2\sigma)^2 = 2^2\sigma$$

4.5

To counteract this phenomenon, more weight needs to be put on observations. It might be done by increasing scaling factor accompanying matrix R, namely the λ in R$_{more\, weights} = \lambda R$

4.6

Tracking Venus Express might have delivered data regarding atmospheric properties of Venus, solar wind characteristics in the Vicinity of Venus as well as would've allowed for technology presentation and experiments regarding tracking or navigation techniques.

Conclusion

In this exam, in my opinion, questions are not put in a clear way. Presented solutions are concise, but long, elaborate descriptions (making perfect sense) would be possible as well. It is important to mind the notation and proper nomenclature (proper names of specific matrices and parameters). Moreover, it seems like year by year the questions are regarding more or less the same stuff (as indicated in the name of the exam parts).